MCMT Homework 12

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Exercise 12.1

$\theta(x,y) = -\theta(y,x)$ for every x, y, so $\sum_x \sum_{y\sim x} \theta(x,y) = 0$. We know for every $x \notin \{a,z\}$, $\sum_{y\sim x} \theta(x,y) = 0$. So $\sum_{x\in\{a,z\}} \sum_{y\sim x} \theta(x,y) = 0$. $\sum_{y\sim a} \theta(a,y) = -\sum_{y\sim z} \theta(z,y)$.

Exercise 12.2

Let θ be the unit current flow of the original graph.

(a) $\sum_{y\sim u} \theta(u,y) = \sum_{y\sim u} c(u,y)(\phi(u) - \phi(y)) = \sum_{y\sim u} c(u,y)(\phi(u) - \phi(y)) = (c(e_1) + c(e_2))(\phi(u) - \phi(v)) + \sum_{y\sim u\setminus\{e_1,e_2\}} c(u,y)(\phi(u) - \phi(y)) = 0$.

Delete e_2 and replace $c(e_1)$ with $c(e_1) + c(e_2)$. The equation above is not changed. As c is unchanged for other edges, ϕ is the same. By the definition of $R(a,z) = \phi(a) - \phi(z)$, $R(a,z)$ is the same.

(b) By Kirchoffs node law, $\theta(v,u) = -\theta(v,w)$. So $\theta(u,v) = \theta(v,w)$.

$\phi(u) - \phi(w) = (\phi(u) - \phi(v)) + (\phi(v) - \phi(w)) = \theta(u,v)R(u,v) + \theta(v,w)R(v,w) = \theta(u,v)(R(u,v) + R(v,w))$.

Replace $R(u,v) + R(v,w)$ with $R(u,w)$, $\phi(u) - \phi(w)$ is the same. Same as the reasoning in previous question, $R(a,z)$ is the same.