

Numerical Linear Algebra –Project one

Due on Oct 04

Requirements:

- i) Output all theoretical proofs, statements, numerical results and conclusions in a tex file. Use the template on Sakai.
- ii) Compress all related files (including .tex file, .pdf file, .m file, and .mat file) into a zip file and upload the zip file to the dropbox.

1. Theoretical Part:

- 3.1, 3.2, 3.6, 5.3, 5.4, 6.3, 7.5
- Let $A = \hat{U}\hat{\Sigma}\hat{V}^*$ be the reduced SVD of the m-by-n matrix ($m \geq n$). If A has full rank, the solution of $\min_x \|Ax - b\|_2$ is $x = \hat{V}\hat{\Sigma}^{-1}\hat{U}^T b$

2. Numerical Experiments:

- Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix} \in \mathbb{R}^{4 \times 3},$$

where $\varepsilon \in \mathbb{R}$ is some sufficiently small number. Letting $\varepsilon = 1, 10^{-3}, 10^{-5}, 10^{-8}$ respectively, compute the QR factorization of A by using the classic and modified Gram-Schmidt algorithm. Numerically check the orthogonality of columns of Q s. State the numerical observations and conclusions. (You need to show that the numerical results support your conclusions)

- 10.2, 10.3