MCMT Homework 8

Shun Zhang

Exercise 8.1

For any non-negative constant c, define $\{\tau \leq t\} = 1_{|(X_0,\cdots,X_t)|=c}$. Then $\tau = c$ is a stopping time.

Suppose there are two stopping times τ_1 and τ_2. We can make $\{X_0, \cdots, X_s\}$ satisfy $\{\tau_1 \leq s\}$ and $\{X_s, \cdots, X_t\}$ satisfy $\{\tau_2 \leq t-s\}$. That is, $\{\tau_1 + \tau_2 \leq t\} = \{\tau_1 \leq s\} \wedge \{\tau_2 \leq t-s, X_0 = y\}$, where y is the state when $\{\tau_1 \leq s\}$ is true. So $\tau_1 + \tau_2$ is a stopping time.

Exercise 8.2

Let A be the set of vetices in one complete graph except the shared vertex. Then $P(A, V \setminus A) = \dfrac{1}{2n-1}$.

Start with any state in A, $\mathbb{P}(X_t \in A) \geq (1 - \dfrac{1}{2n-1})^t \geq 1 - \dfrac{t}{2n-1}$. As $\pi(A) < \dfrac{1}{2}$, we want $\mathbb{P}(X_t = A) < \dfrac{1}{2} + \dfrac{1}{4}$ for $t = t_{mix}$.

So $1 - \dfrac{t}{2n-1} < \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$, $\dfrac{t}{2n-1} > \dfrac{1}{4}$, $t > \dfrac{n}{2}$.

That is, $t_{mix} > \dfrac{n}{2}$.