MATH 358/658 Assignment 1
Due in class on Wednesday, January 22

1. Suppose X follows an Exponential(β) distribution, with density function

$$f(x;\beta) = \beta e^{-\beta \cdot x}, \quad x \geq 0, \beta > 0.$$

   (a) Find the cumulative distribution function $F(x)$
   (b) Compute the median (i.e. the value m such that $P(X \leq m) = 0.5$
   (c) Compute the mode (i.e. the value x which maximimizes $f(x;\beta)$
   (d) Find the moment generating function $\psi(t)$, and use it to find $E(X^2)$

2. Suppose X follows a Poisson(λ) distribution, with probability mass function

$$p_k = P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, ...$$

   (a) Find P($2 \leq X \leq 5$) in terms of unknown parameter λ
   (b) Show that the moment generating function $\psi(t) = e^{\lambda(e^t - 1)}$, and use it to find $E(X)$. Hint: recall the formal definition of the exponential function,

$$e^a = \sum_{k=0}^{\infty} \frac{a^k}{k!}$$

3. A gamma distribution has density function

$$f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta \cdot x}, \quad x \geq 0, \alpha > 0, \beta > 0.$$

   Derive the mode of the gamma distribution when $\alpha > 1$.

4. Throughout the semester we will work a bit with statistical software `R`. Download the program `R` on your computer. You do this by visiting `http://cran.us.r-project.org/` and selecting the file appropriate for your operating system. For this assignment, I only want to make sure you each download and can run the software appropriately by answering a few easy questions.

   Once you've downloaded `R` and opened it up, type `x=rgamma(n=10000,2,2)` into the command line and press enter. You've just made an object (called x) which contains 10,000 simulated values from a Gamma distribution with $\alpha = 2$ and $\beta = 2$. If you next type `x`, you'll see all the values. The commands `mean(x)` and `var(x)` compute the mean and variance of your sample. The command `hist(x)` makes a histogram of your sample, and the command `plot(density(x))` attempts to draw a smooth distribution.

   (a) Copy the histogram and density plots "as metafile", paste into a word document, and print off that page.
   (b) About what is the mode of your simulated sample? What are you expecting?
   (c) What is the mean and variance of your sample? How close are these to the mean and variance you were expecting?