Problems: RX Filtering

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1. *Effective discrete-time channel.* Consider a digital transmission and reception performed in the following steps:

$$\begin{aligned} u(t) &= \sum s[n] p_{tx}(t - nT), \\ y(t) &= h_{\rm chan}(t) * u(t) \\ r[n] &= v(nT), \quad v(t) = p_{\rm rx}(t) * y(t) \end{aligned} \tag{1}$$

   Suppose that

$$p_{\rm rx}(t) = p_{\rm tx}(t) = \frac{1}{\sqrt{T}} \mathrm{sinc}(t/T),$$

   and

$$h_{\rm chan}(t) = G_1 \delta(t - \tau_1) + G_2 \delta(t - \tau_2).$$

   (a) Find $g(t) = p_{\rm rx}(t) * p_{\rm tx}(t) * h_{\rm chan}(t)$, the channel impulse response with filtering.

   (b) Find the effective discrete-time impulse response, $h[n]$, such that

$$r[n] = \sum_k h[k] s[n-k].$$

   (c) Use MATLAB to plot $h[k]$ when $G_1 = 1$, $G_2 = 0.1$, $\tau_1 = 0$, $\tau_2 = 20$ ns and $1/T = 1$ GHz.

2. *Compute a time-domain response*: Consider a digital transmission and reception performed in the steps in (1) with

$$p_{\rm rx}(t) = \delta(t), \quad p_{\rm tx}(t) = \delta(t - \tau),$$

   and

$$\frac{dy(t)}{dt} = \frac{a}{T}(u(t) - y(t)),$$

   for some $a > 0$.

   (a) Find $h_{\rm chan}(t)$, the baseband impulse response from $u(t)$ to $y(t)$.

   (b) Find $g(t) = p_{\rm rx}(t) * p_{tx}(t) * h_{\rm chan}(t)$, the baseband impulse response with filtering.

   (c) Use MATLAB to plot $h[n]$ the effective discrete-time channel response when $\tau = 3.5T$, $a = 0.1$.

   (d) Use MATLAB to plot the response $r[n]$ when

$$s[n] = \begin{cases} 1 & n = 0, 1, \ldots, 9 \\ 0 & \text{else.} \end{cases}$$

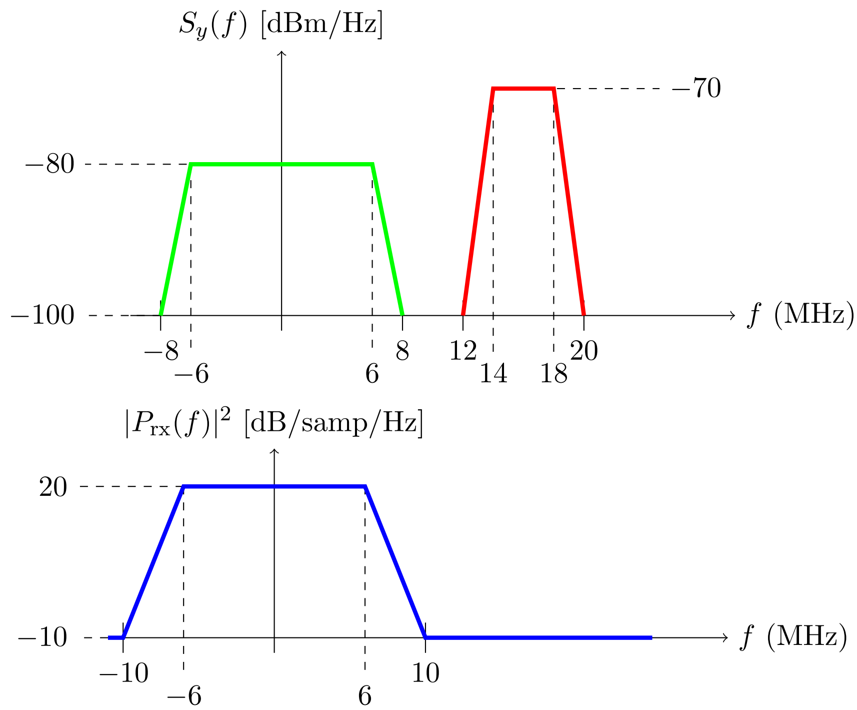

Figure 1: Figures for Problem 4. Top: PSD of the input the RX filter $y(t)$. Bottom: RX filter frequency response.

3. *Sampling in frequency-domain.* Suppose that $v(t)$ has a Fourier Transform,

$$V(f) = \begin{cases} \sqrt{C} & \text{if } f \in [0, 10] \text{ MHz} \\ 0 & \text{else} \end{cases},$$

   where $C = 5(10)^{-6}$ mJ/Hz.

   (a) What is the energy of the signal in dBmJ?

   (b) Find $v(t)$.

   (c) If $r[n] = v(nT)$ with $1/T = 15$ MHz, draw $R(\Omega)$.

4. *RX PSD.* A received signal $y(t)$ has two components with PSD shown in the top of Fig. 1: A desired signal (green) and interfering signal (red). The signal is filtered $v(t) = p_{\mathrm{rx}}(t) * y(t)$ with the frequency response in the bottom panel of Fig. 1.

   (a) Draw the PSD, $S_v(f)$, of the filtered signal $v(t)$. The units of $S_v(f)$ is dBm/Hz^2/sample.

   (b) Draw the discrete-time PSD of the sampled signal $r[n] = v(nT)$ for a sample rate $\frac{1}{T} = 40$ MHz.

   (c) For the component of the signal $y(t)$ with $|f| \le 6$ MHz, what is the energy per sample in $r[n]$? State your answer in dBmJ/sample.