

Question – 1: Finding memory bugs

The fixed and commented codes are shared in the repo.

Question – 2: Optimizing Matrix-matrix multiplication

```
// * Using MMult0 as a reference, implement MMult1 and try to rearrange loops to
// maximize performance. Measure performance for different loop arrangements and
// try to reason why you get the best performance for a particular order?
```

Re-arranging the loops in the order $(jj, ii, kk) ; (kk, ii, jj) ; (ii, jj, kk) ; (kk, jj, ii) ; (ii, kk, jj) ; (jj, kk, ii)$ gave the following run-times below, respectively. The following code is shared in the repo named “hw2_qs2_alt_loops”.

It is seen from below, that for a particular combination of the loops, the Gflops/sec is the lowest at 0.25 approx. This is due to the repetitive data-load from slow-memory to cache. For multiplication of matrices, one element each of both the matrices needs to be loaded from slow-memory. As we know, that data load operations are not done element-wise, rather a whole line of data chunk is loaded into cache, called cache-lines. For a particular combination, the data (cache-line) that is being loaded once (and stored since the loop is designed to use the previously fetched data), are simultaneously used twice for the matrix operations, thereby reducing the run-time and cache-misses. As, memory operations are always slower, this increases the runtime significantly and reduces the number of per-second operations.

```
ss15592@access2 homework2]$ g++ -std=c++11 -o3 -march=native hw2_qs2_alt_loops.cpp -o hw2_qs2_alt_loops && ./hw2_qs2_alt_loops
Dimension    Time      Gflop/s      GB/s      Error
    20      4.825277    0.414487    1.740847    0.000000e+00
   120      4.860625    0.411680    1.660444    0.000000e+00
   220      4.866574    0.411342    1.652845    0.000000e+00
   320      4.951757    0.410282    1.646256    0.000000e+00
   420      5.010580    0.414017    1.660010    0.000000e+00
ss15592@access2 homework2]$ g++ -std=c++11 -o3 -march=native hw2_qs2_alt_loops.cpp -o hw2_qs2_alt_loops && ./hw2_qs2_alt_loops
Dimension    Time      Gflop/s      GB/s      Error
    20      4.822713    0.414708    1.741772    0.000000e+00
   120      4.774076    0.419144    1.690546    0.000000e+00
   220      4.817275    0.415551    1.669760    0.000000e+00
   320      5.201263    0.390600    1.567284    0.000000e+00
   420      5.270847    0.393573    1.578041    0.000000e+00
ss15592@access2 homework2]$ g++ -std=c++11 -o3 -march=native hw2_qs2_alt_loops.cpp -o hw2_qs2_alt_loops && ./hw2_qs2_alt_loops
Dimension    Time      Gflop/s      GB/s      Error
    20      4.885395    0.409387    1.719424    0.000000e+00
   120      4.879018    0.410128    1.654185    0.000000e+00
   220      4.891771    0.409223    1.644332    0.000000e+00
   320      4.952552    0.410216    1.645992    0.000000e+00
   420      5.063243    0.409711    1.642744    0.000000e+00
ss15592@access2 homework2]$ g++ -std=c++11 -o3 -march=native hw2_qs2_alt_loops.cpp -o hw2_qs2_alt_loops && ./hw2_qs2_alt_loops
Dimension    Time      Gflop/s      GB/s      Error
    20      7.352356    0.272024    1.142500    0.000000e+00
   120      7.694731    0.260051    1.048873    0.000000e+00
   220      7.775783    0.257443    1.034454    0.000000e+00
   320      7.849233    0.258830    1.038555    0.000000e+00
   420      8.019245    0.258686    1.037206    0.000000e+00
ss15592@access2 homework2]$ g++ -std=c++11 -o3 -march=native hw2_qs2_alt_loops.cpp -o hw2_qs2_alt_loops && ./hw2_qs2_alt_loops
Dimension    Time      Gflop/s      GB/s      Error
    20      4.835772    0.413588    1.737068    0.000000e+00
   120      4.852172    0.412398    1.663337    0.000000e+00
   220      4.928615    0.406164    1.632039    0.000000e+00
   320      5.244058    0.387413    1.554494    0.000000e+00
   420      5.311945    0.390528    1.565832    0.000000e+00
ss15592@access2 homework2]$ g++ -std=c++11 -o3 -march=native hw2_qs2_alt_loops.cpp -o hw2_qs2_alt_loops && ./hw2_qs2_alt_loops
Dimension    Time      Gflop/s      GB/s      Error
    20      7.273149    0.274986    1.154942    0.000000e+00
   120      7.632174    0.262183    1.057470    0.000000e+00
   220      7.671330    0.260949    1.048540    0.000000e+00
   320      7.846246    0.258928    1.038950    0.000000e+00
   420      8.046003    0.257825    1.033757    0.000000e+00
```

```
// * Experiment with different values for BLOCK_SIZE (use multiples of 4) and
// measure performance. What is the optimal value for BLOCK_SIZE?
```

The processor used for the calculations is: AMD EPYC Processor x86_64, 4 cores @2.894 GHz clock speed. The code is shared in the repo. The screenshots are shown below. The optimal “BLOCK_SIZE” is 16.

Run-times *without parallelization* and block multiplication:

BLOCK_SIZE=8					BLOCK_SIZE=16				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
8	6.759871	0.295864	1.331387	0.000000e+00	16	7.202599	0.277678	1.180131	0.000000e+00
152	6.758311	0.296188	1.192546	0.000000e+00	160	7.243784	0.277071	1.115209	0.000000e+00
296	6.849886	0.295316	1.185253	0.000000e+00	304	7.335308	0.275762	1.106678	0.000000e+00
440	7.018622	0.291285	1.167786	0.000000e+00	448	7.847024	0.275005	1.102475	0.000000e+00
584	8.172254	0.292468	1.171874	0.000000e+00	592	7.519455	0.275917	1.105533	0.000000e+00
728	7.860370	0.294512	1.179665	0.000000e+00	736	8.669604	0.275921	1.105185	0.000000e+00
872	9.150523	0.289843	1.160703	0.000000e+00	880	9.848925	0.276770	1.108338	0.000000e+00
1016	7.161732	0.292882	1.172682	0.000000e+00	1024	8.028918	0.267469	1.070919	0.000000e+00
1160	10.867751	0.287253	1.150002	0.000000e+00	1168	11.653263	0.273471	1.094820	0.000000e+00
1304	15.228948	0.291201	1.165697	0.000000e+00	1312	16.720131	0.270142	1.081391	0.000000e+00
1448	21.482806	0.282647	1.131370	0.000000e+00	1456	22.677586	0.272218	1.089621	0.000000e+00
1592	28.505623	0.283093	1.133082	0.000000e+00	1600	30.697630	0.266861	1.068111	0.000000e+00
1736	37.185316	0.281389	1.126206	0.000000e+00	1744	39.072870	0.271515	1.086683	0.000000e+00
1880	47.457799	0.280024	1.120694	0.000000e+00	1888	49.316129	0.272927	1.092287	0.000000e+00
BLOCK_SIZE=32					BLOCK_SIZE=64				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
32	7.472941	0.267636	1.103998	0.000000e+00	64	7.638595	0.261849	1.063762	0.000000e+00
160	7.533830	0.266404	1.072275	0.000000e+00	192	7.735411	0.259860	1.044852	0.000000e+00
288	7.502853	0.267442	1.073484	0.000000e+00	320	8.219634	0.247166	0.991754	0.000000e+00
416	7.661602	0.263099	1.054924	0.000000e+00	448	8.349225	0.258463	1.036161	0.000000e+00
544	8.498182	0.265215	1.062812	0.000000e+00	576	9.052067	0.253338	1.015113	0.000000e+00
672	9.135633	0.265741	1.064547	0.000000e+00	704	9.413979	0.222380	0.890784	0.000000e+00
800	7.710900	0.265598	1.063720	0.000000e+00	832	9.240342	0.249311	0.998444	0.000000e+00
928	12.110456	0.263963	1.056991	0.000000e+00	960	14.635824	0.241800	0.968208	0.000000e+00
1056	8.942599	0.263365	1.054457	0.000000e+00	1088	10.887802	0.236579	0.947186	0.000000e+00
1184	12.550361	0.264502	1.058901	0.000000e+00	1216	13.899342	0.258724	1.035746	0.000000e+00
1312	17.098637	0.264162	1.057453	0.000000e+00	1344	20.304780	0.239127	0.957222	0.000000e+00
1440	22.600968	0.264235	1.057674	0.000000e+00	1472	26.071119	0.244677	0.979374	0.000000e+00
1568	29.164111	0.264374	1.058172	0.000000e+00	1600	31.815650	0.257483	1.030577	0.000000e+00
1696	37.269085	0.261793	1.047791	0.000000e+00	1728	39.917135	0.258525	1.034697	0.000000e+00
1824	46.268830	0.262311	1.049818	0.000000e+00	1856	49.023534	0.260831	1.043886	0.000000e+00
1952	56.141500	0.264963	1.060395	0.000000e+00	1984	60.426581	0.258480	1.034441	0.000000e+00
BLOCK_SIZE=128					BLOCK_SIZE=256				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
128	7.729577	0.258835	1.043427	0.000000e+00	256	7.831222	0.257082	1.032345	0.000000e+00
256	7.843192	0.256690	1.030769	0.000000e+00	512	8.488915	0.252975	1.013877	0.000000e+00
384	7.917942	0.257445	1.032460	0.000000e+00	768	10.741305	0.253033	1.013451	0.000000e+00
512	8.532276	0.251689	1.008724	0.000000e+00	1024	8.570712	0.250561	1.003222	0.000000e+00
640	8.185634	0.256199	1.026398	0.000000e+00	1280	16.856231	0.248828	0.996090	0.000000e+00
768	10.641130	0.255415	1.022992	0.000000e+00	1536	30.179596	0.240154	0.961242	0.000000e+00
896	11.111584	0.258945	1.036937	0.000000e+00	1792	46.475189	0.247641	0.991117	0.000000e+00
1024	8.474607	0.253402	1.014598	0.000000e+00					
1152	12.007441	0.254646	1.019468	0.000000e+00					
1280	16.478169	0.254537	1.018944	0.000000e+00					
1408	21.801711	0.256063	1.024981	0.000000e+00					
1536	29.435709	0.246223	0.985534	0.000000e+00					
1664	35.991691	0.256028	1.024728	0.000000e+00					
1792	45.194920	0.254656	1.019194	0.000000e+00					
1920	55.609153	0.254558	1.018764	0.000000e+00					

```
// * Now parallelize your matrix-matrix multiplication code using OpenMP.
//
//
// * What percentage of the peak FLOP-rate do you achieve with your code?
```

The peak Flop-rate is given by the following equation:

$$GFlops = (CPU \text{ speed in GHz}) \times (\text{number of CPU cores}) \times (\text{CPU instruction per cycle}) \\ \times (\text{number of CPUs per node}) = 2.894 \times 4 \times 4 \times 4 = 185.22 \text{ Gflops/sec}$$

My code achieves $\left(\frac{1.03}{185.22}\right) * 100 = 0.556\%$ of peak flop-rate.

Run-times **with parallelization** and block multiplication:

BLOCK_SIZE=8					BLOCK_SIZE=16				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
8	172.744586	0.011578	0.052100	0.000000e+00	16	8.018076	0.249437	1.060106	0.000000e+00
152	6.721463	0.297812	1.199084	0.000000e+00	160	2.352411	0.853184	3.434067	0.000000e+00
296	3.396872	0.595512	2.390096	0.000000e+00	304	2.043319	0.989959	3.972861	0.000000e+00
440	2.981231	0.685762	2.749284	0.000000e+00	448	2.143371	1.006811	4.036233	0.000000e+00
584	3.356268	0.712136	2.853423	0.000000e+00	592	2.262472	0.917026	3.674302	0.000000e+00
728	2.927587	0.790744	3.167319	0.000000e+00	736	2.392691	0.999765	4.004495	0.000000e+00
872	3.511945	0.755200	3.024263	0.000000e+00	880	2.733455	0.997232	3.993460	0.000000e+00
1016	2.787324	0.752530	3.013081	0.000000e+00	1024	3.363131	0.638537	2.556643	0.000000e+00
1160	3.793907	0.822843	3.294211	0.000000e+00	1168	3.267773	0.975229	3.904256	0.000000e+00
1304	5.604305	0.791300	3.167626	0.000000e+00	1312	4.375481	1.032299	4.132345	0.000000e+00
1448	7.691680	0.789432	3.159907	0.000000e+00	1456	6.222646	0.992062	3.970975	0.000000e+00
1592	9.687657	0.832991	3.334058	0.000000e+00	1600	8.292278	0.987907	3.954098	0.000000e+00
1736	12.692682	0.824377	3.299407	0.000000e+00	1744	10.555623	1.005045	4.022486	0.000000e+00
1880	16.473545	0.806708	3.228549	0.000000e+00	1888	13.124344	1.025554	4.104387	0.000000e+00

BLOCK_SIZE=32					BLOCK_SIZE=64				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
32	83.082303	0.024073	0.099300	0.000000e+00	64	7.737181	0.258513	1.050207	0.000000e+00
160	5.562734	0.360801	1.452224	0.000000e+00	192	2.806624	0.716206	2.879744	0.000000e+00
288	3.221167	0.622936	2.500396	0.000000e+00	320	3.211804	0.632547	2.538093	0.000000e+00
416	2.978142	0.676850	2.713909	0.000000e+00	448	2.448690	0.881275	3.532969	0.000000e+00
544	3.097747	0.727577	2.915657	0.000000e+00	576	3.234400	0.709014	2.840981	0.000000e+00
672	3.287613	0.738443	2.958169	0.000000e+00	704	2.359502	0.887256	3.554064	0.000000e+00
800	2.763042	0.741212	2.968554	0.000000e+00	832	2.801758	0.822241	3.292919	0.000000e+00
928	4.114917	0.776860	3.110789	0.000000e+00	960	3.729400	0.948931	3.799679	0.000000e+00
1056	3.490060	0.674821	2.701842	0.000000e+00	1088	3.171034	0.812299	3.252182	0.000000e+00
1184	4.704740	0.705585	2.824725	0.000000e+00	1216	3.842152	0.935958	3.746909	0.000000e+00
1312	5.759189	0.784278	3.139504	0.000000e+00	1344	5.502265	0.882442	3.532395	0.000000e+00
1440	7.601570	0.785623	3.144674	0.000000e+00	1472	6.854012	0.930698	3.725319	0.000000e+00
1568	9.867642	0.781366	3.127459	0.000000e+00	1600	9.145759	0.895716	3.585102	0.000000e+00
1696	12.866893	0.758287	3.034938	0.000000e+00	1728	10.893742	0.947293	3.791363	0.000000e+00
1824	15.182666	0.799386	3.199296	0.000000e+00	1856	14.093171	0.907309	3.631191	0.000000e+00
1952	18.204809	0.817115	3.270135	0.000000e+00	1984	16.359457	0.954742	3.820894	0.000000e+00

BLOCK_SIZE=128					BLOCK_SIZE=256				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
128	20.377484	0.098181	0.395792	0.000000e+00	256	8.507532	0.236645	0.950278	0.000000e+00
256	11.607644	0.173443	0.696483	0.000000e+00	512	4.411258	0.486819	1.951079	0.000000e+00
384	5.489758	0.371315	1.489130	0.000000e+00	768	3.733971	0.727887	2.915339	0.000000e+00
512	4.799280	0.447460	1.793334	0.000000e+00	1024	3.090033	0.694971	2.782599	0.000000e+00
640	6.254335	0.335312	1.343343	0.000000e+00	1280	8.614641	0.486881	1.949045	0.000000e+00
768	8.382410	0.324240	1.298647	0.000000e+00	1536	15.858389	0.457030	1.829310	0.000000e+00
896	5.719933	0.503029	2.014362	0.000000e+00	1792	17.697162	0.650340	2.602811	0.000000e+00
1024	3.918727	0.548005	2.194162	0.000000e+00					
1152	12.107239	0.252547	1.011065	0.000000e+00					
1280	9.249480	0.453464	1.815272	0.000000e+00					
1408	15.606056	0.357721	1.431901	0.000000e+00					
1536	16.153178	0.448689	1.795925	0.000000e+00					
1664	18.685557	0.493156	1.973808	0.000000e+00					
1792	27.219417	0.422829	1.692261	0.000000e+00					
1920	18.086576	0.782668	3.132301	0.000000e+00					

Question – 3: Finding OpenMP bugs: The fixed and commented codes are shared in the repo.

Question – 4: OpenMP version of 2D Jacobi/Gauss-Seidel smoothing

The variation of runtimes and the Speedup achieved with various N and no. of threads for **Jacobi** is shown below. The performance and speedup decrease with increase in N and the number of threads.

N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
3	1	0.006000	1.000000	23	1	0.015000	1.000000
3	3	0.034000	29.411834	23	3	0.037000	27.026896
3	5	0.042000	23.809492	23	5	0.044000	22.727319
3	7	0.047000	21.276519	23	7	0.048000	20.833291
3	9	0.050000	20.000019	23	9	0.055000	18.181796
3	11	0.056000	17.857145	23	11	0.061000	16.393413
3	13	0.065000	15.384658	23	13	0.068000	14.705865
3	15	0.071000	14.084535	23	15	0.073000	13.698594
3	17	0.065000	15.384602	23	17	0.078000	12.820502
3	19	0.084000	11.904746	23	19	0.082000	12.195119
3	21	0.083000	12.048201	23	21	0.090000	11.111092
3	23	0.092000	10.869564	23	23	0.096000	10.416671
3	25	0.093000	10.752696	23	25	0.101000	9.901007
3	27	0.102000	9.803922	23	27	0.110000	9.090918
3	29	0.109000	9.174315	23	29	0.100000	9.999986
3	31	0.121000	8.264459	23	31	0.121000	8.264459
3	33	0.120000	8.333341	23	33	0.127000	7.874025
3	35	0.128000	7.812513	23	35	0.132000	7.575759
3	37	0.141000	7.092209	23	37	0.128000	7.812499
3	39	0.135000	7.407408	23	39	0.154000	6.493505
3	41	0.152000	6.578939	23	41	0.146000	6.849308
3	43	0.145000	6.896553	23	43	0.160000	6.249997
3	45	0.153000	6.535943	23	45	0.164000	6.097569
3	47	0.170000	5.882350	23	47	0.156000	6.410261
3	49	0.167000	5.988022	23	49	0.163000	6.134974
3	51	0.172000	5.813948	23	51	0.170000	5.882350
3	53	0.188000	5.319143	23	53	0.176000	5.681814
3	55	0.183000	5.464478	23	55	0.188000	5.319150
3	57	0.186000	5.376341	23	57	0.194000	5.154639
3	59	0.200000	5.000005	23	59	0.196000	5.102038

N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
43	1	0.032000	1.000000	63	1	0.060000	1.000000
43	3	0.040000	25.000024	63	3	0.051000	19.607798
43	5	0.047000	21.276627	63	5	0.047000	21.276627
43	7	0.051000	19.607889	63	7	0.056000	17.857145
43	9	0.059000	16.949148	63	9	0.061000	16.393413
43	11	0.062000	16.129023	63	11	0.066000	15.151519
43	13	0.067000	14.925393	63	13	0.070000	14.285728
43	15	0.075000	13.333325	63	15	0.076000	13.157899
43	17	0.080000	12.500012	63	17	0.083000	12.048201
43	19	0.086000	11.627912	63	19	0.088000	11.363629
43	21	0.093000	10.752668	63	21	0.098000	10.204076
43	23	0.100000	10.000010	63	23	0.099000	10.101012
43	25	0.104000	9.615398	63	25	0.098000	10.204076
43	27	0.109000	9.174315	63	27	0.107000	9.345785
43	29	0.107000	9.345785	63	29	0.117000	8.547018
43	31	0.121000	8.264459	63	31	0.124000	8.064511
43	33	0.130000	7.692301	63	33	0.120000	8.333325
43	35	0.126000	7.936497	63	35	0.139000	7.194235
43	37	0.145000	6.896553	63	37	0.146000	6.849319
43	39	0.136000	7.352946	63	39	0.149000	6.711413
43	41	0.141000	7.092197	63	41	0.152000	6.578950
43	43	0.152000	6.578950	63	43	0.162000	6.172842
43	45	0.157000	6.369424	63	45	0.167000	5.988022
43	47	0.160000	6.250006	63	47	0.160000	6.250006
43	49	0.175000	5.714287	63	49	0.183000	5.464485
43	51	0.170000	5.882350	63	51	0.192000	5.208336
43	53	0.185000	5.405407	63	53	0.199000	5.025129
43	55	0.191000	5.235596	63	55	0.193000	5.181351
43	57	0.187000	5.347593	63	57	0.187000	5.347593
43	59	0.193000	5.181345	63	59	0.206000	4.854367

N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
83	1	0.099000	1.000000	103	1	0.155000	1.000000
83	3	0.064000	15.624968	103	3	0.090000	11.111122
83	5	0.053000	18.867934	103	5	0.079000	12.658228
83	7	0.064000	15.625027	103	7	0.075000	13.333325
83	9	0.066000	15.151519	103	9	0.077000	12.987029
83	11	0.069000	14.492752	103	11	0.081000	12.345702
83	13	0.074000	13.513535	103	13	0.080000	12.500012
83	15	0.095000	10.526313	103	15	0.082000	12.195119
83	17	0.083000	12.048166	103	17	0.089000	11.235957
83	19	0.089000	11.235957	103	19	0.095000	10.526313
83	21	0.097000	10.309265	103	21	0.100000	9.999986
83	23	0.103000	9.708745	103	23	0.103000	9.708745
83	25	0.109000	9.174315	103	25	0.099000	10.101012
83	27	0.099000	10.101012	103	27	0.117000	8.547018
83	29	0.116000	8.620677	103	29	0.123000	8.130087
83	31	0.128000	7.812499	103	31	0.125000	8.000000
83	33	0.117000	8.547001	103	33	0.123000	8.130072
83	35	0.132000	7.575759	103	35	0.138000	7.246376
83	37	0.128000	7.812513	103	37	0.142000	7.042244
83	39	0.138000	7.246376	103	39	0.143000	6.993012
83	41	0.161000	6.211180	103	41	0.152000	6.578950
83	43	0.158000	6.329114	103	43	0.154000	6.493505
83	45	0.158000	6.329114	103	45	0.166000	6.024092
83	47	0.170000	5.882350	103	47	0.177000	5.649716
83	49	0.179000	5.586588	103	49	0.179000	5.586595
83	51	0.184000	5.434782	103	51	0.193000	5.181345
83	53	0.178000	5.617978	103	53	0.187000	5.347593
83	55	0.188000	5.319150	103	55	0.200000	4.999999
83	57	0.190000	5.263156	103	57	0.199000	5.025123
83	59	0.199000	5.025123	103	59	0.210000	4.761904
N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
123	1	0.206000	1.000000	143	1	0.284000	1.000000
123	3	0.089000	11.235927	143	3	0.123000	8.130087
123	5	0.081000	12.345665	143	5	0.091000	10.989030
123	7	0.087000	11.494267	143	7	0.099000	10.101012
123	9	0.084000	11.904746	143	9	0.094000	10.638313
123	11	0.086000	11.627912	143	11	0.098000	10.204076
123	13	0.090000	11.111122	143	13	0.108000	9.259256
123	15	0.096000	10.416671	143	15	0.100000	9.999986
123	17	0.099000	10.101012	143	17	0.108000	9.259256
123	19	0.105000	9.523808	143	19	0.113000	8.849564
123	21	0.120000	8.333325	143	21	0.115000	8.695651
123	23	0.111000	9.009004	143	23	0.113000	8.849545
123	25	0.108000	9.259276	143	25	0.123000	8.130072
123	27	0.116000	8.620677	143	27	0.145000	6.896553
123	29	0.127000	7.874010	143	29	0.146000	6.849308
123	31	0.125000	8.000000	143	31	0.151000	6.622516
123	33	0.138000	7.246376	143	33	0.145000	6.896553
123	35	0.135000	7.407408	143	35	0.142000	7.042256
123	37	0.154000	6.493505	143	37	0.142000	7.042256
123	39	0.149000	6.711413	143	39	0.152000	6.578950
123	41	0.159000	6.289311	143	41	0.161000	6.211180
123	43	0.158000	6.329114	143	43	0.161000	6.211180
123	45	0.171000	5.847953	143	45	0.167000	5.988022
123	47	0.180000	5.555561	143	47	0.169000	5.917155
123	49	0.177000	5.649716	143	49	0.184000	5.434782
123	51	0.186000	5.376348	143	51	0.191000	5.235603
123	53	0.193000	5.181345	143	53	0.203000	4.926112
123	55	0.193000	5.181345	143	55	0.212000	4.716984
123	57	0.211000	4.739337	143	57	0.219000	4.566213
123	59	0.211000	4.739337	143	59	0.209000	4.784686

The variation of runtimes and the Speedup achieved with various N and no. of threads for **Gauss-Siedel** is shown below. The performance and speedup decrease with increase in N and the number of threads.

N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
3	1	0.006000	1.000000	23	1	0.015000	1.000000
3	3	0.036000	27.777767	23	3	0.036000	27.777767
3	5	0.052000	19.230752	23	5	0.041000	24.390310
3	7	0.044000	22.727319	23	7	0.052000	19.230752
3	9	0.051000	19.607889	23	9	0.057000	17.543884
3	11	0.058000	17.241424	23	11	0.063000	15.873025
3	13	0.064000	15.625027	23	13	0.071000	14.084535
3	15	0.068000	14.705865	23	15	0.078000	12.820541
3	17	0.077000	12.987029	23	17	0.083000	12.048201
3	19	0.083000	12.048166	23	19	0.091000	10.989001
3	21	0.090000	11.111122	23	21	0.098000	10.204101
3	23	0.091000	10.989030	23	23	0.101000	9.900983
3	25	0.093000	10.752696	23	25	0.106000	9.433967
3	27	0.103000	9.708722	23	27	0.110000	9.090898
3	29	0.110000	9.090898	23	29	0.111000	9.009004
3	31	0.122000	8.196723	23	31	0.109000	9.174315
3	33	0.123000	8.130087	23	33	0.120000	8.333341
3	35	0.125000	8.000000	23	35	0.123000	8.130072
3	37	0.139000	7.194248	23	37	0.137000	7.299266
3	39	0.135000	7.407408	23	39	0.130000	7.692315
3	41	0.137000	7.299278	23	41	0.151000	6.622516
3	43	0.150000	6.666662	23	43	0.147000	6.802729
3	45	0.164000	6.097560	23	45	0.153000	6.535943
3	47	0.160000	6.249997	23	47	0.166000	6.024092
3	49	0.170000	5.882359	23	49	0.163000	6.134965
3	51	0.184000	5.434789	23	51	0.167000	5.988022
3	53	0.186000	5.376348	23	53	0.182000	5.494508
3	55	0.184000	5.434782	23	55	0.195000	5.128201
3	57	0.199000	5.025123	23	57	0.199000	5.025123
3	59	0.199000	5.025129	23	59	0.196000	5.102044

N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
43	1	0.038000	1.000000	63	1	0.079000	1.000000
43	3	0.047000	21.276519	63	3	0.067000	14.925393
43	5	0.048000	20.833291	63	5	0.068000	14.705917
43	7	0.058000	17.241353	63	7	0.075000	13.333325
43	9	0.063000	15.873025	63	9	0.072000	13.888884
43	11	0.068000	14.705865	63	11	0.078000	12.820502
43	13	0.075000	13.333325	63	13	0.085000	11.764701
43	15	0.080000	12.500012	63	15	0.090000	11.111122
43	17	0.087000	11.494267	63	17	0.095000	10.526339
43	19	0.094000	10.638313	63	19	0.101000	9.901007
43	21	0.098000	10.204076	63	21	0.108000	9.259256
43	23	0.099000	10.101012	63	23	0.113000	8.849545
43	25	0.111000	9.009004	63	25	0.118000	8.474574
43	27	0.113000	8.849564	63	27	0.126000	7.936497
43	29	0.113000	8.849545	63	29	0.129000	7.751941
43	31	0.124000	8.064527	63	31	0.135000	7.407408
43	33	0.127000	7.874010	63	33	0.142000	7.042256
43	35	0.141000	7.092197	63	35	0.148000	6.756757
43	37	0.132000	7.575759	63	37	0.150000	6.666673
43	39	0.152000	6.578950	63	39	0.145000	6.896553
43	41	0.147000	6.802729	63	41	0.151000	6.622526
43	43	0.152000	6.578950	63	43	0.160000	6.250006
43	45	0.168000	5.952373	63	45	0.177000	5.649716
43	47	0.173000	5.780344	63	47	0.182000	5.494508
43	49	0.182000	5.494508	63	49	0.181000	5.524862
43	51	0.185000	5.405407	63	51	0.188000	5.319143
43	53	0.192000	5.208329	63	53	0.193000	5.181351
43	55	0.198000	5.050506	63	55	0.204000	4.901961
43	57	0.203000	4.926107	63	57	0.200000	4.999999
43	59	0.210000	4.761904	63	59	0.206000	4.854367

N	#Threads	Time elapsed(s)	Speedup	N	#Threads	Time elapsed(s)	Speedup
83	1	0.125000	1.000000	103	1	0.191000	1.000000
83	3	0.086000	11.627912	103	3	0.114000	8.771924
83	5	0.079000	12.658228	103	5	0.094000	10.638313
83	7	0.094000	10.638286	103	7	0.123000	8.130072
83	9	0.094000	10.638286	103	9	0.130000	7.692301
83	11	0.093000	10.752696	103	11	0.112000	8.928572
83	13	0.100000	10.000010	103	13	0.109000	9.174295
83	15	0.104000	9.615376	103	15	0.121000	8.264476
83	17	0.103000	9.708745	103	17	0.114000	8.771924
83	19	0.111000	9.009023	103	19	0.134000	7.462683
83	21	0.112000	8.928572	103	21	0.130000	7.692315
83	23	0.109000	9.174315	103	23	0.141000	7.092197
83	25	0.117000	8.547018	103	25	0.133000	7.518789
83	27	0.122000	8.196723	103	27	0.136000	7.352946
83	29	0.133000	7.518803	103	29	0.153000	6.535953
83	31	0.140000	7.142852	103	31	0.144000	6.944453
83	33	0.143000	6.993001	103	33	0.167000	5.988022
83	35	0.155000	6.451614	103	35	0.164000	6.097560
83	37	0.160000	6.250006	103	37	0.175000	5.714287
83	39	0.161000	6.211180	103	39	0.179000	5.586595
83	41	0.171000	5.847953	103	41	0.178000	5.617978
83	43	0.180000	5.555561	103	43	0.193000	5.181345
83	45	0.183000	5.464478	103	45	0.197000	5.076141
83	47	0.177000	5.649716	103	47	0.193000	5.181345
83	49	0.193000	5.181345	103	49	0.208000	4.807688
83	51	0.192000	5.208329	103	51	0.217000	4.608295
83	53	0.198000	5.050506	103	53	0.223000	4.484304
83	55	0.205000	4.878051	103	55	0.227000	4.405286
83	57	0.221000	4.524888	103	57	0.230000	4.347826
83	59	0.248000	4.032260	103	59	0.239000	4.184099

The processor used for the above runs:

```

System Type:                x64-based PC
Processor(s):                1 Processor(s) Installed.
                             [01]: Intel64 Family 6 Model 165 Stepping 2 GenuineIntel ~2592 Mhz
BIOS Version:                Dell Inc. 1.12.0, 12/09/2021
Windows Directory:           C:\WINDOWS
System Directory:             C:\WINDOWS\system32
Boot Device:                  \Device\HarddiskVolume1
System Locale:                 en-us;English (United States)
Input Locale:                 00004009
Time Zone:                    (UTC-05:00) Eastern Time (US & Canada)
Total Physical Memory:        7,986 MB
Available Physical Memory:    1,237 MB
Virtual Memory: Max Size:     32,562 MB
Virtual Memory: Available:    16,155 MB
Virtual Memory: In Use:       16,407 MB

```