CSC/MTH 753 Project 1 (Individual Project)

Hard copy due by Tuesday February 19

- Do your own work, individually, without help from others, **or me on problems in the project**. You can use any book or web resources. If you have questions about any of the problems or what is expected of you in solving the problems, ask them to me in class, so **everyone** can hear my replies. This is the fair way to proceed.
- Provide any Matlab codes that you write, and names of the algorithms your codes use. Display your output. Use **optimtool** if you like, but display your iterations.
- Except for the two students taking the class remotely, give me **hard copy rather than using email**. With a very large class like ours, that approach will simplify the process.
- Turn in a **concise, short, neat, stapled paper, with answers clearly identified**.

1. Consider the function

$$f(x) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

that we discussed in class.

(a) Find **all the local minimizers** x_* of $f(x)$. You might start by examining the graphics of $f(x)$ in order to identify where the local minimizers lie in the plane, see your homework paper on Matlab graphics.

(b) A variety of methods can be used to determine the local minimizers - it's your choice. For example you can use **fminunc** with appropriate starting values determined from your graphics. You could also use **fsolve** to solve the gradient equations set to zero. See also **lsqnonlin** which can be used to minimize a sum of squares on nonlinear functions. Use the doc command to see how to use these m-functions.

(c) Finally, show the Hessian is positive definite at the minimizers you determine. Is there a global minimizer?

2. Consider the following function of 4 variables:

$$f(x) = 100(x_1^2-x_2)^2+(1-x_1)^2+90(x_3^2-x_4)^2+(1-x_3)^2+10.1((1-x_2)^2+(1-x_4)^2)+19.8(1-x_2)(1-x_4)$$

(a) Determine the gradient and Hessian for f.

(b) Determine whether or not f is a convex function. Review Chapters 2 and 3 in the online reference text: "Convex Optimization", Stephen Boyd and Lieven Vandenberghe.

(c) Show that $x_* = (1, 1, 1, 1)$ is a local minimizer of f and run **fminunc** from Matlab to approximate x_*, with $x_0 = (-3, -1, -3, -1)$, and interpret your results. Also, try $x_0 = (0, 0, 0, 0)$, and perhaps other x_0 to test convergence.

3. (Short question). Show that the function $f(x) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$ has only one critical point where the gradient is the zero vector, but that point is neither a local minimizer or maximizer (called a saddle point).