A Survey on Analysis and Design of Model-Based Fuzzy Control Systems

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***Abstract*—Fuzzy logic control was originally introduced and developed as a model free control design approach. However, it unfortunately suffers from criticism of lacking of systematic stability analysis and controller design though it has a great success in industry applications. In the past ten years or so, prevailing research efforts on fuzzy logic control have been devoted to model-based fuzzy control systems that guarantee not only stability but also performance of closed-loop fuzzy control systems. This paper presents a survey on recent developments (or state of the art) of analysis and design of model based fuzzy control systems. Attention will be focused on stability analysis and controller design based on the so-called Takagi–Sugeno fuzzy models or fuzzy dynamic models. Perspectives of model based fuzzy control in future are also discussed.**

***Index Terms*—Adaptive control, control theory, fuzzy control, fuzzy models, fuzzy systems, Lyapunov functions, robustness, stability, stabilization, Takagi–Sugeno (T–S) fuzzy models.**

I. Introduction

FUZZY sets and systems have gone through substantial development since the introduction of fuzzy set theory by Zadeh [331]–[335] about four decades ago. They have found a great variety of applications ranging from control engineering, qualitative modelling, pattern recognition, signal processing, information processing, machine intelligence, decision making, management, finance, medicine, motor industry, robotics, and so on [7], [11], [14], [16], [65], [154], [164], [165], [210], [241], [248], [258], [261], [277], [289], [328], [334], [348]. In particular, fuzzy logic control (FLC), as one of the earliest applications of fuzzy sets and systems, has become one of the most successful applications. In fact, FLC has proven to be a successful control approach to many complex nonlinear systems or even nonanalytic systems. It has been suggested as an alternative approach to conventional control techniques in many cases.

The first fuzzy logic control system is developed by Mamdani and Assilian [199], [200], where control of a small steam engine is considered. The fuzzy control algorithm consists of a set of heuristic control rules, and fuzzy sets and fuzzy logic are used, respectively, to represent linguistic terms and to evaluate the rules. Since then, fuzzy logic control has attracted great attention from both academic and industrial communities. Many people have devoted a great deal of time and effort to both theoretical research and application techniques of fuzzy logic controllers. This can be witnessed by a number of excellent books and tutorial articles on the topic; see, for example, [7], [164], [165], [210], [236], [240], [241], [251], [259], [269], [299], [300], and [318]. Much success has also been achieved in applying FLC to various areas including power systems [1], [88], [99], [149]; telecommunications [5], [45], [49], [131], [169], [343]; mechanical/robotic systems [8], [10], [18], [40], [102], [109], [118], [138], [139], [180], [182], [204], [247], [262], [284], [289], [292], [294], [314], [319]; automobile [16], [102], [116], [185], [205], [218], [222], [260]; industrial/chemical processes [22], [41], [90], [111], [129], [137], [146], [153], [162], [199], [200], [229], [248], [258], [271], [279], [288]; aircrafts [58], [73], [130], [161]; motors [9], [100], [143]; medical services [158], [248], [345]; consumer electronics [106], [156], [170], [172], [219], [255], [263], [311]; and other areas such as chaos control [52], [183] and nuclear reactors [17], [217].

The basic structure of a fuzzy control system consists of four conceptual components: knowledge base, fuzzification interface, inference engine, and defuzzification interface [164], [165]. Fig. 1 shows the block diagram of a fuzzy control system.

The knowledge base contains all the controller knowledge and it comprises a fuzzy control rule base and a data base. The data base is the declarative part of the knowledge base which describes definition of objects (facts, terms, concepts) and definition of membership functions used in the fuzzy control rules. The fuzzy control rule base is the procedural part of the knowledge base which contains information on how these objects can be used to infer new control actions. The inference engine is a reasoning mechanism which performs inference procedure upon the fuzzy control rules and given conditions to derive reasonable control actions. It is the central part of a fuzzy control system. The fuzzification interface (or fuzzifier) defines a mapping from a real-valued space to a fuzzy space, and the defuzzification interface (or defuzzifier) defines a mapping from a fuzzy space defined over an output universe of discourse to a real-valued space. The fuzzifier converts a crisp value to a fuzzy number while the defuzzifier converts the inferred fuzzy conclusion to a crisp value.

Based on the differences of fuzzy control rules and their generation methods, approaches to fuzzy logic control can be roughly classified into the following categories: i) *Conventional fuzzy control*; ii) *fuzzy proportional-integral-derivative (PID) control*; iii) *neuro-fuzzy control*; iv) *fuzzy-sliding mode control*; v) *adaptive fuzzy control*; and vi) *Takagi–Sugeno (T–S) model-based fuzzy control*. However, it should be noted that the overlapping among these categories is inevitable. For example,

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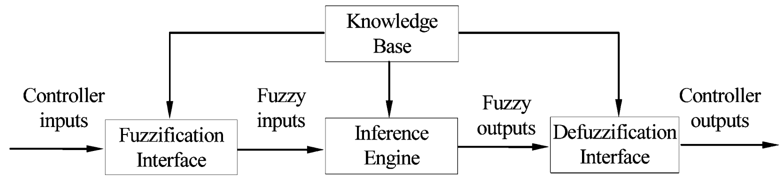

Fig. 1. Basic structure of fuzzy control systems.

conventional fuzzy control can be adaptive, fuzzy PID control can be tuned by neuro-fuzzy systems, or neuro-fuzzy control is adaptive in nature in many cases. Publications on the topic of fuzzy logic control are so huge that an exhaustive list is impossible. Instead, only a very selective list, in fact a small portion of them, is given in the end of this paper. Many excellent works are unfortunately missed. Moreover, this survey paper is not able to cover all these categories of fuzzy logic control in detail. Instead it will briefly review all these categories in the next section and then focus on T–S model-based fuzzy control in more detail in the rest of this paper. Therefore, the main purpose of this paper is to survey state of the art of approaches to systematic analysis and design of model based, in particular, T–S model-based fuzzy control systems which have been developed during the last few years.

The rest of the paper is organized as follows. Section II briefly reviews the general approaches to fuzzy logic control. Section III formulates T–S fuzzy models and discusses their universal function approximation capability. Section IV summarizes main results on stability analysis of T–S fuzzy systems. Sections V–VII present control design approaches based on common (or global) quadratic Lyapunov functions, piecewise quadratic Lyapunov functions, and fuzzy (or non-quadratic) Lyapunov functions, respectively. Concluding remarks, perspectives and challenges of model based fuzzy control in future are discussed in Section VIII.

II. Brief Review of Fuzzy Logic Control

A. Conventional Fuzzy Control (Mamdani Type Fuzzy Control)

Mamdani and Assilian's fuzzy control [199], [200], which is classified as Type-I fuzzy control systems by Sugeno [259], has been replicated for many different control processes. For example, the authors in [137] develop a fuzzy control algorithm for a warm water plant. Ostergaard [229] presents results of experiments with a fuzzy control algorithm for a small scale heat exchanger. There are many other applications of conventional fuzzy control, including robot [10], [289], [314], [319], stirred tank reactor [146], traffic junction [237], steel furnace [153], cement kilns [288], automobile [16], [218], [260], wastewater treatment [279], aircraft [58], [161], missile autopilot [73], motor [100], network traffic management and congestion control [131], [169], bioprocesses [111], fusion welding [15], and so on. In addition, fuzzy control has been widely used in various consumer electronic devices such as video cameras, washing machines, TV, and sound systems in the late 1980s and early 1990s [106], [156], [170], [172], [219], [255], [263], [311].

These methods of conventional fuzzy control are essentially heuristic and model free. The fuzzy control "IF-THEN" rules are obtained based on an operator's control action or knowledge. It is obvious that the design method works well only in the case where an operator plays an important role in controlling the system. Even though the performance of such control scheme is generally satisfactory, stability issue of the closed loop fuzzy control system is often criticized in the earlier development of these methods though the authors in [20] provide a stability analysis of fuzzy control systems via a heuristic approach. Moreover, design of such control systems suffers from lack of systematic and consistent approaches. Thus great efforts have been devoted to stability analysis and controller design issues of conventional fuzzy control systems, and various approaches have been developed. The key idea of these approaches is to regard a fuzzy controller as a nonlinear controller and embed the stability and/or control design problem of fuzzy control systems into conventional nonlinear system stability theory. The typical approaches include describing function approach [136], cell-state transition [132], Lure's system approach [59], [208], Popov's theorem [91], circle criterion [226], [244], [252], conicity criterion [69], sliding mode control [120], and hyperstability [21], [226], among others. However, a general systematic theory for stability analysis and control design of conventional fuzzy control systems is still out of reach. Additional references on the topic of conventional fuzzy control can be found in [5], [62], [66], [88], [99], [108], [158], [160]–[162], [164], [165], [173], [176], [233], [247], [278], [325], and [343].

B. Fuzzy PID Control

Conventional PID controllers are still the most widely adopted method in industry for various control applications, due to their simple structure, ease of design, and low cost in implementation. However, PID controllers might not perform satisfactorily if the system to be controlled is of highly nonlinear and/or uncertain nature. On the other hand, conventional fuzzy control has long been known for its ability to handle nonlinearities and uncertainties through use of fuzzy set theory. It is thus believed that by combining these two techniques together a better control system can be achieved.

The name of fuzzy PID control has been widely used in literature with all sorts of different meanings. For example, the authors in [114] suggest that if a fuzzy controller is designed (or implied equivalently) to generate control actions within PID concepts like a conventional PID controller, then it is called the fuzzy PID controller. In this aspect, the conventional fuzzy controller developed by Mamdani and Assilian [199], [200] is in fact a two-input fuzzy PI controller. Moreover, this conventional fuzzy controller can be further classified as the "direct-action" type [201] of fuzzy PID controllers, since its fuzzy inference deduces a control action output directly to control a system. In contrast with "direct-action," another type of fuzzy PID controllers is classified as "gain-scheduling" [107], [344], for the reason that controller gains change as operating condition or dynamics of a system varies.

Generally the "direct-action" type of fuzzy PID controllers is able to do as well as conventional PID controllers. However, high cost of setting up a fuzzy control system would usually discourage replacing a conventional PID controller with a "direction-action" fuzzy PID controller. As suggested by Chiu [57], it is the "gain-scheduling" type of fuzzy PID controllers that is more likely to gain acceptance from industry. In addition, it is shown that many fuzzy PID controllers are nonlinear PID controllers and perform better than conventional PID controllers in most cases [53], [57], [107], [113], [114], [179], [202], [203], [215], [270], [316], [344].

Other topics of interest related to fuzzy PID control include tuning of fuzzy PID parameters [202], [215], [310], [316], optimal fuzzy PID controller based on genetic algorithm [113], [270], realization of conventional PID controllers by fuzzy control method [213], improved robust fuzzy-PID controller with optimal fuzzy reasoning [179], and stability of fuzzy PID controllers [54], [253]. The author in [53] gives excellent overview on fuzzy PID controllers in general, including adaptive fuzzy PID control and applications of fuzzy PID control. One major limitation of fuzzy PID control is the difficulty of its systematic design with consistent and guaranteed performance. Additional references on the topic of fuzzy PID can be found in [8], [108], [182], [211], [216], [230], [250], [254], [262], and [325].

C. Neuro-Fuzzy Control

Neuro control, more precisely neural network control, and fuzzy control are two of the most popular intelligent control techniques. They are similar in many ways. For example, both of them are basically model-free control techniques, both are able to store knowledge and use it to make control decisions, and both are able to provide robustness of control to certain extent with respect to system variations and external disturbances. However, the two techniques are different in their ways to obtain knowledge. Neuro control acquires knowledge mainly through data training (or learning). This could be an advantage as it lets the data "speak" for itself, but sometimes a disadvantage if the training data set does not fully represent the domain of interest. Fuzzy control, in particular conventional fuzzy control, on the other hand mainly obtains qualitative and imprecise knowledge via an operator or expert's perspective.

As the two control techniques complement to each other, that is, neuro control providing learning capabilities and high computation efficiency in parallel implementation, and fuzzy control providing a powerful framework for expert knowledge representation, the combination or integration of the two techniques have attracted lots of attention from control community. A typical combination of these two techniques is the so-called neuro-fuzzy control, which is basically a fuzzy control augmented by neural networks to enhance its characteristics like flexibility, data processing capability, and adaptability [17], [63], [72], [90], [123], [124], [138], [163], [177], [178], [186], [187], [193], [205], [209], [217], [271], [294], [305], [306], [342]. The process of fuzzy reasoning is realized by neural networks, whose connection weights correspond to the parameters of fuzzy reasoning [38], [123], [124], [135], [187], [220], [231], [232], [264]. Using back-propagation type, or reinforcement type, or any other type neuro network learning algorithms, a neuro-fuzzy control system can identify fuzzy control rules and learn (tune) membership functions of the fuzzy reasoning, and thus realize the neuro-fuzzy control. An excellent survey is given in [212] for neuro-fuzzy rule generation in a more general setting of soft computation. Other topics of interest related to this class of control scheme include tuning parameters in neuro-fuzzy controller via genetic algorithm [72], [209], [249], [306], tuning PID controllers via fuzzy neural networks [250], self-organizing or adaptive neuro-fuzzy control [63], [177], [178], [193], [217], [294], and input–output stability analysis based on small gain theorem [89]. Additional references on the topic of neuro-fuzzy control can be found in [64], [89], [93], [149], [174], [185], [189], [222], [223], [227], [292], and [329]. It should be noted that the T–S fuzzy model is one of the general fuzzy systems used to realize the neuro-fuzzy control in this category, for example, see [129], [287], [292], and [329].

One of the main advantages of neuro-fuzzy control is that it does not basically require information on the mathematical model of a system to be controlled. Thus this class of fuzzy control offers a new avenue in solving many difficult control problems in real life where the mathematical model of a system might be hard, if not impossible, to obtain. However, one of its major limitations is the systematic analysis of stability of the closed loop control systems and convergence of the learning algorithms in the context of the closed loop control systems.

D. Fuzzy Sliding-Mode Control

It is well known that sliding-mode control provides a robust approach to controlling nonlinear systems with uncertainties [290], [349]. Its salient features include good control performance for nonlinear systems, applicability to multiple-input–multiple-output (MIMO) systems, and most importantly, robustness to parameter changes and/or external disturbances. It however often results in chattering phenomena due to its discontinuous switching which arises from its digital implementation. Although a fuzzy controller is shown to be similar to a modified sliding mode controller [234], the key idea of fuzzy sliding model control is to combine or integrate fuzzy control and sliding mode control in such a way that the advantages of both techniques can be realized. One approach is that a sliding mode controller is equipped with capability of handling fuzzy linguistic qualitative information [50], [94], [235]. A direct benefit of such control is that fuzzy logic can effectively eliminate chattering through construction of fuzzy boundary layers which replace crisp switching surfaces [94], [101], [121]. Another approach is to design fuzzy control systems in a way of conventional fuzzy control, fuzzy PID control, or model based fuzzy control, and then to add a supervisory sliding model controller to not only guarantee stability but also improve robust performance of the closed-loop control systems [80], [206], [298].

Another important advantage of fuzzy sliding mode control is that stability analysis and controller design issue of fuzzy control systems can be addressed within the framework of sliding mode control [50], [55], [120], [235], [257], and the well developed techniques of sliding mode control can be applied [290], [349]. Other topics of interest in fuzzy sliding mode control include using genetic algorithms to tune fuzzy membership functions of such controllers [50], [188], decoupling of the high-dimensional systems into subsystems with lower dimensionality [196], use of adaptive fuzzy systems in parameter tuning of sliding-mode controllers [68], and adaptive fuzzy sliding mode control [13], [55], [63], [67], [116]–[118], [274], [280]. In addition, the authors in [134] present an excellent survey on the fusion of computationally intelligent methodologies, including fuzzy logic, and sliding model control. Additional references on the topic of fuzzy sliding mode control can be found in [9], [119], [180], [184]–[186], [305], [309], and [313].

E. Adaptive Fuzzy Control

Adaptive control refers to the control of partially known systems with some kind of adaptation mechanism. Most works in adaptive control are based on the assumption of linear or simplified non-linear mathematical models of systems to be controlled. In fact, adaptive control of linear systems and certain special classes of nonlinear systems has been well developed from the late 1970s to the 1990s [96], [122], [155], while adaptive control of general nonlinear systems still presents a challenge to control community. Nevertheless, mathematical models might not be available for many complex systems in practice, and the adaptive control problem of these systems is far from being satisfactorily resolved.

Following the similar idea in neural networks [246] for their universal function approximation capability, it is shown [301] that a fuzzy system is capable of approximating any smooth nonlinear functions over a convex compact region. Other excellent works on the topic of function approximation of fuzzy systems can be found in [326] and [336]–[339]. Based on this function approximation capability of fuzzy systems, the author in [298] presents an adaptive fuzzy controller for affine nonlinear systems with unknown functions. Fuzzy basis function based fuzzy systems are used to represent those unknown nonlinear functions. The parameters of the fuzzy systems including membership functions characterizing linguistic terms in fuzzy rules are updated according to some adaptive laws which are derived based on Lyapunov stability theory. Since then, a great number of works on adaptive fuzzy control have been reported, see for example, [4], [18], [23], [41], [44], [70], [87], [97], [103], [150], [166], [171], [243], [256], [280], [291], [320], [321], [340], [341], and [346]. The key idea of these works is to use fuzzy systems to approximate unknown nonlinear functions in nonlinear systems and to represent the fuzzy systems in the form of linear regression with respect to unknown parameters and then to apply the well developed adaptive control techniques. However, it should be noted that some kinds of robust approaches have to be adopted for adaptive fuzzy control due to the inherent approximation errors between the approximating fuzzy systems and the original nonlinear functions, and most likely only semiglobal stabilization can be achieved if no supplementary control strategy is employed.

Other topics of interest include improved adaptive fuzzy control schemes with smaller number of tuning parameters or better performance [87], [320], [321], robust adaptive fuzzy controller with various kinds of performances with respect to external disturbances [39], [44], [97], fuzzy model reference adaptive control [95], [150], [324], using genetic algorithms to adaptively tuning membership functions [190], and self-organizing schemes to tune fuzzy membership functions [4], [189]. Fusion of adaptive techniques and sliding mode control techniques are presented in [13], [55], [63], [67], [116]–[118], [274], and [280]. Comparison of adaptive fuzzy control to conventional adaptive control is reported in [228]. Additional references on the topic of adaptive fuzzy control can be found in [1], [5], [49], [64], [68], [93], [96], [139], [174], [193], [205], [217], [222], [223], [227], [238], [242], [257], [281], [306], [315], [330], and [345].

F. T–S Model-Based Fuzzy Control

T–S fuzzy model [265], also called the Type-III fuzzy model by Sugeno [259], is in fact a fuzzy *dynamic* model [25], [28], [29]. This model is based on using a set of fuzzy rules to describe a global nonlinear system in terms of a set of local linear models which are smoothly connected by fuzzy membership functions. This fuzzy modelling method offers an alternative approach to describing complex nonlinear systems [28], [71], [127], [269], [326], [336], and drastically reduces the number of rules in modelling higher order nonlinear systems [259]. Consequently, T–S fuzzy models are less prone to the curse of dimensionality than other fuzzy models. More importantly, T–S fuzzy models provide a basis for development of systematic approaches to stability analysis and controller design of fuzzy control systems in view of powerful conventional control theory and techniques.

A great number of theoretical results on function approximation, stability analysis, and controller synthesis have been developed for T–S fuzzy models during the last ten years or so. T–S fuzzy models are shown to be universal function approximators in the sense that they are able to approximate any smooth nonlinear functions to any degree of accuracy in any convex compact region [28], [71], [127], [269], [326], [336]. This result provides a theoretical foundation for using T–S fuzzy models to represent complex nonlinear systems approximately. Based on the differences of design approaches, the methods for stability analysis and control design of T–S fuzzy systems can be roughly classified into the following six categories: i) *simple local controller design and stability checking*; ii) *stabilization with/without various performance indexes such as H_∞ and H_2 control based on a nominal linear model and a single quadratic Lyapunov function*; iii) *stabilization with/without various performance indexes based on a common quadratic Lyapunov function*; iv) *stabilization with/without various performance indexes based on a piecewise quadratic Lyapunov function*; v) *stabilization with/without various performance indexes based on a fuzzy Lyapunov function*; and vi) *adaptive control when parameters of T–S fuzzy models are unknown*.

The first category of methods is proposed in the earlier stage of developments [24], [25], [30], [80], [133]. Its basic idea is to design a feedback controller for each local model, to obtain a global controller by combining the local controllers in certain way, and then to use some stability criteria to check stability of the resulting closed-loop fuzzy control system. Unfortunately, this kind of methods suffers from a problem that the design process is not constructive in general and many steps of trial and error might be needed before an acceptable controller design can be obtained. The main idea of the second category of methods is to represent a T–S fuzzy model as a *nominal* linear model with uncertainties around the equilibrium of the system, which include all the nonlinearities of the T–S fuzzy model, and then to recast the control problem as a robust linear control problem with uncertainties [74], [83], [145]. In this way, many available robust control synthesis approaches can be directly applied to or further developed for the T–S fuzzy systems. However, this kind of methods tends to be conservative since one nominal model has to be assumed which might not be the case for many complex highly nonlinear systems, thus has not become a mainstream of research efforts in model based fuzzy control.

The basic idea of categories iii)–v) of methods is to design a feedback controller for each local model and to construct a global controller from the local controllers in such a way that global stability with/without various performance indexes of the closed-loop fuzzy control system is guaranteed. The major techniques that have been used include quadratic stabilization, linear matrix inequalities (LMIs), Lyapunov stability theory, bilinear matrix inequalities, and so on. The third category of methods is most popular to date [2], [3], [6], [12], [33]–[37], [40], [42], [46], [47], [56], [61], [109], [110], [112], [115], [119], [125], [130], [140]–[144], [147], [148], [151], [159], [167], [168], [175], [181], [183], [191], [192], [195], [197], [198], [206], [214], [221], [239], [267]–[269], [273], [275], [276], [282]–[286], [293], [302]–[304], [312], [317], [322], [323], [327]. It, however, requires that a common quadratic Lyapunov function can be found for all the local subsystems in a T–S fuzzy model, and this proves to be conservative in many cases. As a less conservative alternative, the fourth category of methods, at the same time, has also been well developed [26], [27], [29], [51], [52], [74], [76]–[78], [82]–[86], [104], [105], [128], [157], [224], [272], [295]–[297]. The fifth category of methods has attracted some attention recently but it presents more challenges or difficulties [60], [98], [266], [307], [347]. The sixth category of methods is to deal with control of T–S fuzzy systems when parameters of T–S fuzzy models are unknown. The most works to date however are quite preliminary in the sense that they only consider unknown parameters in local linear models by assuming that the number of fuzzy rules and membership functions are all known *a priori* [75], [81], [126], [144], [238].

All these results on various approaches to fuzzy logic control, in particular on approaches to T–S model-based fuzzy control demonstrate that these methods provide systematic tools for analysis and design of fuzzy control systems, and that conventional linear system control theories can be suitably utilized and developed for analysis and design of model based fuzzy control systems. In the next few sections, the more detailed survey on the T–S fuzzy model based approaches will be presented. For the sake of presentation simplicity only developments of discrete time T–S fuzzy systems will be focused in this paper. However, it should be noted that the developments of continuous time counterparts have also been widely reported in literature.

III. T–S Model and Universal Function Approximation

T–S fuzzy models or so-called fuzzy dynamic models can be used to represent complex MIMO systems with both fuzzy inference rules and local analytic linear dynamic models as follows:

$$\begin{aligned} R^l :\ & \text{IF} \quad z_1 \text{ is } F_1^l \text{ and } \cdots z_\nu \text{ is } F_\nu^l \\ & \text{THEN } x(t+1) = A_l x(t) + B_l u(t) + a_l \\ & \qquad y(t) = C_l x(t), \\ & l \in L := \{1, 2, \ldots, m\} \end{aligned} \tag{3.1}$$

where R^l denotes the lth fuzzy inference rule, m the number of inference rules, $F_j^l (j = 1, 2, \ldots, \nu)$ are the fuzzy sets, $x(t) \in \Re^n$ the state vector, $u(t) \in \Re^p$ the input vector, $y(t) \in \Re^q$ the output vector, and (A_l, B_l, a_l, C_l) the matrices of the lth local model, and $z(t) := [z_1, z_2, \ldots, z_\nu]$ some measurable variables of the system, for example, the state variables. It is also assumed without loss of generality that the origin is the equilibrium of the T–S fuzzy system (3.1).

It is noted that the local model in terms of (A_l, B_l, a_l, C_l) in (3.1) only represents the properties of the system in a local region and thus is referred to as the fuzzy local model.

By using a standard fuzzy inference method, that is, using a singleton fuzzifier, product fuzzy inference, and center-average defuzzifier, the T–S fuzzy model in (3.1) can be rewritten as [269]

$$\begin{aligned} x(t+1) &= A(\mu)x(t) + B(\mu)u(t) + a(\mu) \\ y(t) &= C(\mu)x(t) \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} A(\mu) &= \sum_{l=1}^{m} \mu_l A_l \quad B(\mu) = \sum_{l=1}^{m} \mu_l B_l \\ a(\mu) &= \sum_{l=1}^{m} \mu_l a_l \quad C(\mu) = \sum_{l=1}^{m} \mu_l C_l \end{aligned} \tag{3.3}$$

$\mu_l(z)$ is the normalized membership function satisfying

$$\begin{aligned} \mu_l &= \frac{\xi_l(z)}{\sum_{i=1}^{m} \xi_i(z)} \\ \xi_l(z) &= \prod_{i=1}^{v} F_i^l(z_i) \\ \mu_l &\geq 0 \quad \sum_{l=1}^{m} \mu_l = 1 \end{aligned} \tag{3.4}$$

and $F_i^l(z_i)$ is the grade of membership of z_i in the fuzzy set F_i^l.

It should be noted that the previous model is a nonlinear model in nature since the membership functions are nonlinear functions of the premise variables which contain some or all of the state variables in general. The previous T–S fuzzy model is in fact the state space fuzzy model. Similarly, the input–output fuzzy model can also be defined [28].

Remark 3.1: T–S fuzzy models include two kinds of knowledge: one is qualitative knowledge represented by fuzzy IF–THEN rules, and the other is quantitative knowledge represented by local linear models. T–S fuzzy models have a compatible structure with a two level control system with the lower level providing basic feedback control and the higher level providing supervisory control or scheduling. By using T–S fuzzy models, one can formulate these two kinds of knowledge into a unified mathematical framework. This framework provides a possibility for developing a systematic analysis and design method for complex nonlinear control systems.

Remark 3.2: T–S fuzzy models are, to certain extent, similar to the concept of typical piecewise linear approximation methods in nonlinear control. Control of a nonlinear system by piecewise linearization is approached by linearizing the system around a number of nominal operating points, and then applying linear feedback control methods to each local linear model [152]. However, analysis of the resulting closed-loop control system is in general difficult and stability or performance of the system can hardly be guaranteed due to the approximation. On the other hand, T–S fuzzy models consist of a set of local linear models smoothly connected by membership functions yielding global models of the systems. Thus, T–S fuzzy models provide a way of designing controllers based on local linear models and analysing stability or performance based on the global nonlinear model, and this also provides a framework to consolidate the general industrial practice of nonlinear control system designs such as gain scheduling control.

Remark 3.3: When only the constant term in the consequent part of (3.1) is present, the fuzzy model is called Type II fuzzy model by Sugeno in [259]. It is a simplified special case of more general T–S fuzzy models. Sugeno [259] presents a detailed stability study for this type of fuzzy models by using Lyapunov stability theory, and also gives an excellent survey on the stability issues of more general fuzzy control systems.

Identification of T–S fuzzy models has attracted great attention from control community and a number of results have been obtained [7], [28], [127], [214], [276], [287], [329]. There are basically two kinds of approaches, one is to linearize the original nonlinear system in a number of operating points when the model of the system is known, which is straightforward, and the other is based on the data generated from the original nonlinear system when its model is unknown (or in the form of black box). The authors in [28] present an approach to identification of T–S fuzzy models, including identification of the number of fuzzy rules (or the number of local linear models) and parameters of fuzzy membership functions by using a fuzzy clustering method, and identification of parameters of local linear models by using a least squares method. The objective is to minimize the global nonlinear prediction error between T–S fuzzy models and the corresponding original nonlinear systems. The authors in [127] present a study of interpretation capability of T–S fuzzy models and propose a method for their identification. The key idea is to achieve not only accurate global nonlinear prediction but also at the same time accurate local models in the sense that the local models are close approximations to the local linearization of the nonlinear system. The latter is particularly important in control design. However, this becomes a difficult multiobjective identification problem. It has been shown that constrained and regularized identification methods may improve interpretability of constituent local models as local linearizations, and locally weighted least squares method may explicitly address the tradeoff between the local and global accuracy of T–S fuzzy models.

Before we present a survey on approaches to stability analysis and controller synthesis of T–S fuzzy systems in the next few sections, we first give a result on universal function approximation of T–S fuzzy models.

A. Universal Function Approximation

Consider a general nonlinear discrete-time system described by a state–space model of the form

$$x(t+1) = f(x(t), u(t)) \tag{3.5}$$

where $x(t) \in \Re^n$ is the state vector and $u(t) \in \Re^p$ the input vector of the system. The function $f(x(t), u(t))$ satisfies the following assumption.

Assumption 3.1: There exists an equilibrium $x_0 = 0 \in \Re^n$ such that $f(0,0) = 0$ and $f \in C^2$, that is, f has the second-order continuous derivative with respect to x and u.

Let Σ_n be the set of all systems of the form (3.5) satisfying the *Assumption 3.1*. Let Σ_{fm} be the set of all T–S fuzzy models of the form (3.1) or (3.2). It is also assumed that $z(t) = x(t)$ here, that is, the membership functions depend on the system state variables. It has been shown [28], [31] that T–S fuzzy models are universal function approximators in the sense that given any $f(x,u) \in \Sigma_n$ there exists a fuzzy model $\hat{f}(x,u) = A(\mu)x + B(\mu)u + a(\mu) \in \Sigma_{\mathrm{fm}}$ that will approximate $f(x,u)$ to any degree of accuracy in any convex compact region. More accurately, let X and U be compact sets in $\Re^n$ and $\Re^p$, respectively, and

$$d_\infty(f_1(x,u) - f_2(x,u)) = \sup_{x \in X, u \in U} (\|f_1(x,u) - f_2(x,u)\|) \tag{3.6}$$

be the sup-metric, then $(X \times U, d_\infty)$ is a metric space. The following theorem shows that $(\Sigma_{\mathrm{fm}}, d_\infty)$ is dense in $(C^2(X \times U), d_\infty)$.

Theorem 3.1 [28], [31]: For any given $f(x,u) \in \Sigma_n$ on the compact set $X \times U \subset \Re^n \times \Re^p$ and arbitrary $\varepsilon > 0$, there exists an $\hat{f}(x,u) \in \Sigma_{\mathrm{fm}}$ such that

$$\begin{aligned} d_\infty(f(x,u) - \hat{f}(x,u)) \\ = \sup_{x \in X, u \in U} (\|f(x,u) - \hat{f}(x,u)\|) < \varepsilon. \end{aligned} \tag{3.7}$$

Remark 3.4: It is noted that there are many other results on universal function approximation property of other kinds of fuzzy systems, for example, the fuzzy systems with fuzzy basis functions [301], [337]–[339]. It is also recently proved that the more commonly used T–S fuzzy model in (3.1) with $a_l \equiv 0$ is also a universal function approximator [269].

Remark 3.5: It should be noted that the result in Theorem 3.1 only concerns the approximation between two *static* nonlinear functions, that is, $f(x,u)$ and $f(x,u)$. However, the error between the states of two dynamic systems described in (3.5) and (3.2), which are corresponding to $f(x,u)$ and $\hat{f}(x,u)$, might grow as time goes. Therefore, much care has to be taken in dealing with the approximation between two dynamic systems instead of two static functions.

For the ease of presentation, we will mainly use the more commonly used T–S fuzzy models, that is, the model (3.1) with $a_l \equiv 0$ in the next few sections, though it is shown that T–S fuzzy models with affine terms has much improved function approximation capabilities [71].

IV. Stability Analysis

Consider the T–S fuzzy model in (3.1) with $u \equiv 0$ and $a_l \equiv 0$ as follows:

$$\begin{aligned} R^l:\ & \text{IF } z_1 \text{ is } F_1^l \text{ and } \ldots z_\nu \text{ is } F_\nu^l \\ & \text{THEN } x(t+1) = A_l x(t), \\ & l \in L := \{1, 2, \ldots, m\}, \end{aligned} \tag{4.1}$$

which can also be described by

$$x(t+1) = \sum_{l=1}^{m} \mu_l(z) A_l x(t). \tag{4.2}$$

Stability analysis of T–S fuzzy systems has been pursued mainly based on Lyapunov stability theory but with different Lyapunov functions. One of them is the so-called *common* (or *global*) quadratic Lyapunov functions, another one is the so-called *piecewise* quadratic Lyapunov functions, and the third one is the so-called *fuzzy* (or *non-quadratic*) Lyapunov functions. In the rest of this section we will present stability analysis results of T–S fuzzy systems based on these Lyapunov functions, respectively.

A. Analysis Based on Common Quadratic Lyapunov Functions

One of the first results on stability analysis based on common quadratic Lyapunov functions is suggested in [268], and since then numerous modifications and improved methods have been proposed. By defining a Lyapunov function candidate as

$$V(x) = x^T P x \tag{4.3}$$

where the matrix P is positive definite, the following result can be readily obtained [268].

Theorem 4.1: The T–S fuzzy system (4.1), or equivalently, (4.2) is globally exponentially stable

i) if there exists a positive definite matrix P such that the following linear matrix inequalities (LMIs) are satisfied:

$$A_l^T P A_l - P < 0, \qquad l \in L \tag{4.4}$$

or, equivalently;

ii) if there exists a positive–definite matrix X such that the following LMIs are satisfied:

$$\begin{bmatrix} -X & X A_l^T \\ A_l X & -X \end{bmatrix} < 0, \qquad l \in L. \tag{4.5}$$

Remark 4.1: The equivalence of (4.4) and (4.5) can be easily established by using the Schur complement together with $X = P^{-1}$ [19]. However, the form of (4.5) is more suitable to controller synthesis which can be observed in the next section. It is also noted that (4.5) implies that its feasible solution of X is positive definite. However, the term of "positive definite" instead of "symmetric" will still be used in subsequent presentation of theorems to avoid any possible confusion.

Remark 4.2: Conditions (4.4) or (4.5) are linear matrix inequalities in the variable P or X, respectively. The feasibilities of these LMIs, as well as other LMIs in the rest of this section and the subsequent sections, are easy to be tested by the available software package *LMI Toolbox* [92].

Remark 4.3: It has been noted that common quadratic Lyapunov functions tend to be conservative, and even worse, might not exist for many complex highly nonlinear systems as demonstrated in [77], and [128]. This is one of the main limitations of this kind of approaches.

B. Analysis Based on Piecewise Quadratic Lyapunov Functions

Due to the drawback of common quadratic Lyapunov functions, it is thus desirable to develop less conservative stability results for T–S fuzzy systems. Piecewise quadratic Lyapunov functions are one of the options available. In order to facilitate development of approaches based on piecewise quadratic Lyapunov functions, one needs partition of the premise variable space, or partition of the state space in the case of $z(t) = x(t)$. The following partition will be referred to be the *first* kind in sequel [26], [27], [29].

Define m regions in the premise variable space as follows:

$$S_l = \{z \,|\, \mu_l(z) > \mu_i(z), \qquad i = 1, 2, \ldots, m, i \neq l\}, \quad l \in L. \tag{4.6}$$

Then the global model of the T–S fuzzy system (4.2) can be expressed in each local region as

$$x(t+l) = (A_l + \Delta A_l(\mu)) x(t), \qquad z(t) \in S_l, \quad l \in L \tag{4.7}$$

where

$$\Delta A_l(\mu) = \sum_{i=1, i\neq l}^{m} \mu_i \Delta A_{li} \quad \Delta A_{li} = A_i - A_l.$$

Remark 4.4: It is noted that the number of regions in this kind of partition is the same as the number of fuzzy rules or the number of local linear models, and that the fuzzy model (4.7) is different from the local model in the T–S fuzzy model (4.1) because it considers all interactions among the local models of

(4.1) in terms of uncertainty ΔA_l and is in fact the global fuzzy model (4.2) expressed in the local region S_l.

For purpose of stability analysis and stabilization, we introduce the following upper bounds for the uncertainty term of the fuzzy model (4.7):

$$[\Delta A_l(\mu)]^{\mathrm{T}}[\Delta A_l(\mu)] \leq E_{lA}^{\mathrm{T}} E_{lA}, \qquad l \in L. \tag{4.8}$$

Remark 4.5: It is noted that there are many ways to obtain these upper bounds, the interested readers can refer to [26], [27], and [29] for details.

In addition, we define a set Ω that represents all possible system transitions among regions, that is

$$\Omega := \{(l,j)|z(t) \in S_l, z(t+1) \in S_j, \forall l,j \in L, l \neq j\}. \tag{4.9}$$

Then, we are ready to present a stability result based on the following piecewise quadratic Lyapunov function candidate,

$$V(x) = x^T P_l x, \quad z \in S_l, \quad l \in L. \tag{4.10}$$

Theorem 4.2 [77]: The T–S fuzzy system (4.1), or equivalently (4.7) is globally exponentially stable

i) if there exists a set of positive–definite matrices $P_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} A_l^T P_l A_l - P_l + E_{lA}^T E_{lA} & A_l^T P_l \\ P_l A_l & -(I - P_l) \end{bmatrix} < 0, \quad l \in L, \tag{4.11}$$

$$\begin{bmatrix} A_l^T P_j A_l - P_l + E_{lA}^T E_{lA} & A_l^T P_j \\ P_j A_l & -(I - P_j) \end{bmatrix} < 0, \qquad l,j \in \Omega \tag{4.12}$$

or, equivalently;

ii) if there exists a set of positive–definite matrices $X_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X_l & X_l A_l^T & X_l E_{lA}^T \\ A_l X_l & -(X_l - I) & 0 \\ E_{lA} X_l & 0 & -I \end{bmatrix} < 0, \qquad l \in L \tag{4.13}$$

$$\begin{bmatrix} -X_l & X_l A_l^T & X_l E_{lA}^T \\ A_l X_l & -(X_j - I) & 0 \\ E_{lA} X_l & 0 & -I \end{bmatrix} < 0, \qquad l,j \in \Omega. \tag{4.14}$$

Remark 4.6: Conditions (4.11) and (4.12) or, equivalently, (4.13) and (4.14) are LMIs in the variables $P_l, l \in L$ or, equivalently, $X_l (= P_l^{-1}), l \in L$. A solution to these inequalities ensures $V(x) = x^T P_l x, z(t) \in S_l$, or equivalently $V(x) = x^T X_l^{-1} x, z(t) \in S_l$ to be a Lyapunov function for the system. The LMIs in (4.11) or (4.13) guarantee that the function decreases along all system trajectories within each region. The LMIs in (4.12) or (4.14) guarantee that the function decreases when the system transits from one region to another.

Remark 4.7: It is noted that the uncertainty terms are introduced in (4.7) for the partition of the premise variable space defined in (4.6), and this would normally lead to conservatism of the resulting stability analysis since the worst case of uncertainties is considered as shown in (4.8). Authors in [26], [27], and [29] present some approaches to searching for approximate upper bounds instead of the worst case bounds for these uncertainties.

Another approach to space partition, which is referred to be the *second* kind in sequel, is suggested by [128]. The partition is based on the natural induction of the fuzzy system (4.1) into a number of polyhedral regions $\{S_l\}_{l \in \bar{L}} \subseteq \Re^v$ of the premise variable space. The regions consist of crisp (operating) and fuzzy (interpolation) regions. The crisp region is defined as the region where $\mu_l(z) = 1$ for some $l \in \bar{L}$, all other membership functions evaluate to zero. The system dynamic of a crisp region is given by one of local models of the fuzzy system (4.1). On the other hand, the fuzzy region is defined as the region where $0 < \mu_l(z) < 1$ and the system dynamic is given by a convex combination of several local linear models. In the extreme case where all the regions of a T–S fuzzy system are crisp, that is, $\mu_l(z) = 1$ for some l and all other membership functions are equal to zero, then the global fuzzy model (4.2) becomes a piecewise linear system

$$x(t+1) = A_l x(t), \qquad z(t) \in S_l, \quad l \in L. \tag{4.15}$$

Remark 4.8: It is noted that the number of regions in the set $\bar{L}$ for the second kind of space partition is different from the number of the local linear models in general which is the number of regions in the first kind of space partition approach.

With such a partition, we can rewrite the global fuzzy model (4.2) in each region as a convex combination of linear models

$$x(t+1) = \sum_{k \in K(l)} \mu_k(z)\{A_k x(t)\}, \; z(t) \in S_l, \; l \in \bar{L} \tag{4.16}$$

with $0 \leqslant \mu_k(z) \leqslant 1, \sum_{k \in K(l)} \mu_k(z) = 1$. For each region S_l, the set $K(l)$ contains the indexes for the system matrices used in the interpolation within that region. For a crisp region, $K(l)$ contains a single element.

It is noted that in comparison with the global fuzzy model (4.2), the fuzzy model (4.16) is described in each local region. This is similar to (4.7) for the first kind of space partition approach.

Similar to (4.9), we define a set $\bar{\Omega}$ that represents all possible transitions among regions of the system (4.16), that is,

$$\bar{\Omega} := \{(l,j)|z(t) \in S_l, z(t+1) \in S_j, \forall l,j \in \bar{L}, l \neq j\}. \tag{4.17}$$

Then, we are ready to present a stability result based on a similar piecewise quadratic Lyapunov function candidate defined in (4.10).

Theorem 4.3 [295]: The T–S fuzzy system (4.1), or equivalently (4.16) is globally exponentially stable

i) if there exists a set of positive–definite matrices $P_l, l \in \bar{L}$ such that the following LMIs are satisfied:

$$A_k^T P_l A_k - P_l < 0, \quad l \in \bar{L}, \; k \in K(l) \tag{4.18}$$

$$A_k^T P_j A_k - P_l < 0, \qquad (l,j) \in \bar{\Omega}, \quad k \in K(l) \tag{4.19}$$

or, equivalently;

ii) if there exists a set of positive–definite matrices $X_l, l \in \bar{L}$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X_l & X_l A_k^T \\ A_k X_l & -X_l \end{bmatrix} < 0, \quad l \in \bar{L}, \quad k \in K(l) \tag{4.20}$$

$$\begin{bmatrix} -X_l & X_l A_k^T \\ A_k X_l & -X_j \end{bmatrix} < 0, \qquad (l,j) \in \bar{\Omega}, \quad k \in K(l). \tag{4.21}$$

Remark 4.9: It is noted that when the positive–definite matrices in (4.18) and (4.19) [or, equivalently, (4.20) and (4.21)] is chosen as a common one, that is, $P_1 = P_2 = \cdots = P_m = P$ (or, equivalently, $X_1 = X_2 = \cdots X_m = X = P^{-1}$), then the result of Theorem 4.3 reduces to that of Theorem 4.1. It thus can be easily seen that common quadratic Lyapunov functions are a special case of the more general piecewise quadratic Lyapunov functions, and the latter is less conservative. However, computation cost of the latter would be higher in general.

C. Analysis Based on Fuzzy Lyapunov Functions

In addition to stability results based on common or piecewise quadratic Lyapunov functions, the authors in [60], [98], [266], [307], and [347] present stability results based on the so-called nonquadratic or fuzzy Lyapunov functions defined as

$$V(x) = \sum_{l=1}^{m} \mu_l(z) x^T P_l x. \tag{4.22}$$

Theorem 4.4 [347]: The T–S fuzzy system (4.1), or equivalently (4.2) is globally exponentially stable

i) if there exists a set of positive–definite matrices $P_l, l \in L$ such that the following LMIs are satisfied:

$$A_l^T P_j A_l - P_l < 0, \qquad j,l \in L \tag{4.23}$$

or, equivalently;

ii) if there exists a set of positive–definite matrices $X_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X_l & X_l A_l^T \\ A_l X_l & -X_j \end{bmatrix} < 0, \qquad j,l \in L. \tag{4.24}$$

Remark 4.10: It is noted that (4.24) implies that its feasible solution of X_l is positive definite. It is also noted that when the positive definite matrices in (4.22) and (4.23) [or, equivalently, (4.24)] is chosen as a common one, that is, $P_1 = \cdots = P_m = P$ (or, equivalently, $X_1 = \cdots = X_m = X = P^{-1}$), then the result of Theorem 4.4 reduces to that of Theorem 4.1. It thus can be easily seen that common quadratic Lyapunov functions are a special case of the more general fuzzy Lyapunov functions, and the latter is less conservative. However, similar to piecewise quadratic Lyapunov functions, computation cost of the latter would be much higher.

Remark 4.11: In the case of continuous time T–S fuzzy systems, stability analysis via fuzzy Lyapunov functions is much more involved or difficult than that for the discrete time counterparts due to the fact that the membership functions are in general functions of system states and the derivative of fuzzy Lyapunov functions involves derivatives of the membership functions and, thus, the derivatives of the system states. As a result, the derivative of the fuzzy Lyapunov function becomes much more complex, in fact it becomes a nonlinear function in terms of the system matrices which thus leads to difficulty in stability analysis.

Stability analysis of T–S fuzzy models with affine terms, that is, $a_l \neq 0$ for some l in (4.1), is much more involved. The analysis based on common quadratic Lyapunov functions is suggested in [140] and [141], and the analysis based on piecewise quadratic Lyapunov functions is suggested in [77], [128], and [295]. Here, for illustration, we present a result based on the second kind of space partition and piecewise quadratic Lyapunov functions.

In this case, we define $L_0 \subseteq \bar{L}$ as the set of indexes for the regions that contain the origin and $L_1 \subseteq \bar{L}$ the set of indexes for the regions that do not contain the origin. For convenient notation, we also define

$$\bar{A}_k = \begin{bmatrix} A_k & a_k \\ 0 & 1 \end{bmatrix} \quad \bar{x} = \begin{bmatrix} x \\ 1 \end{bmatrix} \tag{4.25}$$

where it is noted that $a_k = 0$ for all $k \in K(l)$ with $l \in L_0$. Then, using this notation, similar to (4.16), the fuzzy system (4.1) with $u \equiv 0$ can be expressed as

$$\bar{x}(t+1) = \sum_{k \in K(l)} \mu_k(z) \bar{A}_k \bar{x}(t), \qquad z(t) \in S_l, \quad l \in \bar{L}. \tag{4.26}$$

The following Lyapunov function candidate is used,

$$V(x) = \begin{cases} x^T P_l x, & z \in S_l, \ \ l \in L_0 \\ \bar{x}^T \bar{P}_l \bar{x}, & z \in S_l, \ \ l \in L_1. \end{cases} \tag{4.27}$$

This function combines the power of quadratic Lyapunov functions near the equilibrium point with the flexibility of piecewise affine functions in the large.

Since the matrix P_l or $\bar{P}_l$ is only used to describe the Lyapunov function in the local region S_l, the S -procedure [19] can be used to reduce conservatism of stability results. To this end, the characteristics or information of the local regions can be utilized. As partitions of premise variable space induced from fuzzy membership functions are polyhedra, the matrices $\bar{E}_l, l \in \bar{L}$ can be constructed for each region such that

$$\bar{E}_l \bar{x} \geq 0, \qquad z \in S_l, \quad l \in \bar{L}$$

where $\bar{E}_l = [E_l \ e_l], l \in \bar{L}$ with E_l being an $n \times n$ matrix, e_l being an $n \times 1$ vector, and moreover $e_l = 0_{n\times 1}$ for $l \in L_0$. It should be noted that the above vector inequality is defined as element-wise, that is, each entry of the vector is nonnegative.

Remark 4.12: A systematic procedure for constructing these matrices $\bar{E}_l, l \in L$ for a given T–S fuzzy system can be found in [128]. The procedure is directly based on the information in the fuzzy rule base.

Then, we are ready to present the following stability result [77], [296].

Theorem 4.5: The T–S fuzzy system (4.1) with $u \equiv 0$ or, equivalently, (4.26) is globally exponentially stable, if there exist symmetric matrices, $P_l, l \in L_0, \bar{P}_l, l \in L_1$, and symmetric matrices U_l, Q_{lk}, Q_{ljk} such that U_l, Q_{lk}, Q_{ljk} have nonnegative entries, and the following LMIs are satisfied:

$$0 < P_l - E_l^T U_l E_l, \qquad l \in L_0 \tag{4.28}$$
$$A_k^T P_l A_k - P_l + E_l^T Q_{lk} E_l < 0, \qquad l \in L_0, \quad k \in K(l) \tag{4.29}$$
$$0 < \bar{P}_l - \bar{E}_l^T U_l \bar{E}_l, \qquad l \in L_1 \tag{4.30}$$
$$\bar{A}_k^T \bar{P}_l \bar{A}_k - \bar{P}_l + \bar{E}_l^T Q_{lk} \bar{E}_l < 0, \qquad l \in L_1, \quad k \in K(l), \tag{4.31}$$
$$A_k^T P_j A_k - P_l + E_l^T Q_{ljk} E_l < 0, \qquad (l,j) \in \bar{\Omega}, \quad l,j \in L_0, k \in K(l) \tag{4.32}$$
$$\bar{A}_k^T \bar{P}_j \bar{A}_k - \bar{P}_l + \bar{E}_l^T Q_{ljk} \bar{E}_l < 0, \qquad (l,j) \in \bar{\Omega}, \quad l,j \in L_1, \quad k \in K(l), \tag{4.33}$$
$$\bar{A}_k^T \hat{P}_j \bar{A}_k - \bar{P}_l + \bar{E}_l^T Q_{ljk} \bar{E}_l < 0, \qquad (l,j) \in \bar{\Omega}, \quad l \in L_1, \quad j \in L_0, \quad k \in K(l) \tag{4.34}$$
$$A_k^T \tilde{P}_j A_k - P_l + E_l^T Q_{ljk} E_l < 0, \qquad (l,j) \in \bar{\Omega}, \quad l \in L_0, \quad j \in L_1, \quad k \in K(l) \tag{4.35}$$

where $\hat{P}_j = [I_{n\times n} \quad 0_{n\times 1}]^T P_j [I_{n\times n} \quad 0_{n\times 1}]$ for (4.34), $\tilde{P}_j = [I_{n\times n} \quad 0_{n\times 1}] \bar{P}_j [I_{n\times n} \quad 0_{n\times 1}]^T$ for (4.35).

Remark 4.13: The previous conditions are linear matrix inequalities in the variables $P_l, \bar{P}_l, U_l, Q_{lk}$, and Q_{ljk}. A solution to these inequalities ensures $V(x)$ defined in (4.27) to be a Lyapunov function for the T–S fuzzy system. The LMIs in (4.28) and (4.30) for each region guarantee that the function is positive definite, the LMIs in (4.29) and (4.31) guarantee that the function decreases within each region, and the LMIs in (4.32)–(4.35) guarantee that the function decreases when the system transits from one region to another. In addition, $E_l^T U_l E_l, \bar{E}_l^T U_l \bar{E}_l, E_l^T Q_{lk} E_l, \bar{E}_l^T Q_{lk} \bar{E}_l, E_l^T Q_{ljk} E_l$, and $\bar{E}_l^T Q_{ljk} \bar{E}_l$ in these LMIs are the terms of the S-procedure used to reduce conservatism of the stability result. It should also be noted that the matrices $P_l, l \in L_0, \bar{P}_l, l \in L_1$ are not required to be positive definite.

Remark 4.14: It is noted that most results on controller synthesis of T–S fuzzy systems with *affine* local models as in (4.1) can only be cast as a solution to bilinear matrix inequalities due to the S-procedure [78], [82], [140], [141], which is not a convex programming problem and might lead to difficulty in searching for a feasible solution. Some attempts have been made to cast the controller synthesis as a solution to linear matrix inequalities but the results appear to be very restrictive in the sense that the matrices to characterize the regions for the S-procedure has to be a square and invertible matrix [76]. In other words, if the local models of T–S fuzzy systems are affine instead of linear, the controller synthesis warrants further study.

In the following sections, three categories of controller synthesis approaches will be presented for T–S fuzzy systems with local *linear* (instead of *affine*) models based on common quadratic Lyapunov functions, piecewise quadratic Lyapunov functions, and fuzzy Lyapunov functions respectively. For sake of clarity of presentation, only stabilization controller synthesis will be considered.

V. Stabilization Based on Common Quadratic Lyapunov Functions

Controller synthesis of T–S fuzzy systems based on common quadratic Lyapunov functions has been well developed during the last few years. The problem can be efficiently solved by LMI techniques. In this section, some basic stabilization results will be presented.

Consider the T–S fuzzy system without affine terms in (3.1) rewritten as

$$\begin{aligned} R^l:\ & \text{IF } z_1 \text{ is } F_1^l, \text{ AND}, \cdots z_\nu \text{ is } F_\nu^l \\ & \text{THEN } x(t+1) = A_l x(t) + B_l u(t) \\ & \quad y(t) = C_l x(t), \\ & \quad l \in L := \{1, 2, \ldots, m\} \end{aligned} \tag{5.1}$$

or, equivalently

$$\begin{aligned} x(t+1) &= A(\mu)x(t) + B(\mu)u(t) \\ y(t) &= C(\mu)x(t) \end{aligned} \tag{5.2}$$

where $A(\mu), B(\mu)$, and $C(\mu)$ are defined in (3.3).

Two kinds of control schemes are mainly used, one is the smooth fuzzy control scheme defined as

$$\begin{aligned} C^l:\ & \text{IF } z_1 \text{ is } F_1^l, \text{ AND}, \cdots z_\nu \text{ is } F_\nu^l \\ & \text{THEN } u(t) = K_l x(t) \\ & \quad l \in L := \{1, 2, \ldots, m\} \end{aligned} \tag{5.3}$$

which can be rewritten as

$$u(t) = \sum_{l=1}^{m} \mu_l(z) K_l x(t) \tag{5.4}$$

and the other is the switching control scheme defined as

$$u(t) = K_l x(t), \qquad z(t) \in S_l, \quad l \in L \tag{5.5}$$

where the space partition of the first kind is adopted here. The first control scheme is often called the parallel distributed compensation while the second one is called the local compensation.

The closed-loop fuzzy control system consisting of the T–S fuzzy system (5.2) and the smooth controller (5.4) can be described as

$$x(t+1) = \sum_{j=1}^{m} \sum_{l=1}^{m} \mu_j \mu_l (A_l + B_l K_j) x(t). \tag{5.6}$$

By defining a Lyapunov function candidate as

$$V(x) = x^T X^{-1} x \tag{5.7}$$

where the matrix X is positive definite, one can easily obtain the following stabilization result [267], [269].

Theorem 5.1: The closed-loop fuzzy control system (5.6) is globally exponentially stable, if there exist a positive–definite matrix X and a set of matrices $Q_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X & XA_l^T + Q_j^T B_l^T \\ A_l X + B_l Q_j & -X \end{bmatrix} < 0, \qquad l, j \in L. \tag{5.8}$$

Moreover, the controller gains are given by

$$K_l = Q_l X^{-1}, \qquad l \in L. \tag{5.9}$$

Remark 5.1: It is noted that a number of improved results have been obtained which aim either to reduce the number of LMIs in (5.8), or to reduce its conservatism, or both [142], [192], [198], [267], [275], [286], [304]. One typical result is given as follows [142].

Theorem 5.2: The closed-loop fuzzy control system (5.6) is globally exponentially stable, if there exist a positive–definite matrix X, a set of matrices $Q_l, l \in L$, a set of positive–definite matrices $\Phi_l, l \in L$, and a set of symmetric matrices $\Phi_{lj}, l, j \in L, l < j$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X & XA_l^T + Q_l^T B_l^T \\ A_l X + B_l Q_l & -X \end{bmatrix} < -\Phi_l, \qquad l \in L \tag{5.10}$$

$$\begin{bmatrix} -X & XA_l^T + Q_j^T B_l^T \\ A_l X + B_l Q_j & -X \end{bmatrix} + \begin{bmatrix} -X & XA_j^T + Q_l^T B_j^T \\ A_j X + B_j Q_l & -X \end{bmatrix} < -\Phi_{lj}, \qquad l, j \in L, \quad l < j \tag{5.11}$$

$$\Phi := \begin{bmatrix} 2\Phi_1 & \Phi_{12} & \cdots & \Phi_{1m} \\ \Phi_{12} & 2\Phi_2 & \cdots & \Phi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{1m} & \Phi_{2m} & \cdots & 2\Phi_m \end{bmatrix} > 0. \tag{5.12}$$

Moreover, the controller gains are given by

$$K_l = Q_l X^{-1}, \qquad l \in L. \tag{5.13}$$

On the other hand, with the switching control law (5.5) the closed-loop fuzzy control system can be described as

$$x(t+1) = (A_l + \Delta A_l(\mu) + (B_l + \Delta B_l(\mu))K_l)x(t), \qquad z(t) \in S_l, \quad l \in L. \tag{5.14}$$

By defining the following upper bounds for the uncertainties $\Delta B_l, l \in L$ as those in (3.8) for ΔA_l,

$$[\Delta B_l(\mu)]^{\mathrm{T}} [\Delta B_l(\mu)] \le E_{lB}^{\mathrm{T}} E_{lB}, \qquad l \in L \tag{5.15}$$

one has the following result.

Theorem 5.3: The closed-loop fuzzy control system (5.14) is globally exponentially stable, if there exist a positive–definite matrix X and a set of matrices $Q_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X & XA_l^T + Q_l^T B_l^T & XE_{lA}^T & Q_l^T E_{lB}^T \\ A_l X + B_l Q_l & -(X - I) & 0 & 0 \\ E_{lA} X & 0 & -\frac{1}{2} I & 0 \\ E_{lB} Q_l & 0 & 0 & -\frac{1}{2} I \end{bmatrix} < 0, \qquad l \in L. \tag{5.16}$$

Moreover, the controller gain for each local region is given by

$$K_l = Q_l X^{-1}, \quad l \in L. \tag{5.17}$$

Remark 5.2: By comparing the results in Theorems 5.1–5.3, one notices that the numbers of LMIs in (5.8), (5.10)–(5.12), and (5.16) are $m \times m, (m(m+1)+2)/2$, and m, respectively. It can be easily seen that the computation cost of (5.16) is smaller than that of (5.10)-(5.12), and much smaller than that of (5.8). However, it should also be noted that the price of such computation efficiency in (5.16) is the introduction of uncertainty terms as described in (5.14) which might lead to some extent of conservatism. In addition, the control law used in Theorem 5.3 is a switching control law while that for Theorems 5.1 and 5.2 is a continuous control law.

Remark 5.3: The stabilization results in Theorems 5.1–5.3 have been extended to: i) robust controller designs of T–S fuzzy systems with various performance indexes such as H_∞ and H_2 with respect to external disturbances [34], [36], [46], [47], [110], [168], [192], [197], [283], [312], [317]; ii) controller designs of uncertain T–S fuzzy systems with or without various performance indexes [6], [34], [167], [168], [317]; and iii) controller designs of time-delay T–S fuzzy systems with or without various performance indexes [33], [42], [56], [112], [168], [317], [323].

Remark 5.4: Observer designs, filter designs, and output feedback control designs with or without various performance indexes have also been well developed for T–S fuzzy systems based on common quadratic Lyapunov functions and LMIs [3], [6], [12], [45], [47], [56], [151], [168], [192], [197], [198], [221], [267], [275], [282], [285], [312], [317].

A typical result on observer design of T–S fuzzy systems based on common quadratic Lyapunov functions is given below [198].

Consider the following observer rule:

$$\begin{aligned} O^l: \ &\text{IF } z_1 \text{ is } F_1^l, \text{ AND, } \cdots z_\nu \text{ is } F_\nu^l \\ &\text{THEN } \hat{x}(t+1) = A_l \hat{x}(t) + B_l u(t) + H_l(\hat{y}(t) - y(t)) \end{aligned} \tag{5.18}$$

$$\hat{y}(t) = C_l \hat{x}(t), \qquad l \in L := \{1, 2, \ldots, m\}$$

which can be rewritten as

$$\hat{x}(t+1) = \sum_{l=1}^{m} \mu_l (A_l \hat{x}(t) + B_l u(t) + H_l(\hat{y}(t) - y(t))) \tag{5.19}$$

where $\hat{x}(t)$ is the estimated state vector, $\hat{y}(t)$ the estimated output vector, $H_l, l \in L$ the observer gains to be determined, and the rest of the variables and matrices are as defined in (3.1).

Then, the fuzzy observer error dynamic equation consisting of (5.1) and (5.18) can be given as

$$\tilde{x}(t+1) = \sum_{j=1}^{m} \sum_{l=1}^{m} \mu_j \mu_l (A_l + H_l C_j) \tilde{x}(t) \tag{5.20}$$

where $\tilde{x}(t) = \hat{x}(t) - x(t)$, and the following result is readily obtained [198].

Theorem 5.4: The fuzzy observer error system (5.20) is globally exponentially stable, if there exist a positive–definite matrix P and a set of matrices $Q_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -P & A_l^T P + C_l^T Q_l^T \\ P A_l + Q_l C_l & -P \end{bmatrix} < 0, \qquad l \in L \tag{5.21}$$

$$\begin{bmatrix} -P & A_l^T P + C_j^T Q_l^T \\ P A_l + Q_l C_j & -P \end{bmatrix} + \begin{bmatrix} -P & A_j^T P + C_l^T Q_j^T \\ P A_j + Q_j C_l & -P \end{bmatrix} < 0, \quad l, j \in L, \quad l < j. \tag{5.22}$$

Moreover, the observer gains are given by

$$H_l = P^{-1} Q_l, \qquad l \in L. \tag{5.23}$$

Remark 5.5: It has been shown in [198] that the *separation principle* does hold for T–S fuzzy systems in the case of common quadratic Lyapunov functions. That is, controller design and observer design can be independently carried out based on common quadratic Lyapunov functions, and the resulting closed-loop fuzzy control system, with estimated state variables to be used for state feedback control, will be asymptotically stable.

Most works to date on controller synthesis of various T–S fuzzy systems have been mainly based on common quadratic Lyapunov functions as in the case of stability analysis. However, it has been noted that common quadratic Lyapunov functions tend to be conservative and might not exist for many highly nonlinear systems. Thus, piecewise quadratic Lyapunov functions have been suggested to be an alternative approach which has demonstrated to be less conservative [77], [128]. In the next section, stabilization approaches to T–S fuzzy systems based on piecewise quadratic Lyapunov functions will be outlined.

VI. Stabilization Based on Piecewise Quadratic Lyapunov Functions

In this section, we will present stabilization methods based on piecewise quadratic Lyapunov functions which are suggested in [29], [52], [76], [77], [78], and [85]. With space partition of the first kind and the switching controller defined as

$$u(t) = K_l x(t), \qquad z(t) \in S_l, \quad l \in L \tag{6.1}$$

the closed-loop fuzzy control system can be described by

$$x(t+1) = (A_l + \Delta A_l(\mu) + (B_l + \Delta B_l(\mu))K_l)x(t), \quad z(t) \in S_l \tag{6.2}$$

where the upper bounds for ΔA_l and ΔB_l are given in (4.8) and (5.15), respectively. By using a piecewise quadratic Lyapunov function candidate of the form

$$V(x) = x^T X_l^{-1} x, \qquad z(t) \in S_l \tag{6.3}$$

one has the following stabilization result for the T–S fuzzy system described in (5.1).

Theorem 6.1 [52]: The closed-loop fuzzy control system (6.2) is globally exponentially stable, if there exist a set of positive–definite matrices $X_l, l \in L$ and a set of matrices $Q_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X_l & X_l A_l^T + Q_l^T B_l^T & X_l E_{lA}^T & Q_l^T E_{lB}^T \\ A_l X_l + B_l Q_l & -(X_l - I) & 0 & 0 \\ E_{lA} X_l & 0 & -\frac{1}{2} I & 0 \\ E_{lB} Q_l & 0 & 0 & -\frac{1}{2} I \end{bmatrix} < 0, \quad l = 1, 2, \ldots, m \tag{6.4}$$

$$\begin{bmatrix} -X_l & X_l A_l^T + Q_l^T B_l^T & X_l E_{lA}^T & Q_l^T E_{lB}^T \\ A_l X_l + B_l Q_l & -(X_j - I) & 0 & 0 \\ E_{lA} X_l & 0 & -\frac{1}{2} I & 0 \\ E_{lB} Q_l & 0 & 0 & -\frac{1}{2} I \end{bmatrix} < 0, \quad l, j \in \Omega. \tag{6.5}$$

Moreover, the controller gain for each local region is given by

$$K_l = Q_l X_l^{-1}, \quad l \in L. \tag{6.6}$$

With space partition of the second kind and the switching controller defined as

$$u(t) = K_l x(t), \quad z(t) \in S_l, \quad l \in \bar{L} \tag{6.7}$$

the closed-loop fuzzy control system can be described by

$$x(t+1) = \sum_{k \in K(l)} \mu_k(z(t))(A_k + B_k K_l)x(t), \quad z(t) \in S_l \tag{6.8}$$

where the set $K(l)$ is defined in (4.16). By using the same piecewise quadratic Lyapunov function candidate as in (6.3) one has the following stabilization result for the T–S fuzzy system described in (5.1).

Theorem 6.2 [295]: The closed-loop fuzzy control system (6.8) is globally exponentially stable, if there exist a set of positive–definite matrices $X_l, l \in \bar{L}$ and a set of matrices, $Q_l, l \in \bar{L}$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -X_l & X_l A_k^T + Q_l^T B_k^T \\ A_k X_l + B_k Q_l & -X_l \end{bmatrix} < 0, \quad l \in \bar{L}, \quad k \in K(l) \tag{6.9}$$

$$\begin{bmatrix} -X_l & X_l A_k^T + Q_l^T B_k^T \\ A_k X_l + B_k Q_l & -X_j \end{bmatrix} < 0, \quad (l, j) \in \Omega, \quad k \in K(l). \tag{6.10}$$

Moreover, the controller gain for each local region is given by

$$K_l = Q_l X_l^{-1}, \qquad l \in \bar{L}. \tag{6.11}$$

The stabilization results in Theorems 6.1 and 6.2 can be further improved in the sense of less conservatism by introducing extra slack variables $G_l, l \in \bar{L}$ in LMIs as in [225]. It is noted that the matrices $G_l, l \in \bar{L}$ are not even required to be symmetric. One improved result for Theorem 6.2 is summarized in the following theorem.

Theorem 6.3: The closed-loop fuzzy control system (6.8) is globally exponentially stable, if there exist a set of positive–definite matrices $X_l, l \in \bar{L}$, sets of matrices $Q_l, l \in \bar{L}$, and $G_l, l \in \bar{L}$ such that the following LMIs are satisfied:

$$\begin{bmatrix} X_l - G_l^T - G_l & G_l^T A_k^T + Q_l^T B_k^T \\ A_k G_l + B_k Q_l & -X_l \end{bmatrix} < 0, \quad l \in \bar{L}, \quad k \in K(l) \tag{6.12}$$

$$\begin{bmatrix} X_l - G_l^T - G_l & G_l^T A_k^T + Q_l^T B_k^T \\ A_k G_l + B_k Q_l & -X_j \end{bmatrix} < 0, \quad (l,j) \in \bar{\Omega}, \quad k \in K(l). \tag{6.13}$$

Moreover, the controller gain for each local region is given by

$$K_l = Q_l G_l^{-1}, \quad l \in \bar{L}. \tag{6.14}$$

Remark 6.1: The conservatism and the number of LMIs in Theorems 6.1–6.3 can be further reduced by employing the similar technique as in [142, Th. 5.2].

Remark 6.2: Similar to the common quadratic Lyapunov function case, stabilization controller designs based on piecewise quadratic Lyapunov functions have been extended to: i) robust controller designs of T–S fuzzy systems with various performance indexes such as H_∞ and H_2 with respect to external disturbances [27], [52], [78], [82], [84], [105], [295], [296]; ii) controller designs of uncertain T–S fuzzy systems [74], [83]; and iii) controller designs of time-delay T–S fuzzy systems [51]. Observer designs, filter designs, and output feedback control designs with or without various performance indexes have also been developed for T–S fuzzy systems based on piecewise quadratic Lyapunov functions and linear matrix inequalities [51], [52], [79], [104].

A typical result on observer design of T–S fuzzy systems based on piecewise quadratic Lyapunov functions is given below.

Consider the following piecewise fuzzy observer of the form:

$$\begin{aligned} \hat{x}(t+1) &= \sum_{k \in K(l)} \mu_k(z(t)) \{A_k \hat{x}(t) + B_k u(t) \\ &\quad + H_l(\hat{y}(t) - y(t))\} \\ \hat{y}(t) &= \sum_{k \in K(l)} \mu_k(z(t)) C_k \hat{x}(t), \\ &\qquad z(t) \in S_l, \quad l \in \bar{L} \end{aligned} \tag{6.15}$$

where $\hat{x}(t)$ is the estimated state vector, $\hat{y}(t)$ the estimated output vector, $H_l, l \in \bar{L}$ the observer gain for each region S_l, and the rest of the variables and matrices are as defined in (3.1) and (4.16).

Then the fuzzy observer error dynamic equation can be described as

$$\tilde{x}(t+1) = \sum_{k \in K(l)} \mu_k(z(t))(A_k + H_l C_k), \quad z(t) \in S_l, \quad l \in \bar{L} \tag{6.16}$$

where $\tilde{x}(t) = \hat{x}(t) - x(t)$, and the following result is readied obtained.

Theorem 6.4: The fuzzy observer error system (6.16) is globally exponentially stable, if there exist a set of positive definite matrices $P_l, l \in \bar{L}$ and sets of matrices $G_l, Q_l, l \in \bar{L}$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -P_l & A_k^T G_l^T + C_k^T Q_l^T \\ G_l A_k + Q_l C_k & P_l - G_l - G_l^T \end{bmatrix} < 0, \quad l \in \bar{L}, \quad k \in K(l) \tag{6.17}$$

$$\begin{bmatrix} -P_l & A_k^T G_l^T + C_k^T Q_l^T \\ G_l A_k + Q_l C_k & P_j - G_l - G_l^T \end{bmatrix} < 0, \quad (l,j) \in \bar{\Omega}, \quad k \in K(l). \tag{6.18}$$

Moreover, the observer gain for each region is given by

$$H_l = G_l^{-1} Q_l, \qquad l \in \bar{L}. \tag{6.19}$$

Remark 6.3: Similar to the case of common quadratic Lyapunov functions, it has been shown in [297] that the *separation principle* does hold for T–S fuzzy systems in the case of piecewise quadratic Lyapunov functions. That is, controller design and observer design can be independently carried out based on piecewise quadratic Lyapunov functions, and the resulting closed loop fuzzy control system, with estimated state variables to be used for state feedback control, will be asymptotically stable.

VII. Stabilization Based on Fuzzy Lyapunov Functions

In this section, we will present methods based on fuzzy (or nonquadratic) Lyapunov functions which is suggested in [60], [98], [266], and [307]. With a controller defined in (5.3) and rewritten as

$$u(t) = \sum_{l=1}^{m} \mu_l K_l x(t), \tag{7.1}$$

the closed loop fuzzy control system consisting of the T–S fuzzy system (5.2) and the smooth controller (7.1) can be described as

$$x(t+1) = \sum_{j=1}^{m} \sum_{l=1}^{m} \mu_j \mu_l (A_l + B_l K_j) x(t). \tag{7.2}$$

By defining a Lyapunov function candidate as

$$V(x) = \sum_{l=1}^{m} \mu_l(z) x^T X_l^{-1} x \tag{7.3}$$

where the matrix X_l is positive definite, one has the following stabilization result for the T–S fuzzy system described in (5.1).

Theorem 7.1: The closed loop fuzzy control system (7.2) is globally exponentially stable, if there exist a set of positive–definite matrices $X_l, l \in L$, sets of matrices $Q_l, l \in L$, and $G_l, l \in L$ such that the following LMIs are satisfied:

$$\begin{bmatrix} X_l - G_j^T - G_j & G_j^T A_l^T + Q_j^T B_l^T \\ A_l G_j + B_l Q_j & -X_i \end{bmatrix} < 0, \qquad i, j, l \in L. \tag{7.4}$$

Moreover, the controller gains are given by

$$K_j = Q_j G_j^{-1}, \qquad j \in L. \tag{7.5}$$

It is noted that the number of LMIs and the conservatism in Theorem 7.1 can be reduced by following the similar idea as in [142, Th. 6.2]. The result is given here.

Theorem 7.2 [98]: The closed-loop fuzzy control system (7.2) is globally exponentially stable, if there exist a set of positive–definite matrices $X_l, l \in L$, sets of matrices $Q_l, l \in L, G_l, l \in L$, a set of positive definite matrices $\Phi_l^i, l, i \in L$, and a set of symmetric matrices $\Phi_{lj}^i, l, j, i \in L, l < j$ such that the following LMIs are satisfied:

$$\begin{bmatrix} X_j - G_j^T - G_j & G_j^T A_j^T + Q_j^T B_j^T \\ A_j G_j + B_j Q_j & -X_i \end{bmatrix} < -\Phi_j^i, \quad i, j \in L \tag{7.6}$$

$$\begin{bmatrix} X_l - G_j^T - G_j & G_j^T A_l^T + Q_j^T B_l^T \\ A_l G_j + B_l Q_j & -X_i \end{bmatrix} + \begin{bmatrix} X_j - G_l^T - G_l & G_l^T A_j^T + Q_l^T B_j^T \\ A_j G_l + B_j Q_l & -X_i \end{bmatrix} < -\Phi_{lj}^i, \quad l, j, i \in L, \quad l < j \tag{7.7}$$

$$\Phi^i := \begin{bmatrix} 2\Phi_1^i & \Phi_{12}^i & \cdots & \Phi_{1m}^i \\ \Phi_{12}^i & 2\Phi_2^i & \cdots & \Phi_{2m}^i \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{1m}^i & \Phi_{2m}^i & \cdots & 2\Phi_m^i \end{bmatrix} > 0, \quad i \in L. \tag{7.8}$$

For the observer design, consider the smooth observer defined in (5.18) and the resulting closed-loop observer error system defined in (5.20). Then the following result on observer design of T–S fuzzy systems based on fuzzy Lyapunov functions can be easily established.

Theorem 7.3: The fuzzy observer error system (5.20) is globally exponentially stable, if there exist a set of positive–definite matrices $P_l, l \in L$, sets of matrices $Q_l, l \in L, G_l, l \in L$, a set of positive–definite matrices $\Phi_l^i, l, i \in L$, and a set of symmetric matrices $\Phi_{lj}^i, l, j, i \in L, l < j$ such that the following LMIs are satisfied:

$$\begin{bmatrix} -P_i & A_j^T G_j^T + C_j^T Q_j^T \\ G_j A_j + Q_j C_j & P_j - G_j - G_j^T \end{bmatrix} < -\Phi_j^i, \qquad i, j \in L \tag{7.9}$$

$$\begin{bmatrix} -P_i & A_l^T G_l^T + C_j^T Q_l^T \\ G_l A_l + Q_l C_j & P_l - G_l^T - G_l \end{bmatrix} + \begin{bmatrix} -P_i & A_j^T G_j^T + C_l^T Q_j^T \\ G_j A_j + Q_j C_l & P_j - G_j^T - G_j \end{bmatrix} < -\Phi_{lj}^i, \quad l, j, i \in L, \quad l < j \tag{7.10}$$

$$\Phi^i := \begin{bmatrix} 2\Phi_1^i & \Phi_{12}^i & \cdots & \Phi_{1m}^i \\ \Phi_{12}^i & 2\Phi_2^i & \cdots & \Phi_{2m}^i \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{1m}^i & \Phi_{2m}^i & \cdots & 2\Phi_m^i \end{bmatrix} > 0, \qquad i \in L. \tag{7.11}$$

Remark 7.1: Not like the common or piecewise quadratic Lyapunov function case, the controller synthesis based on fuzzy Lyapunov functions has not been well studied yet though approaches to H_∞ controller synthesis have been presented in [60] and [347]. By comparing (5.8) and (7.4), one notices that the number of LMIs in (7.4) would be much larger than that of (5.8) when the fuzzy rule number m is large. This may lead to difficulty in many practical situations.

Remark 7.2: Similar to the case of common/piecewise quadratic Lyapunov functions, it is expected that the *separation principle* also holds for T–S fuzzy systems in the case of fuzzy Lyapunov functions though its proof is not available yet in open literature.

Remark 7.3: As indicated in Section IV, the main difficulty in using fuzzy Lyapunov functions arises when the continuous time systems are considered. It appears that the difficulty cannot be easily overcome and will present a considerable challenge for community of fuzzy logic control.

VIII. Conclusion

This paper presents a brief survey on analysis and design methods of model based fuzzy control systems. The particular attention is given to the so-called T–S models or fuzzy dynamic models which consist of a family of local linear models smoothly combined with fuzzy membership functions. The issues of universal function approximation, stability analysis, and controller synthesis of T–S fuzzy models have been addressed. It is shown that T–S fuzzy models are able to approximate any smooth nonlinear functions to any degree of accuracy in any convex compact region. Approaches to stability analysis and controller synthesis of T–S fuzzy systems based on either common quadratic Lyapunov functions, piecewise Lyapunov functions, or fuzzy (or nonquadratic) Lyapunov functions are then presented. It is believed that these methods provide a systematic approach to analysis and design of model based fuzzy control systems, and they probably suggest an efficient alternative way to solve more difficult general nonlinear control problems.

The approaches based on common quadratic Lyapunov functions have been well developed for both discrete time and continuous time T–S fuzzy systems, while the approaches based on piecewise quadratic Lyapunov functions have had more challenges. The early works [27], [29], [74], [83], [84], [104], and [105] on controller synthesis based on piecewise quadratic Lyapunov functions suffer from a drawback of some extra restrictive boundary conditions which are in general hard to test or satisfy. Those restrictive conditions have been removed from the most recent works on stability analysis and controller synthesis based on novel piecewise quadratic Lyapunov functions [51], [52], [76]–[78], [82], [128], [295], [296]. However, many issues still need to be addressed for these approaches based on piecewise quadratic Lyapunov functions or fuzzy Lyapunov functions. For example, if local models are affine for T–S fuzzy systems, the best available stabilization results are given by solving a set of bilinear matrix inequalities which is more difficult to solve and computationally much more expensive. Is it possible to cast the problem as one by solving a set of linear matrix inequalities instead? How less conservative are the approaches based on piecewise quadratic Lyapunov functions or fuzzy Lyapunov functions compared to the approaches based on common quadratic Lyapunov functions? Which kind of approaches, the approaches based on piecewise quadratic Lyapunov functions or the approaches based on fuzzy Lyapunov functions, is less conservative? How to further reduce the conservatism of these approaches?

Moreover, there are many other challenges which have not been well studied in model based fuzzy control.

- What kind of nonlinear systems can be well represented by T–S fuzzy models? How to identify T–S fuzzy models to facilitate both accurate approximation and effective controller synthesis? The authors in [127] and [339] have shed some light on the issues.
- Are there any other better techniques which can be used for stability analysis and controller synthesis of T–S fuzzy systems in addition to common/piecewise quadratic Lyapunov functions or fuzzy Lyapunov functions?
- How can one use as much information of T–S fuzzy models as possible to achieve more effective controller synthesis and/or better performance? It is known that much information on fuzzy membership functions have not been used in the approaches based on common/piecewise quadratic Lyapunov functions or fuzzy Lyapunov functions. In fact, only little information of membership functions has been used in space partitions for the approaches based on piecewise quadratic Lyapunov functions.
- How can one design adaptive controllers for T–S fuzzy systems if information/parameters of the models are not known *a priori*, including information on the number of fuzzy rules, the shape and parameters of membership functions, and the parameters of local models? The issue of adaptive control becomes prohibitively difficult if all the information has to be identified online. Some preliminary results have been obtained where only the parameters of local models are assumed unknown [75], [81], [126], [144], [238].
- Whether can fuzzy controllers, which are designed to stabilize T–S fuzzy models, stabilize the original nonlinear systems, even if the T–S fuzzy models universally approximate the original nonlinear systems? If it is possible, how can one design the fuzzy controller to achieve such stabilization of the original nonlinear systems?
- Whether does there exist a fuzzy controller to stabilize a given nonlinear system if it can be stabilized by a smooth controller? This is usually called the *universal fuzzy control* problem. How can it be designed if it exists? Some preliminary results have been obtained in [31] and [32].
- Whether can stability analysis and controller synthesis results on deterministic T–S fuzzy systems be extended to stochastic T–S fuzzy systems? Several authors have made attempts to address these issues [43], [48], [308]. However, to make such results useful one needs to define fuzzy sets, membership functions, fuzzy inferences, etc., in the context of stochastic variables and systems. Some results on probabilistic fuzzy logic and probabilistic fuzzy systems provide a possible way to address the issues on this topic [194], [207].

In addition to the aforementioned challenges, the authors in [245] have also listed a number of open problems and perspectives in a more general setting of fuzzy systems and control, such as fuzzy system identification, adaptive fuzzy control, expert control, supervision and diagnosis, as well as the integration of fuzzy logic techniques and other artificial intelligence techniques for the more sophisticated intelligent control systems. All these challenges also provide great opportunities for our fuzzy logic control community.

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