MCMT Homework 2

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Exercise 2.1

Let Z be a uniform random variable on $[0, 1]$. Let $Z_1, Z_2, \cdots$ to be independent copies of Z.

Define $f(x_t, z_t) = y$ iff $\sum_{i=1}^{y-1} P(x_t, i) \leq z_t < \sum_{i=1}^{y} P(x_t, i)$ (assume that $\sum_{i=1}^{0} \cdot = 0$).

Then $\mathbb{P}(X_{t+1} = y | X_t = x) = P(x, y) = \mathbb{P}(f(x, z_t) = y)$.

Exercise 2.2

No. Assume there are only two distinct distributions π_1 and π_2, so that $\pi_1 P = \pi_1$, $\pi_2 P = \pi_2$. Then for $0 < \lambda < 1$, consider the distribution $\pi = \lambda\pi_1 + (1-\lambda)\pi_2$. $(\lambda\pi_1 + (1-\lambda)\pi_2)P = \lambda\pi_1 + (1-\lambda)\pi_2$. So $\pi P = \pi$. Contradiction.