

Math 355/655: Introduction to Numerical Methods

Homework #6

Due: October 19, 2012

Read Sections 3.5, 3.6

1. Let

$$S(x) = \begin{cases} S_0(x) &= a + bx + cx^2 + dx^3, & \text{if } 0 \leq x \leq 1, \\ S_1(x) &= 1 + (x-1) + (x-1)^2 + (x-1)^3, & \text{if } 1 \leq x \leq 2, \end{cases}$$

be a cubic spline on $[0,2]$ satisfying the boundary conditions $S'(0) = -1$ and $S'(2) = 6$ with the nodes at $x = 0, 1, 2$. Find a, b, c, d .

2. Determine whether S is a cubic spline with nodes $x = -1, 0, 1, 2$:

$$S(x) = \begin{cases} S_0(x) &= 2(x+1) + (x+1)^3, & \text{if } -1 \leq x \leq 0, \\ S_1(x) &= 3 + 5x + 3x^2, & \text{if } 0 \leq x \leq 1, \\ S_2(x) &= 11 + (x-1) + 3(x-1)^2 + (x-1)^3, & \text{if } 1 \leq x \leq 2, \end{cases}$$

3. (MATLAB) For this problem, we will use MATLAB to find a cubic spline that goes through the points $(0,0)$, $(1,0)$, $(3,2)$, and $(4,2)$.

To do this, let $X=[0 \ 1 \ 3 \ 4]$ and $Y=[0 \ 0 \ 2 \ 2]$. Set $x = \text{linspace}(0, 4, 200)$. Then, use the command `plot(x, ppval (spline(X,Y),x))`. To understand what each of these commands do, try typing `help spline` or `help linspace` at the MATLAB command line, as needed. Print out a copy of your spline and attach it to your homework. Which type of cubic spline is this?

4. Derive the formula for the Bézier curve $B(t)$ such that $B(0) = P_1$ and $B(1) = P_5$ that uses control points P_2, P_3 , and P_4 .
5. A Bézier curve is given by

$$\begin{cases} x(t) &= 1 + 6t^2 + 2t^3 \\ y(t) &= 1 - t + t^3 \end{cases}.$$

Find the endpoints and control points.

6. In class we used a 3-piece Bézier curve to form most of a triangle with nodes $(1,2)$, $(3,4)$ and $(5,1)$. Find the final line that we did not find; that is, find the Bézier curve that goes from $(1,2)$ to $(5,1)$. Also, find the line of the form $y = mx + b$ that this parametric curve represents. (Show that this line does in fact represent the curve.)