

MCMT Homework 5

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Exercise 6.1

$$\begin{aligned} 2\|\mu P^t - \pi\|_{TV} &= \sum_{y \in \Omega} |\mu P^t(y) - \pi(y)| \\ &= \sum_{y \in \Omega} \left| \sum_{x \in \Omega} \mu(x) P^t(x, y) - \sum_{z \in \Omega} \pi(z) P^t(z, y) \right| \\ &= \sum_{y \in \Omega} \left| \sum_{z \in \Omega} \pi(z) \left(\sum_{x \in \Omega} \mu(x) P^t(x, y) - P^t(z, y) \right) \right| \\ &\leq \sum_{y \in \Omega} \sum_{z \in \Omega} \pi(z) \left| \sum_{x \in \Omega} \mu(x) P^t(x, y) - P^t(z, y) \right| \\ &= \sum_{y \in \Omega} \sum_{z \in \Omega} \pi(z) \left| \sum_{x \in \Omega} \mu(x) (P^t(x, y) - P^t(z, y)) \right| \\ &\leq \sum_{y \in \Omega} \sum_{z \in \Omega} \pi(z) \sum_{x \in \Omega} \mu(x) |P^t(x, y) - P^t(z, y)| \\ &= \sum_{x \in \Omega} \mu(x) \sum_{y \in \Omega} \sum_{z \in \Omega} \pi(z) |P^t(x, y) - P^t(z, y)| \end{aligned}$$