Multiplexing

CSC 343·643

Wake Forest University
Department of Computer Science

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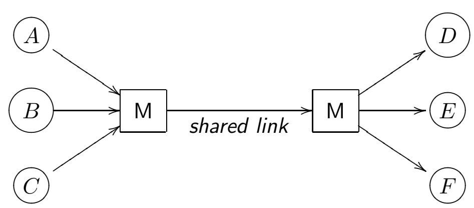
Multiplexing

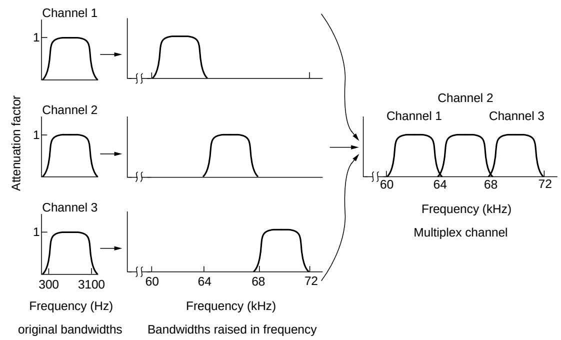
- Often two machines can not utilize the full capacity of a link
- Need to share the channel among multiple users (multiplexing)



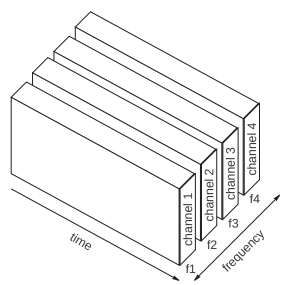
- Types of multiplexing
  - Frequency-division multiplexing
  - Time-division multiplexing
  - Code-division multiplexing
  - Statistical time-division multiplexing

Frequency Division Multiplexing

- Given a medium with bandwidth larger than a single signal
- Divide bandwidth to allow multiple carrier signals
  - Carrier frequencies may *not* overlap (analog signaling)
  - Signal raised in frequency to particular carrier



  - Multiple signals can be sent **simultaneously**



- Telephone system
  - Consider backbone links carrying multiple connections
  - Twelve 4 kHz channels multiplexed together
  - Multiplexed into the 60 to 108 kHz band, called a **group**

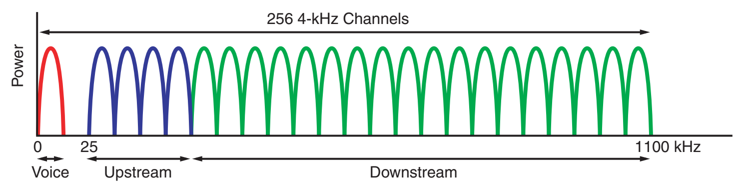
What are other examples of FDM?

Digital Subscriber Line

Using the existing telephony network for high-speed data and voice

- Local loop twisted pair has a 1.1 MHz bandwidth potential
  - However only a fraction is actually used
  - Voice requires 300 Hz to 3400 Hz
  - Remove (filter) anything below 300 Hz and above 3400 Hz
- Asymmetric Digital Subscriber Line (ASDL) has larger bandwidth
  - Asymmetric since higher bit rates downstream than upstream
  - Line is not filtered, 1.1 MHz is available

- Divides 1.1 MHz bandwidth into 256 channels (three bands)
  1. 0 to 4 kHz telephone service (POTS) (1 channel)
  2. 25 kHz to 200 kHz upstream (5 channels)
  3. 250 kHz to 1000 kHz for downstream (remaining channels)

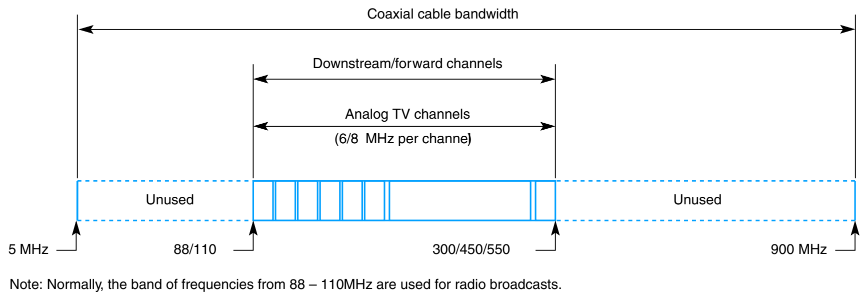


- Each channel modulation is similar to V.34
  - QAM used, 15 bits/baud, 4000 baud
  - Actual rate also depends on line length and quality

What is the bit rate if 224 channels are available?

Cable TV Networks

- Uses FDM to transmit multiple channels concurrently
  - Each channel is allocated 6 Mhz (8 MHz in Europe)

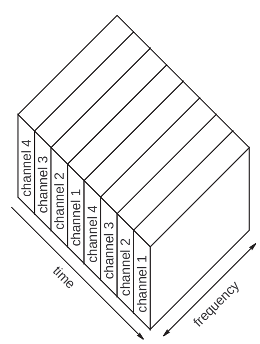


- Upstream is 5 - 42 MHz and downstream is above 550 Hz
  - Use QAM-64 per channel

What is the maximum data rate per channel?

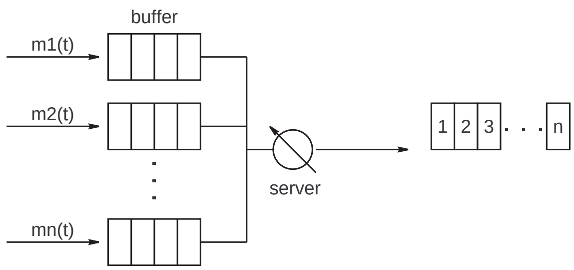
Time Division Multiplexing

- FDM requires analog circuits (not amenable to computers)
- Time Division Multiplexing (TDM) is used by digital electronics
  - However, *local loops* are analog
- TDM works by interleaving multiple signals over time



Generic TDM Description

- A n signals $m_i(t), i = 1...n$ are to be multiplexed
- Input data may be *briefly* buffered
- Buffers scanned to produce a composite digital stream



- Data rate of the composite stream

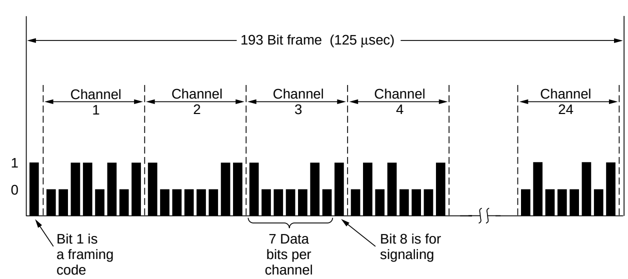
$$r_c \geq \sum_{i=1}^{n} r_i$$

Digital Carrier Systems

- TDM is used in telephone networks, voice is 4 kHz
- 8000 PCM samples are taken per second at 8 bits/sample therefore a voice channel is 64 kbps

  What is the time per sample?

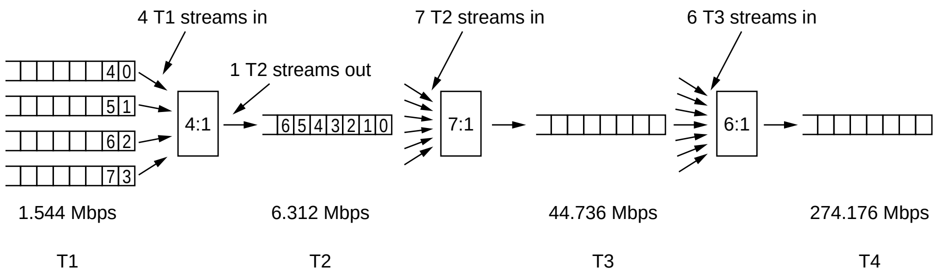
- 24 voice channels are multiplexed together (T1)



- T1 frame consists of $24 \times 8 = 192$ bits plus 1 extra bit for framing, total of 193 bits every 125 μsec

  What is the bit rate of a T1?

- If a T1 is used for data (non-voice), then only 23 channels are used, 24th is for synchronization
- T1 can be multiplexed into higher carriers



TDM Link Control

The T1 description does not have headers, trailers, CRCs, etc...

- Control mechanisms (data-link) are not needed
- Flow control
  - Multiplexed data rate is fixed
  - Multiplexer and demultiplexer operate at same rate
  - Therefore flow control is not needed

    What if a "user" transmits more or less than subscribed?

- Error control
  - Why retransmit an entire T1 frame?
- Flow and error control are handled on a per channel basis

Code Division Multiplexing

- Code Division Multiplexing (CDM) is a form of spread spectrum
  - A narrow band signal is spread over a larger frequency band
  - Users can share simultaneously, and more tolerant of interference
  - Also called Code Division Multiple Access (CDMA)
- Lounge analogy, assume multiple conversations between paris of people
  - TDM - pairs take turns talking, only one pair at a time
  - FDM - pairs use different pitches, multiple pairs at once
  - CDM - pairs use different languages, multiple pairs at once
- Given CDM uses different "codes" per conversation, important that the codes can be separated once received

CDM Chips

- Each bit time is divided into m short intervals called **chips**
  - Typically 64 or 128 chips per bit (*let's use 8 as an example*)
- Each station is assigned a unique m-chip code called a chip squence
  - Assume a bipolar encoding is used for the sequences (−1 and +1)
- Consider the chip sequence (−1 −1 −1 +1 +1 −1 +1 +1)
  - To transmit a 1, then transmit the sequence, to transmit a 0 then send the inverted sequence

Example CDM Sequences

- Assume 4 stations A, B, C, and D with the following sequences

A :	(-1 -1 -1 +1 +1 -1 +1 +1)
B :	(-1 -1 +1 -1 +1 +1 +1 -1)
C :	(-1 +1 -1 +1 +1 +1 -1 -1)
D :	(-1 +1 -1 -1 -1 -1 +1 -1)

- All the chip sequences are pairwise orthogonal (*a requirement*)
  - Normalized inner product of any 2 sequences S and T is zero

$$S \cdot T = \frac{1}{m} \sum_{i=1}^{m} S_i T_i = 0$$

  - For example,

$$\begin{aligned} A \cdot C \quad = \quad & (-1 \cdot -1) + (-1 \cdot +1) + (-1 \cdot -1) + (+1 \cdot +1) \\ & +(+1 \cdot +1) + (-1 \cdot +1) + (+1 \cdot -1) + (+1 \cdot -1) \\ = 0 & \end{aligned}$$

  - Sequences with this property can be found using Walsh codes
- A few interesting properties
  - $S \cdot \overline{T} = 0$
  - $S \cdot S = 1$
  - $S \cdot \overline{S} = -1$

CDM Transmission Example

- Assume all stations are synchronized in time
  - Each can send a 1 (transmitting their sequence), a 0 (negated sequence), or nothing
- If two or more stations transmit simultaneously, then add the signals
  - Assume the chip sequences for A, B, C, and D

Who transmits	**Resulting signal S_i would be**
C	$S_1 =$(-1 +1 -1 +1 +1 +1 -1 -1)
$B + C$	$S_2 =$(-2 0 0 0 +2 +2 0 -2)
$A + \overline{B}$	$S_3 =$(0 0 -2 +2 0 -2 0 +2)
$A + \overline{B} + C$	$S_4 =$(-1 +1 -3 +3 +1 -1 -1 +1)
$A + B + C + D$	$S_5 =$(-4 0 -2 0 +2 0 +2 -2)
$A + B + \overline{C} + D$	$S_6 =$(-2 -2 0 -2 0 -2 +4 0)

  - Remember this is only transmitting a single bit

Recovering the Bit from C

- Receiving station must know the sequence of the source
  - Computer the normalized inner product of the received signal and the source's chip sequence
- Assume you want to recover C (*transmitted a 1*)

Who transmits	**Resulting signal would be**	$C \cdot S_i$
C	(-1 +1 -1 +1 +1 +1 -1 -1)	$(1 + 1 - 1 + 1 + 1 + 1 - 1 - 1)/8 = 1$
$B + C$	(-2 0 0 0 +2 +2 0 -2)	$(2 + 0 + 0 + 0 + 2 + 2 + 0 + 2)/8 = 1$
$A + \overline{B} + C$	(-1 +1 -3 +3 +1 -1 -1 +1)	$(1 + 1 + 3 + 3 + 1 - 1 + 1 - 1)/8 = 1$
$A + B + C + D$	(-4 0 -2 0 +2 0 +2 -2)	$(4 + 0 + 2 + 0 + 3 + 0 - 2 + 2)/8 = 1$

- Note, an important assumption is synchronization of senders
  - Not possible for all types of applications (mobile phones)

Mobile Phones, a brief (one slide, US-oriented) history

- First Generation (1G), analog communications
  - Advanced Mobile Phone System (AMPS) divides geography into cells, each with a set of frequencies
  - Phones used Frequency Division Duplex (FDD) to share bandwidth
- Second Generation (2G), digital communications (voice and slow data)
  - Digital AMPS (DAMPS) uses TDM to host multiple phones within a frequency, popular in the US, but most have moved to GSM
  - Global System for Mobile communications (GSM) uses FDM and TDM like DAMPS, is the most widely used
  - CDMA was introduced, but became widely accepted in 3G
- Third Generation (3G), digital voice and data
  - Wideband CDMA (WCDMA) and CDMA2000 use code division multiplexing

Multiplexing Analogy

3-lane highway used by employees of 3 companies (IBM, Cisco, EMC)

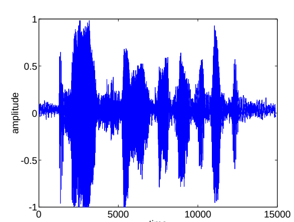
- FDM
  - Assign a lane to each company
  - Employees must use lane assigned to their company
  - Can use assigned lane at any time
- TDM
  - Assign specific times for highway use
    1. IBM employees 5:00am - 6:00am
    2. Cisco employees 6:01am - 7:00am
    3. EMC employees 7:01am - 8:00am
  - Employees use entire highway, but only during assigned time

Any problems with FDM or TDM?

- STDM
  - Any employee can use any lane at any time
  - Works only if there is no *peak* time, based on averages
  - If all employees use highway at 8:00am ...

Statistical Time Division Multiplexing

- Computer traffic (data) is often bursty, not a constant bit stream
  - On/off oriented (consider telephone conversations)

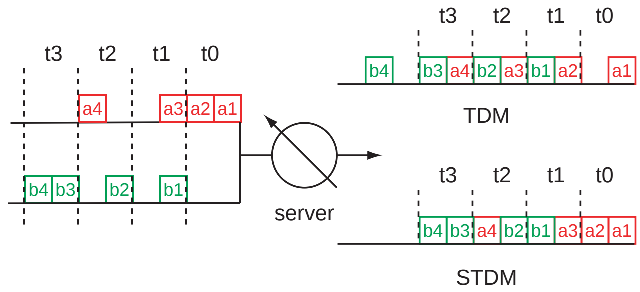


  - *Using TDM, if a station is idle transmit what?*
- Statistical TDM exploits the fact that digital data is often bursty
  - Dynamically allocate time-slots (channels) on demand

What is the disadvantage?

Generic STDM Description

- Slots are *not* assigned to a particular source



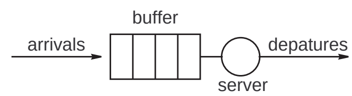
- Assume arriving frames join a *queue*
  - Frames will be buffered until transmitted
  - Addresses are required ⇒ more overhead

  Why are addresses required?

STDM Performance

- Sum of the *average* input rates is less than the multiplexed line
  - However, the sum of the input *peaks* is greater
  - If all inputs transmit at the peak...
- Typically buffering is done on the input side
  - This temporarily stores the data until it is served
  - *Smoothes* the data stream by removing burstiness
  - Buffering does add possible transmission delays
- Analysis is based on *queueing theory*
  - Interested in the average *packet* delay
  - Delay is based on the arrivals and the server

A Simple Model



- Packets arrive at the system according to *arrival pattern*
  - Packets can be from one or multiple sources (multiplexer)
- If the buffer is finite and full, the packet is dropped
- Packets are serviced in a FIFO order
- We are interested in a relationship between the following
  - $N(t)$ the number of packets in the system at time t
  - $\alpha(t)$ the number of arrivals in interval 0 to t
  - T_i the time spent in the system by packet i

Little's Theorem

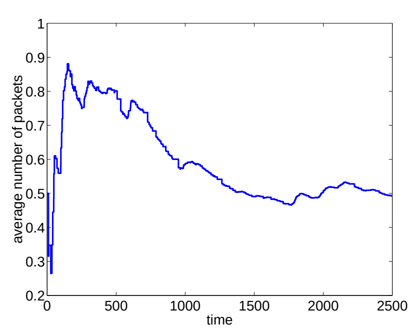
- An intuitive notion of the *typical* number of packets in the system

$$N_t = \frac{1}{t}\int_0^t N(\tau)d\tau$$

  the time average of $N(\tau)$ up to time t

- N_t will change over time, but it will approach a steady state

$$N = \lim_{t\to\infty} N_t$$



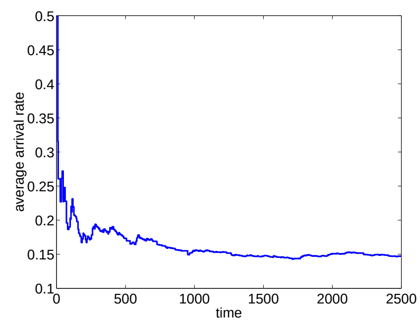
- Similarly, we can make the same claim about the arrivals

$$\lambda_t = \frac{\alpha(t)}{t}$$

the average arrival rate up to time t

- The steady state arrival rate is

$$\lambda = \lim_{t \to \infty} \lambda_t$$



- The time average of the packet delay up to time t is

$$T_t = \frac{\sum_{i=1}^{\alpha(t)} T_i}{\alpha(t)}$$

the average time spent in the system per packet up to time t

- The steady state time average packet delay is

$$T = \lim_{t \to \infty} T_t$$

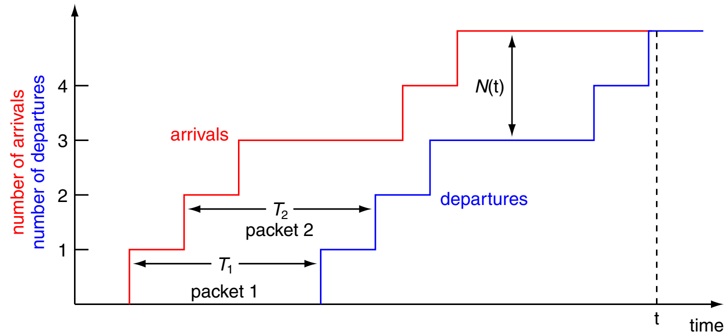
- These quantities (N, λ, and T) are related by a simple formula, also known as **Little's Theorem**

$$\boxed{N = \lambda T}$$

  – It expresses the *natural* idea that crowded systems (large N) are associated with long delays (large T)
  – *Valid for different types of queueing systems...*

Graphical Proof

- Little's Theorem is really an accounting identity



- Suppose the system is initially empty $N(0) = 0$
- Let $\alpha(t)$ be the number of arrivals and $\beta(t)$ be the number of departures at time t
  - The number in the system at time t is $N(t) = \alpha(t) - \beta(t)$

- The area between the graphs of $\alpha(t)$ and $\beta(t)$ is

$$\int_0^t N(\tau) \tag{1}$$

- Let t be any time at which the system is empty $N(t) = 0$, then the **same area** is equal to

$$\sum_{i=1}^{\alpha(t)} T_i \tag{2}$$

- Setting equation (1) equal to equation (2) and dividing by t

$$\frac{1}{t}\int_0^t N(\tau) = \frac{1}{t}\sum_{i=1}^{\alpha(t)} T_i = \frac{\alpha(t)}{t}\frac{\sum_{i=1}^{\alpha(t)} T_i}{\alpha(t)}$$

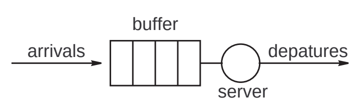
- This equals

$$N_t = \lambda_t T_t$$

Applications

- The significance of Little's Theorem is its generality
  - Holds for almost any queueing system that reaches steady state
  - However, it may require information we don't know
- Given the statistics about the arrivals and departures, we can derive the wait time as well as the queue length
  - *Given a certain arrival and departure process, we want the average delay...*

M/M/1 Queue



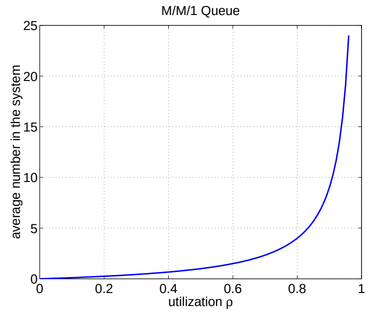
- Consider a single buffer and server, assume
  - Interarrival times are exponential with rate $\lambda > 0$
  - Processing times are exponential with rate $\mu > 0$
  - Buffer space is infinite
- Average delay is

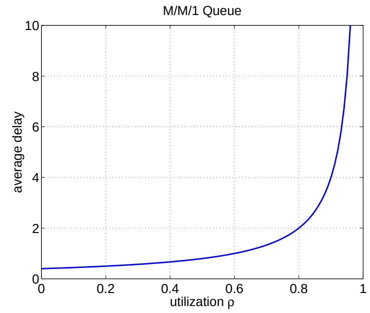
$$T = \frac{1}{\mu - \lambda}$$

- Can determine the delay given the average arrival and processing

M/M/1 Performance

- As the utilization approaches 1, performance decreases (unstable)
- System is stable when $\rho < 1$ (underload)



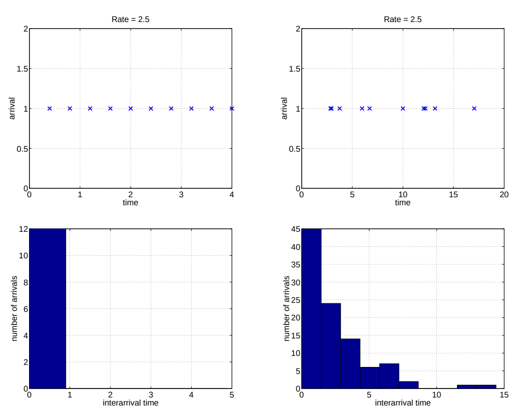


Why does queueing increase dramatically when $\lambda = \mu$?

Why does queueing exist when $\lambda < \mu$ (underload) condition?

Average Rate

- The *arrival pattern* of packets (customers) is important
  - We need to **also describe the variation**



So what? Request CSC 790 High-Speed Networks and find out more...

Back to TDM and STDM Performance

Results for the M/M/1 queue can be applied multiplexing systems

- TDM consists of dividing the capacity into n channels
  - Each channel has rate $\frac{r}{n}$
  - Each stream can be modeled as an M/M/1 queue with
    * Arrival rate λ
    * Transmission rate μ
  - Using Little's Theorem, delay per packet is

$$T_{TDM} = \frac{1}{\mu - \lambda}$$

- STDM buffers packets from n streams
  - Assume multiplexed in one FIFO buffer
  - Total arrival rate is $n \cdot \lambda$
  - System is an M/M/1 queue with transmission rate $n \cdot \mu$
  - Average delay is

$$T_{STDM} = \frac{1}{n \cdot \mu - n \cdot \lambda} = \frac{T_{TDM}}{n}$$

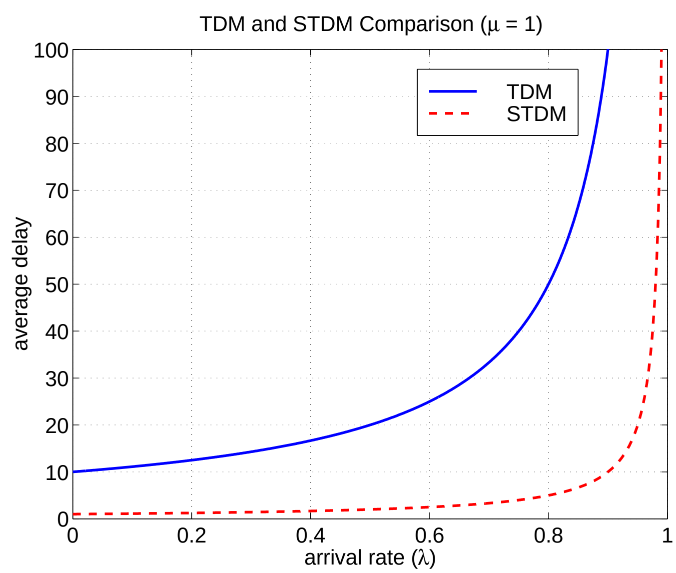
therefore the STDM system delay is a fraction ($\frac{1}{n}$) of TDM

- Example, channel with 128kbps data rate
  - 10 users that transmit packets with independent exponentially distributed lengths of 1000 bits.
  - Each packet is transmitted according to a Poisson process with average rate 10 packets per second.
  - TDM, each user gets a 12.8kbps channel

$$T_{TDM} = \frac{1}{12.8 - 10} = 360\text{msec}$$

  - STDM all users share the link

$$T_{STDM} = \frac{T_{TDM}}{10} = 36\text{msec}$$



Multiplexing Results

- Preceding example indicates
  - TDM (separate channel) multiplexing results in poor delay
  - Performance is even worse if channels are not assigned proportionally to arrival rate
  - Arrival rates tend to change over time...
- However, models presented here rely on assumptions that may not be applicable to actual traffic
- TDM/FDM is still used in telephony since voice traffic *tends* to be *regular* rather than Poisson