Problem Solving and Search

Problem-solving agents

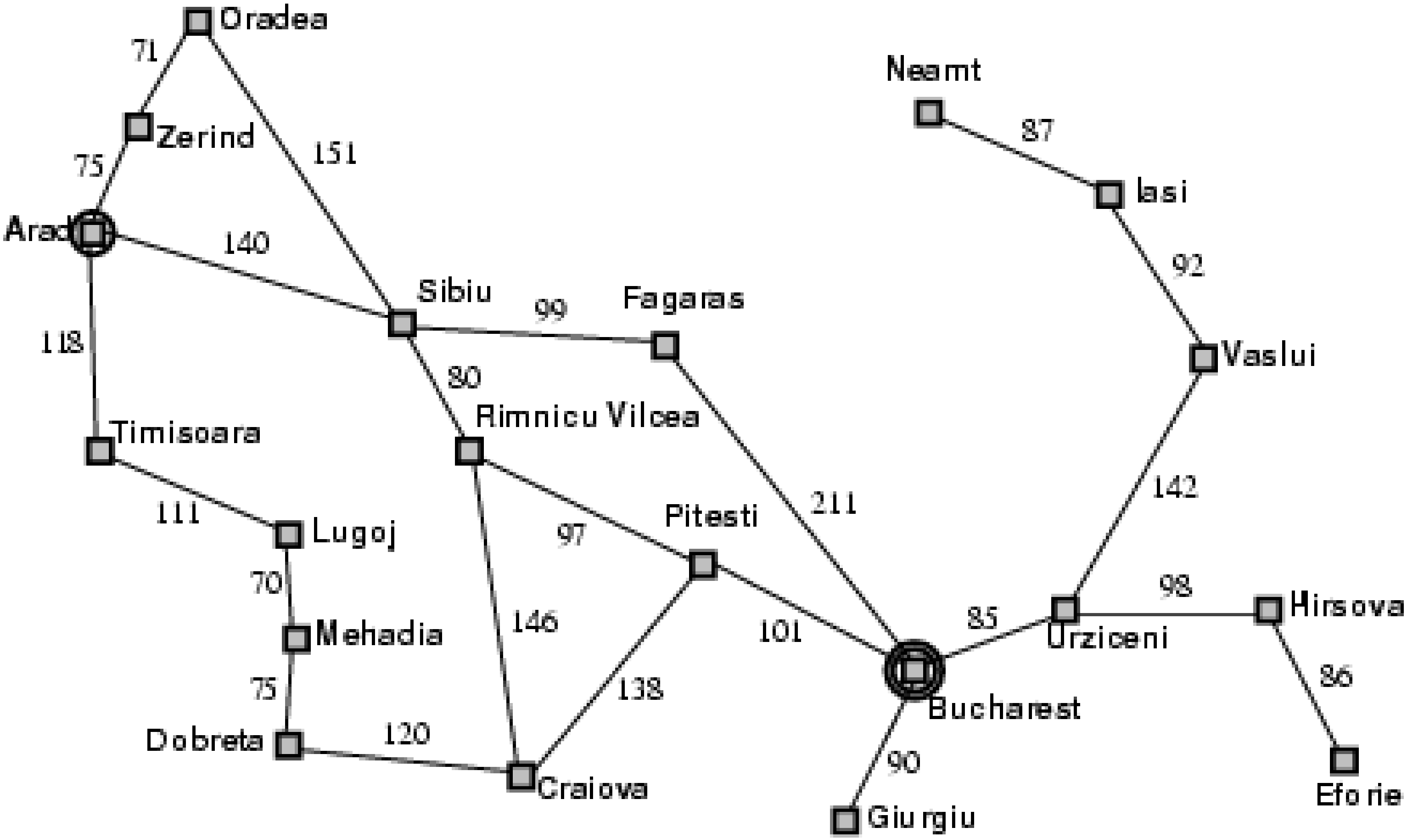
```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then do
       goal ← FORMULATE-GOAL(state)
       problem ← FORMULATE-PROBLEM(state, goal)
       seq ← SEARCH( problem)
   action ← FIRST(seq)
   seq ← REST(seq)
   return action
```

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest

- Formulate goal:
  – be in Bucharest
- Formulate problem:
  – states: various cities
  – actions: drive between cities
- Find solution:
  – sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania

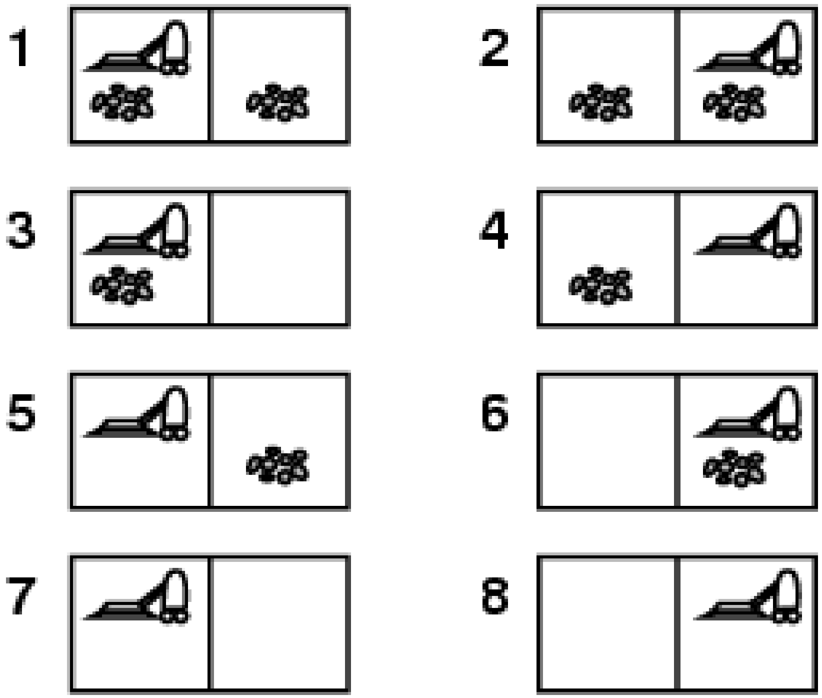


Problem Types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave} search, execution
- Unknown state space → exploration problem

Example: vacuum world

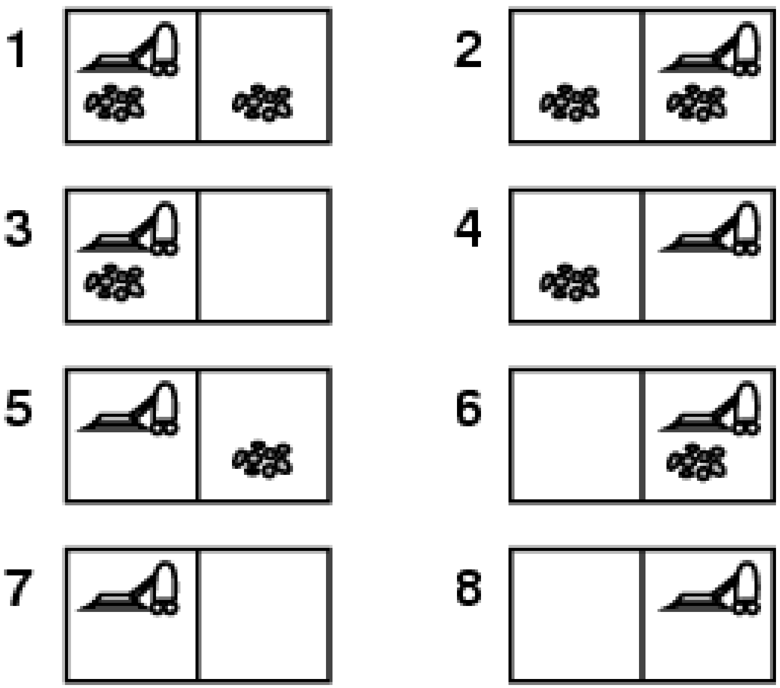
- Single-state, start in #5.
  Solution?



Example: vacuum world

- Single-state, start in #5.
  Solution? *[Right, Suck]*

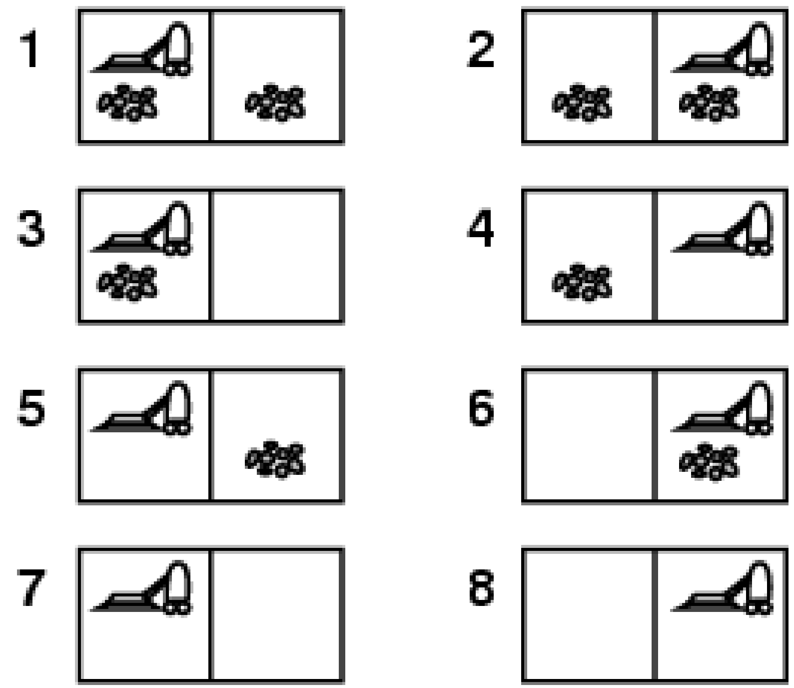
- Sensorless, start in
  {*1,2,3,4,5,6,7,8*} e.g.,
  Right goes to {*2,4,6,8*}
  Solution?



Example: vacuum world

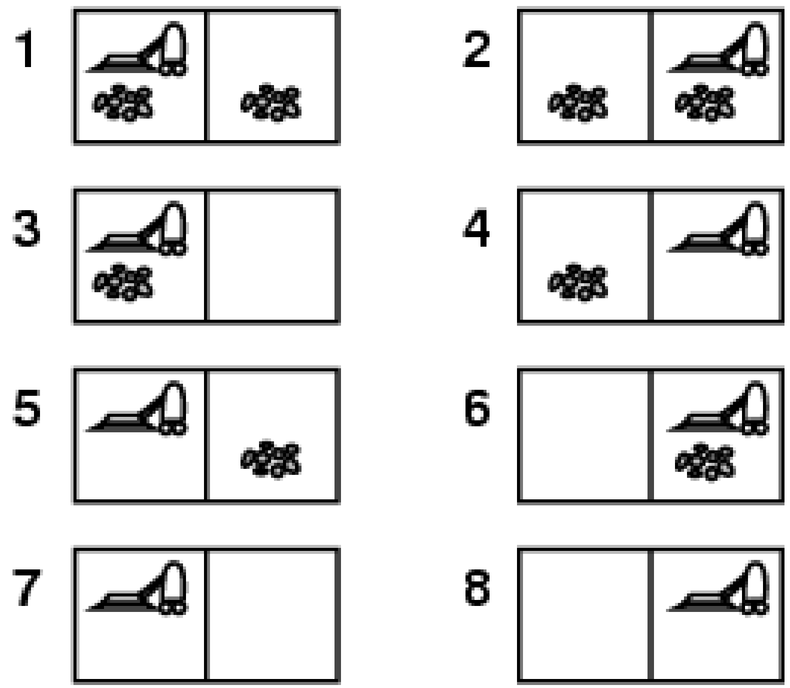
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution?
  [Right,Suck,Left,Suck]

- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7
    Solution?



Example: vacuum world

- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution? *[Right,Suck,Left,Suck]*



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7 Solution? *[Right,* ***if*** *dirt* ***then*** *Suck]*

Problem formulation – Romania

A problem is defined by four items:

1. initial state e.g., "at Arad“

2. actions or successor function $S(x)$ = set of action–state pairs
   – e.g., $S(Arad) = \{<Arad \rightarrow Zerind, Zerind>, \ldots\}$

3. goal test, can be
   – explicit, e.g., x = "at Bucharest"
   – implicit, e.g., $Checkmate(x)$

4. path cost (additive)
   – e.g., sum of distances, number of actions executed, etc.
   – $c(x,a,y)$ is the step cost, assumed to be ≥ 0

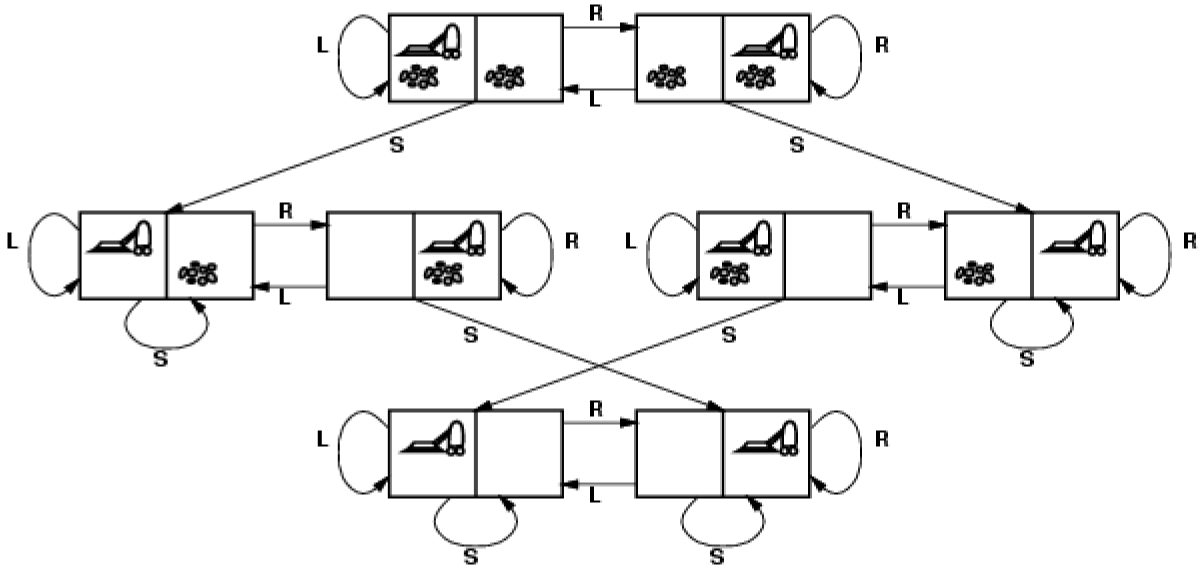
A solution is a sequence of actions leading from the initial state to a goal state

This is still a model i.e., a mathematical description of the problem (ignores many real world complications such as scenery, traffic congestion, bad drivers etc. We abstracted the problem away from the details.

Abstraction of state spaces

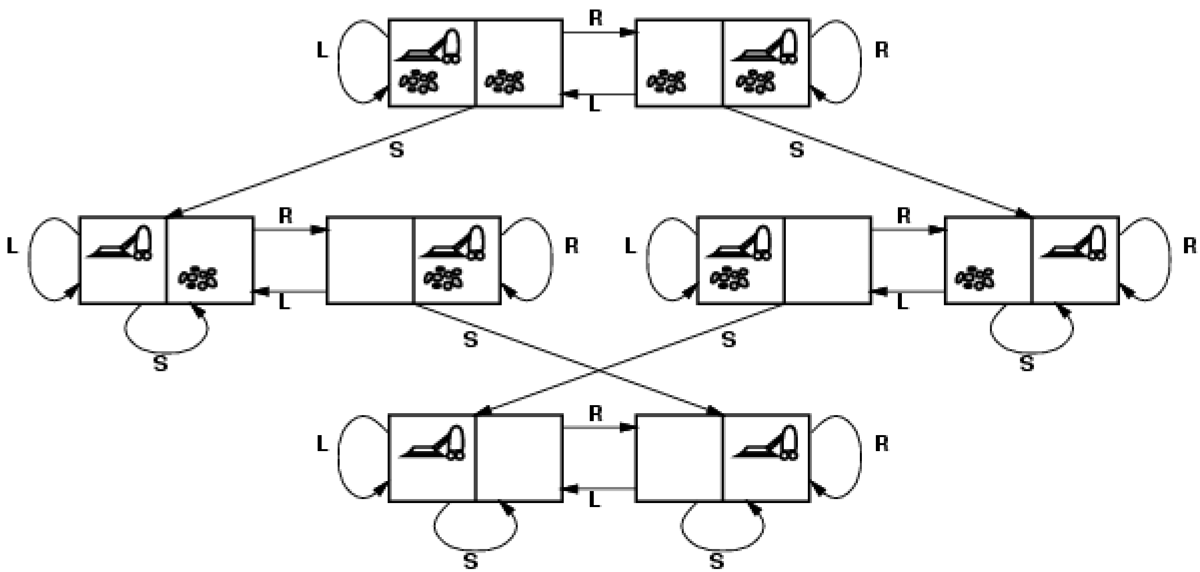
- Real world is absurdly complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states(i.e., details removed)
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind“
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

Vacuum world state space graph



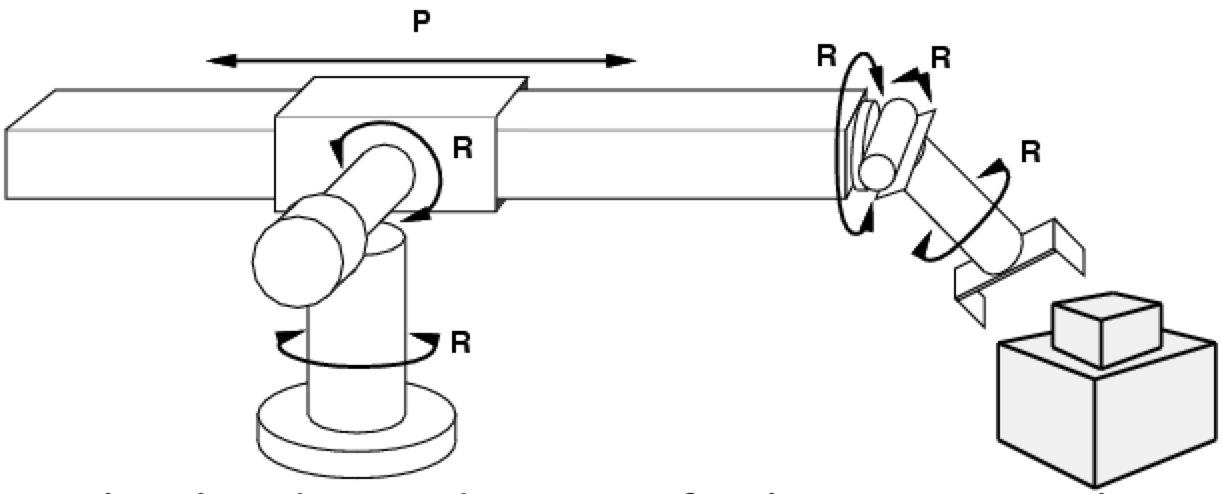
- states?
- actions?
- goal test?
- path cost?

Vacuum world state space graph



- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

A more realistic example: robotic assembly



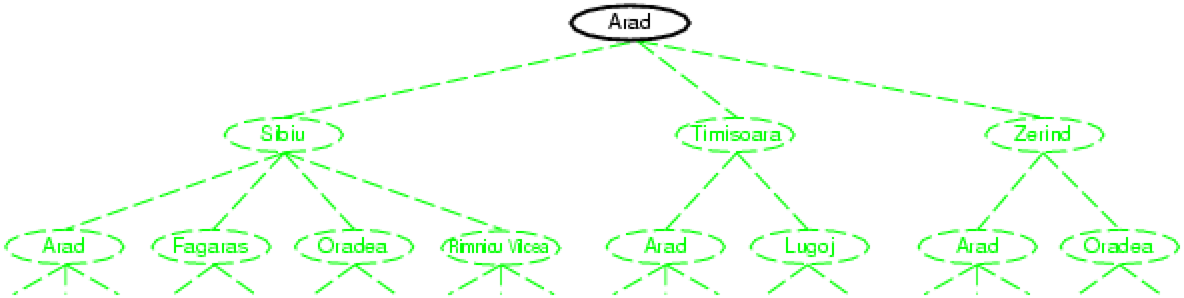
- states?: real-valued coordinates of robot joint angles and parts of the object to be assemble
- actions?: continuous motions of robot joints
- goal test?: complete assembly
- path cost?: time to execute

Tree search algorithms

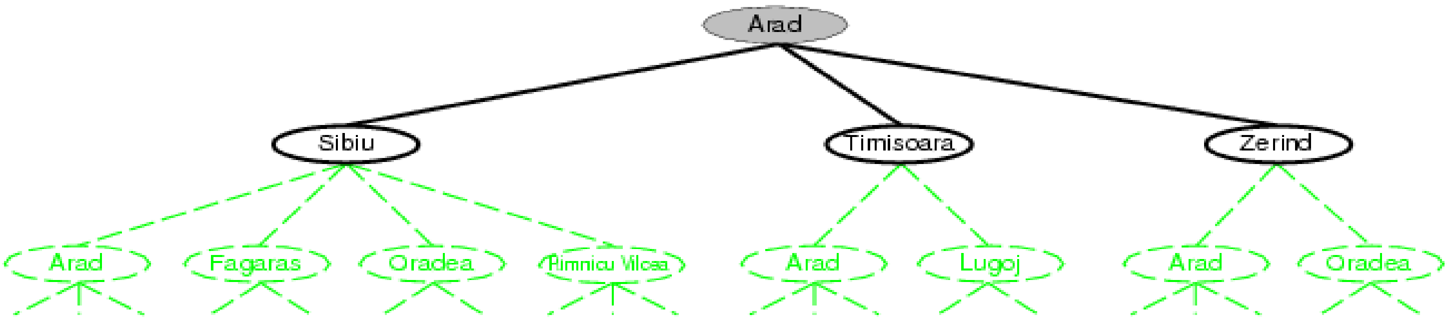
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)
  - Consider the search space over the set of states as a tree
  - Initial state becomes the root
  - Expand each state by adding legal actions to create more nodes (states) in lower levels

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
       if there are no candidates for expansion then return failure
       choose a leaf node for expansion according to strategy
       if the node contains a goal state then return the corresponding solution
       else expand the node and add the resulting nodes to the search tree
```

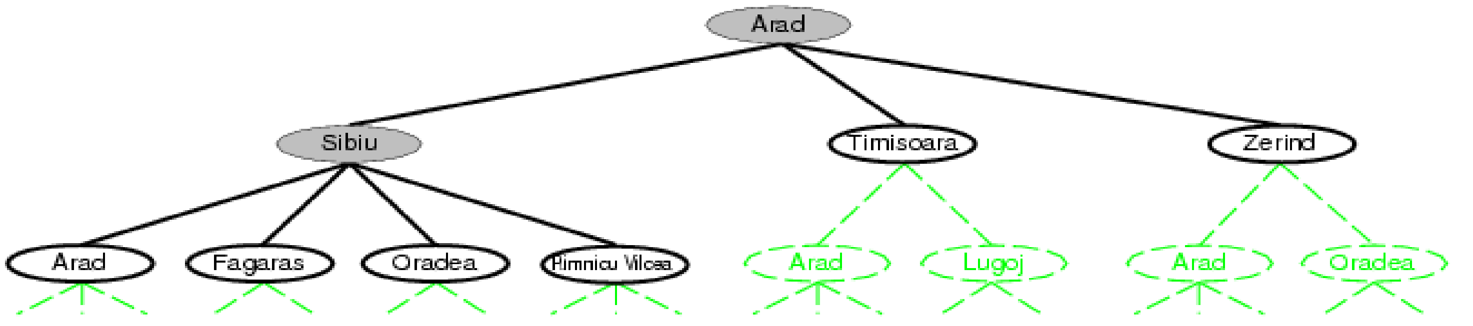
Tree search example – Romania



Tree search example

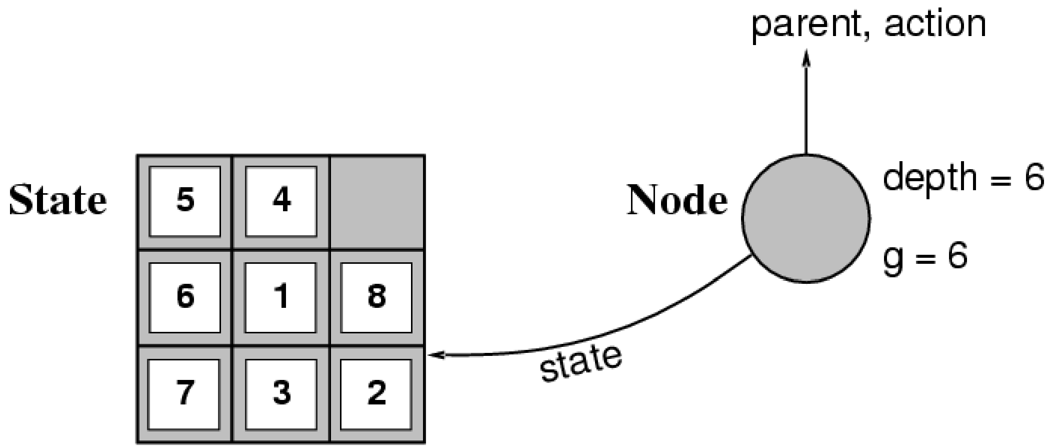


Tree search example



Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Uninformed search strategies

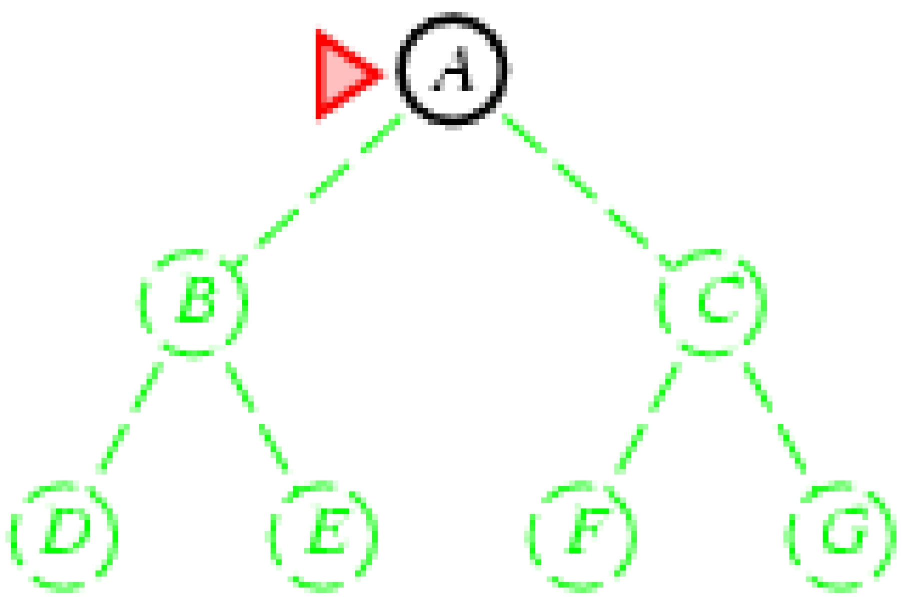
- Uninformed search strategies use only the information available in the problem definition

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node

- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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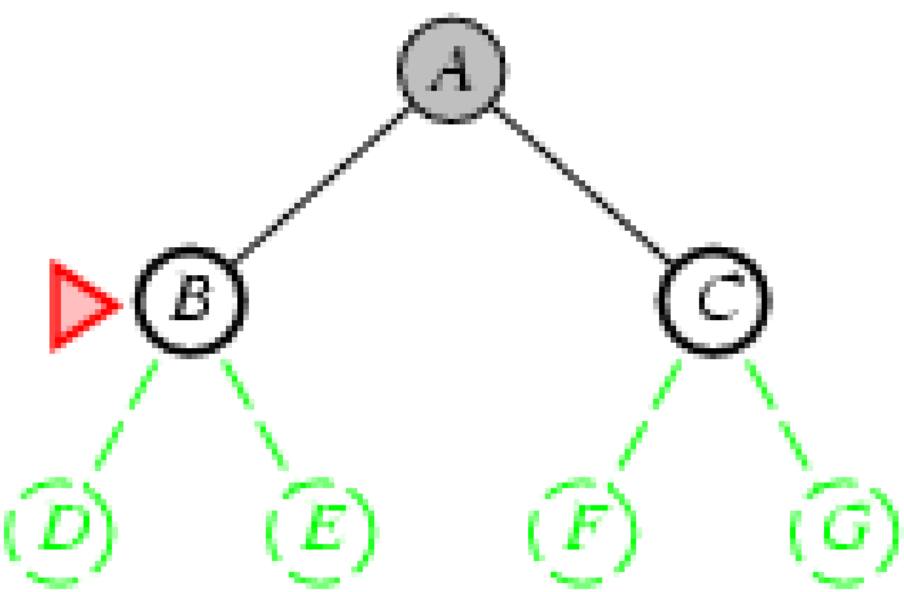

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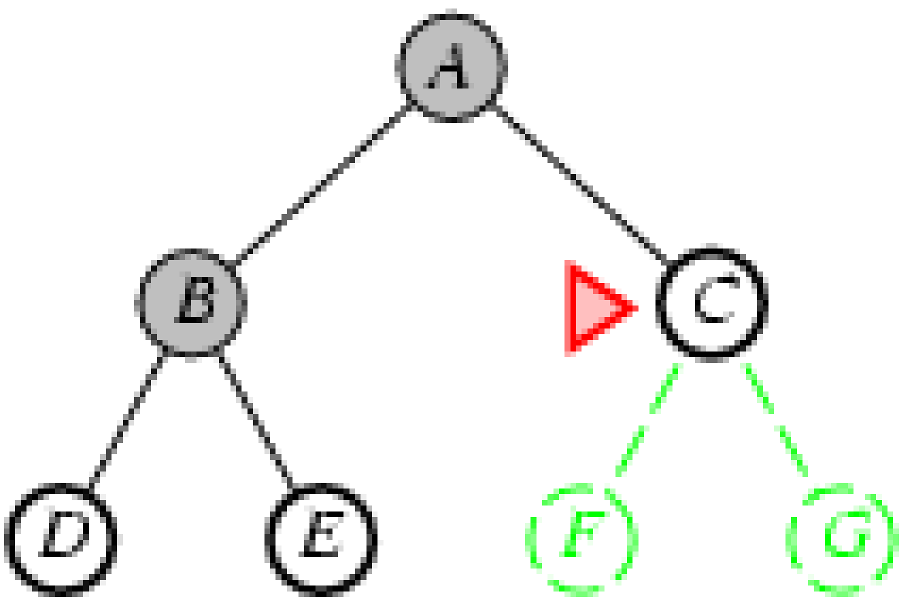

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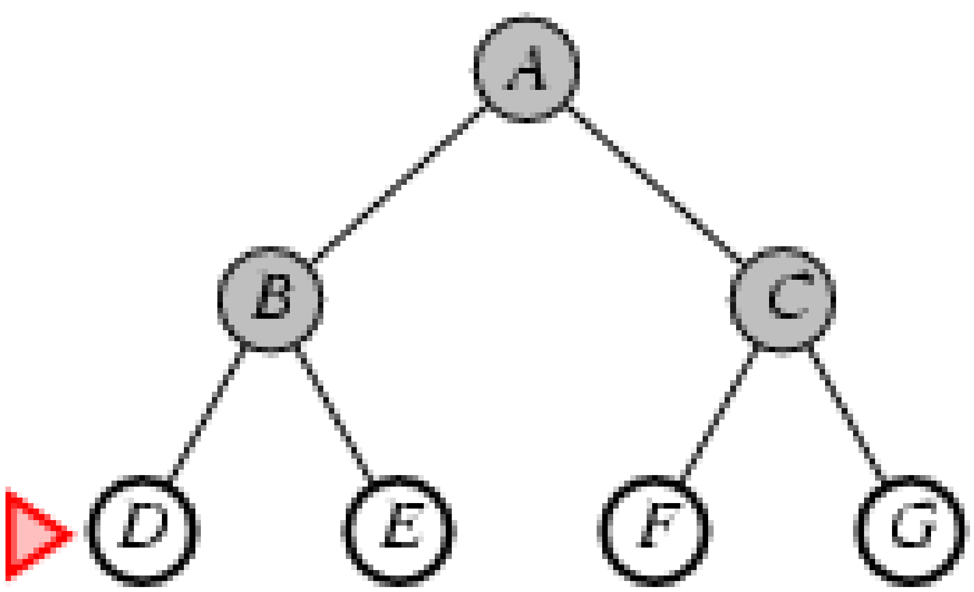


Properties of breadth-first search

- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots +b^d + b(b^d-1)$ = $O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)

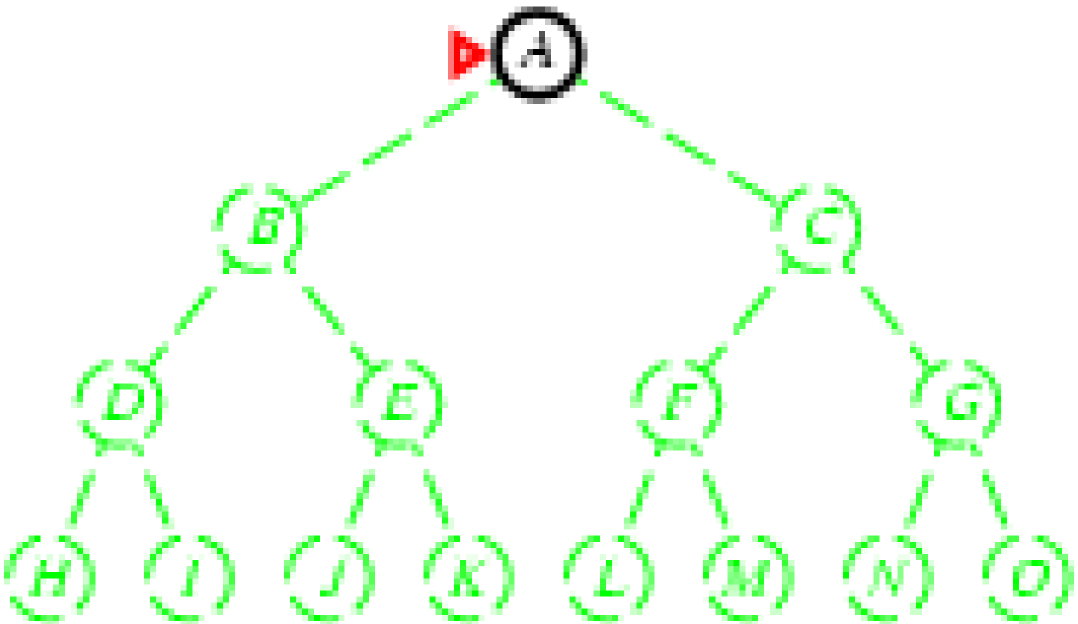
- Space is the bigger problem (more than time)

Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of $g(n)$

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



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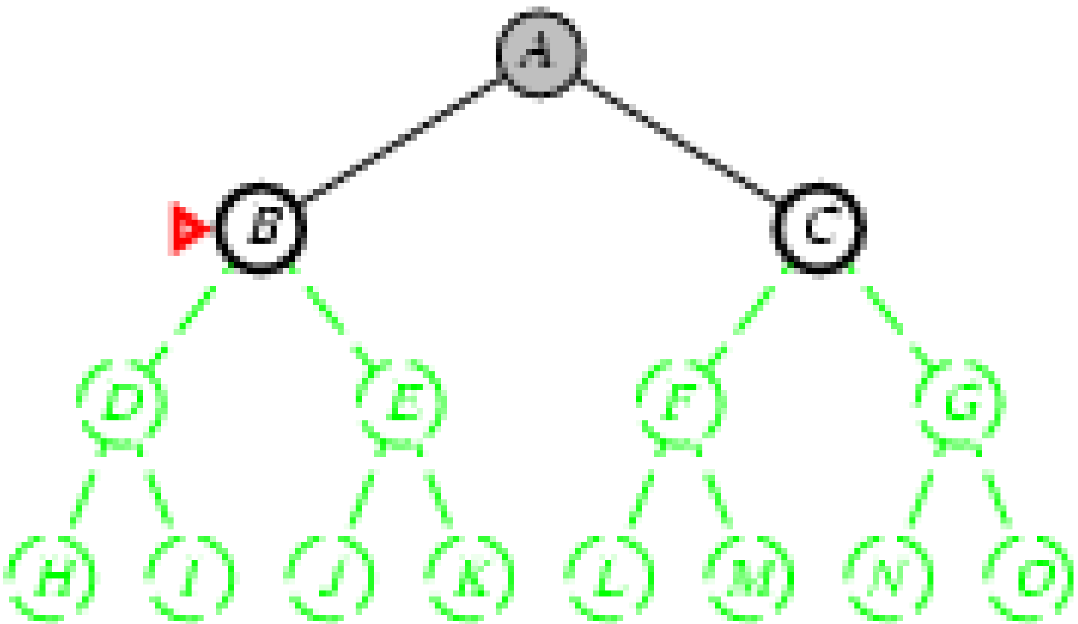

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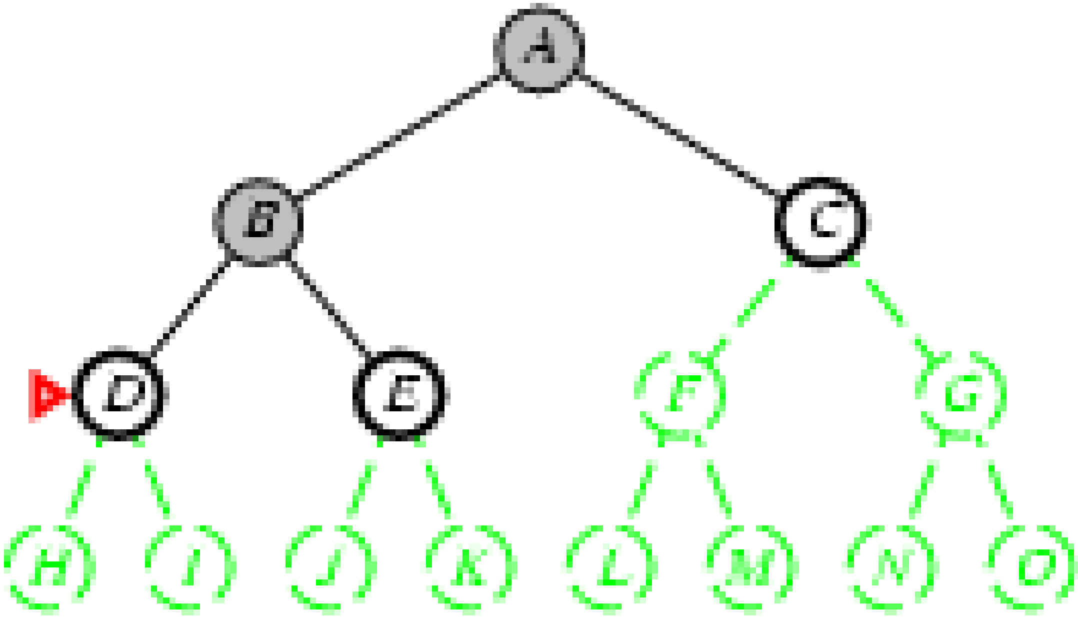

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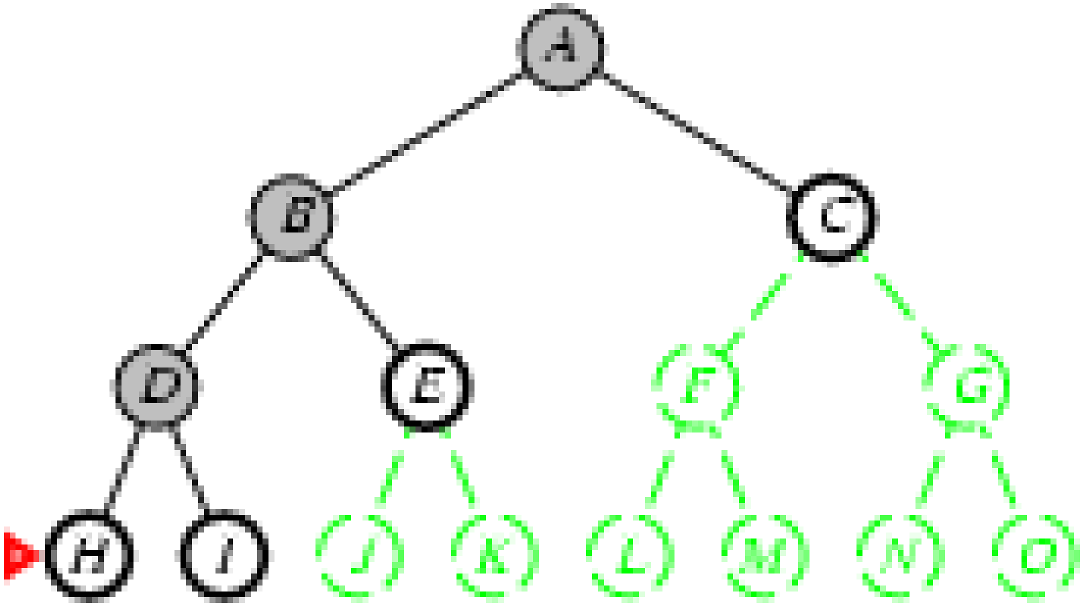

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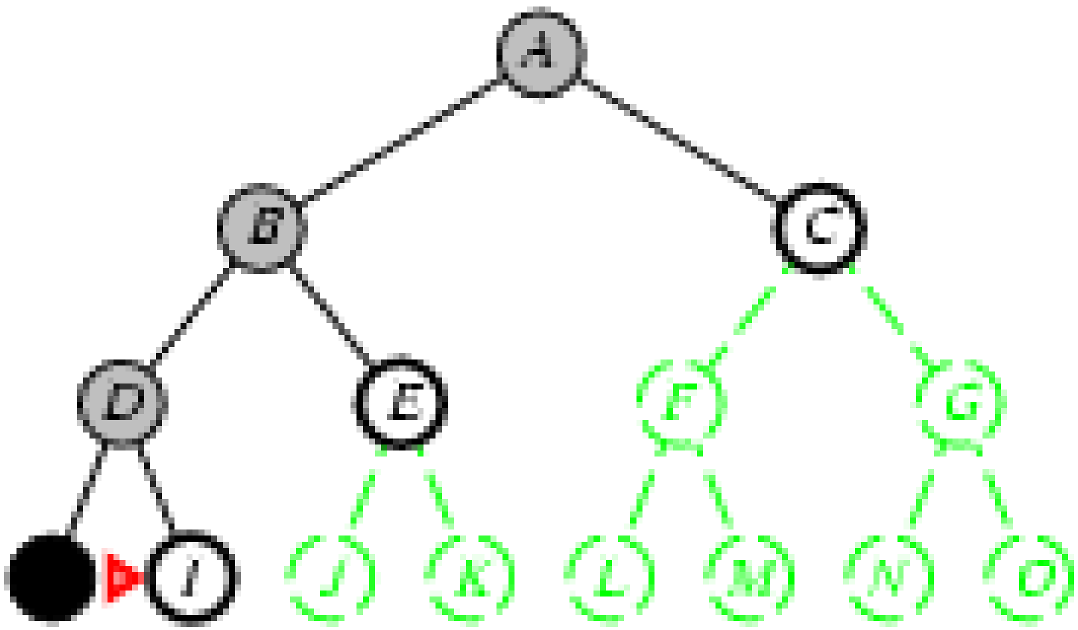

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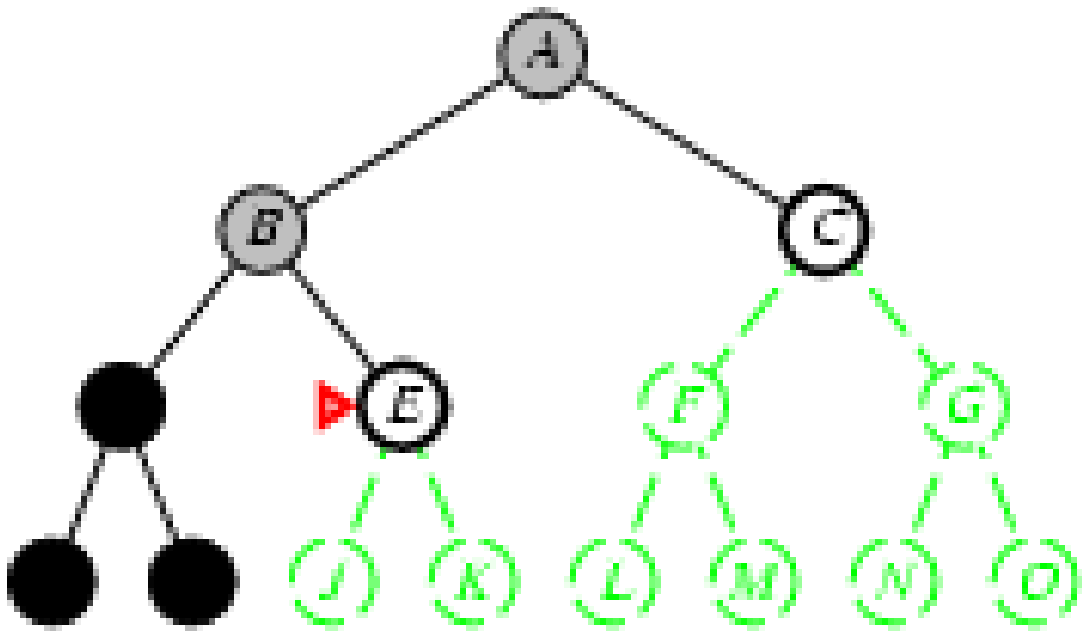

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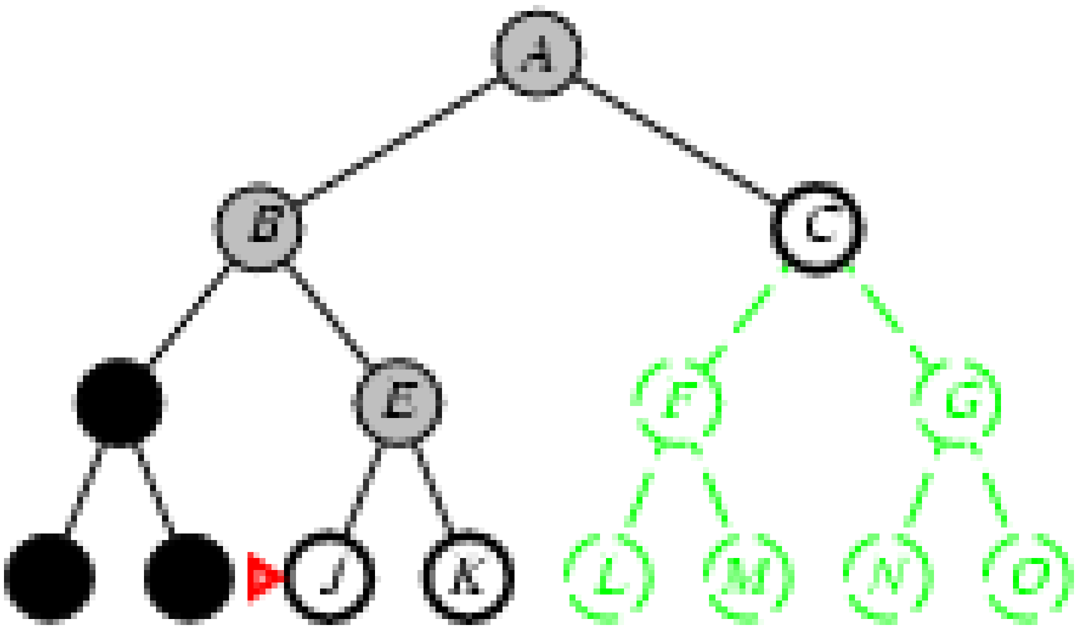

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-
- Implementation:
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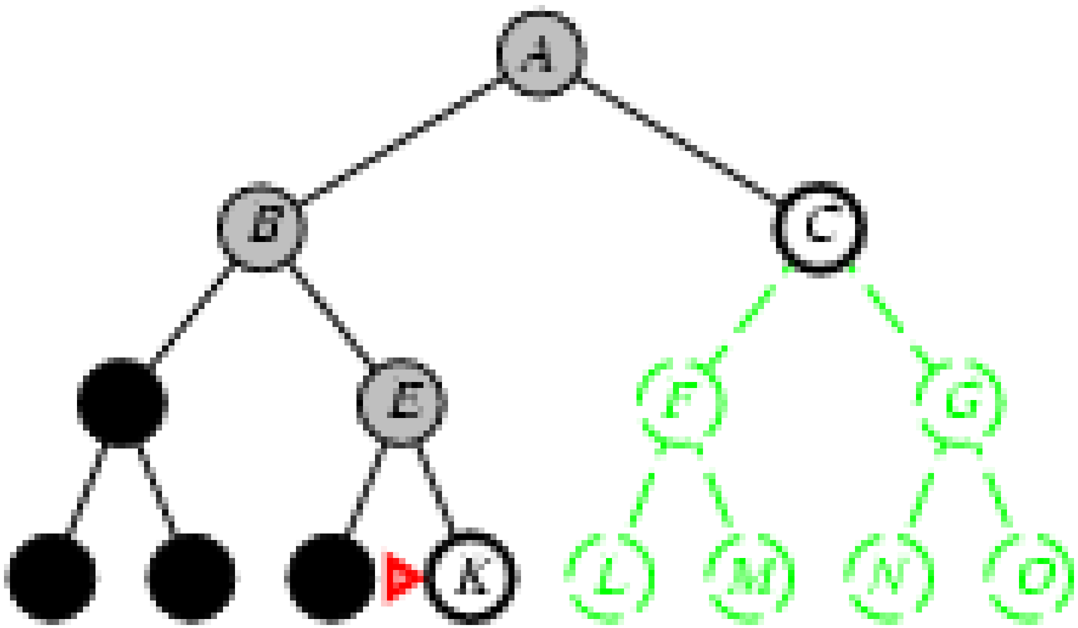
  -



Depth-first search

- Expand deepest unexpanded node
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- Implementation:
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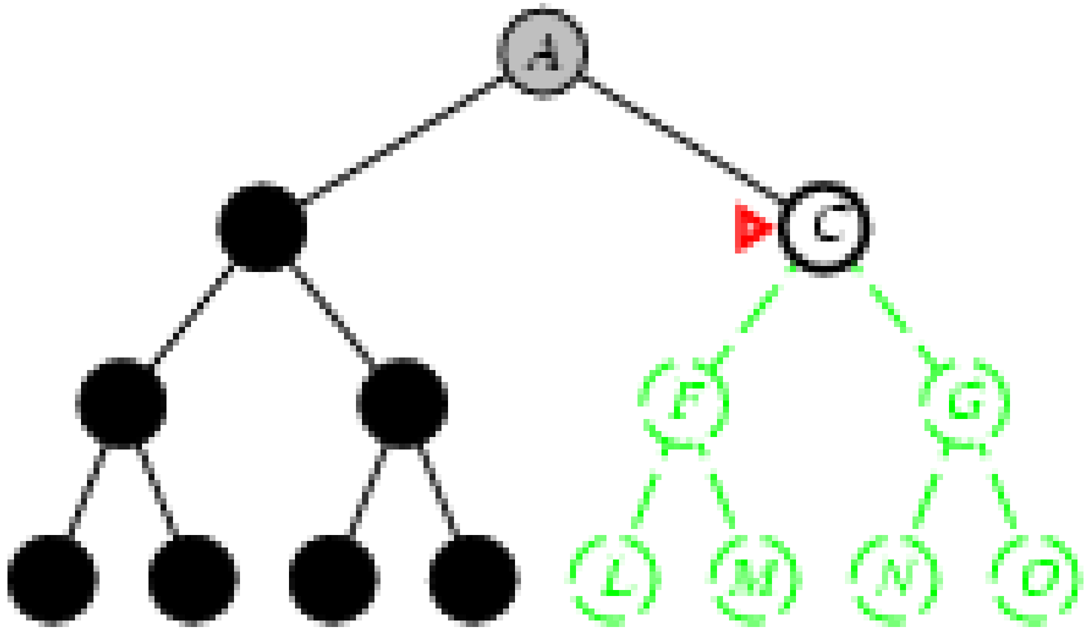
  -



Depth-first search

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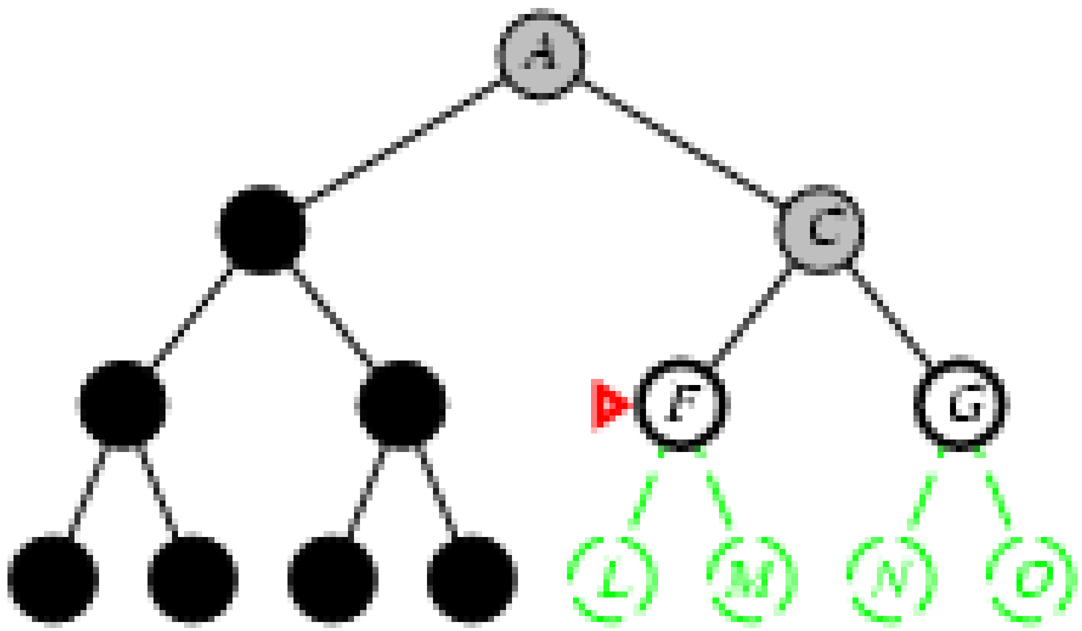
  -



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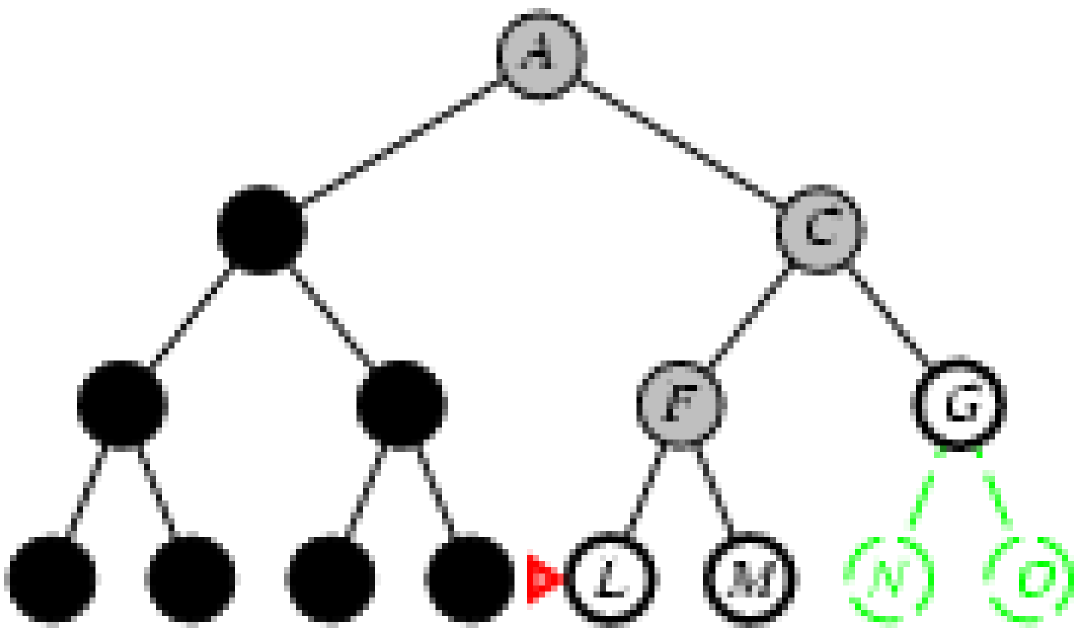

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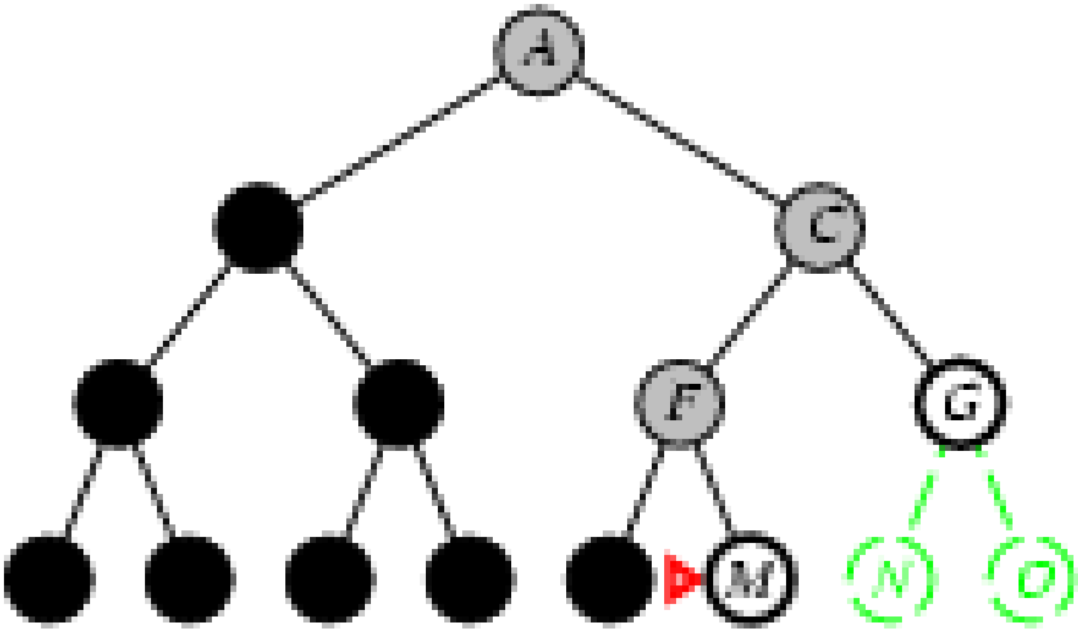


Properties of depth-first search

- <u>Complete?</u> No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
  - Complete in finite spaces
- <u>Time?</u> $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- <u>Space?</u> $O(bm)$, i.e., linear space!
- <u>Optimal?</u> No

Depth-limited search

Depth-first search with depth limit *l*, i.e., nodes at depth *l* have no successors

- **function** DEPTH-LIMITED-SEARCH(*problem*, *limit*) **returns** soln/fail/cutoff
  RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[*problem*]), *problem*, *limit*)

  function RECURSIVE-DLS(*node*, *problem*, *limit*) **returns** soln/fail/cutoff
  cutoff-occurred? ← false
  if GOAL-TEST[*problem*](STATE[*node*]) **then return** SOLUTION(*node*)
  else if DEPTH[*node*] = *limit* **then return** *cutoff*
  else for each *successor* **in** EXPAND(*node*, *problem*) **do**
  result ← RECURSIVE-DLS(*successor*, *problem*, *limit*)
  if *result* = *cutoff* **then** *cutoff-occurred?* ← true
  else if *result* ≠ *failure* **then return** *result*
  if *cutoff-occurred?* **then return** *cutoff* **else return** *failure*

Iterative deepening search

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution, or failure

inputs: *problem*, a problem

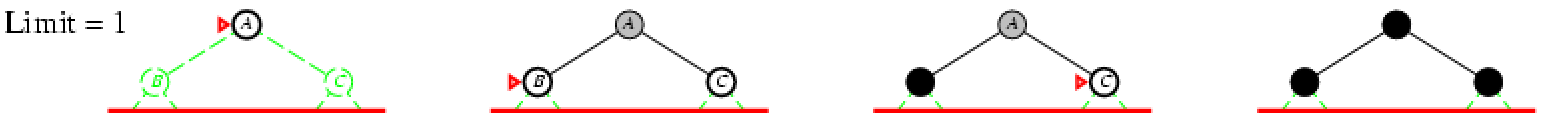
for *depth* $\leftarrow$ 0 **to** ∞ **do**

result $\leftarrow$ DEPTH-LIMITED-SEARCH(*problem*, *depth*)

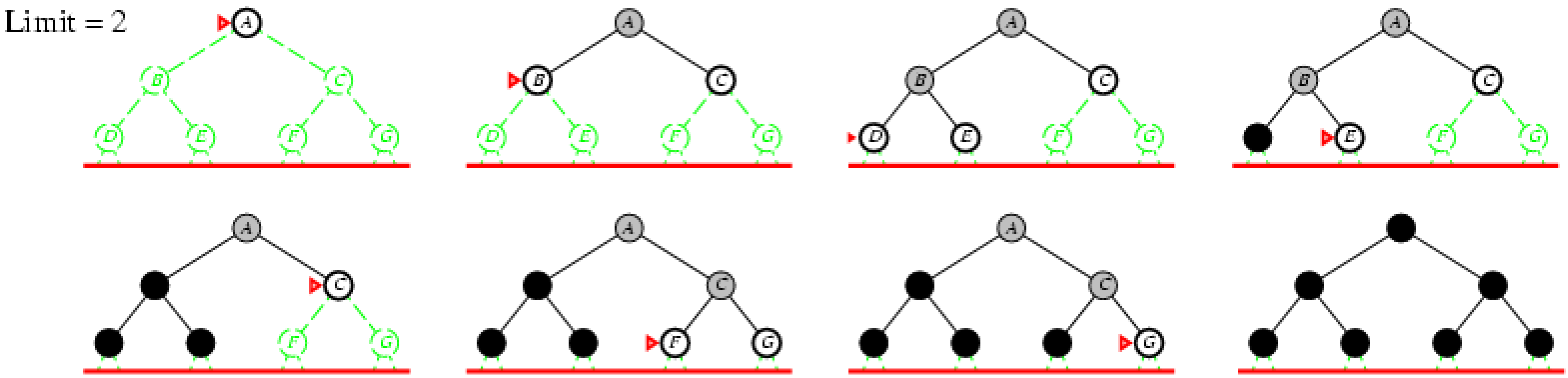
if *result* $\neq$ cutoff **then return** *result*

Iterative deepening search *l* =0

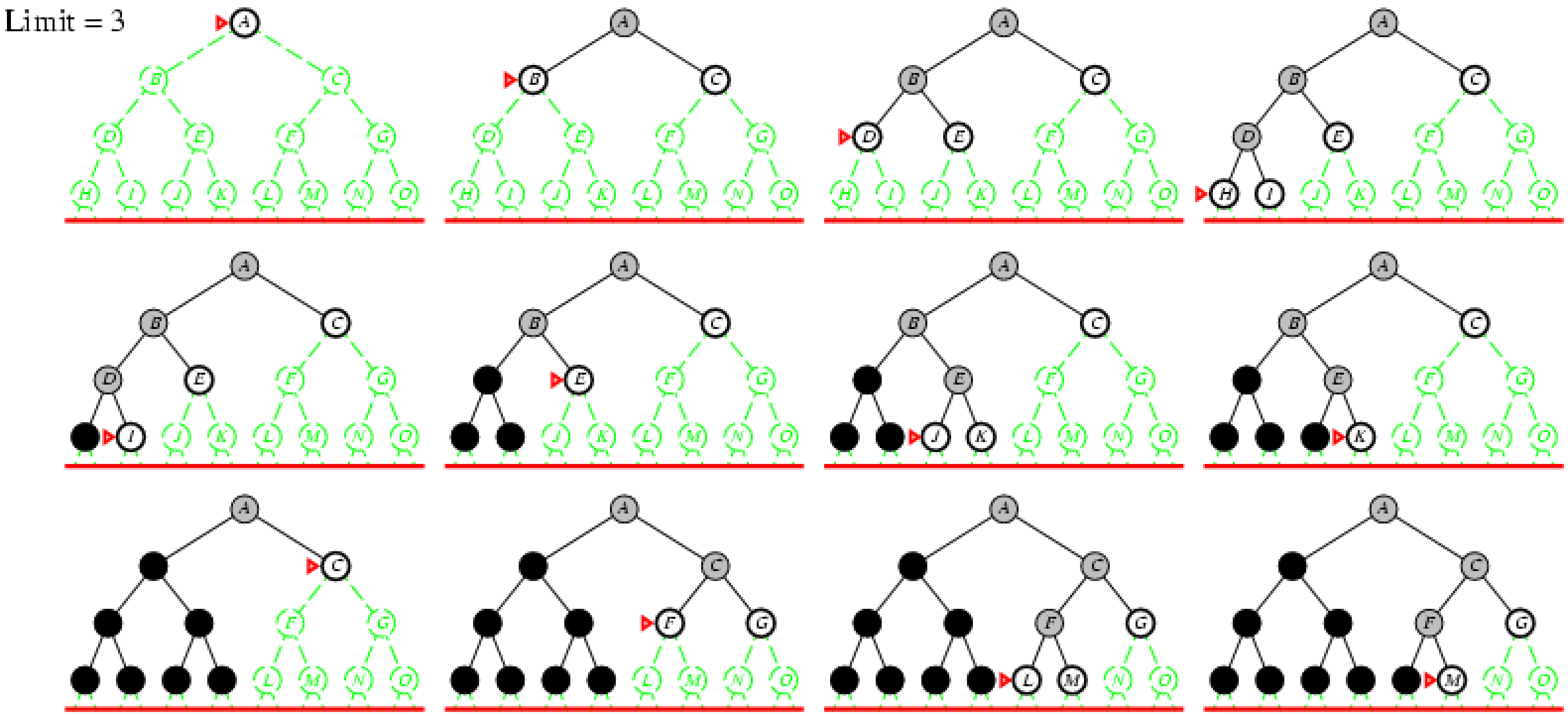
Iterative deepening search *l* =1



Iterative deepening search *l* =2



Iterative deepening search *l* =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\ b\text{^}^1 + (d-1)b\text{^}^2 + \ldots + 3b^{d-2} +2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,
-
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$
- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes