Nonlocal multiphysics for heterogeneous materials, anomalous diffusion, and hydraulic fracturing

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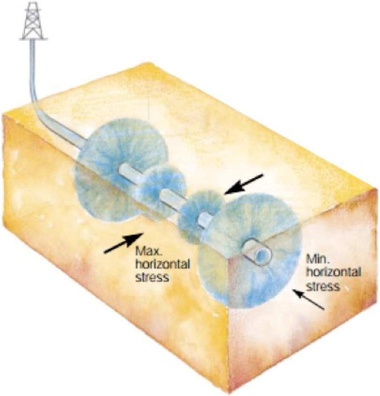
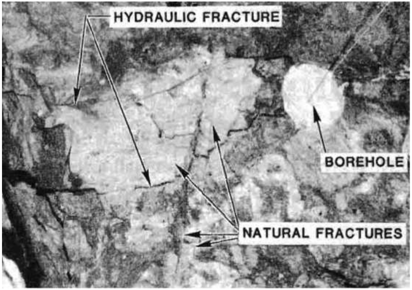
Outline

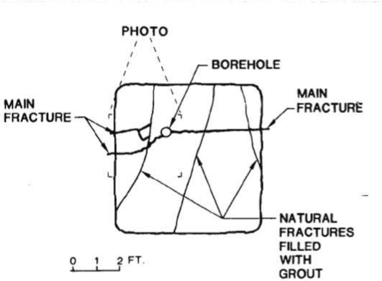
Hydraulic fracturing



The role of natural fractures

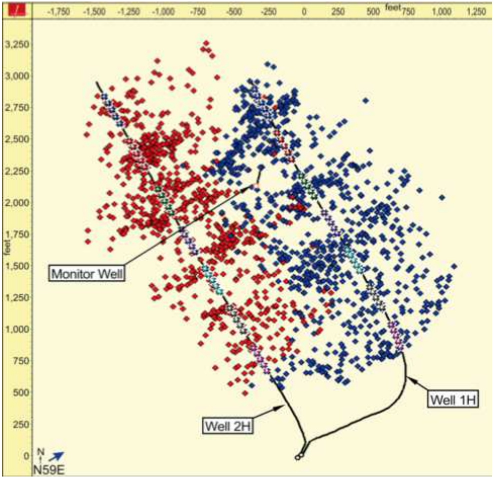
- Most active shale plays contain significant natural fractures.
- The activation of these natural fractures plays an important role in the production of tight gas/shale reservoirs.





N.R. Warpinski and L.W. Teufel: Influence of Geologic Discontinuities on Hydraulic Fracture Propagation (includes associated papers 17011 and 17074). In: *Journal of Petroleum Technology* 39.2 (1987), pp. 209–220. DOI: 10.2118/13224-PA.

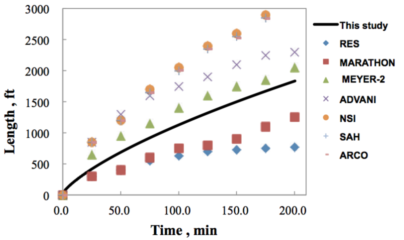
Microsiesmic evidence

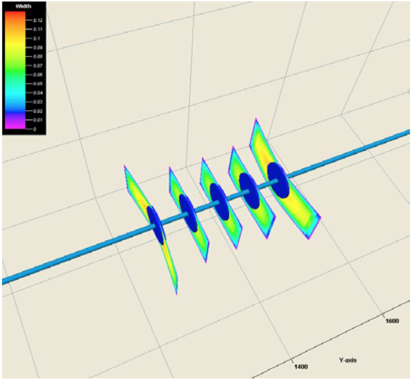


N.R. Warpinski: *Understanding Hydraulic Fracture Growth, Effectiveness, and Safety Through Microseismic Monitoring, Effective and Sustainable Hydraulic Fracturing.* Ed. by R. Jeffery. InTech, 2013. DOI: 10.5772/55974.

Current Modeling Techniques

- P-3D
- BEM
- DEM
- FEM/X-FEM





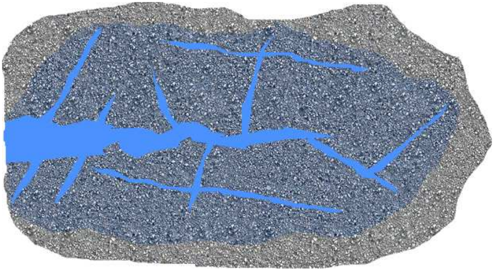
R. Wu, O. Kresse, X. Weng, C. Cohen, and H. Gu: Modeling of Interaction of Hydraulic Fractures in Complex Fracture Networks. In: *SPE Hydraulic Fracturing Technology Conference.* 2012.

J. Olson and K. Wu: Sequential vs. Simultaneous Multizone Fracturing in Horizontal Wells: Insights From a Non-Planar, Multifrac Numerical Model. In: *SPE Hydraulic Fracturing Technology Conference.* SPE 152602. 2012.

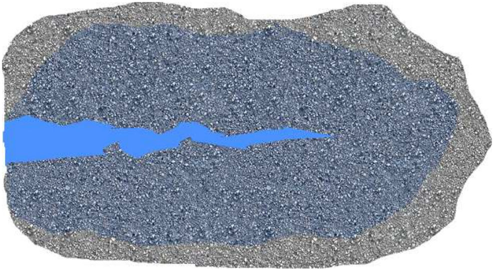
Characteristics for a HF model

- Solid mechanics
  - heterogeneity (material properties, *in situ* stress field, natural fractures, etc.)
  - non-planar fracture models
- Fluid mechanics
  - heterogeneity (permeability, porosity)
  - flow in fractured reservoirs
- Other
  - Ease in building complex realizations
  - Computational efficiency

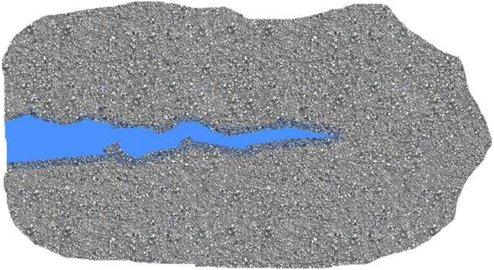
Modeling decisions. . .



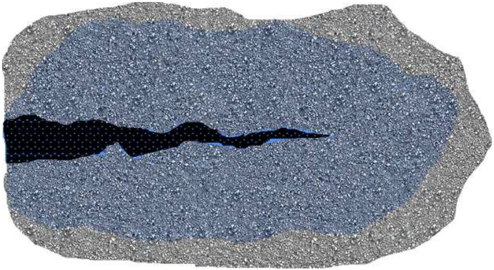
Modeling decisions. . .



Modeling decisions. . .



Modeling decisions. . .



Classical diffusion

Fick's second law

$$\frac{\partial u(\mathbf{x})}{\partial t} = \nabla \cdot (\mathbb{D}(\mathbf{x}) \nabla u(\mathbf{x})) + f(\mathbf{x})$$

Let

$$F(\mathbf{x}) = f(\mathbf{x}) - \frac{\partial u(\mathbf{x})}{\partial t}$$

Develop the variational problem

$$\underbrace{\int_\Omega \nabla \delta u(\mathbf{x}) \cdot (\mathbb{D}(\mathbf{x}) \nabla u(\mathbf{x}))\, \mathrm{d}V_{\Omega_\mathbf{x}}}_{B(\delta u, u)} - \underbrace{\int_\Omega \delta u(\mathbf{x}) F(\mathbf{x}) \mathrm{d}V_{\Omega_\mathbf{x}}}_{l(\delta u)} = 0$$

Quadratic functional

Exploit bilinear symmetry of B

$$\delta I = \frac{1}{2}\delta B(u,u) - \delta l(u) = 0$$

Infer I

$$I = \int_\Omega \underbrace{\frac{1}{2}\nabla u(\mathbf{x}) \cdot (\mathbb{D}(\mathbf{x})\nabla u(\mathbf{x}))}_{Z(u)} \mathrm{d}V_{\Omega_\mathbf{x}} - \int_\Omega u(\mathbf{x})F(\mathbf{x})\mathrm{d}V_{\Omega_\mathbf{x}}$$

or

$$I = \int_\Omega Z(u)\mathrm{d}V_{\Omega_\mathbf{x}} - \int_\Omega u(\mathbf{x})F(\mathbf{x})\mathrm{d}V_{\Omega_\mathbf{x}}$$

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Remove restrictions on Z

- **Locality**
  - Allow Z to depend on points remote from $\mathbf{x}$, i.e. $\mathbf{x}'$
- **Continuity**
  - Allow Z to admit discontinuities in u

$$
\begin{aligned}
Z \equiv \bar{Z} &= \bar{Z}(u(\mathbf{x}'), u(\mathbf{x}), \mathbf{x}', \mathbf{x}) \\
&= \bar{Z}(u', u, \mathbf{x}', \mathbf{x}) \\
&= \bar{Z}(u' - u, \mathbf{x}' - \mathbf{x}) \\
&= \bar{Z}(\underline{U}(\mathbf{x}))\langle \boldsymbol{\xi} \rangle
\end{aligned}
$$

with

$$\underline{U}(\mathbf{x}) = u' - u \quad \text{and} \quad \boldsymbol{\xi} = \mathbf{x}' - \mathbf{x}$$

Minimize I with nonlocal $\bar{Z}$

$$\delta I = \int_\Omega \delta \bar{Z}(\underline{U}) \mathrm{d}V_{\Omega_\mathbf{x}} - \int_\Omega \delta u(\mathbf{x}) F(\mathbf{x}) \mathrm{d}V_{\Omega_\mathbf{x}}$$

Use property of Fréchet derivative for $\delta Z(\underline{U})$

$$\delta I = \int_\Omega \int_\Omega \underline{\nabla} \bar{Z}(\underline{U}) \delta \underline{U} \mathrm{d}V_{\Omega_{\mathbf{x}'}} \mathrm{d}V_{\Omega_\mathbf{x}} - \int_\Omega \delta u(\mathbf{x}) F(\mathbf{x}) \mathrm{d}V_{\Omega_\mathbf{x}}$$

where $\underline{\nabla}$ is the Fréchet derivative operator. Further manipulation leads to

$$\delta I = \int_\Omega \left(- \int_\Omega \underline{\nabla} \bar{Z}(\underline{U}(\mathbf{x})) - \underline{\nabla} \bar{Z}(\underline{U}(\mathbf{x}')) \mathrm{d}V_{\Omega_{x'}} - F(\mathbf{x}) \right) \delta u(\mathbf{x}) \mathrm{d}V_{\Omega_\mathbf{x}}$$

Minimize I with nonlocal $\bar{Z}$

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Minimize I with nonlocal $\bar{Z}$ (cont.)

I has a stationary value at $\delta I = 0$, therefore

$$0 = \int_{\Omega} \underline{\nabla} \bar{Z}(\underline{U}(\mathbf{x})) - \underline{\nabla} \bar{Z}(\underline{U}(\mathbf{x}')) \mathrm{d}V_{\Omega_{x'}} + F(\mathbf{x})$$

$$0 = \int_{\Omega} \underline{Q}(\underline{U}(\mathbf{x})) - \underline{Q}(\underline{U}(\mathbf{x}')) \mathrm{d}V_{\Omega_{x'}} + F(\mathbf{x})$$

For $\underline{Q}(\cdot) = \underline{\nabla} \bar{Z}(\cdot)$. Now using $F(\mathbf{x}) = f(x) - \frac{\partial u(\mathbf{x})}{\partial t}$

$$\frac{\partial u(\mathbf{x})}{\partial t} = \int_{\Omega} \underline{Q}(\underline{U}(\mathbf{x})) - \underline{Q}(\underline{U}(\mathbf{x}')) \mathrm{d}V_{\Omega_{x'}} + f(\mathbf{x})$$

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Minimize I with nonlocal $\bar{Z}$ (cont.)

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Observation #1

- Nonlocal diffusion model converges to the classical theory in the “local limit”

Let

$$\underline{Q}(\mathbf{x}) = \begin{cases} \mathbb{D}\overline{\nabla u}(\underline{U}(\mathbf{x}))\boldsymbol{\xi} & \text{if } \|\boldsymbol{\xi}\| < \delta \\ 0 & \text{otherwise} \end{cases}$$

with

$$\overline{\nabla u}_i = \int_{\mathcal{H}_\delta} \underline{U}\xi_j \mathrm{d}V_{\mathbf{x}'} K_{ij}^{-1}$$

and

$$K_{ij} = \int_{\mathcal{H}_\delta} \xi_i \xi_j \mathrm{d}V_{\mathbf{x}'}$$

Substitute and Simplify

Taylor expand about $\mathbf{x}' = \mathbf{x}$

$$\overline{\nabla u}(\mathbf{x}')_i = \overline{\nabla u}(\mathbf{x})_i + \frac{\partial \overline{\nabla u}(\mathbf{x})_i}{\partial x_k} \xi_k + \mathcal{O}(\|\boldsymbol{\xi}\|^2)$$

$$\begin{aligned}
\frac{\partial u(\mathbf{x})}{\partial t} =& \int_{\mathcal{H}_\delta} D_{ij} \overline{\nabla u}_i(\mathbf{x}) K_{jl}^{-1} \xi_l + \left[D_{ij} \overline{\nabla u}_i(\mathbf{x}) + D_{ij} \frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k} \xi_k + \mathcal{O}(\|\boldsymbol{\xi}\|^2) \right] K_{jl}^{-1} \xi_l \mathrm{d}V_{\mathbf{x}'} \\
=& 2 \int_{\mathcal{H}_\delta} D_{ij} \overline{\nabla u}_i(\mathbf{x}) K_{jl}^{-1} \xi_l \mathrm{d}V_{\mathbf{x}'} + D_{ij} \int_{\mathcal{H}_\delta} \left[\frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k} \xi_k + \mathcal{O}(\|\boldsymbol{\xi}\|^2) \right] K_{jl}^{-1} \xi_l \mathrm{d}V_{\mathbf{x}'} \\
=& D_{ij} \frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k} K_{jl}^{-1} \int_{\mathcal{H}_\delta} \xi_l \xi_k \mathrm{d}V_{\mathbf{x}'} + \mathcal{O}(\delta) \\
=& D_{ij} \frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k} K_{jl}^{-1} K_{lk} + \mathcal{O}(\delta) \\
=& D_{ij} \frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k} \delta_{jk} + \mathcal{O}(\delta) \\
=& D_{ij} \frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_j} + \mathcal{O}(\delta)
\end{aligned}$$

Substitute and Simplify

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=& 2\int_{\mathcal{H}_\delta} D_{ij}\overline{\nabla u}_i(\mathbf{x})K^{-1}_{jl}\xi_l \mathrm{d}V_{\mathbf{x}'} + D_{ij}\int_{\mathcal{H}_\delta}\left[\frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k}\xi_k + \mathcal{O}(\|\boldsymbol{\xi}\|^2)\right]K^{-1}_{jl}\xi_l \mathrm{d}V_{\mathbf{x}'} \\
=& D_{ij}\frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k}K^{-1}_{jl}\int_{\mathcal{H}_\delta}\xi_l\xi_k \mathrm{d}V_{\mathbf{x}'} + \mathcal{O}(\delta) \\
=& D_{ij}\frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k}K^{-1}_{jl}K_{lk} + \mathcal{O}(\delta) \\
=& D_{ij}\frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_k}\delta_{jk} + \mathcal{O}(\delta) \\
=& D_{ij}\frac{\partial \overline{\nabla u}_i(\mathbf{x})}{\partial x_j} + \cancel{\mathcal{O}(\delta)}
\end{aligned}$$

0 as $\delta \to 0$

Observation #2

- Nonlocal diffusion model recovers the fractional Laplacian as a special case

Let

$$\underline{Q}(\mathbf{x}) = \frac{1}{2} C_{n,s} \frac{\underline{U}(\mathbf{x})}{|\boldsymbol{\xi}|^{n+2s}}, \quad 0 < s < 1$$

with

$$C_{n,s} = \frac{2^{2s-1} 2s \Gamma((n+2s)/2)}{\pi^{n/2} \Gamma(1-s)}$$

the R.H.S. of nonlocal diffusion equation then becomes

$$C_{n,s} \int_{\Omega} \frac{u(\mathbf{x}) - u(\mathbf{x}')}{|\boldsymbol{\xi}|^{n+2s}} \mathrm{d}V_{\mathbf{x}'}$$

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$$C_{n,s} \int_{\Omega} \frac{u(\mathbf{x}) - u(\mathbf{x}')}{|\boldsymbol{\xi}|^{n+2s}} \mathrm{d}V_{\mathbf{x}'} = \boxed{(-\Delta)^{2s} u}$$

Nonlocal mass transport

Classical:

$$\rho c \frac{\partial p(\mathbf{x})}{\partial t} = \nabla \cdot \left(\frac{\rho}{\mu} \mathbb{K}(\mathbf{x}) \nabla p(\mathbf{x})\right) + f(\mathbf{x})$$

Nonlocal:

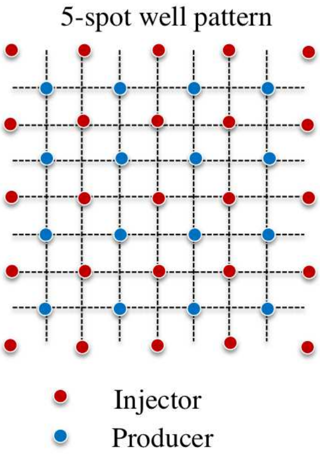
$$\rho c \frac{\partial p(\mathbf{x})}{\partial t} = \int_{\mathcal{H}_\delta} \left(\underline{Q}(\mathbf{x})\langle \boldsymbol{\xi} \rangle - \underline{Q}(\mathbf{x}')\langle -\boldsymbol{\xi} \rangle \right) \mathrm{d}V_{\mathbf{x}'} + f(\mathbf{x})$$

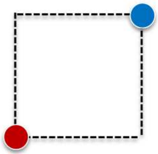
with (in 2D)

$$\underline{Q}(\mathbf{x})\langle \boldsymbol{\xi} \rangle = \frac{\rho}{\mu} \frac{4}{\pi \delta^2} \frac{\boldsymbol{\xi} \cdot \left(\mathbb{K}(\mathbf{x}) - \frac{1}{4} \mathrm{tr}\left(\mathbb{K}(\mathbf{x})\right) \mathbf{I} \right) \cdot \boldsymbol{\xi}}{\|\boldsymbol{\xi}\|^4} \left(p(\mathbf{x}') - p(\mathbf{x}) \right)$$

A. Katiyar, J.T. Foster, H. Ouchi, and M.M. Sharma: A peridynamic formulation of pressure driven convective fluid transport in porous media. In: *Journal of Computational Physics* 261 (Mar. 2014), pp. 209–229. ISSN: 0021-9991. DOI: 10.1016/j.jcp.2013.12.039.

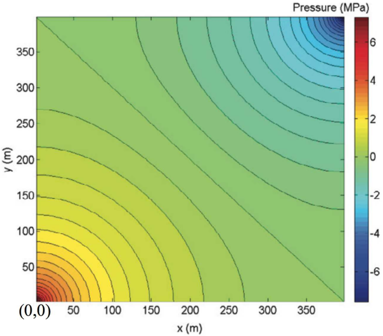
Problem

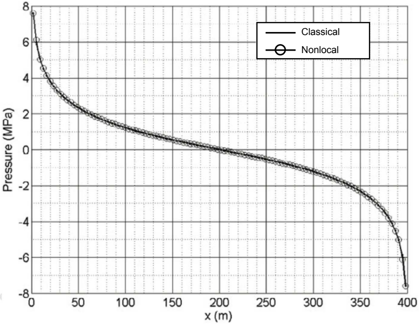




Katiyar, Foster, Ouchi, and Sharma 2014.

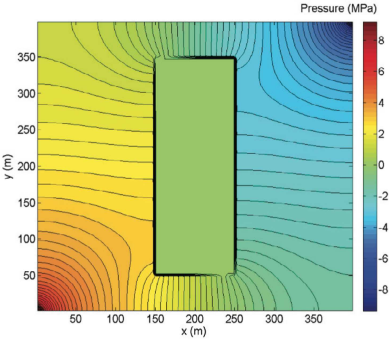
Validation Problem (cont.)

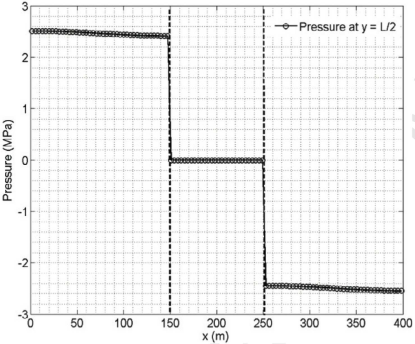




Katiyar, Foster, Ouchi, and Sharma 2014.

Heterogeneity





Katiyar, Foster, Ouchi, and Sharma 2014.

Solid Mechanics

Nonlocal momentum balance

Using the Hamiltonian

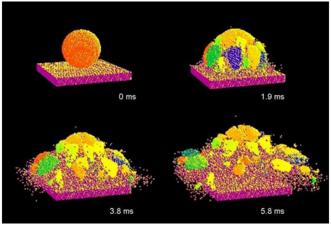
$$H_{\mathbf{u}} = \int_0^\infty \int_\Omega \left(\bar{W}(\underline{\mathbf{Y}}) - \frac{\rho \dot{\mathbf{u}} \cdot \dot{\mathbf{u}}}{2} + \mathbf{b} \cdot \mathbf{u} \right) \mathrm{d}V_{\mathbf{x}} \mathrm{d}t$$

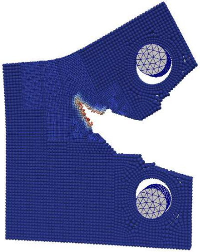
with $\underline{\mathbf{Y}} = \mathbf{u}(\mathbf{x}') - \mathbf{u}(\mathbf{x}) + \mathbf{x}' - \mathbf{x}$ and allowing discontinuous $\mathbf{u}$. The Euler-Lagrange equation associated with this Hamiltonian, is the *peridynamic* equation of motion

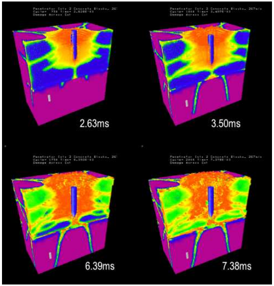
$$\rho \ddot{\mathbf{u}} = \int_\Omega \left(\underline{\mathbf{T}}(\underline{\mathbf{Y}}(\mathbf{x})) - \underline{\mathbf{T}}(\underline{\mathbf{Y}}(\mathbf{x}')) \right) \mathrm{d}V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x})$$

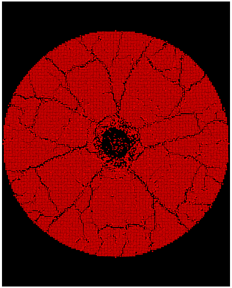
where $\underline{\mathbf{T}} = \underline{\nabla} \bar{W}$.

Examples



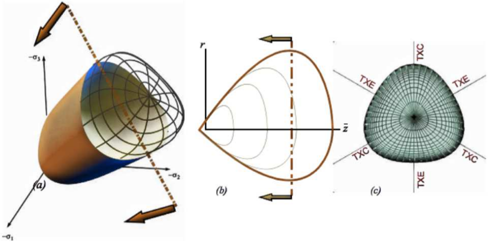






Constitutive modeling

$$\bar{W}(\underline{\mathbf{Y}}) \leftrightarrow W(\mathbf{F})$$



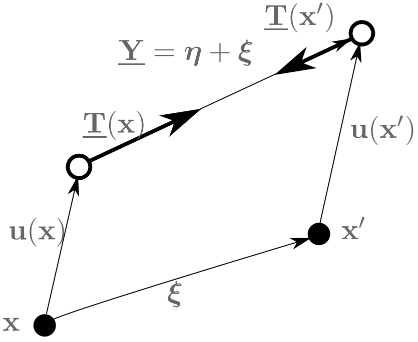
Experiment

Simulation

R.M. Brannon, A.F. Fossum, and O.E. Strack: *Kayenta: Theory and User's Guide*. Tech. rep. Sandia National Laboratories, 2009.

J.T. Foster, S.A. Silling, and W.W. Chen: Viscoplasticity using peridynamics. In: *International Journal for Numerical Methods in Engineering* 81.10 (2010), pp. 1242–1258. ISSN: 1097-0207. DOI: 10.1002/nme.2725.

A fracture model



$$w_{\boldsymbol{\xi}} = \int_0^{\boldsymbol{\eta}(t_f)} \left(\underline{\mathbf{T}}(\underline{\mathbf{Y}}(\mathbf{x})) - \underline{\mathbf{T}}(\underline{\mathbf{Y}}(\mathbf{x}'))\right) \cdot \mathrm{d}\boldsymbol{\eta}$$

J.T. Foster, S.A. Silling, and W. Chen: An energy based failure criterion for use with peridynamic states. In: *International Journal of Multiscale Computational Engineering* 9.6 (2011), pp. 675–688. DOI: 10.1615/IntJMultCompEng.2011002407.

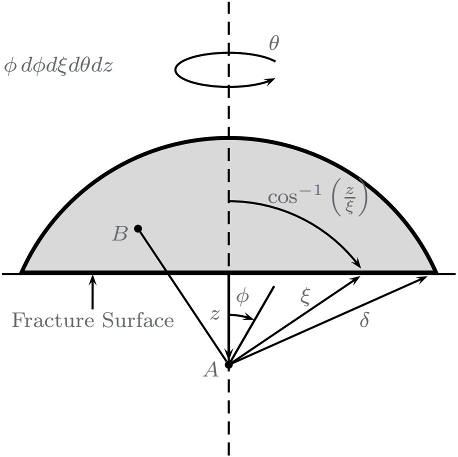
A fracture model (cont.)

$$G = \int_0^\delta \int_0^{2\pi} \int_z^\delta \int_0^{\cos^{-1} z/\xi} w_c \xi^2 \sin\phi \, d\phi d\xi d\theta dz$$

$$w_c = \frac{4G}{\pi\delta^4}$$

$$G = \frac{K_{Ic}^2(1-\nu^2)}{E}$$

$$w_c = \frac{4K_{Ic}^2(1-\nu^2)}{\pi E \delta^4}$$



$$\text{if } w_{\boldsymbol{\xi}} > w_c \text{ then } \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle = \underline{\mathbf{T}}\langle-\boldsymbol{\xi}\rangle = 0$$

Foster, Silling, and Chen 2011.

Coupling

- Constitutive Model

$$\underline{\mathbf{T}} = \left(\frac{-3\bar{\theta}}{m} \underline{\omega x} + \frac{15\mu}{m} \underline{\omega e}^{d} \right) \frac{\underline{\mathbf{Y}}}{\|\underline{\mathbf{Y}}\|}$$

with

$$\bar{\theta} = k\theta + \alpha p$$

where α is the Biot parameter and p is the fluid pressure.

- Damage modified permeability.

$$\mathbb{K} = (1-d)\mathbb{K}_O + d\mathbb{K}_f(w)$$

S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari: Peridynamic States and Constitutive Modeling. In: *J Elasticity* 88 (2007), pp. 151–184. DOI: 10.1007/s10659-007-9125-1.

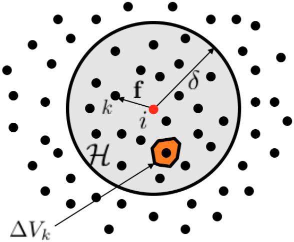
H. Ouchi, J.R. York, A. Katiyar, J.T. Foster, and M.M. Sharma: A Peridynamics Model of Fully-Coupled Porous Flow and Geomechanics for Hydraulic Fracturing. In: *Computational Mechanics* 55.3 (Mar. 2105), pp. 561–576. DOI: 10.1007/s00466-015-1123-8.

Discretization

- Replace integrals with sums...

$$\rho \ddot{\mathbf{u}}(\mathbf{x}_i) = \sum_{k \epsilon \mathcal{H}} \left(\underline{\mathbf{T}}(\mathbf{x}_i) - \underline{\mathbf{T}}(\mathbf{x}_k) \right) \Delta V_k + \mathbf{b}_i$$

$$\rho c \dot{u}(\mathbf{x}_i) = \sum_{k \epsilon \mathcal{H}} \left(\underline{Q}(\mathbf{x}_i) - \underline{Q}(\mathbf{x}_k) \right) \Delta V_k + f_i$$

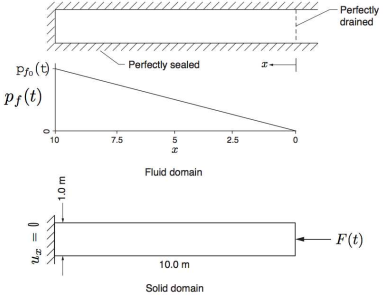


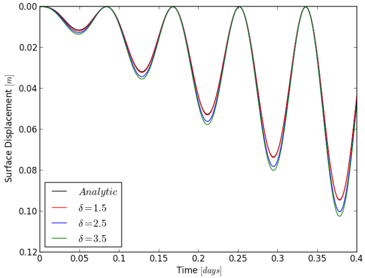
- Meshless (particle method)
  - Related to EFG, RKPM, under certain assumptions. See Bessa et al.
- Currently implemented in DOE Codes
  - Emu, LAMMPS, *Sierra/SM*, *Peridigm*

S.A. Silling and E. Askari: A meshfree method based on the peridynamic model of solid mechanics. In: *Computers and Structures* 83.17-18 (2005), pp. 1526–1535. DOI: 10.1016/j.compstruc.2004.11.026.

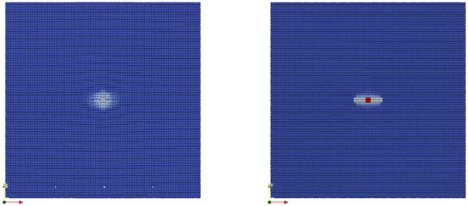
M. Bessa, J.T. Foster, T. Belytschko, and W.K. Liu: A Meshfree Unification: Reproducing Kernel Peridynamics. In: *Computational Mechanics* 53.6 (2014), pp. 1251–1264. DOI: 10.1007/s00466-013-0969-x.

Validation - Time dependent consolidation

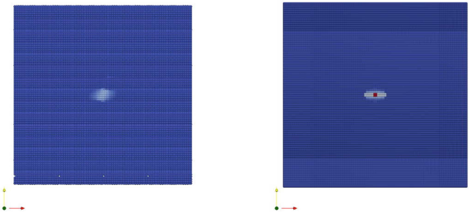




Simple Examples

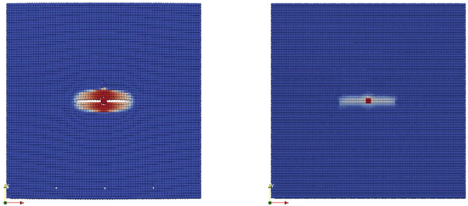


Isotropic permeability

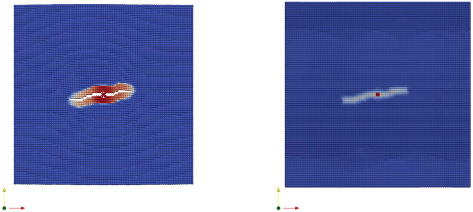


Anisotropic permeability

Simple Examples

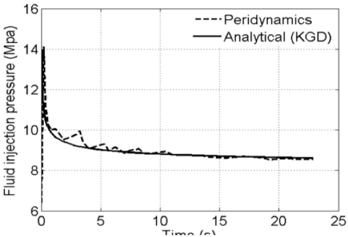


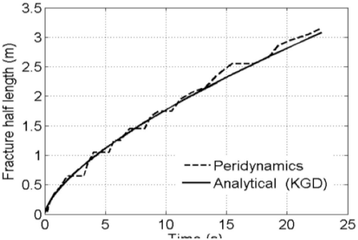
Isotropic permeability



Anisotropic permeability

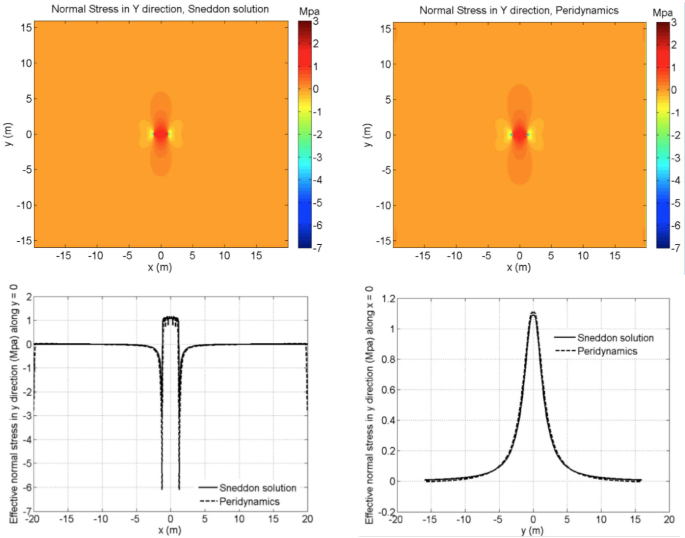
KGD Solution





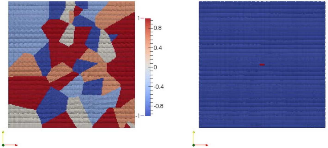
Ouchi, York, Katiyar, Foster, and Sharma 2105.

Sneddon Solution

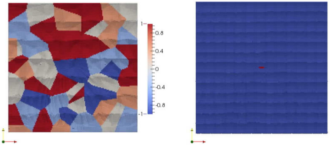


Ouchi, York, Katiyar, Foster, and Sharma 2105.

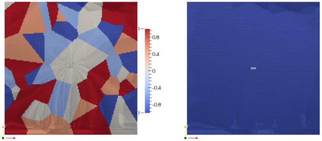
Heterogenuous permeability



Realization 1

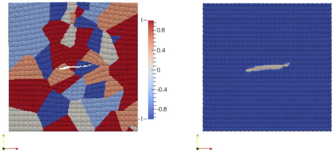


Realization 2

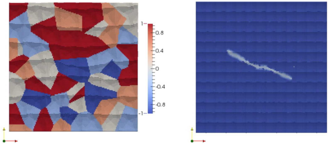


Realization 3

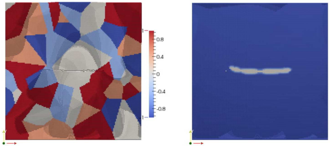
Heterogenuous permeability



Realization 1

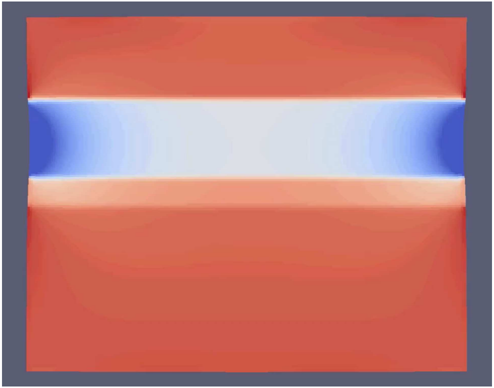


Realization 2

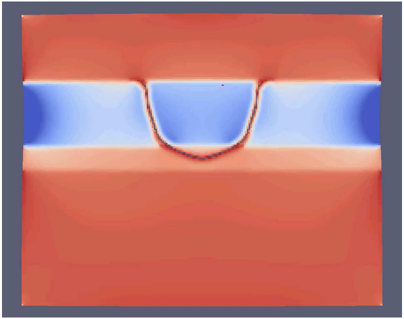


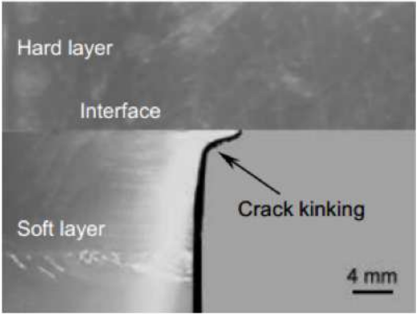
Realization 3

Heterogenuous elastic properties



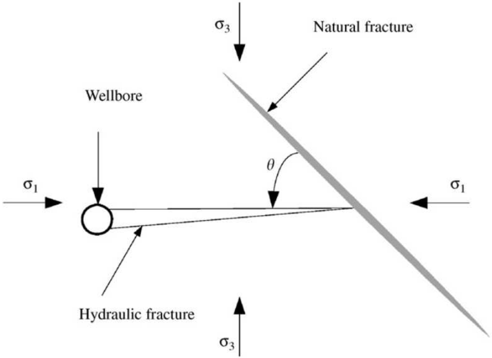
Heterogenuous elastic properties





H Wu, A Chudnovsky, JW Dudley, GK Wong, et al.: A map of fracture behavior in the vicinity of an interface. In: *Gulf Rocks 2004 the 6th North America Rock Mechanics Symposium (NARMS)*. American Rock Mechanics Association. 2004.

Natural fracture interaction

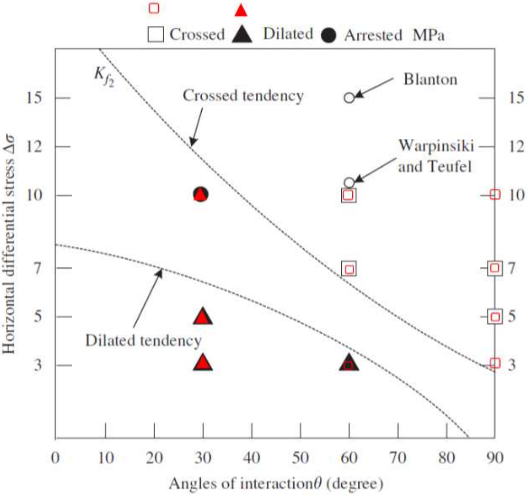


Jian Zhou, Mian Chen, Yan Jin, and Guang-qing Zhang: Analysis of fracture propagation behavior and fracture geometry using a tri-axial fracturing system in naturally fractured reservoirs. In: *International Journal of Rock Mechanics and Mining Sciences* 45.7 (2008), pp. 1143–1152.

Natural fracture interaction

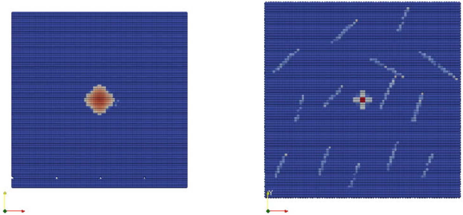
H. Ouchi, A. Katiyar, J.T. Foster, and M.M. Sharma: A Peridynamics Model for the Propagation of Hydraulic Fractures in Heterogeneous, Natrually Fractured Reservoirs. In: *SPE Hydraulic Fracturing Technology Conference*. SPE-173361-MS. Society of Petroleum Engineers. Feb. 2015. DOI: 10.2118/173361-MS.

Natural fracture interaction

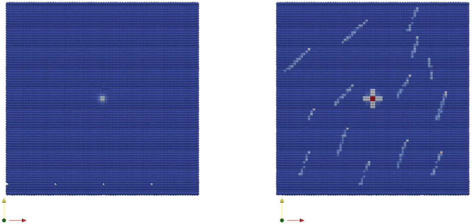


Ouchi, Katiyar, Foster, and Sharma 2015.

Natural fracture interaction

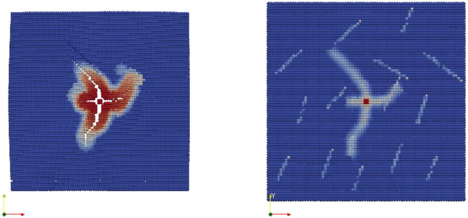


Slow injection rate

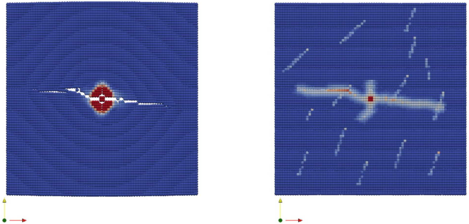


Fast injection rate

Natural fracture interaction

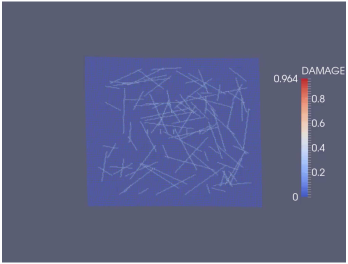


Slow injection rate

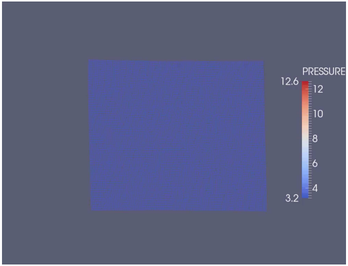


Fast injection rate

Multiple injection sites

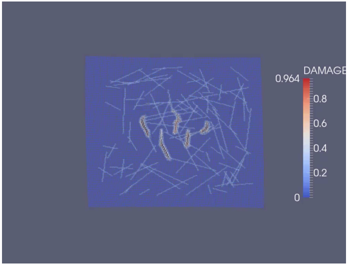


Damage

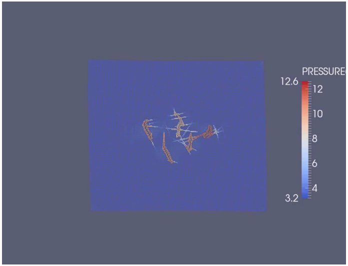


Pressure

Multiple injection sites

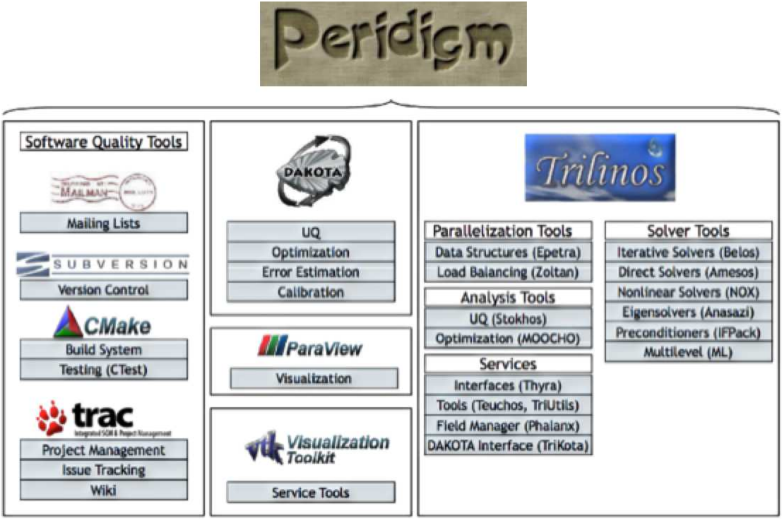


Damage

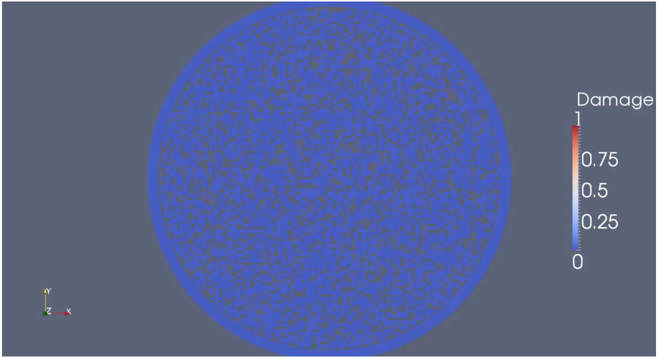


Pressure

HPC Code


Peridigm
Software Quality Tools
Mailing Lists
SUBVERSION
Version Control
CMake
Build System
Testing (CTest)
trac
Project Management
Issue Tracking
Wiki
DAKOTA
UQ
Optimization
Error Estimation
Calibration
ParaView
Visualization
Visualization Toolkit
Service Tools
Trilinos
Parallelization Tools
Data Structures (Epetra)
Load Balancing (Zoltan)
Analysis Tools
UQ (Stokhos)
Optimization (MOOCHO)
Services
Interfaces (Thyra)
Tools (Teuchos, TriUtils)
Field Manager (Phalanx)
DAKOTA Interface (TriKota)
Solver Tools
Iterative Solvers (Belos)
Direct Solvers (Amesos)
Nonlinear Solvers (NOX)
Eigensolvers (Anasazi)
Preconditioners (IFPack)
Multilevel (ML)

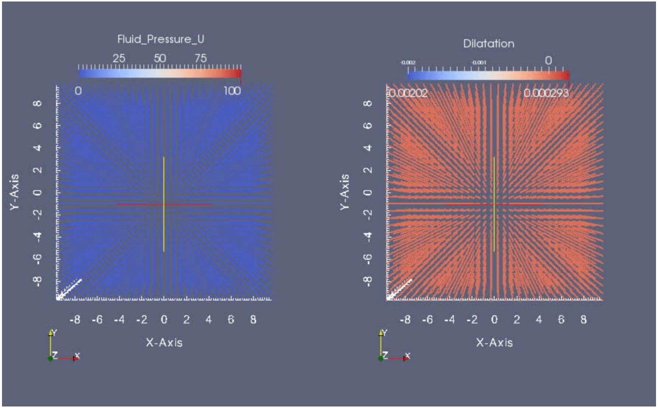
Large-scale *Peridigm* simulation



Acknowledgement:

Sandia National Laboratories

Sneak-peak


Fluid_Pressure_U
25
50
75
0
100
Y-Axis
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
X-Axis
Dilatation
0
0.00202
0.000293
Y-Axis
X-Axis

Ongoing Research

- Multiphase fluids
- Slurry transport
- Fracture mechanics of competing fractures
- HPC
  - Hybrid parallelization
  - Accelerated kernel development for nonlinear solvers

Acknowledgement:

NETL

Questions?