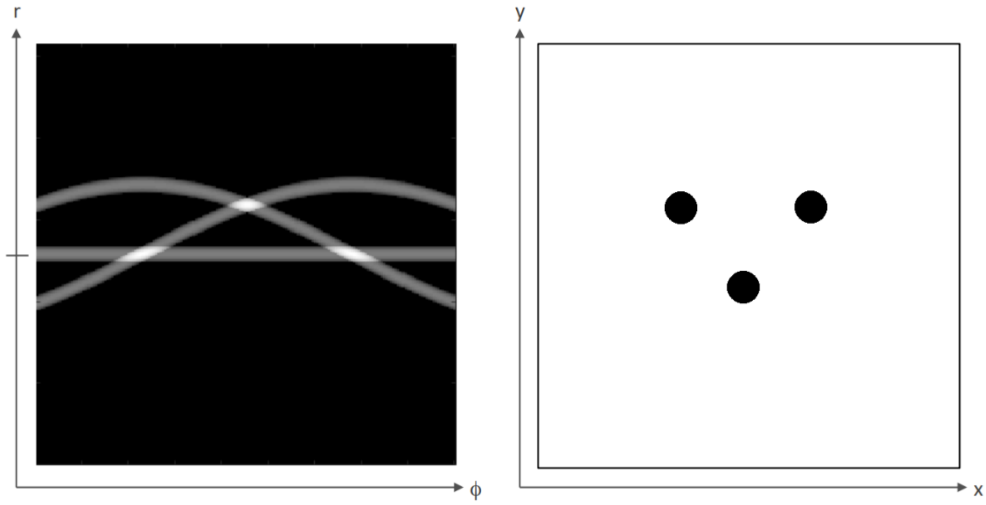
Exercise XCT #2 - Sinogram and CT Image Reconstruction

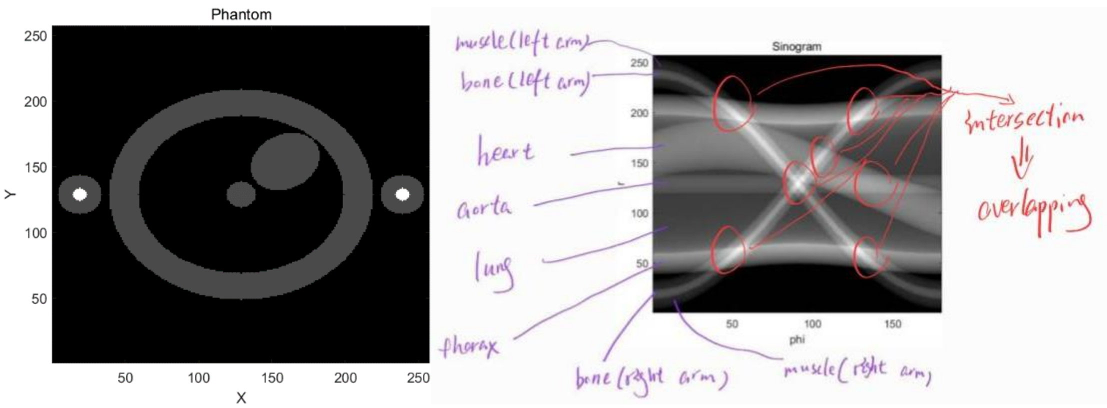
Shuo Li; Yitong Li

Task 2.1

Using the sinogram to reconstruct the object:

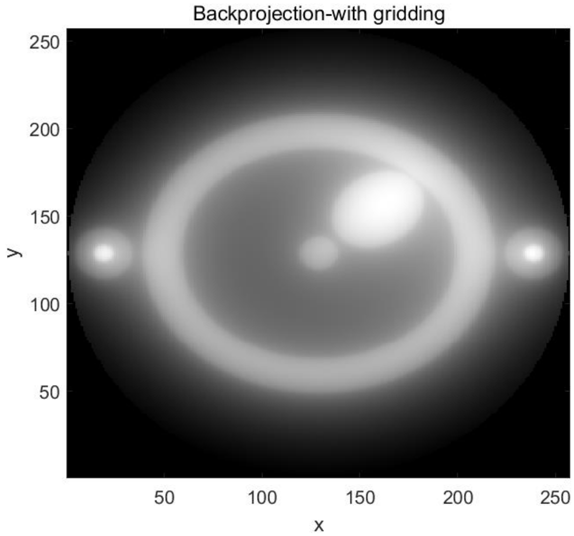


Original object and the corresponding sinogram:



From the result of the sinogram, we get 7 curves which correspond to 7 different structures. If there are different structures overlapping in a specific angle and position, an intersection will appear on the sinogram. For instance, when $\varphi \approx 50°$, the arms (bone & muscle) overlap with the thorax.

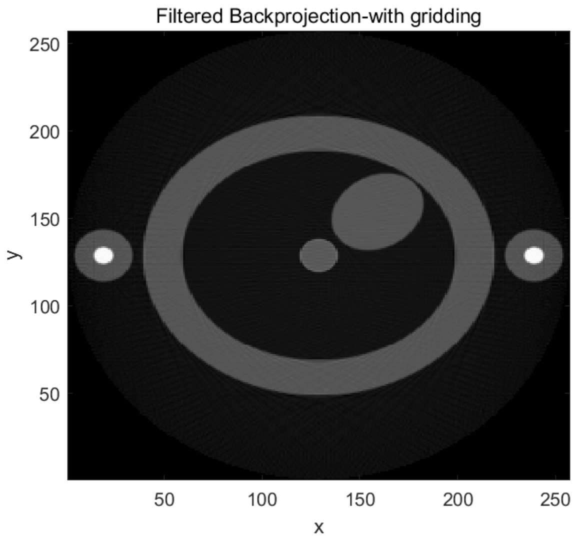
Task 2.2



The reason why the resulting image is blurred is that the result is based on overlapping of projection images in different angles. In this way, there will be still be some intensities in the regions where there are no objects. Set r is the distance away from the target object, the intensity decreases when r increases.

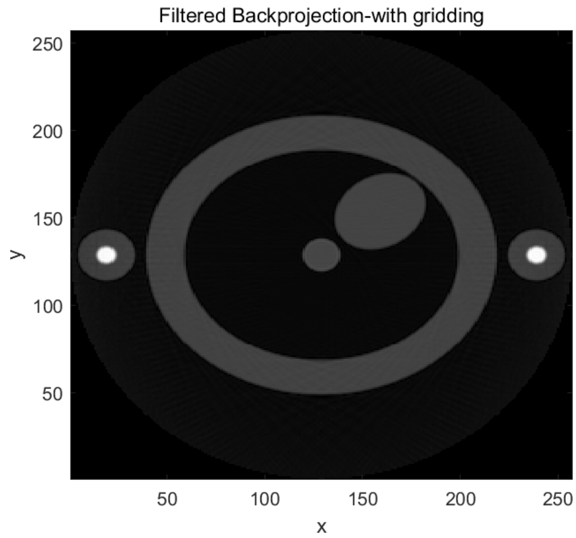
Task 2.3

Result after implementing an ideal high-pass filter:

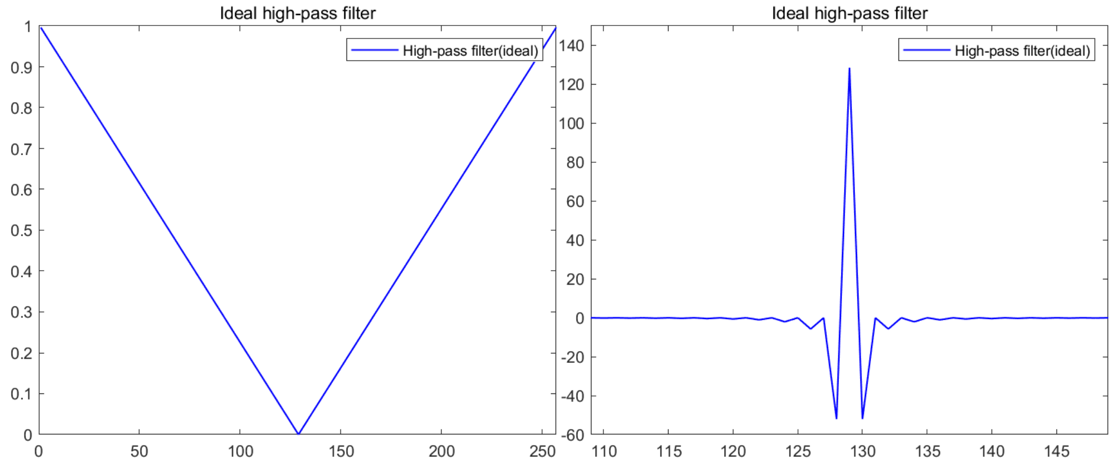


The result turns out that there are some "streak" artifacts. These artifacts correspond to higher frequencies because they are dense and thin. To correct this kind of artifact, we can modify our filter to make it be able to pass through more low-frequency components to offset the high-frequency artifacts.

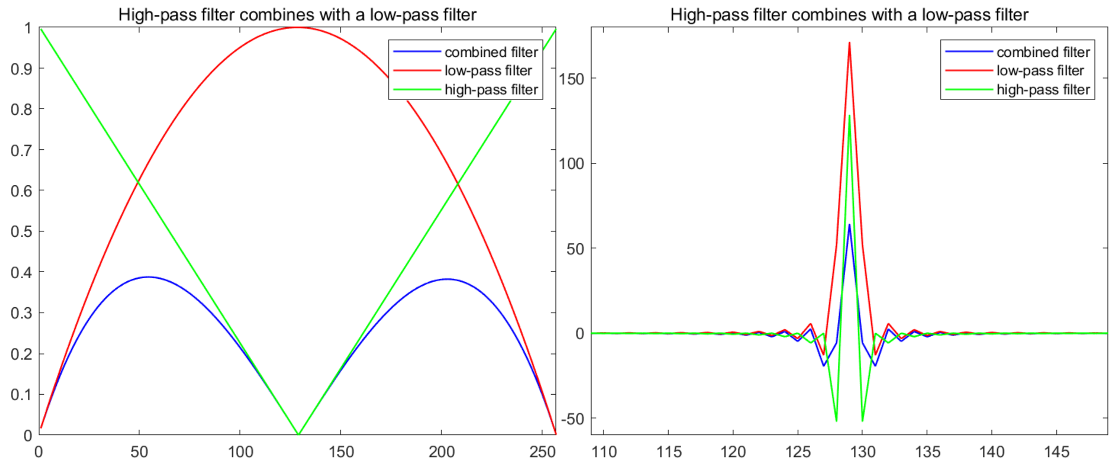
Result after combing the ideal high-pass with a low-pass filter:



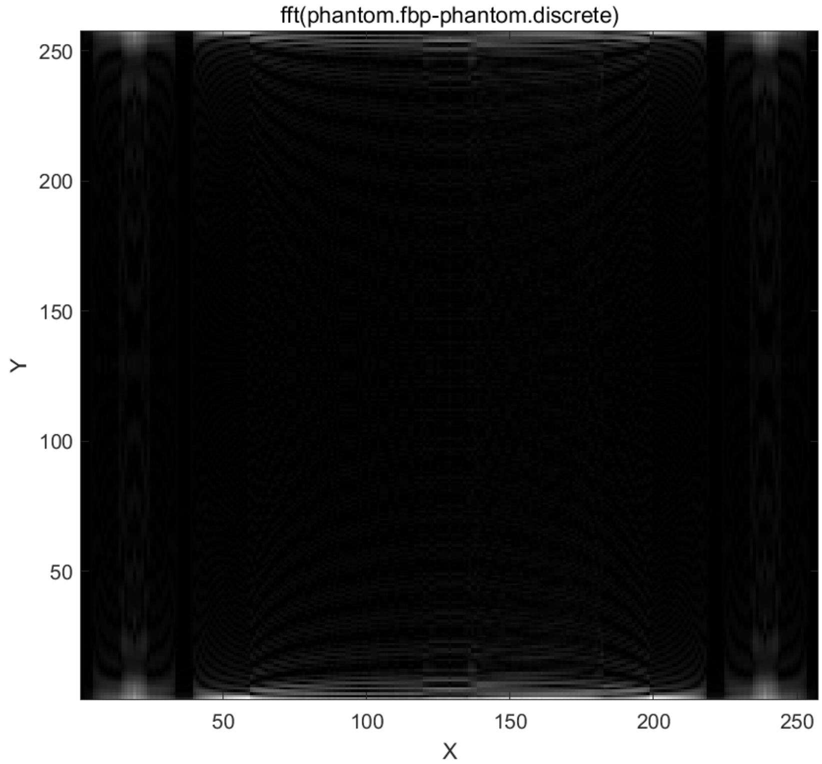
Fourier transform pair of the ideal filter:



Fourier transform pair of the modified filter:



In the figure of the Fourier transform pair of the ideal filter, we can see that the amplitude is intensively near the central frequency. This kind of "vibration" leads to the "streak" artifacts. High-frequency components of the original image are magnified by the ideal high-pass filter. In this way, the imaging system is very sensitive to high-frequency noises. The figure below shows the Fourier transform result of the difference between the filtered backprojection image and the original phantom image. It verifies that the "streak" artifacts mainly correspond to high-frequency components, because the intensities are distributed in the edges. In order to deal with the problem, we can use modified filters to reduce the high-frequency components. But there are also some trade-offs. Although the modified filters can deal with "streak" artifacts, they will also filter out some useful components which also correspond to high frequencies. Our results will be smoothed by these modified filters. So, the balance between filtering out "streak" artifacts and preserving image details is quite important, especially for images which are much more complex than the image in this experiment.



Task 2.4

Fourier-Slice theorem:

Fourier - slice theorem:

The results of the following two calculations are equal:

* Take a two-dimensional function $f(r)$, project it onto a line, and do a Fourier transform of that projection

* Take the same function, but do a two-dimensional Fourier transform first, and then slice it through its origin, which is parallel to the projection line.

Proof: $f(r) = u(s, r)$

① $P_\varphi(r) = \int_{-\infty}^{+\infty} u(s, r)\, ds$

$$F(P_\varphi(r)) = \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} u(s,r) \cdot ds \right) \cdot e^{-iur} dr$$

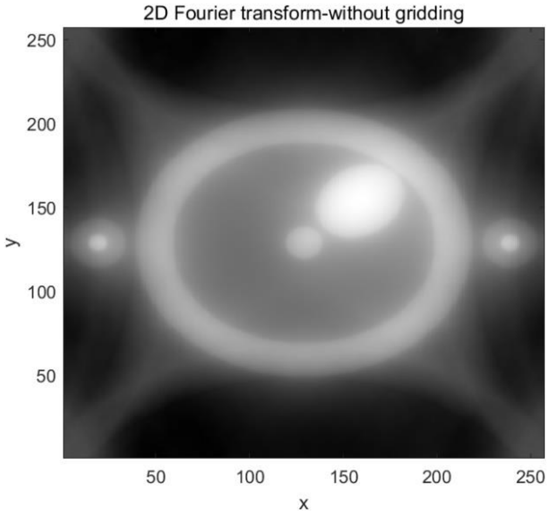
$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(s,r)\, e^{-iur}\, dr \cdot ds$$

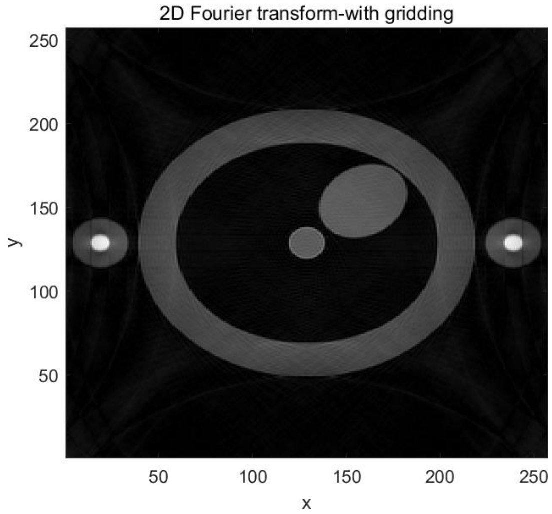
② $$F(u(s,r)) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(s,r) \cdot e^{-iur} \cdot e^{-ivs}\, dr \cdot ds$$

slice it through its origin, which is parallel to the projection line.

$\Rightarrow \quad v = 0.$

$$F(u(s,r))|_{v=0} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(s,r) \cdot e^{-iur}\, dr\, ds$$





The reason why there are some image artifacts is mainly because of our approximation method. Integer coordinates in one coordinate system often correspond to decimal coordinates in another coordinate system. But in the MATLAB implementation, we simply use integers to approximate the decimal coordinates. And that will lead to some errors. In addition, we implement 180 backprojections in this experiment. This may lead to some problems of sparse sampling, which means there will be some data lacking at some coordinates. In order to address this problem, we can use the data with decimal coordinates to fit a surface function and then estimate the value in integer coordinates which can be expressed in matrices. The result turns out that this method can correct most of the image artifacts.

Using the 2D Fourier transform with data gridding, our results are shown above. There is no doubt that this result is much better than the result obtained in Task 2.2. The advantage of the 2D FT method compared with FBP in Task 2.3 is that it can be implemented without having to design specific filters to deal with blurring problems. For reconstruction of images containing a high degree of complexity, this advantage will be more important because the balance between filtering out “streak” artifacts and preserving image details in FBP becomes more difficult.