Problems: Demodulation

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1. *SNR.* Suppose that the receive power is -80 dBm, the symbol rate is 1 MHz and the noise power is -170 dBm/Hz.

   (a) What is the symbol SNR? If the system uses 16-QAM what is the data rate and the SNR per bit (E_b/N_0)?

   (b) What if a second noise source is added at a level of -100 dBm? What is the resulting symbol SNR and SNR per bit?

2. *SNR.* Consider a wireless communication system where the transmit power is 10 dBm, the noise power is -170 dBm/Hz and the data rate is 10 Mbps. What is the maximum path loss (transmit - receive power in dB) for E_b/N_0= 10 dB.

3. *Detection theory.* Suppose that we attempt to detect a signal from the received power y. Let $x = 1$ if a signal is present and $x = 1$ if it is present. So, we want to detect x from y. Assume that y is exponential with power P_i if $x = i$, $i = 0, 1$.

   (a) Write the log likelihood ratio,

   $$L(y) = \ln \frac{p(y|x=1)}{p(y|x=0)}.$$

   (b) Consider the likelihood ratio test detector,

   $$\hat{x} = \begin{cases} 1 & \text{if } L(y) \geq \gamma \\ 0 & \text{if } L(y) < \gamma. \end{cases}$$

   Find the probability of missed detection and false alarm in terms of γ,

   $$P_{MD} = P(\hat{x} = 0|x = 1), \quad P_{FA} = P(\hat{x} = 1|x = 0).$$

   (c) Find γ so that the $P_{FA} = (10)^{-3}$. Plot the missed detection rate P_{MD} at this γ as a function of the SNR P_1/P_0. Your plot should have P_1/P_0 in dB and P_{MD} in log scale.

4. *Bit error rate.*

   (a) Assuming Gray coding, derive the formula for the BER for 16-QAM in terms of the symbol SNR $\gamma_s = E_s/N_0$ and bit SNR $\gamma_b = E_b/N_0$.

   (b) Find an approximate expression for the BER for large γ_b.

5. *Vector-valued channel.* Suppose a transmitted symbol is $x \in \{-1, 1\}$ and the received vector in complex signal space is $\mathbf{r} = (r_1, r_2)^T$ with

$$r_1 = h + w_1, \quad r_2 = hx + w_2,$$

where h represents some unknown channel gain and $\mathbf{w} = (w_1, w_2)$ is i.i.d. Gaussian noise $w_i \sim \mathcal{C}N(0, N_0)$. You can think of the first symbol as being transmitted with a reference and the second symbol as carrying the data.

(a) What is the likelihood $p_{\mathbf{r}|h,x}(\mathbf{r}|h, x)$?

(b) Compute the ML estimate for x based on the most likely channel

$$\widehat{x} = \arg\max_{x \in \{-1,1\}} \max_{h \in \mathbb{C}} p_{\mathbf{r}|h,x}(\mathbf{r}|h, x).$$

Describe the decision regions for $\widehat{x}$.

(c) What is the error rate as a function of the SNR $\gamma = |h|^2/N_0$.