Math 355/655: Introduction to Numerical Methods
Homework #9
Due: November 16, 2012 at 2pm

Read Section 6.2, 7.1, and 7.2. For more practice with GEPP, try some additional problems at the end of Section 6.2 in the book.

1. Use Gaussian Elimination with partial pivoting to solve the following system of equations. Show your work and circle the pivots you use in each matrix.

$$\begin{aligned} 2x_1 + x_2 + x_3 &= \quad 5 \\ 4x_1 - 6x_2 &= -2 \\ -2x_1 + 7x_2 + 2x_3 &= \quad 9 \end{aligned}$$

2. Consider the following system of equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= -1 \\ 2x_1 + 2x_2 + 5x_3 &= -8 \\ 4x_1 + 4x_2 + 8x_3 &= -14 \end{aligned}$$

   (a) Find a solution using Gaussian Eliminiation with partial pivoting to solve the following system of equations. What happens in this case? Can you still find a solution?

   (b) What if we adjust the right-hand side of the third equation by changing -14 to -12? Can you find a solution using Gaussian Elimination with partial pivoting?

   (c) When Gaussian Elimination with partial pivoting fails, what does this seem to mean based on parts (a) and (b) of this problem?

3. Suppose

$$x = \begin{bmatrix} 2 \\ 3 \\ -1 \\ -4 \end{bmatrix}.$$

   Compute $\|x\|_1$, $\|x\|_2$, $\|x\|_\infty$, and $\|x\|_F$.

4. Show that if $D = \text{diag}(\mu_1, \ldots, \mu_k) \in \mathbb{R}^{m \times n}$ with $k = \min\{m, n\}$, then $\|D\|_p = \max |\mu_i|$. (Note: "diag" denotes a diagonal matrix; for example, type the following in matlab: "diag([1, 2, 3, 4])".)

5. Suppose $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$. Show that if $E = uv^T$ then $\|E\|_F = \|E\|_2 = \|u\|_2\|v\|_2$ and that $\|E\|_\infty \leq \|u\|_\infty\|v\|_1$.