Difference-in-differences Part III

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BW Econometrics Reading Group (08/17/2023)

DID in a nutshell

A method to estimate the effect of a policy (treatment) on an outcome by comparing over time groups experiencing different evolutions to their exposure to the policy

- a treatment can be binary or continuous
- a treatment can be endogenous
- different groups can receive treatments at different times

A somewhat general framework

- units adopt a **binary** treatment of interest at different times
  - no unit is treated at time $t = 1$
- absorbing state: once a unit is treated, it remains treated
- observed data
  - $y_{i,t}$: outcome variable
  - $x_{i,t}$: exogenous covariates
  - $D_{i,t}$: treatment status
  - $G_i := \min\{t : D_{i,t} = 1\}$
    - the earliest period in which i is treated
    - if i is never treated in the data sample: $G_i = \infty$
    - under absorbing state: $D_{i,t} = 1$ for all $t \geq G_i$

Potential outcome framework

- $y_{i,t}(\mathbf{0}_{g-1}, \mathbf{1}_{T-g+1})$: unit i's potential outcome at t if it is first treated at g
- $y_{i,t}(\mathbf{0}_T)$: potential outcome at t if never treated
- under absorbing state: the entire path of potential outcomes can be summarized by g
  - $y_{i,t}(g) \equiv y_{i,t}(\mathbf{0}_{g-1}, \mathbf{1}_{T-g+1})$
  - $y_{i,t}(\infty) \equiv y_{i,t}(\mathbf{0}_T)$
- treatment effect at t for i that are treated at g:

$$\tau_{i,t}(g) := y_{i,t}(g) - y_{i,t}(\infty)$$

Fundamental problem of causal inference

If a unit is treated at t, unobservable untreated potential outcome

- impose assumptions to identify/estimate parameters of interest
- assumptions differ in identifying power and credibility

Assumptions for DID

Parallel trend

In the absence of the treatment, both the treated and the control groups would have experienced the same outcome evolution

No anticipatory effects

The treatment has no causal effect prior to its implementation

Stable unit treatment value assumption (SUTVA)

The potential outcomes for any unit do not vary with the treatments assigned to other units (no spillovers)

Two–way fixed effect specifications

Static:

$$y_{i,t} = \alpha_i + \delta_t + D_{i,t}\beta + \varepsilon_{i,t}$$

- β: overall treatment effects across groups and time periods

Dynamic:

$$y_{i,t} = \alpha_i + \delta_t + \sum_{r \neq 0} \mathbf{1}[R_{i,t} = r]\beta_r + \varepsilon_{i,t}$$

- $R_{i,t} = t - G_i + 1$: the time relative to the treatment
- G_i: the earliest period in which i is treated
- β_r: dynamic treatment effects after r periods of treatment
- β_{-r}: testing for pre-trend

Pitfalls of static TWFE estimator

- de Chaisemartin and D'Haultfœuille (2020, AER): TWFE estimator tends to assign negative weights to periods where a large fraction of groups are treated, and to groups treated for many periods

$$\widehat{\beta} = \frac{\sum_{i,t}(D_{i,t} - \widehat{D}_{i,t})y_{i,t}}{\sum_{i,t}(D_{i,t} - \widehat{D}_{i,t})^2}$$

- $\widehat{D}_{i,t} = \overline{D}_i + \overline{D}_t - \overline{D}$ ▸ Some intuition
  - $\overline{D}_i = \sum_t D_{i,t}/T$: time average of treatment status for i
  - $\overline{D}_t = \sum_i D_{i,t}/n$: cross sectional average at time t
  - $\overline{D} = \sum_{i,t} D_{i,t}/nT$: broad average

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- when $\tau_{i,t}(g) = \tau$: homogenous treatment effect
  - population regression coefficient β is τ

Negative weights and forbidden comparison

Negative weights imposes challenges on causal interpretation

- weight determined by OLS

OLS implicitly does three comparisons

- newly treated units to never-treated units
- newly treated units to not-yet-treated units
- forbidden comparison: newly treated units to already-treated units who do not change treatment status
  - involves treatment effects dynamics

More challenges of dynamic TWFE estimators

Proposition 3 in Sun and Abraham (2021):

- $\widehat{\beta}_r$ picks up cohort average treatment effects at other time periods
- $CATT_{g,l}$ (cohort ATT): average treatment effect l periods from the initial treatment for cohort g

$$CATT_{g,l} = \mathbb{E}[y_{i,g+l} - y^{\infty}_{i,g+l} | G_i = g]$$

- $y^{\infty}_{i,g+l}$: never-treated counterfactual
- poses challenges for using $\beta_{-r} = 0$ as pre-trend test
- eventstudyweights

Structure of the presentation

- Alternative estimators
  - Callaway and Sant'Anna (2021):
    - R package: did
  - Borusyak, Jaravel and Spiess (2023)
    - Stata command: did_imputation; event_plot
  - Sun and Abraham (2021)
    - R command: sunab in fixest
    - Stata command: eventstudyinteract

Common themes of new estimators

- Impose (conditional) parallel trend assumptions
  - condition on a rich enough set of covariates
- Presence/absence of never-treated groups
- User-specified weights

Callaway and Sant'Anna (2021)

Based on a "Never-Treated" group ($\forall g = 2 \ldots T, t = 2 \ldots T$ with $t \geq g$)

$$\mathbb{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|G_i = g] = \mathbb{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|C = 1]$$

- C: an indicator that denotes never-treated group
- without absence of treatment, both the group first treated at g and the never–treated group would have experienced the same outcome evolution in all post-treatment periods $t \geq g$
- **When can we impose?**
  - is the never-treated group sizeable?
  - is the never-treated group similar to eventually treated groups?

▸ Conditional version

Another parallel trend assumption

Based on "Not-Yet-Treated" groups ($\forall g = 2 \dots T$; $s, t = 2 \dots T$ with $g \leq t \leq s$)

$$\mathbb{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty) | G_i = g] = \mathbb{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty) | D_{i,s} = 0, G_i > g]$$

- conditional parallel trends between group g and groups that are "not-yet-treated" by time t
- use all not-yet-treated units as valid comparison groups
- seems to restrict pre-treatment trends across groups

▸ Conditional version

Callaway and Sant'Anna (2021): group-time ATT

- Average treatment effect at time t for the group first treated in time g

$$ATT(g, t) = \mathbb{E}[y_{i,t}(g) - y_{i,t}(\infty) \mid G_i = g]$$

Callaway and Sant'Anna (2021): group-time ATT

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$$ATT(g,t) = \mathbb{E}[y_{i,t}(g) - y_{i,t}(\infty) \mid G_i = g]$$

- Identification (Theorem 1)

$$ATT(g,t) = \mathbb{E}[y_{i,t} - y_{i,g-1} | G_i = g] - \mathbb{E}[y_{i,t} - y_{i,g-1} | G_i \in \mathcal{G}_{control}]$$

Two choices of $\mathcal{G}_{control}$

  - Never treated group $\mathcal{G}_{control} = \{\infty\}$
  - All not-yet treated groups $\mathcal{G}_{control} = \{g' : g' > t\}$

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- Estimation

$$\widehat{ATT}(g,t) = \frac{1}{N_g} \sum_{i:G_i=g} (y_{i,t} - y_{i,g-1}) - \frac{1}{N_{\mathcal{G}_{control}}} \sum_{i:G_i \in \mathcal{G}_{control}} (y_{i,t} - y_{i,g-1})$$

Aggregating group-time average treatment effects

General form

$$\theta = \sum_{g \in \mathcal{G}} \sum_{t=2}^{T} w(g, t) \cdot ATT(g, t)$$

- $w(g, t)$: researcher-specified weight
- Example: how do average treatment effects vary with length of exposure to the treatment?
  - average treatment effects e periods after the treatment adoption across all groups

$$\theta_{es}^{bal}(e) = \sum_{g \in \mathcal{G}} \mathbf{1}\{t \leq T\} Pr(G = g | G + e \leq T) ATT(g, g + e)$$

Borusyak, Jaravel and Spiess (2023)

Imputation estimator:

- using **untreated** observations, run the following specification

$$y_{i,t} = \alpha_i + \delta_t + x_{i,t}\beta + \varepsilon_{i,t}$$

- for each **treated** observation, impute but-for outcome (without treatment) using the coefficient estimates in the first stage and $x_{i,t}$ in the treated period

$$\widehat{y}_{it} = \widehat{\alpha}_i + \widehat{\delta}_t + x_{i,t}\widehat{\beta}$$

  compute $\widehat{\tau}^*_{it} = y_{it} - \widehat{y}_{it}$

- estimate the target τ_w using a weighted sum $\sum_{it\in\Omega_1} w_{it}\widehat{\tau}^*_{it}$

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Does it look familiar?

Detour: forecasting method of damage calculation

- In Re: Vitamins Antitrust Litigation (Bernheim, 2002)

  To estimate overcharges, I proceeded in three steps. First, I identified and collected data on factors ("explanatory variables") that would be expected to explain changes in vitamin prices. Second, I used generally accepted statistical tools (multivariate regression) to analyze the actual impact of these variables on vitamin prices during non-conspiratorial time periods. Third, I used these historical statistical relationships along with the observed values of the explanatory variables during the conspiracy period to predict the prices that would have prevailed but for the conspiracy.

- Determine X_t (not affected by the conduct)
- Run regression $y_t = \alpha + \beta X_t + \varepsilon_t$ using the pre-conduct period data
- Construct but-for price during the conduct period, using the X_t during the conduct period

$$\widetilde{y}_t^{D_t=1} = \widehat{\alpha} + \widehat{\beta} X_t$$

- Compute overcharges:

$$OC = \sum_{t \in \text{conduct period}} Q_t \times (y_t - \widetilde{y}_t^{D_t=1})$$

Comparison

BJS impose a stronger parallel trend assumption

- for all $t' \neq t$ and $g \neq g'$

$$\mathbb{E}[y_{i,t}(\infty) - y_{i,t'}(\infty) | G_i = g] = \mathbb{E}[y_{i,t}(\infty) - y_{i,t'}(\infty) | G_i = g']$$

- if the treatment had not occurred, the average outcomes for all adoption groups would have evolved in parallel

BJS use all pre-treatment observations for comparison

- CS use the last pre-treatment observation for comparison

Sun and Abraham (2021)

- application: economic consequence of hospitalizations
  - $D_{i,t}$: treatment of ever being hospitalized
  - G_i: Health and Retirement Survey (HRS) survey wave of initial hospitalization
  - 4 year panel
- $CATT_{g,l}$: average treatment effect l periods after a negative health shock for cohort g, as opposed to never being hospitalized

$$CATT_{g,l} = \mathbb{E}[y_{i,g+l} - y^{\infty}_{i,g+l} | G_i = g]$$

  - $y^{\infty}_{i,g+l}$: never-treated counterfactual
- in this application, no never-treated group, use last-treated group as control instead

Interaction weighted estimator

Economic consequences of hospitalization

- estimate $CATT_{g,l}$ using the following specification

$$y_{i,t} = \alpha_i + \delta_t + \sum_{g \in \{1,2,3\}} \sum_{l \neq -3, \neq -1}^{2} \lambda_{g,l} (\mathbf{1}\{G_i = g\} \cdot D_{i,t}^l) + \varepsilon_{i,t}$$

  - $\mathbf{1}\{G_i = g\}$: group indicator
  - $D_{i,t}^l$: an indicator for unit i being l periods away from the treatment at calendar time t
  - G: groups that are excluded from the regression (comparisons)
    - latest-treated group
  - drop $t = 4$ because everyone is treated, no control cohort

Interaction weighted estimator

- estimate the share of each group g across groups that experience at least l periods relative to hospitalization

$$Pr\{G_i = g | G_i \in [-l, T - l]\}$$

- take a weighted average of $\widehat{\lambda}_{g,l}$ with sample cohort shares (as weights)

Stay tuned: DID part four

de Chaisemartin and D'Haultfœuille (2023)

- non–binary treatment
- non absorbing treatment
- outcome may be affected by treatment lags

statistical inference

- clustered standard errors
- bootstrap

Explanation slides

- early-treated group gets a negative weight in its longer run treatment effect
- $\overline{D}_i \approx 1$: i has been treated most of the periods
- $\overline{D}_t \approx 1$: almost all i's are treated at t
- $\overline{D} < 1$: there is a fraction of non-treated at some time
- $D_{i,t} - \widehat{D}_{i,t} < 0$

◂ Back

A cond parallel trend assumption

Based on a "Never-Treated" group $\forall g = 2 \ldots T, t = 2 \ldots T$ with $t \geq g$)

$$\mathbb{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|X, G_i = g] = \mathbb{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|X, C = 1]$$

- conditional on X, the average outcomes for the group first treated in period g and for the "never-treated" group would have followed parallel paths in the absence of treatment
- intuitively, among units with $X = x$, parallel trend holds
- possible covariates: age, gender

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- intuitively, among units with $X = x$, parallel trend holds
- possible covariates: age, gender
- covariates cannot be affected by the treatment (Appendix A in Borusyak et al., 2023)
  - price-fixing cases (White, Marshall and Kennedy, 2005)
- including lagged outcomes in X?

◂ Back

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- conditional parallel trends between group g and groups that are "not-yet-treated" by time t

Which one should we impose?

- is the never-treated group is sizeable
- is the never-treated group similar to treated groups

◂ Back