Tsunami

Shallow-water equation

$$
\begin{aligned}
\partial_t^2 P &= \nabla \cdot (v^2 \nabla P) \\
&= \partial_x v^2 \partial_x P + \partial_y v^2 \partial_y P + v^2(\partial_x^2 P + \partial_y^2 P) \qquad \text{[general form]} \\
&= v^2(\partial_x^2 P + \partial_y^2 P) \qquad \text{[homogeneous form]}
\end{aligned}
$$

$P = P(x, y, t)$: height of tsunami waves above sea level

$v = \sqrt{gH(x, y)}$: wave speed

ocean depth

gravity acceleration

Shallow-water equation

FD discretization

2nd order time & space schemes:

$$
\begin{aligned}
\partial_t^2 P_{i,j}^n &\approx \frac{P_{i,j}^{n+1} - 2P_{i,j}^n + P_{i,j}^{n-1}}{\Delta t^2} \\
\partial_x^2 P_{i,j}^n &\approx \frac{P_{i+1,j}^n - 2P_{i,j}^n + P_{i-1,j}^n}{\Delta x^2} \\
\partial_y^2 P_{i,j}^n &\approx \frac{P_{i,j+1}^n - 2P_{i,j}^n + P_{i,j-1}^n}{\Delta y^2}
\end{aligned}
$$

2nd order time & 4th order space schemes:

$$
\begin{aligned}
\partial_t^2 P_{i,j}^n &\approx \frac{P_{i,j}^{n+1} - 2P_{i,j}^n + P_{i,j}^{n-1}}{\Delta t^2} \\
\partial_x^2 P_{i,j}^n &\approx \frac{-\frac{1}{12}P_{i+2,j}^n + \frac{4}{3}P_{i+1,j}^n - \frac{5}{2}P_{i,j}^n + \frac{4}{3}P_{i-1,j}^n - \frac{1}{12}P_{i-2,j}^n}{\Delta x^2} \\
\partial_y^2 P_{i,j}^n &\approx \frac{-\frac{1}{12}P_{i,j+2}^n + \frac{4}{3}P_{i,j+1}^n - \frac{5}{2}P_{i,j}^n + \frac{4}{3}P_{i,j-1}^n - \frac{1}{12}P_{i,j-2}^n}{\Delta y^2}
\end{aligned}
$$

Shallow-water equation

FD discretization

4th order space scheme:

$$P_{i+1} = P_i + \Delta x P_i' + \frac{1}{2}(\Delta x)^2 P_i'' + \frac{1}{6}(\Delta x)^3 P_i''' + \frac{1}{24}(\Delta x)^4 P_i'''' + \frac{1}{120}(\Delta x)^5 P_i''''' + O[(\Delta x)^6]$$

$$P_{i-1} = P_i - \Delta x P_i' + \frac{1}{2}(\Delta x)^2 P_i'' - \frac{1}{6}(\Delta x)^3 P_i''' + \frac{1}{24}(\Delta x)^4 P_i'''' - \frac{1}{120}(\Delta x)^5 P_i''''' + O[(\Delta x)^6]$$

$$P_{i+2} = P_i + 2\Delta x P_i' + \frac{1}{2}(2\Delta x)^2 P_i'' + \frac{1}{6}(2\Delta x)^3 P_i''' + \frac{1}{24}(2\Delta x)^4 P_i'''' + \frac{1}{120}(2\Delta x)^5 P_i''''' + O[(\Delta x)^6]$$

$$P_{i-2} = P_i - 2\Delta x P_i' + \frac{1}{2}(2\Delta x)^2 P_i'' - \frac{1}{6}(2\Delta x)^3 P_i''' + \frac{1}{24}(2\Delta x)^4 P_i'''' - \frac{1}{120}(2\Delta x)^5 P_i''''' + O[(\Delta x)^6]$$

➔ $$16(P_{i+1} + P_{i-1}) - (P_{i+2} + P_{i-2}) = 30P_i + (16 - 1 * 4)(\Delta x)^2 P_i'' + O[(\Delta x)^6]$$

➔ $$\boxed{P_i'' = \frac{-P_{i+2} + 16P_{i+1} - 30P_i + 16P_{i-1} - P_{i-2}}{12(\Delta x)^2} + O[(\Delta x)^4]}$$

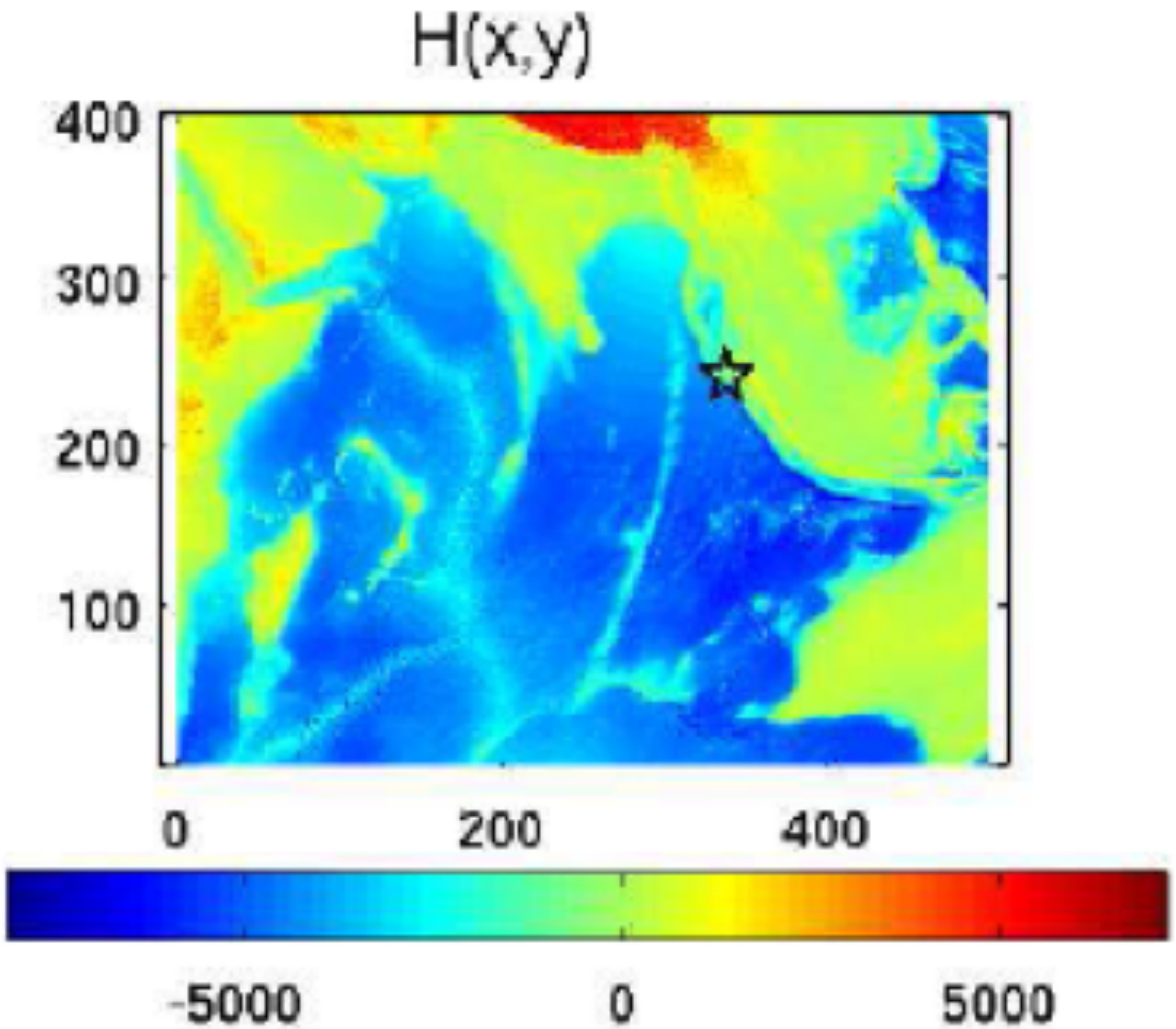
Tsunami

Countries most affected

<u>Model setup:</u>
2004 Mw 9.2 Sumatra earthquake

Below seafloor earthquake
(30 km beneath sea level)
⇒ Strong vertical displacement
⇒ Tsunami

Tsunami



Source: point source,
distributed on 9 points of the grid

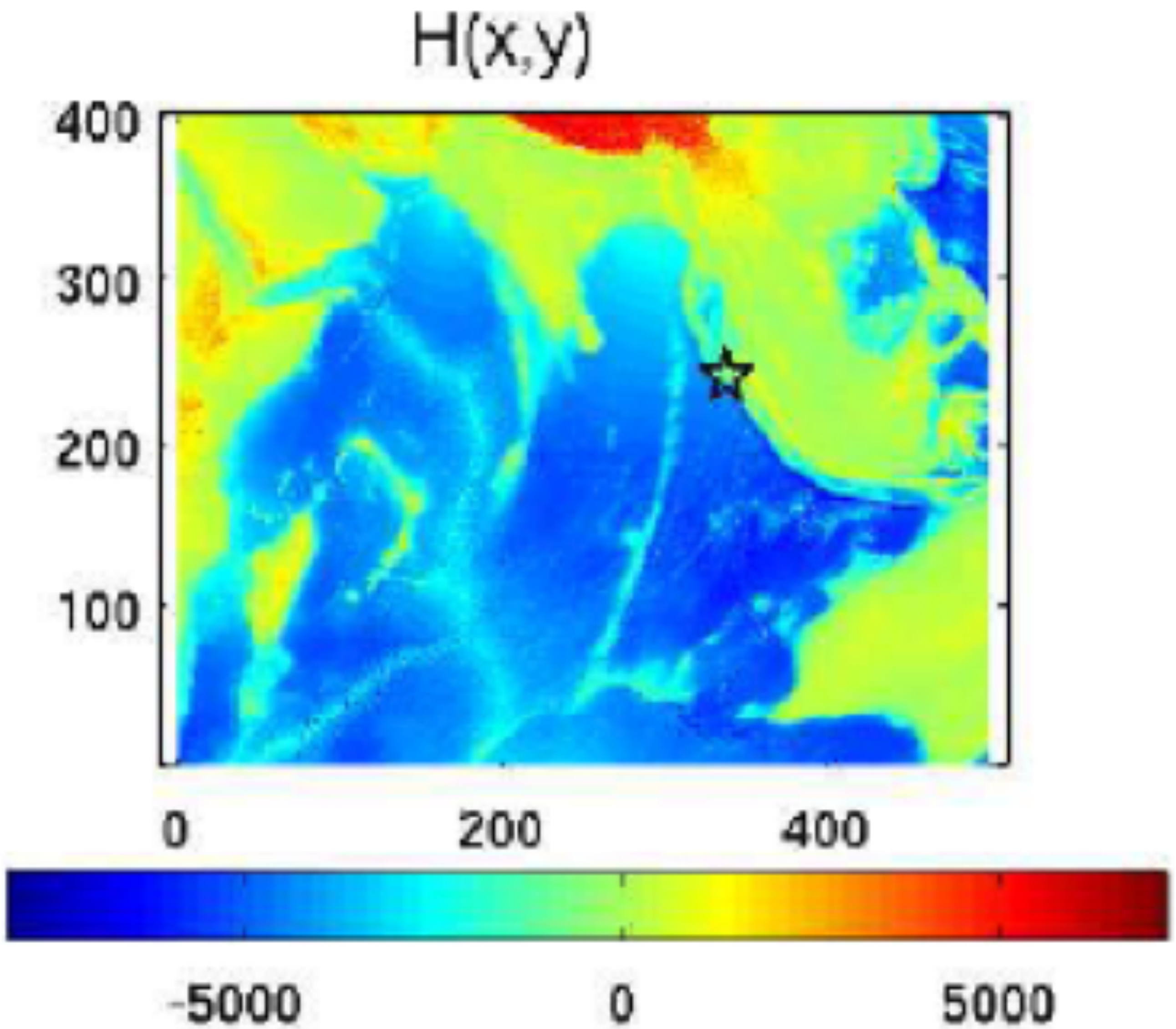
Dirichlet boundary conditions:

$$P(x, y) \quad = \quad 0$$

seafloor: $H(x, y)$

(bathy.out)

Tsunami



FD discretization

$$P^n_{i,j} \equiv P(x = i \times \Delta x, y = j \times \Delta y, t = n \times dt)$$

Consider

$$\Delta x = \Delta y = 10$$

$$dt = 10$$

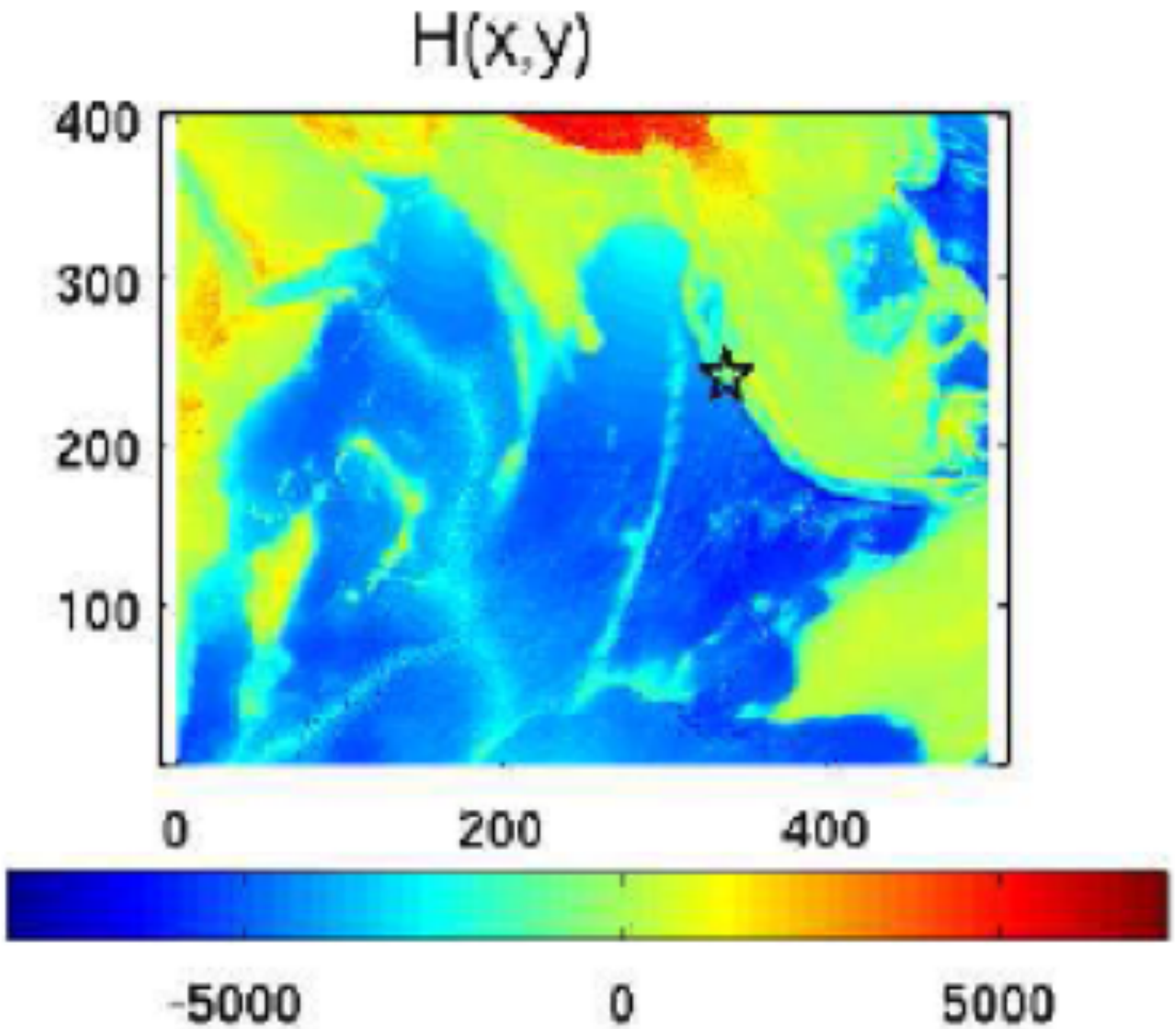
$$nx = 1000 \quad km$$

$$ny = 800 \quad km$$

$$nt = 3000$$

$$v_{max} = 0.2956 \quad km/s$$

Tsunami



Solver: C-code class_tsunami.c

Input file: bathy.out => $H(x, y)$

Output file: slices.out => $P(x, y, t)$
every 100 time steps