MTH326/626 Final **due on Dec 15**

Requirements:
i) Output all theoretical proofs, statements, numerical results and conclusions in a tex file. Use the template on Sakai.
ii) Compress all related files (including .tex file, .pdf file, .m file, and .mat file) into a zip file and upload the zip file to the dropbox.
iii) Late assignment will not be accepted

- **24.1(c)(f), 24.4, 25.1(a)**
- Let A be a m-by-n real matrix ($m > n$). What is the condition number of the coefficient matrix (consider the case of $\|\cdot\|_2$ only) in terms of the singular values of A?
- Let $A \in \mathbf{R}^{n\times n}$ be positive definite. Two vectors u_1 and u_2 are called A-orthogonal if $u_1^T A u_2 = 0$. If $U \in \mathbf{R}^n$ and $U^T A U = I$, then the columns of U are said to be A-orthonormal. Show that every subspace of $\mathbf{R}^n$ has an A-orthonormal basis.
- Let A_k be a best rank-k approximation of the matrix A (A is a m-by-n matrix. $A = \hat{U}\hat{\Sigma}V^* = \sum_{i=1}^n \sigma_i u_i v_i^*$ and $A_k = \sum_{i=1}^k \sigma_i u_i v_i^*$, where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$). Show that A_k is unique if $\sigma_k > \sigma_{k+1}$.
- Let A be a m-by-n full rank matrix. Show that $X = A^+$ (the Moore-Penrose pseudoinverse) minimizes $\|AX - I\|_F$ over all n-by-m matrices X. What is the value of this minimum?
- (Numerical Part) Read through Lecture 27 to finish the given experiments. Use the symmetric matrix A defined as in Example 27.1 (pp. 208) and let $v^{(0)} = (1,1,1)^T/\sqrt{3}$ be initial eigenvector estimate.
(i) Let $\mu = 5$. Use the Inverse Iteration (Algorithm 27.2) to compute estimations of an eigenvalue and an eigenvector of A. Output the results at the iteration $k = 2, 5, 10, 15, 20, 30, 50, 100$.
(ii) Let $\lambda^{(0)} = 5$. Use the Rayleigh Quotient Iteration (Algorithm 27.3) to compute estimations of an eigenvalue and an eigenvector of A. Output the results at the iteration $k = 2, 5, 10, 15, 20, 30, 50, 100$.

(iii) Let k run from 1 to 50. Plot the eigenvalues results, computed by Algorithm 27.2 and Algorithm 27.3 respectively, in the same graph (horizontal axis is the the values of k and vertical axis is the values of $\lambda^{(k)}$). Give your comments on such comparison.