Homework 3

Due 10/18/2012 2 PM

1. (3 points): **Question 8.3** The sentence $\exists x, y \quad x = y$ is valid. A sentence is valid if it is true in every model. An existentially quantified sentence is true in a model if it holds under any extended interpretation in which its variables are assigned to domain elements. According to the standard semantics of FOL as given in the chapter, every model contains at least one domain element, hence, for any model, there is an extended interpretation in which x and y are assigned to the first domain element. In such an interpretation, $x = y$ is true.

2. (5 points): **Question 8.9**

a. $In(Paris, France) \wedge In(Marseilles, France)$ is correct.

b. $\exists c Country(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$ is correct.

c. $\forall c Country(c) \wedge Border(c, Ecuador) \rightarrow In(c, SouthAmerica)$ is correct. So is $\forall c Country(c) \rightarrow [Border(c, Ecuador) \rightarrow In(c, SountAmerica)]$. Both are equivalent.

d. $\neg[\exists c, d In(c, SouthAmerica) \wedge In(d, Europe) \wedge Borders(c, d)]$ is correct. So is $\forall c, d[In(c, SouthAmerica) \wedge In(d, Europe)] \rightarrow \neg Borders(c, d)]$.

e. $\forall x, y \quad \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee \neg(MapColor(x) = MapColor(y))$ is correct. $\forall x, y \quad (Country(x) \vee Country(y) \vee Borders(x, y) \vee \neg(x = y)) \rightarrow \neg(MapColor(x) = MapColor(y))$ is also correct although the inequality is unnecessary

3. (7 points): **Question 8.10**

a. $O(E, S) \vee O(E, L)$

b. $O(J, A) \wedge \exists p \qquad p \neq A \wedge O(J, P)$

c. $\forall O(p, S) \rightarrow O(p, D)$

d. $\neg \exists p \quad C(J, p) \wedge O(p, L)$

e. $\exists p B(p, E) \wedge O(p, L)$

f. $\exists p O(p, L) \wedge \forall q \quad C(q, p) \rightarrow O(q, D)$

g. $\exists p O(p, S) \rightarrow \exists q \quad O(q, L) \wedge C(p, q)$

4. (4 points): **Question 9.4** This is an easy exercise to check that the student understands unification.

a. $\{x/A, y/B, z/B\}$ (or some permutation of this)

b. No unifier (x cannot bind to both A and B).

c. $\{y/John, x/John\}$.

d. No unifier (because the occurs-check prevents unification of y with $Father(y)$).

5. (3 points): **Question 9.6 a,b,c**

a. $Horse(x) \rightarrow Mammal(x)$
$Cow(x) \rightarrow Mammal(x)$
$Pig(x) \rightarrow Mammal(x)$

b. $Offspring(x, y) \wedge Horse(y) \rightarrow Horse(x)$.

c. $Horse(Bluebeard)$.

6. (8 points): **Question 9.9 a,b**

Goal G0: $7 \leq 3 + 9$	Resolve with (8) $\{x1/7, z1/3 + 9\}$.
Goal G1: $7 \leq y1$	Resolve with (4) $\{x2/7, y1/7 + 0\}$. Succeeds.
Goal G2: $7 + 0 \leq 3 + 9$.	Resolve with (8) $\{x3/7 + 0, z3/3 + 9\}$
Goal G3: $7 + 0 \leq y3$	Resolve with (6) $\{x4/7, y4/0, y3/0 + 7\}$ Succeeds.
Goal G4: $0 + 7 \leq 3 + 9$	Resolve with (7) $\{w5/0, x5/7, y5/3, z5/9\}$.
Goal G5: $0 \leq 3$.	Resolve with (1). Succeeds.
Goal G6: $7 \leq 9$.	Resolve with (2). Succeeds.
G4 succeeds	
G2 succeeds.	
G0 succeeds.	

Figure 1: Solution for 9.9.a

From (1),(2), (7) $\{w/0, x/7, y/3, z/9\}$ infer
(9) $0 + 7 \leq 3 + 9$.
From (9), (6), (8) $\{x1/0, y1/7, x2/0 + 7, y2/7 + 0, z2/3 + 9\}$ infer
(10) $7 + 0 \leq 3 + 9$.
($x1, y1$ are renamed variables in (6). $x2, y2, z2$ are renamed variables in (8).)
From (4), (10), (8) $\{x3/7, x4/7, y4/7 + 0, z4/3 + 9\}$ infer
(11) $7 \leq 3 + 9$.
($x3$ is a renamed variable in (4). $x4, y4, z4$ are renamed variables in (8).)

Figure 2: Solution for 9.9.b