Blind iterative restoration of images with spatially-varying blur: with an overview of blind deconvolution

John Bardsley[1], Stuart Jefferies[2], James Nagy[3] and Robert Plemmons[4]

1) Dept. of Mathematical Sciences, Univ. of Montana
2) Institute for Astronomy, Univ. Hawaii
3) Dept. of Mathematics and Computer Science, Emory Univ.
4) Depts. of Mathematics & Computer Science, Wake Forest Univ.

PRET AFOSR grant annual review, Maui HI, January 2011

Outline

- Quick overview of blind deconvolution, i.e spatially invariant case
  - From non-convex to convex
  - Recursive inverse filtering, and phase diversity
- Spatially-variant case
  - What causes spatially-varying blur?
  - Correcting for spatially-varying blur
  - Blind restoration algorithm
  - Results from new approach
  - Summary
- If time: Compressing hyperspectral data

Image Formation Model

The mathematical model for image formation is given by the linear operator equation

$$d = Sf + \eta,$$

where d is the blurred, noisy image, f is the unknown true image, η represents noise, and S is the blurring operator. In the case of spatially invariant blurs, Sf can be written as a convolution of the associated point spread function (PSF) s and the object f; that is,

$$Sf(u,v) = s \star f(u,v) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(u-\xi, v-\gamma) f(\xi,\eta) d\xi d\gamma.$$

In the **blind deconvolution case** both s and f are unknown.

Overview of Blind Deconvolution (not multiframe)

- The joint minimization problem:

$$\min_{s,f} \|s * f - d\|_2^2 + \alpha_1 J(f) + \alpha_2 J(s)$$

- A solution approach: alternating iterations for s and f. α_1 and α_2 regularization parameters. Functionals $J(f)$ and $J(s)$ impose regularization constraints.

- Problem is nonconvex.

Extensive Bibliography on Image Restoration, and more

- Web based bibliography by Keith Price, USC. Thousands of references.
- http://iris.usc.edu/Vision-Notes/bibliography/contents.html
- Includes over 200 papers on BD, May 1966 through December 2005, 9 on spatially-variant (adaptive) restoration.
- Last updated January 14, 2006.

For Example: Papers by T. Chan & C.K. Wong

- IEEE Trans. IP (1998). BD using Total variation regularization. Method has been patented (other BD algorithms have also been patented).

- LAA (2000). Alternating iterations convergence results under strong assumptions.

Book: “Blind Image Deconvolution: Theory and Applications”, Edited by Campisi & Egiazarian CRC Press 2006

1. Katsaggelos, Malona, T. Chan. “Blind image deconvolution: An Overview”
2. Campisi, et al. “Multichannel blind deconvolution using bussgang techniques: Applications to texture synthesis”
3. Likas, Galatsanos. “Baysian methods based on variational approximations for blind image deconvolution”
4. Kundar. “Blind image restoration using recursive inverse filtering”
5. Katkovnik, et al. “Frequency domain blind deconvolution for multiframe imaging”

BD Book: Continued

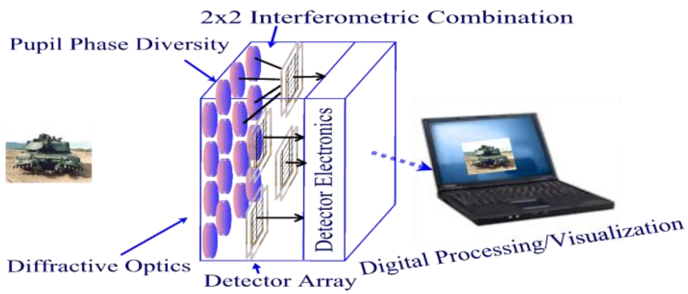
7. Zerubia, et al. “Deconvolution of satellite and aerial images”
8. Adam, et al. “Deconvolution of medical images”
9. Sroubek, Flusser. “Multiframe blind deconvolution with frame registration and resolution enhancement”
10. Hatzinakos, et al., “Blind reconstruction of multiframe imagery based on fusion and classification”
11. Ng, Ple. “Blind deconvolution and structured total least squares computations with applications to array imaging”

Advanced Imaging System- PERIODIC

PIs -- Sudhakar Prasad & Robert Plemmons
Mar 1, 2005 – Present

Program Objective

• To analyze, optimize, simulate, design, and fabricate a beta prototype, integrated optical-digital array-based imaging system.



Expected Outcomes

• Design, fabrication, and testing of high resolution, array imaging, inexpensive camera with large field of view, large depth of field, with a short focus and with a thin profile. The camera is expected to be useful in providing a dramatic increase in image information.

• Prototype thin camera based on compact array of refractive and/or diffractive lenslets.

Approach

• Research and development studies involving mathematical and engineering technology to optimize and fabricate an array camera system with low cost, and enhanced reconfigurability.

• Images to be combined optically and digitally with super-resolution reconstruction methods to achieve a primary image with high resolution, dynamic range, and focal volume.

Accomplishments / Milestones

• Three productive workshops held in 2005.

• Excellent DoD participation and interest in this work by ARO, ARL, DARPA.

• 2005 Milestones: information-based noise analysis, computer simulation studies, and camera prototype fabrication. An overview of the current status of PERIODIC was given at an OSA Conference in October.

Advanced Imaging System- PERIODIC

Challenges
- Optical design of lenslets to provide sufficient resolution over the field of view.
- Refinement of registration algorithms to enable digital resolution enhancement.

Plan for FY06
- Three additional workshops scheduled through August 2006, to study and develop high resolution array image reconstruction, system sampling rate and noise considerations, and diffractive array camera design plans.
- Final 4x4 array imaging prototypes are targeted for August 2006.

Financial Update
- ((As of 31 Dec 2005, burned XYZ amount; ABC remains. Avg burn per month is XXX.))

Customer: Intelligence Community. Government Champion is Dr. Michael King, michack@ucia.gov . Applications include Iris-based biometrics for ID and verification; and for possible use as compact, thin, helmet, vehicle, or aircraft mountable conformal cameras.

What is the Warfighter payoff? PERIODIC array imagers are a key step toward enabling ubiquitous battlefield imaging. The potential for low cost, compactness, and reconfigurability points to the ability to provide a dramatic increase in battlefield image information without overwhelming Warfighter command centers with data.

Nonconvex to Convex – Eliminate Object from Objective Function

- Recursive (inverse) filtering approaches
  - Kundar & Hatzinakos, IEEE Trans. SP (1998)
  - Ng & Plemmons, IEEE Trans. IP (2000)

- Phase diversity approach (details later)
  - Gonsalves: Opt. Eng. (1982)
  - Vogel, Chan, Ple.: SPIE Kona (1998)
  - Gilles, Vogel, Bardsley: Inverse Problems (2002)
  - Others…

“Blind iterative restoration of images with spatially-varying blur”
Bardsley/Jefferies/Nagy/Plemmons

→ Spatially-variant case
- What causes spatially-varying blur?
- Correction of spatially-varying blur
- Blind restoration algorithm
- Results from new approach
- Summary

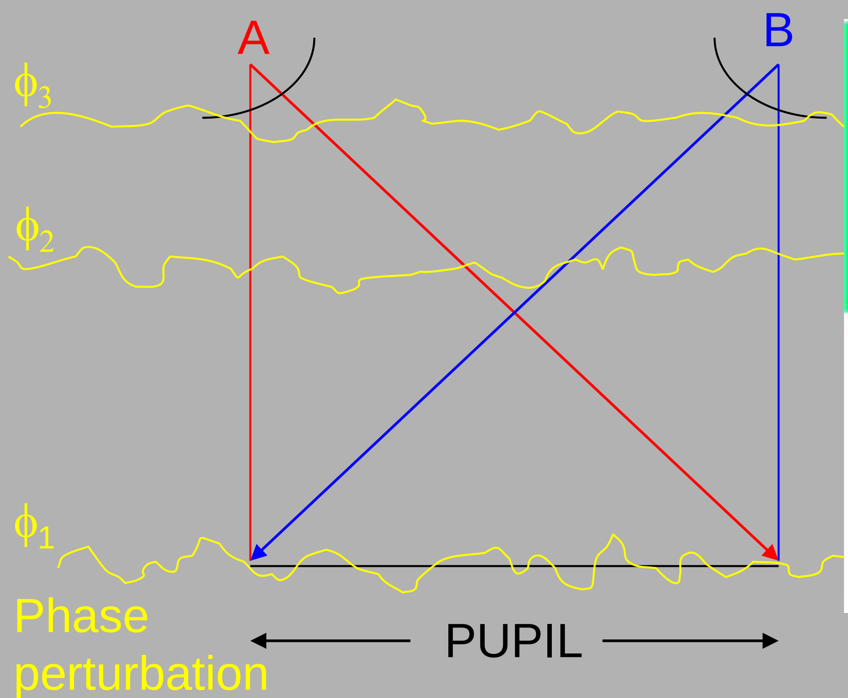
Phase

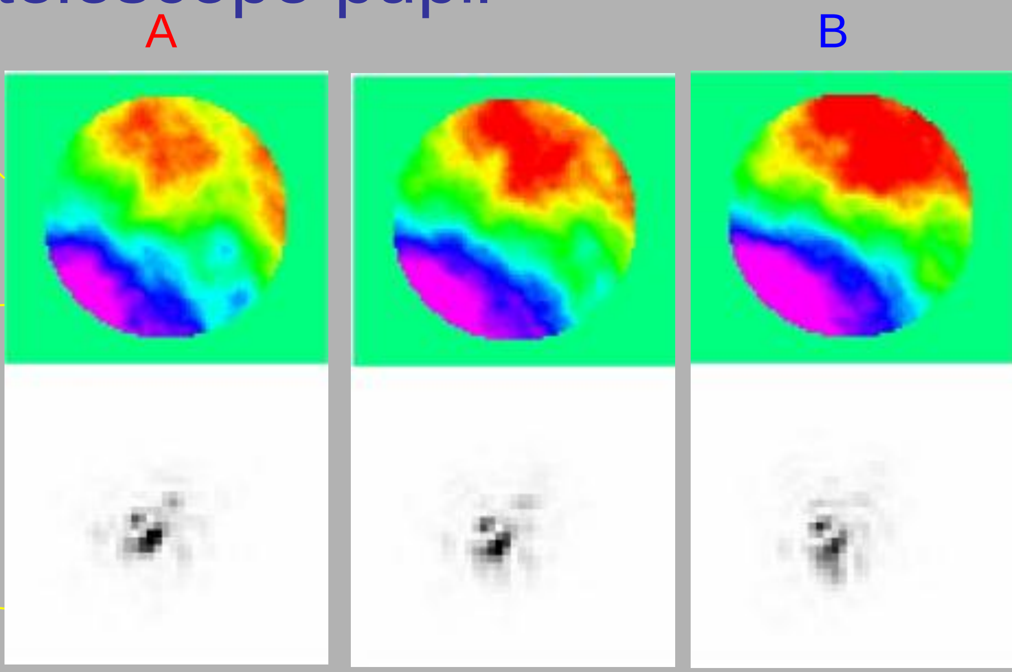
- In atmospheric optics, phase $\phi(x,y)$ denotes the deviation of the wave front from a reference planar wave front.
- Deviation caused by variations in the index of refraction (wave speed) along light ray paths.
- Strong dependence of index of refraction on atmospheric air temperature.
- Because of atmospheric turbulence, phase varies with time and position in space. Often modeled as a stochastic process.

Q. What causes spatially-varying blur?

A. Many sources, e.g.,

- objects moving at different velocities, directions
- distortions in telescope optics
- turbulence outside the telescope pupil





PSF: $s[\phi] = |\mathcal{F}^{-1}(pe^{\imath\phi})|^2$

Q. How can we correct for spatially-varying blur?

A1. Estimate phase screens at multiple heights in forward model

Computationally expensive

A2. - Partition image into segments over which blur is spatially invariant

- restore each segment
- “sew” restored segments back together to form final image.

Approach leaves artifacts in final image (region boundaries)

A3. - Segment image

- estimate point spread function (PSF) for each segment
- determine PSF at each location in image by interpolating between phase diversity measured PSFs,
- restore image globally.

Mathematics of Phase Diversity

- The k^{th} diversity image

  $$d_k = s[\phi+\theta_k] ¤ f + \eta_k$$

- d_k is the observed image, f is the true image
- s is the point spread function (PSF)
- ¤ represents convolution product
- η_k represents noise
- ϕ is the phase function
- θ_k is the k^{th} phase diversity function

Diversity Functions θ_k

- By placing beam splitters in light path and modifying phase differently in each of the resulting paths, we obtain more data.
- Phase diversity functions θ_k represent these deliberate phase modifications applied after light is collected by primary mirror.
- Easiest to implement is defocus blur, modeled by quadratic

  $\theta_k(x,y) = c_k\ (x^2 + y^2)$.

  Parameters c_k are determined by defocus lengths.

PSF Estimation: PD Minimization Problem

- To estimate the phase ϕ, and the object f from data we consider the least squares fit-to-data functional

$$J_{data}[\phi,f] = \sum_k ||s[\phi + \theta_k] ¤ f - d_k||^2.$$

- In Fourier space: $J_{data}[\phi,F] = \sum_k ||S_k[\phi]F - D_k||^2$
- Full minimization functional:

$$J_{full}[\phi, f] = J_{data}[\phi, f] + \gamma J_{object}[f] + \alpha J_{phase}[\phi]$$

Reduced Minimization Functional

- By setting the gradient with respect to the object equal to zero, one obtains the object at a minimizer for J_{full}.
- One obtains the Fourier representation for the minimizing object, $F = P[\phi]^*/Q[\phi]$ where

  $P[\phi] = \sum_k D_k^* S_k[\phi]$, $Q[\phi] = \gamma + \sum_k |S_k[\phi]|^2$
- Leads to reduced minimization functional

$J[\phi] = J_{reduced}[\phi] + \alpha J_{phase}[\phi]$, where

$$J_{reduced}[\phi] = \sum_k ||D_k||^2 - \left\langle \frac{P[\phi]}{Q[\phi]}, P[\phi] \right\rangle .$$

Blind restoration algorithm

- Use phase diversity scheme - two data channels, one with additional (known defocus) perturbation
- Assume isoplanatic imaging model and minimize J(ϕ): || data - model ||^2 + regularization term

Regularization: power spectrum of phases has form

$$\Phi(\omega) = \frac{C_1}{(C_2 + |\omega|^2)^{11/6}}$$

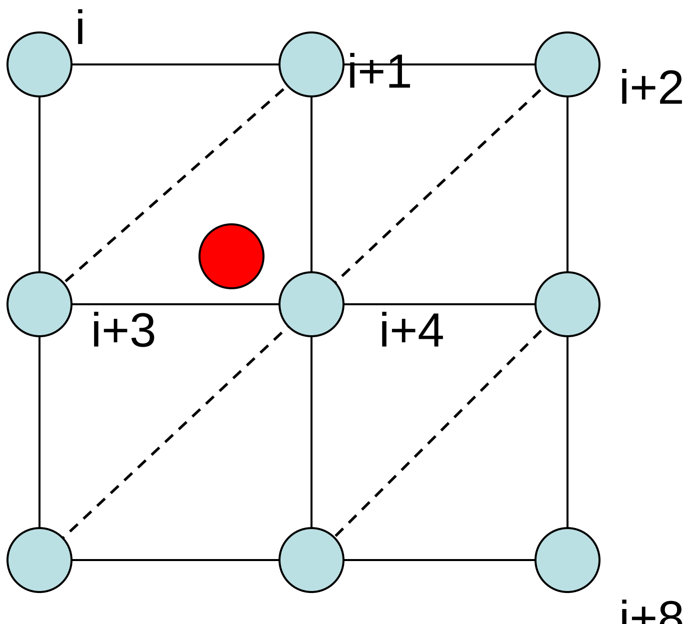
- Limited memory BFGS algorithm used
- Determine PSFs from phase estimates

$$s[\phi] = |\mathcal{F}^{-1}(pe^{\imath\phi})|^2$$

Model for spatially-varying blur

- Problem to solve: $g = Sf$
- Model blurring matrix using: $S = \Sigma_i D_i S_i$

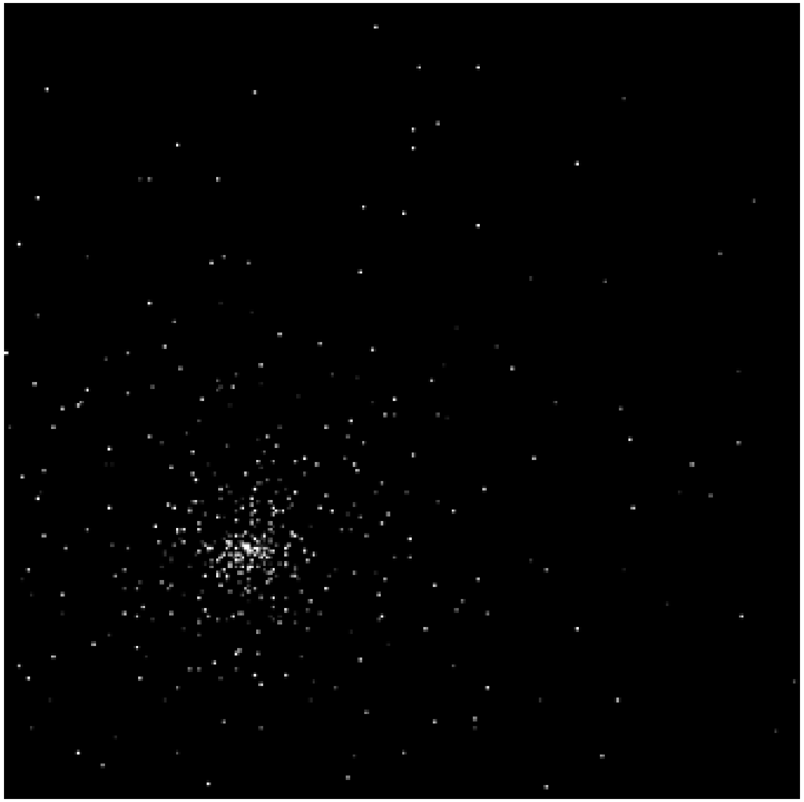
  - S_i is matrix representing PSF for region (i)
  - D_i is non-negative diagonal matrix: $\Sigma_i D_i = I$
  - use piecewise linear interpolation to "sew" individual PSFs



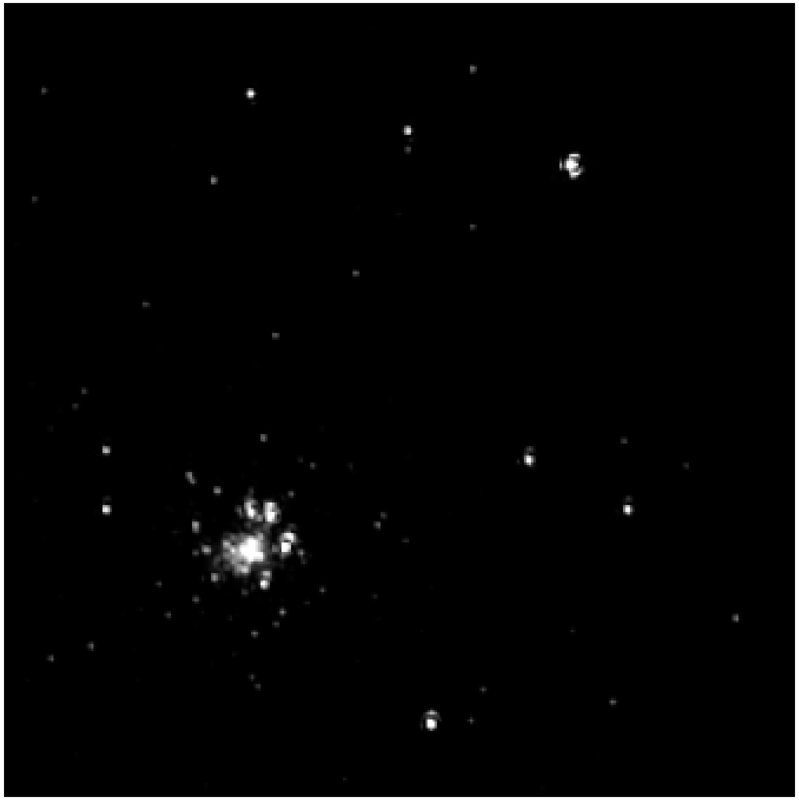
- observed PSF
- S depends on obser. PSFs at i+1, i+3 & i+4

Test of Algorithm

- Use simulated star cluster from Space Telescope. Sci. Inst.

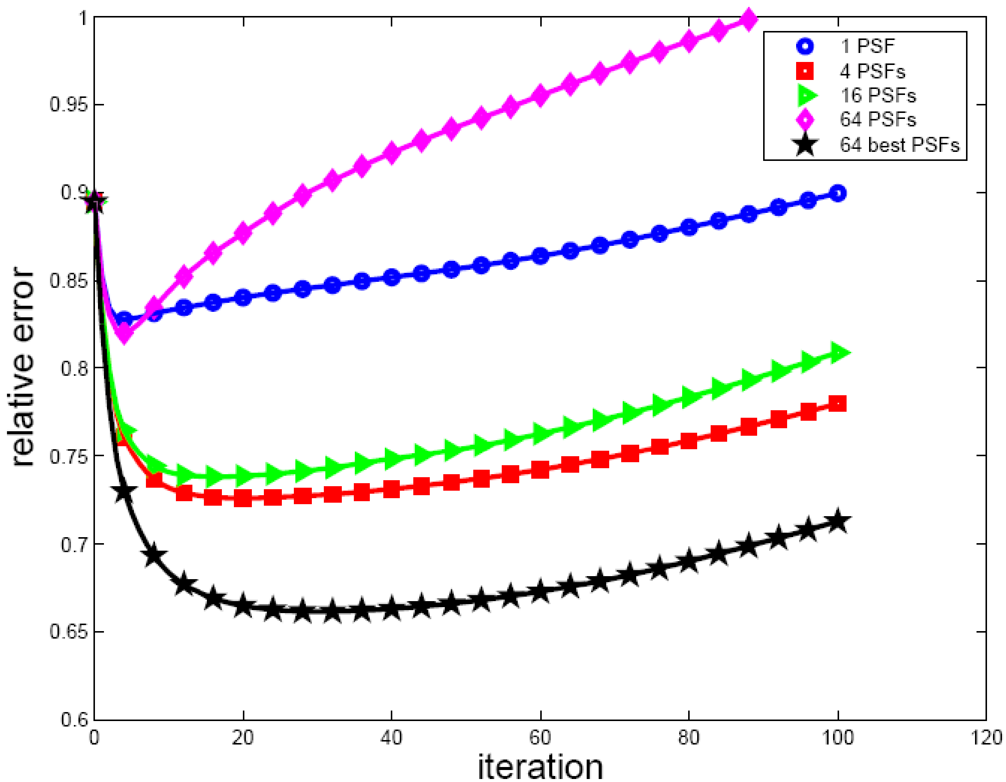


Truth



Blurred + noise

Results

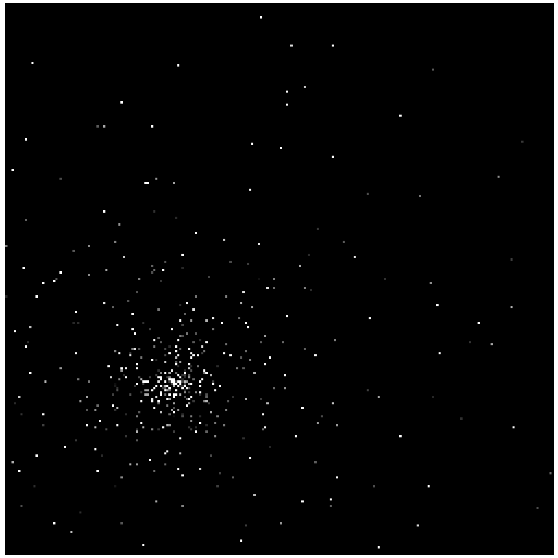


Relative error = $$\frac{||f_{\text{true}} - f_k||_2}{||f_{\text{true}}||_2}$$

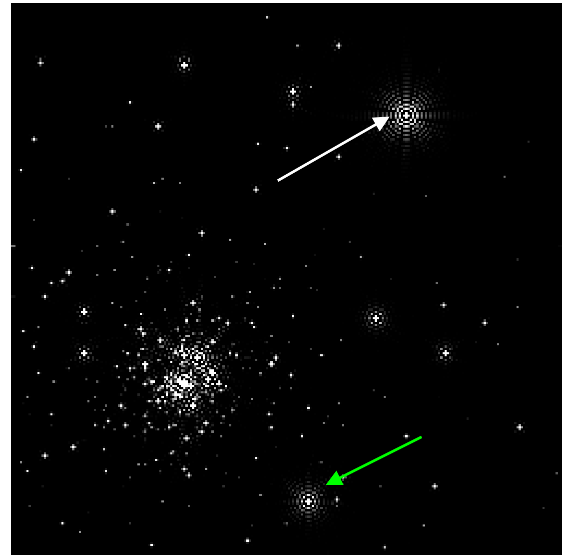
- isoplanatic restoration leaves residual “carpet” of speckles - reduces useful dynamic range
- error increases for large number of segments due to regions with little information
- PSF “mixing” - select segment size for best PSF approximation – small to larger regions

Results

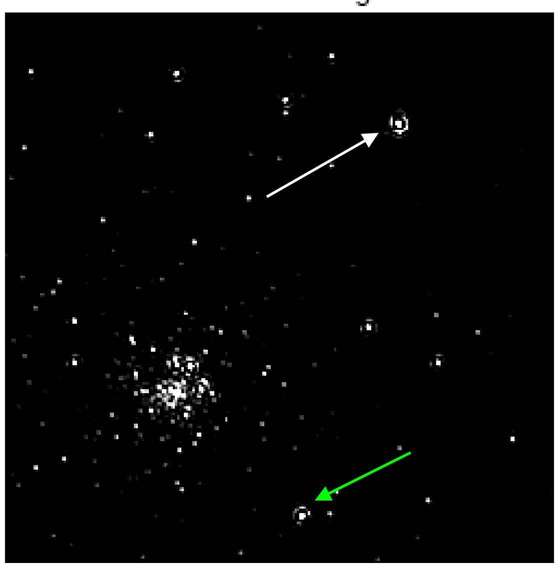
True image.



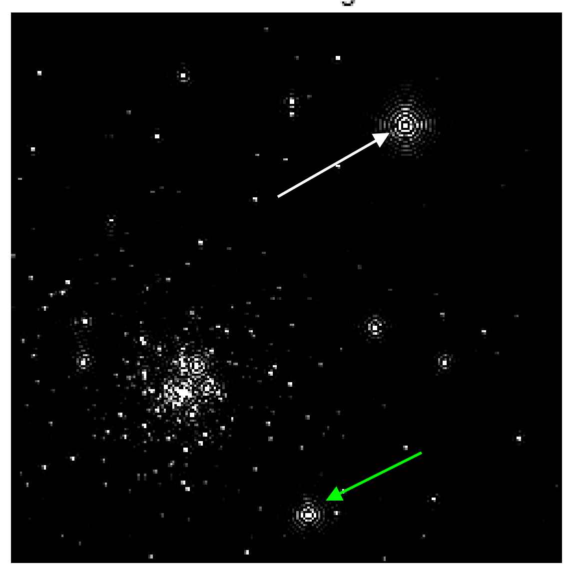
Diffraction limited true image.



Reconstruction using 1 PSF



Reconstruction using mix PSFs



Summary

- Overview of blind image restoration

- Efficient algorithm for removing spatially-varying blur, when blur is unknown

- Significantly better restorations than when treating blur as spatially invariant

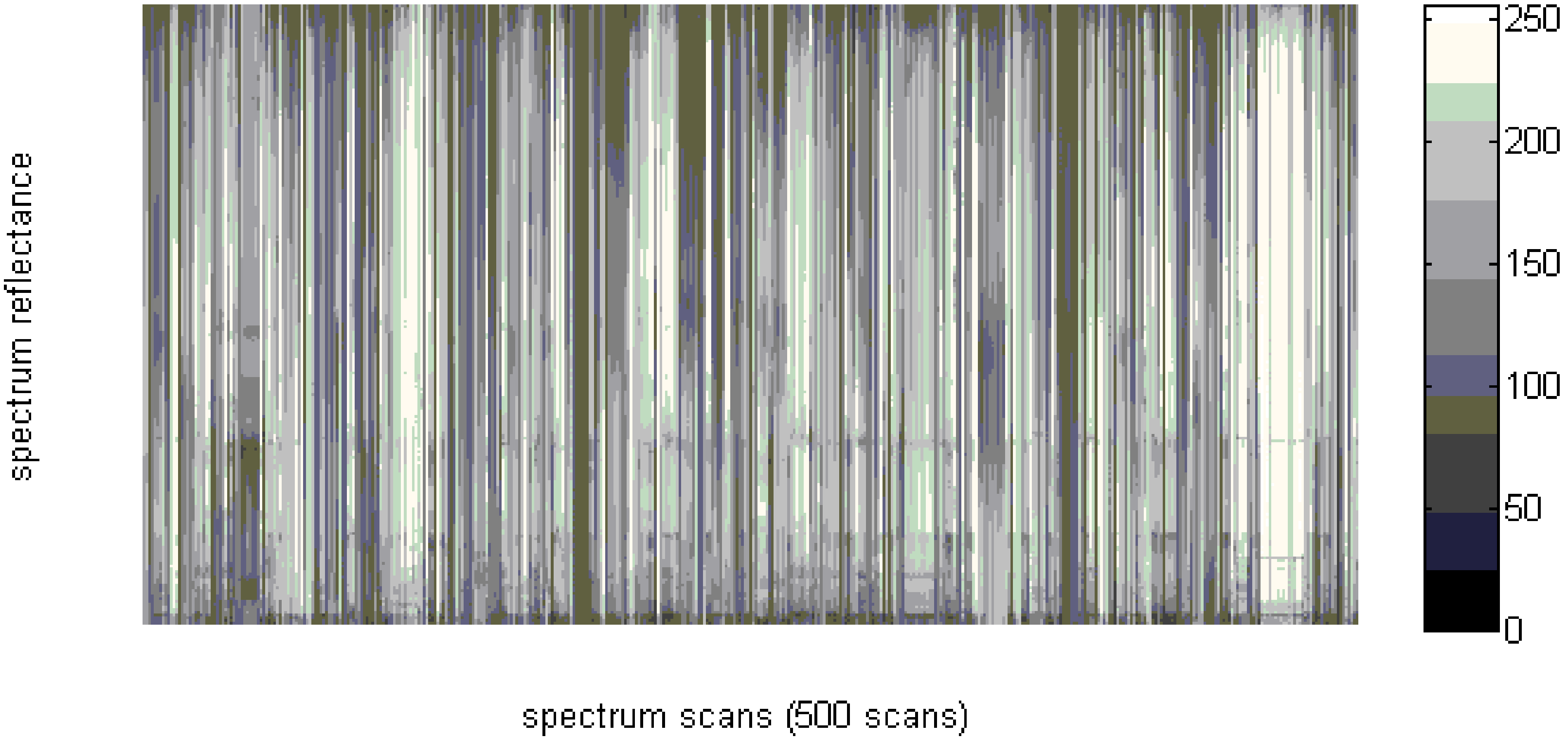
- Next step: extended objects

Acknowledgements: F49620-02-1-107 (AFOSR); DMS-05-11454 (NSF); DAAD19-00-1-0540 and W911NF-05-1-0402 (ARO); DE-AC05-00OR22725 (DTA)

ARL Project: Preliminary Results on using Perron-Frobenius Theory in Compressing Hyperspectral Sensor Data

- Looking down
- 500 spectral scans obtained from U.S. Geological (USGS) digital library.
- Combinations of reflectance data from minerals, buildings, rocks, soils, vegetation, chemical vapors, etc.
- Initialize nonnegative factors using SVD
- Nonnegativity constraints based on P-F theory
- Method can be improved – work with Elad at Technion in Israel

USGS Matrix of Hyperspectral Images



Nonnegative Factorization with Double SVD (NNDSVD)

- Objective function changed to hypersurface function used by Ben Hamza and Dave Brady (IEEE Trans SP, 06):

$$f(W,H) = \sqrt{(1 + ||Y - WH||^2)} - 1$$

- Known to be resistant to outliers (robust)
- Quadratic when argument is small, linear when large
- Used in anisotropic diffusion methods by Sapiro, et al.
- Influence function f’ is differentiable and bounded

Nonnegative Double SVD (NNDSVD)

- By Perron-Frobenius theory,

  if $A \in R_{+}^{m\times n}$ then the singular vectors $u^{(1)}$ and $v^{(1)}$ for the largest singular value σ_1 are non-negative.

- Leads to approximate NMF using SVD applied to blocks of columns of A.
- NNDSVD: SVD based initialization (seeding) of W and H.

The following lemma is trivial for a nonnegative data matrix A.

Lemma. Let $A \in R_{+}^{m \times n}$ be partitioned into blocks as $A = [A_1, \cdots, A_k]$. Let $(\sigma_1^{(j)}, u_1^{(j)}, v_1^{(j)})]$ be the maximum singular triplet of A_j. Set $U = [u_1^{(1)}, \cdots, u_1^{(k)}]$ and $V = [v_1^{(1)}, \cdots, v_1^{(k)}]$. Then U and V are nonnegative.

We use this Lemma to compute nonnegative initial approximations to W and H in approximating A by WH.

The following algorithm (NNDSVD) summarizes the idea

Input: Matrix $A \in \mathbb{R}_{+}^{m \times n}$, integer $k < \min(m, n)$.

Output: Rank-k nonnegative factors $W \in \mathbb{R}_{+}^{m \times k}$, $H \in \mathbb{R}_{+}^{k \times n}$.

1. Compute the largest k singular triplets of A: $[U, S, V] = \mathrm{svds}(A, k)$

2. Initialize $W(:,1) = U(:,1)$ and $H(1,:) = S(1,1)V(:,1)'$

for $j = 2 : k$,

  3. $C = U(:,j) * V(:,j)'$; $C = C.*[C >= 0]$ % sets to 0 the negative part of $C^{(j)}$;

  4. $[u, s, v] = \mathrm{svds}(C)$; % compute the maximum singular triplet of $C_{+}^{(j)}$;

  5. $W(:,j) = u$ and $H(j,:) = S(j,j)s(j,j)v'$;

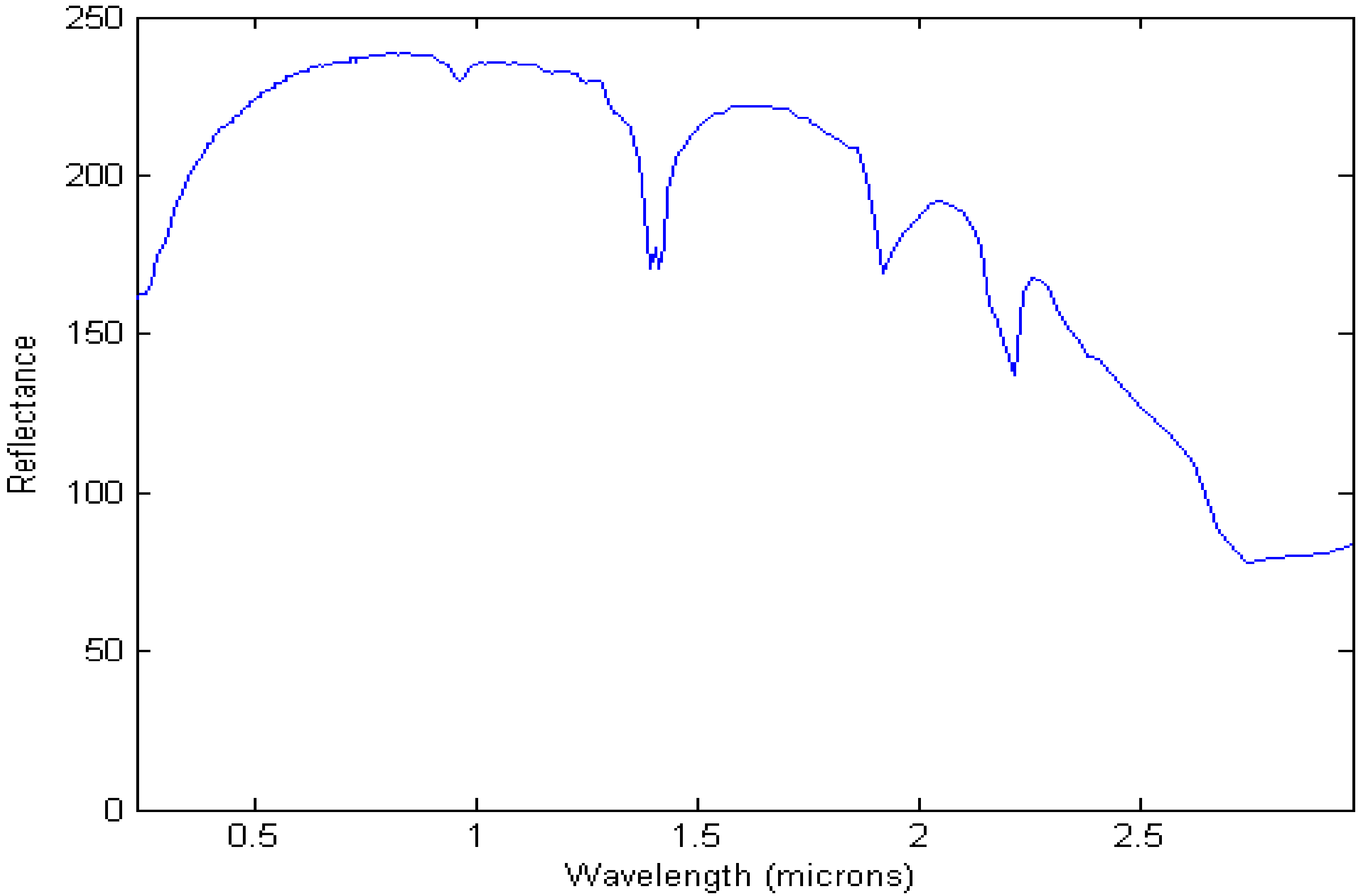
Compression and Reconstruction

- Need compression – large matrices or cubes of hyperspectral data transmitted (satellite to earth, etc.)
- Need faithful and rapid reconstruction for blind source separation of material components, and abundancies
- Reconstruction based on simple application of nonnegative least squares: Hyperspectral data matrix Y ≈ WH using NMF initialized by NNDSVD. j^{th} column y_j ≈ j^{th} column WH.

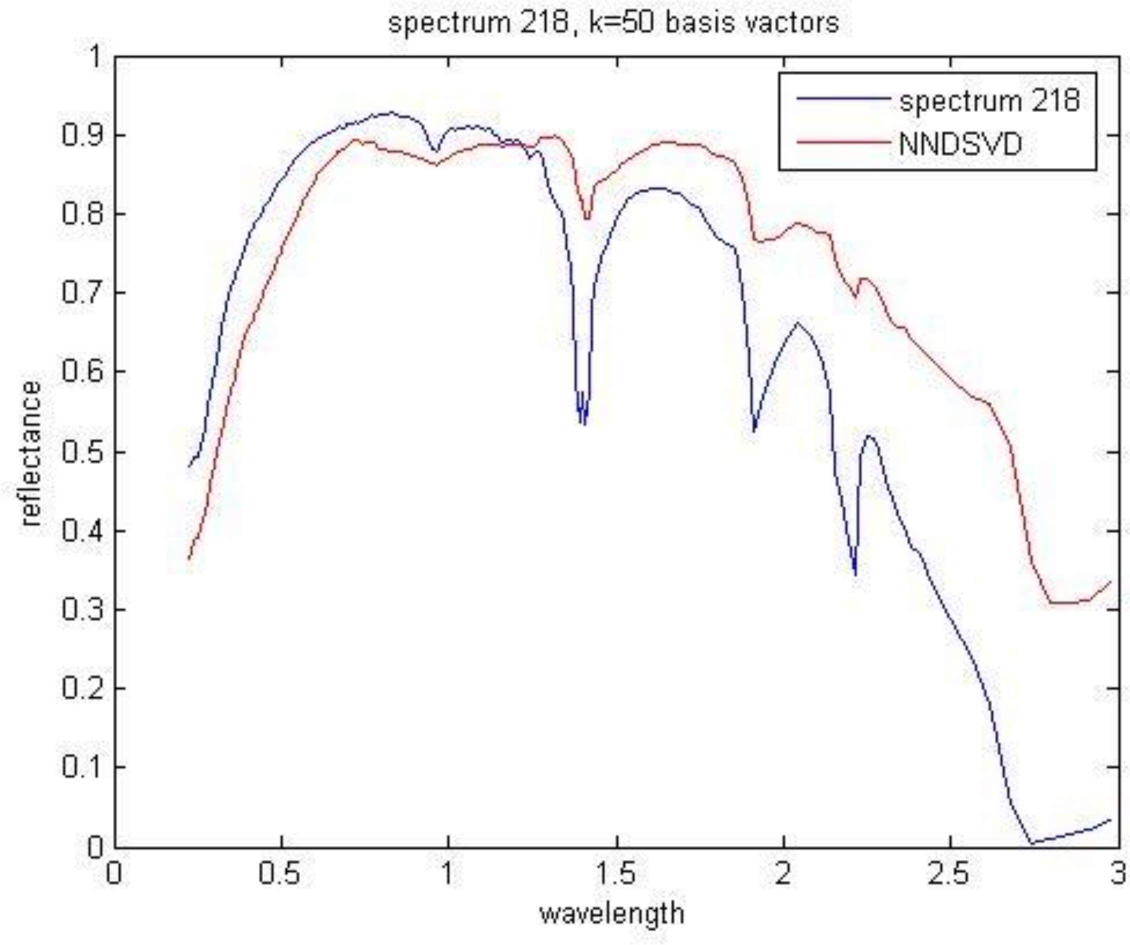
Compute new vectors $\hat{y}_j$ and $\hat{h}_j$. $\min_{h_j \geq 0} ||Wh_j - y_j||_2$.

Set $\hat{y}_j = W\hat{h}_j$. Then $\hat{y}_j \approx y_j$.

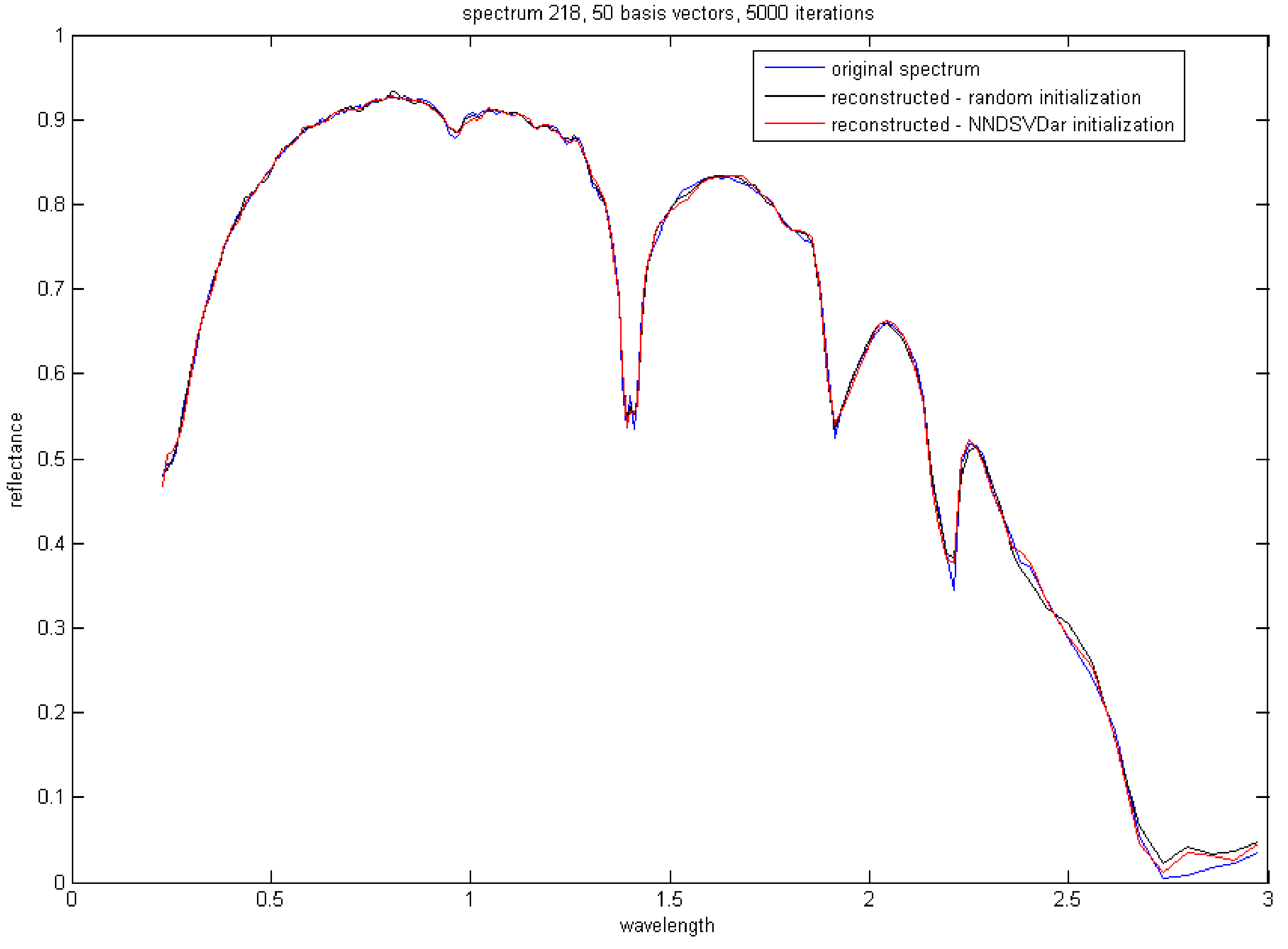
Typical Scan (# 218)



No Iterations



Scan 218 with Iterative Reconstructions
NMF: 1/10 Compression Ratio of Data Matrix



Speedup in NMF Iterations with Initialization, Using Perron-Frobenius Based NNDSVD