Math 355/655: Introduction to Numerical Methods
Homework #8
Due: November 9, 2012 at 2pm

Read Sections 4.2–4.4.

1. Consider the 2-point forward difference formula that has the following expansion:
$$f'(x_0) = \left[\frac{f(x_0+h) - f(x_0)}{h}\right] - \frac{f''(x_0)}{2}h - \frac{f'''(x_0)}{6}h^2 + \mathcal{O}(h^3).$$
Use Richardson extrapolation with the result above to derive an $\mathcal{O}(h^3)$ formula for $f'(x)$.

2. Let
$$I(f) = \int_0^1 f(x)\ dx$$
Consider a numerical quadrature formula of the form
$$Q(f) = w_1 f(1/3) + w_2 f(2/3) + w_3 f(1).$$
Find the weights w_i to maximize the order.

3. Find the order of the following quadrature rule:
$$\int_{-1}^{1} f(x)\ dx \approx \frac{1}{9}\left[5f(\sqrt{0.6}) + 8f(0) + 5f(-\sqrt{0.6})\right].$$

4. Derive the composite midpoint rule.

5. Apply both the composite trapezoidal and composite Simpson's rule with n=6 to approximate $\int_0^{\pi/2} \sin(x)$. Compare both approximations to the correct value of 1. Which one is better?

6. In the previous problem, what interval h would be required for the composite trapezoidal rule to have an error no greater than 0.000005. Repeat for composite Simpson's rule.