Control of An Inverted Pendulum Using Grey Prediction Model

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Abstract – The system with partial unknown structure, parameters and characteristics is called a grey system. The grey theory can be employed to improve the control performance of system without sufficient information or with highly nonlinear property. In this paper, the grey prediction model combined with PD controller is proposed to balance an inverted pendulum which is a classic example of an inherently nonlinear unstable system. The control objective is to swing up the pendulum from the stable position to the unstable position and bring its slider back to the origin of the track. The overall control algorithm is decomposed into two separate grey model controllers for swinging up and balancing based upon the angular and velocity values of the pendulum. The actuator is a Nippon Seiko Co. (NSK) linear motor. The experimental results show that this grey model controller is able to swing up and balance the inverted pendulum and guide its slider to the center of the track. It also has the robustness to balance the inverted pendulum in suffering an external impact acting on the pendulum.

I. INTRODUCTION

The dynamics of balancing a pendulum at the unstable position can be employed in the applications of controlling walking robot and rocket thruster etc. A lot of control design techniques had been used to investigate the control properties of inverted pendulum. The successful application of classical controller design techniques required considerable knowledge of the accurate system dynamic model and desired system behaviors with the expression of an objective function. However, the mathematical model derived from physical relationships or identified from experimental results is only an approximated model. Generally, it is the local linearized result. If the operating range is too large, the control performance of the traditional controller is not acceptable. The highly nonlinear and unstable properties of an inverted pendulum are the behaviors of traditional controller difficult to overcome.

Many previous researchers had applied the neural network theory to control the inverted pendulum systems [1–7]. Most of them were focused on the control problem of how to maintain an inverted pendulum at the neighborhood of unstable equilibrium position. This kind of balancing problem is linear in nature. It can not illustrate the strength of those controllers at handling the hard nonlinear problems. In this paper, the control task is to swing up a pendulum mounted on the slider of a linear motor from its stable position (vertically down) to the zero state (up right) and to keep it there in spite of the disturbance. An additional requirement is that the displacement of the slider is confined to within a preset limit during the swinging up action, and it will be brought to the origin of the track finally. The approach of using neural network control need a long period of learning and appropriate teaching signals. That will introduce larger computation and data base for extracting suitable control knowledge on the decision of control rules. Since this sliding inverted pendulum is a one control input and two output variables (angle and slider position), the control object is more difficult than the traditional inverted pendulum.

The grey system theory is a newly developed one and has widely been used in many fields [8]. With the developed of grey theory and microprocessor, the grey control theory is increasingly applied in the fields of engineering [9–11]. In this paper a grey prediction model combined with a PD controller is proposed to control a sliding inverted pendulum to swing up it from downward position to up right position and guide its slider to the center of the track.

II. GREY SYSTEM PREDICTION

If the internal structure, parameters and characteristic of a system is totally unknown, it can only be observed from the external behaviors, this system is defined as a black box means lack of information. On the contrary, the internal properties are completely well known is called a white system. A system with partial known information and certain unknown information is defined as a grey system. The grey theory treats any random variation as the variation of grey value in a certain range and the random process is considered as a time varying grey process in a certain range. In spite of using statistic regulation, the grey theory employed the method of data generating to obtain more regular generating sequence from those initial random data.

The grey prediction is to establish a grey model extending from the past information to the future based upon the past and present known or un–determinate information. Then the grey model can be used to predict the future variation trend of the system. The specific feature of establishing a grey model is employing the discrete time sequence data to build up the first order ordinary differential equation. During this operation, the accumulated generating operation (ATO) and inverse accumulated generating operation (IAGO) are the basic tools for searching the grey model. If $\{y^0(i)\}$, $y^0(i) \geq 0$ $i = 1, 2, ..., n$ is a time sequence data, then the accumulated generating operation is

$$y^1(k) = \sum_{i=1}^{k} y^0(i) \qquad (1)$$

where $y^0(k)$ is a time sequence data and $y^1(k)$ is an accumulated generating sequence, it is monotony increasing. The operation of IAGO for $y^1(k)$ is

$$\begin{aligned} y^0(0) &= y^1(0) \\ y^0(i) &= y^1(i) - y^1(i-1) \end{aligned} \qquad (2)$$

That means $y^0(i)$ is the inverse accumulated generating sequence of $y^1(k)$.

The trend of this generating time sequence can be approximated by an exponential function. Its dynamic behavior likes a first order differential equation. Hence the first order ordinary differential grey model [8] wanted be established is

$$\frac{dy}{dt} + ay = u \qquad (3)$$

The parameters a and u can be estimated by using least square scheme based upon the accumulate generating data sequence $y^1(k)$ which is accumulated from the past and present output information. If the sampling interval is one unit, then the differential of the generating sequence y^1 can be described as the discrete time sequence.

$$\begin{aligned} \frac{dy^1}{dt} &= y^1(k+1) - y^1(k) \\ &= y^0(k+1) \end{aligned} \qquad (4)$$

and the second term of the first order grey model can be represented as the average of $y^1(k+1)$ and $y^1(k)$. Then the first order differential equation can be described as:

$$y^0(k+1) = a[-\frac{1}{2}(y^1(k+1) + y^1(k))] + u(k+1) \qquad (5)$$

Where the values of parameters a and $u(k+1)$ can be estimated from the time sequence data $y^0(i)$ and the generating data sequence $y^1(i)$. In order to obtain an approximate growing curve for y^1, four or five sets of data are required to extract the trend of this grey model. Hence this identification problem becomes an contradict solution due to extra equation sets. The least square technique is employed to find the optimal average solution between them.

$$\hat{\phi} = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y_n \qquad (6)$$

where $Y_n = [y^0(2)\ y^0(3)\ \dots\ y^0(n)]^T \qquad (7)$

$$B = \begin{bmatrix} -\frac{1}{2}(y^1(1) + y^1(2)) & 1 \\ -\frac{1}{2}(y^1(2) + y^1(3)) & 1 \\ \vdots & \\ -\frac{1}{2}(y^1(n-1) + y^1(n)) & 1 \end{bmatrix} \qquad (8)$$

Then the prediction output can be calculated by substituting the estimated parameters into the differential equation. The digital form is

$$\hat{y}^{1}(k+1) = (y^{0}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a} \qquad (9)$$

Therefore, the prediction output at k+1 step can be estimated by taking derivative of the above equation as

$$\hat{y}^{0}(k+1) = (1 - e^{a})\cdot[y^{0}(1) - \frac{u}{a}]e^{-ak} \qquad (10)$$

or

$$\hat{y}^{0}(k+1) = \hat{y}^{1}(k+1) - \hat{y}^{1}(k) \qquad (11)$$

In this study the most resent five output data y(n–4), y(n–3), y(n–2), y(n–1) and y(n) are accumulated to predict the next step output y(n+1) by using the grey model. This principle is similar to the forgetting factor of the identification algorithm.

III. GREY PREDICTION CONTROL

Since the control task in this paper is to swing up a pendulum from its stable position (vertically down) to the zero state (up right), it is a large range nonlinear control problem. The slider position must also be controlled, it becomes a very complicated one–input and two–output system control problem. In addition, the swinging up and the balancing are two completely different control objectives. Hence, the overall control objective is divided into two steps. In the first step, the pendulum is swing up from the stable position (vertically down) to over the horizontal line ($-90^\circ = \theta$ or $\theta = 90^\circ$). The control law in this range is

$$\bar{u} = \begin{cases} K_1(e^{-m\dot{\theta}} - n|180^\circ - \hat{\theta}|) & \text{for } \dot{\theta} > 0 \\ -K_1(e^{m\dot{\theta}} - n|180^\circ - \hat{\theta}|) & \text{for } \dot{\theta} < 0 \end{cases} \qquad (12)$$

The secondary step is to control the pendulum to the inverted unstable position (up right $\theta = 0^\circ$) and move the slider to the origin of the linear motor track. The PD controller is employed to improve the robustness of this system in suffering external disturbance.

$$\bar{u} = K_p(\theta^* - \hat{\theta}_{n+1}) + K_d(\dot{\theta}^* - \hat{\dot{\theta}}_{n+1}) \qquad (13)$$

where θ^* is the an objective function used for guiding the slider back to origin.

$$\theta^*(t) = -\theta_{max}[\frac{2}{1+\exp(b(x(t)+\dot{x}(t))} - 1] \qquad (14)$$

$$\dot{\theta}^*(t) = 0 \qquad (15)$$

where b is a constant coefficient and θ_{max} is the maximum objective value of the inverted pendulum inclination angle. $x(t)$ and $\dot{x}(t)$ are the distance away from the origin and the velocity of the slider respectively. If x deviates from the target position, the virtual target position of θ is set to bring it to the target position. When the slider is at the target position (origin), the virtual target position θ^* of the pendulum becomes zero according to equation (14). Hence, the pendulum is inverted when the slider is at the target position. The grey prediction controller adjusts the input frequency of the linear motor in order to move the pendulum angle θ to the virtual target position θ^* based on the difference between the target position and the predictive position of the pendulum.

IV. EXPERIMENTAL RESULTS

The experimental layout and block diagram are shown in Fig. 1. The actuator is a Nippon Seiko Co. (NSK) megathrust linear servo motor (HA2) with $4\mu m$/pulse driver (EMLA20C13–04). The sliding bed is 100 cm. The length and weight of pendulum is 40 cm and 0.4 kg respectively. The encoder of the pendulum is RI 58 with resolution 1/20000 per revolution. The controller is implemented on a IBM PC/AT 80486–33. The control input is the input pulse frequency of the linear motor. The outputs are the angle and angular velocity of the pendulum and the position and velocity of the slider of the linear motor.

The number of sequential data used in establishing grey model is 5. The parameters in the first control step of swinging up the inverted pendulum are $K_1 = 30000$, $m = 0.002$ and $n = 0.000015$. The parameters used in the second step grey prediction control are $K_p = 22200$, $K_d = 1600$, $b = 0.00001$ and $\theta_{max} = 10^\circ$. The dynamic response of the angle and angular velocity are shown in Fig. 2(a) and 2(b) respectively. The corresponding input frequency of the linear motor, the response of position and velocity of the slider of linear motor are shown in Fig. 3(a), 3(b) and 3(c). The pendulum reaches the specified equilibrium position (up right) within 2 second. The slider returns to the specified origin within 4 seconds with a small variation.

In order to verify the robustness of this neural controller, the pendulum is impacted by an external force at the time 9.5 sec. The dynamic response of the angle and angular velocity are shown in Fig. 4(a) and 4(b) respectively. The corresponding input frequency of the linear motor, the response of position and velocity of the slider of linear motor are shown in Fig. 5(a), 5(b) and 5(c). It can observed that the fast moving of the slider of linear motor in response to the external impact acting on the inverted pendulum in order to reduce the influence of external disturbance. In addition, the on–line quicker model matching of grey prediction control can real time response to the dynamic variation of the pendulum due to external impact. Hence it can quickly control the inverted pendulum and maintain at stable situation. The corresponding phase plane trajectory of inverted pendulum is shown in Fig. 6. The angular dynamic response of inverted pendulum with and without grey prediction model are shown in Fig. 7 for comparison. The solid line exhibits the response of a PD control with grey prediction model, and the dashed line depicts the response of a pure PID control. It can be observed that the grey prediction model has effectively improved the control performance.

V. CONCLUSION

In this paper a grey model prediction control approach is proposed to solve the nonlinear control problem of an inverted pendulum. The pendulum located on the slider of a linear motor is swing up from the stable position (point down) to the unstable equilibrium position (up right). The overall processes include first order model prediction and a PD controller for regulating the output variables to the objective function. The experimental results show that the control performance is very good. The grey prediction control method has quickly on–line matching speed and robust stabilizing ability for external impact acting on the pendulum. This controller is based on the on–line first order grey model prediction without the knowledge of system dynamic model. It needs only five initial data set of the output angular position to begin the grey model prediction operation.

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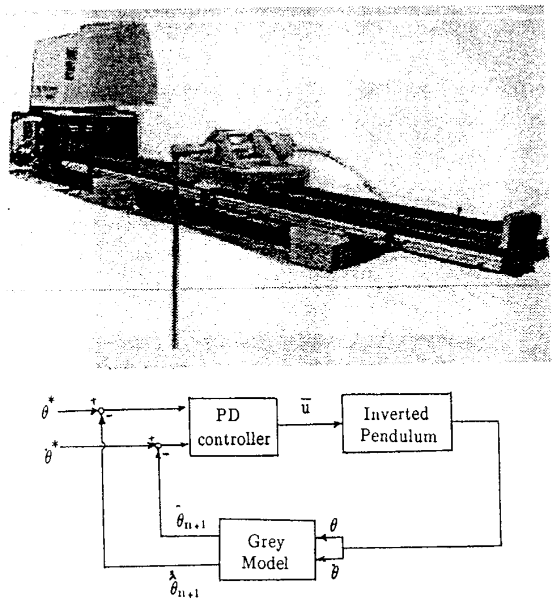

Fig. 1 Control block diagram of inverted pendulum.

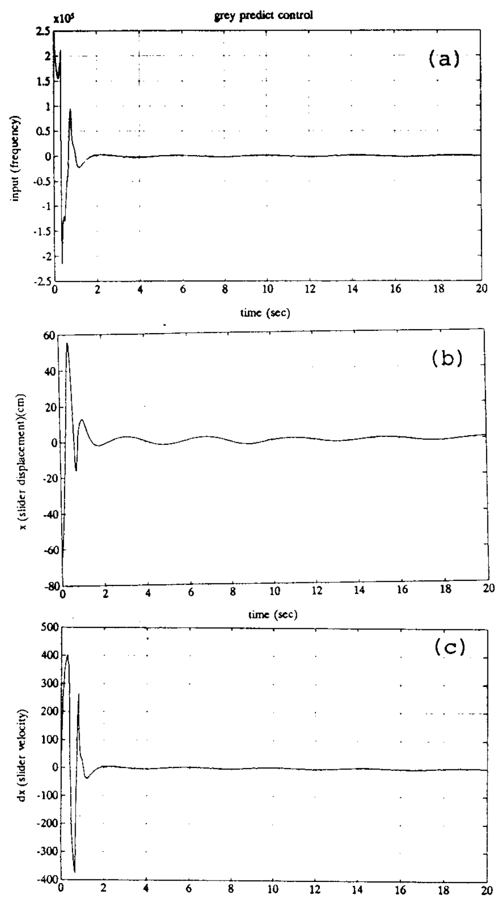

Fig. 3 (a) The input frequency of linear motor, (b) the position response of the base and (c) the velocity of the base.

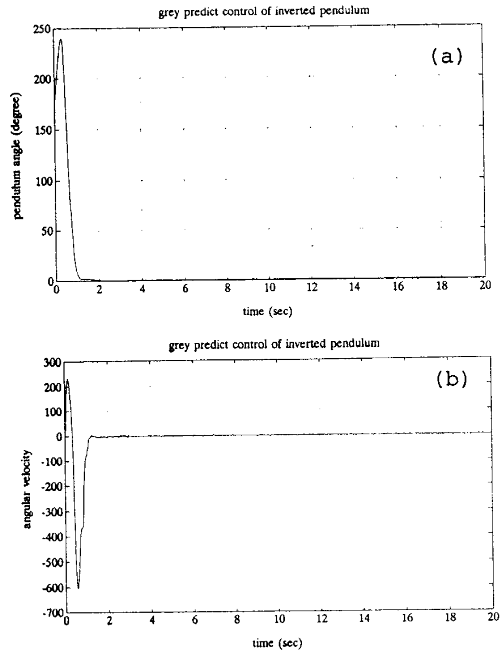

Fig. 2 The dynamic response of the inverted pendulum:(a)angle and (b)angular velocity.

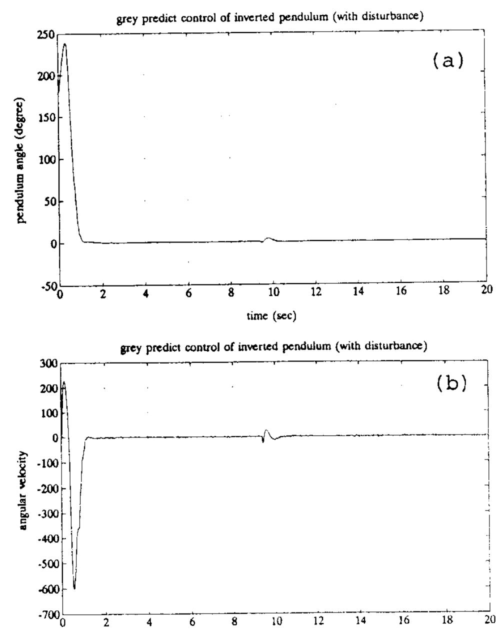

Fig. 4 Dynamic response of the inverted pendulum with external disturbance: (a) angle and (b) angular velocity.

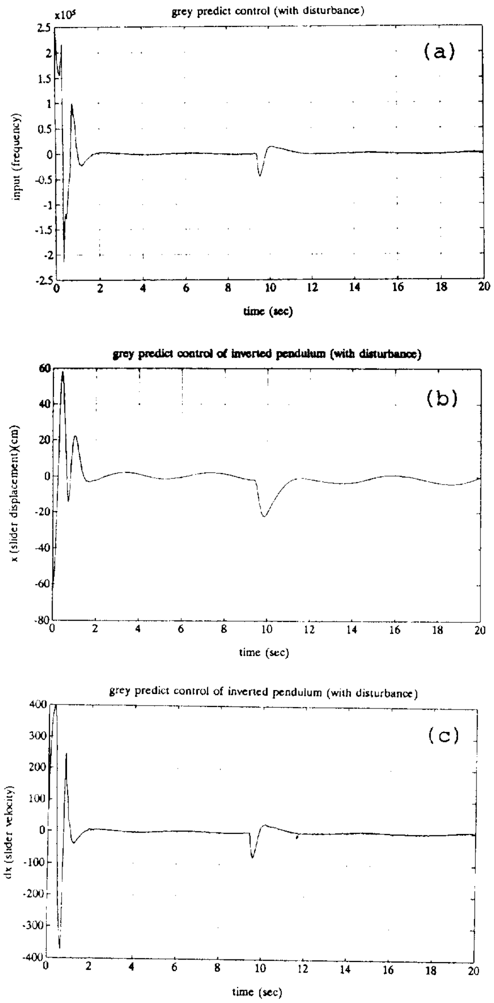

Fig. 5 (a) The input frequency of linear motor, (b) the position response of the base and (c) the velocity of the base with external disturbance.

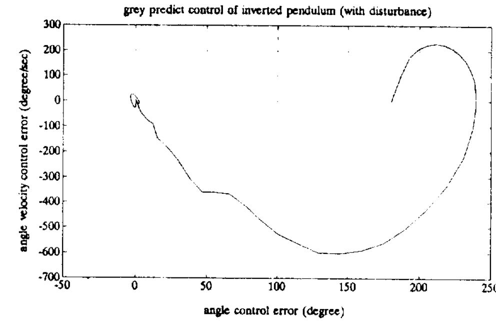

Fig. 6 Phase plane trajectory of inverted pendulum.

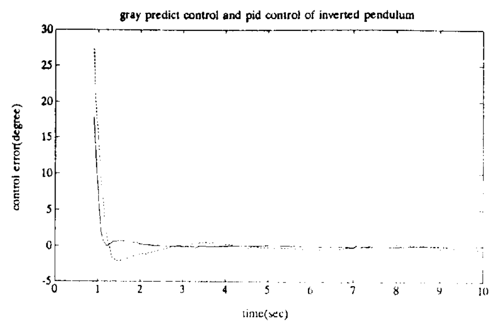

Fig. 7 PD controller with grey prediction model (solid line) and without GPM (dashed line).