First-Order Logic

Chapter 8

Pros and cons of propositional logic

☺ Propositional logic is declarative

☺ Propositional logic allows partial/disjunctive/negated information
(unlike most data structures and databases)

☺ Propositional logic is compositional:
meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

☺ Meaning in propositional logic is context-independent
(unlike natural language, where meaning depends on context)

☹ Propositional logic has very limited expressive power
(unlike natural language)

- E.g., cannot say "pits cause breezes in adjacent squares“
  - except by writing one sentence for each square

First-order logic

- Whereas propositional logic assumes the world contains facts, first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Connectives	$\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
• Equality	=
• Quantifiers	$\forall, \exists$

Atomic sentences

Atomic sentence = *predicate* ($term_1$,...,$term_n$)
or $term_1 = term_2$

Term = *function* ($term_1$,...,$term_n$)
or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart) > (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives

$$\neg S,\ S_1 \wedge S_2,\ S_1 \vee S_2,\ S_1 \Rightarrow S_2,\ S_1 \Leftrightarrow S_2,$$

E.g. $Sibling(KingJohn,Richard) \Rightarrow Sibling(Richard,KingJohn)$

$>(1,2) \vee \leq (1,2)$

$>(1,2) \wedge \neg >(1,2)$

Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for

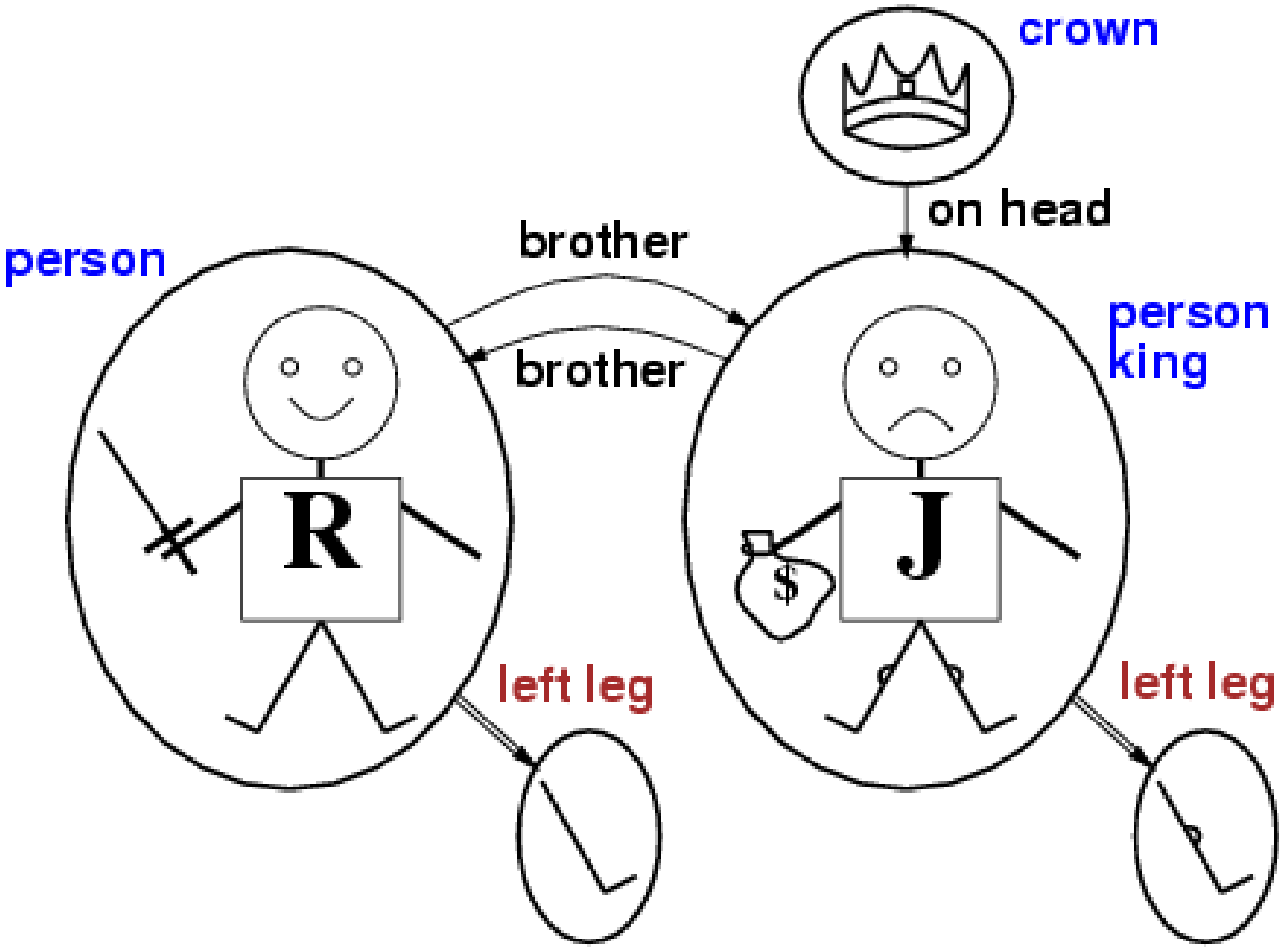
constant symbols → objects

predicate symbols → relations

function symbols → functional relations

- An atomic sentence *predicate(term*$_1$*,...,term*$_n$*)* is true
  iff the objects referred to by *term*$_1$*,...,term*$_n$
  are in the relation referred to by *predicate*

Models for FOL: Example



Universal quantification

Syntax: $\forall$<*variables*> <*sentence*>

Everyone at WFU is smart: $\forall x\ At(x,WFU) \Rightarrow Smart(x)$

- $\forall x\ P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model
- Roughly speaking, equivalent to the conjunction of instantiations of *P*

$$
\begin{array}{ll}
 & At(KingJohn,WFU) \Rightarrow Smart(KingJohn) \\
\wedge & At(Richard, WFU) \Rightarrow Smart(Richard) \\
\wedge & At(Carolina, WFU) \Rightarrow Smart(Carolina) \\
\wedge & \ldots
\end{array}
$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

$\forall$x At(x,WFU) $\wedge$ Smart(x)

means “Everyone is at WFUand everyone is smart”

Existential quantification

- $\exists$*<variables>* *<sentence>*
- There exists someone at WFU who is smart:

$\exists x$ At(x,WFU) $\wedge$ Smart(x)

- $\exists x\ P$ is true in a model *m* iff *P* is true with *x* being some possible object in the model
- Roughly speaking, equivalent to the disjunction of instantiations of *P*

$$
\begin{array}{ll}
 & \text{At(KingJohn,WFU)} \Rightarrow \text{Smart(KingJohn)} \\
\vee & \text{At(Richard, WFU)} \Rightarrow \text{Smart(Richard)} \\
\vee & \text{At(Carolina, WFU)} \Rightarrow \text{Smart(Carolina)} \\
\vee & \ldots
\end{array}
$$

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \text{ At}(x,\text{WFU}) \Rightarrow \text{Smart}(x)$$

is true if there is anyone who is not at WFU and is smart!

Properties of quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$
- $\exists x\ \forall y$ is not the same as $\forall y\ \exists x$
- $\exists x\ \forall y$ Loves(x,y)
  - "There is a person who loves everyone in the world"
- $\forall y\ \exists x$ Loves(x,y)
  - "Everyone in the world is loved by at least one person"

- Quantifier duality: each can be expressed using the other
- $\forall x$ Likes(x,IceCream) $\qquad \neg\exists x\ \neg$Likes(x,IceCream)
- $\exists x$ Likes(x,Broccoli) $\qquad \neg\forall x\ \neg$Likes(x,Broccoli)

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

- E.g., definition of *Sibling* in terms of *Parent*:

  $\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \land\ \exists m,f\ \neg\ (m = f) \land \text{Parent}(m,x)$
  $\land\ \text{Parent}(f,x) \land \text{Parent}(m,y) \land\ \text{Parent}(f,y)]$

Using FOL

The kinship domain:

- Brothers are siblings

  $\forall x,y\ Brother(x,y) \Leftrightarrow Sibling(x,y)$

- One's mother is one's female parent

  $\forall m,c\ Mother(c) = m \Leftrightarrow (Female(m) \land Parent(m,c))$

- “Sibling” is symmetric

  $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x)$

Using FOL

The set domain:

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}\ ) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- $\neg\exists x,s\ \{x|s\} = \{\}$
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- $\forall x,s\ x \in s \Leftrightarrow [\ \exists y,s_2\}\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:
  `Tell`(KB,Percept([Smell,Breeze,None],5))
  `Ask`(KB,∃a BestAction(a,5))
- I.e., does the KB entail some best action at *t=5*?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution σ,
  *S*σ denotes the result of plugging σ into *S*; e.g.,
  S = Smarter(x,y)
  σ = {x/Hillary,y/Bill}
  *S*σ = Smarter(Hillary,Bill)
- `Ask`(KB,S) returns some/all σ such that KB ⊨ σ

Knowledge base for the wumpus world

- Perception
  - $\forall t,s,b\ \text{Percept}([s,b,\text{Glitter}],t) \Rightarrow \text{Glitter}(t)$
- Reflex
  - $\forall t\ \text{Glitter}(t) \Rightarrow \text{BestAction}(\text{Grab},t)$

Deducing hidden properties

- $\forall x,y,a,b\ \mathit{Adjacent}([x,y],[a,b]) \Leftrightarrow$
  $[a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$

Properties of squares:

- $\forall s,t\ \mathit{At}(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$

Squares are breezy near a pit:

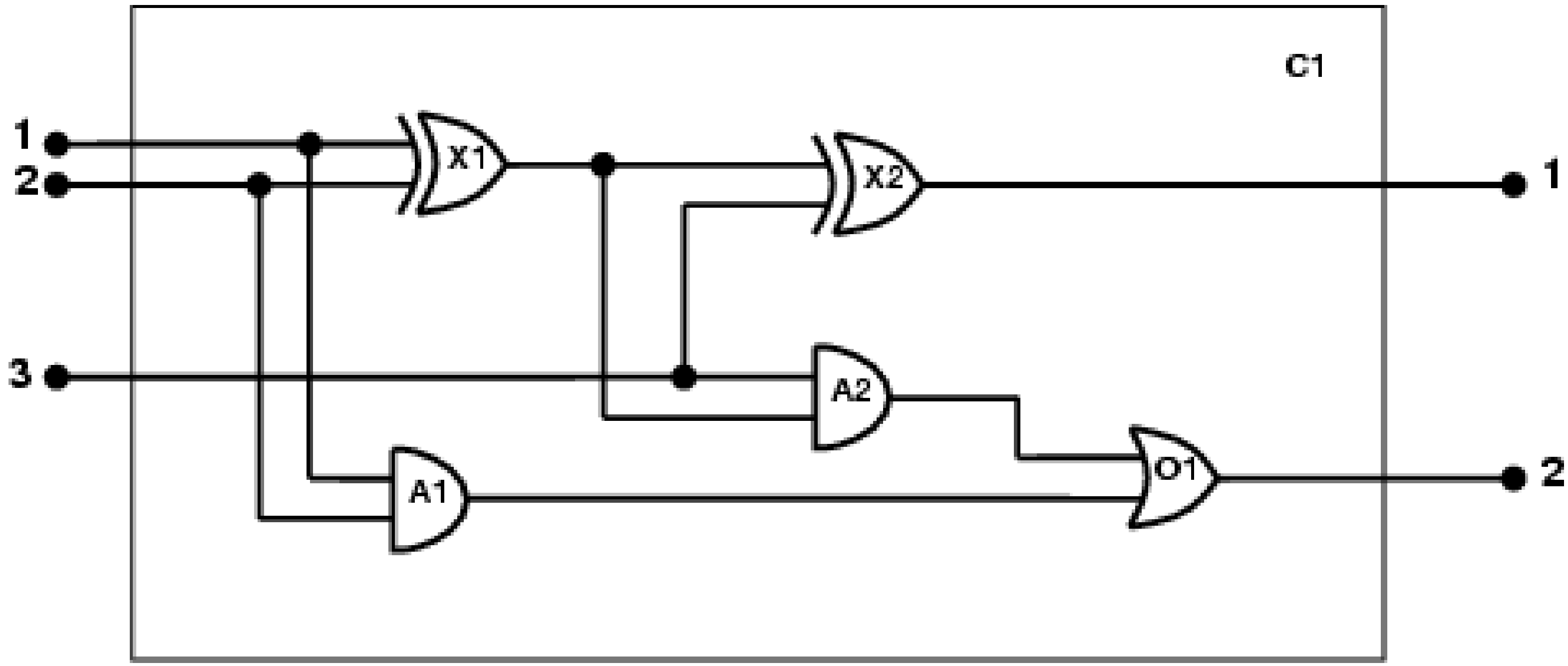
- Diagnostic rule---infer cause from effect
  $\forall s$ Breezy(s) $\Rightarrow$ \Exi{r} Adjacent(r,s) $\wedge$ Pit(r)$
- Causal rule---infer effect from cause
  $\forall r$ Pit(r) $\Rightarrow$ [$\forall s$ Adjacent(r,s) $\Rightarrow$ Breezy(s)$]

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

One-bit full adder



The electronic circuits domain

1. Identify the task
   – Does the circuit actually add properly? (circuit verification)

2. Assemble the relevant knowledge
   – Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   – Irrelevant: size, shape, color, cost of gates

3. Decide on a vocabulary alternatives:

   $Type(X_1) = XOR$

   $Type(X_1, XOR)$

   $XOR(X_1)$

The electronic circuits domain

4. Encode general knowledge of the domain

- $\forall t_1,t_2$ Connected(t_1, t_2) $\Rightarrow$ Signal(t_1) = Signal(t_2)
- $\forall$t Signal(t) = 1 $\vee$ Signal(t) = 0
- 1 ≠ 0
- $\forall t_1,t_2$ Connected(t_1, t_2) $\Rightarrow$ Connected(t_2, t_1)
- $\forall$g Type(g) = OR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ $\exists$n Signal(In(n,g)) = 1
- $\forall$g Type(g) = AND $\Rightarrow$ Signal(Out(1,g)) = 0 $\Leftrightarrow$ $\exists$n Signal(In(n,g)) = 0
- $\forall$g Type(g) = XOR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ Signal(In(1,g)) ≠ Signal(In(2,g))
- $\forall$g Type(g) = NOT $\Rightarrow$ Signal(Out(1,g)) ≠ Signal(In(1,g))

The electronic circuits domain

5. Encode the specific problem instance

$Type(X_1) = XOR$ $\quad$ $Type(X_2) = XOR$

$Type(A_1) = AND$ $\quad$ $Type(A_2) = AND$

$Type(O_1) = OR$

$Connected(Out(1,X_1),In(1,X_2))$ $\quad$ $Connected(In(1,C_1),In(1,X_1))$

$Connected(Out(1,X_1),In(2,A_2))$ $\quad$ $Connected(In(1,C_1),In(1,A_1))$

$Connected(Out(1,A_2),In(1,O_1))$ $\quad$ $Connected(In(2,C_1),In(2,X_1))$

$Connected(Out(1,A_1),In(2,O_1))$ $\quad$ $Connected(In(2,C_1),In(2,A_1))$

$Connected(Out(1,X_2),Out(1,C_1))$ $\quad$ $Connected(In(3,C_1),In(2,X_2))$

$Connected(Out(1,O_1),Out(2,C_1))$ $\quad$ $Connected(In(3,C_1),In(1,A_2))$

The electronic circuits domain

6. Pose queries to the inference procedure

What are the possible sets of values of all the terminals for the adder circuit?

$\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1,C_1)) = $i_1 \wedge$ Signal(In(2,C_1)) = $i_2 \wedge$ Signal(In(3,C_1)) = $i_3 \wedge$ Signal(Out(1,C_1)) = $o_1 \wedge$ Signal(Out(2,C_1)) = o_2

7. Debug the knowledge base

May have omitted assertions like $1 \neq 0$