

Conventional Fuzzy Control and Its Enhancement

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Abstract—Conventional fuzzy control can be considered mainly composed of fuzzy two-term control and fuzzy three-term control. In this paper, more systematic analysis and design are given for the conventional fuzzy control. A general robust rule base is proposed for fuzzy two-term control, and leave the optimum tuning to the scaling gains, which greatly reduces the difficulties of design and tuning. The digital implementation of fuzzy control is also presented for avoiding the influence of the sampling time. Based on the results of previous fuzzy two-term controllers, a simplified fuzzy three-term controller is proposed to enhance performance. A two-level tuning strategy is also planned, which first tries to set up the relationship between fuzzy proportional/integral/derivative gain and scaling gains at the high level, and optionally tunes the control resolution at low level. Simulation of different order models show the characteristics of fuzzy control, effectiveness of the new design methodologies, and advantages of the enhanced fuzzy three-term control.

I. INTRODUCTION

Fuzzy control was first introduced [8] and applied [9] in the 1970's in an attempt to design controllers for systems that are structurally difficult to model. Since then, fuzzy control has become one of the most active and fruitful research areas in fuzzy set theory, and many practical applications to industrial processes as well as studies of the theory itself, have been reported [1], [2].

Just like the conventional *nonfuzzy* control which has two-term and three-term control, the conventional fuzzy control also has two-term and three-term control. The fuzzy two-term control has two different types: one is Fuzzy-Proportional-Derivative (FZ-PD) type control which generates control output from error and change in error and is a position type control; another is Fuzzy-Proportional-Integral (FZ-PI) type control which generates incremental control output from error and change in error and is a velocity type control. The fuzzy three-term control is Fuzzy-Proportional-Integral-Derivative (FZ-PID) type control. The existing one generates incremental control output from error, change in error and acceleration error [3].

FZ-PI type control is known to be more practical than FZ-PD type because it is difficult for the FZ-PD to remove steady state error. The FZ-PI type control is, however, known to give poor performance in transient response for higher order process due to the internal integration operation. To improve the performance of FZ-PI type control, a method with resetting capability has been proposed [4]. However, the method needs another rule-base for resetting capability, which makes the design task more complex. Theoretically, FZ-PID type control should enhance the performance a lot. However, the existing FZ-PID type control [3] needs three inputs, which will expand the rule-base greatly and make the design more difficult. Although some approximations on acceleration error can reduce the difficulties, the performance is not improved much over FZ-PI because of the small influence of acceleration error in general. Thus, an enhanced FZ-PID appears necessary.

Fuzzy control design is involved with two important stages: 1) knowledge base design, and 2) control tuning. The control rules are normally extracted from practical experience, which makes the design

more difficult. Multiple simultaneous adjustments (rules, membership functions, and gains) makes the optimum tuning more difficult. A phase plane technique has been used for designing rule base for FZ-PI type control [5], which reduces the design difficulties. However, it still needs to be extended to other type fuzzy control. Finally, implementation of fuzzy control in computer should be less sensitive to the influence of the sampling time.

Based on the above points, more systematic analysis and design for conventional fuzzy control are presented in this paper. Attention is focused on the following approaches:

- 1) *A more systematic design for fuzzy two-term control* In spite of different applications, more standard and robust rule bases are proposed for fuzzy two-term control by the phase plane technique. The tuning of fuzzy control, which is normally a multiple and simultaneous adjustment, reduces to tuning of the scaling gains. A digital structure for fuzzy control is also presented to avoid the influence of the sampling time.
- 2) *A simplified fuzzy three-term control* To enhance performance, a simplified FZ-PID control is proposed based on the results of fuzzy two-term control. Compared with the existing FZ-PID, it is simple in structure, easy in implementation, fast in calculation and better in performance.
- 3) *A two-level tuning methodology* A two-level tuning methodology is planned for tuning fuzzy control. A relationship between scaling gains and fuzzy proportional/integral/derivative gain is discussed at high level; an optional tuning of control resolution is proposed at low level. A heuristic tuning method is also presented.
- 4) *Performance comparison by simulation* Simulation is done to compare fuzzy controls with their nonfuzzy counterparts, and each other.

II. FUZZY TWO-TERM CONTROL

A. Basic Algorithm

A nonfuzzy PI control algorithm can be expressed as

$$\begin{aligned} U^{PI} &= K_p e + K_I \int e dt \\ &= K_p \int \left\{ \dot{e} + \frac{e}{T_i} \right\} dt \end{aligned} \quad (1)$$

and a nonfuzzy PD control algorithm can be expressed as

$$\begin{aligned} U^{PD} &= K_p e + K_D \dot{e} \\ &= K_p \{ e + T_d \dot{e} \} \end{aligned} \quad (2)$$

where e is the error (set point-output), \dot{e} is the derivatives of error, $T_i = K_I/K_p$, and $T_d = K_D/K_p$. If variables e and \dot{e} are fuzzy variables, (1) and (2) will become FZ-PI type and FZ-PD type control respectively.

The continuous-time algorithm of fuzzy two-term controllers, FZ-PI type and FZ-PD type control, are shown in Fig. 1. K_e and K_d are input scaling gains. K_1 and K_2 are the output scaling gains. The fuzzy input parts are the same for both FZ-PI type and FZ-PD type control. The main differences lie in the rule base and fuzzy output processing. The FZ-PI type control contains an integral action in the output. The output of FZ-PI type control is \dot{u} , which can be called as the velocity type output. The output of FZ-PD type control is u . Their rule base design and digital implementation of data base are also slightly different.

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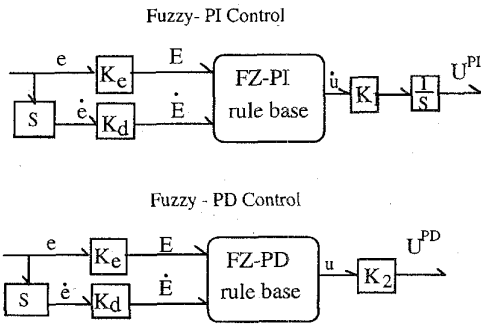


Fig. 1. The continuous-time structure of fuzzy two-term control.

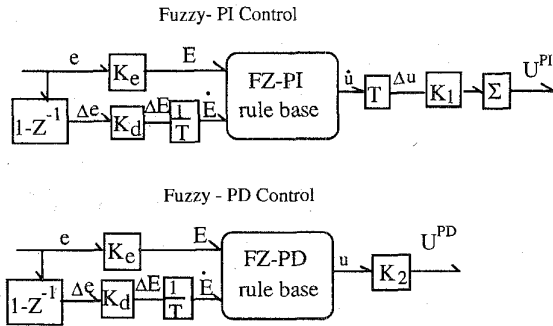


Fig. 2. The digital structure of fuzzy two-term control.

B. Digital Implementation

The fuzzy control in Fig. 1 is actually a continuous-time control law. To implement such control law in digital computer, it is necessary to approximate the integral and derivatives that appears in the control law.

The integral action in the FZ-PI output can be approximated by

$$U_k^{PI} = U_{k-1}^{PI} + K_1 \Delta u$$

$$\Delta u = \dot{u}T \quad (3)$$

and the derivative action in the fuzzy inputs can be approximated by the forward difference

$$\dot{e} \approx \frac{\Delta e}{T} \quad (4)$$

where T is the sampling time and Δe is the change in the error. Then the digital implementation of fuzzy two-term controllers are shown in Fig. 2. This digital structure is insensitive to the sampling time T .

C. Knowledge Base

To design a fuzzy control is actually to design its knowledge base which is composed of data base and rule base as shown in Fig. 3.

Data Base: The data base is to do with the definition of control variables, membership functions (MF's) and scaling gains.

The control input variables in fuzzy two-term control are chosen as error (e) and derivatives of error (\dot{e}); the output variable is chosen as velocity/position output (\dot{u}/u). Each variable is decomposed into a set of fuzzy regions, which are called labels. The most popular labels are negative large (NL), negative medium (NM), negative small (NS), zero (ZR), positive small (PS), positive medium (PM), and positive large (PL). The function that relates the grade of each label and the variable is called the membership function (MF). One of the most popular MF for each control variable is the triangle.

The resolution of each variable mainly depends on the fuzziness of its MF's, which can be controlled by its scaling gain [5]. The relation

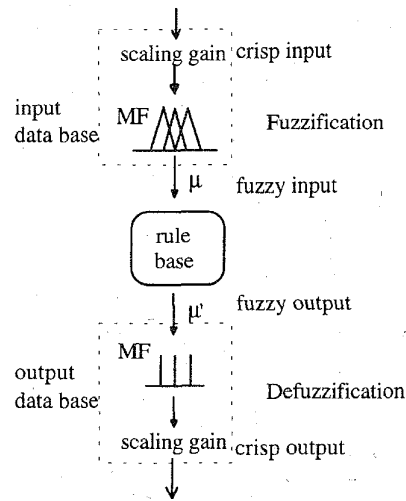


Fig. 3. The architecture of the fuzzy control knowledge base.

between two input scaling gains can be approximated as a constant α [7]. Then $K_d = \alpha K_e$. The output of FZ-PI and FZ-PD in Fig. 2 can be expressed as

$$U^{PI} = K_1 \sum F \left\{ K_e e, \frac{K_d}{T} \dot{e} \right\}$$

$$U^{PD} = K_2 F \left\{ K_e e, \frac{K_d}{T} \dot{e} \right\}. \quad (5)$$

Each of them has two sets of gains which are called as fuzzy proportional and integral/derivative gains (K_p and K_I/K_D) respectively so as to be analogous to that of its nonfuzzy counterpart. Thus the fuzzy K_p and K_I of FZ-PI can be expressed as

$$K_p = K_1 F \left\{ \frac{K_d}{T} \right\} T$$

$$K_I = K_1 F \{ K_e \} T = K_1 F \left\{ \frac{K_d}{\alpha} \right\} T. \quad (6)$$

and, the fuzzy K_p and K_D of FZ-PD can be expressed as

$$K_p = K_2 F \{ K_e \}$$

$$K_D = K_2 F \left\{ \frac{K_d}{T} \right\} = K_2 F \left\{ \frac{\alpha}{T} K_e \right\} \quad (7)$$

where $F\{ \}$ represents the fuzzy operation.

Rule Base: The rule base is to do with the fuzzy inference rules. Strictly speaking, the rule base should be different for FZ-PI and FZ-PD control because of their different control characteristics.

In fuzzy control, the error state space (phase plane) can act as a bridge between system performance and the rule base [5]. The step response of the system can be roughly divided into four areas $A_1 \sim A_4$ and two sets of points: cross-over $\{b_1, b_2\}$ and peak-valley $\{c_1, c_2\}$ as shown in Fig. 4. The system equilibrium point is the origin of the phase plane.

a) The sign of rules: The sign of the rule base can be determined by following metarules:

- 1) If both E and \dot{E} are zero, then maintain present control setting.
- 2) If conditions are such that E will go to zero at a satisfactory rate, then maintain present control setting.
- 3) If e is not self-correcting, then the sign of the rule can be determined by five sub-criterion.

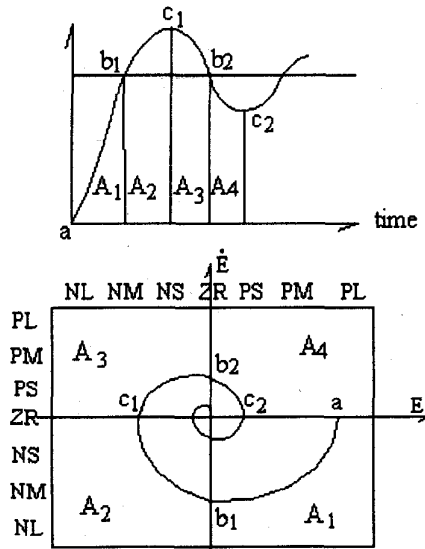


Fig. 4. Mapping from time domain to rule base.

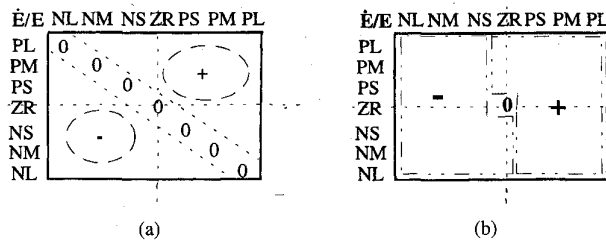


Fig. 5. (a) The rule base frame for FZ-PI control. (b) The rule base frame for FZ-PD control.

- Rules for cross-over $\{b_1, b_2\}$ should prevent the overshoot in A_2/A_4 ;
- Rules for peak-valley $\{c_1, c_2\}$ should speed up the response.
- Rules for area A_1/A_3 should speed up the response when e is large and prevent the overshoot in A_2/A_4 when e is close to zero.
- Rules for area A_2 should decrease the overshoot around the peak;
- Rules for area A_4 should decrease the overshoot around the valley;

The sign of the rule for FZ-PI control can be obtained as below and pictured in Fig. 5(a):

- $\dot{u} = 0$, when in equilibrium, or when E will go to zero at a satisfactory rate (in area A_1 and area A_3); (Rule 1, Rule 2)
- $\text{Sign}(\dot{u}) = \text{Sign}(\dot{E})$, when $E = 0$; (Rule 3.1)
- $\text{Sign}(\dot{u}) = \text{Sign}(E)$, when $\dot{E} = 0$; (Rule 3.2)
- $\dot{u} > 0$, when in area A_4 , or $E \gg 0$ in area A_1 , or E will be close to zero in area A_3 ; (Rule 3.3, Rule 3.5)
- $\dot{u} < 0$, when in area A_2 , or $E \ll 0$ in area A_3 , or E will be close to zero in area A_1 ; (Rule 3.3, Rule 3.4).

The sign of the rule for FZ-PD control can be obtained as below and pictured in Fig. 5(b)

- $u = 0$, when in equilibrium; (Rule 1)
- $u_k = u_{k-1}$, when E will go to zero at a satisfactory rate; (Rule 2)

TABLE I
A GENERAL FZ-PI RULE BASE

\dot{E}/E	NL	NM	NS	ZR	PS	PM	PL
PL		ps	pm	pl	pl	pl	pl
PM	ns		ps	pm	pl	pl	pl
PS	nm	ns		ps	pm	pl	pl
ZR	nl	nm	ns		ps	pm	pl
NS	nl	nl	nm	ns		ps	pm
NM	nl	nl	nl	nm	ns		ps
NL	nl	nl	nl	nl	nm	ns	

- $\text{Sign}(u) = \text{Sign}(\dot{E})$, when $E = 0$; (Rule 3.1)
- $\text{Sign}(u) = \text{Sign}(E)$, when $\dot{E} = 0$; (Rule 3.2)
- $u > 0$, when in area A_1 and A_4 ; (Rule 3.3, 3.5)
- $u < 0$, when in area A_2 and A_3 ; (Rule 3.3, 3.4)

b) The magnitude of rules: The magnitude of rules can be figured out from the following heuristic steps:

- Determine rules \dot{u}_0/u_0 for peak-valley points $\{c_1, c_2\}$

$$\begin{aligned} \dot{u}_0 &= E, & \text{when } \dot{E} &= 0, & \text{for FZ-PI} \\ u_0 &= E, & \text{when } \dot{E} &= 0, & \text{for FZ-PD.} \end{aligned}$$

- Determine the rest of rules. As \dot{E} has the same sign as the output at the state $E = 0$ (Rule 3.1), it should have similar effect on the rest of states. Therefore, a heuristic method can be used as

For FZ-PI

$$\dot{u} = \dot{u}_0 + \dot{E} = E + \dot{E}.$$

For FZ-PD

- Cross-over area

$$\begin{aligned} u &= PS, & \text{when } \dot{E} &> 0 \\ u &= NS, & \text{when } \dot{E} &< 0. \end{aligned}$$

Small rules are used for preventing any overshoot in area A_2/A_4 .

- Area A_1 and A_3

$$\begin{aligned} u &= \min\{(u_0 + \dot{E}), PS\} \\ &\quad \text{when in area } A_1 \text{ (rules must be positive)} \\ u &= \max\{(u_0 + \dot{E}), NS\} \\ &\quad \text{when in area } A_3 \text{ (rules must be negative).} \end{aligned}$$

- Area A_2 and A_4

$$u = u_0 + \dot{E}.$$

The above heuristic method can build general rule bases, which are shown in Tables I and II. Theoretically, the rule base can be tuned slightly for a optimum performance. Practically, these general rule bases are robust enough for a wide range of applications. The most distinguishing difference between FZ-PI and FZ-PD rule base is the switching line at which the sign of rules change; one is the diagonal line (when $E = \dot{E}$) for FZ-PI, the other one is around the set point (when $E = 0$) for FZ-PD.

The rule base designed by the phase-plane technique is easier to modify. For instance, the rule base can be updated easily for a long time delayed process by including the time delay information in the rule base [5]. This delayed rule base plays a similar role to the Smith-predictor in conventional nonfuzzy control.

TABLE II
A GENERAL FZ-PD RULE BASE

\dot{E}/E	NL	NM	NS	ZR	PS	PM	PL
PL	ns	ns	ns	ps	pl	pl	pl
PM	ns	ns	ns	ps	pl	pl	pl
PS	nm	ns	ns	ps	pm	pl	pl
ZR	nl	nm	ns	ps	pm	pl	pl
NS	nl	nl	nm	ps	ps	pm	pm
NM	nl	nl	nl	ps	ps	ps	ps
NL	nl	nl	nl	ps	ps	ps	ps

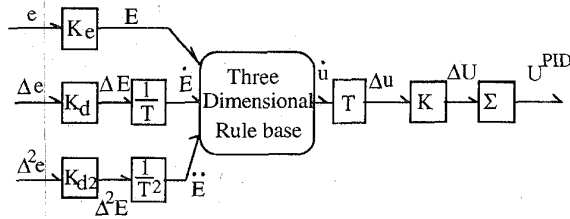


Fig. 6. Digital structure of a velocity type Fuzzy-PID control.

III. FUZZY THREE-TERM CONTROL (FZ-PID)

It is well known that FZ-PI type control give poor performance in transient response for higher order process due to the internal integration operation and FZ-PD type control give large steady state error. To improve performance, a FZ-PID type control is needed.

A. Velocity-Type Algorithm

The existing fuzzy three-term control is a velocity type PID control [3], which can be shown in Fig. 6. Its knowledge base is quite different with fuzzy two-term control.

As there are three inputs, the rule base is a three dimensional one and should have n^3 rules (n is the number of MF's) for a complete rule base. A large number of rules makes the knowledge base design more difficult. On the other hand, the acceleration error term has little influence on the performance because of limited measurement and computer resolution.

B. A Hybrid Velocity/Position-Type Algorithm

To enhance performance, a hybrid velocity/position type PID algorithm is presented for practical application as in the equation below

$$U_k^{\text{PID}} = U_k^{\text{PI}} + U_k^{\text{PD}} \quad (8)$$

where U_k^{PI} is the velocity type PI control and U_k^{PD} is the position type PD control.

$$\begin{aligned} U_k^{\text{PI}} &= U_{k-1}^{\text{PI}} + \Delta U_k^{\text{PI}} \\ \Delta U_k^{\text{PI}} &= \dot{u}_k T \end{aligned} \quad (9)$$

$$\begin{aligned} \dot{u}_k &= K_I e_k + K_P \dot{e}_k \\ U_k^{\text{PD}} &= K_P e_k + K_D \dot{e}_k. \end{aligned} \quad (10)$$

If the e_k and \dot{e}_k are fuzzy variables, (8) becomes a hybrid velocity/position FZ-PID type control. This FZ-PID type control can be implemented by combining FZ-PI and FZ-PD as shown in Fig. 7.

The rule base used is two dimensional instead of a three dimensional one. The design of a PID rule base becomes designing a PI and a PD rule base. The difficulties to design a three dimensional

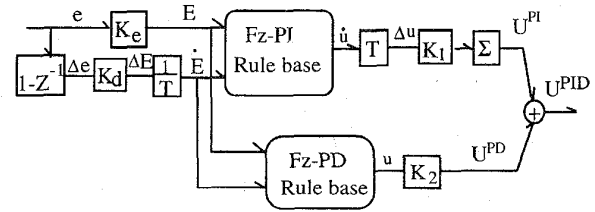


Fig. 7. Digital structure of a hybrid velocity/position type Fuzzy-PID.

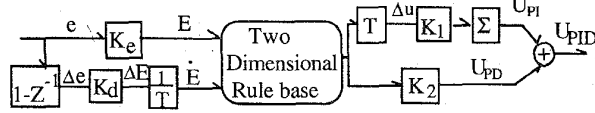


Fig. 8. Digital structure of the simplified Fuzzy-PID type control.

rule base is avoided. Two rule bases share the same inputs, which also reduces the tuning complexity.

To further reduce the complexity of the rule-base design and increase efficiency, a simplified FZ-PID type control is proposed in Fig. 8, by sharing a common rule-base for both FZ-PI and FZ-PD parts. Practically, this simplified FZ-PID can achieve the similar performance as the actual FZ-PID shown in Fig. 7. The simplified FZ-PID is simple in structure, easy in implementation and fast in computation [10]. A PI rule base is chosen here, as PI control is normally more important for steady state performance. The fuzzy K_P , K_I , and K_D of this velocity/position type fuzzy-PID control can be expressed in the equation below

$$\begin{aligned} K_P &= T K_1 F\left\{\frac{\alpha}{T} K_e\right\} + K_2 F\{K_e\} \\ K_I &= T K_1 F\{K_e\} \\ K_D &= K_2 F\left\{\frac{\alpha}{T} K_e\right\} \end{aligned} \quad (11)$$

$F\{\}$ represents the fuzzy operation.

IV. FUZZY CONTROL TUNING

A. Tuning Philosophy

Tuning fuzzy control is a very difficult task as it has more parameters to be tuned than its nonfuzzy counterparts.

- 1) *Rule tuning* Changes of rules may affect the performance. However, it is not so easy to tune the rule base.
- 2) *MF tuning* Changes of MF's may not affect the performance very much. It is also not very convenient to tune MF's.
- 3) *Gain tuning* Changes of gain affects the performance greatly. As it is easier to tune the gain than the rule base and MF's, so gain tuning is the most common way for tuning fuzzy control.

Based on the above reasons, the general and robust rule base discussed before, and standard MF's can be used for different applications, leaving the optimum tuning to the scaling gains. A multiple and simultaneous adjustment reduces to a simple gain tuning.

As the simplified FZ-PID type control is composed of FZ-PI and FZ-PD, therefore tuning fuzzy two-term control is more fundamental and essential. The gain structure of fuzzy control are different with its nonfuzzy counterparts. Fuzzy control actually has two levels of gain. The scaling gains (K_e/K_d , α and K_1/K_2) are in the lower level; the fuzzy proportional and integral/derivative gains, which are formed by coupling scaling gains, are in the higher level.

The high-level coarse tuning can follow the tuning strategy of its nonfuzzy counterparts [6]—try to reach the stable performance by controlling K_p , K_I and K_D . After that, a low-level fine tuning may be needed to enhance the performance by adjusting the resolution of control variables.

B. Coarse (High-Level) Tuning for Stability and Robustness Requirement

The proportional and integral effects, or proportional and derivative effects, are coupled together in fuzzy control. Their couplings are defined by α . Equations (6) and (7) can be transformed into a new form, so that the relationship between fuzzy gains (K_p , K_I and K_D) and scaling gains can be seen more easily

The gains of FZ-PI

$$\begin{aligned} K_p &= K_1 F\{K_d\}T \\ K_I &= K_p \frac{F\{\frac{K_d}{\alpha}\}}{F\{K_d\}}T. \end{aligned} \quad (12)$$

The gains of FZ-PD

$$\begin{aligned} K_p &= K_2 F\{K_e\} \\ K_D &= K_p \frac{F\{\frac{\alpha}{T}K_e\}}{F\{K_e\}}. \end{aligned} \quad (13)$$

The Relationship Between Scaling and Fuzzy Gains

- 1) The output scaling gain K_1/K_2 has more direct influence on fuzzy proportional gain K_p .
- 2) The input scaling K_e/K_d has also some influence on the fuzzy proportional gain K_p of FZ-PD/FZ-PI. However, its influence is more indirect and fuzzy compared with K_1/K_2 because of the fuzzy operations.
- 3) The ratio α affects the couplings of PI, or PD because changes of α will affect the influence of \dot{e} to the output. In other words, it may have similar influence as T_d/T_i in nonfuzzy PD/PI control. However, its influence is more fuzzy similarly because of the fuzzy operations.

The Heuristic Method for Tuning

- 1) Tuning FZ-PD/FZ-PI
 - a) Tuning K_e : Try to spread E all over the controlled range for the best resolution, and do not let E out of the range too much in order to keep the control of it.
 - b) Determining a correct α : As the α can be approximated as a constant [7] during the control operation, try to find the appropriate one which gives a proper coupling between proportional and integral, or proportional and derivative action. Be careful do not let \dot{E} out of the range too much.
 - c) Tuning output scaling K_1/K_2 : As FZ-PI gives a velocity type control, so K_1 is usually smaller; while K_2 is larger because of position type control from FZ-PD. Tuning output scaling is equivalent to tuning fuzzy proportional gain.

Repeat Steps 1–3 until the performance is satisfactory. Step 1 is normally a first step for tuning, while Steps 2–3 are mixed up. After getting the proper α , Step 2 can be omitted with only two gains to be tuned.

2) Tuning the simplified FZ-PID

- a) Tuning fuzzy two-term control first: Depending on the type of process, either FZ-PI or FZ-PD are used only. Tune this two-term control first to get a reasonable performance.
- b) Tuning output scaling K_1/K_2 : After adding another two-term control, keep input gains unchanged. Adjusting the output gains is normally enough to get a good result.

TABLE III
GAIN SCHEDULING STRATEGY

$K_I = \alpha K_e$		
functions\gain	K_e	K_1/K_2
coarse control	small	large
fine control	large	small

- c) Fine tuning all gains: Based on the above result, re-tuning gains may get closer to optimum result.

The influence of fuzzy gains K_p , K_I , and K_D in fuzzy control on the performance is similar to that of its nonfuzzy counterpart. By adjusting K_p , K_I , and K_D properly through scaling gains, a stable performance can be achieved. However, the tuning of fuzzy control is more difficult than its nonfuzzy counterpart because of the two level coupled gains. There is still no systematic tuning method like Ziegler–Nichol method for fuzzy control.

C. Fine (Low-Level) Tuning for Performance Enhancement

Practically, it may be hard to get an optimum performance by coarse tuning only because different transient and steady state needs different control resolution. In transient state, large errors need a coarse control which employs coarser MF's; while in steady state, small errors need a fine control which employs finer MF's. The relationship between coarser and finer MF's is actually a constant and can be adjusted by the scaling gains [5]. Thus the gain scheduling strategy in Table III can help to achieve closer to optimum performance after the coarse tuning. Practically, by using a smaller output scaling gain in the steady state period, better performance can be achieved [5].

V. SIMULATION AND PERFORMANCE ANALYSIS

The performance of fuzzy control and its nonfuzzy counterpart on different order of linear system are compared. The proper power limitation is considered. The anti-windup technique is used for nonfuzzy controllers. The general rule bases shown in Tables I and II are used for fuzzy two-term control. Only coarse tuning is used for the simulation.

The quantitative criteria for measuring the performance is chosen as IAE (Integral of Absolute Error) and ITAE (Integral of Time Absolute Error)

$$IAE = \int |e|dt \quad (14)$$

$$ITAE = \int t|e|dt. \quad (15)$$

IAE accounts mainly for error at the beginning of the response and to a lesser degree for the steady state duration. ITAE keeps account of errors at the beginning but also emphasises the steady state.

The simulation is made by DCS—a simulation language developed by Control Laboratory, Department of Electrical Engineering, University of Auckland. The numerical integration method used is fourth-order Runge–Kutta method. The integration interval T is chosen as 0.01 s. The simulation duration is 30 s.

A. Simulation on a Second-Order Linear Model

A second-order linear model is chosen as

$$\frac{2.2}{1 + 0.5s + s^2}. \quad (16)$$

After more or less optimum tuning, the simulation results of FZ-PD and PD, FZ-PI and PI, and FZ-PID and PID are shown in Figs. 9–11.

TABLE IV-A
PERFORMANCE COMPARISON FOR PD AND FZ-PD ON SECOND-ORDER MODEL

	K_p/T_d	$(K_2/K_e)/\alpha$	IAE	ITAE
PD	300/0.1		0.87	1.19
FZ-PD		(100/1)/1	0.97	2.93

TABLE IV-B
PERFORMANCE COMPARISON FOR PI AND FZ-PI ON SECOND-ORDER MODEL

	K_p/T_i	$(K_1/K_e)/\alpha$	IAE	ITAE
PI	10/11		2.37	11.04
FZ-PI		(0.13/2.5)/1	3.82	11.87

TABLE IV-C
PERFORMANCE COMPARISON FOR PID AND FZ-PID ON SECOND-ORDER MODEL

	$K_p/T_i/T_d$	$(K_1/K_2/K_e)/\alpha$	IAE	ITAE
PID	2/1/0.5		0.89	0.83
FZ-PID		(2.2/1/1)/1	1.25	1.99

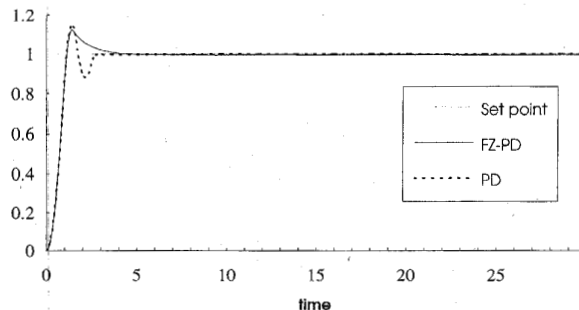


Fig. 9. Performance comparison between PD and FZ-PD on the second-order model.

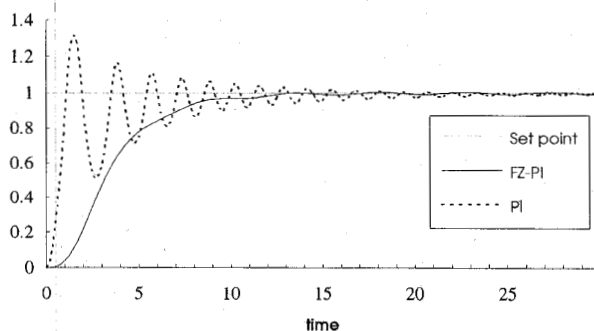


Fig. 10. Performance comparison between PI and FZ-PI on the second-order model.

Their performance index comparison are given in Table IV. The nonfuzzy control is generally better than the fuzzy control on this simple process. By large proportional gain, FZ-PD can achieve quite good steady state performance. FZ-PI is slow in the transient period. Just like its nonfuzzy counterpart, fuzzy three-term control is still better than fuzzy two-term control in overall performance.

B. Simulation on a Third-Order Linear Model

A third-order linear model is chosen as

$$\frac{5}{s^3 + 4.5s^2 + 5.5s + 15} \quad (17)$$

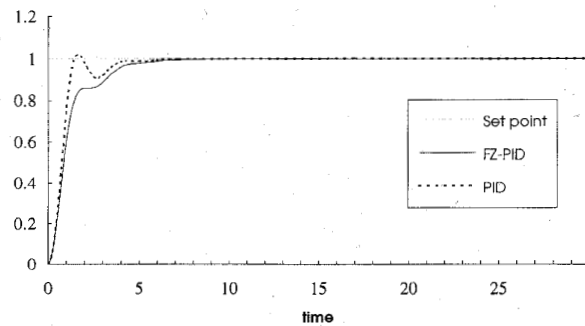


Fig. 11. Performance comparison between PID and FZ-PID on the second-order model.

TABLE V-A
PERFORMANCE COMPARISON FOR PD AND FZ-PD ON THIRD-ORDER MODEL

	K_p/T_d	$(K_2/K_e)/\alpha$	IAE	ITAE
PD	15/0.5		5.41	75.2
FZ-PD		(5.5/7)/4	2.11	18.8

TABLE V-B
PERFORMANCE COMPARISON FOR PI AND FZ-PI ON THIRD-ORDER MODEL

	K_p/T_i	$(K_1/K_e)/\alpha$	IAE	ITAE
PI	0.05/0.06		3.99	15.81
FZ-PI		(0.5/0.02)/0.01	4.08	14.04

TABLE V-C
PERFORMANCE COMPARISON FOR PID AND FZ-PID ON THIRD-ORDER MODEL

	$K_p/T_i/T_d$	$(K_1/K_2/K_e)/\alpha$	IAE	ITAE
PID	5/1.5/0.6		1.56	3.94
FZ-PID		(3/4/3)/0.5	1.28	1.69

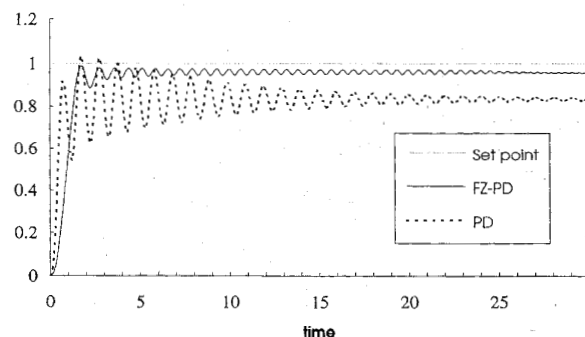


Fig. 12. Performance comparison between PD and FZ-PD on the third-order model.

After more or less optimum tuning, the simulation results of FZ-PD and PD, FZ-PI and PI, and FZ-PID and PID are shown in Figs. 12–14. The performance index comparison are given in Table V. Fuzzy control seems better than its nonfuzzy counterparts on this more complex process. As the proportional gain can not be too large for FZ-PD, it has large steady state error. FZ-PI is still slow in the transient period. FZ-PID shows a superior performance in all the aspects.

C. Performance Analysis

It is well known that FZ-PI type control can achieve good performance on a first-order system. However, it can not achieve good