SOLUTIONS

MATH 358/658 Assignment 7
Due March 26.

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(4) • Open 'CI Script.R'. You will modify this R script to answer the following two questions.

1. Simulate 1000 samples of size $n = 20$ from a $N(\mu = 10, \sigma = 1)$ density, and for each compute the 95% confidence interval for μ. How many of your samples produce a confidence interval which contains the true μ?
2. Now make a one-sided 90% confidence interval for μ as $(-\infty, c)$ for 1000 simulated samples of size $n = 20$ from a $N(\mu = 10, \sigma = 1)$ density. How many of these contain the true value $\mu = 10$?

Obviously, you will need to modify the existing script to handle all changes related to n, σ, and the number of simulations. When your script functions correctly, save it and e-mail it to me (erhardrj@wfu.edu) so I can verify you've done the problem correctly. An excellent way to verify your script is correct is to save it, close R, re-open R, and simply highlight the entire script and press F5. It should run without error.

HW7 Solutions

- p. 484 #6

Find c such that $P(\bar{X} \le \mu + c \cdot \sigma') = 0.95$

$$P\left(\frac{\bar{X} - \mu}{\sigma'/\sqrt{n}} \le \sqrt{n} \cdot c\right) = 0.95$$

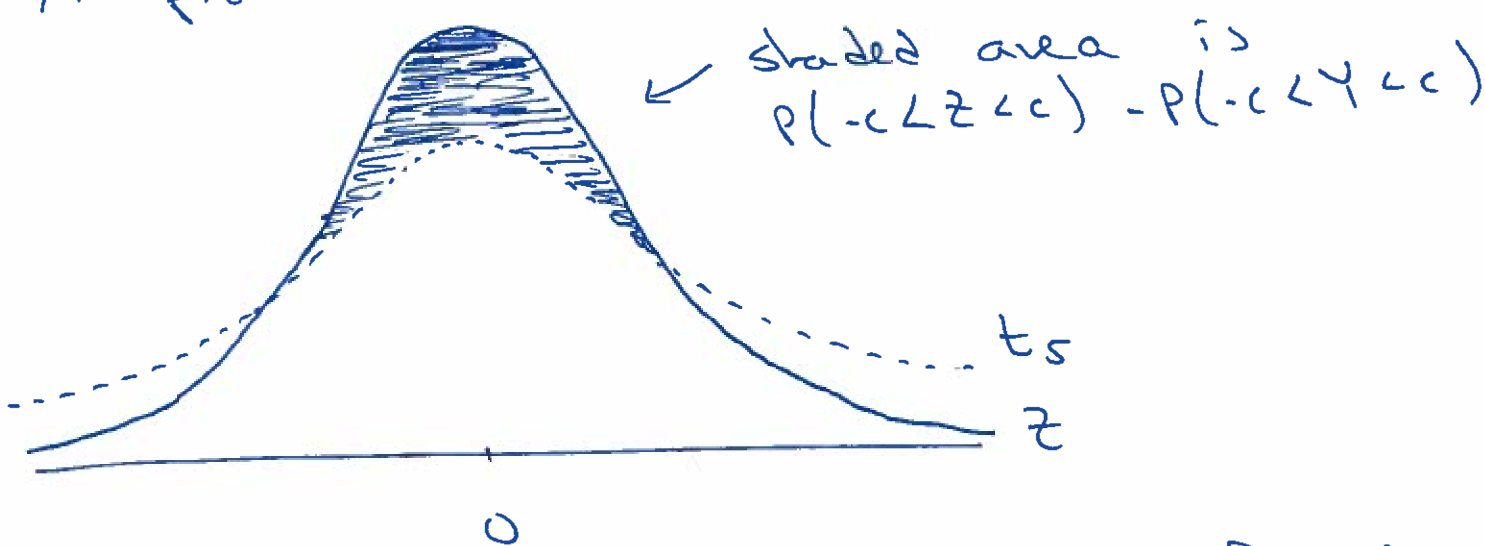
$$P(T \le \sqrt{n} \cdot c) = 0.95$$

$$\Rightarrow \sqrt{n} \cdot c = t_{19, 0.95} = 1.729$$

$$\Rightarrow c = \frac{1.729}{\sqrt{20}} = \boxed{0.3866}$$

- p. 484 #8

A picture is worth 1000 words:



maximized over region where $f_Z(x) \ge f_T(x)$,

which is $(-c, +c) = \boxed{(-1.63, +1.63)}$

p. 494 #2 $\bar{X} = 3.0625, \quad \sigma' = .5125$

(a) $\bar{X} \pm t_{7,.95} \frac{\sigma'}{\sqrt{n}} = \boxed{(2.719, 3.406)}$

(b) $\bar{X} \pm t_{7,.975} \frac{\sigma'}{\sqrt{n}} = \boxed{(2.634, 3.491)}$

(c) $\bar{X} \pm t_{7,.995} \frac{\sigma'}{\sqrt{n}} = \boxed{(2.428, 3.697)}$

#4 $X_1, \ldots, X_n \sim N(\mu, \sigma^2 \text{ (known)})$

95% CI is $\bar{X} \pm z_{.975} \cdot \frac{\sigma}{\sqrt{n}}$

$\text{Length} = \left(\bar{X} + z_{.975} \frac{\sigma}{\sqrt{n}}\right) - \left(\bar{X} - z_{.975} \frac{\sigma}{\sqrt{n}}\right)$

$= 2 \cdot z_{.975} \frac{\cancel{\sigma}}{\sqrt{n}} \leq 0.01 \cancel{\sigma}$

$\Rightarrow$ need $n > \left(\frac{2 \cdot 1.96}{.01}\right)^2 = \boxed{153{,}665}$

#7 $\bar{X} = 156.85, \quad \sigma' = 22.642, \quad n = 20$

90% CI is $\bar{X} \pm t_{19,.95} \cdot \frac{\sigma'}{\sqrt{n}}$

$= \boxed{(148.1, 165.6)}$

[signature]