Math 355/655: Introduction to Numerical Methods
Homework #2
Due: September 12, 2012

This homework is based on the fixed point iteration and Newton's method.

1. Do the MATLAB practice worksheet. (Posted on the homework page.)

2. Let $\phi(x) = (x^2 + 4)/5$. Analytically compute all fixed points. Would the fixed point iteration converge to a fixed point in the interval $[0, 2]$ for all initial guesses $[0, 2]$? Be sure to explain your answer. If the fixed point iteration does not converge for all initial guesses in this interval, find an interval where it will converge for all initial guesses. Be sure to justify (completely) why it will converge.

3. Consider the problem of finding a root of $f(x) = \frac{1}{2}\sin(\frac{x}{2}) - x + \pi$

   (a) Write this as a fixed-point iteration, i.e., find a function g that you would find a fixed point of to solve the root-finding problem.

   (b) Is there is a unique fixed-point on the interval $[0, 2\pi]$?

   (c) Compute the first 4 iterates of the fixed-point method starting with $p_0 = \pi$.

4. Consider the function $f(x) = 1/x - a$, where a is a fixed real number. Write down the Newton iteration for computing the reciprocal of a positive real number a without performing any division. Use this algorithm with $x_0 = 0.3$ to approximate e^{-1}, where $e = 2.7182818284$, by calculating x_1, x_2, x_3, and x_4. Repeat these calculations for $x_0 = 1.0$. Comment on your results.

5. (MATLAB) For this problem, download the MATLAB file `my_newton.m`. This file is set up for use for the following problem: Find a root of $f(x)$ where $f(x) = 1 - e^x$.

   (a) Read through the file newton.m. At the top of the command line window for MATLAB, change the current folder to the folder where you downloaded and saved `my_newton.m`. Type `[x]=my_newton(1.0)` at the command line and watch the program run. Look back over the file `my_newton.m` a couple times and see if you can *generally* make sense of each line. For this part, turn in the result of running the program (you may use the "diary" command or just cut and paste the output into your favorite text editor). Finally, does Newton's method look linearly or quadratically convergent in this case?

   (b) Currently, the file is looking for a point such that $|f(x)| < 1e^{-8}$. Edit the file so it looks for a point such that $|f(x)| < 1e^{-15}$ and rerun the program. Also, instead of starting with $x_0 = 1.0$ (which is the value in parenthesis when we call `my_newton`), try $x_0 = 2.0$. Like in part (a), save the output to turn in.

6. (MATLAB) Edit the `my_newton.m` file so it works for the following function $f(x) = x(1 - e^x)$. (Hint: You may have to google or look on line to figure out the proper syntax if you get an error.) Use this file for part (b) of the following problem:

   (a) Verify that $f(0) = 0$ and $f'(0) = 0$. What is $f''(0)$?

   (b) Keep the termination criteria at $|f(x)| < 1e - 15$. Run this program with $x_0 = 5.0$. Save the output. Does Newton's method look linearly or quadratically convergent in this case?