

HW #11 - math 355/655

$$1. y = F_4 x = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$$

$$0 = c$$

$$4 = d$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ 2-4i \\ 0 \\ 2+4i \end{bmatrix} = \begin{bmatrix} 0 \\ 1-2i \\ 0 \\ 1+2i \end{bmatrix}$$

$$a_0 = 0 = a_2, \quad b_0 = b_2 = 0$$

$$a_1 = 1 = a_3, \quad b_1 = +2, \quad b_3 = 2$$

$$p_n(t) = \frac{2}{2} \left[ a_1 \cos\left(\frac{2\pi(t-0)}{4}\right) - b_1 \sin\left(\frac{2\pi(t-0)}{4}\right) \right] = \cos\left(\frac{\pi}{2}t\right) + 2\sin\left(\frac{\pi}{2}t\right)$$

$$2. y = F_4 x = F_4 \begin{bmatrix} -3 \\ \frac{1}{2} \\ 3 \\ -\frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} -3 \\ \frac{1}{2} \\ 3 \\ -\frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ -6i \\ 0 \\ -6+i \end{bmatrix} = \begin{bmatrix} 0 \\ -3-\frac{1}{2}i \\ 0 \\ -3+\frac{1}{2}i \end{bmatrix}$$

$$c = 0$$

$$d = 2$$

$$a_0 = 0 = a_2, \quad b_0 = b_2 = 0$$

$$b_1 = -\frac{1}{2}, \quad b_3 = \frac{1}{2}, \quad a_1 = -3, \quad a_3 = -3$$

$$p_n(t) = \frac{2}{2} \left[ a_1 \cos\left(\frac{2\pi(t-0)}{2}\right) - b_1 \sin\left(\frac{2\pi(t-0)}{2}\right) \right] = -3\cos(\pi t) + \frac{1}{2}\sin(\pi t)$$