Math 355/655: Introduction to Numerical Methods
Homework #1
Due: September 5, 2012

This homework covers the section on the bisection method. It covers Section 2.1 in the textbook. There is no MATLAB homework this week.

1. Can the bisection method be used to find a root of the following functions using the following intervals? Why or why not?
   (a) $f(x) = \cos(x) + e^x$ with $[0, \pi/2]$
   (b) $f(x) = x^3 + x + 1$ with $[-1, 0]$
   (c) $f(x) = 1/x$ with $[-1, 1]$
2. Consider $f(x) = x(x-1)(x+2)$, which has roots at $x = 0, 1, -2$. Determine which root the bisection method approximates when using the starting interval $[-3, 2]$.
3. Suppose $f(x)$ is a given continuous function in $[-1, 4]$ such that $f(-1)$ and $f(4)$ have different signs.
   (a) Bound the absolute error for the approximation c_{30} generated after 30 iterations of the bisection method.
   (b) Use the bound on the absolute error to determine how many iterations of the bisection method need to be taken to achieve an absolute error less than 10^{-11} for a root of $f(x)$ in $[-1, 4]$.
4. (a) Use the bisection method to generate the first 4 approximations $c_1, \ldots, c_4$ of $2\sqrt{2}$ by finding a positive root of $x^2 - 8$ using the starting interval $[2, 3]$.
   (b) Find the bound of the absolute error for the final approximation and verify that the actual absolute error satisfies this bound.
5. Suppose we modify the bisection method as follows:
   - Approximations are chosen at the midpoint of the interval. The interval is cut into two at the location $(2a + b)/3$.

   Answer the following questions about this modified bisection method:
   (a) Calculate the first 3 approximations c_1, c_2, and c_3 when $f(x) = \cos(x) - x$ with starting interval $[0, \pi/2]$.
   (b) Bound the absolute errors of the nth iteration when the starting interval is $[a, b]$.