CS 188: Artificial Intelligence
Fall 2008

Lecture 18: Decision Diagrams
10/30/2008

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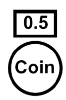
Announcements

- P4 EXTENDED to Tuesday 11/4
- Midterms graded, pick up after lecture
- Midterm course evaluation up on web soon, please fill out!
- Final contest instructions out today!
  - Prizes will be good ☺

Sampling

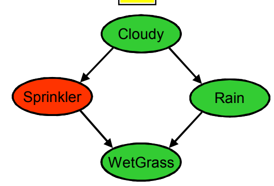
- Basic idea:
  - Draw N samples from a sampling distribution S
  - Compute an approximate posterior probability
  - Show this converges to the true probability P



- Outline:
  - Sampling from an empty network
  - Rejection sampling: reject samples disagreeing with evidence
  - Likelihood weighting: use evidence to weight samples

Prior Sampling

P(C)
.50



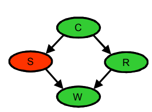
C	P(S|C)
T	.10
F	.50

C	P(R|C)
T	.80
F	.20

S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

Rejection Sampling

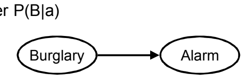
- Let's say we want P(C)
  - No point keeping all samples around
  - Just tally counts of C outcomes
- Let's say we want P(C| s)
  - Same thing: tally C outcomes, but ignore (reject) samples which don't have S=s
  - This is rejection sampling
  - It is also consistent for conditional probabilities (i.e., correct in the limit)



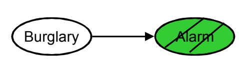
c, ¬s, r, w
c, s, r, w
¬c, s, r, ¬w
c, ¬s, r, w
¬c, s, ¬r, w

Likelihood Weighting

- Problem with rejection sampling:
  - If evidence is unlikely, you reject a lot of samples
  - You don't exploit your evidence as you sample
  - Consider P(B|a)



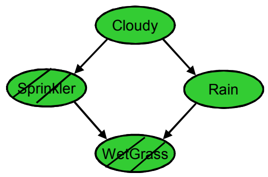
- Idea: fix evidence variables and sample the rest



- Problem: sample distribution not consistent!
- Solution: weight by probability of evidence given parents

Likelihood Sampling

P(C)
.50



C	P(S|C)
T	.10
F	.50

C	P(R|C)
T	.80
F	.20

$w = 1.0 \times 0.1 \times 0.99$

S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

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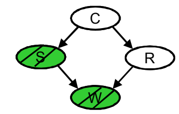
Likelihood Weighting

- Sampling distribution if z sampled and e fixed evidence

$$S_{WS}(\mathbf{z},\mathbf{e}) = \prod_{i=1}^{l} P(z_i|\text{Parents}(Z_i))$$



- Now, samples have weights

$$w(\mathbf{z},\mathbf{e}) = \prod_{i=1}^{m} P(e_i|\text{Parents}(E_i))$$

- Together, weighted sampling distribution is consistent

$$S_{WS}(\mathbf{z},\mathbf{e})w(\mathbf{z},\mathbf{e}) = \prod_{i=1}^{m} P(e_i|\text{Parents}(E_i)) \prod_{i=1}^{m} P(e_i|\text{Parents}(E_i))$$

$$= P(\mathbf{z},\mathbf{e})$$

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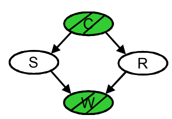
Likelihood Weighting

- Note that likelihood weighting doesn't solve all our problems
- Rare evidence is taken into account for downstream variables, but not upstream ones
- A better solution is Markov-chain Monte Carlo (MCMC), more advanced
- We'll return to sampling for robot localization and tracking in dynamic BNs



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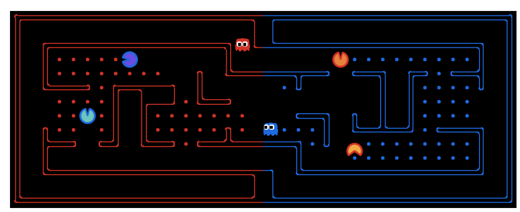
Pacman Contest



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Recap: Inference Example

- Find P(W|F=bad)
  - Restrict all factors

W	P(W)
sun	0.7
rain	0.3

$P(W)$

W	P(F=bad|W)
sun	0.2
rain	0.9

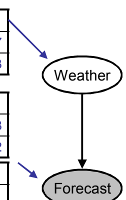
$P(bad|W)$

  - No hidden vars to eliminate (this time!)
  - Just join and normalize

W	P(W)
sun	0.7
rain	0.3

F	P(F|sun)
good	0.8
bad	0.2

F	P(F|rain)
good	0.1
bad	0.9



W	P(W,F=bad)
sun	0.14
rain	0.27

$P(W, bad) = P(W) \times P(bad|W)$

W	P(W | F=bad)
sun	0.34
rain	0.66

$P(W|F = \text{bad})$

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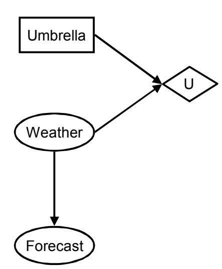
Decision Networks

- MEU: choose the action which maximizes the expected utility given the evidence
- Can directly operationalize this with decision diagrams
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action
- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, must be parents, act as observed evidence)
  - Utilities (depend on action and chance nodes)



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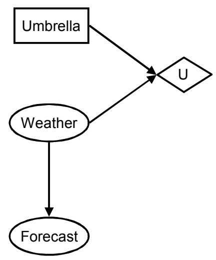
Decision Networks

- Action selection:
  - Instantiate all evidence
  - Calculate posterior over parents of utility node
  - Set action node each possible way
  - Calculate expected utility for each action
  - Choose maximizing action



Example: Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_w P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

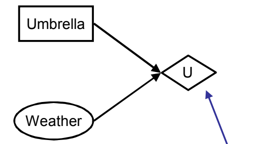
Umbrella = take

$$\mathrm{EU}(\mathrm{take}) = \sum_w P(w)U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$



W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Example: Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{bad}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

Umbrella = take

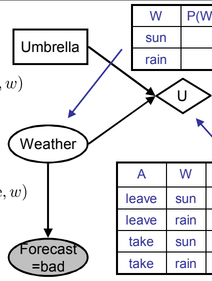
$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

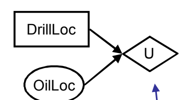
W	P(W|F=bad)
sun	0.34
rain	0.66



A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Value of Information

- Idea: compute value of acquiring each possible piece of evidence
  - Can be done directly from decision network
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - Prior probabilities 0.5 each, mutually exclusive
  - Current price of each block is k/2
  - MEU = 0 (either action is a maximizer)
- Solution: compute value of information
  = expected gain in MEU from observing new information
- Probe gives accurate survey of A. Fair price?
  - Survey may say "oil in a" or "oil in b," prob 0.5 each
  - If we know O, MEU is k/2 (either way)
  - Gain in MEU?
  - VPI(O) = k/2
  - Fair price: k/2



O	P
a	1/2
b	1/2

D	O	U
a	a	k/2
a	b	-k/2
b	a	-k/2
b	b	k/2

Value of Information

- Current evidence E=e, utility depends on S=s

$$\mathrm{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- Potential new evidence E': suppose we knew E' = e'

$$\mathrm{MEU}(e, e') = \max_a \sum_s P(s|e, e')\, U(s,a)$$

- BUT E' is a random variable whose value is currently unknown, so:
  - Must compute expected gain over all possible values

$$\mathrm{VPI}_e(E') = \sum_{e'} P(e'|e) \left(\mathrm{MEU}(e, e') - \mathrm{MEU}(e)\right)$$

- (VPI = value of perfect information)

VPI Example

MEU with no evidence

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

MEU if forecast is good

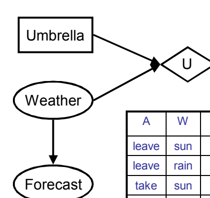
$$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

Forecast distribution

F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95 - 70) + 0.41 \cdot (53 - 70)$$

$$0.59 \cdot (+25) + 0.41 \cdot (-17) = +22$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$\mathrm{VPI}_e(E') = \sum_{e'} P(e'|e) \left(\mathrm{MEU}(e, e') - \mathrm{MEU}(e)\right)$$

VPI Properties

- Nonnegative in expectation

$$\forall E', e : \text{VPI}_e(E') \geq 0$$

- Nonadditive ---consider, e.g., obtaining E_j twice

$$\text{VPI}_e(E_j, E_k) \neq \text{VPI}_e(E_j) + \text{VPI}_e(E_k)$$

- Order-independent

$$\begin{aligned}\text{VPI}_e(E_j, E_k) &= \text{VPI}_e(E_j) + \text{VPI}_{e,E_j}(E_k) \\ &= \text{VPI}_e(E_k) + \text{VPI}_{e,E_k}(E_j)\end{aligned}$$

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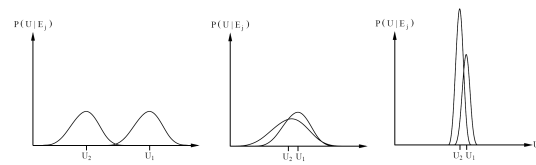
VPI Scenarios

- Imagine actions 1 and 2, for which $U_1 > U_2$
- How much will information about E_j be worth?



Little – we're sure action 1 is better.

A lot – either could be much better

Little – info likely to change our action but not our utility

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Reasoning over Time

- Often, we want to reason about a sequence of observations
  - Speech recognition
  - Robot localization
  - User attention
  - Medical monitoring

- Need to introduce time into our models
- Basic approach: hidden Markov models (HMMs)
- More general: dynamic Bayes' nets

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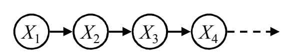
Markov Models

- A Markov model is a chain-structured BN
  - Each node is identically distributed (stationarity)
  - Value of X at a given time is called the state
  - As a BN:



$$P(X_1) \qquad P(X|X_{-1})$$

- Parameters: called transition probabilities or dynamics, specify how the state evolves over time (also, initial probs)

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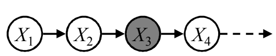
[DEMO: Battleship]

Conditional Independence



- Basic conditional independence:
  - Past and future independent of the present
  - Each time step only depends on the previous
  - This is called the (first order) Markov property

- Note that the chain is just a (growing) BN
  - We can always use generic BN reasoning on it (if we truncate the chain)

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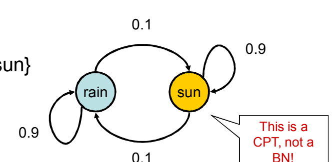
Example: Markov Chain

- Weather:
  - States: X = {rain, sun}
  - Transitions:



  - Initial distribution: 1.0 sun
  - What's the probability distribution after one step?

$$P(X_2 = \text{sun}) = P(X_2 = \text{sun}|X_1 = \text{sun})P(X_1 = \text{sun}) + P(X_2 = \text{sun}|X_1 = \text{rain})P(X_1 = \text{rain})$$

$$0.9 \cdot 1.0 + 0.1 \cdot 0.0 = 0.9$$

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Mini-Forward Algorithm

- Question: probability of being in state x at time t?
- Slow answer:
  - Enumerate all sequences of length t which end in s
  - Add up their probabilities

$$P(X_t = sun) = \sum_{x_1 \ldots x_{t-1}} P(x_1, \ldots x_{t-1}, sun)$$

$$P(X_1 = sun)P(X_2 = sun|X_1 = sun)P(X_3 = sun|X_2 = sun)P(X_4 = sun|X_3 = sun)$$

$$P(X_1 = sun)P(X_2 = rain|X_1 = sun)P(X_3 = sun|X_2 = rain)P(X_4 = sun|X_3 = sun)$$

$$\vdots$$

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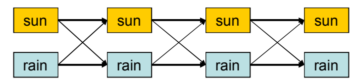
Mini-Forward Algorithm

- Better way: cached incremental belief updates
  - An instance of variable elimination!



$$P(x_t) = \sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1})$$

$$P(x_1) = \text{known}$$

Forward simulation

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Example

- From initial observation of sun

$$\left\langle \begin{matrix} 1.0 \\ 0.0 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.9 \\ 0.1 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.82 \\ 0.18 \end{matrix} \right\rangle \quad \Rightarrow \quad \left\langle \begin{matrix} 0.5 \\ 0.5 \end{matrix} \right\rangle$$

$P(X_1)$ $P(X_2)$ $P(X_3)$ $P(X_\infty)$

- From initial observation of rain

$$\left\langle \begin{matrix} 0.0 \\ 1.0 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.1 \\ 0.9 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.18 \\ 0.82 \end{matrix} \right\rangle \quad \Rightarrow \quad \left\langle \begin{matrix} 0.5 \\ 0.5 \end{matrix} \right\rangle$$

$P(X_1)$ $P(X_2)$ $P(X_3)$ $P(X_\infty)$

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Stationary Distributions

- If we simulate the chain long enough:
  - What happens?
  - Uncertainty accumulates
  - Eventually, we have no idea what the state is!
- Stationary distributions:
  - For most chains, the distribution we end up in is independent of the initial distribution
  - Called the stationary distribution of the chain
  - Usually, can only predict a short time out

[DEMO: Battleship]

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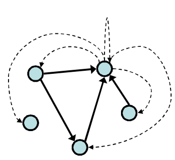
Web Link Analysis

- PageRank over a web graph
  - Each web page is a state
  - Initial distribution: uniform over pages
  - Transitions:
    - With prob. c, uniform jump to a random page (dotted lines)
    - With prob. 1-c, follow a random outlink (solid lines)



- Stationary distribution
  - Will spend more time on highly reachable pages
  - E.g. many ways to get to the Acrobat Reader download page!
  - Somewhat robust to link spam
  - Google 1.0 returned the set of pages containing all your keywords in decreasing rank, now all search engines use link analysis along with many other factors

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Most Likely Explanation

- Question: most likely sequence ending in x at t?
  - E.g. if sun on day 4, what's the most likely sequence?
  - Intuitively: probably sun all four days
- Slow answer: enumerate and score

$$P(X_t = sun) = \max_{x_1 \ldots x_{t-1}} P(x_1, \ldots x_{t-1}, sun)$$

$$P(X_1 = sun)P(X_2 = sun|X_1 = sun)P(X_3 = sun|X_2 = sun)P(X_4 = sun|X_3 = sun)$$

$$P(X_1 = sun)P(X_2 = rain|X_1 = sun)P(X_3 = sun|X_2 = rain)P(X_4 = sun|X_3 = sun)$$

$$\vdots$$

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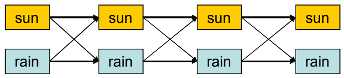
Mini-Viterbi Algorithm

- Better answer: cached incremental updates



- Define: $m_t[x] = \max_{x_{1:t-1}} P(x_{1:t-1}, x)$

$$a_t[x] = \arg\max_{x_{1:t-1}} P(x_{1:t-1}, x)$$

- Read best sequence off of m and a vectors

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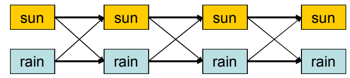
Mini-Viterbi



$$\begin{aligned} m_t[x] &= \max_{x_{1:t-1}} P(x_{1:t-1}, x) \\ &= \max_{x_{1:t-1}} P(x_{1:t-1}) P(x|x_{t-1}) \\ &= \max_{x_{t-1}} P(x_t|x_{t-1}) \max_{x_{1:t-2}} P(x_{1:t-1}) \\ &= \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x] \end{aligned}$$

$$m_1[x] = P(x_1)$$

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