

### Homework 3 Solutions

#### Problem 1

56 kbps unslotted Aloha channel shared by N stations.  
Stations: 1 packet(1000 bits) every 100 seconds.

- **Required data rate**

The required data rate for N stations is:

$$DataRate = N \frac{1000bits}{100s} = 10Nbps$$

- **Available data rate**

The pure Aloha has a maximum efficiency of 18.4 % therefore the maximum available data rate is:

$$AvailableDataRate = 0.184 * 56000bps = 10.304kbps$$

- **Maximum Number of Stations**

The maximum sustainable number of stations is:

$$N = \frac{10304}{10} = 1030$$

#### Problem 2

Slotted aloha channel with 10% of the slots are idle.

a. **What is the channel load g?**

The fraction of idle slots is:  $P(idle)=0.1$

Since  $P(idle) = e^{-g} \rightarrow g = -\ln(P(idle)) = -\ln(0.1) = 2.3$   
**g=2.3**

b. **What is the system throughput in packets/slot?**

The throughput is:

$$throughput = P(success) = ge^{-g} = 23\%$$

c. **Is the channel overloaded or under-loaded?**

The maximum throughput (36.8%) is achieved for  $g=1$ . For  $g>1$  the channel efficiency drops because it is **overloaded**.

### Problem 3

We consider CSMA/CD.

$B=t/D$ , where  $t$  is the average propagation delay, and  $D$  is the average transmission delay. The throughput is  $E=1/[1+5B]$ .

**Given:**

Transmission rate  $C=100\text{Mbps}$ ,

Frame length  $L=2000$  bits,

Cable distance  $d=1$  mile,

Assume that signals travel at the speed of light in the cable.

**a. Find  $t$  and  $D$**

The speed of light is  $3 \times 10^8 \text{ m/s}$

The distance is 1 mile = 1,609.344 m. Therefore  $t = \frac{1,609.344\text{m}}{3 \times 10^8 \text{ m/s}} = 5.36\mu\text{s}$

$$D = \frac{2000\text{bits}}{100 \times 10^6 \text{ bps}} = 20\mu\text{s}$$

**b. Find  $B$  and  $E$**

$$B = \frac{t}{D} = \frac{5.36}{20} = 0.268 \rightarrow E = \frac{1}{1 + 5 \times 0.268} = 42.7\%$$

**c. If  $C=1\text{Gbps}$ , repeat (a) and (b)**

$$t = \frac{1,609.344\text{m}}{3 \times 10^8 \text{ m/s}} = 5.36\mu\text{s} \quad D = \frac{2000\text{bits}}{10^9 \text{ bps}} = 2\mu\text{s}$$

$$B = \frac{t}{D} = \frac{5.36}{2} = 2.68 \rightarrow E = \frac{1}{1 + 5 \times 2.68} = 6.94\%$$

**d. When the transmission rate increases, does  $E$  increase or decrease? Why?**

As the rate increase the transmission delay decreases too. The ratio of the propagation delay over the transmission delay increases dropping the efficiency.

### Problem 4

**a. Summarize key characteristics of Aloha, Slotted Aloha, CSMA/CD**

#### **I. Unslotted Aloha**

Multiple transmitters single receiver configuration.

No synchronization (packets sent immediately)

Variable packet size.

Retry time exponentially distributed.

$$MaxThroughput = \frac{1}{2e} = .184$$

## II. Slotted Aloha

Multiple transmitters single receiver configuration.

Time divided into slots.

Require synchronization.

Fixed packets length.

Random delay before retransmission of collided packets.

$$MaxThroughput = \frac{1}{e} = .36$$

## III. CSMA/CD

Single media shared by several stations.

Check for busy line before transmission.

If collision detected wait random time before retransmission.

Max time to detect collision is equal to twice the propagation delay between the two stations.

Variable packet size.

### b. For the above protocols, is it possible to achieve a throughput of 1.

Aloha protocols can never achieve a throughput as it is inherently indicated in the throughput formula. The maximum achievable for pure Aloha is 18.4% and 36% for slotted Aloha. The sources transmit independently regardless the state of order source resulting to inevitable high probability of collision.

The throughput formula for CSMA/CD is  $\frac{1}{1 + 5 * \frac{t_p}{D_T}}$

Theoretically the protocol cannot achieve a throughput of 1 since this would mean zero propagation and or infinite transmission delay.

But a throughput of 100% can be approached by making the ratio propagation delay over transmission delay the smallest possible. For a given distance the speed fixes a lower bound for the propagation delay therefore the only degree of freedom is the transmission delay (decrease rate or increase packet size).

## Problem 5

- a. Assume a mean distance between stations of 0.375 km. This is an approximation based on the following observation. For a station on one end, the average distance to any other station is 0.5 km. For a station in the center, the average distance is 0.25 km. With this assumption, the time to send equals transmission time plus propagation time.

$$T = \frac{10^3 \text{ bits}}{10^7 \text{ bps}} + \frac{375 \text{ m}}{200 \times 10^6 \text{ m/sec}} = 102 \mu \text{ sec.}$$

**b.**

$$T_{\text{interfere}} = \frac{375 \text{ m}}{200 \times 10^6 \text{ m/sec}} = 1.875 \mu \text{ sec}$$

$$T_{\text{interfere(bit-times)}} = 10^7 \times 1.875 \times 10^{-6} = 18.75 \text{ bit-times.}$$

### Problem 6

**a.** Again, assume a mean distance between stations of 0.375 km.

$$T = \frac{10^3 \text{ bits}}{10^8 \text{ bps}} + \frac{375 \text{ m}}{200 \times 10^6 \text{ m/sec}} = 12 \mu \text{ sec.}$$

**b.**

$$T_{\text{interfere}} = \frac{375 \text{ m}}{200 \times 10^6 \text{ m/sec}} = 1.875 \mu \text{ sec}$$

$$T_{\text{interfere(bit-times)}} = 10^8 \times 1.875 \times 10^{-6} = 187.5 \text{ bit-times.}$$