MTH 358 HW 9

p. 548 # 1

$X \sim \text{Exp}(\beta)$, $\Omega_0 : \beta \geq 1$ vs. $\Omega_1 : \beta < 1$

δ says reject if $X \geq 1$.

(a.) $\pi(\beta|\delta) = \text{Probability}(\text{reject} \mid \beta)$

$= P(X \geq 1 \mid \beta)$

$= 1 - F(1)$

$= e^{-\beta(1)} = e^{-\beta}$.

(b) $\text{Size} = \max_{\beta \in \Omega_0} \pi(\beta|\delta)$

$e^{-\beta}$ is decreasing in β, so

$\max_{\beta \geq 1} e^{-\beta}$ occurs at $\beta = 1$

$\text{Size} = e^{-1}$.

#2 $X_1, \ldots, X_n \sim U[0, \theta]$, $Y_n = \max(X_1, \ldots, X_n)$

~~We know that~~

$P(Y \leq y) = F(y) = P(X \leq y)^n = \left(\frac{y}{\theta}\right)^n$

(a.) $\pi(\theta|\delta) = P(Y_n \le 1.5 | \theta)$

$$= \left(\frac{1.5}{\theta}\right)^n$$

(b.) size $= \max_{\theta \in \Omega_0} \pi(\theta|\delta)$

$\pi(\theta|\delta)$ is decreasing in θ, so

$$\text{size} = \max_{\theta \geq 2} \pi(\theta|\delta) = \pi(\theta = 2|\delta)$$

$$= \left(\frac{1.5}{2.0}\right)^n .$$

#4 $X_1, \ldots, X_n \sim N(\mu, \sigma^2 = 1)$

$\Omega_0 : \mu = \mu_0$, $n = 25$, reject if $|\bar{X} - \mu_0| \geq c$.

We know $\bar{X} \sim N(\mu, \frac{\sigma^2}{n} = \frac{1}{25})$

$$\Rightarrow \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

Reject if $\left|\frac{\bar{X} - \mu_0}{1/5}\right| \geq z_{.975} = 1.96$

$\Leftrightarrow$ reject if $|\bar{X} - \mu_0| \geq \frac{1.96}{5} = \boxed{0.392}$

#12 p. 586

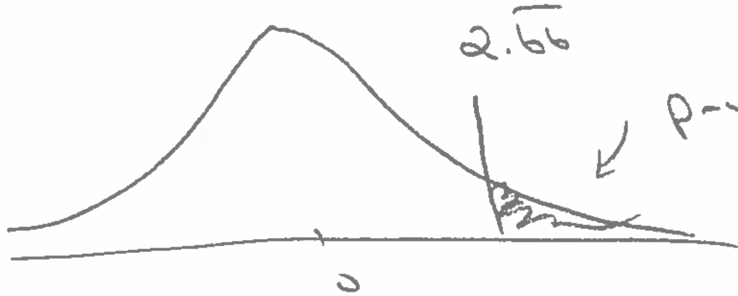
$n=17, \quad \bar{x}=3.2, \quad \frac{1}{n}\sum(x_i-\bar{x})^2 = 0.09$

First, $\sum(x_i-\bar{x})^2 = n\cdot(.09) = 1.53$

$\Rightarrow \sigma'^2 = \frac{1}{16}1.53 = 0.095625$

$H_0 : \mu \le 3$

$$T = \frac{\bar{x}-\mu_0}{\sigma'/\sqrt{17}} = \frac{3.2-3}{.3092325/\sqrt{17}} = \boxed{2.\overline{66}}$$

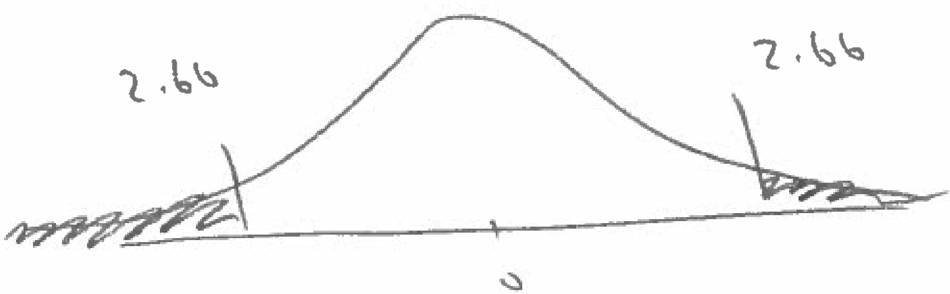


p-val $= \boxed{.00855}$

#14. Same $T = 2.\overline{66}$, but now

p-val $= 2(.00855)$

$= 0.0171$



#4 p.596

$X_1, \dots, X_m \sim N(\mu_1, \sigma_1^2)$

$Y_1, \dots, Y_n \sim N(\mu_2, \sigma_2^2)$

Let $\mu_1 = \mu_2$, $\sigma_2^2 = k \cdot \sigma_1^2$

- First, $\bar{X}_m - \bar{Y}_n \sim N\left(0, \frac{\sigma_1^2}{m} + \frac{k \cdot \sigma_1^2}{n}\right)$

$$\Rightarrow \frac{\bar{X}_m - \bar{Y}_n}{\sigma_1 \left(\frac{1}{m} + \frac{k}{n}\right)^{1/2}} \sim N(0,1)$$

- Second, $S_X^2 = \Sigma (X_i - \bar{X}_m)^2$

$$\frac{S_X^2}{\sigma_1^2} = \Sigma \left(\frac{X_i - \bar{X}_m}{\sigma_1}\right)^2 \sim \chi^2_{m-1}$$

$$\frac{S_Y^2}{k \cdot \sigma_1^2} \sim \Sigma \left(\frac{Y_i - \bar{Y}_n}{\sqrt{k} \cdot \sigma_1}\right)^2 \sim \chi^2_{n-1}$$

$$\frac{S_X^2 + S_Y^2/k}{\sigma^2} \sim \chi^2_{m+n-2}$$

$$\Rightarrow T = \frac{Z}{\left(\frac{W}{m+n-2}\right)^{1/2}} \sim T_{m+n-2}$$

$$= \frac{(\bar{X}_m - \bar{Y}_n)}{\left(\frac{1}{m} + \frac{k}{n}\right)^{1/2}} \cdot \frac{(m+n-2)^{1/2}}{\left(S_X^2 + \frac{S_Y^2}{K}\right)^{1/2}} \sim T_{m+n-2}.$$

10.

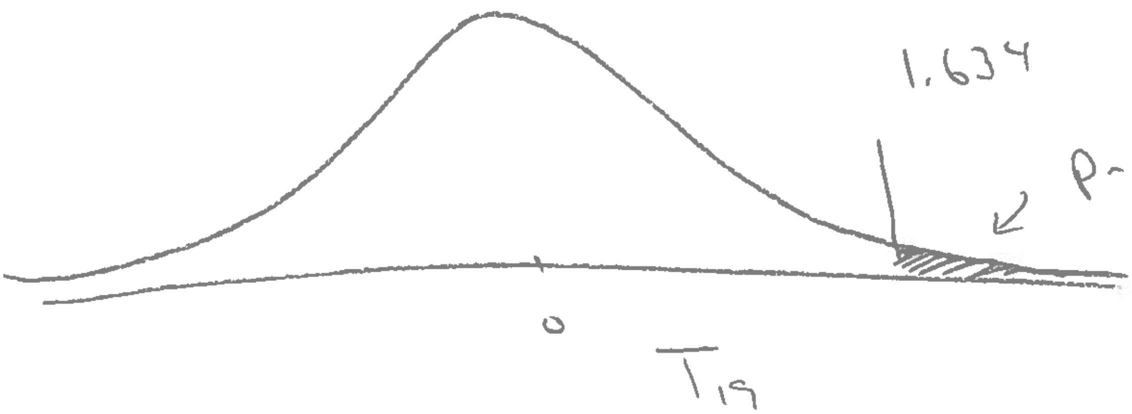
	Calcium	Placebo
sample	10	11
mean	5	-.2727
st. dev	8.7432	5.90069

$\Omega_0 : \mu_c \le \mu_p \qquad \Omega_1 : \Omega_c > \Omega_p$

$$T = \frac{\left[\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\left(\frac{1}{m} + \frac{1}{n}\right)^{1/2}}\right]}{\left[\frac{(S_X^2 + S_Y^2)^{1/2}}{(m+n-2)^{1/2}}\right]}$$

$$= \left[\frac{(5 + .2727) - (0)}{\left(\frac{1}{10} + \frac{1}{11}\right)^{1/2}}\right] \Big/ \left[\frac{9(8.7432)^2 + 10(5.90069)^2}{10 + 11 - 2}\right]^{1/2}.$$

$$= \frac{12.067}{7.3848} = 1.634$$



p-val = 0.0593

Reject at 0.1 level since p-val < .1