# MCMT Homework 4

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## Exercise 4.1

$\mathbb{P}_\pi(X_0 = x_0, X_1 = x_1, \cdots, X_t = x_t)$
$= \mathbb{P}_\pi(X_0 = x_0)\mathbb{P}_\pi(X_1 = x_1|X_0 = x_0)\cdots\mathbb{P}_\pi(X_t = x_t|X_{t-1} = x_{t-1})$
$= \pi(x_0)P(x_0, x_1)\cdots P(x_{t-1}, x_t)$
$= P(x_1, x_0)\pi(x_1)\cdots P(x_{t-1}, x_t)$          Detailed balance on $x_0, x_1$
$= P(x_1, x_0)P(x_2, x_1)\pi(x_2)\cdots P(x_{t-1}, x_t)$      Detailed balance on $x_1, x_2$
$\cdots$
$= P(x_1, x_0)P(x_2, x_1)\cdots P(x_t, x_{t-1})\pi(x_t)$
$= \pi(x_t)P(x_t, x_{t-1})\cdots P(x_2, x_1)P(x_1, x_0)$
$= \mathbb{P}_\pi(X_0 = x_t, X_1 = x_{t-1}, \cdots, X_t = x_0)$

## Exercise 4.2

If $y \in \Omega(x, v)$,
$\quad \pi(x)P(x, y)$
$= \pi(x)\dfrac{1}{|V|}\sum_v \pi^{x,v}(y)$
$= \pi(x)\dfrac{1}{|V|}\sum_v \dfrac{\pi(y)}{\pi(\Omega(x, v))}$
$= \pi(x)\dfrac{1}{|V|}\sum_v \dfrac{\pi(y)}{\pi(\Omega(y, v))}$
$= \pi(y)\dfrac{1}{|V|}\sum_v \dfrac{\pi(x)}{\pi(\Omega(y, v))}$
$= \pi(y)\dfrac{1}{|V|}\sum_v \pi^{y,v}(x)$
$= \pi(y)P(y, x)$