Project .

Consider the following simple data set having $n = 4$ "pairs" of one predicting variable x and one response variable y.

Pair	x	y
1	3	7
2	7	8
3	10	9
4	14	11

Consider simple (with intercept β_0) linear regression of y from x.

Write the model in terms of the y_i, x_i, β_0, β_1, ϵ_i

What assumptions are made on the ϵ_i's.

What is $E(y_i)$, What is $\mathrm{Var}(y_i)$.

Write the above model in terms of vectors and matrices; to start. $\underline{y} = \begin{pmatrix} 7 \\ 8 \\ 9 \\ 11 \end{pmatrix} =$

Identify the matrix "Z" that multiplies the $\underline{\beta}$ vector in the model.

Express $\hat{\underline{\beta}}$ in terms of vectors & matrices.

Do the vector/matrix calculations to find the values for $\hat{\underline{\beta}}$, i.e. for $\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$.

Note: the inverse of a 2×2 matrix of form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\boxed{\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$

Do matrix/vector calcs to find $\hat{\underline{y}} = Z\hat{\underline{\beta}}$ and to find $\hat{\sigma}^2 = \frac{(\underline{y}-\hat{\underline{y}})'(\underline{y}-\hat{\underline{y}})}{n-2}$

Verify your calculations for $\hat{\underline{\beta}}$ and $\hat{\sigma}^2$ by running EXCEL for the above data set (EXCEL) enter x, y in columns; choose Data → Data Analysis → Regression.