

MCMT Homework 4

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Exercise 4.1

$$\begin{aligned} & \mathbb{P}_\pi(X_0 = x_0, X_1 = x_1, \dots, X_t = x_t) \\ &= \mathbb{P}_\pi(X_0 = x_0) \mathbb{P}_\pi(X_1 = x_1 | X_0 = x_0) \cdots \mathbb{P}_\pi(X_t = x_t | X_{t-1} = x_{t-1}) \\ &= \pi(x_0) P(x_0, x_1) \cdots P(x_{t-1}, x_t) \\ &= P(x_1, x_0) \pi(x_1) \cdots P(x_{t-1}, x_t) && \text{Detailed balance on } x_0, x_1 \\ &= P(x_1, x_0) P(x_2, x_1) \pi(x_2) \cdots P(x_{t-1}, x_t) && \text{Detailed balance on } x_1, x_2 \\ &\dots \\ &= P(x_1, x_0) P(x_2, x_1) \cdots P(x_t, x_{t-1}) \pi(x_t) \\ &= \pi(x_t) P(x_t, x_{t-1}) \cdots P(x_2, x_1) P(x_1, x_0) \\ &= \mathbb{P}_\pi(X_0 = x_t, X_1 = x_{t-1}, \dots, X_t = x_0) \end{aligned}$$

Exercise 4.2

Define $V_1 = \{v \in V : y \in \Omega(x, v)\}$. Define $V_2 = V \setminus V_1$.

$$\begin{aligned} & \pi(x) P(x, y) \\ &= \pi(x) \frac{1}{|V|} \sum_v \pi^{x,v}(y) \\ &= \pi(x) \frac{1}{|V|} \sum_{v \in V_1} \frac{\pi(y)}{\pi(\Omega(x, v))} + \sum_{v \in V_2} 0 \\ &= \pi(x) \frac{1}{|V|} \sum_{v \in V_1} \frac{\pi(y)}{\pi(\Omega(y, v))} \\ &= \pi(y) \frac{1}{|V|} \sum_v \frac{\pi(x)}{\pi(\Omega(y, v))} \\ &= \pi(y) \frac{1}{|V|} \sum_v \pi^{y,v}(x) \\ &= \pi(y) P(y, x) \end{aligned}$$