

# Opportunistic $p$ -persistent CSMA in wireless networks

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**Abstract**—In this paper, Opportunistic  $p$ -persistent Carrier Sense Multiple Access (OpCSMA) is proposed to incorporate multiuser diversity into  $p$ -persistent CSMA. In the OpCSMA, each mobile terminal (MT) is assumed to have the knowledge of its channel state information (CSI). At each idle time slot, an MT sends a packet if the corresponding channel gain is above the threshold, which is determined such that the probability of accessing the medium is maintained to be  $p$  for any idle slots. Using the infinite user model, it is proved that the OpCSMA provides significant increase in the expected capacity compared to  $p$ -persistent CSMA. In addition, the expected capacity of the OpCSMA is shown to be proportional to  $\ln G$  at low SNR where  $G$  is the offered load. Simulation results corroborate the above findings in slow fading channels.

## I. INTRODUCTION

Carrier Sense Multiple Access (CSMA) is a medium access control (MAC) protocol being widely used in many popular networks such as 802.3 Ethernet and 802.11 Wireless LAN (WLAN) [1]. The application of the CSMA can be also found in sensor networks where the coordination among sensors is minimized to lower the cost [2]. The 802.11 WLAN MAC standard protocol is shown to be closely approximated by the  $p$ -persistent CSMA even though two protocols have different back-off intervals [3].

In a scalar wireless multiple access channel, the maximum system throughput was achieved by allowing one user with the best channel condition to access the wireless medium [4],[5],[6]. As the number of users rises, the maximum throughput increases due to the multiuser diversity effect. The multiuser diversity was observed in the multiple antenna systems as well [7],[8]. However, in general, a centralized scheduler is required to utilize the multiuser diversity, which would not be feasible for random access channels. Recently in [9], the channel-aware ALOHA was proposed for the fast fading channel. It achieves the multiuser diversity gain by letting each mobile terminal (MT) access the medium only when the channel gain is larger than the threshold. Since [9] assumes independent and identically distributed (i.i.d.) block fading over time slots, a single threshold value is assigned to each MT. Similar protocol in [2] was shown to achieve throughput comparable to that with a centralized scheduler.

However, in a slow fading channel, the block fading assumption is no longer valid and the criterion on setting the threshold becomes more complicated. If the 802.11 WLAN system is considered, the channel is usually slowly varying

and also, the MAC protocol is well approximated by the  $p$ -persistent CSMA. In this paper, we propose a variant of  $p$ -persistent CSMA called Opportunistic  $p$ -persistent CSMA (OpCSMA) that utilizes multiuser diversity in a slow fading channel. Assuming reciprocity of channel gains, each MT can obtain the knowledge of its own channel gain [10]. At each idle time slot, an MT accesses the medium only if the corresponding channel gain exceeds the threshold, which is predefined such that the probability of accessing the medium is maintained to be  $p$  for any idle slots. Different from the fast fading case, each user is assumed to have a constant channel gain for the subsequent time slots. Therefore, the threshold for the current idle slot takes a smaller value if there was no packet transmission in the previous idle slot. Using the infinite user model [1], the OpCSMA is shown to have much larger expected capacity than the conventional  $p$ -persistent CSMA at high traffic loads. Also, the expected capacity of the OpCSMA is proved to be proportional to  $\ln G$  at low SNR where  $G$  is the offered load. This implies that  $G$  is related with the average number of active users that affects the degree of multiuser diversity gain. On the other hand, the multiuser diversity can be utilized to minimize the power consumption of the wireless network, which is discussed in [11].

## II. SYSTEM MODEL AND $p$ -PERSISTENT CSMA

In wireless networks such as the WLAN, a time-varying number of MTs communicate with an access point (AP) through slow fading channels. Assuming only one MT accesses the channel at each scheduling time, the received signal of the AP at time  $t$  is given by

$$y(t) = \sqrt{h_i}x_i(t) + n(t), \quad (1)$$

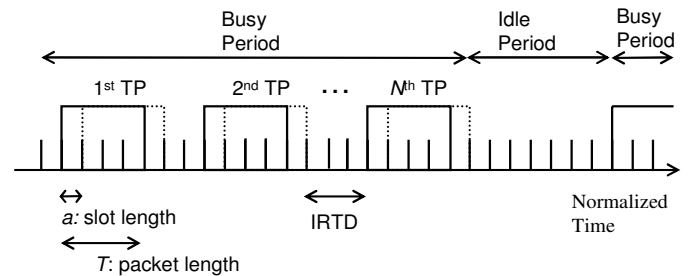


Fig. 1.  $p$ -persistent CSMA time-slot structure. (TP: transmission period, IRTID: initial random transmission delay)

where  $x_i(t)$  is the transmitted signal from MT  $i$  over the bandwidth  $2W$ ,  $h_i$  denotes the channel gain from MT  $i$  to the AP, and  $n(t)$  is an additive white Gaussian noise (AWGN) with the power of  $N_0W$ . Without loss of generality, noise power  $N_0W$  and each user's transmit signal power are assumed to be 1 throughout this paper. Also, each user's signal-to-noise ratio (SNR) is assumed i.i.d. according to the probability density function (pdf)  $f_H(h)$ . For a Rayleigh fading channel,  $f_H(h) = e^{-\frac{h}{P_r}}/P_r$  where  $P_r$  is the average signal power at the receiver.

This paper considers the infinite user model that was first considered by Abramson [12], and used to evaluate the throughput of  $p$ -persistent CSMA [1]. The traffic sources are composed of an infinite number of MTs that collectively form a single Poisson process; thus, each new packet always arrives at a new user. In case of a collision or a back-off, the packet is delayed for a random interval so that it appears as a new arrival in the future. By aggregating arrival process including new and retransmitted packets, the average number of arrivals per unit time,  $G$ , is defined as *offered load*. The number of MTs in the network equals the number of packets waiting for transmission; thus, the effect of time variations in the number of users is considered in the infinite user model. In this paper, it is also assumed that the channel gains of all users are i.i.d. according to the pdf  $f_H(h)$ , and the channel gain is invariant over time. Although this model may appear unnatural at first, it actually lower bounds the performance of a finite-user system since each user's packets are assumed to compete against each other [12]. In practice, these modeling hypotheses approximate a large finite population in which each MT transmits packets infrequently through slowly-varying channels.

Fig. 1 shows the operation of  $p$ -persistent CSMA [1] along a normalized time axis that is finely partitioned into slots of duration  $a$ . The slot duration is usually twice the maximum propagation delay between the AP and MTs. All MTs can start transmitting only at the beginning of a slot and each packet is of constant length  $T$ . To simplify the analysis, time axis is normalized by  $T$  so that the packet length is equal to 1. A beacon signal is placed at the beginning of each slot for channel estimation at the MTs. Provided the wireless channel is reciprocal in time division duplexed (TDD) WLANs, the estimated downlink channel gain can replace the uplink channel gain. When a packet arrives at an MT, the MT first checks whether any other MTs are transmitting a packet. If the channel is found idle, the MT accesses the wireless medium with probability  $p$  to reduce the collision probability. Therefore, even during a busy period, a silent period appears with probability  $1 - p$ , which is called *initial random transmission delay* (IRTD). If an MT starts transmitting a packet at time  $t = 0$ , the other MTs wait until  $t = T + a$  where the  $a$  term is due to the propagation delay. Thus, as illustrated in the dotted boxes of Fig. 1, the duration of a transmission period (TP) is  $(T + a)$ . Once an MT decides to defer transmission, it repeats the same procedure for the following idle slots. When all MTs have no packets to send,

an idle period is maintained until new packets arrive.

The throughput of a random-access protocol has been measured by using the average channel utilization  $S$  that is the number of successfully-transmitted packets per unit time. Using renewal theory and probabilistic analysis, it was shown in [1] that the average channel utilization is

$$S = \frac{(1 - e^{-aG})[P'_s\pi_0 + P_s(1 - \pi_0)]}{(1 - e^{-aG})[a\bar{t}'\pi_0 + a\bar{t}(1 - \pi_0) + 1 + a] + a\pi_0}, \quad (2)$$

where  $P'_s$  and  $P_s$  are respectively the probability of successful transmission of the first and the remaining TPs,  $\pi_0$  is the probability that no packet arrives during a TP, and  $\bar{t}'$  and  $\bar{t}$  are the average duration of an IRTD before the first and the other TPs.

### III. OPPORTUNISTIC $p$ -PERSISTENT CSMA (OPCSMA)

In  $p$ -persistent CSMA, if the channel is sensed idle, each MT generates a Bernoulli random variable with the mean of  $p$  at the beginning of each slot. Then, an MT accesses the wireless medium if its random variable is equal to one. Though the packet collision probability can be controlled by adjusting  $p$ ,  $p$ -persistent CSMA ignores the channel state information in accessing the medium. To incorporate multiuser diversity, the OPCSMA decides whether to access the medium by comparing the channel gain with the predetermined threshold as shown in Fig. 2. Under the OPCSMA, each MT has a set of thresholds related with its channel gain. Since it is assumed that the channel statistics are the same for all MTs, every user shares a single set of thresholds,  $\{T_0, T_1, \dots, T_k, \dots\}$  where the subscript denotes the index of idle slots. Assume that MT  $i$  has a packet to send, but  $k-1$  idle slots have elapsed without its accessing the medium. Then, MT  $i$  sends a packet at the  $k$ th idle slot if its channel gain  $h_i$  exceeds the threshold  $T_k$ . Therefore, it can be easily seen that  $T_{m-1} > T_m$  for all positive integer  $m$ . In the analysis, it is assumed that  $a$  is so small that no packets arrive during an IRTD. As a result,  $k$  simply denotes the idle slot index. Since the channel gains of all MTs are less than  $T_{k-1}$  at the  $k$ th idle slot, the probability of accessing the wireless medium for the MT  $i$  is

$$P(h_i > T_k | h_i < T_{k-1}) = \frac{F_H(T_{k-1}) - F_H(T_k)}{F_H(T_{k-1})}, \quad k = 1, 2, \dots \quad (3)$$

where  $F_H(h) = \int_0^h f_H(x)dx$  is the probability distribution function of a channel gain. To maintain the probability of transmission at each idle slot equal to  $p$ ,  $P(h_i > T_k | h_i < T_{k-1}) = p$  for all positive integer  $k$ . Then the following relation can be obtained from (3)

$$qF_H(T_{k-1}) = F_H(T_k), \quad (4)$$

where  $q = 1 - p$ . In particular, the transmission probability at the 0th idle slot is  $P(h_i \geq T_0) = 1 - F_H(T_0)$ , so the threshold at slot  $k$  is  $T_k = F_H^{-1}(q^{k+1})$  from (4) and the initial condition at  $k = 0$ . As an example, this paper considers a Rayleigh fading channel where  $T_k = -P_r \ln(1 - q^{k+1})$ .

Without a collision, the maximum achievable rate for MT  $i$  is equal to  $0.5 \log_2(1 + h_i)$  if the packet length is much shorter

than the coherence time of the channel. In the absence of new packet arrival during an IRTD, *expected channel capacity* [13] when an MT accesses channel at slot  $k$  is

$$C(k) = \int_{T_k}^{T_{k-1}} f_H(h|T_{k-1} > h \geq T_k) \frac{1}{2} \log_2(1+h) dh$$

$$= \frac{1}{2pq^k \ln 2} \left( e^{\frac{1}{P_r}} \left( E_1 \left( \frac{1+T_k}{P_r} \right) - E_1 \left( \frac{1+T_{k-1}}{P_r} \right) \right) \right.$$

$$\left. - e^{\frac{T_{k-1}}{P_r}} \ln(1+T_{k-1}) + e^{\frac{T_k}{P_r}} \ln(1+T_k) \right), \quad (5)$$

where  $E_1(x) = \int_x^\infty \frac{e^{-u}}{u} du$  is called the *exponential integral function* and  $T_{-1} = \infty$ . If it is assumed that a collision always causes packet errors, the expected capacity  $C$  is expressed as

$$C = \mathbf{E} \left[ \sum_{k=0}^{\infty} P_K(k) C(k) P_s \right] \quad (6)$$

where  $P_K(k)$  denotes the probability that a packet is transmitted at slot  $k$ , and  $P_s$  is the probability of no packet collisions. During the first TP in a busy period (see Fig. 1), the number of packet arrivals is Poisson distributed with a mean of  $\lambda = aG$ . Provided  $a$  is small, only one packet arrives with high probability; thus,  $P_s \approx 1$  and  $P_K(k=1) \approx 1$ . For the following TPs, the number of arrivals,  $n$ , is Poisson distributed with a larger mean of  $\lambda = (1+a)G$ . In other words,  $\pi_n = e^{-\lambda} \frac{\lambda^n}{n!}$  since arrived packets during the previous packet transmission have been backlogged until the channel is available. As a result, the probability of transmitting packets at slot  $k$  is  $P_K(k) = q^{kn}(1-q^n)$  where  $q^{kn}$  is the probability of no transmission before slot  $k$  and  $1-q^n$  is the probability of at least one attempt to access the channel. Similarly,  $P_s(n)$  equals the probability of transmitting only one packet,  $npq^{n-1}$ , divided by the probability of at least one transmission; thus,  $P_s(n) = \frac{npq^{n-1}}{1-q^n}$ . Hence, expected channel capacity is

$$C = \pi_0 \bar{C}_F + (1 - \pi_0) \bar{C}_R, \quad (7)$$

where  $\bar{C}_F$  and  $\bar{C}_R$  denote respectively the expected capacity for the first TP in a busy period and the following TPs.

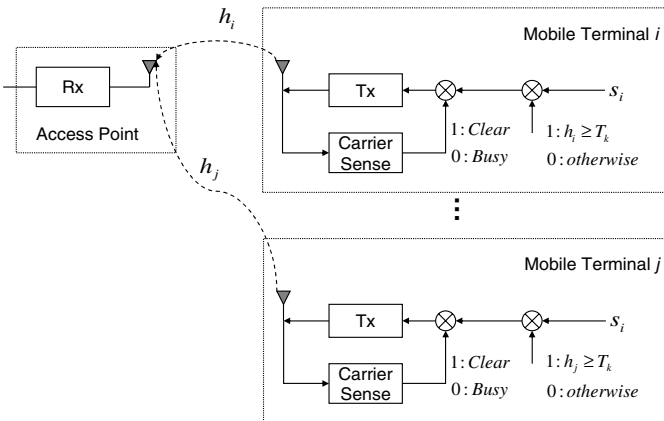


Fig. 2. Block diagram of the OpCSMA

Since only one packet arrives in the network for the first TP with high probability,  $\bar{C}$  equals the expected capacity with only one user in the network, which is

$$\bar{C}_F = \int_0^\infty \frac{1}{2P_r} e^{-h/P_r} \log_2(1+h) dh$$

$$= \frac{e^{\frac{1}{P_r}}}{2 \ln 2} E_1 \left( \frac{1}{P_r} \right). \quad (8)$$

The expected capacity during the other TPs is given as follows:

$$\bar{C}_R = \sum_{n=1}^{\infty} \frac{\pi_n}{1 - \pi_0} P_s(n) \sum_{k=0}^{\infty} P_K(k) C(k), \quad (9)$$

where  $n$  denotes the number of packets in the network, and  $k$  represents the slot index. After changing the order of the summation and using  $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ ,  $\bar{C}_R$  becomes

$$\bar{C}_R = \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}} \sum_{k=0}^{\infty} e^{\lambda q^{k+1}} C(k). \quad (10)$$

Provided  $a$  is small, the analytic result of the expected capacity is obtained by applying (10) into (7).

The expected channel capacity  $C$  is the maximum possible amount of successfully-transmitted data divided by the actual transmission time, but in CSMA systems, no data is transmitted during IRTDs and idle periods. By taking into account such wasted time, we define the *throughput* as the amount of successfully-transmitted data divided by the duration of both the transmission and the wasted time. In a manner similar to (2), the throughput using modified access control is as follows:

$$S_m = \frac{(1 - e^{-aG})C}{(1 - e^{-aG})[a\bar{t}'\pi_0 + a\bar{t}'(1 - \pi_0) + 1 + a] + a\pi_0}. \quad (11)$$

Without modified access control, the throughput is the same as (11) except that  $\bar{C}_R$  in (7) is replaced with  $\bar{C}'$  which is

$$\bar{C}' = \bar{C}_F \sum_{n=1}^{\infty} \frac{\pi_n}{1 - \pi_0} P_s(n), \quad (12)$$

since the channel gain of a successfully-transmitted packet is distributed as  $f_h$  regardless of  $k$  and  $n$ . Proposition 1 demonstrates that proposed access control increases the throughput of  $p$ -persistent CSMA.

**Proposition 1:** For any  $p$  and  $G$ , the expected channel capacity  $\bar{C}_R$  in (9) is larger than  $\bar{C}'$  in (12). Thus, the throughput  $S_m$  is increased by modified access control.

*Proof:* See Appendix. ■

Throughput  $S$  in (2) can be also applied to the OpCSMA in ergodic channels since the probability of accessing the channel is a constant  $p$ . Therefore, little problem may occur even if MTs using the OpCSMA co-exist with MTs employing conventional  $p$ -persistent CSMA.

In study of multiuser diversity, the channel capacity at a low SNR is proportional to  $\ln U$  where  $U$  is the number of users in a network. In proposition 2, as  $p \rightarrow 0$ , the limit of expected capacity at a low SNR is shown to become proportional to  $\ln G$  where  $G$  is the offered load.

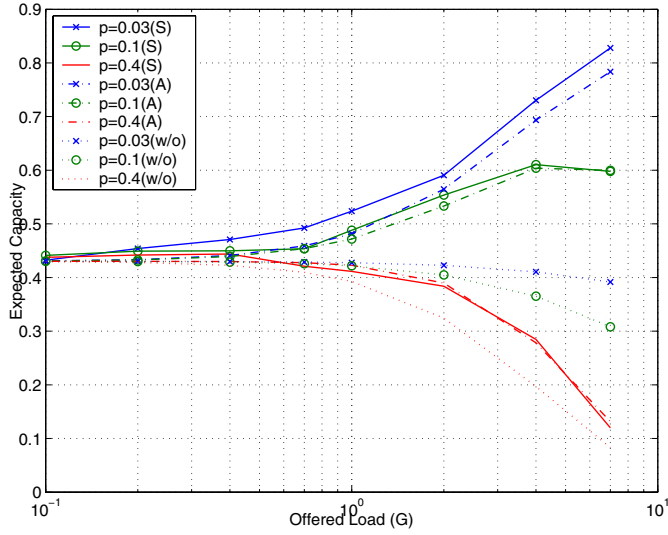


Fig. 3. Comparison of analyzed and simulated expected channel capacity when  $\alpha=0.01$  and average SNR = 0dB. A: Analysis, S: Simulation, w/o: without modified access control.

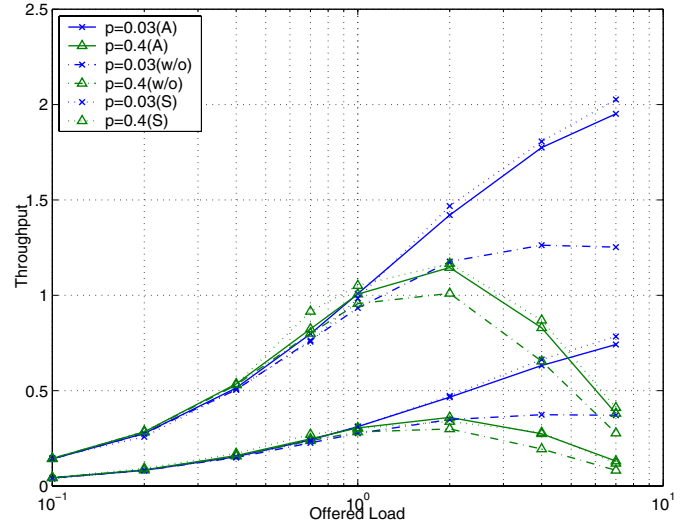


Fig. 5. Expected throughput. A: Analysis, S: Simulation, w/o: without modified access control. Upper lines: average SNR 10dB, Lower lines: average SNR:0dB

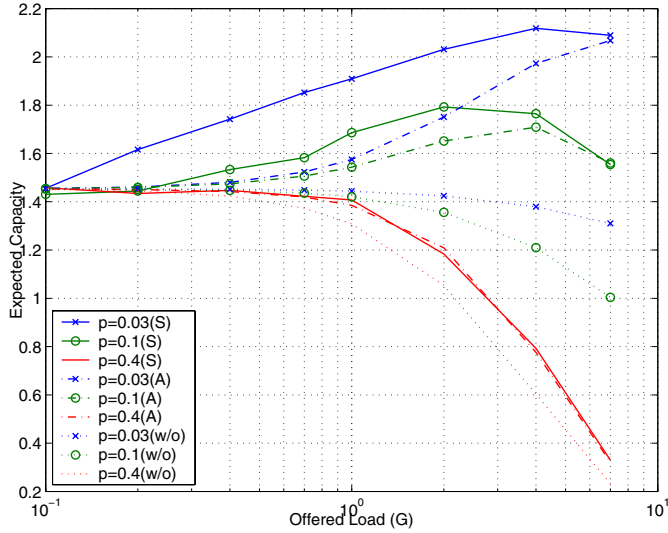


Fig. 4. Comparison of analyzed and simulated expected channel capacity when  $\alpha=0.1$  and average SNR = 10dB. A: Analysis, S: Simulation, w/o: without modified access control.

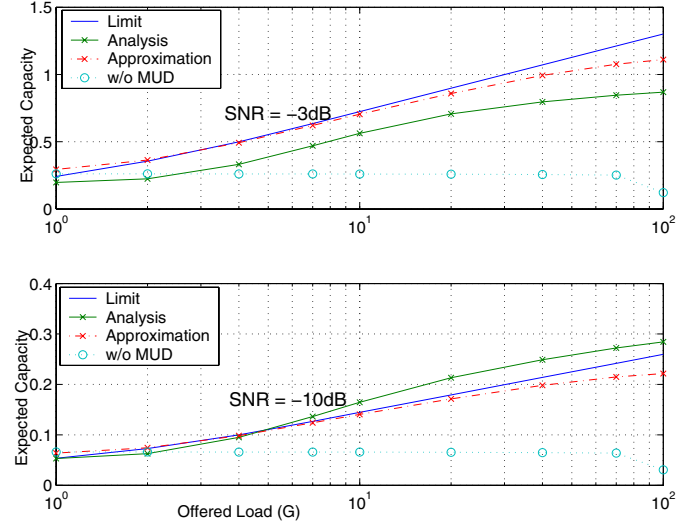


Fig. 6. Comparison of the limit in (13) to the analysis in (10), the approximation in (17), and without modified access control.  $p = 0.001$ ,  $\alpha = 0.01$ , and SNR = -3dB (upper) and -10dB (lower).

**Proposition 2:** For low SNR, the asymptotic expected channel capacity  $C$  as  $p \rightarrow 0$  is

$$C_0 = \lim_{p \rightarrow 0} C \approx \frac{(\gamma + \ln \lambda) P_r}{2}, \quad (13)$$

where  $\gamma = \lim_{n \rightarrow \infty} (\sum_{k=1}^n \frac{1}{k} - \ln n)$  denotes *Euler-Mascheroni* constant.

*Proof:* See Appendix. ■

Under the infinite-user model, the offered load  $G$  approximates the average number of active users in a network which is closely related with the multiuser diversity gain.

#### IV. NUMERICAL RESULTS

In this section, the expected channel capacity and the throughput of  $p$ -persistent CSMA and the OpCSMA are evaluated through simulations in a Rayleigh fading channel. Also, the simulation results are compared to the analytic results provided in Section III. First, a Poisson random variable  $n$  with mean  $\lambda = aG$  is generated at the beginning of each slot during an idle period. This is repeated until  $n > 0$ , then a busy period begins by re-setting  $k = 0$  where  $k$  is the slot index. Second, random variables with the pdf  $f_H(h)$  are assigned to channel gains for newly arrived packets. Then, one of the following three operations is performed depending on  $l$ , the number of packets whose channel gain exceeds  $T_k$ ;  $l = 0$ ,

increase  $n$  by a Poisson random variable with mean  $\lambda = aG$ , increment  $k$ , then repeat the second step;  $l = 1$ , increase  $R$ , the amount of successfully-transmitted data, by  $0.5 \log_2(1 + h_i)$ ;  $l > 1$ , declare a collision. Third, a Poisson random variable  $n$  with mean  $\lambda = (1 + a)G$  is generated at the end of a packet transmission. If  $n$  is zero, the algorithm returns to the first step, otherwise, to the second step. At the end of the simulation, the expected capacity and the throughput are obtained by dividing  $R$  by the number of successfully-transmitted packets, and the total simulation time, respectively. Contrary to the analysis, the simulation allows new packet arrivals during IRTDs. Thus, simulated expected capacity is usually larger than the analytic result because the channel gains for newly arrived packets can be even greater than  $T_{k-1}$ .

The analyzed expected capacity is compared to the simulation result at  $a = 0.01$  and average SNR=0dB in Fig. 3. At small  $p$ , the analysis tends to underestimate expected capacity compared to the simulation. With smaller  $p$ , the duration of an IRTD is longer, which allows more packet arrivals during an IRTD. However, the analytic result well matches to the simulation result when  $p$  is large. Compared to the conventional method, the OpCSMA almost doubles the expected capacity for large  $G$  due to the multiuser diversity gain. However, when  $G < 0.4$ , the gain in the expected capacity disappears because: 1) only one packet has arrived before transmission with high probability; and 2) a busy period consists of only one TP.

In Fig. 4, the analytic results become less accurate as  $a$  increases to 0.1 since the duration of an IRTD is ten-times as long as that in Fig. 3. However, the analysis is still accurate at large  $G$  since, even with small  $p$ , the length of an IRTD decreases as  $G$  increases. The proposed access control also provides the increased expected capacity at reasonably high SNR. As an example, when the average SNR is equal to 10dB and  $G = 7$ , the OpCSMA shows 60% improvement in the expected capacity.

Similarly, the throughput  $S_m$  is significantly enhanced by employing the OpCSMA as shown in Fig. 5. Provided  $G = 7, p = 0.03$ , and SNR=10dB, the proposed method improves the throughput from 1.25 bits/dim to 2.02 bits/dim. At lower average SNR, the relative gain exceeds 100% if the other conditions are maintained. The merit of incorporating multiuser diversity is reduced as  $G$  decreases below 0.5.

At smaller SNR, (17) is an approximation of the expected capacity. Also, Proposition 2 shows the limit of the expected capacity as  $p \rightarrow 0$ . To verify the accuracy of these analysis, approximation results are compared to analytic results in Fig. 6 provided  $p = 0.001, a = 0.01$ . At -10dB SNR, both analysis and limit curves show that the expected capacity linearly increases with  $\ln G$ . In addition, the deviation of the limit curve from the analytic one becomes very small for  $G \leq 10$ . Compared to the expected capacity using the  $p$ -persistent CSMA, the relative gain is as high as 900% at large  $G$ . With higher average SNR of -3dB, similar linear improvement in the expected capacity results from multiuser diversity. In addition, the analytic result no longer exhibits

linear improvement with high  $G$ , which is in part due to drastic increase in the collision probability with high  $G$ .

## V. CONCLUSION

This paper proposes a variant of  $p$ -persistent CSMA called Opportunistic  $p$ -persistent CSMA (OpCSMA) that achieves multiuser diversity gain in a slow fading channel. Under the infinite user model, the expected capacity and the throughput of a  $p$ -persistent CSMA network are significantly improved due to the multiuser diversity gain which the OpCSMA provides. Analytic results demonstrate that the expected channel capacity increases proportional to  $\ln G$  at low SNR where  $G$  is the offered load. Thus,  $G$  may be considered as the effective number of users in the network that is useful in analyzing the degree of multiuser diversity. In particular, the OpCSMA is a promising protocol for 802.11 WLAN since its MAC protocol is well approximated by  $p$ -persistent CSMA as well as the typical WLAN channel is slowly varying. Under the OpCSMA, the probability of accessing the wireless medium is still equal to  $p$ . Therefore, the new systems using the OpCSMA may be deployed without hurting the existing systems with conventional  $p$ -persistent CSMA. However, if the channels are not i.i.d. or the delay constraint of the service is tight, MTs with larger average channel gain tend to utilize the wireless medium for the most of the time, so the proposed method needs to take into account the fairness issues.

## APPENDIX PROOF OF PROPOSITION 1

Using the OpCSMA, MT  $i$  transmits a packet successfully if and only if  $T_{k-1} > h_i \geq T_k$  and  $h_j < T_k$  for  $j \neq i$ . Suppose that the number of packets in the network is  $n$ . From the above property, the expected channel capacity given  $n$  is expressed as

$$\bar{C}_n = E[\max(0.5 \log_2(1 + h_1), \dots, 0.5 \log_2(1 + h_n))], \quad (14)$$

where  $h_i$  is an i.i.d. random variable with a pdf  $f_H(h_i)$  and obviously  $\bar{C}_n \geq \bar{C}_F$  for all  $n$ . From the ergodicity of a channel and (9),

$$\bar{C}_R = \sum_{n=1}^{\infty} \frac{\pi_n}{1 - \pi_0} P_s(n) \bar{C}_n. \quad (15)$$

Comparing (15) and (12),  $\bar{C}_R < \bar{C}''$  since  $\bar{C}_n \geq \bar{C}_F$  for all  $n$ .

## APPENDIX PROOF OF PROPOSITION 2

$C(k)$  is lower bounded as follows since  $\log_2(1 + h)$  is monotonically increasing:

$$\begin{aligned} C(k) &= \int_{T_k}^{T_{k-1}} \frac{e^{-h/P_r}}{2P_r P(T_k \leq h < T_{k-1})} \log_2(1 + h) dh \\ &\geq 0.5 \log_2(1 - P_r \ln(1 - q^{k+1})). \end{aligned} \quad (16)$$

Then, insertion of  $C(k)$  in (16) into (9), and use of  $\log_2(1+x) \approx x$  and  $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ , produces the following approximation for  $\bar{C}_R$ :

$$\bar{C}_R \approx \frac{-P_r p \lambda e^{-\lambda}}{2 - 2e^{-\lambda}} \sum_{k=0}^{\infty} q^k \ln(1 - q^{k+1}) e^{\lambda q^{k+1}}. \quad (17)$$

When  $p$  is small, the summation can be approximated by an integration as the following

$$\begin{aligned} \bar{C}_R &\approx \frac{-P_r p \lambda e^{-\lambda}}{2 - 2e^{-\lambda}} \int_0^{\infty} q^x \ln(1 - q^{x+1}) e^{\lambda q^{x+1}} dx \\ &= \frac{-P_r p e^{-\lambda} (e^{\lambda} (E_1(\lambda) - E_1(\lambda p)) - e^{\lambda q} \ln p)}{(2 - 2e^{-\lambda}) q \ln q}. \end{aligned} \quad (18)$$

For small  $x$ ,  $E_1(x) \approx -\gamma - \ln x$  where  $\gamma = \lim_{n \rightarrow \infty} (\sum_{k=1}^n \frac{1}{k} - \ln n)$  is *Euler-Mascheroni* constant. In addition,  $\lim_{p \rightarrow 0} \frac{p}{q \ln q} = -1$  and  $\lim_{p \rightarrow 0} e^{\lambda q} = e^{\lambda}$ . Applying these approximations into (18) leads to limit of  $\bar{C}_R$  as follows:

$$\lim_{p \rightarrow 0} \bar{C}_R = \frac{(\gamma + \ln \lambda) P_r}{2 - 2e^{-\lambda}}. \quad (19)$$

Finally, for sufficiently large  $G$ ,  $\pi_0 \approx 0$ , then  $C$  at small  $p$  is

$$C_0 = \lim_{p \rightarrow 0} (e^{-\lambda} \bar{C}_F + (1 - e^{-\lambda}) \bar{C}_R) = \frac{(\gamma + \ln \lambda) P_r}{2}. \quad (20)$$

#### REFERENCES

- [1] L. Kleinrock and F. A. Tobagi, "Packet switching in radio channels: Part 1- carrier sense multiple-access modes and their throughput-delay characteristics," *IEEE Trans. on commun.*, vol. com-23, pp. 1400–1416, Dec. 1975.
- [2] P. Venkatasubramanian, S. Adireddy, and L. Tong, "Sensor networks with mobile access: Optimal random access and coding," *IEEE J. Select. Areas Comm.*, vol. 22, pp. 1058–1068, Aug. 2004.
- [3] R. Bruno, M. Conti, and E. Gregori, "Dynamic tuning of the IEEE 802.11 protocol to achieve a theoretical throughput limit," *IEEE/ACM Trans. on Networking*, vol. 8, pp. 785–799, Dec. 2000.
- [4] R. Knopp and P. Humblet, "Information capacity and power control in single cell multiuser communications," *Proc. IEEE ICC*, pp. 331–335, Nov. 1995.
- [5] W. Yu, W. Rhee, and J. M. Cioffi, "Optimal power control in multiple access fading channels with multiple antennas," in *Proc. IEEE ICC*, pp. 575–579, June 2001.
- [6] P. Viswanath, D. Tse, and V. Anantharam, "Asymptotically optimal water-filling in vector multiple-access channels," *IEEE Trans. on Info. Theory*, vol. 47, pp. 241–268, Jan. 2001.
- [7] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. on Info. Theory*, vol. 48, pp. 1277–1294, June 2002.
- [8] J. Chung, C. S. Hwang, K. Kim and Y. K. Kim, "A random beamforming technique in mimo systems exploiting multiuser diversity," *IEEE J. Select. Areas Comm.*, vol. 21, pp. 848–855, June 2003.
- [9] X. Qin and R. Berry, "Exploiting multiuser diversity for medium access control in wireless networks," *Proc. of IEEE INFOCOM*, pp. 1084–1094, 2003.
- [10] A. Paulraj, R. Nabal and D. Gore, *Introduction to space-time wireless communications*. Cambridge university press, 2003.
- [11] C. S. Hwang, K. Seong and J. M. Cioffi, "Power efficient opportunistic p-persistent csma for wireless networks," *Proc. IEEE ICC*, Nov. 2006.
- [12] A. Ephremides and B. Hajek, "Information theory and communication networks: An unconsumed union," *IEEE Trans. on Info. Theory*, vol. 44, pp. 2416–2434, Oct. 1998.
- [13] E. Biglieri, J. Proakis, and S. Shamai, "Fading channel: information theoretic and communication aspects," *IEEE Trans. on Info. Theory*, vol. 44, pp. 2619–2698, Oct. 1998.