CS 371/671 Mid-term Study Guide

To be discussed in class on November 13

1 First Order Logic Representation

The goal of this section is to ensure that you understand how to convert knowledge (or statements in English) into formal FOL representation.

1. Represent each of these sentences in first-order predicate calculus (FOPC)
   - a. Mary passed every college class she took.
   - b. There is a book in Wendt Library written by a Canadian.
   - c. John has not read every book in Wendt Library but someone has.
   - d. Sues neighbor is Freds brothers wifes friend
2. Provide a formal interpretation that shows that the following translation from English to FOL is incorrect. Be sure to explain your answer formally using your interpretation:

   "Every cat is owned by David"

   $$\forall x[cat(x) \wedge owner(x, David)]$$

3. For each pair of FOPC wffs below, state its most-general unifier (mgu) or say none exists. Show your work. Universally quantified variables are indicated by ?s
   - a. P(?x,3) P(1,2,3)
   - b. P(?x, ?y, ?z) P(1, f(g(b, a)), f(g(a, b))
   - c. P(?x, f(3), ?x) P(g(1,2), ?z, g(?z, 2))
   - d. P(?x) Q(?x)
   - e. P(?x, ?y, ?x) P(f(?a, ?b), ?b, f(?b, ?b))
4. Do the following sentences in FOL capture the English meaning? Explain why or why not.
   - "all bears have hair": $\forall x, Bear(x) \rightarrow Hair(x)$
   - "there is a bear that has white hair": $\exists x, Bear(x) \rightarrow Hair(x)$
   - "There is an anteater that eats bread": $\exists x, Anteater(x) \rightarrow EatBread(x)$
   - "All anteaters eat ants": $\forall x, Anteater(x) \wedge EatAnts(x)$
5. In your answers for the question below, do not use any function, constant or predicate symbols, other than the ones given at the end of the questions. Translate the below sentences using ONLY constants l and s for logic and Professor Smith respectively, unary predicate symbols I and H for is-instructor and is-happy respectively, and binary predicates C and L where C(x; y) stands for x is child of y, and L(x; y) stands for x likes y.
   1. Each instructor is happy if some of his/her children like logic.
   2. Some instructor is happy if all of his/her children like logic.
   3. Professor Smith is happy if some of her children like logic.

6. Translate the sentences below using the following constants, $c3$, $m2$, c, and l (for $CISC304$, $MATH210$, $CISC$, and $LING$ respectively); unary predicate symbols, S and I (for *is-student*, and *is-instructor* respectively); and binary predicate symbols, C, $T1$, $T2$, and M where

$$C(x;y) \text{ stands for course } x \text{ is offered by department } y,$$
$$T1(x;y) \text{ corresponds to course} x \text{ is taught by } y,$$
$$T2(x;y) \text{ corresponds to course } x \text{ is taken by } y, \text{ and}$$
$$M(x;y) \text{ corresponds to } x \text{ is the major of } y$$

   1. Any student who takes $CISC304$ will also take $MATH210$.
   2. Some instructor of a $CISC$ course has taught a $LING$ course.
   3. Some $CISC$ major has taken all $CISC$ courses.
   4. No instructor who teaches $CISC304$ has taught a course taken by a $LING$ student.
   5. Every $LING$ major has taken a course taught by some instructor who has taught $CISC304$.

7. (8.23) For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it.

   1. No two people have the same social security number.

$$\neg\exists x, y, n \qquad Person(x) \wedge Person(y) \rightarrow [HasSS\#(x,n) \wedge HasSS\#(y,n)].$$

   2. John's social security number is the same as Mary's.

$$\exists n \qquad HasSS\#(John,n) \wedge HasSS\#(Mary,n).$$

   3. Everyone's social security number has nine digits.

$$\forall x, n \qquad Person(x) \rightarrow [HassSS\#(x,n) \wedge Digits(n,9)].$$

   4. Rewrite each of the above (uncorrected) sentences using a function symbol $SS\#$ instead of the predicate $HasSS\#$.

8. (8.24) Represent the following sentences in first-order logic, using a consistent vocabulary:

   1. Some students took French in spring 2011
   2. Every student who takes French passes it
   3. Only one student took Greek in spring 2011
   4. The best score in Greek is always higher than the best score in French
   5. Every person who buys a policy is smart
   6. No person buys an expensive policy
   7. There is an agent who sells policies only to people who are not ensured

   Look at the other parts of the question as well. They are important too.

2 Inference in First-Order Logic

1. Q.9.10 A popular riddle is, "Brothers and sisters have I none, but that man's father is my father's son". Use rules from page 301 to show who the man is.

2. For each of the following atomic sentences give the most general unifier, if it exists :

   1. P(A,B,F(B)), P(x,y,z)
   2. Q(y,G(A,B)), Q(G(x,x), y)
   3. Younger(y,Mother(y)), Younger(John,Mother(x))
   4. Likes(Brother(y),y), Likes(x,x)

3. The following is a knowledge base.

   1. Mick is a IS student
   2. Amy is an CS student
   3. CS students have better jobs than IS students

   Provide a proof that Amy has a better job than Mick.

4. The following is a knowledge base.

   1. Mick is a mouse
   2. Amy is an elephant
   3. An elephant is always afraid of a mouse

   Provide a proof that Amy is afraid of Mick.

5. Convert the following First Order Logic sentence into its Conjunctive Normal Form :

$$\forall x \quad (AIK(x) \leftrightarrow \exists c(AIC(c) \wedge TakenBy(c, x)))$$

6. The following is a story that you will need to translate into a FOL knowledge base (its sentences numbered and described by FOL Clauses) : The college handbook says that it is an offense meriting expulsion from the college when a student copies HW materials from online sites. The online site SOMESITE offers various prepared answers to selected questions from various academic fields. Joe Doe is a student all of whose HWs were directly copied from the site. Prove that Joe Doe should be expelled from the college.

7. What is the difference between forward and backward chaining?

3 Probabilistic Reasoning

Make sure you know the following terms well: Chain Rule, Conditional Distribution, Prior distribution, Posterior Distribution, Marginal Distribution, Conditional Independence, Bayes Rule, evidence, query, inference etc. Also, learn how variable elimination, prior sampling, likelihood weighting and rejection sampling work.

1. Show from first principles that $P(a|b \wedge a) = 1$.

2. Consider the Bayes net shown below. Write out the formula to compute $P(A = T|D = F)$. Compute the value of $P(A = T|D = F)$.

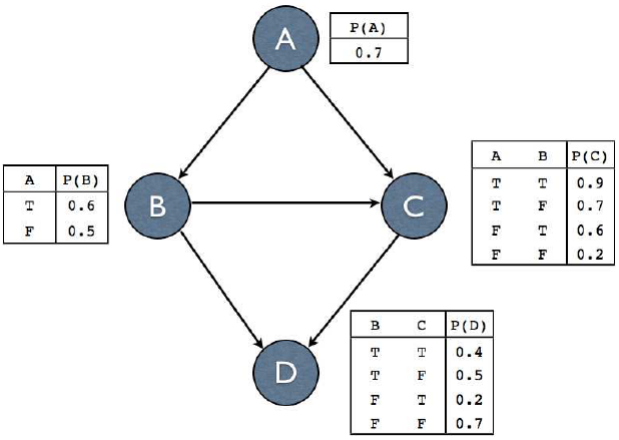

Figure 1:

3. Consider the following Bayes net: $A \rightarrow B \rightarrow C \leftarrow D$

    1. Write out the joint distribution as a product of the conditionals for this network.
    2. How many independent parameters are needed to fully define this network (assuming that all are binary variables)?
    3. How many independent parameters would we need to de
       ne the joint distribution $P(A, B, C, D)$ if we made no assumptions about independence or conditional independence?

4. Consider the following Bayes net: $A \rightarrow B \rightarrow C$. Give an expression for $P(B = 1|C = 0)$ in terms of the parameters of this network. Use notation like $P(C = 1|B = 0)$ to represent individual Bayes net parameters.

5. Consider the belief network below, which extends the electrical domain to include an overhead projector. Answer the following questions about how knowledge of the values of some variables would affect the probability of another variable:

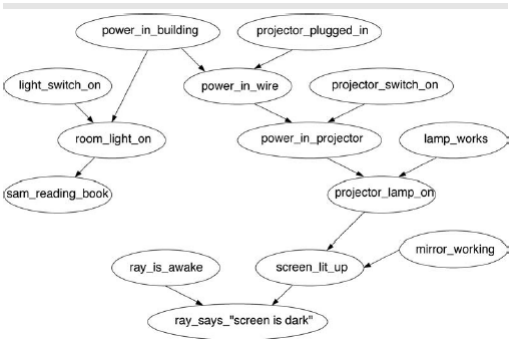

Figure 2:

    1. Can knowledge of the value of Projector plugged in affect your belief in the value of Sam reading book? Explain.
    2. Can knowledge of Screen lit up affect your belief in Sam reading book? Explain.
    3. Can knowledge of Projector plugged in affect your belief in Sam reading book given that you have observed a value for Screen lit up? Explain.
    4. Which variables could have their probabilities changed if just Lamp works was observed?
    5. Which variables could have their probabilities changed if just Power in projector was observed?