MCMT Homework 10

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Exercise 10.1

$\mu((1, 2, \cdots, k) \to (2, 3, \cdots, k, 1)) = \frac{1}{n}$.
$\mu(\cdot) = 0$, for other permutations.

Exercise 10.2

1. Consider when $\tau = 3$.

   When $X_3 = X_0$, we may swap a card with itself at each time, and swap three different cards for $t = 1, 2, 3$. There are $3 * 2 = 6$ ways. We can also swap two cards back and forth for the first two times, and swap the third card with itself. There are $3 * 2 = 6$ ways. So there are 12 ways to get $X_3 = X_0$.

   When $X_3' = (213)X_0$, which we stop at swaping two cards on the top. We can swap any card with itself in first two rounds, other than 1 and 2, in first two rounds, and then swap 1 and 2 in the third time. We exclude the case of "1 and 2" in the first two rounds, because otherwise 3 won't be marked. There are $5 * 2 = 10$ ways to do so. We can also swap the third card with either of the top cards back and forth for the first two times, and swap 1 and 2 in the third time. There are $2 * 2 * 2 = 8$ ways to do so. There are 18 ways for this case.

   However, $\pi(X_3) = \pi(X_3')$ but $P(\tau = 3, X_3) \neq P(\tau = 3, X_3')$. This is not a strong stationary time.

2. Consider when $\tau = 3$.