Inverse Problems Take-Home Final Exam Exam

Work any 4 of the 5 problems.

Due by Friday May 4 (cannot take late papers). You can either email me a pdf file put hard copy to my office door box, Man. 341, or in my mailbox in Math or CSC. Have surgery scheduled Monday May 7. Thanks.

- Note on individual help! Please ask questions in class, not privately. Providing individual help is not fair to the rest of the class.
- Resources: Use any resources you choose, except other classmates and faculty. You may find information in books, papers, and on the internet.
- **Explain your answers for each problem**, show any Matlab code that you wrote and used in a problem, and use Matlab graphics when appropriate.
- Turn in a compact, documented exam paper. Turning in a lot of Matlab output is not necessary.
- I'll be emailing you some files and providing some handouts to help with the exercises.

1. See the Notes by Hansen, Discrete Inverse Problems. You will need to use the Hansen Notes and the Regularization Tools and Manual that you downloaded from the Hansen web page, along with the image processing toolbox.

   Work exercise 6.5.4 on page 89. This exercise illustrates the regularizing effects of conjugate gradient iterations.

2. Noise in Images.

   (a) Describe various types of noise that can be present in observed images.

   (b) How is the SNR for images defined in terms of Decibels (dB)? This is probably in your notes. If not, you can look up "decibels, SNR, definition" on Google?

   (c) What are generally the sources of the following types of noise: Gaussian additive, Poisson multiplicative, salt and pepper, impulse?

   (d) In Matlab, type doc imnoise to see how to run an example or so of adding noise to an image you select from Matlab, and removing the noise. See the image processing toolbox demos for Noise Reduction Filtering to determine how to use noise reduction filters, such as the Wiener, Median, etc., filters. Show your results. You might use the image display commands demonstrated in class and/or see Exercise 1 for display commands and the online image processing toolbox demos.

3. Toeplitz matrices, circulant matrices and the FFT.

   Refer to Project 1 and the code gravity.m with $n = 30$, example 1, and depth $d = .5$. Generate the coefficient matrix A, x and b using: $[A, b, x] = gravity(30, 1, 0, 1, .5)$; A will be 30×30 and Toeplitz.

   Toeplitz matrix times vector using the FFT.

   Find $bc = Ax$, with the true x using Matlab's fft and ifft commands. Recall you first need to extend A to a 64×64 circulant matrix C. "Remember from class, and the handouts", you need only form the first column of C. Compare your result bc with b obtained from $[A, b, x] = gravity(30, 1, 0, 1, .5)$. Show all work.

4. See the handout on the Conjugate Gradient Algorithm. You can use the m-file I emailed you, conjgrad.m.

   (a) Use the same $[A, b, x] = gravity(30, 1, 0, 1, .5)$ as in the previous problem to generate A, b and x. Replace b with bn = b + p*randn(30,1), and $p = 10^{-4}$.

   Run your code conjgrad.m with A, bn, x0 = bn, and the number of iterations k, [xc] = conjgrad(A,bn,x0,k). Try k = 1, 2, 3, 4, 5, 10, 15, 30. Also try the large value $k = 1000$. For which value of k is your xc most accurate? The least accurate? Why? Also vary the p a bit. Explain your results.

   (b) Using the FFT in conjugate gradient iterations.

   Modify conjgrad.m by replacing all the matrix times vector operations by FFT operations, using Matlab's fft and ifft commands. Essentially, you will be using result in problem 3 above. Test your modified code using a simple part of problem 3, just to make sure it runs correctly.

5. Some Project 2 Short Answer Questions.

   (a) Topic 1A.

   Does $\mathcal{P}$ correspond to unknown image of an MRI? If not, then what other component is needed along with $\mathcal{P}$?

   i. Yes! $\mathcal{P}$ is the unknown image.
   ii. No! The sensitivity kernels are also needed.
   iii. No! The image cannot be found.
   iv. No! The frequencies of the ultrasound waves are needed.

   (b) Topic 1B.

   What are the main applications of Computed Tomography in medical imaging, and what are the primary drawbacks to its use?

   (c) Topic 2.

   Derive the equation $Ls = t$ for travel time in seismic tomography. From the example in our group's presentation, solve it using a least-squares approach. Set:

   L = [0 10 10 10;0 0 10 10;0 0 10 10;10 10 10 10]

   t = $[30\ 22\ 22\ 42]^T$

(d) Topic 3.

What is the L-Curve Criterion based on?

i. The balance between the solution norm and the residual norm
ii. The balance between the regularization error and the perturbation error
iii. The balance between the TSVD method and the Tikhonov method
iv. The balance between the SVD coefficients and the solution coefficients

(e) Topic 4.

What regularization approach is used to solve the thickness determination problem in textile material design, and why is it effective?

(f) Topic 5.

What is the technique that enables us to visualize the interior from data measured on the ground surface for geophysical applications?

i. electrical impedance tomography
ii. Tikhonov regularization
iii. anomaly detection
iv. dual reciprocity boundary element modeling

(g) Topic 6.

What ϕ-function will give us Tikohnov regularization and why is it we don't want to use just this regularization?