CS 371/671 Mid-term Study Guide

To be discussed in class on November 13

1 First Order Logic Representation

The goal of this section is to ensure that you understand how to convert knowledge (or statements in English) into formal FOL representation.

1. Represent each of these sentences in first-order predicate calculus (FOPC)

   - a. Mary passed every college class she took.
     $\forall c[CollegeClass(c) \wedge Took(Mary, c) \rightarrow Passed(Mary, c)]$
   - b. There is a book in Wendt Library written by a Canadian.
     $\exists b, y[Book(b) \wedge In(b, WedntLibrary) \wedge Wrote(y, b) \wedge Canadian(y)]$
   - c. John has not read every book in Wendt Library but someone has.
     $\exists b[Book(b) \wedge In(b, WendtLibrary) \wedge \neg HasRead(John, b)] \wedge \exists y \forall x[Person(y) \wedge (Book(x) \wedge In(x, WendtLibrary) \rightarrow HasRead(y, x))]$
   - d. Sue's neighbor is Fred's brother's wife's friend.
     $\exists x, y, z[Neighbor(Sue, x) \wedge Friend(x, y) \wedge Wife(y, z) \wedge Brother(z, Fred)]$

2. Provide a formal interpretation that shows that the following translation from English to FOL is incorrect. Be sure to explain your answer formally using your interpretation:

"Every cat is owned by David"

$$\forall x[cat(x) \wedge owner(x, David)]$$

Ans: In order to prove that this translation is incorrect, we need to describe a world in which the English sentence holds, but the logic sentence does not. If we can describe even one such world, then the two cannot be logically equivalent.

Consider a world where there are only three objects, *Felix*, *Rover* and *David*. In English, we'll say that Felix is a cat, Rover is a dog, and David is a human who owns both pets. In logic, here are the formulas that describe our world:

$$cat(Felix) = True \; owner(Felix, David) = True \; cat(Rover) = False \; owner(Rover, David) = True$$
$$cat(David) = False \; owner(David, David) = False$$

This world clearly satisfies the English sentence, because there is only one cat in the world and it is owned by David. Now let us turn to the logic sentence. Since the logic sentence contains a universal quantifier, let us consider all possible bindings of variables in the logic sentence with objects in our world. Thus, the

Binding	WFF	Result
$\{x/Felix\}$	$cat\,(Felix) \wedge owner\,(Felix, David)$	$True$
$\{x/Rover\}$	$cat\,(Rover) \wedge owner\,(Rover, David)$	$False$
$\{x/David\}$	$cat\,(David) \wedge owner\,(David, David)$	$False$

logic translation does not hold, and since the original English sentence does hold, we can conclude that the translation is incorrect.

3. For each pair of FOPC wff's below, state its most-general unifier (mgu) or say none exists. Show your work. Universally quantified variables are indicated by ?s

   - a. P(?x,3) P(1,2,3)
     Since the two P functions do not have the same arity (number of arguments), there is no way to make the two sides match. Therefore, no mgu exists.
   - b. P(?x, ?y, ?z) P(1, f(g(b, a)), f(g(a, b))
     The mgu is $\{x/1; y/f(g(b,a)), z/f(g(a,b))\}$
   - c. P(?x, f(3), ?x) P(g(1,2), ?z, g(?z, 2))
     No Mgu exists. You could see quickly that the subformulas $g(1,2)$ and g(f(3),2) contain no variables to assign and do not match
   - d. P(?x) Q(?x)
     P and Q are function names and not variables, and they do not match, so we conclude that no mgu exists.
   - e. P(?x, ?y, ?x) P(f(?a, ?b), ?b, f(?b, ?b))
     The mgu is $\{x/f(a,b), y/b, a/b\}$

4. Do the following sentences in FOL capture the English meaning? Explain why or why not.

   - "all bears have hair": $\forall x, Bear(x) \rightarrow Hair(x)$ True
   - "there is a bear that has white hair": $\exists x, Bear(x) \rightarrow Hair(x)$ False
   - "There is an anteater that eats bread": $\exists x, Anteater(x) \rightarrow EatBread(x)$ False
   - "All anteaters eat ants": $\forall x, Anteater(x) \wedge EatAnts(x)$ False

5. In your answers for the question below, do not use any function, constant or predicate symbols, other than the ones given at the end of the questions. Translate the below sentences using ONLY constants l and s for logic and Professor Smith respectively, unary predicate symbols I and H for is-instructor and is-happy respectively, and binary predicates C and L where C(x; y) stands for x is child of y, and L(x; y) stands for x likes y.

   1. Each instructor is happy if some of his/her children like logic.

   $$\forall x(I(x) \wedge (\exists y(C(y,x) \wedge L(y,l)) \rightarrow H(x)))$$

   2. Some instructor is happy if all of his/her children like logic.

   $$\exists x(I(x) \wedge (\forall y(C(y,x) \rightarrow L(y,l)) \rightarrow H(x)))$$

   3. Professor Smith is happy if some of her children like logic.

   $$\exists x(C(x,s) \wedge L(x,l) \rightarrow H(s))$$

6. Translate the sentences below using the following constants, $c3$, $m2$, c, and l (for $CISC304$, $MATH210$, $CISC$, and $LING$ respectively); unary predicate symbols, S and I (for *is-student*, and *is-instructor* respectively); and binary predicate symbols, C, $T1$, $T2$, and M where

   $C(x;y)$ stands for course x is offered by department y,
   $T1(x;y)$ corresponds to course$x$ is taught by y,
   $T2(x;y)$ corresponds to course x is taken by y, and
   $M(x;y)$ corresponds to x is the major of y

   1. Any student who takes $CISC304$ will also take $MATH210$.

   $$\forall x S(x) \wedge T2(c3,x) \rightarrow T2(m2,x)$$

   2. Some instructor of a $CISC$ course has taught a $LING$ course.

   $$\exists x((I(x) \wedge \exists y(T1(y,x) \wedge C(y,c))) \wedge \exists z(T1(z,x) \wedge C(z,l)))$$

   3. Some $CISC$ major has taken all $CISC$ courses.

$$\exists x(M(c,x) \wedge \forall y(C(c,y) \rightarrow T2(y,x)))$$

4. No instructor who teaches $CISC304$ has taught a course taken by a $LING$ student.

$$\neg\exists x(T1(c3,x) \wedge \exists y(T1(y,x) \wedge \exists z(M(l,z) \wedge T2(y,z))))$$

5. Every $LING$ major has taken a course taught by some instructor who has taught $CISC304$.

$$\forall x(M(l,x) \wedge \exists y(T2(y,x) \wedge \exists z(T1(y,z) \wedge T1(c3,z))))$$

7. (8.23) For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it.

   1. No two people have the same social security number.

$$\neg\exists x,y,n \qquad Person(x) \wedge Person(y) \rightarrow [HasSS\#(x,n) \wedge HasSS\#(y,n)].$$

   Ans: This uses $\rightarrow$ with $\exists$. It also says that no person has a social security number because it doesn't restrict itself to the cases where x and y are not equal. Correct version

$$\neg\exists x,y,n \qquad Person(x) \wedge Person(y) \wedge \neg(x=y) \wedge [HasSS\#(x,n) \wedge HasSS\#(y,n)].$$

   2. John's social security number is the same as Mary's.

$$\exists n \qquad HasSS\#(John,n) \wedge HasSS\#(Mary,n).$$

   Ans: OK.

   3. Everyone's social security number has nine digits.

$$\forall x,n \qquad Person(x) \rightarrow [HassSS\#(x,n) \wedge Digits(n,9)].$$

   Ans: This says that everyone has every number. $HasSS\#(x,n)$ should be in the premise:

$$\forall x,n \qquad Person(x) \wedge HassSS\#(x,n) \rightarrow [Digits(n,9)].$$

   4. Rewrite each of the above (uncorrected) sentences using a function symbol $SS\#$ instead of the predicate $HasSS\#$.
   Ans: Here $SS\#(x)$ denotes the social security number of x. Using a function enforces the rule that everyone has just one.

$$\neg\exists x,yPerson(x) \wedge Person(y) \rightarrow [SS\#(x) = SS\#(y)]$$
$$SS\#(John) = SS\#(Mary)$$
$$\forall xPerson(x) \rightarrow Digits(SS\#(x),9)$$

8. (8.24) Represent the following sentences in first-order logic, using a consistent vocabulary:

   1. Some students took French in spring 2011

$$\exists xStudent(x) \wedge Takes(x,F,Spring2011).$$

   2. Every student who takes French passes it

$$\forall x,sStudent(x) \wedge Takes(x,F,s) \rightarrow Passes(x,F,s).$$

   3. Only one student took Greek in spring 2011

$$\exists xStudent(x) \wedge Takes(x,G,Spring2011) \wedge \forall y \quad y\neg x \rightarrow \neg Takes(y,G,Spring2001).$$

   4. The best score in Greek is always higher than the best score in French

$$\forall s\exists x\forall yScore(x,G,s) > Score(y,F,s).$$

   5. Every person who buys a policy is smart

$$\forall xPerson(x) \wedge (\exists y,zPolicy(y) \wedge Buys(x,y,z)) \rightarrow Smart(x).$$

   6. No person buys an expensive policy

$$\forall x,y,zPerson(x) \wedge Policy(y) \wedge Expensive(y) \rightarrow \neg Buys(x,y,z).$$

   7. There is an agent who sells policies only to people who are not ensured

$$\exists xAgent(x) \wedge \forall y,zPolicy(y) \wedge Sells(x,y,z) \rightarrow (Person(z) \wedge \neg Insured(z))$$

Look at the other parts of the question as well. They are important too.

2 Inference in First-Order Logic

1. Q.9.10 A popular riddle is, "Brothers and sisters have I none, but that man's father is my father's son". Use rules from page 301 to show who the man is.
   Ans: Please refer to $cs.wfu.edu/\ snataraj/AI/Misc/9.10\_Answer.docx$ for the answer

2. For each of the following atomic sentences give the most general unifier, if it exists :

   1. P(A,B,B), P(x,y,z) **Ans:** $\{x/A, y/B, z/B\}$
   2. Q(y,G(A,B)), Q(G(x,x), y) **Ans:** No unifier
   3. Younger(y,Mother(y)), Younger(John,Mother(x)) **Ans:** $\{y/John, x/John\}$
   4. Likes(Brother(y),y), Likes(x,x) **Ans:** No unifier

3. The following is a knowledge base.

   1. Mick is a IS student
   2. Amy is an CS student
   3. CS students have better jobs than IS students

   Provide a proof that Amy has a better job than Mick.
   Ans: We can construct the following sentences: $IS(Mick), CS(Amy)$ and $\forall x, y \quad CS(x) \wedge IS(y) \rightarrow Betterjob(x, y)$. Now using these, we can apply resolution and simply get the modes ponens to obtain the result.

4. The following is a knowledge base.

   1. Mick is a mouse
   2. Amy is an elephant
   3. An elephant is always afraid of a mouse

   Provide a proof that Amy is afraid of Mick. **Ans:** We can construct the following sentences: $Mouse(Mick)$, $Elephant(Amy)$, and $\forall x, yMouse(x) \wedge Elephant(y) \rightarrow Afraid(y, x)$. Combine 1 and 2 and do a substitution of $x/Mick$ and y/Amy. Now apply Modes ponens to get $Afraid(Amy, Mick)$

5. Convert the following First Order Logic sentence into its Conjunctive Normal Form :

$$\forall x \quad (AIK(x) \leftrightarrow \exists c(AIC(c) \wedge TakenBy(c, x)))$$

$$(\neg AIK(x) \vee AIC(CourseTakenBy(x))) \wedge (\neg AIK(x) \vee TakenBy(CourseTakenBy(x), x))$$
$$\wedge(\neg AIC(z) \vee \neg TakenBy(z, x) \vee AIK(x))$$

6. The following is a story that you will need to translate into a FOL knowledge base (its sentences numbered and described by FOL Clauses) : The college handbook says that it is an offense meriting expulsion from the college when a student copies HW materials from online sites. The online site SOMESITE offers various prepared answers to selected questions from various academic fields. Joe Doe is a student all of whose HWs were directly copied from the site. Prove that Joe Doe should be expelled from the college.

   1. $Student(x) \wedge HWMat(y) \wedge OnLineSite(z) \wedge Copied(x, y, z) \rightarrow Exp(x)$
   2. $OnlineSite(PapersEasy) \wedge Offers(PapersEasy, ANS) \wedge Answer(ANS)$
   3. $Student(JoeDoe)$
   4. $(\forall x)HWMat(x) \rightarrow HWSub(JoeDoe, x)$
   5. $(\forall x)(HWMat(x) \wedge HWSub(JoeDoe, x) \rightarrow Copied(JoeDoe, x, PapersEasy))$
   6. $(\forall x)(Answer(x) \rightarrow HWMat(x))$
   7. $Exp(JoeDoe)$

   Substituting x/ANS and doing a Generalized Modes Ponens on 6 and 2 yields $HWMat(ANS)$ (let us call it 10). Similarly with the same substitution, doing GMP 10 and 4, $HWSub(JoeDoe, ANS)$ (let us call this 11). GMP on 5,10,11,4 with substitution x/ANS yields $Copied(JoeDoe, ANS, PapersEasy)$ (let us call it 12). Now doing GMP on 1, 3, 10, 8, and 12 with $x/JoeDoey/ANSz/PapersEasy$ yields $Exp(JoeDoe)$.

7. What is the difference between forward and backward chaining?

3 Probabilistic Reasoning

Make sure you know the following terms well: Chain Rule, Conditional Distribution, Prior distribution, Posterior Distribution, Marginal Distribution, Conditional Independence, Bayes Rule, evidence, query, inference etc. Also, learn how variable elimination, prior sampling, likelihood weighting and rejection sampling work.

1. Show from first principles that $P(a|b \wedge a) = 1$.
Ans: The first principles needed here are the definition of conditional probability, $P(X|Y) = P(X \wedge Y)/P(Y)$, and the definitions of the logical connectives. It is not enough to say that if $B \wedge A$ is giventhen A must be true! From the definition of conditional probability, and the fact that $A \wedge A \leftrightarrow A$ and that conjunction is commutative and associative, we have,

$$P(A|B \wedge A) = \frac{P(A \wedge (B \wedge A)}{P(B \wedge A)} = \frac{P(B \wedge A)}{P(B \wedge A)} = 1$$

2. Consider the Bayes net shown below. Write out the formula to compute $P(A = T|D = F)$. Compute the value of $P(A = T|D = F)$. **Ans:** Let A denote $A = T$ and D' denote $D = F$.

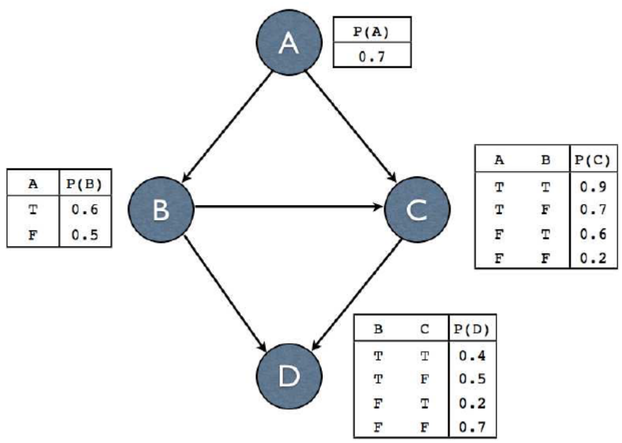

Figure 1:

$$\begin{aligned}
P(A|D') = \frac{P(A \wedge D'}{P(D')}) \\
&= \frac{P(A \wedge D')}{P(A \wedge D') + P(A' \wedge D')} \\
&= \frac{P(A)P(D'|A)}{P(A)P(D'|A) + P(A')P(D'|A')} \\
&= \frac{P(A)\sum_b P(b|A)\sum_c P(c|A,b)P(D|b,c)}{\sum_a P(a)\sum_b P(b|a)\sum_c P(c|a,b)P(D|b,c)}
\end{aligned} \quad (1)$$

Putting the numbers in, the answer is 0.749

3. Consider the following Bayes net: $A \to B \to C \leftarrow D$

   1. Write out the joint distribution as a product of the conditionals for this network.
   $P(A, B, C, D) = P(A)P(B|A)P(D)P(C|B, D)$
   2. How many independent parameters are needed to fully define this network (assuming that all are binary variables)?
   We need 8 independent parameters

3. How many independent parameters would we need to define the joint distribution $P(A, B, C, D)$ if we made no assumptions about independence or conditional independence?
$2^4 - 1$

4. Consider the following Bayes net: $A \to B \to C$. Give an expression for $P(B = 1|C = 0)$ in terms of the parameters of this network. Use notation like $P(C = 1|B = 0)$ to represent individual Bayes net parameters.
Ans: Using Bayes' Rule, we have

$$P(B = 1|C = 0) = \frac{P(C=0|B=1)P(B=1)}{P(C=0)} = \frac{(1-P(C=0|B=1))P(B=1)}{P(C=1)}$$

Equations for P(B=1) and P(C=0) can be derived by

$$P(B = 1) = P(B = 1|A = 0)P(A = 0) + P(B = 1|A = 1)P(A = 1)$$
$$P(C = 0) = 1 - P(C = 1) = 1 - (P(C = 1|B = 0)P(B = 0) + P(C = 1|B = 1)P(B = 1))$$

5. Consider the belief network below, which extends the electrical domain to include an overhead projector. Answer the following questions about how knowledge of the values of some variables would affect the probability of another variable:

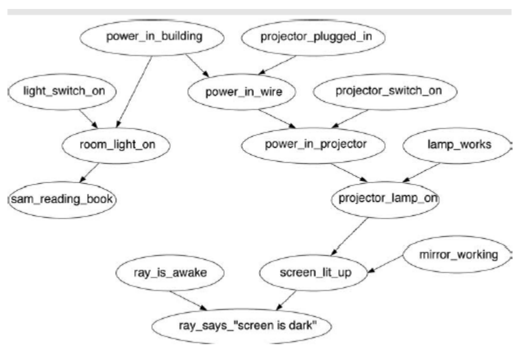

Figure 2:

1. Can knowledge of the value of Projector plugged in affect your belief in the value of Sam reading book? Explain.
Ans: No. whether the projector is plugged in is independent of whether Sam is reading.
2. Can knowledge of Screen lit up affect your belief in Sam reading book? Explain.
Ans: Yes. It can affect our belief in whether there is power in the building.
3. Can knowledge of Projector plugged in affect your belief in Sam reading book given that you have observed a value for Screen lit up? Explain.
Ans: Yes. For example, knowing that the projector is not plugged in can explain why the screen is not lit up, which would then change out belief as to whether there was power in the building (which might have been a cause the the screen was not lit up).
4. Which variables could have their probabilities changed if just Lamp works was observed?
Ans: Observing that Lamp works can affect our belief in $Projector\_lamp\_on$, $Screen\_lit\_up$ and $Ray\_says\_$"$screen\_is\_dark$"
5. Which variables could have their probabilities changed if just Power in projector was observed?
Ans: Observing just Power in projector can affect our belief in all variables except $Light\_switch\_on$, $Lamp\_works$, $Mirror\_working$ and Ray_is_awake.