Problems: Symbol Mapping and TX Filtering

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1. *Symbol mapping.* Suppose a sequence of bits $b[k]$ are mapped to a 16-QAM constellation shown in Fig. 1 where the constellation points are $s[n] = s_I[n] + js_Q[n]$ with $s_c[n], s_s[n] \in \{-3, -1, 1, 3\}$. Then, the symbols are modulated to produce a complex baseband signal $u(t)$,

$$u(t) = \sum_{n=-\infty}^{\infty} b[n]p(t - nT),$$

for some pulse shape $p(t)$.

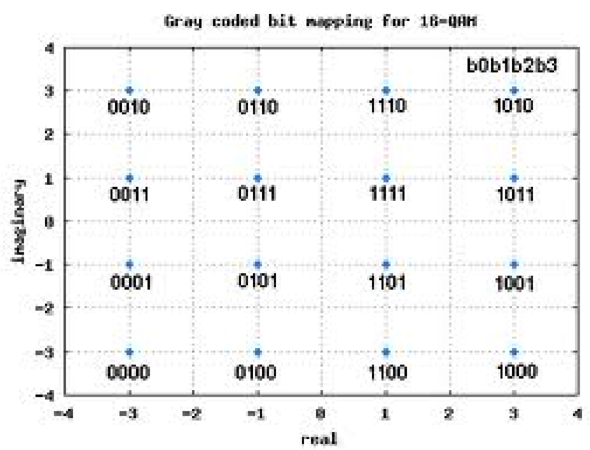

Figure 1: 16-QAM constellation with gray coded bit labels.

(a) For an information rate of 80 Mbps, what should the value of T be? State your units.

(b) Suppose the sequence of bits is $\mathbf{b} = (0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0)$ starting at $b[0]$. Assume the bits are read from left to right so that

$$(b[4n], b[4n + 1], b[4n + 2], b[4n + 3]) \mapsto (b_0, b_1, b_2, b_3)$$

where (b_0, b_1, b_2, b_3) are shown as the labels in the figure. What are the sequence of symbols $s[n]$ resulting from the bits $b[k]$?

(c) Suppose that the pulse shape is

$$p(t) = A \max \{0, 1 - |t/T|\}$$

Draw $p(t)$.

(d) Draw the real and imaginary parts of $u(t)$ for the above symbols and pulse shape.

2. *PSD.* Suppose we use linear modulation,

$$u(t) = \sum_n s[n]p(t - nT),$$

where

- The symbol rate is $\frac{1}{T} = 20$ Msym/s;
- The pulse shape is $p(t) = \sqrt{P_0}e^{-t/T}I_{[0,\infty)}(t)$, where $P_0 = 10$ mW.
- The PSD of the symbols $s[n]$ is $S_s(\Omega) = C\, Rect(\Omega/\Omega_0)$ where $\Omega_0 = 1.6\pi$.

(a) If $\mathbb{E}|s[n]|^2 = 1$, find C.

(b) Draw the PSD, $S_u(f)$ of $u(t)$. Make sure you label your axes.

(c) What is the power in the main lobe (in dBm).

(d) What is the power in the first sidelobe (in dBm)? Only compute the power of the sidelobe in the right frequencies.

3. *Upsampling filter design.* A sequence of symbols $s[n]$ have a symbol rate of $1/T = 30$ MHz and PSD, $S(\Omega)$ shown in the top panel of Fig. 2. Assume the signal of interest is contained in $|\Omega| < 0.8\pi$.

(a) The signal is up-sampled by a factor of $M = 2$ to create a new signal $v[k]$. Assume the upsampling uses zero insertion. What is the sample rate of $v[k]$? Draw the PSD, $S_v(\Omega)$. Make sure you label the axes.

(b) The upsampled signal $v[k]$ is then filtered to produce $w[k] = h[k] * v[k]$ where the power gain of $|H(f)|^2$ is shown in the middle panel of Fig. 2. Assume $\Omega_0 = 0.4\pi$ and $\Omega_1 = 0.6\pi$. Draw the PSD $S_w(\Omega)$.

(c) The filtered signal $w[k]$ is then linearly modulated to produce $u(t) = \sum_k w[k]p(t - kT/2)$ where the power gain of the pulse shape is shown in the bottom panel of Fig. 3. Select the parameters of the pulse shape filters, f_0, f_1, G_0, G_1 such that:

- The PSD $S_u(f)$ is flat in the region of the signal of interest (i.e. $|f| \leq 12$ MHz).
- The PSD $S_u(f)$ is at least 40 dB below the peak PSD for all $|f| > 18$ MHz.
- The total power in the signal of interest is 15 dBm.
- The filter has the widest transition bandwidth $(f_1 - f_0)$ and minimum stopband rejection $(G_0 - G_1)$ as possible.

Draw the resulting PSD $S_u(f)$.

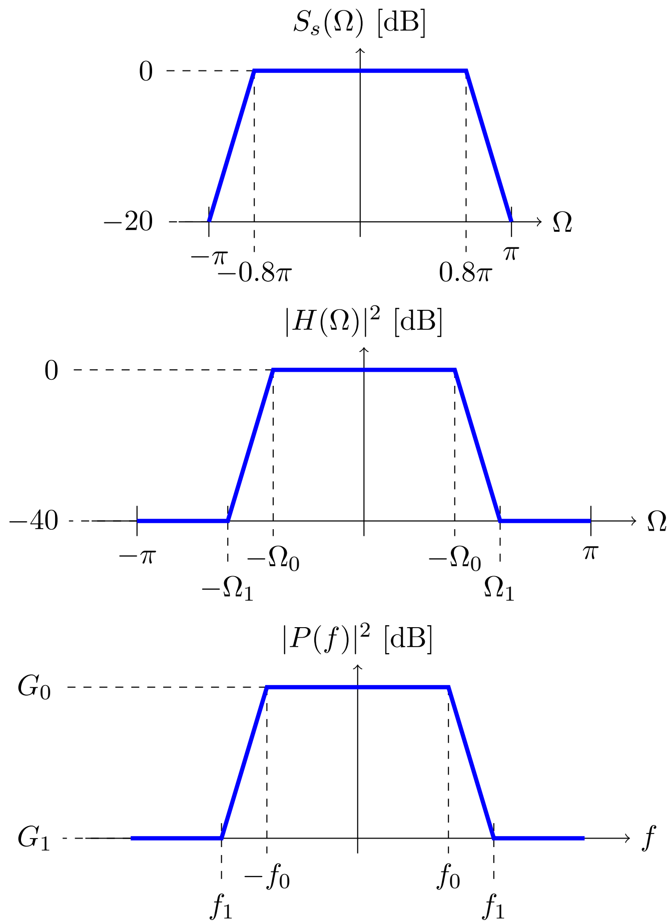

Figure 2: PSDs and filter responses for Problem 3. Note values are in dB scale.