MCMT Homework 5

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Exercise 6.1

$2||\mu P^t - \pi||_{TV} = \sum_{y\in\Omega} |\mu P^t(y) - \pi(y)|$
$= \sum_{y\in\Omega} |\sum_{x\in\Omega} \mu(x)P^t(x,y) - \pi(y)|$
$= \sum_{y\in\Omega} |\sum_{x\in\Omega} \mu(x)P^t(x,y) - \sum_{x\in\Omega} \mu(x)\pi(y)|$
$= \sum_{y\in\Omega} |\sum_{x\in\Omega} \mu(x)(P^t(x,y) - \pi(y))|$
$\leq \sum_{y\in\Omega} \sum_{x\in\Omega} \mu(x)|P^t(x,y) - \pi(y)|$
$= \sum_{x\in\Omega} \mu(x)2||P^t(x,\cdot) - \pi||_{TV}$
$\leq sup_x 2||P^t(x,\cdot) - \pi||_{TV}$

In the last step, let $\mu = \delta_x$ so that $2||P^t(x,\cdot) - \pi||_{TV}$ is maximized. Thefore, $sup_\mu||\mu P^t - \pi||_{TV} = d(t)$.