MATH 358/658 Assignment 2

Due in class on Wednesday, January 29

1. Suppose X is a random variable with $P(X \geq 0) = 1$ and $P(X \geq 10) = 0.3$. Prove $E(X) \geq 3$.

2. Suppose that X is a random variable with $E(X) = 10, P(X \leq 6) = 0.2$, and $P(X \geq 14) = 0.3$. Prove that $Var(X) \geq 8$.

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6. Let X follow a uniform distribution on [0,10]. Define $Y_n = \max(X_1, ..., X_n)$.

   (a) Show that the pdf of Y_n is

   $$f(y) = \frac{n}{10^n} \cdot y^{n-1}, \;\; 0 \leq y \leq 10.$$

   (Hint: first find the cdf $P(Y_n \leq t)$, and then take the derivative).

   (b) Show that $Y_n \to 10$ in probability.

   (c) What is $\lim_{n\to\infty} E(Y_n)$?

   (d) What is $\lim_{n\to\infty} E({Y_n}^k)$ for all $k > 1$?

7. Do this in `R`. A file called `HW2Script.R` is also on Sakai. To open it, you first open `R`, then go to *File*, *Open Script*, and open the file. You can run individual lines from the script by pressing F5 when the cursor is in that line. Keep pressing F5 to run more lines. You can also highlight multiple lines and press F5.

   A Bernoulli random variable has pmf

   $$P(X = k) = p^k(1-p)^{1-k}, \;\; k = 0, 1.$$

   Since the variance is finite, we expect $\bar{X}_n \to p$ by the Law of Large Numbers. Write code to make a set of 1000 sample averages, each taken from n draws from a Bernoulli density with $p = 0.25$. Use these to make 4 histograms, for $n = 4, 10, 100, 1000$. Print off the four histograms (remember, you can copy "as metafile" and paste into Microsoft word). Be sure they all have the same horizontal range. I did the $n = 4$ case for you.