Problems: Passband Modulation

Prof. Sundeep Rangan

1. *Passband conversion.* Let $u(t)$ be a complex baseband signal with the real and imaginary parts of the spectrum (Fourier Transform) shown in Fig. 1. The constants are $f_0 = 5$ MHz, $f_1 = 10$ MHz, $A = 8$ and $B = 10$.

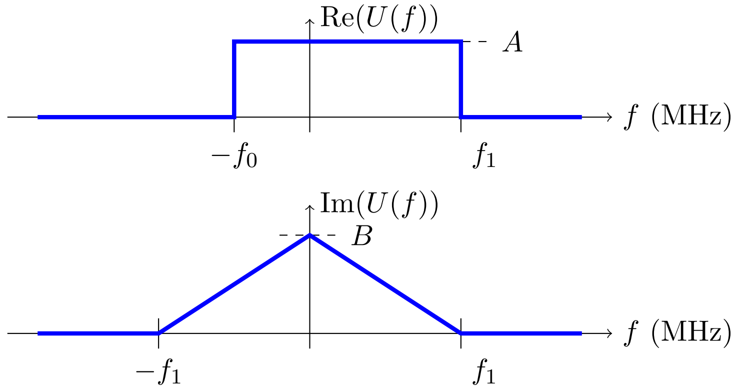

Figure 1: Real and imaginary parts of complex baseband signal $U(f)$

   (a) Suppose that we create a real passband signal $u_p(t) = \mathrm{Re}(u(t)e^{2\pi i f_c t})$ for a carrier frequency $f_c = 800$ MHz. Draw the spectrum of $U_p(f)$. Show both the real and imaginary parts and show both the positive and negative frequencies.

   (b) Is $u(t)$ an energy signal or power signal? What is its energy or power (in linear scale)? Leave your answer in terms of A, B, f_0 and f_1. You do not need to convert to dB scale.

   (c) A receiver attempts to downcovert the signal with a two step process:

$$v(t) = 2u(t)e^{-2\pi i f_c t}, \quad \hat{u}(t) = h_{LPF}(t) * v(t),$$

   where $h_{LPF}(t)$ has a frequency response,

$$H_{LPF}(f) = \begin{cases} C & \text{if } |f| < f_{LPF} \\ 0 & \text{if } |f| \geq f_{LPF}. \end{cases}$$

   For what values of C and f_{LPF} is $\hat{u} = u(t)$?

2. *Baseband equivalent filter.* Consider a communication system with three steps:

   - A complex baseband signal $u(t)$ is upconverted $u_p(t) = \mathrm{Re}(u(t)e^{2\pi i f_c t})$ for some f_c.

- The real passband channel is passed through a linear filter,

$$\frac{dy_p(t)}{dt} = bu_p(t) - ay_p(t),$$

with constants a and $b > 0$.

- The received signal is downconverted, $v(t) = 2y_p(t)e^{-2\pi i f_c t}$ and $y(t) = h_{\rm LPF}(t) * v(t)$ where $h_{\rm LPF}(t)$ is an ideal low-pass filter.

(a) What is the real passband frequency response, $H_p(f) = \frac{Y_p(f)}{U_p(f)}$?

(b) What is the effective baseband frequency response $H(f) = \frac{Y(f)}{U(f)}$?

(c) Find a_1 and b_1 such that

$$\frac{dy(t)}{dt} = b_1 x(t) - a_1 y(t).$$

(d) Suppose that $2\pi f_c \gg a$, what is the power gain of $H(0)$ in dB?

3. *PSD and RX filtering.* Suppose that a real passband signal has two components:

$$x(t) = x_0(t) + x_1(t),$$

where $x_0(t)$ is a desired signal, and $x_1(t)$ is an interfering signal. They have PSD $S_i(f) = A_i\, Rect((f - f_i)/W_i)$, $i = 0, 1$ with parameters:

- Desired signal: $f_0 = 2.50$ GHz, $W_0 = 20$ MHz, total receive power P_0 = -100 dBm.
- Interfering signal: $f_1 = 2.53$ GHz, $W_1 = 10$ MHz, total receive power P_1 = -80 dBm.

(a) Find A_i from P_i using reasonable approximations. State the units of A_i.

(b) Draw $S_0(f)$ and $S_1(f)$.

(c) A signal is downconverted with mixing $v(t) = 2x(t)e^{2\pi i f_c t}$ and $u(t) = h(t) * v(t)$. Find f_c and a filter magnitude response $|H(f)|^2$ such that:

- The component from desired signal is centered at 0 and amplified to -60 dBm.
- The component from interfering signal attenuated to below -110 dBm.

There is no single correct answer. Draw $|H(f)|^2$ and the PSD of $u(t)$.