MCMT Homework 11

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Exercise 11.1

It is sufficient to show τ is an on-to mapping to show it is bijection. For $x \in S_n$, $\tau^{-1}x$ exists. So $P(\tau\sigma) = P(\sigma)$. If σ follows uniform distribution, so does $\tau\sigma$.

Exercise 11.2

Suppose there are n cards in total and m cards in the left hand. Consider the next card to be dropped from the bottom of left-right deck or the right-hand deck.

Following the first model, the number of cases that the next card is from the left hand is $\binom{n}{m} - \binom{n-1}{m} = \binom{n-1}{m-1}$. The number of cases that the next card is from the right hand: $\binom{n-1}{m}$. The ratio of probability of the next card being from the left hand versus the right hand is $\binom{n-1}{m-1}/\binom{n-1}{m} = m/(n-m)$.

Following the second model, the ratio of probability of the next card being from the left hand versus the right hand is $\frac{m}{n}/\frac{n-m}{n} = m/(n-m)$.

So they have the same probability distribution on the next card to be dropped. This is applied to each step. So these two characterizations are equivalent.