MCMT Homework 8

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Exercise 8.1

For any non-negative constant c, define $\{\tau \leq t\} = 1_{|(X_0,\cdots,X_t)|=c}$. Then $\tau = c$ is a stopping time.

Suppose there are two stopping times τ_1 and τ_2. We can make $\{X_0, \cdots, X_s\}$ satisfy $\{\tau_1 \leq s\}$ and $\{X_s, \cdots, X_t\}$ satisfy $\{\tau_2 \leq t-s\}$. That is, $\{\tau_1 + \tau_2 \leq t\} = \{\tau_1 \leq s\} \wedge \{\tau_2 \leq t-s, X_0 = y\}$, where y is the state when $\{\tau_1 \leq s\}$ is true. So $\tau_1 + \tau_2$ is a stopping time.

Exercise 8.2

Consider a Markov chain of three states. x_1 represents one complete graph except the shared vertex. x_2 represents the other complete graph except the shared vertex. x_3 is the shared vertex. Then $P(x_1, x_3) = \frac{1}{2n-1}$.

Start with δ_1 (which means starting with arbitrary distribution that only covers the first complete graph except the shared vertex in the original graph).

$\mathbb{P}(X_t = x_1) \geq (1 - \frac{1}{2n-1})^t \geq 1 - \frac{t}{2n-1}$. As $\pi(x_1) < \frac{1}{2}$, we want $\mathbb{P}(X_t = x_1) < \frac{1}{2} + \frac{1}{4}$ for $t = t_{mix}$.

So $1 - \frac{t}{2n-1} < \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, $\frac{t}{2n-1} > \frac{1}{4}$, $t > \frac{n}{2}$.

That is, $t_{mix} > \frac{2}{n}$.