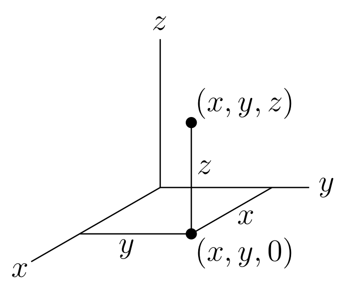
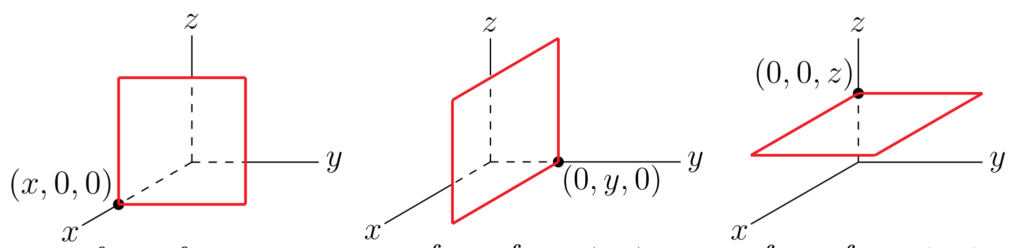
# A.6 3d Coordinate Systems

## A.6.1 Cartesian Coordinates

Here is a figure showing the definitions of the three Cartesian coordinates $(x, y, z)$

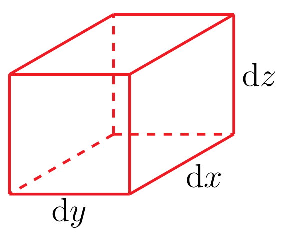



and here are three figures showing a surface of constant $x$, a surface of constant $x$, and a surface of constant $z$.




surface of constant $x$ (a plane) | surface of constant $y$ (a plane) | surface of constant $z$ (a plane)

Finally here is a figure showing the volume element $\mathrm{d}V$ in cartesian coordinates.

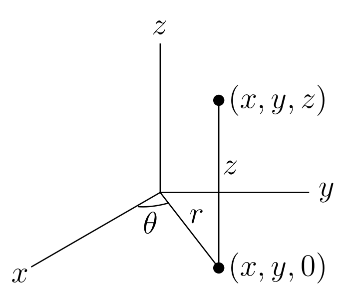



volume element $\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

## A.6.2 Cylindrical Coordinates

Here is a figure showing the definitions of the three cylindrical coordinates

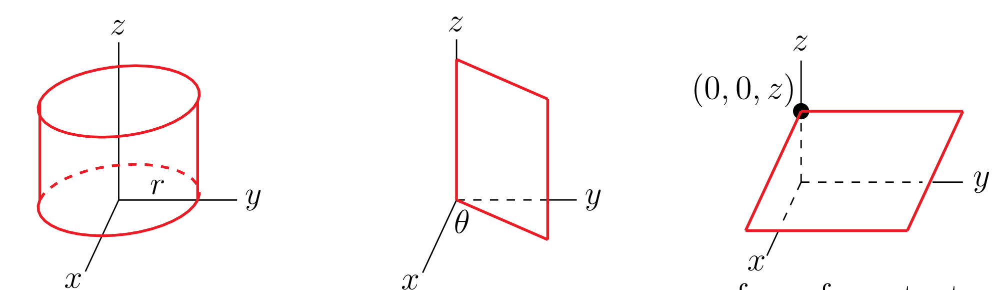
$r =$ distance from $(0, 0, 0)$ to $(x, y, 0)$
$\theta =$ angle between the the $x$ axis and the line joining $(x, y, 0)$ to $(0, 0, 0)$
$z =$ signed distance from $(x, y, z)$ to the $xy$-plane




The cartesian and cylindrical coordinates are related by

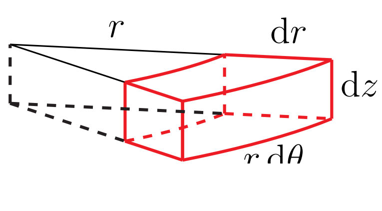
$$
\begin{array}{lll}
x = r\cos\theta & y = r\sin\theta & z = z \\
r = \sqrt{x^2 + y^2} & \theta = \arctan\dfrac{y}{x} & z = z
\end{array}
$$

Here are three figures showing a surface of constant $r$, a surface of constant $\theta$, and a surface of constant $z$.




surface of constant $r$ (a cylindrical shell) | surface of constant $\theta$ (a plane) | surface of constant $z$ (a plane)

Finally here is a figure showing the volume element $\mathrm{d}V$ in cylindrical coordinates.



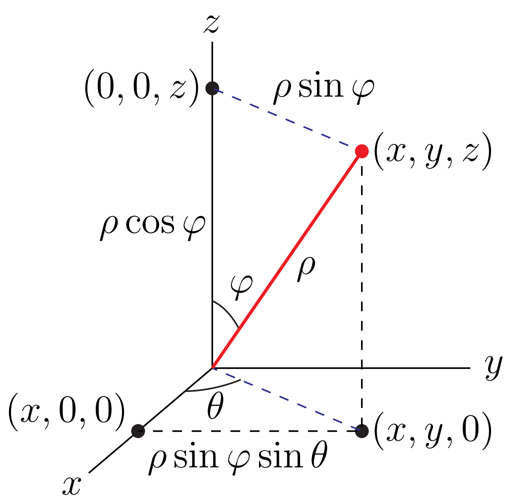
$r\,\mathrm{d}\theta$

volume element $\mathrm{d}V = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$

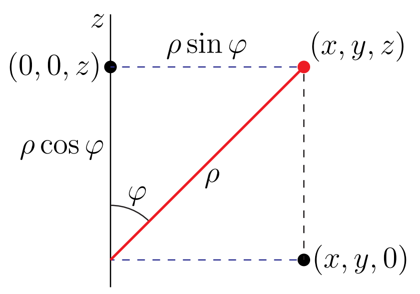
### A.6.3 Spherical Coordinates

Here is a figure showing the definitions of the three spherical coordinates

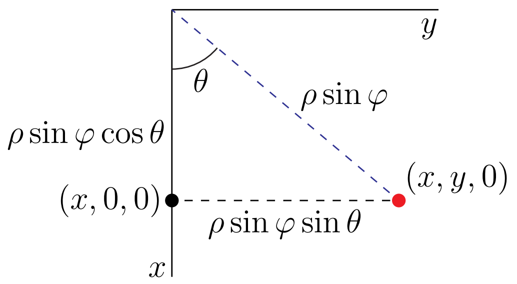
$\rho =$ distance from $(0,0,0)$ to $(x,y,z)$
$\varphi =$ angle between the $z$ axis and the line joining $(x,y,z)$ to $(0,0,0)$
$\theta =$ angle between the $x$ axis and the line joining $(x,y,0)$ to $(0,0,0)$




and here are two more figures giving the side and top views of the previous figure.




side view



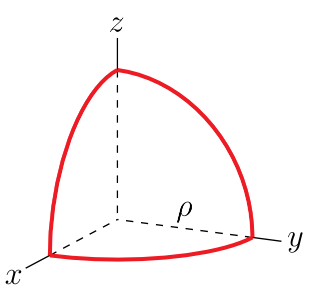

top view

The cartesian and spherical coordinates are related by

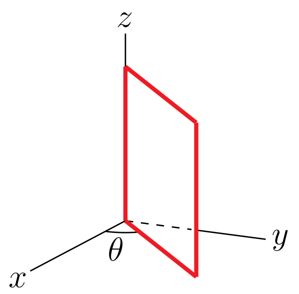
$$x = \rho\sin\varphi\cos\theta \qquad y = \rho\sin\varphi\sin\theta \qquad z = \rho\cos\varphi$$

$$\rho = \sqrt{x^2+y^2+z^2} \qquad \theta = \arctan\frac{y}{x} \qquad \varphi = \arctan\frac{\sqrt{x^2+y^2}}{z}$$

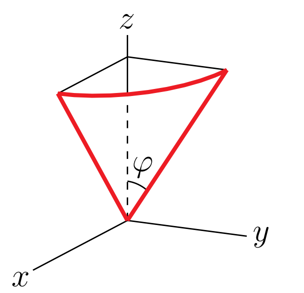
Here are three figures showing a surface of constant $\rho$, a surface of constant $\theta$, and a surface of constant $\varphi$.




surface of constant $\rho$
(a sphere)

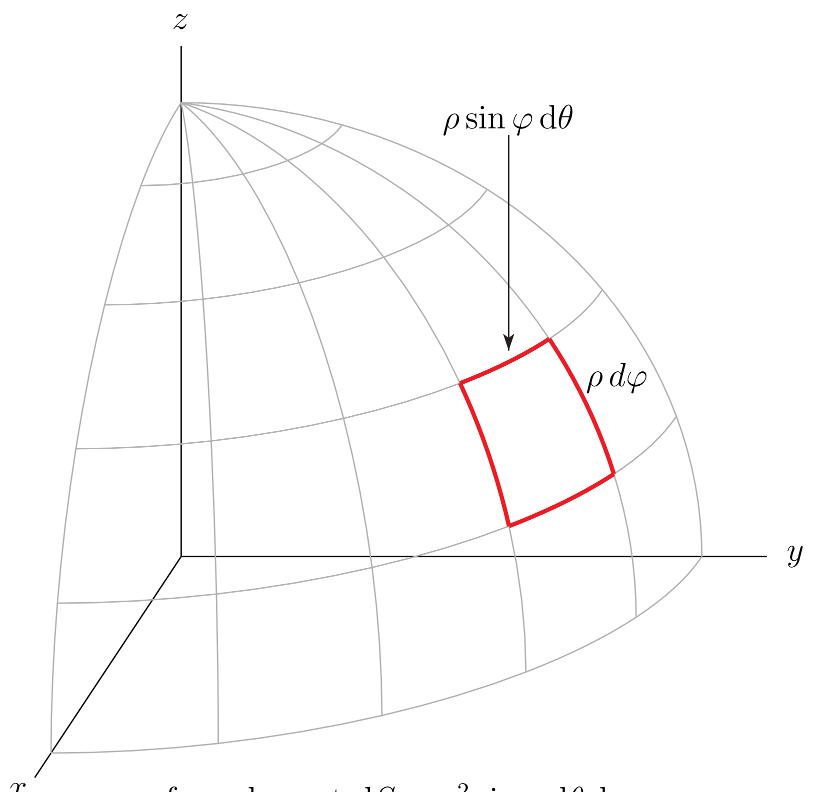



surface of constant $\theta$
(a plane)



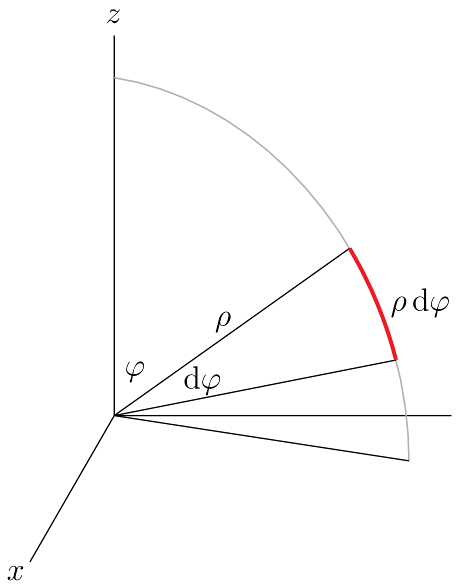

surface of constant $\varphi$
(a cone)

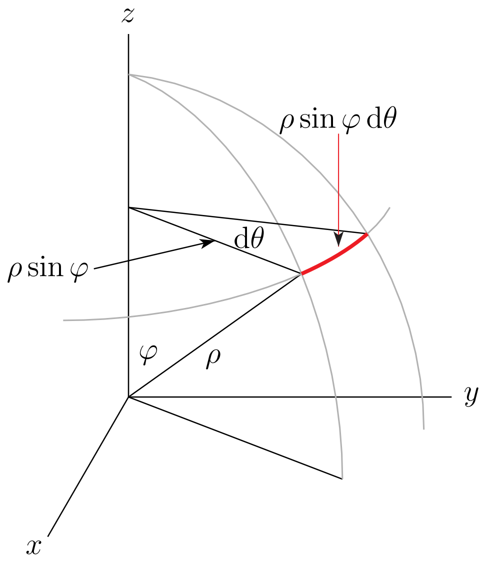
Here is a figure showing the surface element $\mathrm{d}S$ in spherical coordinates



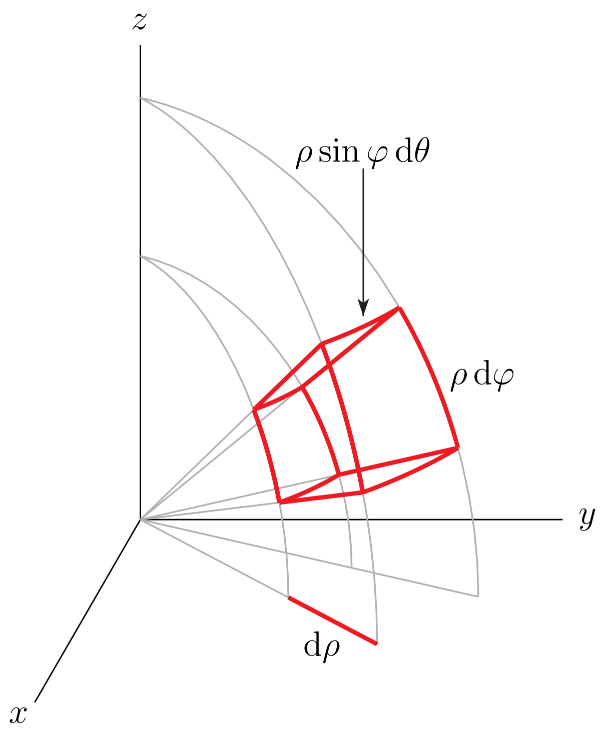

surface element $\mathrm{d}S = \rho^2\sin\varphi\,\mathrm{d}\theta\,\mathrm{d}\varphi$

and two extracts of the above figure to make it easier to see how the factors $\rho\,\mathrm{d}\varphi$ and $\rho\sin\varphi\,\mathrm{d}\theta$ arise.




Finally, here is a figure showing the volume element $\mathrm{d}V$ in spherical coordinates




volume element $\mathrm{d}V = \rho^2 \sin\varphi\; \mathrm{d}\rho\, \mathrm{d}\theta\, \mathrm{d}\varphi$