Math 355/655: Introduction to Numerical Methods
Homework #5
Due: October 12, 2012

Read Section 3.3, 3.4

1. Consider the data points (-2,1), (-1,4), (0,11), (1,16), (2,13), (3,-4).
   (a) Write down the divided difference table associated to this data.
   (b) Determine from the table, the degree of the interpolating polynomial of least degree passing through these data points.

2. Simplify $f[x_0, x_1, x_2]$ and $f[x_2, x_0, x_1]$ and verify they are the same.

3. Consider the data points (-2,-1), (0,1), (-1,3).
   (a) Write down the interpolating polynomial for this data using a divided difference table.
   (b) Add the data point (1,-1) and write down the new interpolating polynomial.
   (c) Evaluate the polynomial you obtained in part (b) and its derivative at $x = 1/2$.

4. Given $x_0 = -1$, $x_1 = 1$ and $f(x_0) = 0, f(x_1) = 2$, and $f'(x_0) = 1, f'(x_1) = -1$:
   (a) Find the Hermite interpolating polynomial using divided differences.
   (b) Add the information $x_2 = 0$ and $f(x_2) = 1$ and $f'(x_2) = 2$ and find the resulting Hermite interpolating polynomial.
   (c) For your answer for part (b), verify that the Hermite interpolating polynomial both interpolates the data and interpolates the derivative at each data point.

5. Let $S(x)$ be a natural cubic spline on $[0, 4]$ interpolating the data points (0,3), (2,4), (4,2). Find the cubic spline. (Hint: You may use MATLAB but be sure to write down on your paper the linear system you plug into MATLAB.)

6. Suppose we have data points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$. Instead of using cubics to build a spline, suppose we use polynomials of degree less than or equal to 5. We require the spline to interpolate the data points, i.e., $S(x_i) = y_i$. In addition, we require continuity of the spline up to and including the 4th derivative on $[x_0, x_n]$ (i.e., $S^{(iv)}$ is continuous). How many degrees of freedom remain? (In other words, how many more equations are there than constraints?).