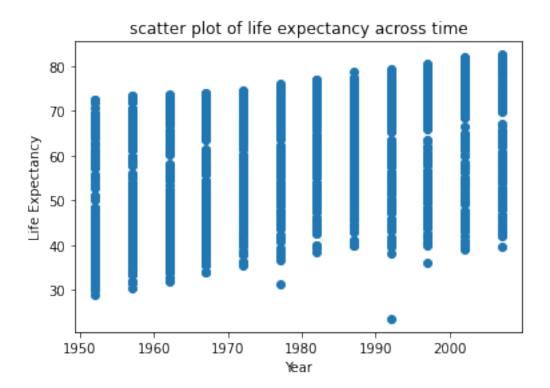
р3

April 29, 2022

```
[310]: import pandas as pd
       import matplotlib.pyplot as plt
       import numpy as np
[311]: data = pd.read_csv("mirror/08_gap-every-five-years.tsv", sep='\t')
       data.head()
       data
[311]:
                 country continent year
                                          lifeExp
                                                              gdpPercap
                                                        pop
       0
                                           28.801
                                                    8425333 779.445314
             Afghanistan
                              Asia 1952
       1
             Afghanistan
                              Asia 1957
                                           30.332
                                                    9240934
                                                             820.853030
       2
             Afghanistan
                              Asia 1962
                                           31.997
                                                   10267083
                                                             853.100710
             Afghanistan
       3
                              Asia 1967
                                           34.020
                                                   11537966
                                                             836.197138
       4
             Afghanistan
                              Asia 1972
                                           36.088
                                                   13079460 739.981106
       1699
                Zimbabwe
                            Africa 1987
                                           62.351
                                                    9216418 706.157306
       1700
                Zimbabwe
                            Africa 1992
                                           60.377
                                                   10704340 693.420786
       1701
                Zimbabwe
                            Africa 1997
                                           46.809
                                                   11404948 792.449960
       1702
                Zimbabwe
                            Africa 2002
                                           39.989
                                                   11926563 672.038623
       1703
                Zimbabwe
                            Africa 2007
                                                   12311143 469.709298
                                           43.487
       [1704 rows x 6 columns]
      Part 1
      Exercise 1
[312]: subdata = data[['year','lifeExp','continent']].sort_values(by=['year'])
       x = subdata['year']
       y = subdata['lifeExp']
       plt.scatter(x,y)
       plt.title('scatter plot of life expectancy across time')
       plt.xlabel('Year')
       plt.ylabel('Life Expectancy')
       plt.show()
```



Q1: Is there a general trend (e.g., increasing or decreasing) for life expectancy across time? Is this trend linear?

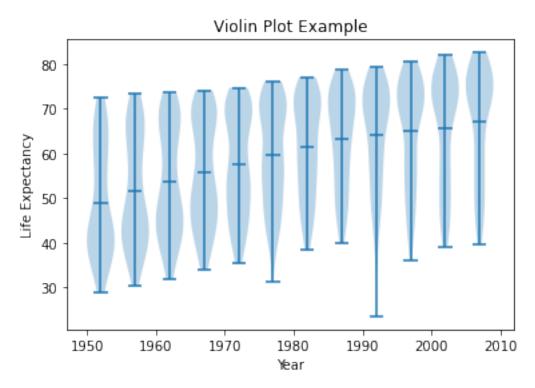
Ans: Yes, there is a general increasing positive trend, which means the life expectancy increases as time passes. It is approximately a linear trend.

```
[313]: Max_year = subdata['year'].max()
Min_year = subdata['year'].min()
life_exp_per_year = []
years = []

for y in range(Min_year,Max_year+1):
    col = []
    years.append(y)
    for x in range(0,len(subdata)):
        if subdata.iat[x,0] == y:
            col.append(subdata.iat[x,1])
    if col!=[]:
        life_exp_per_year.append(col)
    else:
        years.remove(y)
```

```
[314]: fig, ax = plt.subplots()
```

```
ax.violinplot(life_exp_per_year,years,widths=4,showmeans=True)
ax.set_xlabel("Year")
ax.set_ylabel("Life Expectancy")
ax.set_title("Violin Plot Example")
fig.savefig("violin.png")
```



Q2: How would you describe the distribution of life expectancy across countries for individual years? Is it skewed, or not? Unimodal or not? Symmetric around it's center?

Ans: For the first two distributions, they are right-skewed unimodal distributions and not symmetric around their centers. For the third, fourth and fifth distributions, they are skewed bimodal ditribution and they are not perfectly symmetric about their centers. For the rest ditributions, they are left-skewed unimodal ditribution and not symmetric around their centers. Overall, the distributions of life expectancy across countries for individual years are skewed, mostly unimodal and not symmetric.

Q3: Suppose I fit a linear regression model of life expectancy vs. year (treating it as a continuous variable), and test for a relationship between year and life expectancy, will you reject the null hypothesis of no relationship?

Ans: Based on the Violin Plot, though there is not enough information indicating a relationship, there is a clear coorelation between those two variables. Thus, I would reject the null hypothesis.

Q4: What would a violin plot of residuals from the linear model in Question 3 vs. year look like?

Ans: It would be a linear distribution (a horizontal line) since the residuals are relatively same for

a linear distribution.

Q5: According to the assumptions of the linear regression model, what should that violin plot look like? That is, consider the assumptions the linear regression model you used assumes (e.g., about noise, about input distributions, etc); do you think everything is okay?

Ans: It should be a linear distribution centered around 0

Exercise 2

```
[315]: from sklearn.linear_model import LinearRegression import statsmodels.api as sm
```

```
[316]: x = np.array(subdata['year']).reshape(-1, 1)
y = np.array(subdata['lifeExp'])

model = LinearRegression().fit(x, y)
r_sq = model.score(x,y)
print('Coefficient of determination:', r_sq)
print('Intercept:', model.intercept_)
print('Slope:', model.coef_)
print('Regression Line: y = {} * x {}'.format(model.coef_,model.intercept_))
```

Coefficient of determination: 0.18975713852188825

Intercept: -585.6521874415398

Slope: [0.32590383]

Regression Line: y = [0.32590383] * x -585.6521874415398

```
[317]: lm = sm.OLS.from_formula('lifeExp ~ year', subdata)
  result = lm.fit()
  result.summary()
```

[317]: <class 'statsmodels.iolib.summary.Summary'>

OLS Regression Results

			=======================================
Dep. Variable:	lifeExp	R-squared:	0.190
Model:	OLS	Adj. R-squared:	0.189
Method:	Least Squares	F-statistic:	398.6
Date:	Thu, 28 Apr 2022	Prob (F-statistic):	7.55e-80
Time:	23:13:19	Log-Likelihood:	-6597.9
No. Observations:	1704	AIC:	1.320e+04
Df Residuals:	1702	BIC:	1.321e+04
Df Model:	1		
Covariance Type:	nonrobust		
=======================================			

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-585.6522	32.314	-18.124	0.000	-649.031	-522.273

year	0.3259	0.016	19.965	0.000	0.294	0.358
Omnibus:		386.		bin-Watson:		1.875
<pre>Prob(Omnibus) Skew:</pre>	:	-0.:		que-Bera (JB): b(JB):		90.750 1.97e-20
Kurtosis:		2.	004 Con	d. No.		2.27e+05

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.27e+05. This might indicate that there are strong multicollinearity or other numerical problems.
- Q6: On average, by how much does life expectancy increase every year around the world?

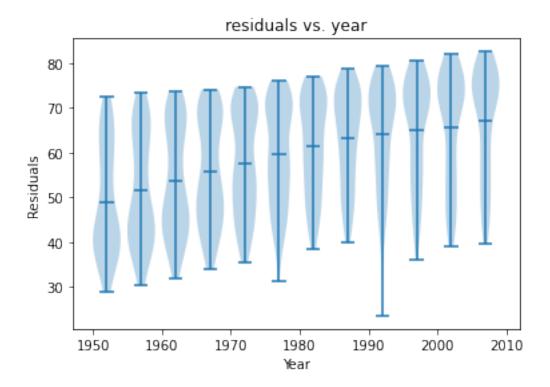
Ans: 0.32590383

Q7: Do you reject the null hypothesis of no relationship between year and life expectancy? Why?

Ans: Yes, we reject the null hypothesis. The p-value is 7.55e-80 which is small enough to reject the null hypothesis

Exercise 3

```
[318]: subdata['residuals'] = subdata['lifeExp'] - (model.coef_ * subdata['year'] +
        →model.intercept_)
       res_years = subdata['year']
       residuals = []
       for y in range(Min_year, Max_year+1):
           col = []
           for x in range(0,len(subdata)):
               if subdata.iat[x,0] == y:
                   col.append(subdata.iat[x,1])
           if col!=[]:
               residuals.append(col)
       fig, ax = plt.subplots()
       ax.violinplot(residuals, years, widths=4, showmeans=True)
       ax.set_xlabel("Year")
       ax.set_ylabel("Residuals")
       ax.set_title("residuals vs. year")
       fig.savefig("violin2.png")
```



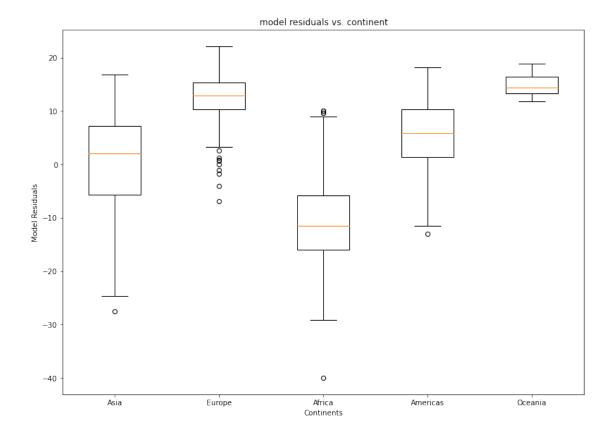
Q8: Does the plot of Exercise 3 match your expectations (as you answered Question 4)?

Ans: Yes, it matches. It is a relatively linear distribution (a horizontal line) since the residuals are relatively same for a linear distribution.

Excise 4

```
[321]: continent = data['continent'].unique()
    residuals2 = []
    for y in continent:
        col = []
        for x in range(0,len(subdata)):
            if subdata.iat[x,2] == y:
                 col.append(subdata.iat[x,3])
        residuals2.append(col)
```

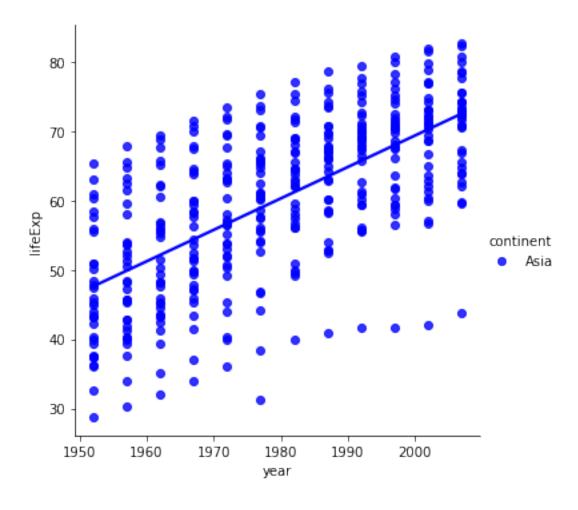
```
[322]: fig = plt.figure(figsize =(10, 7))
    ax = fig.add_axes([0, 0, 1, 1])
    bp = ax.boxplot(residuals2)
    ax.set_xticklabels(continent)
    plt.title("model residuals vs. continent")
    plt.xlabel('Continents')
    plt.ylabel('Model Residuals')
    plt.show()
```

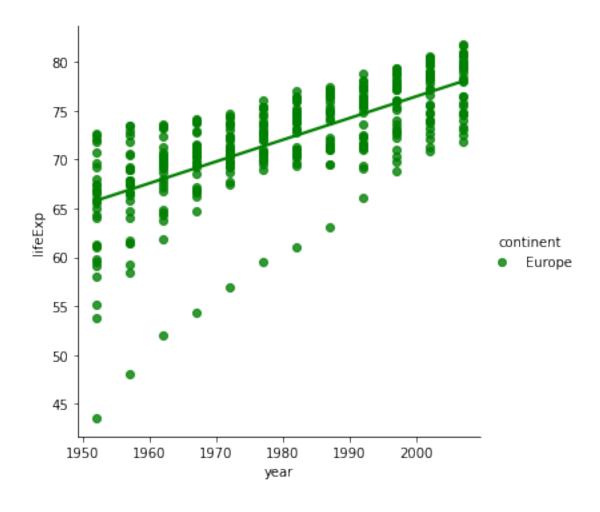


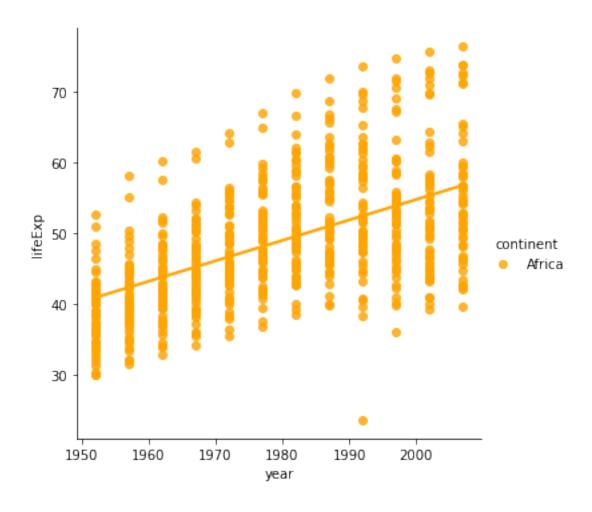
Q9: Is there a dependence between model residual and continent? If so, what would that suggest when performing a regression analysis of life expectancy across time?

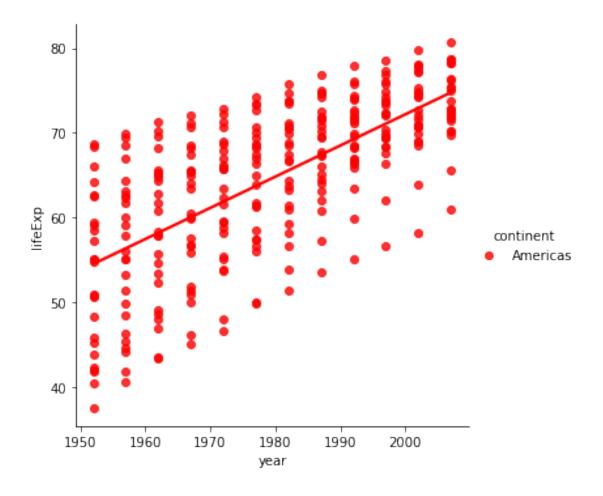
Yes, it seems there is a dependence between model residual and continent since for each continents, the connection between residuals and continents are unclear and each boxgraph is separated and distinct. This suggests that we may need to separate each continents to perform a regression analysis of life expectancy across time.

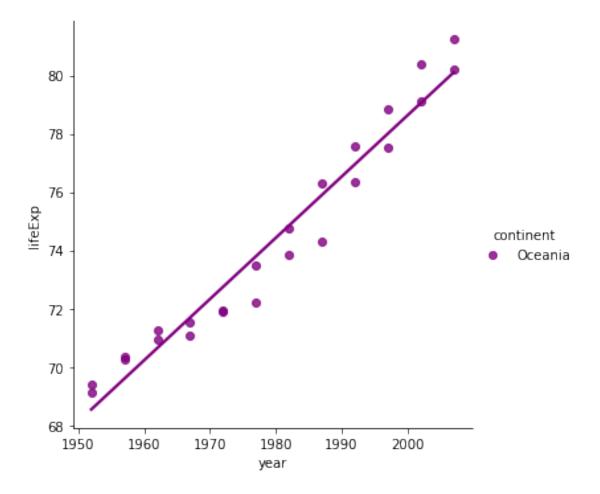
Exercise 5:











Q10: Based on this plot, should your regression model include an interaction term for continent and year? Why?

Ans: Yes, an interaction term is needed since the slopes and intersections of those regression lines are too distinct. It means that including an interaction term for continent and year can make the regression more precise and correct.

Exercise 6

```
[353]: lm2 = sm.OLS.from_formula('lifeExp ~ year * continent', subdata)
result2 = lm2.fit()
result2.summary()
```

[353]: <class 'statsmodels.iolib.summary.Summary'>

OLS Regression Results

Dep. Variable:	lifeExp	R-squared:	0.693
Model:	OLS	Adj. R-squared:	0.691
Method:	Least Squares	F-statistic:	424.3

Date: Fri, 2 Time: No. Observations: Df Residuals: Df Model: Covariance Type:	29 Apr 2022 00:11:23 1704 1694 9 nonrobust	AIC: BIC:		0.00 -5771.9 1.156e+04 1.162e+04
[0.025 0.975]	coef	std err	t	P> t
Intercept -588.911 -459.605	-524.2578	32.963	-15.904	0.000
continent[T.Americas] -252.315 -25.382	-138.8484	57.851	-2.400	0.016
continent[T.Asia] -416.396 -208.870 continent[T.Europe]	-312.6330 156.8469	52.904 54.498	-5.909 2.878	0.000
49.957 263.737 continent[T.Oceania]	182.3499	171.283	1.065	0.287
-153.599 518.298 year	0.2895	0.017	17.387	0.000
0.257 0.322 year:continent[T.Americas]	0.0781	0.029	2.673	0.008
0.021 0.135 year:continent[T.Asia] 0.111 0.216	0.1636	0.027	6.121	0.000
year:continent[T.Europe] -0.122 -0.014	-0.0676	0.028	-2.455	0.014
year:continent[T.Oceania] -0.249 0.090	-0.0793	0.087	-0.916	0.360
Omnibus: Prob(Omnibus): Skew: Kurtosis:	27.121 0.000 -0.121 3.750	Durbin-Wats Jarque-Bera Prob(JB): Cond. No.	(JB):	1.866 44.106 2.65e-10 2.09e+06

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.09e+06. This might indicate that there are strong multicollinearity or other numerical problems.

Q11: Are all parameters in the model significantly (in the p-value sense) different from zero? If

not, which are not significantly different from zero?

Ans: Not all parameters are significantly different from zero. The Oceania is -0.916 which is close to zero.

Q12: On average, by how much does life expectancy increase each year for each continent?

```
[357]: result2.params
[357]: Intercept
                                     -524.257846
       continent[T.Americas]
                                     -138.848447
       continent[T.Asia]
                                     -312.633049
       continent[T.Europe]
                                      156.846852
       continent[T.Oceania]
                                      182.349883
                                        0.289529
       year:continent[T.Americas]
                                        0.078122
       year:continent[T.Asia]
                                        0.163593
       year:continent[T.Europe]
                                       -0.067597
```

-0.079257

Ans: for Americas per year: increase 0.078122; for Oceania per year: increase -0.079257; for Asia per year: increase 0.163593; for Europe per year: increase -0.067597; for Africa per year: increase 0.289529

Exercise 7:

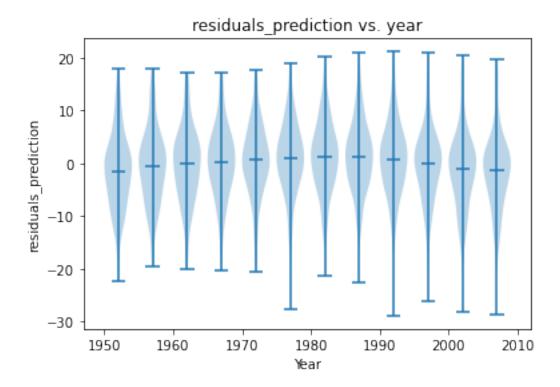
dtype: float64

year:continent[T.Oceania]

```
[372]: subdata['prediction'] = result2.predict()
subdata['residuals2'] = subdata['lifeExp'] - subdata['prediction']

residuals2 = []
for y in years:
    col = []
    for x in range(0,len(subdata)):
        if subdata.iat[x,0] == y:
            col.append(subdata.iat[x,5])
    if col!=[]:
        residuals2.append(col)
```

```
[374]: fig, ax = plt.subplots()
    ax.violinplot(residuals2, years, widths=4, showmeans=True)
    ax.set_xlabel("Year")
    ax.set_ylabel("residuals_prediction")
    ax.set_title("residuals_prediction vs. year")
    fig.savefig("violin4.png")
```



Comments: It fits the assumption that it would be normally distributed because those distributions are unimodal, normal and centered around zero.

Part2

```
[525]: import pandas as pd
       import sklearn
       from sklearn import datasets
       from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
       from sklearn.tree import DecisionTreeClassifier
       from sklearn import tree
[526]: iris = datasets.load_iris()
       df = pd.DataFrame(data = np.c_[iris['data'], iris['target']], columns =__
        →iris['feature_names'] + ['target'])
       df['species'] = pd.Categorical.from_codes(iris.target, iris.target_names)
[526]:
                               sepal width (cm)
                                                 petal length (cm)
                                                                      petal width (cm)
            sepal length (cm)
       0
                          5.1
                                             3.5
                                                                 1.4
                                                                                   0.2
                          4.9
                                             3.0
                                                                                   0.2
       1
                                                                 1.4
       2
                          4.7
                                             3.2
                                                                 1.3
                                                                                   0.2
                          4.6
                                             3.1
                                                                 1.5
                                                                                   0.2
       3
       4
                          5.0
                                                                                   0.2
                                             3.6
                                                                 1.4
```

```
6.7
                                                                 5.2
                                                                                   2.3
       145
                                             3.0
       146
                          6.3
                                             2.5
                                                                 5.0
                                                                                   1.9
                                             3.0
                                                                 5.2
                                                                                   2.0
       147
                          6.5
       148
                          6.2
                                             3.4
                                                                 5.4
                                                                                   2.3
       149
                          5.9
                                             3.0
                                                                 5.1
                                                                                   1.8
            target
                      species
       0
               0.0
                       setosa
       1
               0.0
                       setosa
       2
               0.0
                       setosa
       3
               0.0
                       setosa
               0.0
                       setosa
       145
               2.0 virginica
       146
               2.0 virginica
               2.0 virginica
       147
               2.0 virginica
       148
       149
               2.0 virginica
       [150 rows x 6 columns]
[527]: X = df[['sepal length (cm)', 'sepal width (cm)', 'petal length (cm)', 'petal width
        y = df['species']
      Linear Discriminant Analysis
[528]: clf1 = LinearDiscriminantAnalysis()
       clf1 = clf1.fit(X.values, y.values)
      Decision trees
[529]: clf2 = DecisionTreeClassifier()
       clf2 = clf2.fit(X.values, y.values)
      10-fold cross-validation
[530]: from sklearn.model_selection import train_test_split
       from sklearn.metrics import mean_squared_error
       from math import sqrt
       from sklearn import model_selection
       from sklearn.linear_model import LogisticRegression
       from sklearn.model_selection import KFold
       from sklearn.model_selection import LeaveOneOut
       from sklearn.model_selection import LeavePOut
       from sklearn.model_selection import ShuffleSplit
       from sklearn.model_selection import StratifiedKFold
```

```
[533]: kfold = model_selection.KFold(n_splits=10, random_state=100,shuffle=True)
model_kfold = LinearDiscriminantAnalysis()
results_kfold = model_selection.cross_val_score(model_kfold, X, y, cv=kfold)
print("%0.2f accuracy with a standard deviation of %0.2f" % (results_kfold.

wmean(), results_kfold.std()))
```

0.98 accuracy with a standard deviation of 0.04

0.95 accuracy with a standard deviation of 0.06

WriteUp: For Part 2, I choose Iris dataset. I load the data using load_iris() from the sklearn.datasets.module. The prediction is classification of types of iris. There are three expected outcomes: setosa, versicolour and virginica. The predictors are sepal length (cm)','sepal width (cm)','petal length (cm)','petal width (cm)'.

Then I make my data as a dataframe(code is above). I let X be the predictors and y be the classification. Then using Linear Discriminant Analysis and Decision trees algorithm(code is above). For Decision trees, I use the 'DecisionTreeClassifier ()'. As for performance metric, I use 10-fold cross-validation to get each algrithm's accuracy and std div.

I first the 'model_selection.KFold' function from 'scikit-learn' and creates 10 folds. Models are LinearDiscriminantAnalysis() and DecisionTreeClassifier(). Then I fit the model and generates cross-validation scores. The 'cv' argument specifies the number of cross-validation splits. At last, print the mean accuracy result and std div.

For Linear Discriminant Analysis, 0.98 accuracy with a standard deviation of 0.04. For Decision trees, 0.95 accuracy with a standard deviation of 0.06. Standard arror = $SE = SD/\sqrt{(\text{sample size})}$. Thus, for Decision trees, SE = 0.005. For LDA, SE = 0.003. Thus, Linear Discriminant Analysis is better because of higher accuracy and lower SE.