Mathematical Modeling of Direct Torque Control of BLDC motor

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***Abstract*— Direct torque control method for BLDC motor drives usually represents reduction in high ripples in the torque. The important characteristics observed in DTC is non-linear regulation of stator flux and torque within variables structure by hysteresis band controller. For particular sampling period Ts values of torque and flux are chosen for Hysteresis band . This would help to achieve average sampling frequency equal to constant switching frequency of switches. Abrupt ripples generates in quantities when control of flux and torque occurs at time in allowed limits. This means that voltage vectors are chosen to modify this quantity in time Ts within hysteresis band which would results in reduction in torque ripples. Here PI controller is suitable to adjust reference value responsible for sensitivity of speed error . The obtained simulation results shown where overshoot occurs in speed using PI controller and trajectory of stator flux represented in α-β plane.**

Keywords—BLDC motor, Torque generation in BLDC motor, Mathematical Modeling of Direct torque control method

I. INTRODUCTION

Brushless DC motor or Permanent Magnet Synchronous motor with trapezoidal shaped back EMF is one of best adaptable electric motor finding their varieties of applications in Electric Vehicle. It has high power to weight ratio than other motors. There are many methods implemented for torque ripple reduction in BLDC motors such as PWM control, Current control and Direct Torque control method[5]. Therefore, for utilizing advantages of high torque generation capability, direct torque control technique is applied to modern electric vehicles, military applications etc.

Takahashi[1] and Depenbrock [2] introduced DTC control technique for Induction motor drives during 1980's. It is primarily method of estimating instantaneous torque based on circular characteristics obtained from evaluating stator flux vector. Therefore, to supply inverter voltage proper voltage vectors are selected from look-up table. Earlier case it began to Induction motor drives, Permanent magnet drives (PMSM) and now it has expanded for BLDC motor drives.

Torque ripples generated in such DTC drives since none of the switching space vector of inverter unable to produce exact stator voltage. Therefore generated desired graph of torque and speed characteristics of the motor and which leads to torque ripples and decreases efficiency of drive. The principle of DTC is torque represents ripples or pulsation that relates to amplitude of hysteresis band hence to minimize these issues, PI controller is implemented. DTC scheme implemented for BLDC is not as simple as that of others as its back emf and current waveforms are trapezoidal and square wave in shape respectively. In BLDC motor, stator flux linkage trajectory is not circular exactly , it has sharp dips at every 60 electrical degree due to commutation changes at 60 degree duration.

These sharp dips leads to estimation of torque inaccurate and difficult. To calculate torque parameter, this DTC scheme uses approximate estimation of back emf from look up table and rotor electrical angle followed by back emf and which is constructed based on mathematical analysis of BLDC motor.

This paper focuses on mathematical approach of BLDC motor where motor torque is estimated and further in DTC scheme which represents circular trajectory of stator flux linkages to increase and decrease to flux and torque throughout according to requirement of load.

II. Mathematical modeling of BLDC motor

BLDC motor has many advantages over other motors regarding to their good torque speed characteristics , efficiency, low harmonics generation, long operational property and high speed drives.

A. Generation of Six –steps waveforms

The supply current of PMSM has rectangular current shape and back EMF trapezoidal in shape called BLDC motor. Three Hall sensors are placed at 120 degree electrical apart for smooth and symmetrical operation of motor . It helps to find rotor position. BLDC motor drive dependent on rotor position which comes at particular points at 60degree for six step commutation technique. Switching stator coils at exact sequence ,exact time requires the position of stator field magnets to be known.

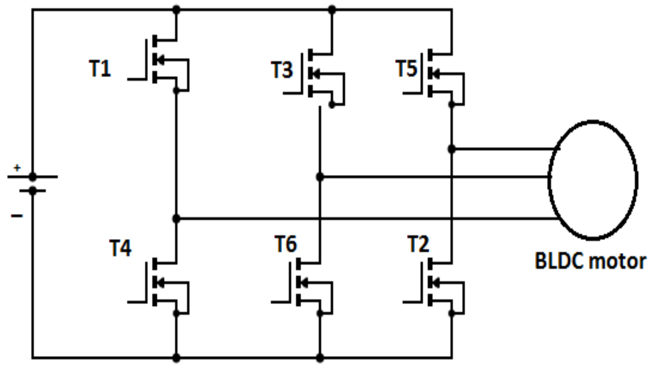


Fig.1 . BLDC motor drive

Optical Encoders or hall sensors are placed on stator to know exact position of rotor field magnet and information is given to controller for switching stator coil from sensor. Brushless DC motor with 120 conduction mode, torque spikes increase at every 60 which causes rotor to pulsate. As, torque is defined as simply multiplication of voltage and excitation current and the voltage leads to changes in spikes wise and causes transition in current due to delay cause in commutation or switching period[6].After detecting, Hall sensors signals generation of six step waveforms is done through switching table shown in following table 1.

Rotor Position	Ha	Hb	Hc	Q1	Q2	Q3	Q4	Q5	Q6
NA	0	0	0	0	0	0	0	0	0
0^0-60^0	1	0	0	1	1	0	0	0	0
60^0-120^0	1	1	0	0	1	1	0	0	0
120^0-180^0	0	1	0	0	0	1	1	0	0
180^0-240^0	0	1	1	0	0	0	1	1	0
240^0-300^0	0	0	1	0	0	0	0	1	1
300^0-360^0	1	0	1	1	0	0	0	0	1
NA	1	1	1	0	0	0	0	0	0

Table 1. Switching sequence for six step generation of waveforms[6]

Hence presented coded value defined position of rotor or motion and stator windings to be energized. Position of Hall sensors signal is high or low depend upon which sensor is close to North or South pole of rotor. Depending upon binary value, signals i.e. MOSFET switches Q1-Q6 are OFF and ON .

B. Torque Production in BLDC motor

Brushless DC motor is called as DC motor because its field and armature mmf are kept perpendicular but field mmf is rotating at synchronous speed with rotor. Here rotor is rotating therefore to keep them perpendicular it is necessary to sense the position of rotor. That is essence of BLDC motor hence field mmf rotating with rotor and stator field also rotating at synchronous speed and this is achieved by switching and the angle between the fields can be controlled. For that purpose there is no need of brushes and commutator but which stator phase switch , that depend on rotor position because these two fields should be 90degree apart to produce maximum torque. So, need to sense rotor position, this action is performed by using hall sensors. There are 3 phases of motor which excited by the means of inverter. The current flowing through inverter is depend upon which inverter is keeping on. To understand stator mmf direction is changed , if consider the coil is in direction say one terminal positive and other terminal negative, called it as plus. So, the mmf is in this direction, if changing the polarities it will be minus and this is in opposite direction. Therefore if connecting A phase at positive polarity and C phase at negative polarity ,current is flowing through switch 1 and going out through other switch then net mmf is along this direction . If C phase is turn off by making switch B with negative polarity ,keeping A phase positive then mmf will switch in this direction. So in this way as rotor is rotating , by switching action stator mmf made to produce its net mmf in switching direction using inverter .

C. Mathematical Approach

Based on above approach ,mathematical analysis of

BLDC motor with voltage and torque equations is represented as follows:

$$V_{ab} = R(i_a - i_b) + L\frac{d}{dt}(i_a - i_b) + e_a - e_b \qquad [1]$$

$$V_{bc} = R(i_a + 2i_b) + L\frac{d}{dt}(i_a + 2i_b) + e_b - e_c \qquad [2]$$

and the complete state space model can be represented as,

$$\begin{bmatrix} ia' \\ ib' \\ wm' \\ \theta m' \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & 0 \\ 0 & -\frac{R}{L} & 0 & 0 \\ 0 & 0 & -\frac{Kf}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} ia \\ ib \\ wm \\ \theta m \end{bmatrix} +$$

$$\begin{bmatrix} \frac{2}{3L} & \frac{1}{3L} & 0 \\ -\frac{1}{3L} & \frac{1}{3L} & 0 \\ 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ab} - e_{ab} \\ V_{bc} - e_{bc} \\ T_e - T_L \end{bmatrix} \qquad [3]$$

Where trapezoidal waveform of back-emf can be given as,

$$F(\theta_e) = \begin{cases} 1, \\ 1 - \frac{6}{\pi}\left(\theta_e - \frac{2\pi}{3}\right) \\ -1 \end{cases}$$

For ($0 < \theta_e < 2\pi/3$)
($2\pi/3 < \theta_e < \pi$)
($\pi < \theta_e < 5\pi/3$)

where R and L are resistance and inductance of motor and J motor inertia, Te and TL is electrical torque and load torque of motor respectively and Kf is friction constant, is rotor electrical position angle, Vab,Vbc, Vac are the voltage between two phases, eab , ebc and eca back emf generated, Wm and are rotor mechanical speed and mechanical angle of motor respectively. Based on above derived equations mathematical modeling of BLDC motor is represented in MATLAB as follows:

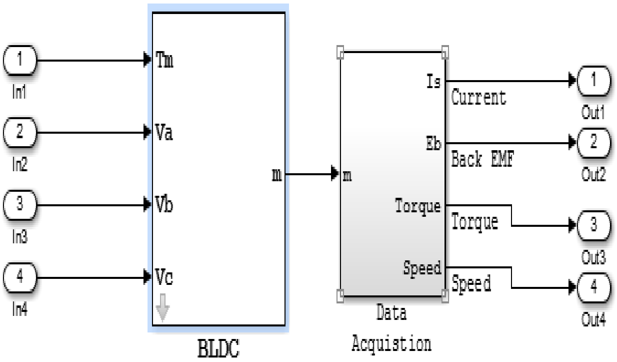


Fig.1. Mathematical Model of BLDC motor

Here BLDC motor modeled based on following parameter which are detailed following:

Sr.No.	No. of Parameter	Rating
1	Stator resistanace	1.43
2	Inductance	9.4e-3
3	Flux	0.2158
4	Rotor inertia	1.5e-3
5	Friction coefficient	2e-3
6	No. of Poles	4

Table 1: Specifications of BLDC motor[1]

III. Direct Torque Control Technique

Basically there are two flux control method for PMSM drive : field oriented or vector control method and Direct torque control method. Flux oriented control method employed for AC drives are used field control which leads to direct flux control. In such cases, rotor rotating flux space vectors is estimated and then controlling its angular speed which is derived from feedback speed sensors and stator current vector. The main disadvantage in this method is that it requires tachometer or encoder or its sensor for driving its flux control method to achieve high accuracy operation of drive which adds indirectly cost to system. DTC method is sensor less technology where information about an actual rotor speed is not essential for torque mode of operation. Therefore only exact estimation of stator flux and torque is important for the working of hysteresis comparator. Here voltage space vectors that is being applied to BLDC motor parameter chosen according to output of hysteresis controller that uses difference between flux and torque reference and their estimated values.

The flux trajectory called as circle or hexagon results depending on selection of voltage vectors.

DTC based techniques do not require complex co-ordinate transformation. The ON/ OFF operation of power inverter switches helps to obtain decoupled nature of non-linear behavior of AC motor drives .In this method states of power switches are determined directly by the calculated and reference torque and flux signals .This is achieved by making use of switching table . The input of which are torque , stator flux error and stator flux calculated error which is divided into six sectors of 60 degree. The output of switching table are input to the switching devices of inverter. The error signals are then digitalized or quantized into two level hysteresis comparator and torque error are given to three level hysteresis comparator as shown in figure. 2.

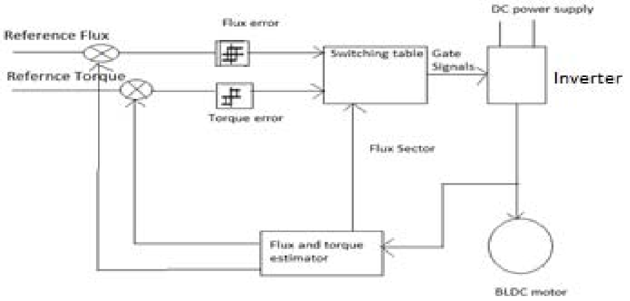


Fig. 2 .Block Diagram of DTC[3]

It is well known that establishing variable speed drives involved energy saving in industry. Earlier installation of DC machines involved decoupled nature of flux and torque control and further its principle employed called Field oriented control and Direct torque control method to induction and synchronous motor drives. The DTC drive involves sensor less operation ,highly precise and accurate control. Main features consist of Direct torque control of flux and torque ,indirect control of stator current and voltages, no coordinate transformation and separate voltage modulated blocks and it require vector control block. Absence of current controller and ability to know only sector in which stator flux linkage space vector is varied rather exact position of vectors which is in case of vector control or field oriented method. As described earlier, principle behind DTC control is to calculate instantaneous parameter of torque which dependent on stator flux vectors which is given by equation (5)- (7) and hence to estimate increase or decrease of stator flux linkage and calculated torque,

$$\psi_{as} = \int (V_{as} - R_{as} i_{as}) dt \qquad [4]$$

$$\psi_{\beta S} = \int (V_{\beta S} - R_{\beta S} i_{\beta s}) dt \quad \text{s} \qquad [5]$$

$$T = \frac{3}{2}\frac{P}{2}\left(\frac{d\psi_{r\alpha}}{d\theta_e} i_{s\alpha} + \frac{d\psi_{r\beta}}{d\theta_e} i_{s\beta}\right) \qquad [6]$$

Where p= pole pairs, θe= electrical rotor angle, Ls= inductance of stator coils and isα and isβ are stator current of α and β axis and , ψr and ψr are α and β stator and rotor flux linkage respectively. The stator space vectors are

Therefore change in switching instant is determined by the equation as

$$\psi_s = \frac{1}{T_n \int_0^t V_s dt} \qquad [7]$$

The rotor flux vectors therefore continues to rotate at synchronous frequency and the angle δ_s changes ,according to that torque changes. Hence to reduce torque ripples and improvement in estimation process more sophisticated theory of space vector modulation is investigated.

A. Estimations of Parameters

In BLDC motor rotor time constants changes slowly comparately stator flux linkage and it can be assumed as constants in magnitude and in speed of rotation during transient period as its rotor time constant is relatively large. When forward active voltage vectors are applied then stator flux linkage vectors moved away from rotor flux vector . This will leads to increment in torque parameter because torque angle increases . Therefore if zero or backward active voltage space vectors are applied then torque angle is reduced. For mathematical analysis purpose primarily mathematical model of BLDC motor is developed using equations [1],[2]. Further current and voltage is converted into park's transformation which is used to calculate stator flux linkage is given by equation [6],[7]. Voltage and current equations wrt to stationary frame of reference.

$$\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 0 & -1/\sqrt{3} \end{bmatrix} \begin{bmatrix} Vs\alpha \\ Vs\beta \end{bmatrix} \qquad [8]$$

$$\begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 0 & -1/\sqrt{3} \end{bmatrix} \begin{bmatrix} Is\alpha \\ Is\beta \end{bmatrix} \qquad [9]$$

Stator Flux Linkages:

$$\psi_{s\alpha} = V_{s\alpha} - I_{s\alpha} R_s \qquad [10]$$

$$\psi_{s\beta} = V_{s\beta} - I_{s\beta} R_s \qquad [11]$$

Rotor flux linkages and position estimator, stator flux linkage position helps to determined variation of vectors within trajectory which is evaluated by following equations:

Rotor flux linkage:

$$\psi_{R\alpha} = \psi_{s\alpha} - L_s\, i_{s\alpha} \qquad [12]$$

$$\psi_{R\beta} = \psi_{s\beta} - L_s i_{s\beta} \qquad [13]$$

And its position in α-β stationary reference frame is given by

Position Estimator:

$$\theta e = \tan^{-1}\left(\frac{\psi_{s\beta} - L_s is\beta}{\psi_{s\alpha} - L_s i_{s\alpha}}\right) \qquad [14]$$

Stator flux linkage Position:

$$\theta_s = \tan^{-1}\left(\frac{\psi_{s\beta}}{\psi_{s\alpha}}\right) \qquad [15]$$

Where $\psi_{R\alpha}, \psi_{R\beta}$ are rotor flux linkages wrt to alpha and beta axis and θe defines rotor position estimator and θs is stator flux linkage position. Estimated parameter are compared in two level and three level flux and torque comparator respectively i.e. torque is varied within limits of 1 and -1 and flux parameters restricted within band limits of -1,0 and 1. Therefore output of comparator and sector selection given to look up table which contains logic (100001, 11000, 101000, 100100, 100010, 011000). Voltage vector is varied within limits of 60 degrees duration. Results generated is fed up to inverter circuit with applied voltage of 220 volts where switching voltage is given by,

$$V_{sa} = \frac{2}{3}V_a - \frac{1}{3}V_b - \frac{1}{3}V_c \qquad [16]$$

$$V_{sb} = \frac{2}{3}V_b - \frac{1}{3}V_a - \frac{1}{3}V_c \qquad [17]$$

$$V_{sc} = \frac{2}{3}V_c - \frac{1}{3}V_b - \frac{1}{3}V_a \qquad [18]$$

Switching table for DTC of BLDC drive is as shown below:

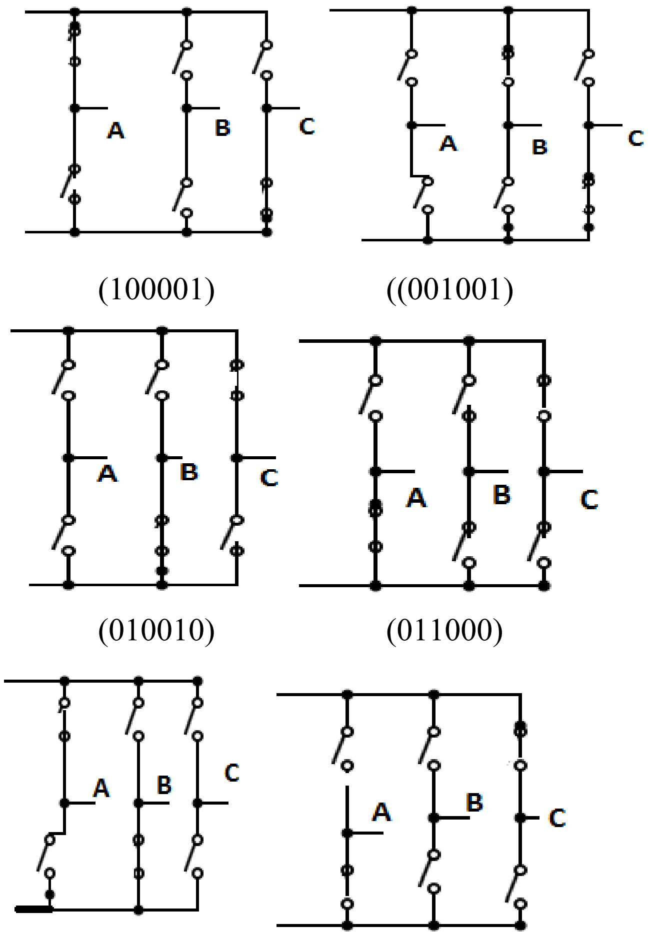
Torque	Flux	Sectors					
		I	II	III	IV	V	VI
1	1	V1	V2	V3	V4	V5	V6
	0	V2	V3	V4	V5	V6	V1
	-1	V3	V4	V5	V6	V1	V2
0	1	V1	V2	V3	V4	V5	V6
	0	V0	V0	V0	V0	V0	V0
	-1	V3	V4	V5	V6	V1	V2

Table II. voltage space vectors[1]

Figure below shows mathematical model of Direct Torque control of BLDC motor in which two different loops regarding magnitude of stator flux and torque is provided. The reference values of stator flux and torque is compared with actual values calculated by motor stationary reference frame model and respective errors are evaluated for further analysis. Resulting errors are fed into flux and torque hysteresis current controller. The results obtained from current controllers are combined together with position of stator flux and is given as input switching table selection table. Here stator flux vectors are divided into different six vectors which are operated for to phase conduction mode for BLDC motor as its operation with Maximum torque generation obtained with 120 degree condition. With reference stator flux and torque tents to be restricted within hysteresis band. Here Function of flux hysteresis band affects stator current distortions of lower order harmonics and torque hysteresis band affects switching frequency. Direct torque control method requires estimation of stator flux and torque which are analyzed by two different phase current and state of inverter. Flux and torque values calculation can be performed using mechanical speed and two phase stator current. Advantages of these scheme it has no co-ordinate transformation ,high efficiency and low losses. Here switching losses are relatively low as transistors are switched only it is need to keep torque and flux with their hysteresis band.

B. Inverter switching Table

Here sectors are all of 60 degree and which are distributed 30 degrees around corresponding voltage space vectors. The voltage vectors selected are (100001, 11000, 101000, 100100, 100010, 011000) which are vary according to switching timings given to space vectors. If the space vectors lies in K^th sector where K= 1,2,3,4,5,6 then its magnitude increased by using varied space vectors are K, K+1, K-1 and its magnitude decreased by K+2, K-2 ,K+3 vectors ,here K+1 and K+2 considered as active forward voltage vectors and K-1 and K-2 denotes active backward vectors. Therefore selected voltage vectors causes the production of torque in BLDC motor, in addition to that switching frequency have great impact on torque ripples. Main idea behind this method is to keep switching frequency as low as possible so that losses associated with minimize also most appropriate vectors is one which required minimum number of switching and it drives simultaneous. While calculating torque parameter, different torque equations are previously defined by considering EMF is directly proportional to rotor speed. Hence six vectors generated for switching sequence for BLDC motor is given as follows .



(100001) ((001001)

(010010) (011000)

(001100) (100100)

Fig3..Swtchinng sequence of Inverter

Since both upper and lower switches in same leg may be simultaneously ON or OFF in case BLDC motor drive irrespective of known state of freewheeling inverter switching i.e. one digit used for each state of s[pace vectors. Therefore two space vectors are represented as V1, V2,V3,V4,V5,V6 which indicates switching condition as (100001) , (001001), (010010) ,(011000), (001100), (100100) respectively from left to right. These are logical values denotes upper and ;lower switching pattern and phases A,B and C. The Zero voltage space vectors is defined by (000000) states.

Therefore further mathematical simulated works to be carried out for Direct torque control method of BLDC motor which is given below. As shown in above diagram, control algorithm includes closed loops system provided by PI controller which required for the wide range of speed control. This PI controller is suitable to adjust reference value of torque with input values of reference value of speed and estimated speed value in steady state. The PI controller is very sensitive to changes in Kp and Ki values ,it needs good tuning for optimal performance.

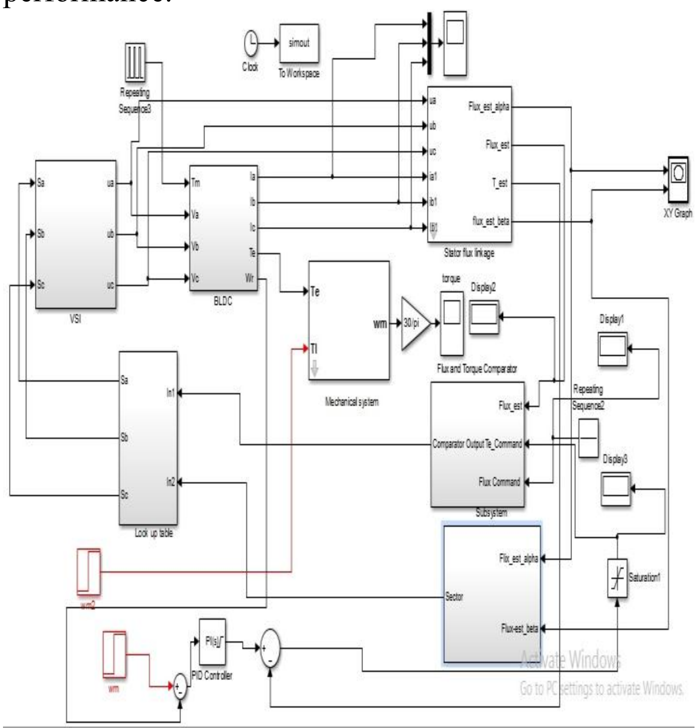


Fig.4. Mathematical Model of DTC of BLDC motor

Therefore initial case BLDC motor is simulated with respective parameters which are discussed in table . Stator flux linkage derived by using basic equations of flux and torque equation of BLDC motor[3], [4] and [5]. Further Comparator block includes two level and three level hysteresis current controller and which is used to compared with original estimated vales of motor.

Output of comparator is given to Lookup table from which values of switching instants is selected for six vectors of sectors selection. Inverter is given V=200V used to give supply voltage to BLDC motor. The flux alpha and flux beta is given to sector selection hence they varied within their defined limits to obtained circular trajectory of Space vectors.

V. Simulated Results

The mathematical model represented of BLDC motor drive is simulated using MATLAB13R(b) to obtained the dynamic performance of motor drive. Phase current waveforms of 5 amp generated from results shown below in fig(4). The DTC method further includes parameter for BLCD drive of 1200rpm motor at 1 Nm load torque. For given input logical values for switching of motor current its back EMF is represented in figure(5) where its shape is trapezoidal in shape.

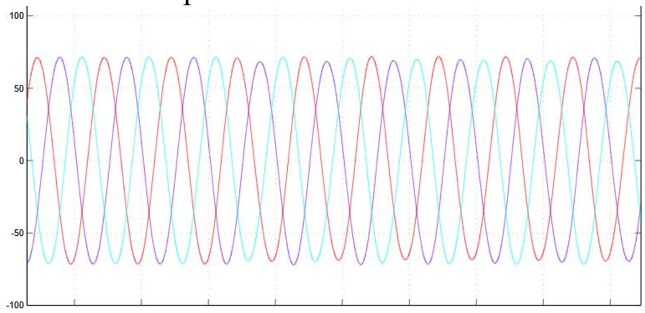


Fig.4 Phase Current

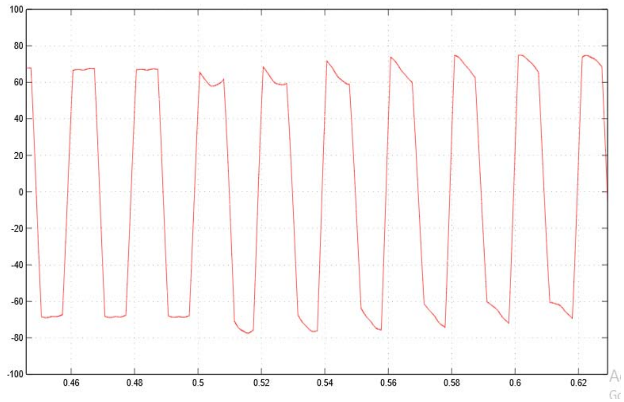


Fig.5. Back EMF

Results obtained from Direct torque control method trajectory of stator flux which are represented by two components in α-β plane is circular manner below in figure 7.

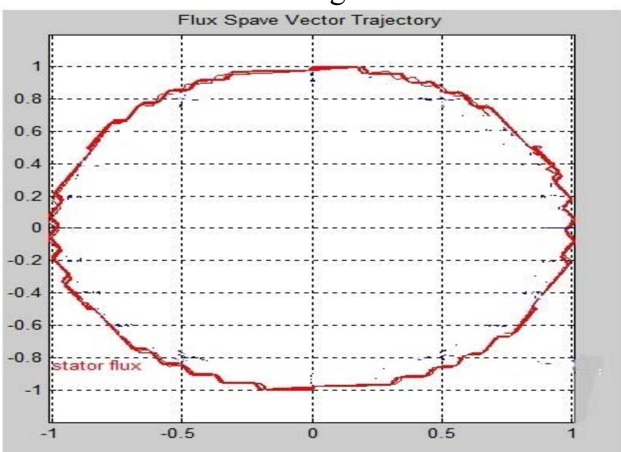


Fig. 6.Circular trajectroy of space vectors

Here load torque of 1.2 Nm and magnitude of hystersis bands for torque and flux are defined for the 0.001 Nm and 0.001Wb respectively. The

Coventional PI controller includes values of proportional gain as Kp= 4 and integral Ki=1. Above figure shows stator flux linage locus which is not exacly in circular shape due to unexcited open phase conduction of BLDC motor and therefore sharp deeps or changes occures due variation of load torque under full load condition. The simulation time for model 3 seconds. These sharp variations are due to changes in commutation time interval between switches. To setting gating signals and power switches real conditions in simulation is as close as actual BLDC motor parameter with R-L element in motor.

Fig. 7. Torque Characteristics

Torque commanded values is reduced from the regulation of BLDC moto speed using PI controller. Here chosen results to present the results relating to roational spped of BLDC motor evolution, electromagnetic torque and flux on α-β space.

As shown in above fig.7. overshoot on torque is limitaed by the means of refrence value. Also,speed track reference with good performance is achieved with PI controller.

Here torque characteristics of motor has been analysed by using state space, conventional PI controller and switching states of inverter. Further results have been verified through the

VI. Conclusion

Results with conventional BLDC motor current controller with Direct torque control method is discussed under rated condition. Due to direct torque control method errors between estimated and actual values of flux and torque, it is possible to control directly different state of inverter tor reduce torque and flux errors. The simulated results helps to determined precise control of estimated torque and hence it leads to reduction in less steady state errors compared with other conventional methods of high performance motor drives.

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