Exercise MRI #1

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Task 1.1

The ratio between up- and down-state spin population follows Boltzman statistics.

$$\frac{n_{us}}{n_{ds}} = e^{-\frac{\Delta E}{k_B T}}$$

$$\Delta E = \hbar \gamma B_0$$

$$\hbar = 1.05457266 \times 10^{-34} \ J \cdot s$$

$$\frac{\gamma}{2\pi} = 42.58 \ MHz/T$$

$$B_0 = 3T = 930 K$$

$$k_B = 1.38064852 \times 10^{-23} \ m^2 kg \ s^{-2} K^{-1}$$

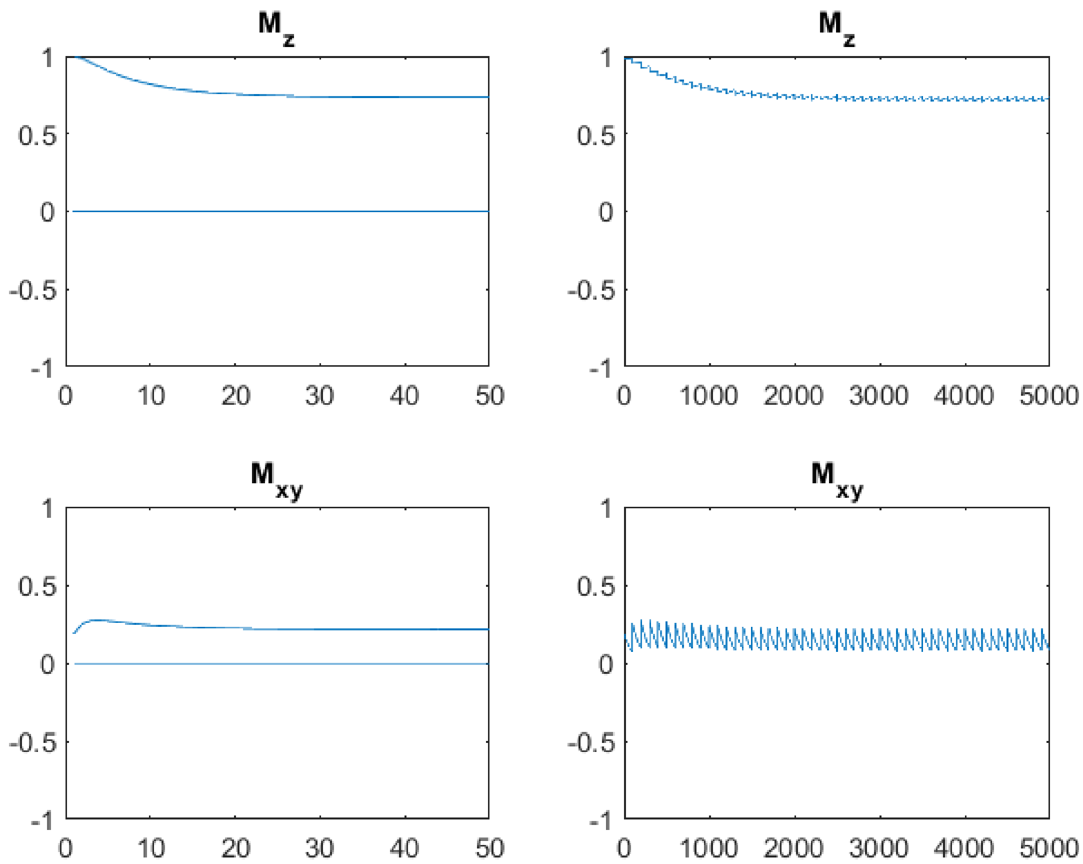
$$\frac{n_{us}}{n_{ds}} = 0.99998$$

$$\frac{\Delta n}{n} = \frac{n_{us} - n_{ds}}{n_{us} + n_{ds}} = 9.888 \times 10^{-6}$$

Task 1.2

- Perform equal excitations of given flip angle θ at a given repetition time T_R

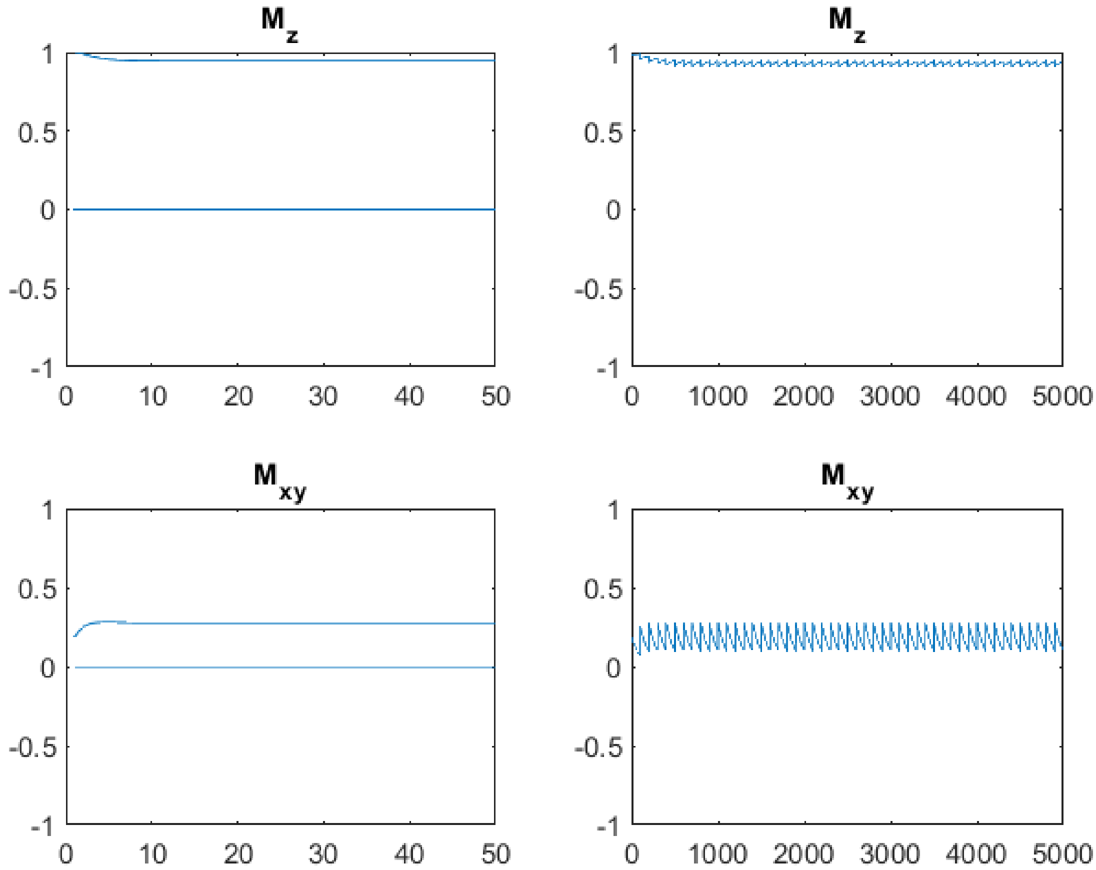
T1 = 500, T2 = 50, angle_flip = 0.06×π, repetition time = 50, number of pulses = 50.



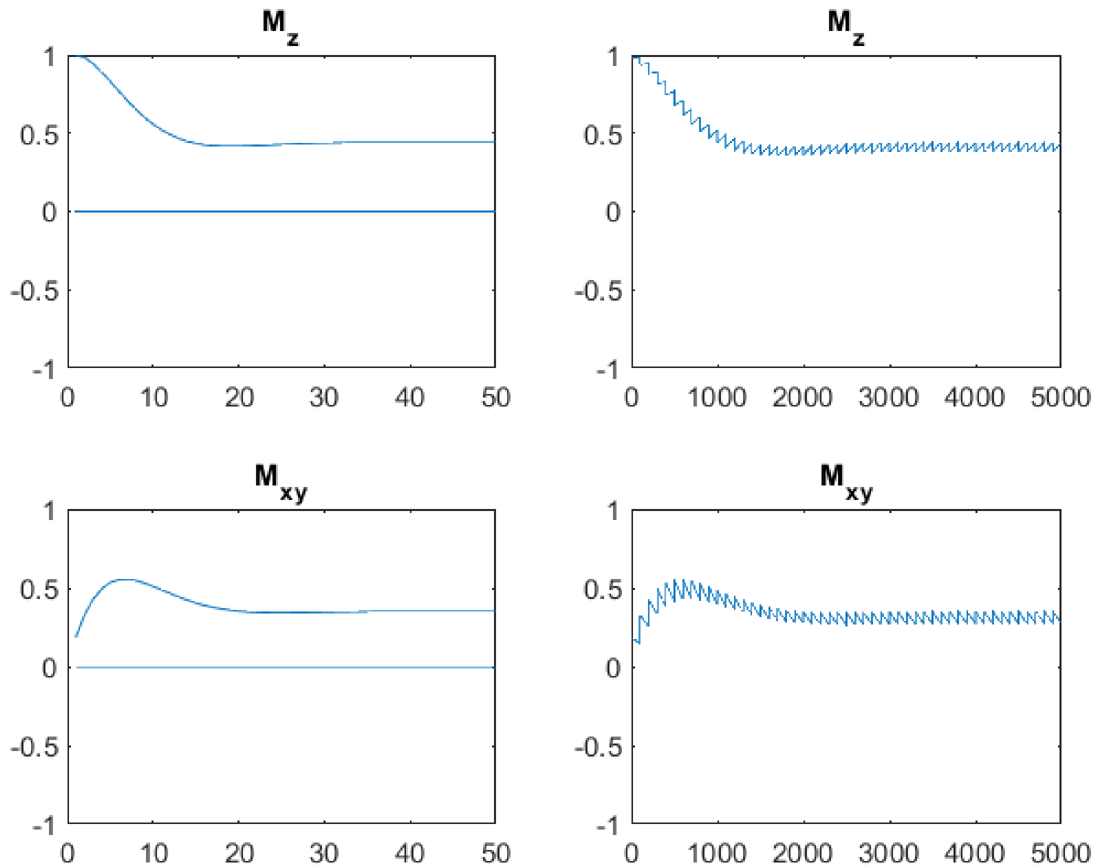
The result shows that the system always converges to a steady state, which keeps oscillating periodically because of repeated excitation.

Vary T1, T2 and the flip angle.

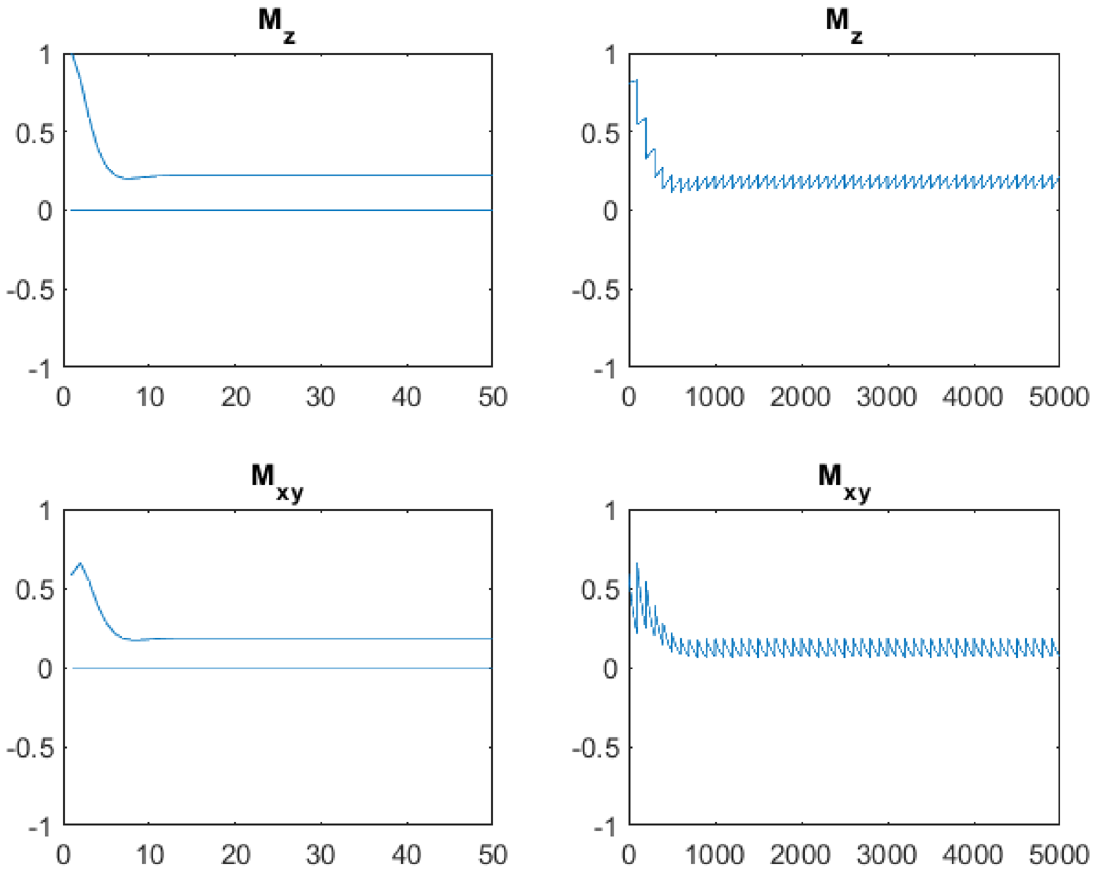
T1 = 100, T2 = 50, angle_flip = 0.06×π, repetition time = 50, number of pulses = 50.



T1 = 500, T2 = 200, angle_flip = 0.06×π, repetition time = 50, number of pulses = 50.



T1 = 100, T2 = 50, angle_flip = 0.2×π, repetition time = 50, number of pulses = 50.



When we vary T1, T2 and the flip angle, we can find out that the time to converge and the magnitudes of oscillation will also vary a lot. They are important parameters which influence the periodic magnetization dynamics.

- Ernst angle

Calculate the optimum flip angle when Tr and T1 have been given.

Immediately after nth Tr: $M_z = M_z(n)$

After RF flip of Φ and spoiling: $M_z = M_z(n)cos\Phi$

After T1 recovery:

$$M_z(n+1) = M_0 - [M_0 - M_z(n)cos\Phi]e^{-T}$$

In equilibrium, $M_z = M_z(n) = M_z(n+1)$

$$M_z(1 - cos\Phi \times e^{-\frac{Tr}{T1}}) = M_0(1 - e^{-\frac{Tr}{T1}})$$

$$M_z = \frac{M_0(1 - e^{-\frac{Tr}{T1}})}{1 - cos\Phi \times e^{-\frac{Tr}{T1}}}$$

Equilibrium transverse magnetization:

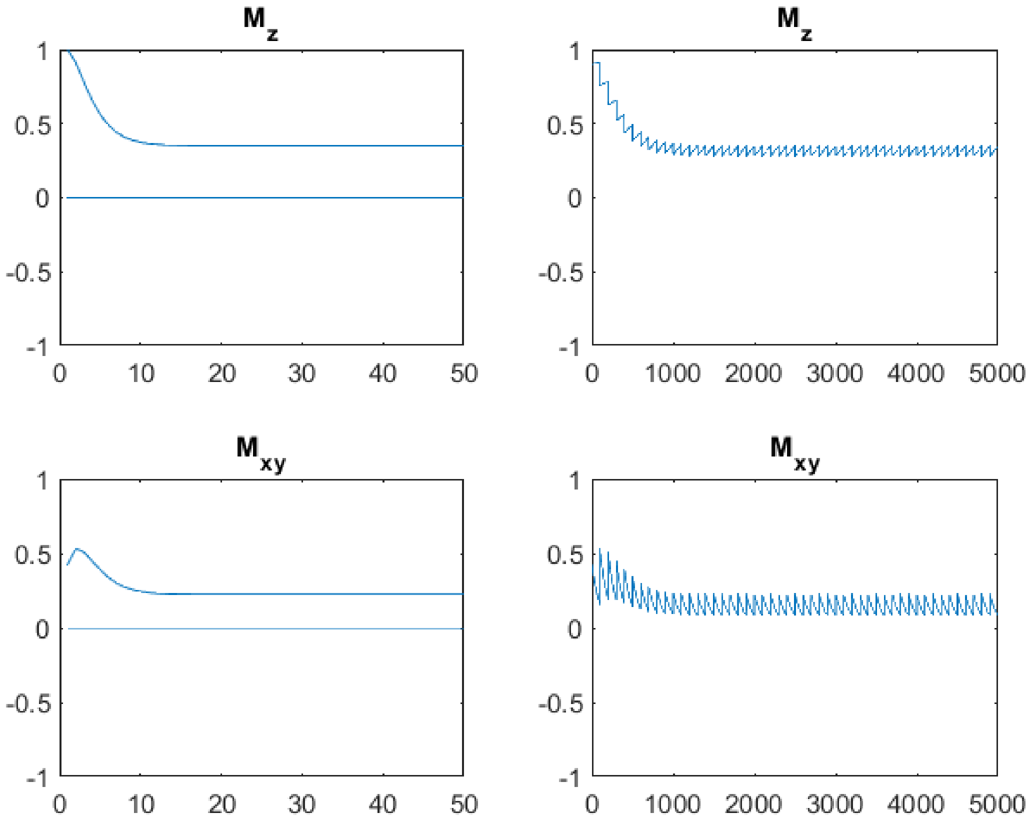
$$M_\perp = M_z = \frac{M_0(1 - e^{-\frac{Tr}{T1}})}{1 - cos\Phi \times e^{-\frac{Tr}{T1}}}$$

The maximum transvers magnetization can be reached when:

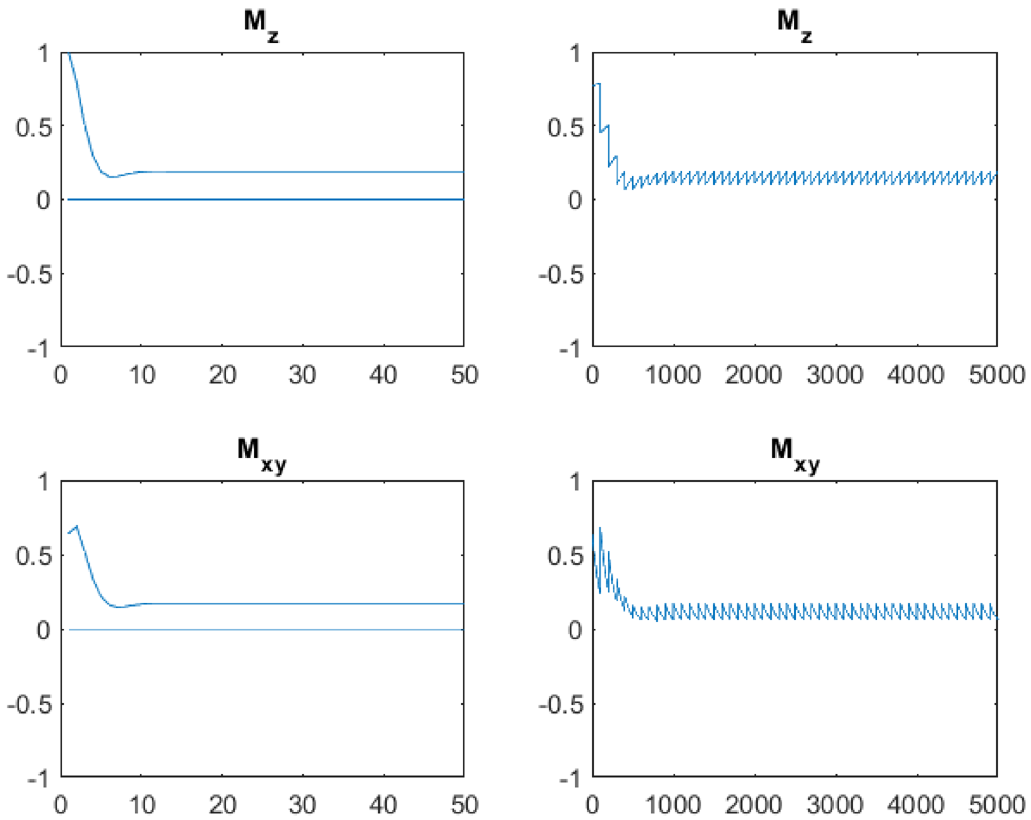
$$cos\Phi_E = e^{-\frac{Tr}{T1}}$$

$$\Phi_E = \arccos\left(e^{-\frac{Tr}{T1}}\right)$$

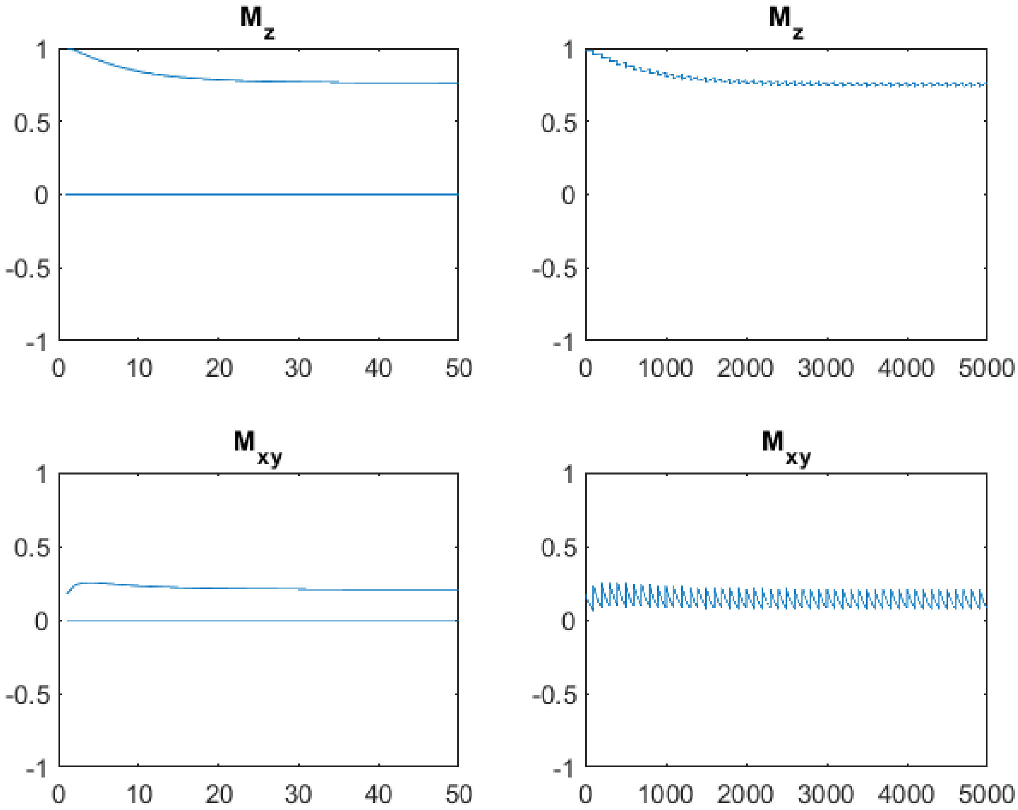
When T1 = 500, T2 = 50, Φ_E = 25.1986°.



When Φ_E = 40°:



When Φ_E = 10°:



We can find that the transverse magnetization all decreased when we change the flip angle to a larger one or a smaller one.