Homework 4 Key

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1 Problem 1

For this problem, we wanted to show the language $L = \{0^n 1^n\}$ was not regular. We also wanted to note what happens when we set $i = 0$ and $i = 1$. Just to rehash. Assume for contradiction that L is regular. Let p be the pumping length given by the pumping lemma. Let $s = 0^p 1^p$. By the pumping lemma, $|s| > p$ thus $s = xyz$ such that $|xy| \leq p, |y| = k > 0, \forall i \geq 0 xy^i z \in L$. Note what happens when $i = 1$ though, so we have $xy^1 z = xyz = s \in L$. So in this case we have no contradiction and thus we can't say L is not regular here. However, when we set $i = 0$ then we have $xy^0 z = xz = 0^{p-k} 1^p$. So xz does not have an equal number of ones and zeros and hence it is not in our language. By the pumping lemma, xz should be in our language. This is a contradiction. Therefore L is not regular.

2 Problem 2.1

For this problem, we wanted to show the parse trees and derivations of the string. For simplicity, I will only show the derivations since the parse trees follow easily from them.

2.1 Part a

$E \Rightarrow T \Rightarrow F \Rightarrow a$

2.2 Part b

$E \Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow F + F \Rightarrow a + F \Rightarrow a + a$

2.3 Part c

$E \Rightarrow E+T \Rightarrow E+T+T \Rightarrow T+T+T \Rightarrow F+T+T \Rightarrow F+F+T \Rightarrow F+F+F \Rightarrow a+F+F \Rightarrow a+a+F \Rightarrow a+a+a$

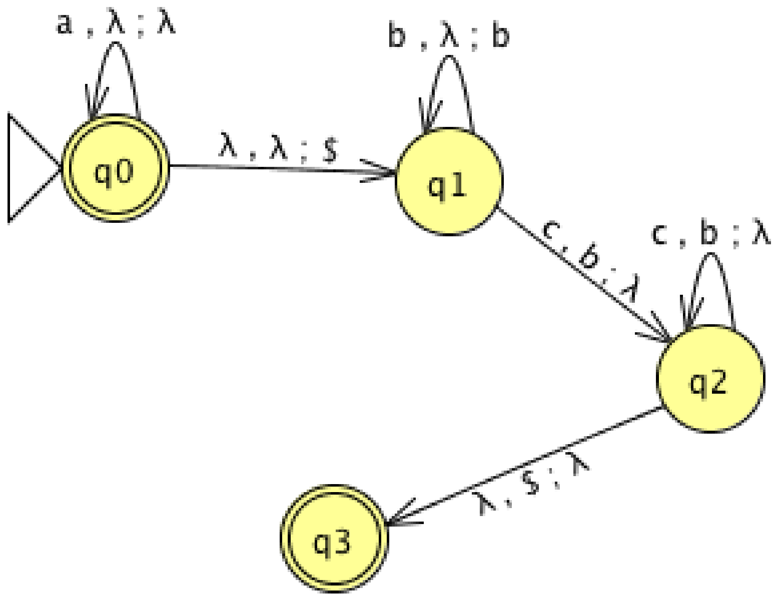
2.4 Part d

$E \Rightarrow T \Rightarrow F \Rightarrow (E) \Rightarrow (T) \Rightarrow (F) \Rightarrow ((E)) \Rightarrow ((T)) \Rightarrow ((F)) \Rightarrow ((a))$

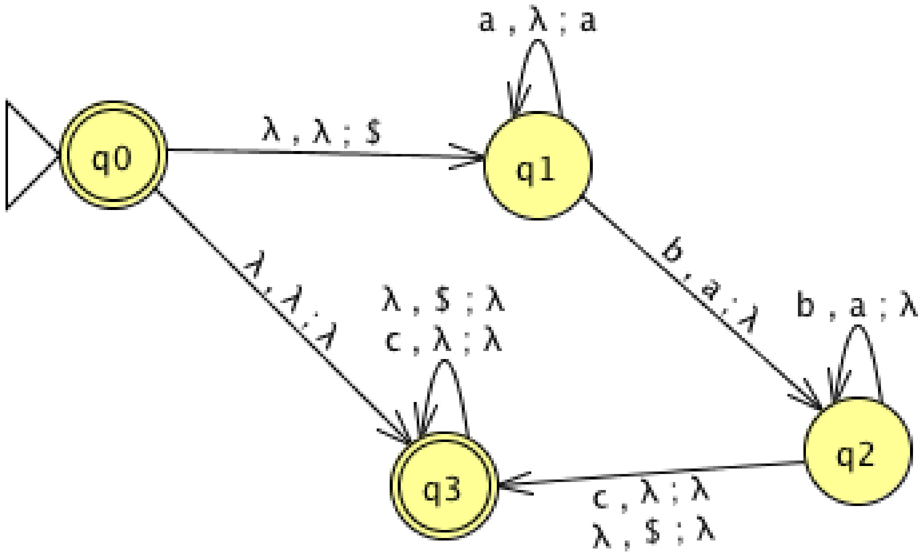
3 Problem 2.2

Please note that the notation along the arcs is a bit different but follows the same format that we did in class. Just remember $\lambda = \epsilon$ and $; = \rightarrow$.

3.1 PDA for A



3.2 PDA for B



3.3 Part a

For this problem, we wish to show that the class of context-free languages is not closed under intersection. Suppose we have context-free languages, A and B as described in the book. The above PDAs recognize A and B respectively thus A and B are context-free languages, by lemma 2.21. Note the intersection of these two languages: $A \cap B = \{a^n b^n c^n | n \geq 0\}$. However, the result of example 3.26 in the book showed that $\{a^n b^n c^n | n \geq 0\}$ was not a context-free language. Therefore we may conclude that the interesection of two context-free languages does not always yield a context-free language. Thus the class of context-free languages are not closed under intersection.

4 Problem 2.4

4.1 Part b

$S \to 0R0|1R1|\epsilon$
$R \to 0R|1R|\epsilon$

4.2 Part c

$S \to 0|1|00S|01S|10S|11S$

5 Problem 2.16

Let $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ be CFLs. Construct a new CFG G_{union} where $L(G_{union}) = L(G_1) \cup L(G_2)$. Let S be a new variablethat is neither in V_1 nor V_2 and assume these sets are disjoint. Let $G_{union} = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{r_0\}, S)$, where r_0 is $S \to S_1|S_2$. Now G_{union} recognizes the union of G_1 and G_2.

To construct the concatentation grammar, similarly to G_{union} except we change r_0 into $S \to S_1S_2$.

To construct the Kleene closure grammar that generates the language $L(G_1)^*$, let S^1 be a new variablenot in V_1 and make it the starting variable. Let r_0 be $S^1 \to S^1S_1|\epsilon$ be a new rule in G.

6 Problem 3.1

I apologize about the poor quality of the image here.

a. $q_1 0, \sqcup q_2 \sqcup, \sqcup\sqcup q_{accept}$

c. $q_1 000, \sqcup q_2 00, \sqcup x q_3 0, \sqcup x 0 q_4 \sqcup, \sqcup x 0 \sqcup q_{reject}$

d. $q_1 000000, \sqcup q_2 00000, \sqcup x q_3 0000\sqcup, \sqcup x 0 q_4 000, \sqcup x 0 x q_3 00, \sqcup x 0 x 0 q_4 0,$
$\sqcup x 0 x 0 x q_3 \sqcup, \sqcup x 0 x 0 q_5 x \sqcup, \sqcup x 0 x q_5 0 x \sqcup, \sqcup x 0 q_5 x 0 x \sqcup, \sqcup x q_5 0 x 0 x \sqcup, \sqcup q_5 x 0 x 0 x \sqcup,$
$q_5 \sqcup x 0 x 0 x \sqcup, \sqcup q_2 x 0 x 0 x \sqcup, \sqcup x q_2 0 x 0 x \sqcup, \sqcup x x q_3 x 0 x \sqcup, \sqcup x x x q_3 0 x \sqcup, \sqcup x x x 0 q_4 x \sqcup,$
$\sqcup x x x 0 x q_4 \sqcup, \sqcup x x x 0 x \sqcup q_{reject}$

$q_7 \sqcup \#1\sqcup, \sqcup q_9 \#1\sqcup, \sqcup\# q_{11} 1\sqcup,$