Nonlinear Optimization Take Home Final Exam 2013

(Please do your own work, without help from others in the class or from me.)

Due by 5:00 pm Tuesday May 7 (you can either email your paper (single PDF file) to me or place stapled hard copy in the box on my office door, rm 341, or in my Math or CSC mailbox, by May 7).

Write up your results for each problem separately. You might organize your results on working the problems by:

- Describing the problem
- Describing your method(s) of solution.
- Interpreting your computed results (if the problem involved a computation).

1. Study of Matlab's fminmax

(a) Investigate the following minmax problem using fminmax.m, where:
f1(x) = x ^ 2 + x + 1
f2(x) = 3
f3(x) = x ^ 4 + 5
f4(x) = -(x^2) + 1
f5(x) = sin(x) + cos(x)

(b) How can this be reduced to a normal unconstrained optimization problem of a single objective function, i.e., how can this problem be easily solved? Hint: Try graphing these equations in MATLAB. Consider which function strictly dominates all the other equations. Using this information, can the problem be solved using only one of the functions?

2. Study of Matlab's fmincon

Problem . Use *fmincon* to minimize

$$f(x_1, x_2) = \sin(x_1 x_2)$$

with the constraints

$$0 \leq x_1 \leq 2 \text{ and } -2 \leq x_2 \leq 0$$

and initial point $[1, -1]$ without having to provide the gradient. (Hint: Use an m-file to define your function. Be sure to account for empty constraint values in the *fmincon* command line.) Be sure to point out which algorithm was used, as well plot the function within the bounds.

3. Overview of Optimization Software

Name the three types of optimization software that have been discussed and explain how these types of software differ. Furthermore, specifically discuss how AMPL, Opensolver, and Maple exemplify these similarities and differences. When and why would one type of software have an advantage over any of the others?

4. Optimization in Medical Research

Specify an objective function (which should have a local minimizer) you are interested in, use the Wolfe Conditions (weak or strong conditions) to find an interval for the step size alpha. And show why the steepest descent method gives a "zig-zag" pattern in iterations. That is, applying the steepest descent method, specify an objective function and find the range of step length which satisfies the Wolfe Conditions. Prove that the iterative step follows a "zig-zag" pattern.

5. Linear Programming

A farmer has orders for 600 hay bales from Nashville and 400 hay bales from Dallas. The farmer has 700 hay bales stored in his farm in Asheville and 800 bales stored in his farm in Statesville. It costs him $5 to ship a hay bale from Asheville to Nashville and $10 to ship it to Dallas. It costs him $4 to ship from Statesville to Dallas and $15 from Statesville to Nashville. How many hay bales should he ship to each location (Dallas and Nashville) from his two different farm locations to fill the order at the least amount of cost?

Part (a): Define your variables and give the objective function and constraint equations.

Part(b): Find the solution using linprog.m (Hint: for the equality equations you may want to have 2 of the same equations where one of the equations is negated in order to imply the condition of greater than and less than to allow for the equals.)

6. Approximation and Fitting

Consider the data points you received following a successful lab experiment:

$$x = (-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3),$$
$$y = (26, 18, 14, 9, 5, 4, 5, 7, 10, 15, 20, 29, 36).$$

Choose an appropriate function and fit it to the data points. Explain your choice. Which Matlab routine is natural to use for minimizing the error in the fit? Plot the data points together with the fit function and comment on how well the function fits the data points.

7. Global Optimization

(a) How does the presence of multiple local minima or maxima in an optimization model effect the type of method that is used to solve the global optimization problem?

(b) Why would mutation in the genetic algorithm help to find the global optimizer?

8. Discrete Optimization

We have a group of *6* single women and *6* single men who are attending the WFU graduate student speed dating event. Each student indicates who among the opposite sex would be acceptable potential partner. This situation can be represented as a bipartite graph. Here, the vertices are the students, and a woman is joined by an edge to a man if they mutually like each other. For example: We could have the women Amber, Brittany, Chrissy, Donna, Eliza, and Fran, and the men Able, Brad, Cane, David, Enoch, and Finn. If Amber liked Able and Brad, (and vice-versa), Brittany liked Cane and Able, Chrissy liked David, Enoch and Finn, Donna liked Brad, Eliza liked Able and David, and Fran liked Finn, what would the bipartite graph look like? In this situation could we match everybody to someone they liked, assuming only one match per person?

9. Statistical Estimation in Optimization

Suppose in any one day, a car dealership can sell 0,1,2,3 or 4 cars. The CEO of ACME Cars noted that for the top dealerships in America, the following probability mass function was true for their daily sales:

X=k	0	1	2	3	4
Pr(X=k)	.10	.25	.45	.15	.05

The CEO implements a different sales strategy each month for 4 months. He is interested in using the strategy with smallest variance in car sales from the top dealerships. The following distributions describe the sales per day under the 4 strategies.

Strategy 1

X=k	0	1	2	3	4
Pr(X=k)	0	.3	.6	.1	0

Strategy 2

X=k	0	1	2	3	4
Pr(X=k)	0	1	0	0	0

Strategy 3

X=k	0	1	2	3	4
Pr(X=k)	.2	.5	.15	.09	.06

Strategy 4

X=k	0	1	2	3	4
Pr(X=k)	.2	.2	.2	.2	.2

Which strategy has the smallest Kullback-Leibler divergence from the top dealerships? Interpret the results. (HINT: Define x=[0 1 2 3 4]' and q=[.1 .25 .45 .15 .05]. Define each strategies probability mass function similiarly).