MCMT Homework 3

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Exercise 3.1

Assume that $\pi(x) = 0$ for a $x \in \Omega$. Because $\pi P = \pi$, $\sum_{i\in\Omega} \pi(i)P(i,x) = \pi(x) = 0$. Consider state y such that $\pi(y) > 0$, then $P(y,x) = 0$. Because π is the stationary distribution for P, so $\pi P^t = \pi$, or $P^t = P$, for arbitrary $t \geq 1$. So $P^t(y,x) = 0$ for arbitrary $t \geq 1$. This chain is not irreducible. Contradiction.

Exercise 3.2

Check the distribution at state x, $\pi P_x = \sum_{i\in adj(x)} \pi(i)P(i,x) = \sum_{i\in adj(x)} \frac{deg(i)}{2|E|}\frac{1}{deg(i)} = \sum_{i\in adj(x)} \frac{1}{2|E|} = \frac{deg(x)}{2|E|} = \pi(x)$, where $adj(x)$ is the neighbor set of x, with the size of $deg(x)$.