MCMT Homework 10

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Exercise 10.1

$\mu((1, 2, \cdots, k) \to (2, 3, \cdots, k, 1)) = \frac{1}{n}$.
$\mu(\cdot) = 0$, for other permutations.

Exercise 10.2

Let $p = 1/(3^2)^3$, and consider the cases when $n = 3$, $\tau = 3$.

1. When $X_3 = X_0$, we may swap a card with itself at each time, and swap three different cards for $t = 1, 2, 3$. The probability is $3!p = 6p$. We can also swap two cards back and forth for the first two steps, and swap the third card with itself. The probability is $\binom{3}{2}2^2p = 12p$. So $\mathbb{P}(X_3 = X_0, \tau = 3) = 18p$.

   When $X_3 = (2\ 1\ 3)X_0$, which we stop at swaping two cards on the top. We can swap any card with itself in the first two rounds, but include 3 at least once, and then swap 1 and 2 in the third step. The probability is $5 * 2p = 10p$. We can also swap the third card with either of the top cards back and forth for the first two steps, and then swap 1 and 2 in the third step. The probability is $2 * 2^3p = 16p$. We can conclude that $\mathbb{P}(X_3 = (2\ 1\ 3)X_0, \tau = 3) \geq 26p$.

   π is a uniform distribution, but $\mathbb{P}(X_3 = X_0, \tau = 3) \neq \mathbb{P}(X_3 = (2\,1\,3)X_0, \tau = 3)$. So this is not a strong stationary distribution.

2. R_t must be distinct for $t = 1, 2, 3$. So $P(\tau = 3) = 3! * 3^3p = 162p$.

   When $X_3 = X_0$, we may swap a card with itself at each time, and swap three different cards for $t = 1, 2, 3$. The probability is $3!p = 6p$. We can also swap two cards back and forth for two steps, and swap the third card with itself in the other step. The probability is $\binom{3}{2}\binom{3}{2}2p = 18p$. So $\mathbb{P}(X_3 = X_0, \tau = 3) = 24p$.

   There are 6 possible outcomes for $\tau = 3$, but $\mathbb{P}(X_3 = X_0, \tau = 3) \neq 162p/6$. So this is not a strong stationary distribution.