exercise Ultrasound 2 SHUC LI

1. $Z_{NFB} = \frac{r^2}{\lambda_{tissue}}$, $r \to 0$, $Z_{NFB} = 0$

lateral resolution $\begin{cases} x = 0 & 2r = 0 \\ x > 0 & 2r = +\infty \end{cases}$

broadening angle $\theta = 90°$

real part

magnitude

2. directionality

n = 1

spherical wave

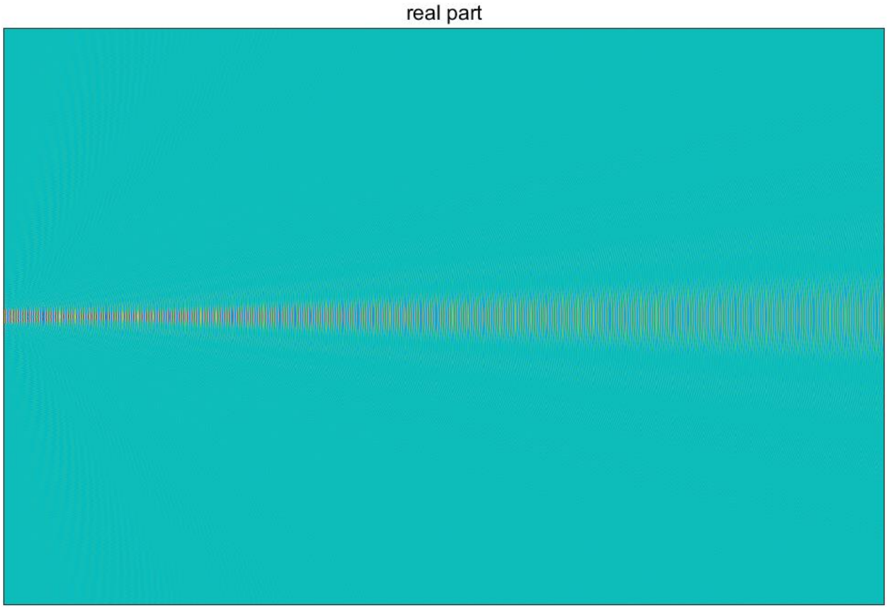
n > 1

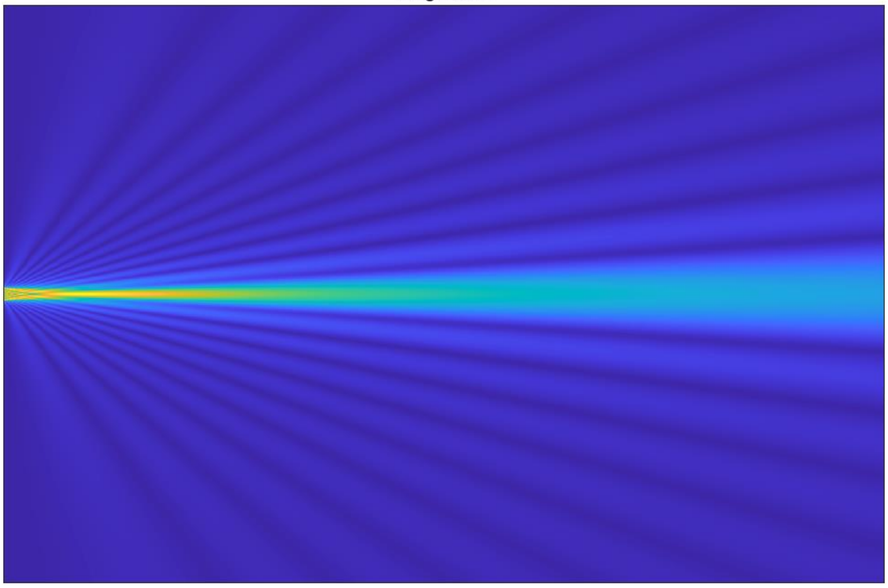
directionality ↑

In the many-element case, the transducer radius r is no longer close to 0. With the increasing number of transducers, the radius r inreases. Z_{NFB} and lateral resolution increase, the broadening angle decrease, which means the directionality gets better.

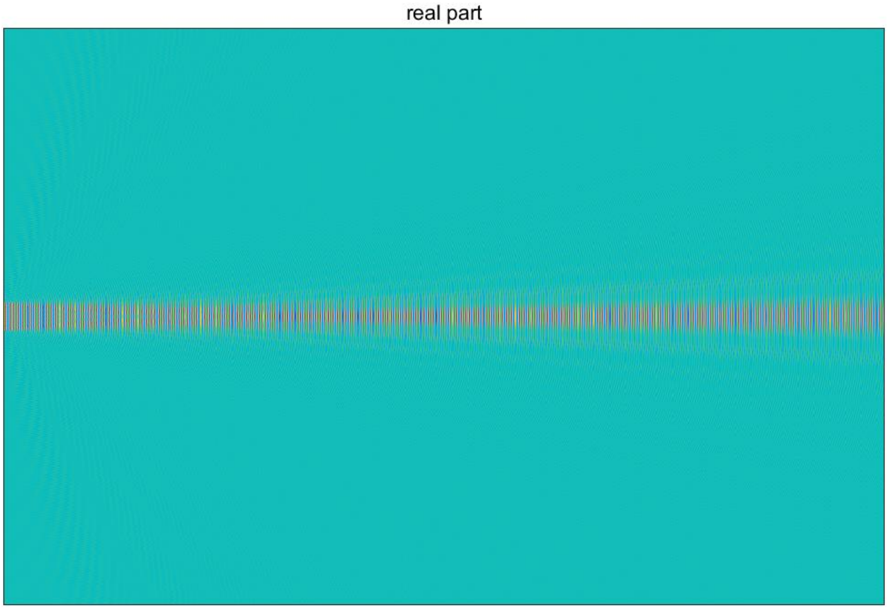
nt = 20

real part

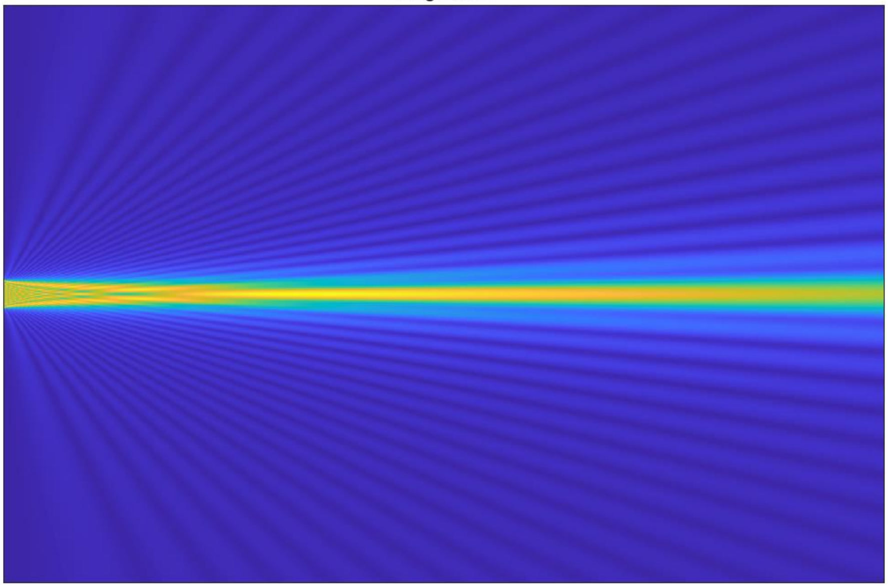




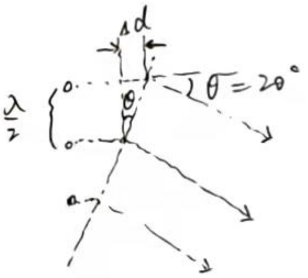
nt = 40



magnitude



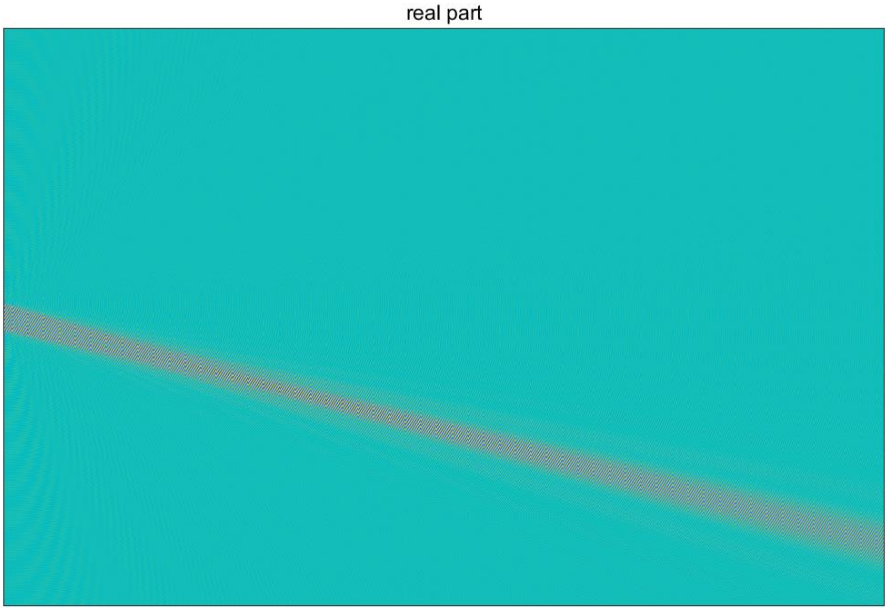
3.



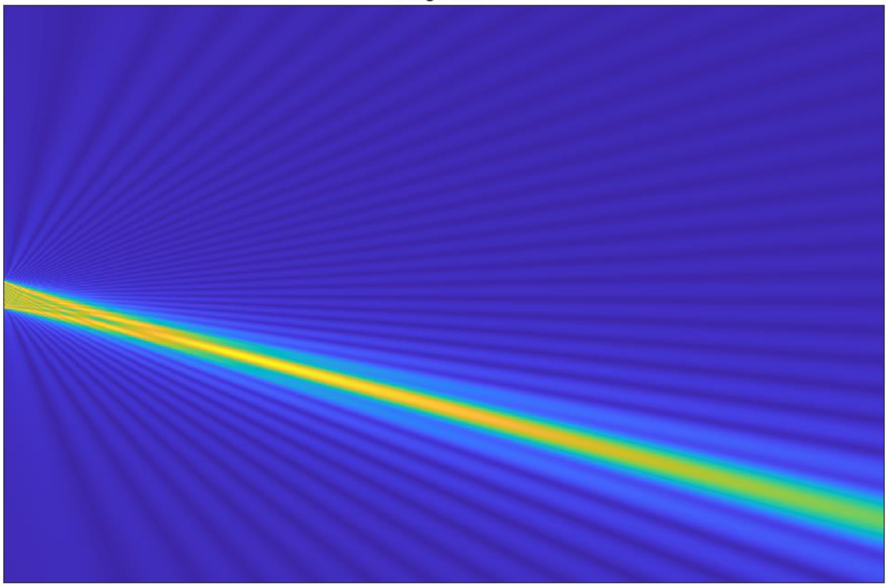
$$\begin{cases} \dfrac{\Delta d}{(\frac{\lambda}{2})} = \tan 20^\circ \\ \Delta d = c\cdot\Delta t = \dfrac{c\cdot\Delta\varphi}{2\pi f} = \dfrac{\lambda\Delta\varphi}{2\pi} \end{cases}$$

$$\Rightarrow \Delta\varphi = \tan 20^\circ \pi = 1.14345 \text{ rad}$$

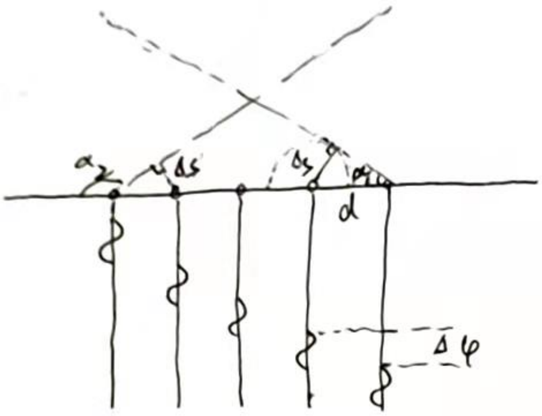
real part



magnitude



4.



phase difference = 0 ① $d\cdot\sin\alpha_1 = c\cdot\Delta t = \frac{\lambda\Delta\varphi}{2\pi}$

$$\sin\alpha_1 = \frac{\lambda\Delta\varphi}{2\pi d}$$

phase difference = π ② $d\cdot\sin\alpha_2 = \frac{\lambda(\Delta\varphi-\pi)}{2\pi}$

$$\sin\alpha_2 = \frac{\lambda(\Delta\varphi-\pi)}{2\pi d}$$

$$\begin{cases} \sin\alpha_1 \in (-1,1) \\ \sin\alpha_2 \in (-1,1) \\ \sin\alpha_1\cdot\sin\alpha_2 < 0 \end{cases}$$

$\Rightarrow \quad \frac{\lambda\pi}{2\pi d} \in (-1,1) \qquad d > \frac{\lambda}{2}$

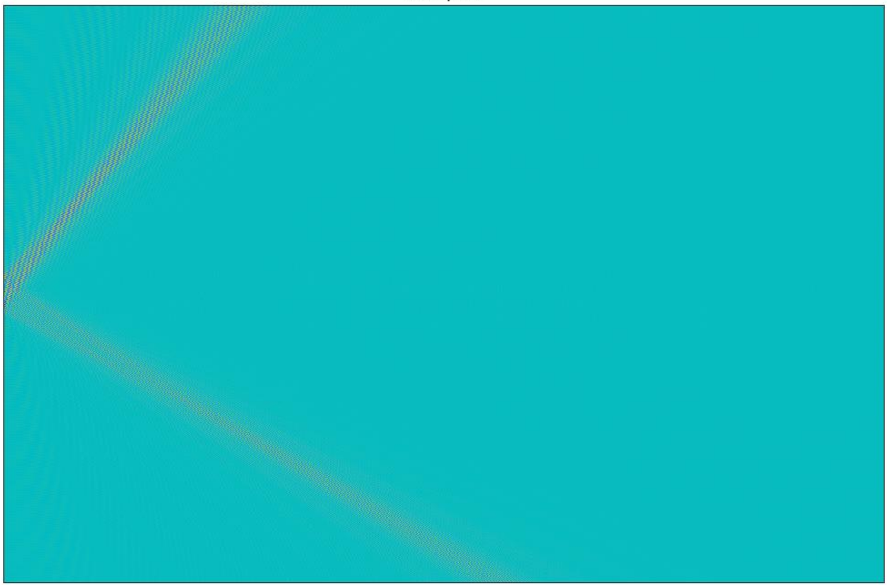
The generation of grating lobe is because of the diffraction which is based on Huygen's principle.

When $d > \frac{\lambda}{2}$, there are multiple solutions for

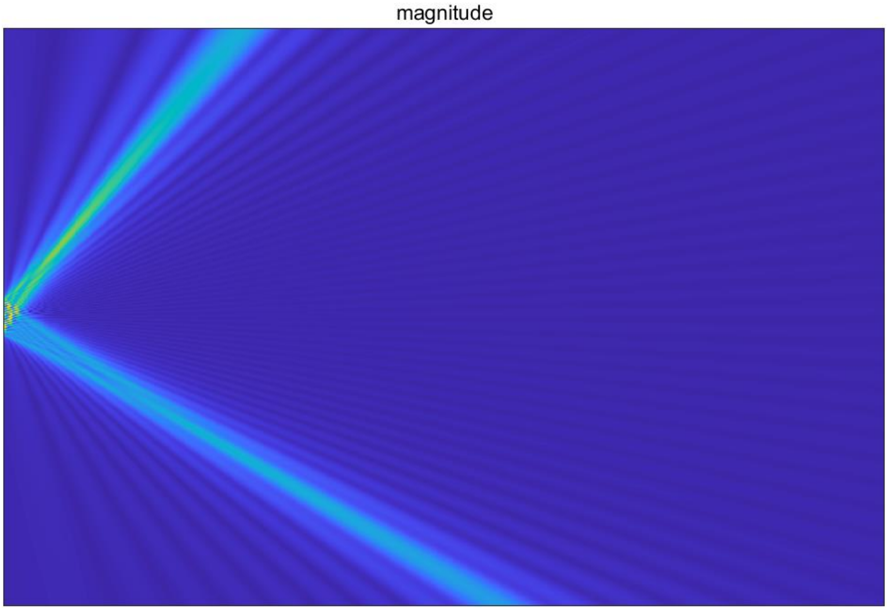
$$\Delta \Phi \bmod 2\pi = kd\sin(\alpha) \bmod 2\pi$$

A grating lobe will be generated.

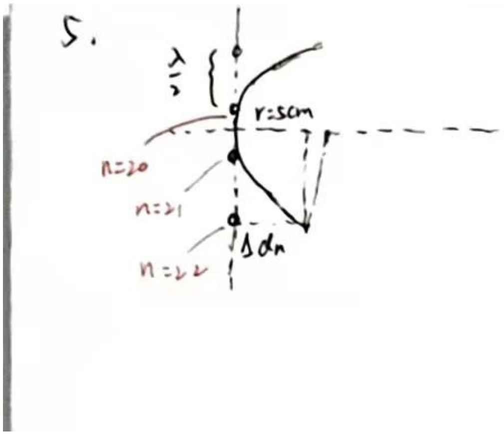
real part



magnitude



5.

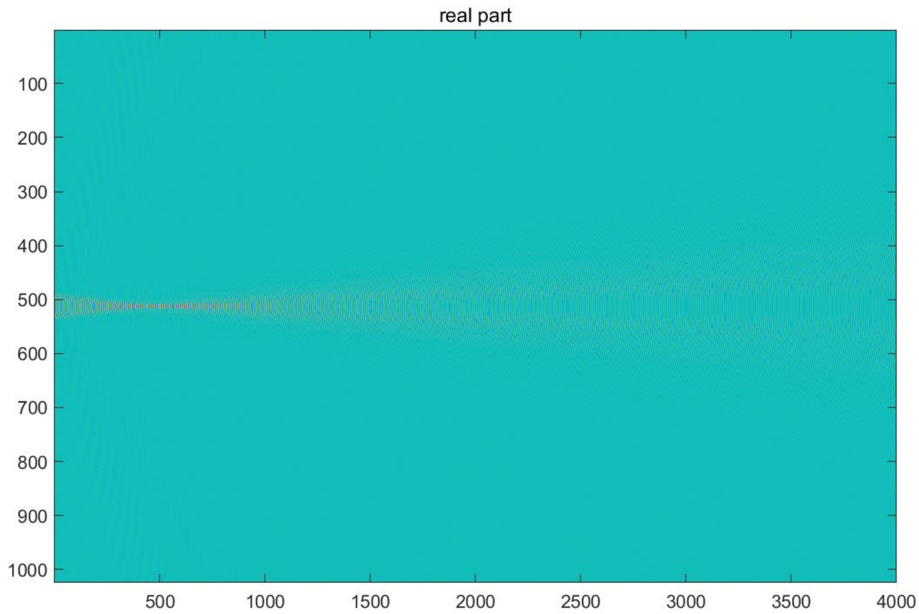


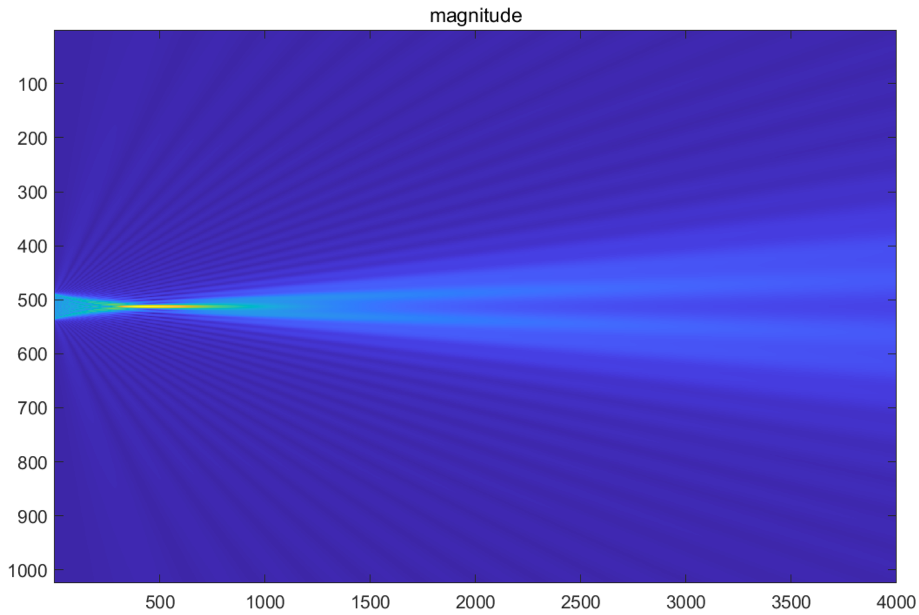
$t_n = 40.$

$$\Delta d_n = r - \sqrt{r^2 - \left[\frac{\left(\frac{t_n+1}{2}-n\right)\lambda}{2}\right]^2}$$

$$\Delta \varphi_n = -k \cdot \Delta d_n = \frac{-2\pi}{\lambda} \cdot \Delta d_n$$

$$= \frac{-10\pi}{\lambda} + \frac{2\pi}{\lambda}\sqrt{5^2 - \left[\frac{(20.5-n)\lambda}{2}\right]^2}$$



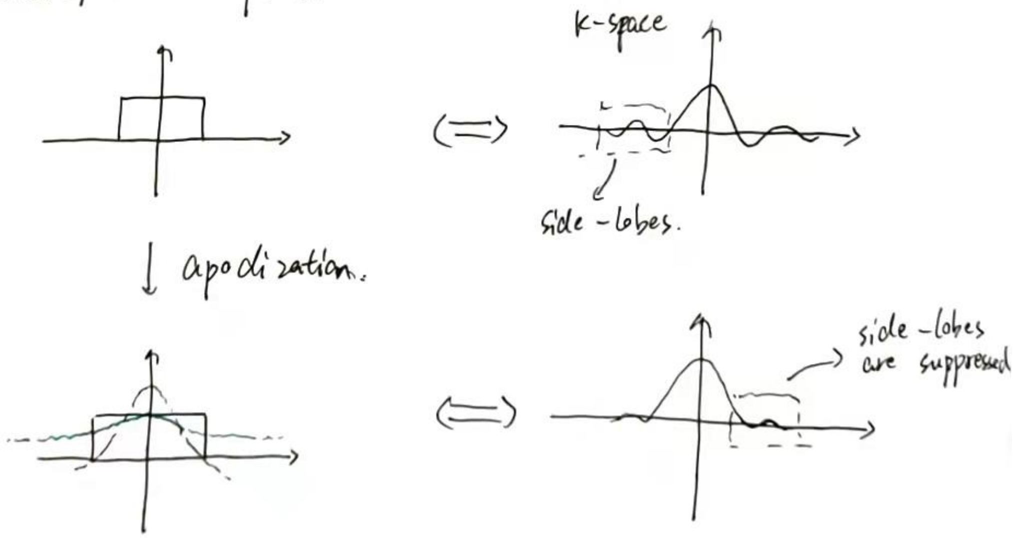


6. $f(x) = e^{-\frac{1}{2}\left(\frac{x-20.5}{\sigma}\right)^2}$

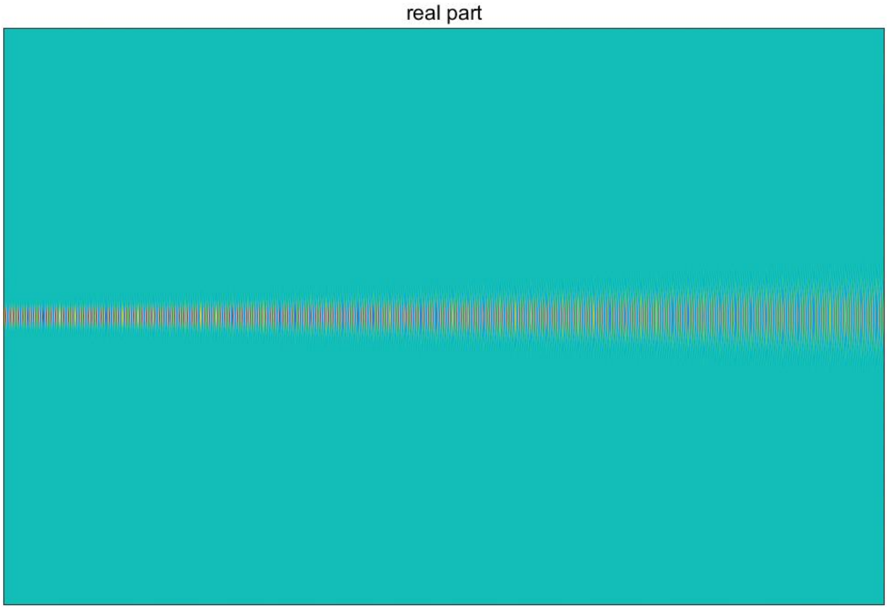
$$f(1) = 0.1 = e^{-\frac{1}{2}\left(\frac{-19.5}{\sigma}\right)^2}$$

$$\Rightarrow \sigma = 9.086817$$

Applying a gaussian filter can help decrease the amplitude of side-lobes. It's similar with the apodization technique in optics.



real part



magnitude