

MCMT Homework 3

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Exercise 3.1

Assume that $\pi(x) = 0$. Because $\pi P = \pi$, $\sum_{i \in \Omega} \pi(i)P(i, x) = \pi(x) = 0$. So $P(i, x) = 0$ for i such that $\pi(i) > 0$. Such i exists as the sum of π is 1. So there is 0 probability for these i to access x . This chain is not irreducible. Contradiction.

Exercise 3.2

$\pi P_x = \sum_{i \in \text{adj}(x)} \frac{\deg(i)}{2|E|} \frac{1}{\deg(i)} = \sum_{i \in \text{adj}(x)} \frac{1}{2|E|} = \frac{\deg(x)}{2|E|} = \pi(x)$, where $\text{adj}(x)$ is the neighbor set of x , with the size of $\deg(x)$.