Similarity and Dissimilarity Measures

Distance or Similarity Measures

- **Many data analytics tasks involve the comparison of objects in terms of their similarities (or dissimilarities)**
  - Clustering
  - Nearest-neighbor search, classification, and prediction
  - Characterization and discrimination
  - Automatic categorization
  - Correlation analysis
- **Many of todays real-world applications rely on the computation similarities or distances among objects**
  - Personalization
  - Recommender systems
  - Document categorization
  - Information retrieval
  - Target marketing

2

Similarity and Dissimilarity

- **Similarity**
  - Numerical measure of how alike two data objects are
  - Value is higher when objects are more alike
  - Often falls in the range [0,1]
- **Dissimilarity (e.g., distance)**
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- **Proximity can refer to a measure of similarity or dissimilarity**

3

Distance or Similarity Measures

- **Measuring Distance**
  - In order to group similar items, we need a way to measure the distance between objects (e.g., records)
  - Often requires the representation of objects as "feature vectors"

An Employee DB

ID	Gender	Age	Salary
1	F	27	19,000
2	M	51	64,000
3	M	52	100,000
4	F	33	55,000
5	M	45	45,000

Feature vector of Employee 2: <M, 51, 64000.0>

Term Frequencies for Documents

	T1	T2	T3	T4	T5	T6
Doc1	0	4	0	0	0	2
Doc2	3	1	4	3	1	2
Doc3	3	0	0	0	3	0
Doc4	0	1	0	3	0	0
Doc5	2	2	2	3	1	4

Feature vector for Document 4: <0, 1, 0, 3, 0, 0>

4

Distance or Similarity Measures

- **Properties of Distance Measures (IMPORTANT)**
  - for all objects A and B, dist(A, B) ≥ 0
  - dist(A, A) = 0
  - for all A, B, dist(A,B) = dist(B,A)
  - dist(A, C) ≤ dist(A, B) + dist (B, C)
- **Representation of objects as vectors:**
  - Each data object (item) can be viewed as an n-dimensional vector, where the dimensions are the attributes (features) in the data
  - Example (employee DB): Emp. ID 2 = <M, 51, 64000>
  - Example (Documents): DOC2 = <3, 1, 4, 3, 1, 2>
  - The vector representation allows us to compute distance or similarity between pairs of items using standard vector operations, e.g.,
    - Cosine of the angle between vectors
    - Manhattan distance
    - Euclidean distance
    - Hamming Distance

5

Data Matrix and Distance Matrix

- **Data matrix**
  - Conceptual representation of a table
    - Cols = features; rows = data objects
  - *n* data points with *p* dimensions
  - Each row in the matrix is the vector representation of a data object

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

- **Distance (or Similarity) Matrix**
  - *n* data points, but indicates only the pairwise distance (or similarity)
  - A triangular matrix
  - Symmetric

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \ldots & \ldots & 0 \end{bmatrix}$$

6

Proximity Measure for Nominal Attributes

- **If object attributes are all nominal (categorical), then proximity measure are used to compare objects**
- **Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)**
- **Method 1: Simple matching**
  - m: # of matches, p: total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- **Method 2: Convert to Standard Spreadsheet format**
  - For each attribute A create M binary attribute for the M nominal states of A
  - Then use standard vector-based similarity or distance metrics

Proximity Measure for Binary Attributes

- **A contingency table for binary data**

		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- **Distance measure for symmetric binary variables**

$$d(i, j) = \frac{r + s}{q + r + s + t}$$

- **Distance measure for asymmetric binary variables**

$$d(i, j) = \frac{r + s}{q + r + s}$$

- **Jaccard coefficient (similarity measure for asymmetric binary variables)**

$$sim_{Jaccard}(i, j) = \frac{q}{q + r + s}$$

Normalizing or Standardizing Numeric Data

- **Z-score:**
  - x: raw value to be standardized, μ: mean of the population, σ: standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the value is below the mean, "+" when above

$$z = \frac{x - \mu}{\sigma}$$

- **Min-Max Normalization**

$$x'_i = \frac{x_i - \min x_i}{\max x_i - \min x_i}(new\max - new\min) + new\min$$

ID	Gender	Age	Salary
1	F	27	19,000
2	M	51	64,000
3	M	52	100,000
4	F	33	55,000
5	M	45	45,000

ID	Gender	Age	Salary
1	1	0.00	0.00
2	0	0.96	0.56
3	0	1.00	1.00
4	1	0.24	0.44
5	0	0.72	0.32

Common Distance Measures for Numeric Data

- **Consider two vectors**
  - Rows in the data matrix

$$X = \langle x_1, x_2, \cdots, x_n \rangle \quad Y = \langle y_1, y_2, \cdots, y_n \rangle$$

- **Common Distance Measures:**
  - Manhattan distance:

$$dist(X,Y) = |x_1 - y_1| + |x_2 - y_2| + \cdots + |x_n - y_n|$$

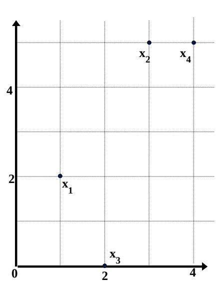
  - Euclidean distance:

$$dist(X,Y) = \sqrt{(x_1 - y_1)^2 + \cdots + (x_n - y_n)^2}$$

  - Distance can be defined as a dual of a similarity measure

$$dist(X,Y) = 1 - sim(X,Y) \qquad sim(X,Y) = \frac{\sum_i (x_i \times y_i)}{\sqrt{\sum_i x_i^2 \times \sum_i y_i^2}}$$

Example: Data Matrix and Distance Matrix



Data Matrix

point	attribute1	attribute2
x1	1	2
x2	3	5
x3	2	0
x4	4	5

Distance Matrix (Manhattan)

	x1	*x2*	*x3*	*x4*
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Distance Matrix (Euclidean)

	x1	*x2*	*x3*	*x4*
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Distance on Numeric Data: Minkowski Distance

- **Minkowski distance: A popular distance measure**

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

  - where $i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ are two p-dimensional data objects, and h is the order (the distance so defined is also called L-h norm)
- **Note that Euclidean and Manhattan distances are special cases**
  - $h = 1$: (L_1 norm) Manhattan distance

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + \ldots + |x_{i_p} - x_{j_p}|$$

  - $h = 2$: (L_2 norm) Euclidean distance

$$d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + \ldots + |x_{i_p} - x_{j_p}|^2)}$$

Vector-Based Similarity Measures

- **In some situations, distance measures provide a skewed view of data**
  - E.g., when the data is very sparse and 0's in the vectors are not significant
  - In such cases, typically vector-based similarity measures are used
  - Most common measure: Cosine similarity

$$X = \langle x_1, x_2, \ldots, x_n \rangle \quad Y = \langle y_1, y_2, \ldots, y_n \rangle$$

  - Dot product of two vectors: $sim(X,Y) = X \bullet Y = \sum_i x_i \times y_i$
  - Cosine Similarity = normalized dot product
  - the norm of a vector X is: $\|X\| = \sqrt{\sum_i x_i^2}$
  - the cosine similarity is: $sim(X,Y) = \dfrac{X \bullet Y}{\|X\| \times \|y\|} = \dfrac{\sum_i (x_i \times y_i)}{\sqrt{\sum_i x_i^2} \times \sqrt{\sum_i y_i^2}}$

Vector-Based Similarity Measures

- **Why divide by the norm?**

$$X = \langle x_1, x_2, \ldots, x_n \rangle \quad \|X\| = \sqrt{\sum_i x_i^2}$$

  - Example:
    - X = <2, 0, 3, 2, 1, 4>
    - $\|X\|$ = SQRT(4+0+9+4+1+16) = 5.83
    - X^* = X / $\|X\|$ = <0.343, 0, 0.514, 0.343, 0.171, 0.686>
  - Now, note that $\|X^*\|$ = 1
  - So, dividing a vector by its norm, turns it into a *unit-length* vector
  - Cosine similarity measures the angle between two unit length vectors (i.e., the magnitude of the vectors are ignored).

Example Application: Information Retrieval

- **Documents are represented as "bags of words"**
- **Represented as vectors when used computationally**
  - A vector is an array of floating point (or binary in case of bit maps)
  - Has direction and magnitude
  - Each vector has a place for every term in collection (most are sparse)

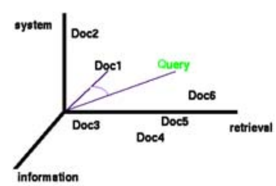
Document Ids

	nova	galaxy	heat	actor	film	role
A	1.0	0.5	0.3			
B	0.5	1.0				
C		1.0	0.8	0.7		
D		0.9	1.0	0.5		
E				1.0		1.0
F					0.7	
G	0.5		0.7			0.9
H		0.6		1.0	0.3	0.2
I			0.7	0.5		0.3

a document vector

$$D_i = w_{d_{i1}}, w_{d_{i2}}, \ldots, w_{d_{it}}$$
$$Q = w_{q1}, w_{q2}, \ldots, w_{qt}$$
$$w = 0 \text{ if a term is absent}$$

Documents & Query in n-dimensional Space



- **Documents are represented as vectors in the term space**
  - Typically values in each dimension correspond to the frequency of the corresponding term in the document
- **Queries represented as vectors in the same vector-space**
- **Cosine similarity between the query and documents is often used to rank retrieved documents**

Example: Similarities among Documents

- **Consider the following document-term matrix**

	T1	T2	T3	T4	T5	T6	T7	T8
Doc1	0	4	0	0	0	2	1	3
Doc2	3	1	4	3	1	2	0	1
Doc3	3	0	0	0	3	0	3	0
Doc4	0	1	0	3	0	0	2	0
Doc5	2	2	2	3	1	4	0	2

Dot-Product(Doc2,Doc4) = <3,1,4,3,1,2,0,1> * <0,1,0,3,0,0,2,0>
0 + 1 + 0 + 9 + 0 + 0 + 0 + 0 = 10

Norm (Doc2) = SQRT(9+1+16+9+1+4+0+1) = 6.4
Norm (Doc4) = SQRT(0+1+0+9+0+0+4+0) = 3.74

Cosine(Doc2, Doc4) = 10 / (6.4 * 3.74) = 0.42

Correlation as Similarity

- **In cases where there could be high mean variance across data objects (e.g., movie ratings), Pearson Correlation coefficient is the best option**
- **Pearson Correlation**

$$corr(x,y) = \frac{\mathrm{cov}(x,y)}{stdev(x) \cdot stdev(y)}$$

- **Often used in recommender systems based on Collaborative Filtering**