

# DISSERTATION TITLE

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A dissertation submitted to  
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in partial fulfilment of the requirements for the degree of  
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AUGUST, 2021

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# DISSERTATION INFORMATION AND STATEMENT

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## STATEMENT:

Unless otherwise noted or referenced in the text, the work described in this dissertation is, to the best of my knowledge and belief, my own work. It has not been submitted, either in whole or in part for any degree at this or any other academic or professional institution.

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Signature of Student

# Abstract

Write your abstract here(no more than one page).

Usually the first paragraph describes briefly background and motivation of your dissertation work/research, and then aim and objectives.

The second paragraph may describe the methods and experiments.  $z = \frac{x1}{y2}$ .

Third paragraph gives the results and conclusions.

# Acknowledgements

I would like to thank my supervisor, Dr. xxx xxx, for his/her many suggestions and constant support during this research, ..... blah, blah, blah.

I would also like to express my gratitude to whoever sponsored my course in full or part.

Finally, I am very grateful to my family (parents, etc.) for their patience and *love*. Particularly, I want to thank my partner (girlfriend, boyfriend, wife, husband, etc.) for putting up with me, blah, balh, blah, .....

Your name here

Norwich, UK.

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# List of Abbreviations

**API** Application Programming Interface. 5, 6

**CMP** School of Computing Sciences. 1

**KDD** Knowledge Discovery form Database. 6

**LOA** List of Abbreviations. 5

**SVM** Support Vector Machine. 6

**UEA** University of East Anglia. 1

**UML** Unified Modelling Language. 6

# Chapter 1

## Notes on how to use the Latex Dissertation template

This LATEX template was created by Dr. Wenjia Wang<sup>1</sup> to be used by the Master students at the School of Computing Sciences (CMP), the University of East Anglia (UEA), to write their dissertation with Latex.

This section gives you the sufficient instructions on how to use the template so you must read it carefully before using it.

NOTE: you should use *TexStudio* as your text editor and Latex compiler to make sure this Latex Template working properly, although it may work with other text editors.

Please let me know if you find any bugs or problems, although I may not have time to resolve them in time.

### 1.1 How to use this Latex Template

#### 1.1.1 Preparation Steps: P1 to P4

P1. Download the template package from the blackboard of Dissertation Module and Unzipped it to an intended working folder on your U (University) drive, or any other drive on your Computer, e.g. Dissertation.

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<sup>1</sup>Created in 2005 based on CMP thesis style file and one from Stanford University.

Previous versions: created in 2005 (v0), updated in 2010(v1) and 2012(v1.1). Major Revision on 06/08/2015-17/08/2015(v2): Added the function of generating the list of Abbreviations (you must follow the instructions carefully and exactly in order to produce a list of Abbreviations). Major revision on 22/11/2016(v3), added notes for how to use the Template. Updated on 15/04/2019: Updated the Instruction notes. Latest update on 24/03/2020(v4), revised latex style file to typeset the dissertation title page in bold, as per the dissertation handbook, and added a confidentiality statement.

Note, you are strongly recommended to use the University's file storage on U drive as your main working folder because you can access it from any PCs on Campus and also your own PC via VPN. In addition, you must have a backup storage to save all your working files and programs developed for your dissertation.

P2. Start TexStudio and open "DissertationTemplate5.tex"

Note: it is a tex file that

(1) uses, i.e. includes all the other files, such as Abstract, Acknowledgement, and each of Chapter files, which are written or edited separately with Latex.

(2) generates a pdf file of your dissertation as a whole.

P3. you need to change/replace/fill few places in this file to suit your need: such as, Your course, Year, Confidentiality, Dissertation Title, your name, markers, etc.

**Note:** If your dissertation is considered to be confidential, you and your supervisor need to decide an end date (month day, year) of confidentiality. Then you need to UNCOMMENT the two lines in the template file, where are indicated in the template tex file, which copied over here to show what they look like.

```
\confidential{} % display the confidentiality statement on the title page
```

```
\setconfidentialdate2{August31,2021} % set the end date to e.g. August 31, 2021
```

Once they are uncommented, and a specific date, e.g. August 31, 2021, is set, the pre-written confidentiality statement and the end date will be displayed in red colour on the title page, as shown below.

### ***CONFIDENTIALITY STATEMENT:***

*The contents of this dissertation remain confidential until August 31, 2021 and should not be discussed or disclosed to any third party without the prior written permission from the School of Computing Sciences, the University of East Anglia.*

P4. Save it with your new file name to a folder specially created for writing your dissertation, e.g. "Wang\_Dissertation2020.tex"

## **1.1.2 Work on each latex file**

Then following the steps below to work on each file to write your dissertation.

1. Write your abstract in a separate tex file and name it as *Abstract.tex*

2. Write your acknowledgement in a separate tex file and name it as *Acknowledgement.tex*

Note: Both *Abstract.tex* and *Acknowledgement.tex* files are already included in the style file. So you must not change their names but only the contents.

3. Write each chapter in a separate tex file and name them as, e.g. Ch1.tex, Ch2.tex, etc. and then use “\include{...}” to include them as shown in this example.

(New notes added on 06/08/2015)

4. If you wish to produce a list of Abbreviations/Acronyms that are used in your dissertation, you must read the notes given in Section 1.4.

5. Using footnote (Wenjia added this on 11/09/2015)

If you have to use footnotes (although you should try to avoid using any) in any chapter of your dissertation, you can use command `\footnote{footnote text}` in where you want, for example, a footnote is included here.<sup>2</sup>

The footnotes will be automatically numbered continuously within a CHAPTER.

6. Added the confidentiality statement and the end date, as described above(24/03/2020).

## 1.2 Making Citations and Citation Styles

You are required to use the Harvard style for citing references in your dissertation.

Specifically, there are two sub-styles to be used in different situations, when using package “natbib”, which is included at the preamble of the template file.

1. Use command `\citep{...}`.

If the authors of a reference are NOT part of your sentence, e.g. “A study (Wang, 2008) has been done to investigate the influence of some factors on the accuracy of an ensemble.”, then use `\citep{...}` in your Latex file, such as “A study `\citep{Wang08}` has been done...”, it then produces the text as: “A study (Wang, 2008) has been done.....”

2 Use command `\citet{...}`.

If the authors of a reference are part of your sentence, e.g. “Wang (2008) studied

---

<sup>2</sup>Your footnote text: If you want to generate a list of Abbreviations, you must follow the instructions given here carefully and exactly, particularly using Command “Makeglossaries” in “Tools” before Compiling.

the factors that can affect the performance of a machine learning ensemble.”, then use `\citet{Wang08}` studied ... . It then produces the text as: “Wang (2008) studied .....”

You can press function key “F8” in TexStudio to compile bibliography, i.e. to pull all the cited references out from your Bibtex file and generate a bib file. Check the message to see if there is any error in this process.

## 1.3 Creating Equations

You can write an equation, as shown in Equation 1.3.1, by using `\begin{equation}` write equation here `\end{equation}`. For example,

$$y = a + b_1x_1 + b_2x_2 \quad (1.3.1)$$

If your equation is too long for a single line, instead of using the above environment, use “`\begin{align}`” command to align an equation of multiple lines at a specified point. Use `\\` to specify a line break, and `&` to indicate where the lines should be aligned.

For example, the following equation, as shown in Equation 1.3.2, is aligned at “=”.

$$\begin{aligned} f(x) &= (x + a)(x + b) \\ &= x^2 + (a + b)x + ab \end{aligned} \quad (1.3.2)$$

Equation 1.3.3 is aligned at the left brace.

$$\begin{aligned} f(x) &= \pi \{ a + b_1x_1 + b_2x_2 + b_3x_3^4 + b_4x_4^3 + b_5x_5^2 \\ &\quad + b_6x_6^5 + b_7x_7^2 + b_8x_8^3 + b_9x_9^3 \} \end{aligned} \quad (1.3.3)$$

Note: “`\{align}`” must not be nested within “`\{equation}`”, it replaces “`\{equation}`”.

If you do not want to automatically number an equation, use `\{equation*` or `\{align*`. For example, the following equation will not be numbered.

$$y = a + b_1x_1 + b_2x_2^2 + b_3x_3^3$$

## 1.4 Define and generate a List of Abbreviations(LoA)

This section tells you how to define Abbreviations, generate a list of them and show the list on the preamble of your dissertation.

I will show you how to use the acronyms defined in a given file named as “acronyms.tex”, which will be then listed in the List of Abbreviations (LOA) if THEY ARE USED in your Dissertation.

### 1.4.1 Define abbreviations/acronyms

( Notes and sample files:

“acronymNotes.tex”: brief introduction on how to make a LoA.

“acronyms.tex”: a sample file where you define abbreviations. )

To define an abbreviation or acronym, open the provided sample file in the template package - “acronym.tex”, in any text editor, e.g. TeXStudio, you can see some abbreviations (or acronyms) already defined in it.

You can simply use the following command *newacronym* to define an abbreviation/acronym in the format: `\newacronym{label}{name}{description}`

For example: `\newacronym{api}{API}{Application Programming Interface}`

### 1.4.2 Use the defined Abbreviations/Acronyms

You can use `\gls`, or `\Gls`, Capital, to insert the abbreviation to anywhere you want in your tex file. Or use `\glspl`, or `\Glspl` for using their plural forms.

In the first time you use it, it will produce the full text of the abbreviation, followed by its abbreviation in (). After that, it will only produce the abbreviation.

For example, `\Gls{api}` will be shown as Application Programming Interface (API), i.e. ‘Application Programming Interface (API)’ (without the quotation marks), and will add a linked page number to where it is used, e.g. ‘1’ in this case, and will be shown in the LOA.

After that, `\Gls{api}` will produce only the abbreviation, i.e. API.

Unified Modelling Language (UML), Support Vector Machine (SVM), Knowledge Discovery from Database (KDD) are some other abbreviation examples I defined in “acronym.tex” file. Their plural format can be produced by using command: `\glspl{}`. e.g. `\glspl{uml}`, `\glspl{svm}`, `\glspl{kdd}`, which produce: UMLs, SVMs, KDDs.

### 1.4.3 Compiling your main tex file with an Abbreviation file

If you have defined your abbreviations or acronyms in file “acronyms.tex”, and use some of them in your text file of other Chapters, such as in this note file, by using the commands given above, you must compile and build your integrating tex file (e.g. DissertationSample.tex) to produce the intended files, e.g. pdf file, with the steps given in the next Section in TeXstudio.

Please note:

- (1) Only the used acronyms will appear in the List of Abbreviations.
- (2) notice the difference in using “gls” and “glspl”

## 1.5 Compiling/Building your tex file

After you have written each of your tex files for your chapters, you need to include each of them in your dissertation main latex file, using a command “`\include{...}`”, as shown in this example.

Then, you compile your main latex file by following the steps below to produce the intended results, i.e. a PDF file of your dissertation with every thing included.

1. Run “Compile” or “Build/View” by clicking their icon, or simply press the shortcut key F5. (note: you may see a pdf file with your text, but it won’t have the list of abbreviations.)

2. Run “Glossary” or “Makeglossaries”:

If you have a list of abbreviations defined in the file “acronyms.tex”, you must do this, otherwise the list won’t be included in your dissertation.

Click “Tools” and then choose “Glossary” or simply press shortcut key F9.



(If it is not working, you can try: Click “Tools”, and “Commands”, and choose “Makeglossaries” to run it.)

Ignore any warning message.

Note: whenever you make any new entry to your “acronym.tex” file, and/or use any abbreviation/acronym in your other tex file, you must do this step to update your generated “.gls” file.

3. Run “Build and View” (or press F5) again. This time the pdf file should contain the actual list of abbreviations after the list of Figures and the title appears in the Table of Content(TOC).

# Chapter 2

## Introduction

This chapter gives a brief introduction to the background of my dissertation topic on Lomonosov function(Lomonosov, 1973), and motivation, defines the aim and objectives of my dissertation.

### 2.1 Background

Lomonosov (1991) conjectured that the adjoint of a bounded operator on a Banach space has a non-trivial closed invariant subspace. In view of the known examples of operators without an invariant subspace (Enflo, 1987; Read, 1985), this is the strongest version of the invariant subspace problem that can possibly have an affirmative answer. In particular, if the Lomonosov conjecture is true, then every operator on a reflexive Banach space has a non-trivial invariant subspace(as shown in Figure 2.1).

Considering the strong influence of Lomonosov's results on the theory of invariant

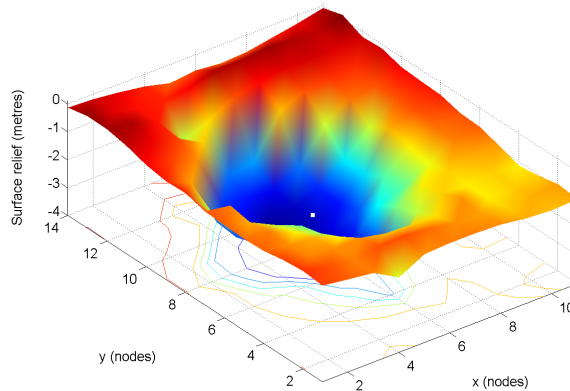


Figure 2.1: Caption of a figure should be placed underneath of the figure. This figure is not associated with the text.

Table 2.1: A sample table. Caption of a Table should be placed on the top of it.

Day	Min Temp	Max Temp	Summary
Monday	11C	22C	A clear day with lots of sunshine. However, the strong breeze will bring down the temperatures.
Tuesday	9C	19C	Cloudy with rain, across many northern regions. Clear spells across most of Scotland and Northern Ireland, but rain reaching the far northwest.
Wednesday	10C	21C	Rain will still linger for the morning. Conditions will improve by early afternoon and continue throughout the evening.

subspaces, it is not surprising that both the conjecture and the techniques developed in the interesting paper (Lomonosov, 1991) received further attention. L. de Branges used this result to obtain a characterization of the invariant subspace problem in terms of density of certain functions. This stimulated another characterization of the invariant subspace problem given by Y.A. Abramovich, C.D. Aliprantis, and O. Burkinshaw in (Abramovich et al., 1995). Section 1.4 presents a more detailed account of this work.

We take a slightly different approach. First we give a constructive proof of the approximation theorem, inspired by the well known Lomonosov construction used in (Lomonosov, 1973; Radjavi and Rosenthal, 1973). This theorem is then applied to give an alternative proof of the main result in (Abramovich et al., 1995). Our proof applies to both real and complex Banach spaces, while the original result was established for complex Banach spaces only. The alternative proof somehow explains the role of compact operators that appear in the characterizations of the invariant subspace problem (Abramovich et al., 1995).

One may notice that the weak\*-compactness of the unit ball in dual Banach spaces plays an important role in (Abramovich et al., 1995; de Branges, 1959, 1993; Lomonosov, 1991), as well as in the applications given in this chapter. In other words,

if the Lomonosov conjecture is true, then the compactness of the unit ball, with respect to the weak\* topology, is likely to be an important ingredient of its proof.

Equation 2.1.1 shows the function of linear regression of  $n$  variables:  $x_1, x_2, \dots, x_n$ .

$$y = a + b_1x_1 + b_2x_2 + b_3x_3 + \dots + b_nx_n = a + \sum_{i=1}^n b_ix_i \quad (2.1.1)$$

This work is more or less self-contained and the notation and terminology used in it is (supposed to be) standard. However, here are a few conventions that hold throughout this chapter:

## 2.2 Aim and Objectives

The aim of this dissertation is to develop a machine learning ensemble system(Wang, 2008) to predict ....

The associated objectives are:

- 1) To study the relevant literature ....
- 2) To design and implement an ensemble system ....
- 3) To test the developed ensemble system with benchmark data and real-world data.
- 4) To compare its performance with the state of the art methods.

## 2.3 Research Project Work Plan

Gives a research plan here.

## 2.4 Structure of Dissertation

describes how your dissertation is structured.

## 2.5 Summary

Summarise what has been presented in this chapter.

# **Chapter 3**

## **Literature Review**

This chapter reviews the related literature to my dissertation topic.

### **3.1 Review on basic methods**

### **3.2 Related Work**

### **3.3 Summary**

Summarise what has been presented in this chapter.

# Chapter 4

## Methodology Design

This chapter describes the methodology designed my dissertation project.

### 4.1 Analysis of the methods

### 4.2 Design of Methodology

### 4.3 Evaluation Methods and Measures

### 4.4 Tools and Resources

### 4.5 Summary

Summarise what has been presented in this chapter.

# Chapter 5

## Conclusion

In this chapter, (if you do not have a Chapter specifically for Evaluation and Discussion), you firstly evaluate your results using statistical significance tests, then discuss the strengths and limits of our methods, and also evaluate your methodology for your dissertation, and finally draw conclusions and make some suggestions for further work.

### 5.1 Evaluation and Discussion

Normally you should have a chapter before conclusion chapter, particularly for describing evaluation and discussion on your work as a whole. If you do not have such a chapter, you should have at least a Section dedicated to this, before giving conclusion and suggestion for further work.

### 5.2 Conclusion

write your conclusions in this section. Every point of your conclusion must have been evaluated and discussed in the previous Chapter or Section

### 5.3 Suggestion for Further Work

# References

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# Appendix A

## A Sample Appendix on Invariant Subspaces

An application of the main result of the previous chapter to the algebra generated by an essentially self-adjoint operator  $A$  yields the existence of nonzero vectors  $x, y \in \mathcal{H}$  such that  $\tau(p) = \langle p(A)x, y \rangle$  is a positive functional on the space of all polynomials on the essential spectrum of  $A$ . This result immediately implies the existence of real invariant subspaces for essentially self-adjoint operators acting on a complex Hilbert space. Elementary convex analysis techniques, applied to the space of certain vector states, yield the existence of invariant subspaces for essentially self-adjoint operators acting on an infinite-dimensional real Hilbert space.

### A.1 Introduction

The existence of invariant subspaces for compact perturbations of self-adjoint operators appears to be one of the most difficult questions in the theory of invariant subspaces Lomonosov (1992). The positive results about the existence of the invariant subspaces for the Schatten-class perturbations of self-adjoint operators, acting on a complex Hilbert space, date back to the late 1950's. For the facts concerning such operators see Chapter 6 in Radjavi and Rosenthal (1973), where a brief history of the problem, together with the references to the related topics is given. The proofs of those results are based on the concept of the separation of spectra. However, Ljubič and Macaev Ljubič and Macaev (1965) showed that there is no general spectral theory by constructing an example of an operator  $A$  such that  $\sigma(A|_{\mathcal{M}}) = [0, 1]$  whenever  $\mathcal{M}$  is a nonzero invariant subspace for  $A$ . This suggests that different techniques might be needed to establish the existence of invariant subspaces for essentially self-adjoint operators.

## A.2 On Real Invariant Subspaces

Recently V.I. Lomonosov (1992) proved that every essentially self-adjoint operator acting on a complex Hilbert space has a nontrivial closed *real* invariant subspace. We give an alternative proof, based on Proposition, and thus introduce the idea that will be later generalized in order to yield the existence of proper invariant subspaces for essentially self-adjoint operators acting on a real Hilbert space.

Recall that a *real subspace* of a complex Hilbert space  $\mathcal{H}$  is a subset that is closed under addition and multiplication by the *real* scalars. A real subspace  $\mathcal{M} \subset \mathcal{H}$  is invariant for an operator  $A \in \mathbf{B}(\mathcal{H})$  if and only if  $\mathcal{M}$  is invariant under all operators in the *real* algebra generated by  $A$ , i.e. the algebra of all real polynomials in  $A$ .

## A.3 The Space of Vector States

In the previous section we applied our machinery only to the imaginary part of an essentially self-adjoint operator  $A$ . An application to the real part yields the existence of “vector states” on the space of all polynomials restricted to the essential spectrum of  $A$ . Before proceeding, we make the following conventions that hold through the rest of this chapter:

## A.4 Invariant Subspaces on a Real Hilbert Space

In this section we use vector states in order to establish the existence of invariant subspaces for essentially self-adjoint operators acting on an infinite-dimensional real Hilbert space. The invariant subspace problem for essentially self-adjoint operator will be translated into an extreme problem and the solution will be obtained upon differentiating certain functions at their extreme. Once again we will employ the differentiability of the Hilbert norm. We start with the following lemma.

*We suggest that further research in this direction is likely going to reveal additional properties of essentially self-adjoint operators and thus contribute to our understanding of how such operators act on the underlying Hilbert space in terms of invariant subspaces.*