Selective Entry in Incomplete Procurement Auctions

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Abstract

This paper studies the entry behaviors in procurement auctions of incomplete contracts held by the California Department of Transportation. Under a selective entry model framework, I construct a model considering both endogenous participation and incompleteness of contracts. The model explains the entry behaviors in incomplete procurement auctions. Based on the model, I structurally estimate the impact on entry behaviors of incompleteness. Furthermore, I also compare the model to models without endogenous participation or incompleteness of contracts. It shows that estimation will be biased when neglecting entry behaviors or incompleteness in procurement auctions.

1 Introduction

Government procurement contracts are often incomplete or ambiguous to some extent. The initial contracts are vague on a number of main features, so modification and renegotiation frequently appear after the contract being assigned. Procurement contracts are often assigned through low-price auctions because there are many well known benefits of procuring projects in the way of auctions. However, auctions on incompleteness contracts make bidding very difficult to study. In addition, entry is also a central feature in the literature of auctions. Selection and endogenous participation are inevitable in the study of markets. In this paper, I study the relationship between the entry behavior of potential bidders and the incompleteness of procurement contracts in the highway procurement auctions held by the California Department of Transportation (CalTrans).

This paper employs a three-stage model to contain both incompleteness of contracts and selective auction entry. It is very important to consider these two features at the same time due to several aspects. First, procurement auctions usually has both endogenous participation and contract incompleteness. For example, in the literature, Caltrans procurement data is considered for both features. Bidding behaviors in incomplete contracts has been investigated in former research in CalTrans procurement auction data. In Bajari, Houghton, and Tadelis(2014)[4], the adaption cost of incomplete contracts and bidding behaviors are well studied. In An and Tang(2019)[1], the equilibrium in the process of renegotiation are modeled and the hold up problem is well analyzed. Meanwhile, it is believed that endogenous participation exists in CalTrans procurement auction. In Bajari, Hong and Ryan(2010)[3], the entry behavior of large firms is anaylzed through a discrete game for CalTrans procurement data. In Gil and Marion(2012)[11], the entry decisions in the CalTrans highway procurement auctions are discussed empirically. Despite the prevalence of the research on incomplete contracts and the endogenous participation of auctions, the relation between the two terms are almost ignored. This paper contributes to combining the two variables and finding the relationship between them.

Second, the relationship between the entry decisions and the incompleteness is obscure. Econometricians can recover private cost distribution of bidders under entry models or incomplete contract models. These structural models can be plausible when there is no relation between endogenous participation and incompletenss of contracts. However, from the reduced form analysis in this paper, the level of incompleteness of contracts has a significant impact on entry decisions of contractors. The incompleteness of contracts, on one hand, will bring uncertainty to the bidders in the preparation of bids which leads to a higher preparation cost. The subsequent holdup problem, on the other hand, will lead to a potential benefit to the bidders. This makes the relationship between entry decisions and incompleteness of contracts ambiguous and makes it very dangerous to ignore either one in a structural model.

Third, a structural model is very needed to resolve the mixture of entry behaviors and contracts' incompleteness. The influence of incompleteness and subsequent holdup depends on not only some actual terms, like the engineer estimate of projects and characteristics of auctions, but also the expected cost of final design of projects and the exptected reimbursement from the government during the renegotiation of project designs. Furthermore, as I will state in section 9.1, there is a non-linear distortion on bidding strategies and entry strategies from the expected holdup of the contracts. To deal with it, a structural model is very necessary to deal with the combined two features.

This paper is closedly related to the literature of selective entry in auction models (e.g., Marmer, Shneyrov, and Xu, 2013[16]; Gentry and Li, 2014[10]; Robers and Sweeting, 2013[18]; Bhattacharya, Robers, and Sweeting, 2014[6]; Roberts and Sweeting, 2016[19]; Gentry et al., 2020[7])). This paper extends selective entry model with consideration of contracts incompleteness. The identification and estimation of the three-stage entry model can be applied as a starting point for other entry models to contain characteristics of interest.

The paper also contributes to the literature on incomplete contracts (e.g., Crocker and Reynolds, 1993[8]; Bajari, McMillan, and Tadelis, 2009[5]; Bajari, Houghton, and Tadelis, 2014[4]; Lewis and Bajari, 2014[12]; De Silva et al., 2015[9]; An and Tang, 2019[1]). The literature on both entry and incomplete contracts is quite rare. Nicta and Vatiero (2009)[17] consider a theory model on contracts incompleteness and entry deterrence. Seidel (2010)[20] accounts for entry of foreign market under incomplete contracts. My paper first introduces endogenous participation to an incomplete contracts auction framework and consider a structural method to account for the combination of these two important features.

In Section 2, the article describes the selective entry model with incomplete contracts. Section 3 provides the identification of the model. Section 4 gives the data description and the way of creating potential bidder set. Section 5 presents the empirical results for the relation between the two main features. Section 6 gives the robustness check under potential bidders sets constructed by different methods. Section 7 characterizes our method of structural estimation. Section 8 provides structural estimation results. Section 9 accounts for the importance of consideration of combination of incompleteness and entry, then it considers potential bias of different structural models when an econometrician only accounts for either of contracts' incompleteness and entry. Finally, section 9 concludes.

2 Model

The model endeavors to contain both endogenous participation and incompleteness of contracts. A selective entry model framework is employed to analyze the feature of endogenous participation. To introduce the feature of incompleteness of contracts, I apply a Nash bargaining procedure at the last stage of our model.

2.1 Timing

There are three stages in the model: pre-auction entry, auction bidding, and post-auction renegotiation. At the first stage, the government observes a signal $\tilde{X}$ which is related to the final design X^* with distribution function $F_{X^*|\tilde{X}}$. Government then announces an initial specification $X \in \mathcal{X}$ for the procurement contract, where the support $\mathcal{X}$ is a real interval in $\mathbb{R}$. The specification is prepared by government engineers. It is a scalar value, and it represents the engineer estimated cost of the project prepared by government engineers.

After the announcement from the buyer, contractors then observe the design specification X and a private signal s_i, which is an indicator of his private cost c_i for the project. The private cost and the private signal are drawn symmetrically across bidders from a joint distribution, denoted by $F(C, S|X)$.

Based on the announced specification, contractors forms a belief about the new specification X^*, which is a specification of project jointly managed by the buyer and auction winner in the renegotiation period. This belief can be described as the distribution function of X^* conditional on X, denoted by $\lambda(X^*|X)$.

Contractors decide whether or not to enter the auction. If entry, the contractors pay an auction preparation cost k. This finishes the pre-auction entry stage.

In the second stage, entry bidders observe their private costs c_i for completing the contract with design specification X. Base on their private costs and their beliefs, bidders make their bids b_i. The auction proceeds in a first-price auction setting, i.e., the bidder with the lowest bid wins the contract.

In the third stage, the government and the auction winner jointly observe the realization of X^*, which is the specification of the new design. They together decide on whether or not to adopt the new design X^*. The division of the net incremental social surplus follows a Nash bargaining game.

Besides the timing and information structure, some utility and cost functions of bidders are defined. I assume that the social surplus from design X is $\pi(X)$. The social surplus increase from change of different specification of projects can be described in a function $\phi(X, X^*) \equiv \pi(X^*) - \pi(X)$. I denote the incremental cost for the winning contractor as $a(X, X^*)$ which is realized during the renegotiation period.

2.2 Equilibrium Strategies

Pure strategies for a contractor consist of two parts, an entry strategy $\bar{s}(X)$, which is a mapping from his information at the first stage, X to the entry threshold he obeys and a bidding strategy $\beta(c_i, X)$ which is a mapping from his information at the second stage (c_i, X) to the bidding price he proposes. The government's pure strategy is a mapping from his signal $\tilde{X}$ to an initial design X.

To form a symmetric pure-strategy Perfect Bayesian Equilibrium(psPBE), the buyer follows a pure strategy α^*, and each contractor follows pure strategies $(\bar{s}^*, \beta^*)$ and holds a belief about the new design $\lambda^*(X^*|X)$ such that,

(a) for any x, the entry threshold strategy $\bar{s}^*$ can be characterized as,

$$\Pi(\bar{s}|\bar{s}, X = x) = k \tag{1}$$

where $\Pi(s_i|\bar{s}, X)$ is the expected utility when bidder i with a private signal s_i decides to enter and all other bidders holds symmetric entry strategy $\bar{s}$. k is the participation cost.

(b) for all (c_i, x),

$$\beta^*(c_i, x) = \arg\max_b \; Prob(\min_{j \neq i} \beta^*(c_j, X) \geq b | X = x; \bar{s})[b - c_i + \delta(x; \lambda^*)] \tag{2}$$

where $\delta(x; \lambda^*) \equiv E_{\lambda^*}[\gamma \Delta u(x, X^*)|X = x]$ is a contractor's expected payoff from the renegotiation period.

(c) the contractor's belief λ^* is consistent with $F_{X^*|\tilde{X}}$ and α^* for all x on the support of $\alpha^*(\tilde{X})$.

(d) for all $\tilde{x}$,

$$\alpha^*(\tilde{x}) = \arg\max\{\pi(x) - \varphi_\lambda(x; \bar{s}^*, \beta^*) + \mu(x, \tilde{x})\} \tag{3}$$

where $\mu(x, \tilde{x}) \equiv E[(1-\gamma)\Delta u(x, X^*)|\tilde{X} = \tilde{x}]$ is the expected holdup benefits calculated from the bargaining stage and $\varphi_\lambda(x; \bar{s}^*, \beta^*)$ is the buyer's expected payment to the winning bidder with initial design x. All contractors follow the strategy $\bar{s}^*$ and β^*.

2.3 Equilibrium Solution

I solve the model backward. At the last stage (renegotiation stage), the Nash bargaining process can be specified as,

$$\begin{aligned} \max \quad & (u_c - d_c)^\gamma (u_b - d_b)^{1-\gamma} \\ s.t. \quad & u_b + u_c \leq u_0 \end{aligned}$$

where $d_c = b - c$ is the utility of winning bidder when the negotiation fails to reach an agreement, $d_b = \pi(X) - b$ is the utility for the buyer for disagreement, $u_0 = \pi(X^*) - a - c$. u_b, u_c are the agreement utility for the winner and buyer. a is the cost of adjustment of the contract that goes into negotiation.

The solution to the maximization problem can be given as,

$$u_c = \gamma(\pi(X^*) - \pi(X) - a) + b - c$$

The utility change of the winning bidder from negotiation of a new design can be derived as,

$$\delta(x, x^*) = \gamma(\pi(X^*) - \pi(X) - a(X, X^*)) \equiv \gamma\Delta u(X, X^*)$$

At the bidding stage, the CDF of cost distribution is,

$$F^*(c;\bar{s}) = \frac{1}{\bar{s}}\int_0^{\bar{s}} F(c|t)dt$$

The probability of winning one rival (when the bidder bids b) is,

$$G(b;\bar{s}) = 1 - \bar{s} + \bar{s}(1 - F^*(\beta^{-1}(b);\bar{s}))$$

The winning probability at stage 2 when bidding b,

$$W(b;\bar{s}) = G(b;\bar{s})^{N-1}$$

Therefor, in stage 2, the bidder with cost c_i and bid b has expected payoff,

$$\pi(c_i;b|\bar{s}) = (b - c_i + \delta(x;\lambda^*))W(b;\bar{s})$$

where $\delta(x;\lambda^*) = \int_{\underline{x}}^{\overline{x}} \gamma\left(\pi(X^*) - \pi(X) - a(X,X^*)\right)\lambda_{X^*|X}(X^*|X)dX^*$ is the expected utility change coming from the renegotiation period.

The bidder is seeking to maximize this payoff, so the first order condition for him is,

$$W(b;\bar{s}) + (b - c_i + \delta(x;\lambda^*))\frac{dW(b;\bar{s})}{db} = 0$$

Under symmetric bidding strategy, $b = \beta(c_i)$, we can calculate the bidding strategy as,

$$\beta(c_i;\bar{s}) = \frac{1}{[1-\bar{s}F^*(c_i;\bar{s})]^{N-1}}\int_{c_i}^{\bar{c}}[t - \delta(x;\lambda^*)]d\{1 - [1-\bar{s}F^*(t;\bar{s})]^{N-1}\} + \bar{c}\frac{(1-\bar{s})^{N-1}}{[1-\bar{s}F^*(c_i;\bar{s})]^{N-1}} \tag{4}$$

Back to stage 1, the expected payoff at stage 1 is,

$$\Pi(s_i|\bar{s}) = \int_0^1 \pi(c_i; \beta^*(c_i)|\bar{s}) f(c_i|s_i) dc_i$$

The entry strategy is characterized as the solution to,

$$\Pi(\bar{s}^*|\bar{s}^*) = k \tag{5}$$

The bidding strategy can be characterized by plugging in the entry strategy $\bar{s}$ to $\beta(c_i; \bar{s})$.

As for the buyer, given the contractor belief λ, the buyer's optimization problem is:

$$\alpha^*(\tilde{x}; \lambda) \equiv \arg\max_{x \in \mathcal{X}} \{\pi(x) - \varphi_\lambda(x; \bar{s}^*, \beta^*) + \mu(x, \tilde{x})\}$$

where $\varphi_\lambda(x; \bar{s}^*, \beta^*)$ is the expected payment for the contractor.

Under appropriate conditions, it can be proved that the psPBE described above exists[1].

3 Identification

In the data, the initial contract specification X, the final specification of the renegotiation X^*, auction bids b_i for entry bidders, and transfer payment from the government to the winning bidder Y are observed. Based on the information, I study whether the three stage model is identified. The objective is to examine whether main model primitives $F(c|s)$, k, and $\bar{s}(z)$ can be recovered uniquely. To do so, I follow the main steps of An and Tang (2019)[1] to handle the identification of the incompleteness part in the second and third stage, including basic model parameters γ, π, a, $F_{X^*,\tilde{X}}$. To deal with the identification of selective entry of auctions, I apply the method of Gentry and Li (2014)[10] to identify entry threshold $\bar{s}(z)$, entry participation cost k and private cost distributions in the first and second stage, $F^*(c; \bar{s})$ and $F(c|s)$.

[1] See in Appendix A

I assume the distribution of $\tilde{X}$ is normalized to a standard uniform distribution in $[0, 1]$. Let $D = 1$ if renegotiation for new design occurs, and 0 otherwise. The buyer's strategy $\alpha^*(\cdot)$ is identified as $\alpha^*(\tau) = x_\tau$ for all $\tau \in [0, 1]$ from the property of monotonicity. x_τ is defined as the τ-th quantile of the initial designs reported in the data. I identify the distribution directly from the data which contain pairs of X and X^*,

$$F_{X^*|s(X,X^*)>0,\tilde{X}=\tau}(x^*) \equiv Prob\{X^* \leq x^*|\tilde{X} = \tau, \Delta u(X, X^*) > 0\} = Prob\{X^* \leq x^*|X = x_\tau, D = 1\}$$

for all x^* and $\tau \in (0, 1)$.

The social surplus $\pi(\cdot)$ can be identified from the equation from the derivation of buyer's problem in An and Tang (2019)[1],

$$\pi'(x) = [1 - p^*(x)]^{-1} \left(\varphi'(x; \bar{s}^*, \beta^*) + \int_{\{\Delta u(s,t)>0\}} y_1(x, t) dF_{X^*|X=x}(t) \right)$$

The right hand side of the equations are all identified in the data, then $\pi(x) = \pi_0 + \int_{\underline{x}}^{x} \pi'(z)dz$ is also identified.

From the derivation in the third stage, it can be shown that,

$$y(x, x^*) = \gamma[\pi(x^*) - \pi(x)] + (1 - \gamma)a(x, x^*)$$

The bargaining power can be identified

I follow the assumption (A1) in An and Tang (2019)[1], i.e., there exist x, ξ, x^*, $\xi^* \in \mathcal{X}$ such that $\Delta u(x, x^*) > 0$, $\Delta u(\xi, \xi^*) > 0$, $a(x, x^*) = a(\xi, \xi^*)$, and $\pi(x^*) - \pi(x) \neq \pi(\xi^*) - \pi(\xi)$. For (x, x^*) and (ξ, ξ^*) that satisfy the assummption, we have,

$$y(x, x^*) - y(\xi, \xi^*) = \gamma[\pi(x^*) - \pi(x) - (\pi(\xi^*) - \pi(\xi))]$$

so that I can uniquely recover the bargaining power γ. After identifying γ and π, the revision cost a is also identified.

The expected holdup $\delta(x;\lambda)$ in the second stage is identified through,

$$\delta(x;\lambda) = E\Delta u = \int_{\underline{x}}^{\bar{x}} \gamma[\pi(X^*) - \pi(X) - a(X, X^*)] dF_{X^*|X}(X^*)$$

In the first stage, the entry strategy can be identified as mentioned in Gentry and Li (2014)[10],

$$\hat{s}_N(z) = \frac{E(n|N, z)}{N}$$

In the bidding stage, the CDF and PDF of bids for the entry bidders, which are defined as $G_0(b|\bar{s})$ and $g_0(b|\bar{s})$, can be directly identified from data. From the first order condition in the bidding stage, the private cost are connected with bids, entry threshold, holdup, and probabilitied as,

$$c_i = b_i + \delta(x;\lambda^*) - \frac{1}{N-1} \frac{1 - \bar{s}G_0(b_i;\bar{s})}{\bar{s}g_0(b_i;\bar{s})}$$

The $\bar{s}$ is identified in first stage and $G_0(b_i|\bar{s})$ and $g_0(b_i|\bar{s})$ can be recovered from the bids in bidding stage. Therefore, the cost c_i and the cost distribution for entry bidders $F^*(c;\bar{s})$ are identified.

The joint distribution of private cost and private signal $F(c|s)$ are identified on the support of $\bar{s}$.

$$F(c|\bar{s}) = \frac{\partial}{\partial \bar{s}} \left(\bar{s}F^*(c;\bar{s})\right)$$

In the first stage, by applying the characterization of the equilibrium entry threshold, I identify the entry cost from the relation,

$$k(z) = \int_0^{\bar{c}} (\beta(c_i) - c_i + \delta(x;\lambda)) G_0^{N-1}(\beta(c_i); \hat{s}_N(z)) f(c_i|\hat{s}_N(z)) dc_i$$

4 CalTrans auctions: Data and Potential bidders

The data I used in this article comes from the paving contracts by Caltrans from 1999 to 2005. The source of the data is the same as Bajari, Houghton, and Tadelis(2013)[4]. The initlal data sample includes $N = 819$ projects and 3661 bids by 349 general contractors.

The entry behaviors is of my interest, so I need the information on potential bidders so that I can observe the entry decisions. In CalTrans auctions, econometricians cannot directly observe the potential bidders. Similar to Li and Zhang(2010[13], 2015[14]) and Li and Zheng(2012)[15], I recover the set of potential bidders by adding all actual bidders who participated in at least one auction in the same district and during 90 days(a quarter) before the bidding date of the auction. For the created potential bidders set, I have 18958 observations and 3661 entry behaviors. The entry probability for the entire potential bidders set is 19.31% and the average entry probability for 819 auctions is 19.32%.

Since I do not have the characteristics of the potential bidders, I need to calculate them to complete the data set. *dist* is the distance between the potential bidder and the project. The sample data set gives addresses of contractors and locations of projects. I use the Python package geopy to transform the locations into a pair of latitude and longitude. I then use function *geodesic* to calculate the direct distance between the two pairs of latitude and longitude. The distance is measured in miles. This variable measures the geographic cost advantage. The contractors with shorter distance is expected to have a lower transportation cost which might be considered as influential to the entry behaviors.

A contractor's production capacity and backlog situation may affect a contractor's entry decision. Following Bajari, Houghton, and Tadelis(2013)[4], I use the data of winning bids, bidding dates and contract days in the sample data set to construct the measure of backlog, capacity and utilization rate. Assuming each project is achived at a constant speed, the *backlog* variable for a contractor at a specific auction is defined as the summation of the remaining value of the projects on which the contractor is working at the bidding date of the auction. The variable *capacity* is defined as the maximum *backlog* of the contractor. The utilization rate *util* is defined as *backlog*/*capacity* which measures the production capability and opportunity cost of a contractor. $fringe$ is defined as a dummy variable for contractors. When a firm wins less than 1% of the value of the contracts, $fringe$ is set to equal to 1.

For each auction, I collect some characteristics from the sample data set as the auction level variables. The specification of initial design of the contract, X is the engineer estimate of the total project. The specification of a new design, X^* is the project cost after the renegotiation. The variable *ContractDays* is the project's contract days described in the contract. The *job* variable includes the information of project construction: $job = 1$ represents major construction and $job = 0$ for minor construction. $transfer$ denotes the transfer payment from CalTrans to contractors for incomplete contracts at the renegotiation period. It is set to 0 when no renegotiation occurs.

For the measure of incompleteness, I create *incr* as the relative difference of new design X^* and the intial design X, $incr \equiv (X^* - X)/X$.

Table 1 and Table 2 show the summary statistics on auction level and contractor level respectively.

As I am curious about the relationship between entry decisions and incompleteness of contracts, the method of constructing potential bidders set is very important. To validate that result does not vary from the way of constructing, I do some robustness check on it.

First, I change the way of constructing potential bidders set by adjusting the requirement of time length. I try 30 days(a month), 45 days, 180 days(half a year), and 365 days(a year) instead of 90 days before the bidding date of the auction to form the potential bidders.

Second, I employ a different way of constructing the potential bidders set - focusing

Table 1: Auction level summary statistics

Variables	mean	standard deviation	median
number of potential bidders	23.14	9.52	22
number of entry bidders	4.47	2.15	4
engineering estimate (1m dollars)	2.88	7.25	0.95
renegotiation value of the project(1m dollars)	2.57	6.47	0.88
job	0.39	0.51	0
ave.fringe	0.68	0.12	0.70
ave.distance (miles)	309.53	496.23	183.27
ave.util	0.10	0.06	0.10
incomplenteness(incr)	-0.08	0.22	0.09
transfer (1m dollars)	0.31	1.30	0.05

Table 2: Contractor level summary statistics

Variables	mean	standard deviation	median
fringe	0.94	0.23	1
distance (miles)	413.45	897.88	88.32
util	0.09	0.25	0
ave.incr	-0.09	0.10	0.09
ave.transfer (1m dollars)	0.35	1.18	0.20
ave.entry	0.23	0.20	0.17

on the large firms. I follow the way of constructing potential bidders in Bajari, Hong and Ryan(2010)[3] and Gil and Marion(2012)[11]. In Bajari, Hong and Ryan(2010)[3], they focus on the entry behaviors on the four largest firms. They select the four largest firms in market share measured by winning bids[2]. In Gil and Marion(2012)[11], they investigate the entry behaviors for the 20 firms with most number of entry. I construct potential bidders set for the 25 firms with most number of entry. I select 25 since first 25 firms enter at least 1% among all the auctions. To validate this way of constructing, I also change the amount of potential bidders to see if the results are robust.

[2]Each of the firm have a market share of at least 5%

5 Reduced Form Results

I employ logit regressions to find whether there is relation between incompleteness of contracts and entry behaviors of contractors. Table 3 reports logit regression results. The variable $rtransfer$ is relative transfer payment which is defined as $transfer/X$. The variable *ave.rdis* is the average of rivals' distances for a contractor in each auction. $ave.fringe$ is defined similarly as the average rivals' fringe type.

A negative effect from *incr* to entry probability is reported in all the three regressions. This confirms the relation between incompleteness of contracts and entry behaviors. The number of potential bidders N also have negative effects on entry decision which could be considered as competition effects. The initial design X, the utilization rate *util* and the firms' type $fringe$ have negative effects because these variables can represent firms' capability limitation. A small firm are less likely to enter a large project, i.e. a $fringe$ firm and a large engineering estimate cost X. In this way, I expect a positive estimate for rivals' $fringe$ values. This is confirmed by the positive estimate of effect of rivals' average $fringe$ value in regression (3). Firms with high utilization rate may not be capable of an additional project since they are working close to their capacity limit. When controlling the size of projects, the *contractdays* variable has a positive effect on entry behavior. Longer contract days reduce the risk of overtime, so the project's requirements are loose and more likely to be achieved.

The $rtransfer$ variable is not significant and this result might come from two reasons. First, bidders consider their benefit from renegotiation as a hold up problem instead of transfer payment only, i.e., bidders also need to consider adjustment costs due to the change of projects. Both adjustment costs and transfer payment are related to the degree of incompleteness of contracts, therefore the information of transfer payment to some extent has been included in the information of *incr*. Furthermore, contractors' entry decisions are more likely determined by the rational expectation of these terms rather than actual terms, so reduced form directly using actual terms might not be able to rationalize bidders' behaviors.

A structural model is needed to deal with this situation.

The variable related to distances, *dist* and *ave.rdis*, are not significant in the regressions. This result may come from the way I construct the potential bidders set. As I define, a potential bidder have entered an auction in the same district during the previous 90 days, so the distance information has been gathered when I create the data set. In addition, it might be because the transportation cost is not crucial in the decision of entry behaviors.

To quantitatively explain the impact on entry behavior from the incompleteness, I calculate the average marginal effect for the logit model (3). The result is reported in Table 4 and in Figure 1. I find that when the relative incompleteness measure increases by one unit, the probability of entry decreases by 8.36%. In the data, the incompleteness measure I construct has a maximum 1.17, so the most incomplete contract hold back the potential bidders from entry with probability decrease 9.86%[3].

6 Robustness Check on Reduced Form Results

I this section, I proceed several robustness checks on the empirical results in order to make sure that the reduced form results in the previous section does not rely on the method of constructing potential bidders set.

6.1 Different Time Length when Constructing Potential Bidders Set

First, I change the way of constructing potential bidders set by adjusting the requirement of time length. I employ 30 days(a month), 45 days, 180 days(half a year), and 365 days(a year) instead of 90 days before the bidding date of the auction to form the potential bidders. Table 5 shows the regression results for these different potential bidders.

I find that the regression estimates of constructed measure of incompleteness are all sig-

[3] $1.17 \times 8.36\% = 9.86\%$

Table 3: Logit regression result of Entry

	Dependent variable:		
	entry		
	(1)	(2)	(3)
N	-0.038^{***}	-0.038^{***}	-0.043^{***}
	(0.002)	(0.002)	(0.003)
engestimate(X)	-0.009^{***}	-0.009^{***}	-0.008^{***}
	(0.003)	(0.003)	(0.003)
job	0.031	0.032	0.042
	(0.039)	(0.039)	(0.039)
fringe	-0.497^{***}	-0.497^{***}	-0.490^{***}
	(0.040)	(0.040)	(0.041)
util	-0.379^{***}	-0.380^{***}	-0.377^{***}
	(0.087)	(0.087)	(0.087)
dist	-0.001	0.0003	-0.049
	(0.028)	(0.028)	(0.045)
contractdays	0.001^{***}	0.001^{***}	0.001^{***}
	(0.0002)	(0.0002)	(0.0002)
rtransfer		-0.237	
		(0.192)	
incr	-0.567^{***}	-0.577^{***}	-0.556^{***}
	(0.087)	(0.087)	(0.087)
ave.rdis			0.0001
			(0.0001)
ave.rfringe			0.541^{**}
			(0.212)
Constant	-0.162^{***}	-0.147^{**}	-0.456^{***}
	(0.060)	(0.061)	(0.127)
Observations	18,958	18,958	18,958
Log Likelihood	$-8,979.571$	$-8,978.799$	$-8,975.255$
Akaike Inf. Crit.	17,977.140	17,977.600	17,972.510
Note:			*p<0.1; **p<0.05; ***p<0.01

Figure 1: Average marginal effect of logit regression

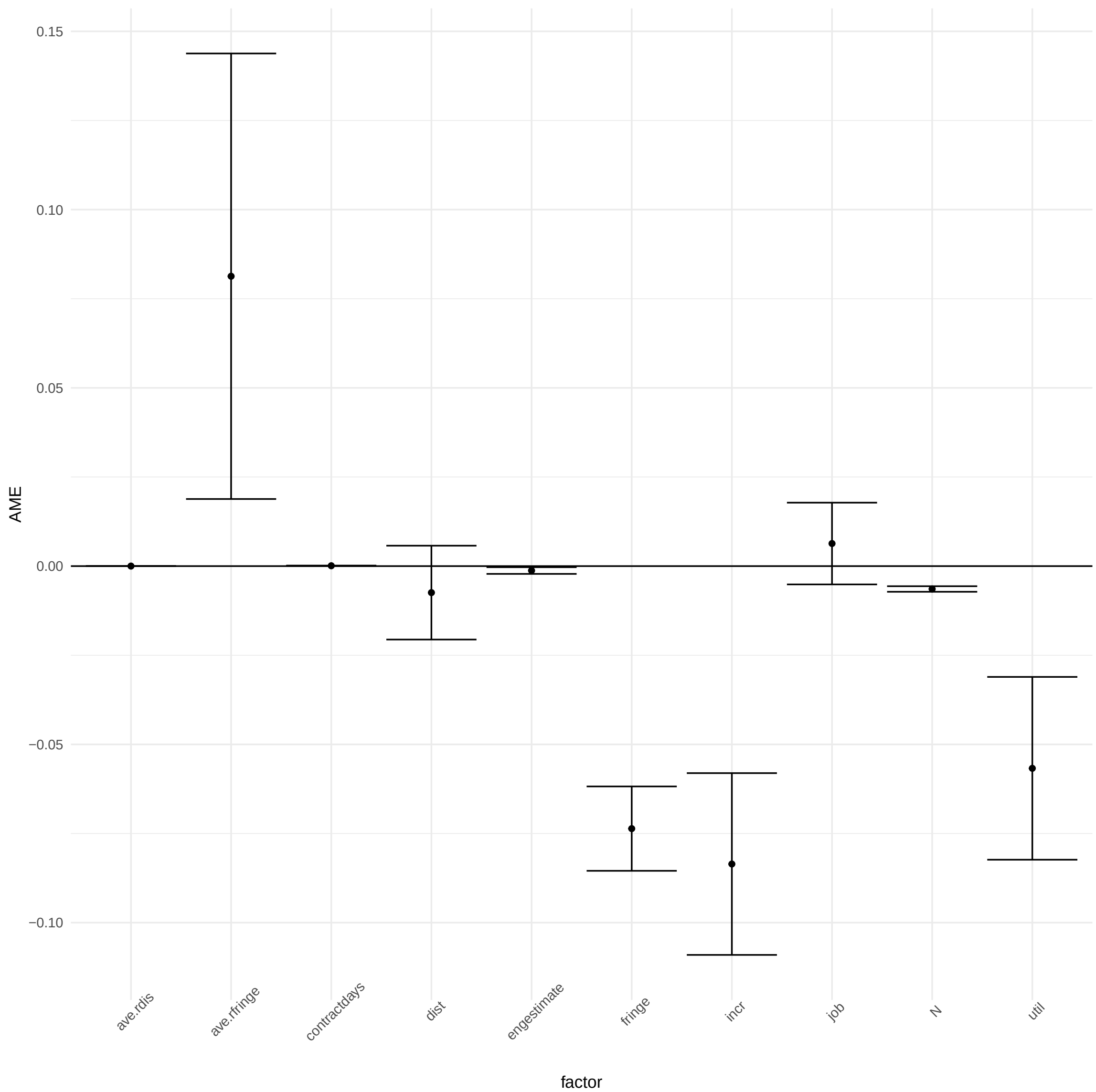


Table 4: Average marginal effect of logit regression

factor	AME	SE	z	p	lower	upper
ave.rdis	0	0	1.5669	0.1171	0	0
ave.rfringe	0.0813	0.0319	2.5504	0.0108	0.0188	0.1438
contractdays	0.0001	0	3.9111	0.0001	0.0001	0.0002
dist	-0.0074	0.0067	-1.1071	0.2683	-0.0206	0.0057
X	-0.0012	0.0005	-2.5952	0.0095	-0.0022	-0.0003
fringe	-0.0736	0.006	-12.195	0	-0.0855	-0.0618
incr	-0.0836	0.013	-6.4234	0	-0.1091	-0.0581
job	0.0063	0.0058	1.0829	0.2788	-0.0051	0.0178
N	-0.0064	0.0004	-16.113	0	-0.0072	-0.0056
util	-0.0567	0.0131	-4.3359	0	-0.0824	-0.0311

nificantly negative. They also have similar magnitudes. Table 6 shows the average marginal effects computed in these different potential bidders set. I can consider the average marginal effects are quite robust relative to the ways we construct the data sets.

6.2 Reduced Form Result on Large Firms

In this part, I follow the idea of studying large firms entry behaviors in Bajari, Hong and Ryan(2010)[3] and Gil and Marion(2012)[11]. I select the firms most frequently participating in the auctions. Table 7 contains the results of different specifications of logit regression using this potential bidders set. I find that the estimates of *incr* is significantly negative. This validates the negative effects of incompleteness on entry decisions.

To validate the robustness of the results from this way of constructing potential bidders set, I also change the number of firms and do the logit regression for different potential bidders set. Figure 2 shows the average marginal effects of the estimate of *incr* for different model specification and potential bidders set with different number of firms[4]. I find that, the estiamtes are robust when we appropriately set the model specification, such as the logit2 and logit3 model case.

[4]The number of firms varies from 15 to 30

Table 5: Logit regression result on different potential bidders set(different time length)

	Dependent variable:				
	entry				
	30days	45days	90days	180days	365days
N	-0.062^{***}	-0.050^{***}	-0.043^{***}	-0.037^{***}	-0.033^{***}
	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)
X	-0.008^{**}	-0.008^{**}	-0.008^{***}	-0.009^{***}	-0.009^{***}
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
job	-0.027	-0.029	-0.042	-0.026	-0.015
	(0.041)	(0.040)	(0.039)	(0.038)	(0.038)
fringe	-0.208^{***}	-0.311^{***}	-0.490^{***}	-0.666^{***}	-0.769^{***}
	(0.043)	(0.042)	(0.041)	(0.040)	(0.040)
util	-0.783^{***}	-0.682^{***}	-0.377^{***}	-0.014	0.157^{*}
	(0.090)	(0.089)	(0.087)	(0.086)	(0.086)
dist	-0.037	-0.051	-0.049	-0.077^{*}	-0.094^{**}
	(0.044)	(0.044)	(0.045)	(0.045)	(0.046)
contractdays	0.001^{***}	0.001^{***}	0.001^{***}	0.001^{***}	0.001^{***}
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
incr	-0.596^{***}	-0.561^{***}	-0.556^{***}	-0.554^{***}	-0.532^{***}
	(0.091)	(0.089)	(0.087)	(0.085)	(0.084)
ave.rdis	0.0001	0.0001	0.0001	0.0001^{**}	0.0001^{**}
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ave.rfringe	0.643^{***}	0.511^{***}	0.541^{**}	0.370	0.572^{**}
	(0.174)	(0.187)	(0.212)	(0.240)	(0.267)
Constant	-0.098	-0.233^{**}	-0.414^{***}	-0.421^{***}	-0.668^{***}
	(0.103)	(0.110)	(0.126)	(0.147)	(0.170)
Observations	13,572	15,322	18,958	23,121	25,537
Log Likelihood	$-7,595.401$	$-8,130.956$	$-8,975.255$	$-9,678.234$	$-10,050.680$
Akaike Inf. Crit.	15,212.800	16,283.910	17,972.510	19,378.470	20,123.360
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 6: Average marginal effect of incr for different potential bidders set

time length	AME	SE	z	p	lower	upper
30days	-0.092	0.014	-6.569	0	-0.120	-0.065
45days	-0.088	0.014	-6.327	0	-0.115	-0.060
90days	-0.084	0.013	-6.423	0	-0.109	-0.058
180days	-0.081	0.012	-6.521	0	-0.105	-0.057
365days	-0.075	0.012	-6.328	0	-0.099	-0.052

Figure 2: Average Marginal Effects of the estimate of incr in different logit specifications with potential bidders set with different number of firms

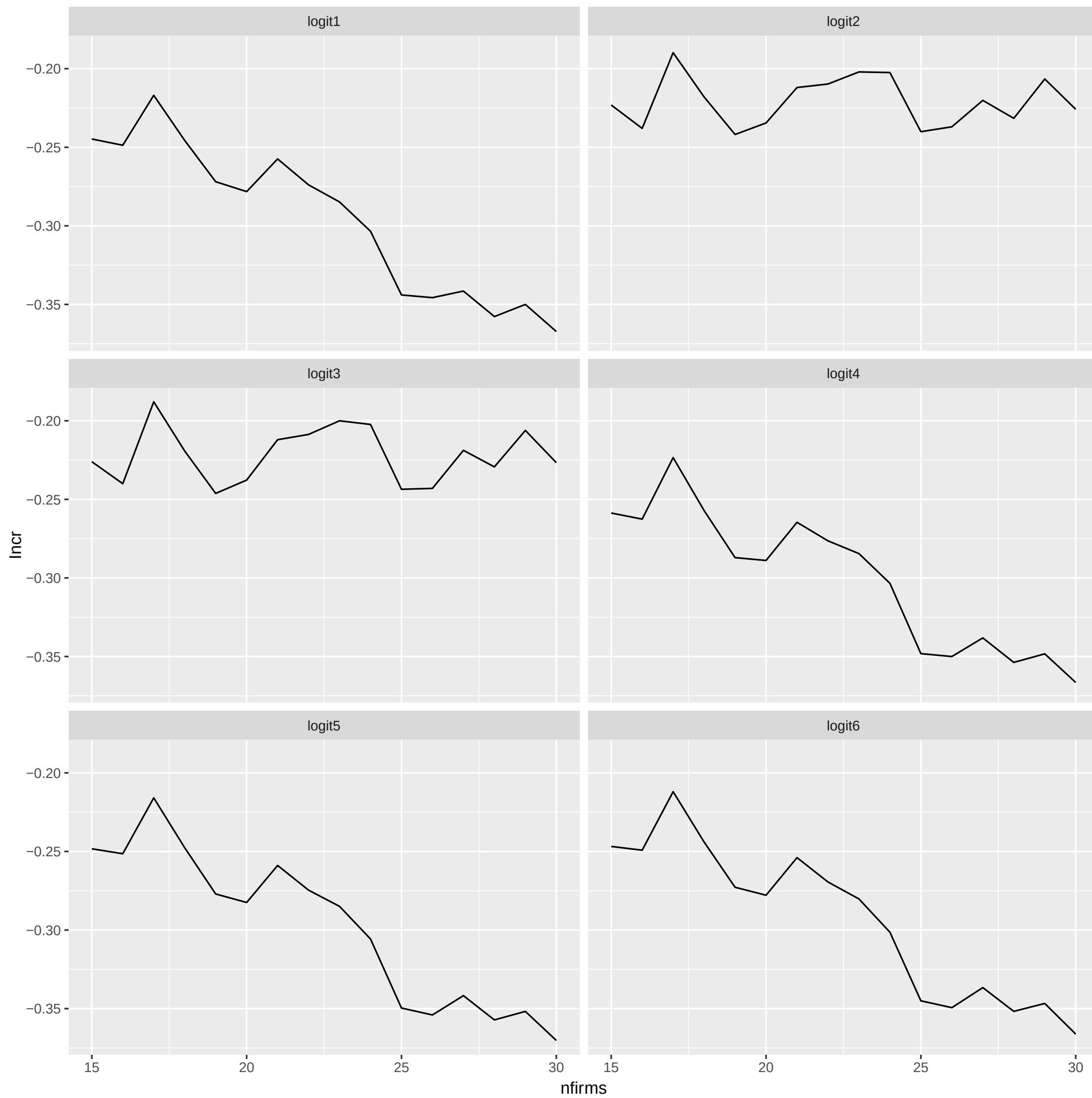


Table 7: Logit regression using potential bidders set constructed by most frequently entry firms(25 firms case)

	Dependent variable:					
	entry					
	(1)	(2)	(3)	(4)	(5)	(6)
dist	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{***}
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
util	0.535^{***}	0.537^{***}	0.227^{**}	0.222^{**}	0.222^{**}	0.210^{*}
	(0.102)	(0.102)	(0.110)	(0.110)	(0.110)	(0.110)
nbids		0.050^{***}	0.051^{***}			
		(0.011)	(0.012)			
contractdays	-0.001^{***}	-0.001^{***}	-0.001^{***}	-0.001^{**}	-0.001^{**}	
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	
incr	-0.344^{***}	-0.240^{**}	-0.244^{**}	-0.348^{***}	-0.350^{***}	-0.345^{***}
	(0.116)	(0.118)	(0.120)	(0.118)	(0.117)	(0.118)
job		0.016	0.017	0.012		
		(0.053)	(0.053)	(0.053)		
fringe			-1.124^{***}	-1.123^{***}	-1.123^{***}	-1.124^{***}
			(0.059)	(0.059)	(0.059)	(0.059)
Constant	-2.093^{***}	-2.317^{***}	-1.819^{***}	-1.596^{***}	-1.589^{***}	-1.646^{***}
	(0.045)	(0.076)	(0.079)	(0.060)	(0.050)	(0.045)
Observations	20,475	20,475	20,475	20,475	20,475	20,475
Log Likelihood	−5,670.282	−5,660.989	−5,454.205	−5,463.705	−5,463.731	−5,467.306
Akaike Inf. Crit.	11,350.560	11,335.980	10,924.410	10,941.410	10,939.460	10,944.610
Note:						*p<0.1; **p<0.05; ***p<0.01

7 Structural Estimation

I employ a parametric model to recover the primitives in our model. The estimation procedure is based on the formula derived in the identification part.

$$c_i = b_i + \delta(x; \lambda) - \frac{1}{N-1} \frac{1 - \bar{s}G_{0k}(b_i; \bar{s})}{\bar{s}g_{0k}(b_i; \bar{s})} \tag{6}$$

To handle the estimation of incompleteness including the expected holdup for bidders, I employ the two-stage parametric specification in An and Tang(2019)[1] so that it can apply to my three stage model. The entry threshold is estimated through a logistic regression from the entry situation in the data. After doin this, I employ a local polynomial regression to recover the distributionn of bids in the second stage. I finally obtain the distribution of private cost in the first stage by differencing the distriubtions in the second stage according to the identification formula.

7.1 Estimation of Social Surplus and Incremental Cost

I employ a parametric form to model the bargaining power of the contractors in contract j as, $\gamma(w; \zeta) = \exp(w'\zeta)/[1+\exp(w'\zeta)]$ where w includes auction level characteristics, contract job type, number of potential bidders, average fringe, average utilization rate and average distance.

In the third stage, post-auction negotiation, The transfer payment from the buyer to the winning contractor is assumed in a form,

$$Y = 1\{\phi - a + \epsilon > 0\} \times [\gamma\phi + (1-\gamma)a + \varepsilon] \tag{7}$$

where (ϵ, ε) jointly follow normal distributioin with zero mean, standard deviation $(\sigma, 1)$ and a correlation coefficient ρ.

The change in social surplus is specified as $\pi(x^*, job; \theta) - \pi(x^*, job; \theta)$, where

$$\pi(x, job; \theta) = \theta_1 x + \theta_2(x \times job) + \theta_3 x^2$$

and the incremental costs for the winning contractor is,

$$a(x, x^*, job; \theta) = \theta_0 + \theta_4(x^* - x) + \theta_5(x^* - x) \times job + \theta_6(x^* - x)^2$$

The parameters $(\theta, \zeta, \rho, \sigma)$ are estimated through a two step estimation. In the first step, a probit model on equation (7) is applied to get estimates for $(\hat{\theta}_1 - \hat{\theta}_4, \hat{\theta}_2 - \hat{\theta}_5, \hat{\theta}_3, \hat{\theta}_6)$. After doing this, I can recover the parameters $\tau \equiv (\theta_1, \theta_2, \zeta, \rho, \sigma)$ using an extremum estimator as An and Tang (2019)[1],

$$\hat{\tau} = \arg\max_{\tau}\{\mathcal{L}_J(\tau) - \mathcal{M}_J(\tau)\}$$

where $\mathcal{L}_J$ is the log-likelihood for the transfers under contract revision and $\mathcal{M}_J$ contains the moment conditions from the first-orded condition for buyer's optimization problem. I give more details in the Appendix B about the extreme estimator.

7.2 Estimation of Expected Holdup

I specify the expected holdup term in equation (6) as,

$$E[Y - a(x_j, X^*, job_j)|x_j, \xi_{j,i}, D_j = 1]Prob(D_j = 1|x_j, job_j)$$

The probability of incompleness of contracts is estimated through a logit regression. The expectation of $Y - a$ is estimated by two parts. The first part, the expecion of Y_j conditional on x_j and a vector of bidders level characteristics $\xi_{j,i}$, is estimated through a local constant method. The second part, the ex-ante adjustment cost a, is estimated through a simulationbased method. Equation (8) reports the formula of estimated value of $E(Y-a)$, $\hat{\delta}_{j,i}$.

$$\hat{\delta}_{j,i} = \hat{E}(Y_j|x_j, \xi_{j,i}, d_j = 1) - S^{-1}\sum_{s=1}^{S} a(x_j, x*_{j,s}, job_j; \hat{\theta})$$

7.3 Estimation of Single Index and Distribution of Bids in the Second Stage

To account for the auction level characteristics, I employ a single index approach so that all the characteristics influence the model primitives only through a scalar $z'\beta$. I assume the contractors' bidding behavior follow the form,

$$\log(b_{j,i}) = z_j'\beta + h_j'\zeta + e_{j,i} \tag{8}$$

where z_j and h_j represents auction level characteristics. The main difference of z_j and h_j is that z_j affects distribution of contractors' private cost which means the private cost distribution can be characterized as $F(c, s|z_j'\beta)$ in the first stage. h_j represents other variables that may also influence a bidder's behavior. h_j contains the difference between the expectation of final specification, EX^*, and initial engineer estimate, X, which directly indicates uncertainty of a contract and the measure of compensation from the revison, $E(Y-a)$, which indirectly benefits the contractors as a result of a holdup problem.

I recover the single index $z_j'\beta$ by regressing the log of sumbitted sealed bids on all the auction level characteristics z_j and other characteristics h_j. At the same time, the bid distributions in the second stage, $g_{0j}(\cdot)$ and $G_{0j}(\cdot)$, can be estimated through the distribution of $e_{j,i}$. I adopt kernel density estimation to get the CDF and PDF of error terms $e_{j,i}$. The distributions of interest can be connected to the distribution of the residuals through,

$$G_{0B|Z,H}(b_{j,i}|z_j, h_j) = Pr\{z_j'\beta + h_j'\zeta + e_{j,i} \leq \log(b_{j,i})\} = F_e(\log(b_{j,i}) - z_j'\beta - h_j'\zeta) \tag{9}$$

and

$$g_{0B|Z}(b_{j,i}|z_j, h_j) = \frac{\partial}{\partial b} F_e(\log(b_{j,i}) - z_j'\beta - h_j'\zeta) = f_e(\log(b_{j,i}) - z_j'\beta - h_j'\zeta)/b_{j,i} \tag{10}$$

7.4 Estimation of Entry Thresholds

I estimate entry thresholds by using a binomial logistic regression with $n_j \sim \mathcal{B}\{N_j, \frac{\exp \beta m_j}{1+\exp \beta m_j}\}$. As I discussed earlier, the entry threshold is affected by the combination of the single index $z_j'\beta$, the incompleteness of a contract which measured in $EX^* - X$, and the benefits from the holdup problem $E(Y - a)$.

7.5 Estimation of Private Cost Distribution in the Second Stage and the first stage

After the estimation in the previous sections, I recover the private costs for the entry bidders in the second stage by adding up the bid, the expected holdup, and a function of distribution terms.

$$c_{j,i} = b_{j,i} + \hat{\delta}_{j,i} - \frac{1}{N-1} \frac{1 - \hat{\bar{s}}\hat{G}_0(b_{j,i}|z_j, h_j)}{\hat{\bar{s}}\hat{g}_0(b_{j,i}|z_j, h_j)}$$

Based on the estimated costs, I follow the local polynomial estimation method in Aryal, Gabrielli and Vuang (2019)[2] to recover the private cost distribution in the second stage, $F^*(c; \bar{s}, z'\beta)$.

To recover the private cost distribution in the first stage, I apply the connection between the two distribution function in the identification section,

$$F(c|\bar{s}, z'\beta) = \frac{\partial}{\partial \bar{s}} \left(\bar{s}F^*(c; \bar{s}, z'\beta)\right)$$

I employ numerical differentiation to finally get the private cost distribution in the first stage.

8 Estimation Results

Table 8 reports the estimation result of the linear regression of natural log of bids on auction level characteristics and the expected incompleteness terms estimated from the stuctural estimation including the expected difference of initial design and final design of contracts ($EX^* - X$) and natural log of expected holdup ($\log(E(Y - a))$). For entry bidders, they will bid more if they expect the difference of two design are large. Their bids will increase by 3.08% when the expected difference of designs increases by 1 million dollars. Meanwhile, they will bid slightly more aggressive when they expect a larger holdup from the negotiation with the government. But this effect is not significant for entry bidders.

Table 9 indicates the marginal effects of factors on the entry behavior in the logistic regression. The difference between the expectation of the initial design and the final design deters entry behaviors of contractors. Entry probability will decrease by 0.64% when bidders expect 1 million dollars increase on the two designs. In addition, I can observe a significant positive effect of expected holdup on entry behaviors. If they expect 1% increase on the expected holdup of contracts, entry probability will increase 4.87%.

Based on the two tables, I could conclude that the effects of incompleteness of contracts is twofold. Expected difference between designs hinders entry of bidders and at the same time will deter these entry bidders from making an aggressive bid. The expected holdup makes bidders more likely to enter the auction, but the effects on entry bidders' bid are not significant and relatively small.

Figure 3 presents the estimated private cost distribution in the first stage where the private signal equals the mean value of entry probability and the single index equals the mean value in data.

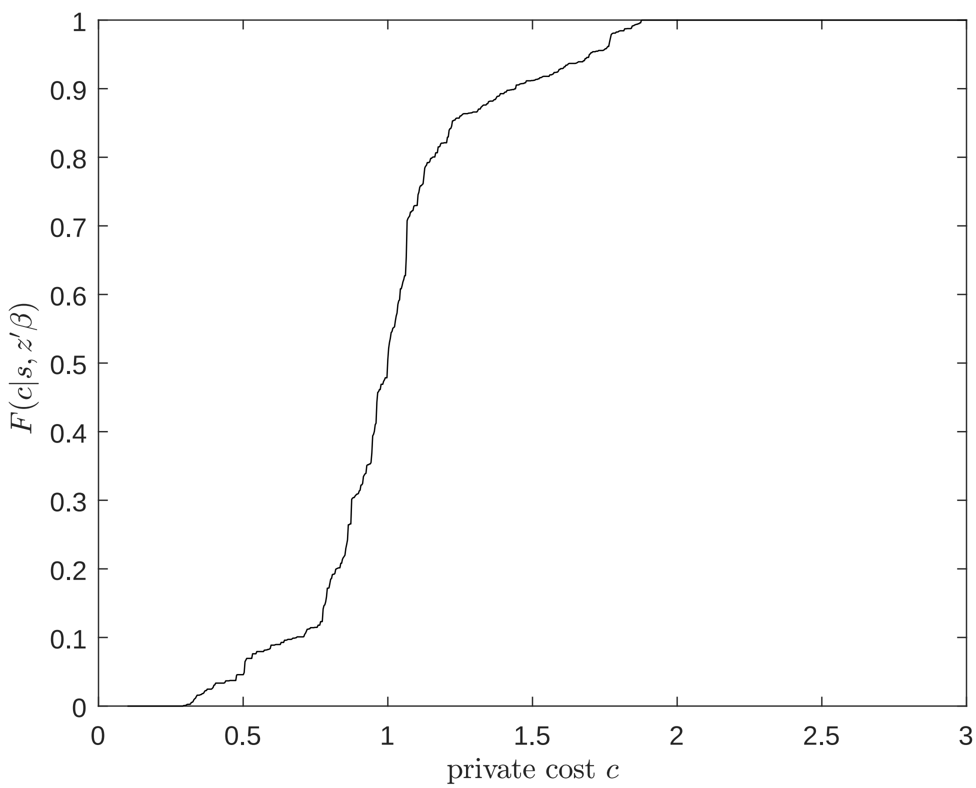
Table 8: Linear Regression of log of bids on auction level characteristics

	Dependent variable:	
	$\log(b)$	
	(1)	(2)
Job	-0.119^{**}	-0.114^{**}
	(0.0535)	(0.0536)
ave.fringe	-0.0938^{**}	-0.0986^{**}
	(0.0467)	(0.0469)
ave.util	0.271^{***}	0.275^{**}
	(0.0794)	(0.0795)
ave.dist	-0.000192^{**}	-0.000203^{***}
	(0.0000897)	(0.0000898)
Job*ave.fringe	0.1011	0.0946
	(0.0635)	(0.0635)
Job*ave.util	0.297^{**}	0.296^{**}
	(0.118)	(0.118)
Job*ave.dist	0.000256^{*}	0.000268^{**}
	(0.000133)	(0.000133)
$\log(X)$	0.976^{***}	0.980^{***}
	(0.00297)	(0.0114)
N	0.000647	0.000629
	(0.000498)	(0.000501)
$EX^* - X$		0.0308^{**}
		(0.0123)
$\log(E(Y-a))$		-0.0166
		(0.0171)
Constant	0.0651	0.0335
	(0.0350)	(0.0469)
Observations	3661	3661
R^2	0.9697	0.9697
$adjusted R^2$	0.9696	0.9696
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 9: Estimates of Average Marginal Effects on Entry Threshold

single index $z'\beta$	-0.0497^{***}
$EX^* - X$	-0.0064^{***}
$\log(E(Y-a))$	0.0487^{***}
N	-0.0090^{***}
Note:	*p<0.1; **p<0.05; ***p<0.01

Figure 3: Estimated Private Cost Distribution in the First Stage $F(c|s, z'\beta)$ at $s = 0.2267$, $z'\beta = 0.193$



9 Importance of Consideration of Both Incompleteness and Entry

9.1 Distortion on bidding strategy from Consideration of Entry

According to An and Tang (2019)[1], econometricians can consider an adjusted cost for a contractor $\tilde{c}_i \equiv c_i - \delta(x; \lambda^*)$ to deal with the structural problem of auctions with incompleteness. However, when it comes to the consideration of both incompleteness and entry, the problem will be much more complicated. Based on the first order condition, the solution can be characterized as,

$$\beta(c_i; \bar{s}) = \frac{1}{[1 - \bar{s}F^*(c_i; \bar{s})]^{N-1}} \int_{c_i}^{\bar{c}} [t - \delta(x; \lambda^*)] d\{1 - [1 - \bar{s}F^*(t; \bar{s})]^{N-1}\} + \bar{c}\frac{(1 - \bar{s})^{N-1}}{[1 - \bar{s}F^*(c_i; \bar{s})]^{N-1}} \tag{11}$$

The effect of holdup on bidding behavior can be measured as,

$$-\frac{1}{[1 - \bar{s}F^*(c_i; \bar{s})]^{N-1}} \int_{c_i}^{\bar{c}} \delta(x; \lambda^*) d\{1 - [1 - \bar{s}F^*(t; \bar{s})]^{N-1}\} = \left\{-1 + \frac{(1 - \bar{s})^{N-1}}{[1 - \bar{s}F^*(c_i; \bar{s})]^{N-1}}\right\} \delta(x; \lambda^*) \tag{12}$$

From this equation, it indicates that bidders will bid less compared to the case when holdup equals to zero. It is natural because bidders will bid more aggressively if they expect a future return from the government. However, this term is not equal to the value of the holdup term so it cannot compensate the bidding behavior. The net effect of expected holdup and corresponding aggressive bidding behavior varies according to the private cost, entry probability and the cost distribution. Therefore, it is very necessary to have a structural model to consider both incompleteness and entry in a procurement auction to handle the highly nonlinear problem.

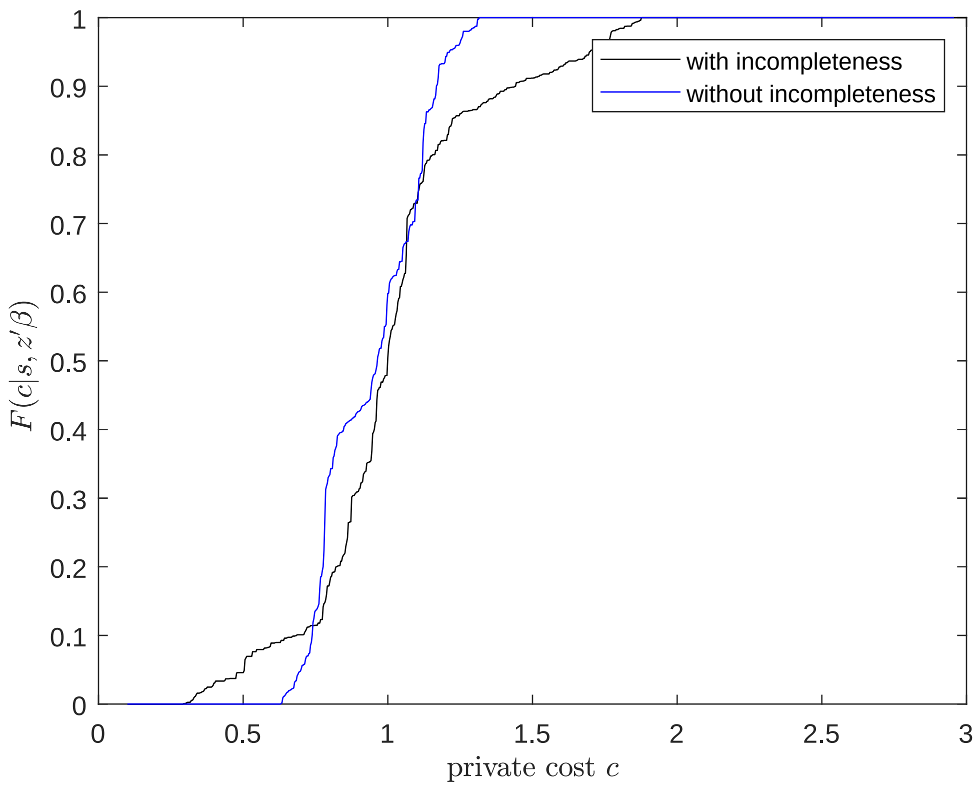
9.2 Comparison between Estimation of Different Models

From this article, I find that when it comes to the structural estimation of a procurement auction with entry problem and contract incompleteness, neglecting either one will give rise to significant estimation bias.

If an econometrician ignores the entry property of an auction and proceeds a structural estimation on it, the estimation result is meaningless if the entry threshold varies much due to heterogeneous auction level characteristics. For procurement auctions with incompletness, this will be worse because the belief of incompleteness of contracts will naturally involve endogenous participation and the effect of incompleteness on entry is ambiguous. To quantitively account for the impact of ignoring endogenous participation, econometricians have to consider not only actual incompleteness of contracts or potential entry situation, but also beliefs of bidders, private cost distribution and expectation of incompleteness and potential holdup problem. Thus, if econometricians neglect the entry property for auctions with incompleteness, the estimated private cost distribution is very hard to tell whether it is overestimated or underestimated.

As mentioned in the previous section, if an econometrician negelects consideration of incompleteness, it will make entry bidders seem to bid more aggressively. It seems that the private cost will be completely underestimated through this process. However, the neglect of incompleteness will affect the estimation of entry threshold and will lead to biased estimation of entry behavior. Since the effects of expected incompleteness problem on entry is two fold, it makes the consequence of ignoring contracts' incompleteness ambiguous. In our data, if I estimated the model without consideration of incompleteness, Figure 4 reports the private cost distribution at mean values of single index and private signal. From the figure, the model without consideration of incompleteness seems to have lower estimated costs compared to the model with incompleteness. However, the distribution does not reveal a simple leftward shift of costs distribution, i.e., a clear stochastic dominance is not observed. Meanwhile, distortion on different value of private cost makes the shape of extreme value of costs very

Figure 4: Estimated Private Cost Distribution in the First Stage $F(c|s, z'\beta)$ with and without consideration of contracts' incompleteness at $s = 0.2267$, $z'\beta = 0.193$



different. In this way, policy makers might be misleaded since very low private costs involve the winner of auctions and very high private costs involve participation rate of auctions.

10 Conclusion

In this paper, I investigate entry behaviors in procurement auctions in an incomplete contracts framework. I empirically examine the impact of incompleteness on the entry decision using the potential bidders set and constructed ex post incompleteness measure. According to the reduced form results, I introduce a structural method to combine the two central features in the literature of auction. Based on the selective entry model (e.g., Gentry and Li, 2014[10]) and incomplete auction models (e.g., Bajari, Houghton, and Tadelis, 2014[4], An and Tang, 2019[1]), I create a three stage model to explain the underlying relationship between the entry decision and the incompleteness. I prove the existence of equilibrium of the model and provide identification for the model. The estimates shows that bidders' behaviors, including entry and bidding strategies, are affected by a combination of contract incompleteness, holdup in procurement auctions, beliefs of incompleteness and benefits from contracts' change, and even their private costs and costs' distribution. This result naturally makes consideration of both entry and incompleteness very important for econometricians and policy makers.

11 Appendix A. Proof of Existence of Equilibrium

At the last stage (negotiation stage), the problem can be specified as,

$$\begin{aligned} \max \quad & (u_c - d_c)^{\gamma}(u_b - d_b)^{1-\gamma} \\ s.t. \quad & u_b + u_c \leq u_0 \end{aligned}$$

where $d_c = b - c$ is the disagreement utility for winning bidder, $d_b = \pi(X) - b$ is the disagreement utility for the buyer, $u_0 = \pi(X^*) - a - c$. u_b, u_c are the agreement utility for the winner and buyer. a is the cost from adjustment of the contract that goes into negotiation.

The solution to the maximization problem can be given as,

$$u_c = \gamma(\pi(X^*) - \pi(X) - a) + b - c$$

So we can get the utility change of the winning bidder from negotiation of a new design,

$$\begin{aligned} \delta(x, x^*) &= \gamma(\pi(X^*) - \pi(X) - a) + b - c - (b - c) \\ &= \gamma(\pi(X^*) - \pi(X) - a(X, X^*)) \\ &= \gamma \Delta u(X, X^*) \end{aligned}$$

At stage 2 (bidding stage), the CDF of cost distribution is,

$$F^*(c; \bar{s}) = \frac{1}{\bar{s}} \int_0^{\bar{s}} F(c|t)dt$$

So the probability of winning one rival when the bidder bids b is,

$$G(b; \bar{s}) = 1 - \bar{s} + \bar{s}(1 - F^*(\beta^{-1}(b); \bar{s}))$$

The winning probability at stage 2 when bidding b,

$$W(b;\bar{s}) = G(b;\bar{s})^{N-1}$$

Therefor, in stage 2, the bidder with cost c_i and bid b has expected payoff,

$$\pi(c_i; b|\bar{s}) = (b - c_i + \delta(x;\lambda^*))W(b;\bar{s})$$

where $\delta(x;\lambda^*) = \int_{\underline{x}}^{\overline{x}} \gamma\left(\pi(X^*) - \pi(X) - a(X,X^*)\right)\lambda_{X^*|X}(X^*|X)dX^*$ is the expected utility change coming from the renegotiation period.

The bidder is seeking to maximize this payoff, the first order condition for it is,

$$W(b;\bar{s}) + (b - c_i + \delta(x;\lambda^*))\frac{dW(b;\bar{s})}{db} = 0$$

Therefore he have,

$$\frac{\frac{dW(b;\bar{s})}{db}}{W(b;\bar{s})} = -\frac{1}{b - c_i + \delta(x;\lambda^*)}$$

Under symmetric bidding strategy, $b = \beta(c_i)$, we have,

$$\begin{aligned}
\frac{(N-1)G(b;\bar{s})^{N-2}(-\bar{s})f^*(\beta^{-1}(b);\bar{s})\frac{d\beta^{-1}(b)}{db}}{G(b;\bar{s})^{N-1}} &= -\frac{1}{b - c_i + \delta(x;\lambda^*)} \\
(-\bar{s})(N-1)G(b;\bar{s})^{N-2}f^*(c_i;\bar{s})\frac{1}{\beta'(c_i)G(b;\bar{s})^{N-1}} &= -\frac{1}{b - c_i + \delta(x;\lambda^*)} \\
\beta'(c_i)G(b;\bar{s})^{N-1} + \beta(c_i)(-\bar{s})(N-1)G(b;\bar{s})^{N-2}f^*(c_i;\bar{s}) &= (c_i - \delta(x;\lambda^*))(-\bar{s})(N-1)G(b;\bar{s})^{N-2}f^*(c_i;\bar{s}) \\
\left[\beta(c_i)(1 - \bar{s}F^*(c_i;\bar{s}))^{N-1}\right]' &= (c_i - \delta(x;\lambda^*))(-\bar{s})(N-1)(1 - \bar{s}F^*(c_i;\bar{s}))^{N-2}f^*(
\end{aligned}$$

Therefore we have,

$$\beta(c_i) = \frac{1}{[1 - \bar{s}F^*(c_i;\bar{s})]^{N-1}}\int_{c_i}^{\bar{c}}[t - \delta(x;\lambda^*)]d\{1 - [1 - \bar{s}F^*(t;\bar{s})]^{N-1}\} \tag{13}$$

Assumption 1 *Cost-signal pairs* (c_i, s_i) *are drawn symmetrically across bidders from a joint distribution such that:*

1. *The support of the random variable* c_i *is a bounded interval* $\mathcal{C} = [0, \bar{c}]$ *and the joint CDF* $F(c, s)$ *is continous in* (c, s)*.*

2. *For each bidder i, the conditional distribution of* c_i *is stochastically ordered in* s_i*:* $s' \geq s$ *implies* $F(c|s') \leq F(c|s)$*.*

3. *The random pairs* (c_i, s_i) *are independent across bidders:* $(c_i, s_i) \perp (c_j, s_j)$ *for all* $j \neq i$*.*

4. *The conditional distribution* $F(c|s)$ *is differentiable in* v *and* s*.*

5. *Without loss of generality, the first-stage signals* s_i *are normalized to have a uniform marginal distribution on* $[0, 1]$*:* $s_i \sim U[0, 1]$*.*

Back to stage 1, the expected payoff at stage 1 is,

$$\Pi(s_i|\bar{s}) = \int_0^1 \pi(c_i; \beta^*(c_i)|\bar{s}) f(c_i|s_i) dc_i$$

Now we need to show the existence of optimal $\bar{s}^*$.

First, the function $\Pi(s_i|\bar{s})$ is decreasing in s_i and $\bar{s}$.

Proof: $\pi(c_i; \beta^*(c_i)|\bar{s})$ is the value function of the bidder maximization problem, so we have,

$$\frac{\partial \pi(c_i; \beta^*(c_i)|\bar{s})}{c_i} = -W(\beta^*(c_i)) < 0$$

Therefore, $\pi(c_i; \beta^*(c_i)|\bar{s})$ is decreasing in c_i. Since we also have stochastic ordering for $F(c_i|\bar{s})$ in $\bar{s}$ from Assumption 1.2, we have $\Pi(s_i|\bar{s})$ is decreasing in s_i.

Similarly, we can get the derivative of the value function $\pi(c_i; \beta^*(c_i)|\bar{s}) = (\beta(c_i) - c_i +$

$\delta(x;\lambda^*))(1-\bar{s}F^*(c;\bar{s}))^{N-1} = (\beta(c_i)-c_i+\delta(x;\lambda^*))(1-\int_0^{\bar{s}} F(c|t)dt)^{N-1}$ with respect to $\bar{s}$.

$$\frac{\partial \pi(c_i;\beta^*(c_i)|\bar{s}))}{\bar{s}} = (\beta(c_i)-c_i+\delta(x;\lambda^*))(N-1)(-F(c|\bar{s})) < 0$$

So function $\Pi(s_i|\bar{s})$ is decreasing in $\bar{s}$.

Second, function $\Pi(s_i|\bar{s})$ is continuous in $(s_i, \bar{s})$.

$$\begin{aligned}
\Pi(s_i|\bar{s}) &= \int_0^1 \pi(c_i;\beta^*(c_i)|\bar{s}))dF(c_i|s_i) \\
&= \pi(c_i;\beta^*(c_i)|\bar{s})F(c_i|s_i) - \int_0^1 F(c_i|s_i)d\pi(c_i;\beta^*(c_i)|\bar{s}) \\
&= \pi(c_i;\beta^*(c_i)|\bar{s})F(c_i|s_i) + \int_0^1 F(c_i|s_i)W(\beta(c_i)|\bar{s})dc_i
\end{aligned}$$

From the assumption, we have the continuity of $F(c|s)$, so we get the continuity of $\beta(\cdot)$. $\pi(c_i;\beta^*(c_i)|\bar{s}) = (\beta(c_i)-c_i+\delta(x;\lambda^*))(1-\int_0^{\bar{s}} F(c|t)dt)$ is continous in $\bar{s}$. So we have the first part $\pi(c_i;\beta^*(c_i)|\bar{s})F(c_i|s_i)$ is continuous in $(s_i,\bar{s})$. In the second term, $W(\beta(c_i)|\bar{s}) = (1-\int_0^{\bar{s}} F(c|t)dt)^{N-1}$ is continuous in $\bar{s}_i$.

Therefore we have the continuity of the function $\Pi(s_i|\bar{s})$.

In the third step, we consider a symmetric equilibrium in threshhold entry strategies $\bar{s}$. If $\Pi(0|0) \leq k$, remaining out is always optimal. If $\Pi(1|1) \geq k$, entry at first stage is always optimal. Otherwise, we have $\Pi(0|0) \geq k$ and $\Pi(1|1) \leq k$. From the continuity and monotonicity of function $\Pi(s_i|\bar{s})$, there exists an interior equilibrium $\bar{s}^* \in (0,1)$ such that a marginal bidder is indifferent to entry:

$$\Pi(\bar{s}^*|\bar{s}^*) = k \tag{14}$$

For the buyer, the expected payment $\varphi_\lambda(x;\bar{s}^*,\beta^*)$ for the contractor can be calculated as,

$$\varphi_\lambda(x; \bar{s}^*, \beta^*) = N \int_0^{\bar{s}} \int_0^{\bar{c}} \beta^*(c, x; \lambda) H(c|\bar{s}(x; \lambda)) f(c|s_i) dc_i ds_i$$

where $H(c|\bar{s}) = [1 - \bar{s} + \bar{s}(1 - F^*(c; \bar{s}))]^{N-1} = [1 - \bar{s}F^*(c; \bar{s})]^{N-1}$.

Given the contractor belief λ, the buyer's optimization problem is:

$$\alpha^*(\tilde{x}; \lambda) \equiv \arg\max_{x \in \mathcal{X}} \{\pi(x) - \varphi_\lambda(x; \bar{s}^*, \beta^*) + \mu(x, \tilde{x})\}$$

Suppose $\alpha^* : \mathcal{X} \to \mathcal{X}$ is the buyer's strategy in a strictly monotone pure strategy PBE, and is differentiable. For the belief on the equilibrium path, we have, $\lambda(\cdot|x) = F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(\cdot)$ for all x on the support of $\alpha^*(\tilde{X})$. Then we have for all $\tilde{x} \in \mathcal{X}$,

$$\pi'(\alpha^*(\tilde{x})) - \varphi'_{\lambda^*}(\alpha^*(\tilde{x}); \bar{s}^*, \beta^*) + \mu_1(\alpha^*(\tilde{x}), \tilde{x}) = 0$$

The function $\varphi_\lambda(x; \bar{s}^*, \beta^*)$ can be derived as,

$$\begin{aligned}
\varphi_\lambda(x; \bar{s}^*, \beta^*) &= N \int_0^{\bar{s}} \int_0^{\bar{c}} \int_{c_i}^{\bar{c}} [t - \delta(x; \lambda)] d\{1 - [1 - \bar{s}F^*(t; \bar{s})]^{N-1}\} f(c|s_i) dc_i ds_i \\
&= N \int_0^{\bar{s}} \int_0^{\bar{c}} \int_{c_i}^{\bar{c}} t d\{1 - [1 - \bar{s}F^*(t; \bar{s})]^{N-1}\} f(c|s_i) dc_i ds_i \\
&\quad - N \int_0^{\bar{s}} \int_0^{\bar{c}} \int_{c_i}^{\bar{c}} \delta(x; \lambda) d\{1 - [1 - \bar{s}F^*(t; \bar{s})]^{N-1}\} f(c|s_i) dc_i ds_i
\end{aligned}$$

The second term can be derived as,

$$
\begin{aligned}
N\delta(x;\lambda)\int_0^{\bar{s}}\int_0^{\bar{c}}[1-\bar{s}F^*(t;\bar{s})]^{N-1}f(c|s_i)dc_i ds_i &= N\delta(x;\lambda)\int_0^{\bar{s}}\int_0^{\bar{c}}[1-\int_0^{\bar{s}}F(c|t)dt]^{N-1}f(c|s_i)dc_i ds_i \\
&= N\delta(x;\lambda)\int_0^{\bar{c}}[1-F(c,s=\bar{s})]^{N-1}\int_0^{\bar{s}}f(c|s_i)ds_i dc_i \\
&= N\delta(x;\lambda)\int_0^{\bar{c}}[1-F(c,s=\bar{s})]^{N-1}dF(c,s=\bar{s}) \\
&= -\delta(x;\lambda)[1-F(c,s=\bar{s})]^{N}\Big|_0^{\bar{c}} \\
&= \delta(x;\lambda)
\end{aligned}
$$

The first term can be derived as,

$$
\begin{aligned}
N&\int_0^{\bar{s}}\int_0^{\bar{c}}\int_{c_i}^{\bar{c}}td\{1-[1-\bar{s}F^*(t;\bar{s})]^{N-1}\}f(c|s_i)dc_i ds_i \\
&= N\int_0^{\bar{c}}\int_0^{\bar{s}}\int_0^{t}tf(c_i|s_i)dc_i ds_i d\{1-[1-\bar{s}F^*(t;\bar{s})]^{N-1}\} \\
&= N\int_0^{\bar{c}}t\int_0^{\bar{s}}F(t|s_i)ds_i d\{1-[1-\bar{s}F^*(t;\bar{s})]^{N-1}\} \\
&= \bar{s}N\int_0^{\bar{c}}tF^*(t;\bar{s})d\{1-[1-\bar{s}F^*(t;\bar{s})]^{N-1}\} \\
&= \bar{s}N\int_0^{\bar{c}}tF^*(t;\bar{s})(N-1)[1-\bar{s}F^*(t;\bar{s})]^{N-2}\bar{s}dF^*(t;\bar{s}) \\
&= N\int_0^{\bar{c}}t[\bar{s}F^*(t;\bar{s})-1+1](N-1)(1-\bar{s}F^*)^{N-2}\bar{s}dF^*(t;\bar{s}) \\
&= N\int_0^{\bar{c}}t(N-1)[1-\bar{s}F^*(t;\bar{s})]^{N-2}\bar{s}dF^*(t;\bar{s})-N\int_0^{\bar{c}}t[1-\bar{s}F^*(t;\bar{s})]^{N-1}(N-1)\bar{s}dF^*(t;\bar{s}) \\
&= N\int_0^{\bar{c}}td\{1-[1-\bar{s}F^*(t;\bar{s})]^{N-1}\}-(N-1)\int_0^{\bar{c}}td\{1-[1-\bar{s}F^*(t;\bar{s})]^{N}\}
\end{aligned}
$$

which directly depends on $\bar{s}$ and other fundamental distributions. The optimal $\bar{s}^*$ is only influenced by the participation cost and the buyer's signal X. Therefore, we can rewrite the expected payment in the form $\varphi_\lambda(x;\bar{s},\bar{\beta})$ as $\sigma(x)-\delta(x;\lambda)$.

Let

$$\Lambda(x,\tilde{x}) \equiv E[\Delta u_+(x,X^*)|\tilde{X}=\tilde{x}] = \int_{\omega(x)} [\pi(x^*) - \pi(x) - a(x,x^*)]dF(x^*|\tilde{x})$$

where $\omega(x) \equiv \{x^* : s(x,x^*) > 0\}$. Then we have $\mu(x,\tilde{x}) = (1-\gamma)\Lambda(x,\tilde{x})$ and $\delta(x;\lambda^*) = \gamma\Lambda(x, \alpha^{*-1}(x))$.

Therefore we have,

$$\varphi'_{\lambda^*}(\alpha^*(\tilde{x}); \bar{s}^*, \beta^*) = \sigma'(\alpha^*(\tilde{x})) - \Lambda_1(\alpha^*(\tilde{x}),\tilde{x}) - \frac{\Lambda_2(\alpha^*(\tilde{x}),\tilde{x})}{\alpha^{*\prime}(\tilde{x})}$$

We finally have the ordinary differential equation,

$$\alpha^{*\prime}(\tilde{x}) = \Gamma(\alpha^*(\tilde{x}),\tilde{x}))$$

where $\Gamma(x,\tilde{x}) = \frac{\gamma\Lambda_2(x,\tilde{x})}{\sigma'(x) - \pi'(x) - \Lambda_1(x,\tilde{x})}$.

We define a contractor's conditional belief λ as ultra-pessimistic at $x \in \mathcal{X}$ if it assigns no probability mass to a new designs which yields positive net surplus change, i.e.,

$$\int 1\{x^* \in \omega(x)\} d\lambda(x^*|x) = 0, \tag{15}$$

where $\omega(x) \equiv \{x^* : \Delta u(x,x^*) > 0\}$. A buyer's objective function when contractors hold an "ultra-pessimistic" belief λ is:

$$H^o(x,\tilde{x}) \equiv \pi(x) + (1-\gamma)\Lambda(x,\tilde{x}) - \sigma(x)$$

Assumption 2 *(Smoothness and Concavity) π, σ and Λ are differentiable and bounded over their domains. For each $\tilde{x} \in \tilde{\mathcal{X}}$, H^o is strictly concave in x over $\mathcal{X}$.*

It follows from the assumption and the theorem of maximum that

$$\mathcal{X} \equiv \{x : x = \arg\max_{z \in \mathcal{X}} H^o(z, \tilde{x}) \text{ for some } \tilde{x} \in \mathcal{X}\}$$

is convex and compact in $\mathcal{X}$. Let x_h^o(and x_l^o) denote the supremum (and infimum) of $\mathcal{X}_o$; x_h (and x_l) denote the supremum (and infimum) of $\mathcal{X}$; and $\tilde{x}_h$ (and $\tilde{x}_l$) denote the supremum (and infimum) of $\tilde{X}$. Define the following ordinary differential equation with an initial condition:

$$\alpha'(\tilde{x}) = \Gamma(\alpha(\tilde{x}), \tilde{x}) \text{ and } \alpha(\tilde{x}_l) = x_l^o. \tag{16}$$

Assumption 3 *The function Γ satisfies,*

1. *Γ is continuous over $\mathcal{X} \times \tilde{\mathcal{X}}$, and there exists $L \in \mathbb{R}_{++}$ such that for all $(\tilde{x}, x, x')$ in $\tilde{\mathcal{X}} \times \mathcal{X} \times \mathcal{X}$, $|\Gamma(x', \tilde{x}) - \Gamma(x, \tilde{x})| \leq L|x' - x|$.*

2. *$M(\tilde{x}_h - \tilde{x}_l \leq x_h - x_l^o)$, where $M \equiv \sup_{\mathcal{X} \times \tilde{\mathcal{X}}} \Gamma(x, \tilde{x}) < \infty$.*

It follow from Assumption 3 and Picard's Local Existence Theorem that there exists a solution to this ODE over $\tilde{X} \equiv [\tilde{x}_l, \tilde{x}_h]$.

Now we need to show the Monotonicity of α^*.

Assumption 4 *(Monotonicity)*

1. *For any pair $\tilde{x}' > \tilde{x}$ on $\mathcal{X}$, $F_{X^*|\tilde{X}=\tilde{x}'}$ first order stochastically dominates $F_{X^*|\tilde{X}=\tilde{x}}$*

2. *For each $x \in \mathcal{X}$, $\Lambda(x, \tilde{x})$ is increasing in $\tilde{x}$.*

3. *For each $\tilde{x} \in \tilde{\mathcal{X}}$, $\pi'(x) + \Lambda(x, \tilde{x})$ is nonincreasing in x.*

Assumption 5 *(Support Condition)*

1. *For any $(x, \tilde{x}) \in \mathcal{X} \times \tilde{\mathcal{X}}$, the integral $\int 1\{x^* \in \omega(x) dF_{X^*|\tilde{X}=\tilde{x}}(x^*) > 0$.*

2. For some α^* that solves (16), $\mathcal{X}_o \subseteq \mathcal{X}_e \equiv \{x : x = \alpha^*(\tilde{x})$ for some $\tilde{x} \in \tilde{\mathcal{X}}\}$.

Proof: From the assumption that $x' \notin \mathcal{X}_e$ implies $x' \notin \mathcal{X}_o$. Thus, there exists $x^o \in \mathcal{X}_o$ such that $H^o(x^o, \tilde{x}) \geq H^o(x', \tilde{x})$. We have $\delta(x; \lambda^*) \geq 0$, where the equality holds under ultra-pessimistic beliefs. Therefore, for any $z \in \mathcal{X}$,

$$H^o(z, \tilde{x}) \leq \pi(z) + \mu(z, \tilde{x}) - \sigma(z) + \delta(z; \lambda^*)$$

where the inequality holds with equality for $z \notin \mathcal{X}_e$ and holds strictly for $z \in \mathcal{X}_e$. Hence, we finally have,

$$\begin{aligned}
&\pi(x^o) + \mu(x^o, \tilde{x}) - \sigma(x^o) + \delta(x^o; \lambda^*) \\
&\quad > H^o(x^o, \tilde{x}) \geq H^o(x', \tilde{x}) \\
&\quad = \pi(x') + \mu(x', \tilde{x}) - \sigma(x') + \delta(x'; \lambda^*)
\end{aligned}$$

This shows that the buyer will not deviate to x' under the belief λ^*.

Appendix B: Details on Extremum Estimator

I follow the extreme estimator in An and Tang (2019)[1]. Though their model is quite different from the model in this article, the first order conditions for the government are the same. Therefore, I can apply their method to deal with the estimation of the incompleteness part in this article. The extreme estimator is specified as,

$$\hat{\tau} = \arg\max_{\tau}\{\mathcal{L}_J(\tau) - \mathcal{M}_J(\tau)\}$$

where $\mathcal{L}_J$ is defined as,

$$\mathcal{L}_J \equiv J^{-1}\sum_j \log\Phi\left(\frac{\hat{I}_{s,j} + \frac{\rho}{\sigma}(y_j - \hat{I}_{o,j})}{\sqrt{1-\rho^2}}\right) + \log\Phi'\left(\frac{y_j - \hat{I}_{o,j}}{\sigma}\right) - \log\sigma - \log\Phi(\hat{I}_{s,j})$$

The $\hat{I}_{o,j} \equiv \gamma_j\hat{\phi}_j + (1-\gamma_j)\hat{a}_j$ and $\hat{I}_{s,j} \equiv \hat{\phi}_j - \hat{a}_j$ can be derived from the probit model setting, where $\gamma_j \equiv \gamma(w_j;\zeta)$, $\phi_j \equiv \pi(x_j^*, w_j; \tau, \hat{\theta}) - \pi(x_j, w_j; \tau, \hat{\theta})$ and $\hat{a}_j \equiv a(x_j, x_j^*, w_j; \tau, \hat{\theta})$. The moment condition term $\mathcal{M}_J$ is defined as,

$$\mathcal{M}_J \equiv J^{-1}\sum_j[\hat{\varphi}_1(x_j, w_j) - \hat{\pi}_1(x_j, job_j; \tau) - \hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)]^2$$

where $\hat{\varphi}_1$, $\hat{\pi}_1$ and $\hat{\mu}_1$ are first order derivates of $\varphi(x, w)$, $\pi(x, job)$ and $\mu(x, \tilde{x}, job)$ with respect to x. I can estimate $\hat{\varphi}_1$ through estimating regression model of auction payment on x, w and interaction terms. $\hat{\varphi}_1$ can be calculated by plugging in these estimates. $\hat{\pi}_1$ can be calculated through the defined function form of π. To calculate $\hat{\mu}_1$, I can estimate it using,

$$\mu_1(x, \tilde{x}, job; \tau) = E[(1-\gamma(W;\zeta))|job] \times E\left[\Phi(\Delta(u(X^*, x, job))\frac{\partial\Delta u(X^*, x, job)}{\partial x}|\tilde{x}, job\right]$$

The first term can be directly estimated by plugging in the parameters and taking conditional sample mean. The second term can be acquired by using simulated observations of x^* drawn from the estimated normal distribution of X^* given $\tilde{X} = \tilde{x}_j$.

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