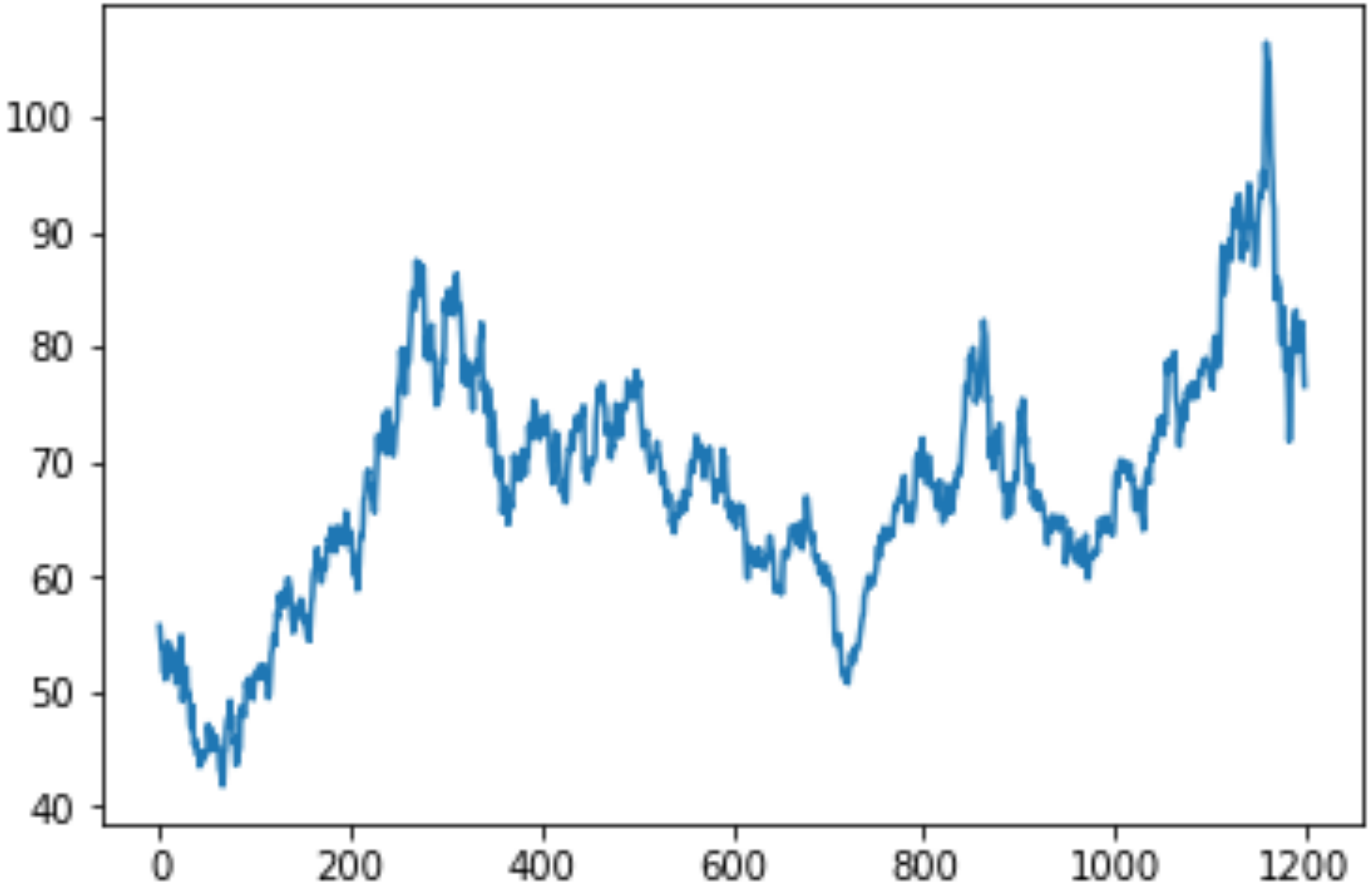
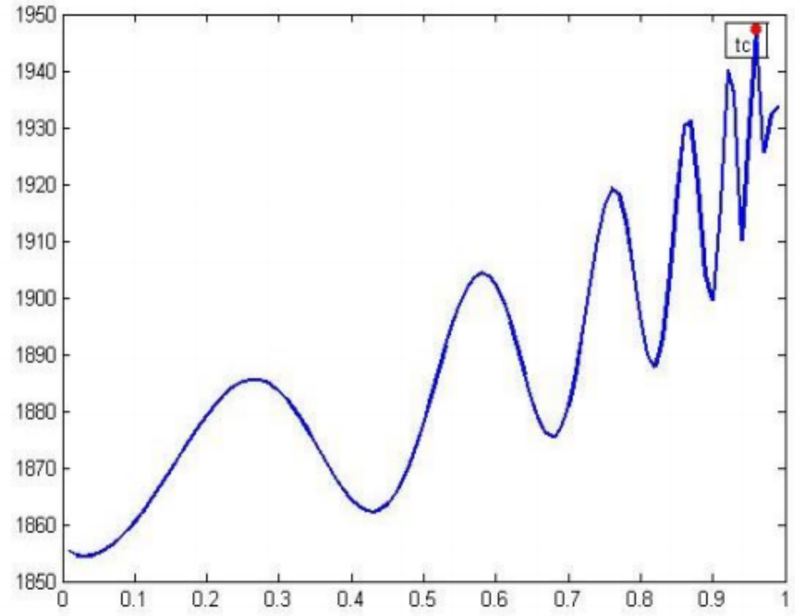
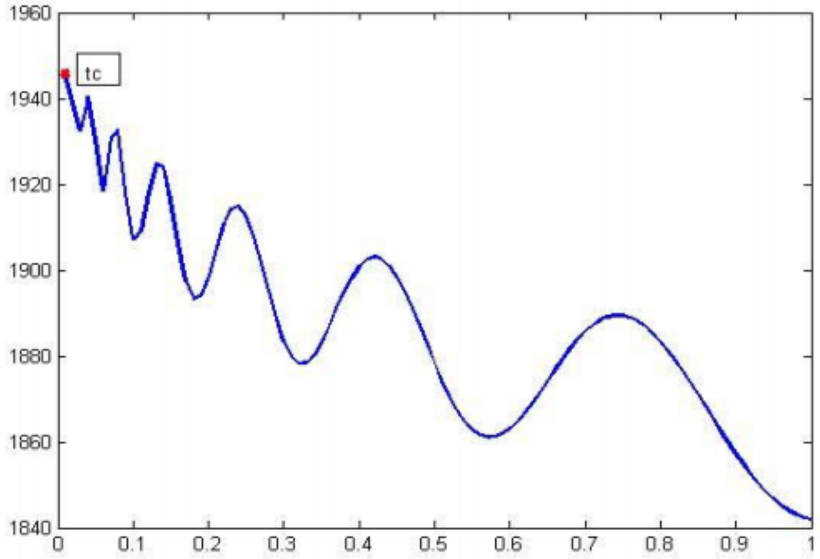
Exercise

2019.10.30

Financial Application (Bubble modeling)







圖一 泡沫

圖二 反泡沫

資料來源：國泰君安證券研究所

log-periodic power laws (LPPL) for bubble modeling

$$\ln[p(t)] \approx A + B(t_c - t)^{\beta}\{1 + C\cos[\omega \ln(t_c - t) + \phi]\}, \tag{12}$$

where $A > 0$ is the value of $[\ln p(t_c)]$ at the critical time, $B < 0$ is the increase in $[\ln p(t)]$ over the time unit before the crash if C were to be close to zero, $C \neq 0$ is the proportional magnitude of the oscillations around the exponential growth, $0 < \beta < 1$ should be positive to ensure a finite price at the critical time t_c of the bubble and quantifies the power law acceleration of prices, and ω is the frequency of the oscillations during the bubble, while $0 < \phi < 2\pi$ is a phase parameter. Expression (12), which is known as the LPPL, is the fundamental equation that describes the temporal growth of prices before a crash and it has been proposed in different forms in various papers (e.g. Sornette 2003a, Lin, Ren, and Sornette 2009 and references therein). We remark that A, B, C and ϕ are just units distributions of betas and omegas, as described in Sornette and Johansen (2001) and Johansen (2003), and do not carry any structural information.

- Two step algorithm
  - Each gene includes 4 non-linear variables t_c, β, ω, Φ
  - Use linear regression to estimate best A, B, C
- For each parameter setting, we can measure the fitness between synthetic signals and real financial time-series data.
- Apply genetic algorithm to approximate the optimal solution by minimizing the average fitness error between time 0 and t_c.
- Homework:
  - LPPL
    - Find the optimal LPPL parameters, suppose t_c in [1151,1166]
    - Plot the synthetic signals and real time-series data with different colors in a figure