Math 355/655: Introduction to Numerical Methods
Homework #3–part 1
Due: September 20, 2012

This is the FIRST part of the homework. More will be posted after Tuesday's lecture.

1. For each sequence below: (i) Determine what the sequence converges to, and (ii) use the **definition** of linear and quadratic convergence to determine if they determine linearly or quadratically:

   (a) $\{x_n\}_{n=1}^{\infty}$ with $x_n = n^{-10}$.

   (b) $\{x_n\}_{n=1}^{\infty}$ with $x_n = 5 + 3^{-2^n}$.

   (c) Newton's method's sequence of approximations on $f(x) = x^k$ with $k > 1$ using $x_0 \neq 0$. Does your answer match the theorem we discussed in class concerning multiple roots?

2. (MATLAB) Last week, you found that Newton's method converged linearly when finding a root of $f(x) = x(1 - e^x)$. This week, we are going to see if we can find a way to obtain quadratic convergence to the root $r = 0$. Consider the following variations of Newton's method:

   - Variation #1: Apply Newton's method instead to the problem of finding a root of $h(x)$ (i.e., find r such that $h(r) = 0$), where $h(x) = f(x)/f'(x)$.
   - Variation #2: Modify Newton's method approximations as follows:

$$x_{k+1} = x_k - s\frac{f(x_k)}{f'(x_k)}.$$

   (a) Modify your `my_newton.m` file so that it uses Variation #1. Run the code with `tol=1e-8` and $x_0 = 0.1$. Does this variation look linearly or quadratically convergent to the root $r = 0$? Save the output as well as your modified `my_newton.m` file and turn these both in. (Hint: Be very careful with parentheses; when in doubt, use them!)

   (b) Start with a fresh copy of `my_newton.m` (i.e., so it uses the regular Newton method and not Variation #1–you may re-download it from the class webpage). Modify the file so it uses Variation #2 to find a root of f. With $s = 2$, `tol=1e-15`, and $x_0 = 0.1$, run Variation #2. Does this variation look linearly or quadratically convergent to the root $r = 0$? Save the output as well as your modified file and turn these both in.

   (c) Comment on your results in parts (a) and (b). For inspiration, you may want to re-live part of the previous homework assignment by running Newton's method (without modifications) on $f(x)$ to find a root with $x_0 = 0.1$.