Exercise XCT #1 - X-ray Projection and Digital Subtraction Angiography
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Task 1.1

ρ-density:

```
rho_blood     = 1.060;        % density blood    [g/cm3]
rho_bone      = 1.920;        % density bone     [g/cm3]
rho_lung      = 1.050;        % density lung/air [g/cm3]
rho_muscle    = 1.050;        % density muscle   [g/cm3]
```

Mass attenuation coefficient:

```
mac_blood(1)   = 0.2278;       % blood @  50 keV   [cm2/g]
mac_blood(2)   = 0.1492;       % blood @ 150 keV   [cm2/g]

mac_bone(1)    = 0.4242;       % bone @  50 keV    [cm2/g]
mac_bone(2)    = 0.1480;       % bone @ 150 keV    [cm2/g]

mac_lung(1)    = 0.2270;       % lung @  50 keV    [cm2/g]
mac_lung(2)    = 0.1493;       % lung @ 150 keV    [cm2/g]

mac_muscle(1)  = 0.2262;       % muscle @  50 keV  [cm2/g]
mac_muscle(2)  = 0.1492;       % muscle @ 150 keV  [cm2/g]
```

The total mass attenuation coefficient is mainly based on two effects: Compton effect and photoelectric interaction.

In photoelectric attenuation, the probability of a photoelectric interaction (P_{pe}) occurring depends on the energy(E) of the incident X-ray, the effective atomic number (Z_{eff}) of the tissue, and the tissue density (ρ):

$$P_{pe} \propto \rho \frac{Z_{eff}^3}{E^3}$$

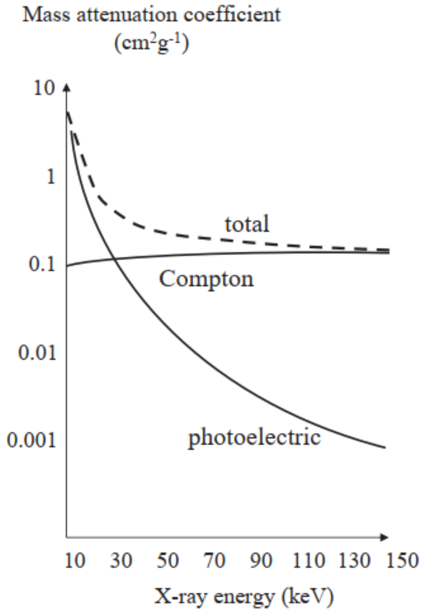
In Compton scattering, the energy of the scattered X-ray is given by:

$$E_{X,scat} = \frac{E_{X,inc}}{1 + (\frac{E_{X,inc}}{mc^2})(1 - cos\theta)}$$

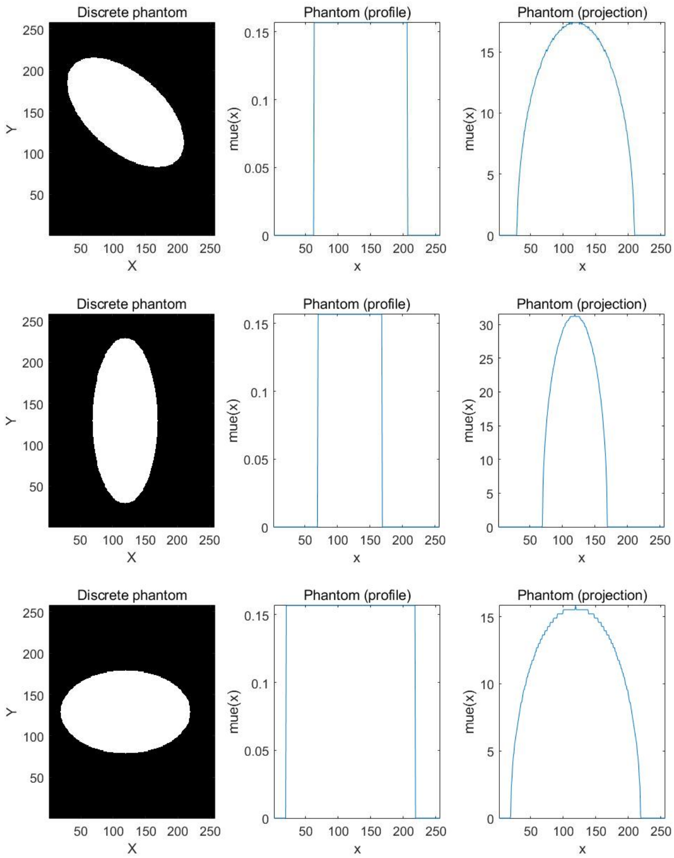
The value of the mass attenuation coefficient is the sum of the individual contributions from photoelectric absorption and Compton scattering.

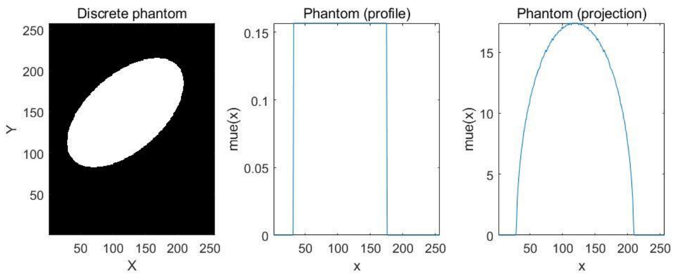
$$\mu(E) = \mu(E)_{photoelectric} + \mu(E)_{Compton}$$

In here, the value of $\mu(E)_{photoelectric}$ has a positive correlation with P_{pe}. Similarly, the value of $\mu(E)_{Compton}$ also has a positive correlation with $E_{X,scat}$. As the figure below shows, the contribution from the photoelectric effect dominates at low X-ray energies, but Compton scatters is the more important term at high energies.



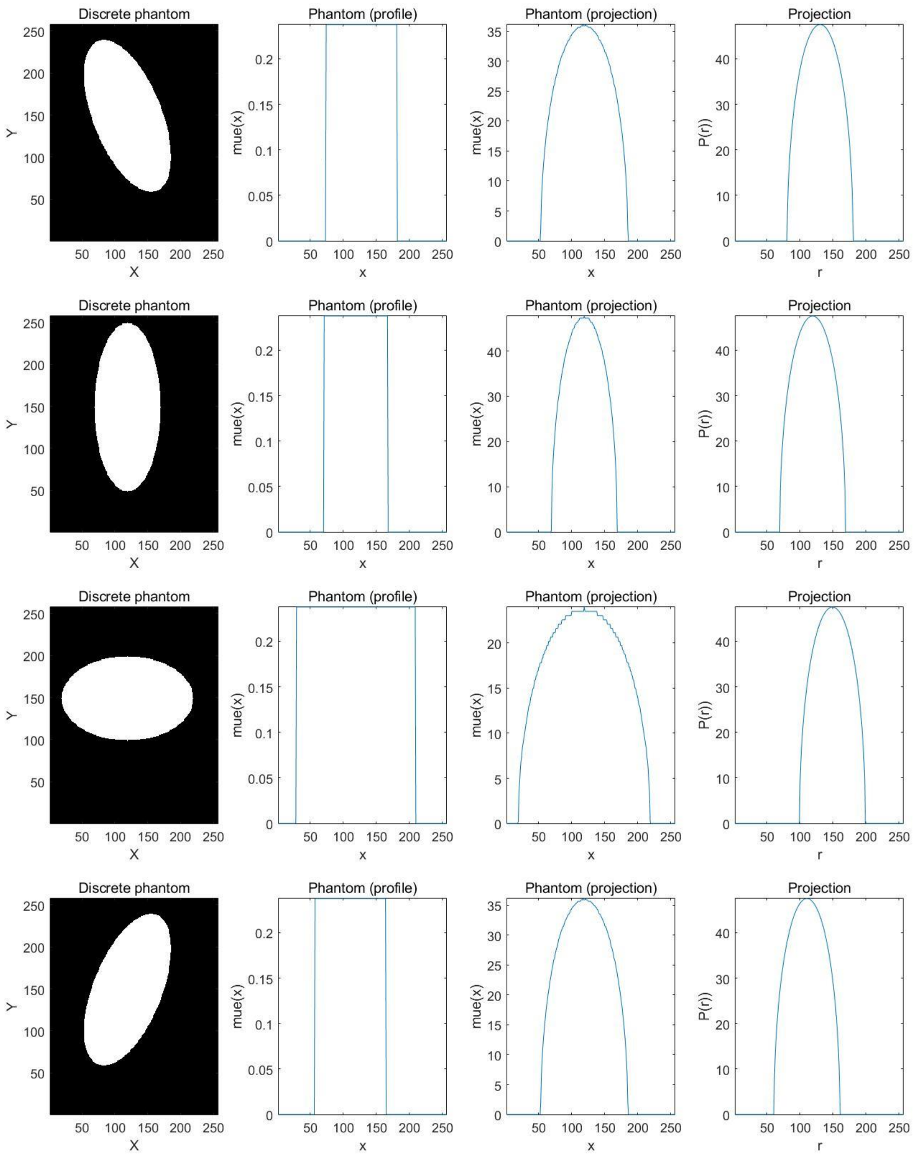
The result of the sum along the vertical direction:





The results turned out that the shapes of the sum along the vertical direction sym are symmetric and determined by the position, size, and shape of the ellipses.

Task 1.2

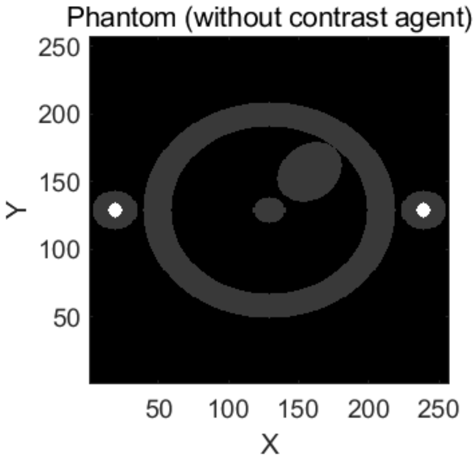


The results turned out that, the sum along the vertical direction can only compute the vertical projection whereas the line integral can compute the projection in arbitrary direction. What's more, even during the computation of vertical projection, the values near the peak are a little bit different. That is because of the discrete property of digital images. So the images are not exactly ellipses. But the results of line integral are computed based on elliptic functions, these results are more accurate.

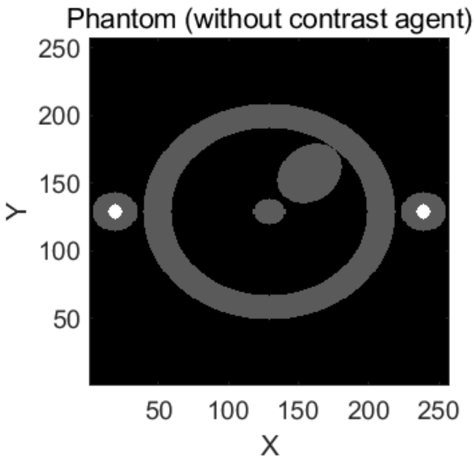
Task 1.3

Horizontal projections for anode voltages of 50 versus 150 keV:

50keV:



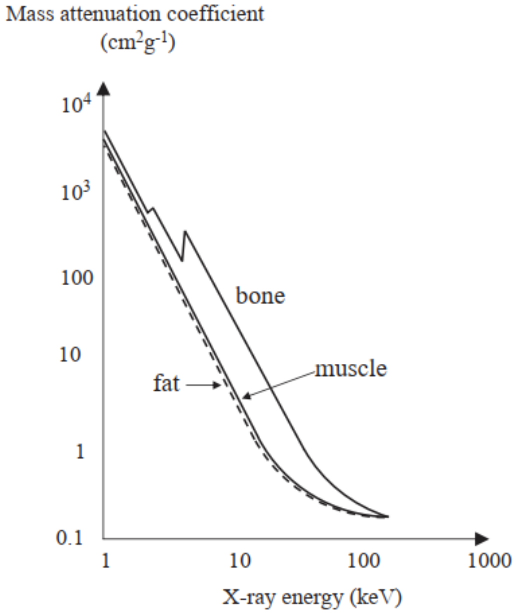
150keV:



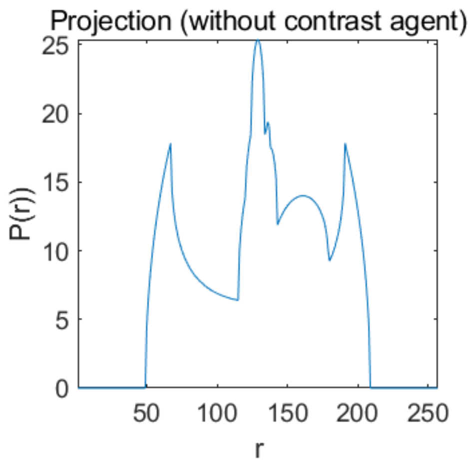
When the anode voltage is equal to 50keV, the contrast between bone and muscle is higher than when the voltage is equal to 150keV. This is because of the change of mass attenuation coefficients under different anode voltages. When the anode voltage is equal to 50keV, the difference of mass attenuation coefficient between bone and muscle is much higher than when the voltage is 150 keV.

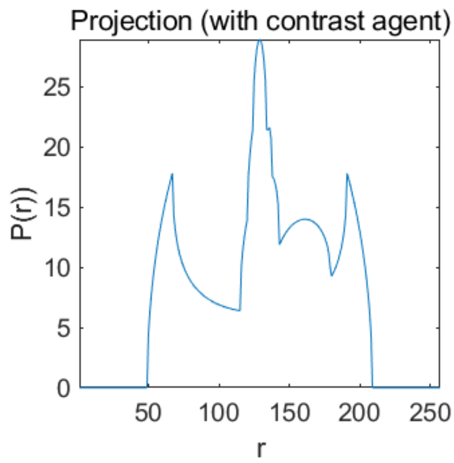
```
mac_bone(1)      = 0.4242;               % bone @  50 keV    [cm2/g]
mac_bone(2)      = 0.1480;               % bone @ 150 keV    [cm2/g]
```

```
mac_muscle(1)    = 0.2262;               % muscle @  50 keV  [cm2/g]
mac_muscle(2)    = 0.1492;               % muscle @ 150 keV  [cm2/g]
```

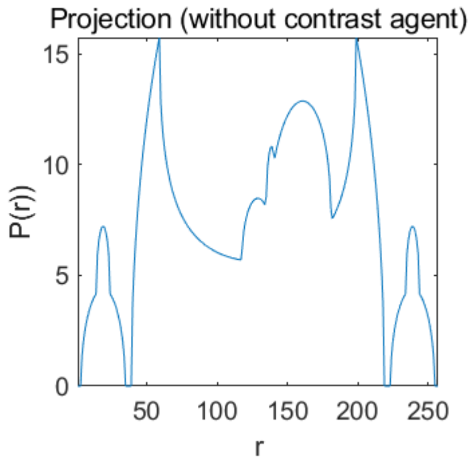


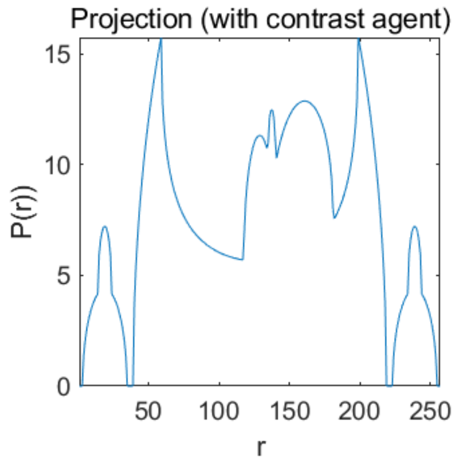
Horizontal projections(50keV, 150keV):



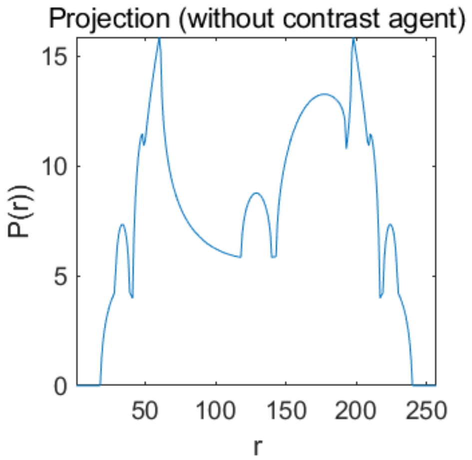


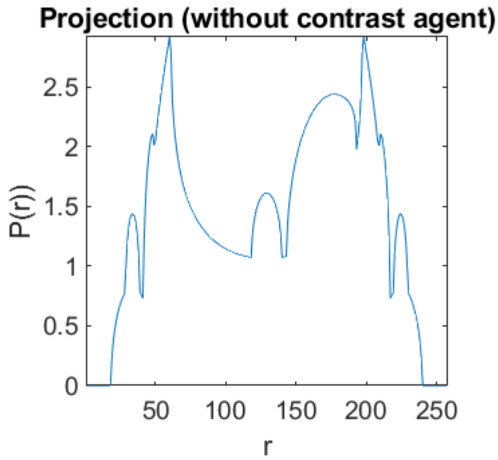
Vertical projections(50keV, 150keV):





Angulated projections(φ=30°, 50keV, 150keV):





The reason why the aorta is not seen well even if under two different voltages is that the mass attenuation coefficient of blood and muscle are very close either when the voltage is equal to 50 keV or when the voltage is equal to 150 keV.

```
mac_blood(1)     = 0.2278;                % blood @  50 keV   [cm2/g]
mac_blood(2)     = 0.1492;                % blood @ 150 keV   [cm2/g]
```

```
mac_muscle(1)    = 0.2262;                % muscle @  50 keV   [cm2/g]
mac_muscle(2)    = 0.1492;                % muscle @ 150 keV   [cm2/g]
```

General equation of blood contrast based on Beer-Lamberts law:

The intensity corresponds to blood:

$$I_{blood} = I_0 * e^{-\mu_{blood}*x}$$

The intensity corresponds to the background matter:

$$I_{back} = I_0 * e^{-\mu_{back}*x}$$

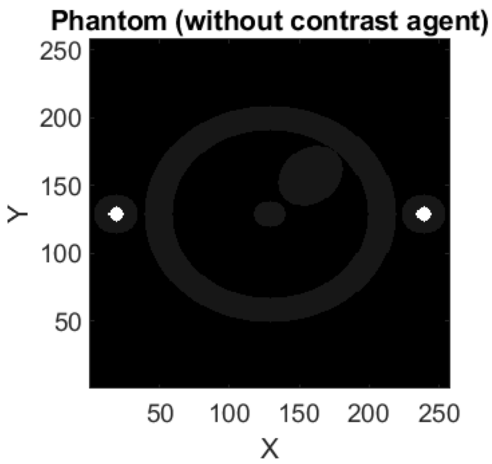
The contrast without a contrast agent:

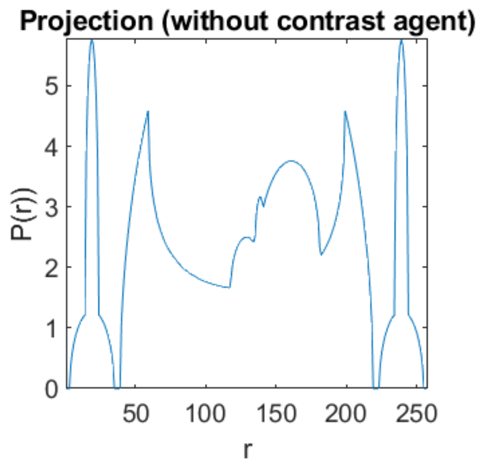
$$C_{without\_agent} = |I_{blood} - I_{back}| = I_0 * e^{-\mu_{back}*x} * \left|1 - e^{(\mu_{back}-\mu_{blood})*x}\right|$$

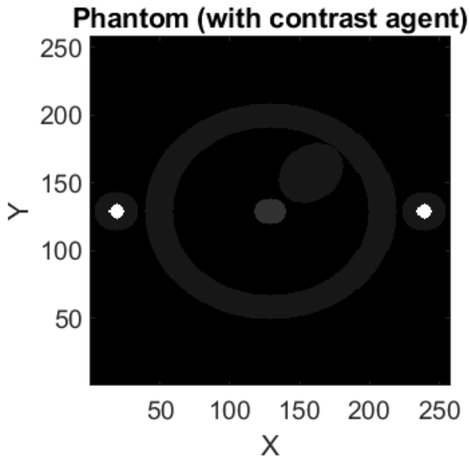
The contrast with a contrast agent:

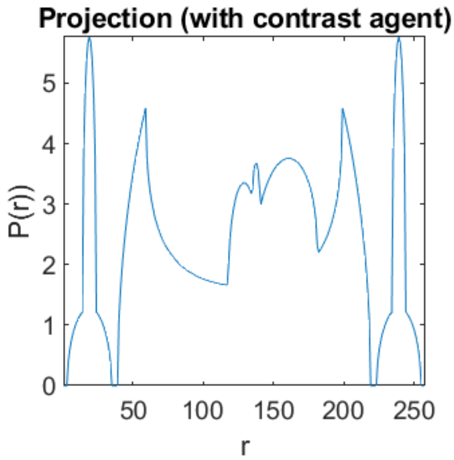
$$C_{with\_agent} = |I_{blood} - I_{back}| = I_0 * e^{-\mu_{back}*x} * \left|1 - e^{(\mu_{back}-\eta*\mu_{blood})*x}\right|$$

DSA implementation (vertical projection):









The result shows that, the contrast has been improved after adding the iodine contrast agent. When $\mu_{back} \approx \mu_{blood}$, $C_{without\_agent}$ is close to 0, which means the contrast is very bad and the blood can't be seen well. After adding the contrast agent, C_{with_agent} will be increased. In this way, the contrast will be improved.