MCMT Homework 17

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Exercise 17.1

1. Consider v, λ so that $Pv = \lambda v$. Let v_k be the component of x with the maximum magnitude. $\lambda|v_k| = |\sum_i P_{ik} v_i| \leq \sum_i P_{ik}|v_i| \leq \sum_i P_{ik}|x_k| = |x_k|$. So $\lambda \leq 1$.

2. It is proved that the nullity of $P - I$ is 1. $(1, 1, \cdots, 1)^T$ is a solution to $Pv = v$, so it is the unique solution.

3. Suppose that $Pv = -v$. Let $v = v^+ - v^-$ where v^+ and v^- have non-negative coordinates and disjoint support.

   Because P perserves the sum of components of v, so
   $\sum Pv^+ = \sum v^+$
   $\sum Pv^- = \sum v^-$
   $\sum Pv = \sum v \Rightarrow \sum P(v^+ - v^-) = \sum(v^- - v^+)$.
   By these linear equations, we have $\sum v^+ = \sum v^-$.

   Because $P(v^+ - v^-) = v^- - v^+$, where Pv^+, Pv^-, v^+, v^- are all non-negative vectors. $\sum Pv^+ = \sum Pv^- = \sum v^+ = \sum v^-$. We have $Pv^+ = v^-, Pv^- = v^+$.

   For a state x in the support of v^+, we know $P(x, x) = 0$ because $Pv^+ = v^-$. Because $P^{2k+1}v^+ = v^-$, $P^{2k+1}(x, x) = 0$. As the transition matrix is irreducible, the period must be a multiple of 2.