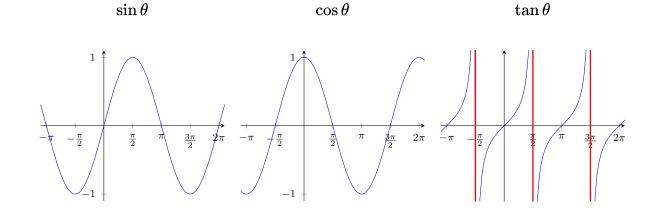
CLP-3 Multivariable Calculus

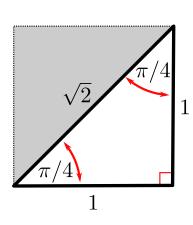
Joel Feldman, Andrew Rechnitzer, Elyse Yeager

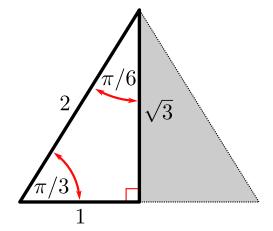
A.1 Trigonometry

A.1.1 Trigonometry — Graphs



A.1.2 Trigonometry — Special Triangles





From the above pair of special triangles we have

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \sin \frac{\pi}{6} = \frac{1}{2} \qquad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{4} = 1 \qquad \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \qquad \tan \frac{\pi}{3} = \sqrt{3}$$

A.1.3 Trigonometry — Simple Identities

• Periodicity

$$\sin(\theta+2\pi)=\sin(heta) \qquad \cos(heta+2\pi)=\cos(heta)$$

• Reflection

$$\sin(-\theta) = -\sin(\theta)$$
 $\cos(-\theta) = \cos(\theta)$

 $\bullet \ \ \text{Reflection around} \ \pi/4$

$$\sin\!\left(rac{\pi}{2} - heta
ight) = \cos heta \qquad \cos\!\left(rac{\pi}{2} - heta
ight) = \sin heta$$

 $\bullet \ \ \text{Reflection around} \ \pi/2 \\$

$$\sin(\pi - \theta) = \sin \theta$$
 $\cos(\pi - \theta) = -\cos \theta$

• Rotation by π

$$\sin(heta+\pi)=-\sin heta \qquad \cos(heta+\pi)=-\cos heta$$

Pythagoras

$$\sin^2 heta + \cos^2 heta = 1 \ an^2 heta + 1 = \sec^2 heta \ 1 + \cot^2 heta = \csc^2 heta$$

 $\bullet \ \sin$ and \cos building blocks

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$

A.1.4 Trigonometry — Add and Subtract Angles

• Sine

$$\sin(lpha\pmeta)=\sin(lpha)\cos(eta)\pm\cos(lpha)\sin(eta)$$

Cosine

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

• Tangent

$$an(lpha+eta) = rac{ anlpha+ aneta}{1- anlpha aneta} \ an(lpha-eta) = rac{ anlpha- aneta}{1+ anlpha aneta}$$

• Double angle

$$egin{aligned} \sin(2 heta) &= 2\sin(heta)\cos(heta) \ \cos(2 heta) &= \cos^2(heta) - \sin^2(heta) \ &= 2\cos^2(heta) - 1 \ &= 1 - 2\sin^2(heta) \end{aligned} \ an(2 heta) &= rac{2\tan(heta)}{1 - an^2 heta} \ \cos^2{ heta} &= rac{1 + \cos(2 heta)}{2} \ \sin^2{ heta} &= rac{1 - \cos(2 heta)}{2} \ an^2{ heta} &= rac{1 - \cos(2 heta)}{1 + \cos(2 heta)} \end{aligned}$$

• Products to sums

$$\sin(lpha)\cos(eta) = rac{\sin(lpha+eta)+\sin(lpha-eta)}{2} \ \sin(lpha)\sin(eta) = rac{\cos(lpha-eta)-\cos(lpha+eta)}{2} \ \cos(lpha)\cos(eta) = rac{\cos(lpha-eta)+\cos(lpha+eta)}{2}$$

• Sums to products

$$\sin lpha + \sin eta = 2 \sin rac{lpha + eta}{2} \cos rac{lpha - eta}{2} \ \sin lpha - \sin eta = 2 \cos rac{lpha + eta}{2} \sin rac{lpha - eta}{2} \ \cos lpha + \cos eta = 2 \cos rac{lpha + eta}{2} \cos rac{lpha - eta}{2} \ \cos lpha - \cos eta = -2 \sin rac{lpha + eta}{2} \sin rac{lpha - eta}{2}$$

A.1.5 Inverse Trigonometric Functions

Domain: $-1 \le x \le 1$

 $\arcsin x$

 $\arctan x$

Domain: $-1 \le x \le 1$

 $\arccos x$

Domain: all real numbers

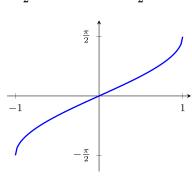
Range:

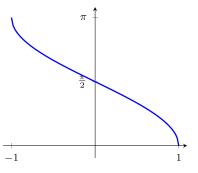
 $-\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}$

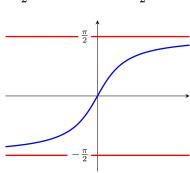
Range: $0 \leq \arccos x \leq \pi$

Range:

 $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$







Since these functions are inverses of each other we have

$$egin{arcsin} rcsin(\sin heta) = heta & -rac{\pi}{2} \leq heta \leq rac{\pi}{2} \ rccos(\cos heta) = heta & 0 \leq heta \leq \pi \ rctan(an heta) = heta & -rac{\pi}{2} \leq heta \leq rac{\pi}{2} \ \end{array}$$

and also

$$\sin(rcsin x) = x \qquad -1 \le x \le 1 \ \cos(rccos x) = x \qquad -1 \le x \le 1 \ \tan(rctan x) = x \qquad ext{any real } x$$

 $\operatorname{arccsc} x$

 $\operatorname{arcsec} x$

 $\operatorname{arccot} x$

Domain: $|x| \geq 1$

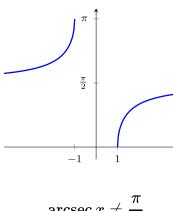
Domain: $|x| \ge 1$

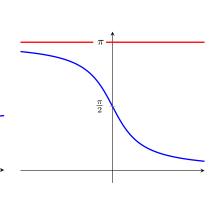
Domain: all real

numbers

Range: $-rac{\pi}{2} \leq x \leq rac{\pi}{2}$

 $\mathsf{Range:}\ 0 \leq x \leq \pi \qquad \mathsf{Range:}\ 0 < \mathrm{arccot}\ x < \pi$





 $\mathrm{arccsc}\,x
eq 0$

 $\operatorname{arcsec} x \neq \frac{\pi}{2}$

Again

$$egin{arcsec} rccsc(\csc heta) = heta & -rac{\pi}{2} \leq heta \leq rac{\pi}{2}, \; heta
eq 0 \ rccsc(\sec heta) = heta & 0 \leq heta \leq \pi, \; heta
eq rac{\pi}{2} \ rccsc(\cot heta) = heta & 0 < heta < \pi \end{aligned}$$

and

$$egin{array}{ll} \csc(rccsc x) = x & |x| \geq 1 \ \sec(rcsec x) = x & |x| \geq 1 \ \cot(rccot x) = x & ext{any real } x \end{array}$$