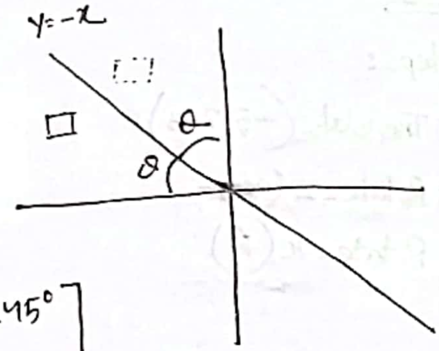
Lecture-4

Integer

2(b)



Transformation matrix for reflection along $y=-x$:

$m = -1$

$\tan\theta = -1$

$\therefore \theta = \tan^{-1}(-1)$

$= -45^\circ$

$M_1 = \text{Rotate}(45^\circ) * \text{Reflect\_Y} * \text{Rotate}(-45^\circ)$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

=

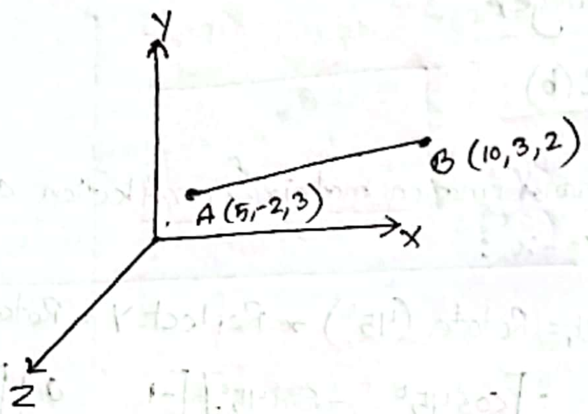
$M_2 = \text{Rotation}(90^\circ) * \text{Reflect\_Y}$

$$= \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

=

$M_1 = M_2$

3(b)



Steps:

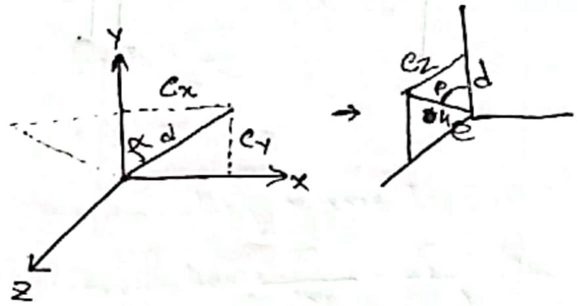
① Translate $(-5,2,-3)$

② Rotate_z (α)

③ Rotate_x (β)

Translate $(-5,2,-3)$:

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$U_e = \frac{B-A}{|B-A|} = e_x, e_y, e_z$$

Now,

$$e_x = \frac{10-5}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$e_y = \frac{3+2}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$e_z = \frac{2-3}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{-1}{\sqrt{51}}$$

$$d = \sqrt{(e_x)^2+(e_y)^2} = \frac{5\sqrt{102}}{51}$$

$$\cos\beta = \frac{d}{u_e} = \frac{5\sqrt{102}}{51}$$

$$\sin\beta = \frac{e_z}{u_e} = \frac{-1}{\sqrt{51}}$$

$$\cos\alpha = \frac{e_y}{d} = \frac{5/\sqrt{51}}{5\sqrt{102}/51} = \frac{1}{\sqrt{2}} \quad ; \quad \sin\alpha = \frac{e_x}{d} = \frac{5/\sqrt{51}}{5\sqrt{102}/51} = \frac{1}{\sqrt{2}}$$

composite transformation matrix:

$$A'B' = \text{Translate}^{-1}(5,-2,3) * \text{Rotate\_z}(\alpha) * \text{Rotate\_x}(-\beta) * AB$$

$$= \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 10 \\ -2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Composite transformation matrix for point A'B':

$$A'B' = \text{Rotate}_{-x}(-\beta) * \text{Rotate}_{-z}(\alpha) * \text{Translate}^{-1}(5,-2,3) * AB$$

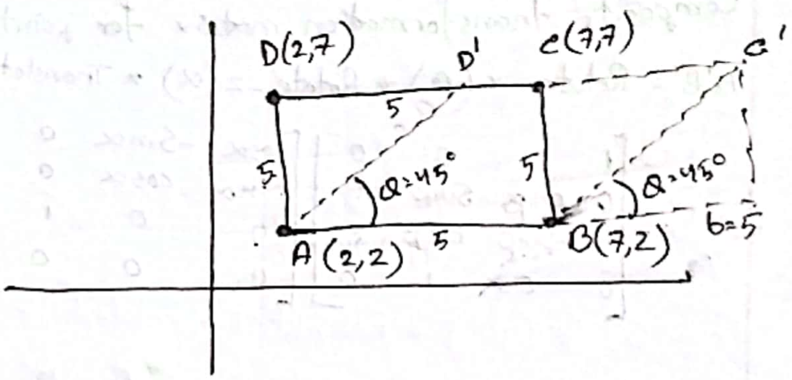
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 10 \\ -2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

=

3(d)

Steps:

1. Translate (-2,-2)
2. Shear x (1)
3. Translate (2,2)



Composite transformation matrix:

$\theta = 45^\circ$

M_1 = Translate (2,2) * Shear x (1) * Translate (-2,-2)

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$\tan\theta = \frac{5}{b}$

$\tan 45^\circ = \frac{5}{b}$

$\therefore b = 5$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Shearing along x-axis by 5 points.

Shear factor: $\frac{5}{\Delta y} = \frac{5}{(7-2)} = \frac{5}{5} = 1$

$$= \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A'B'C'D' = $m_1 \times V$

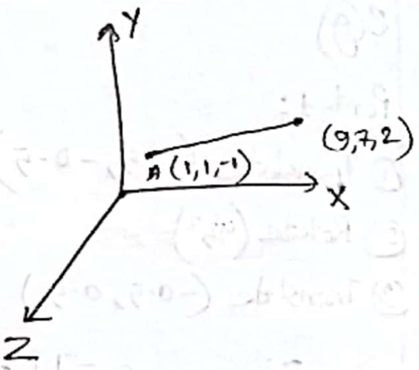
$$= \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 7 & 7 & 2 \\ 2 & 2 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 7 & 12 & 7 \\ 2 & 2 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

A'B'C'D' = (2,2) (7,2) (12,7) (7,7)

Decipher

3(c)



Steps:

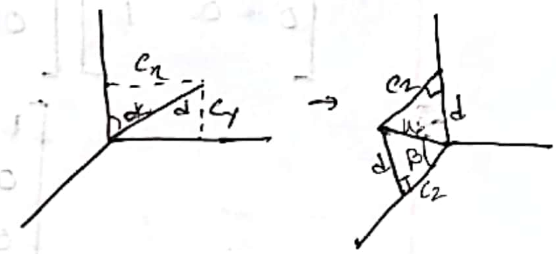
① Translate $(-1,-1,1)$

② Rotate_z (α) (anticlock)

③ Rotate-x (β) (anticlock)

Translate $(-1,-1,1)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$U_e = \frac{B-A}{|B-A|} = c_x, c_y, c_z$$

$$c_x = \frac{9-1}{\sqrt{(9-1)^2+(7-1)^2+(2+1)^2}} = \frac{8}{\sqrt{109}}$$

$$c_y = \frac{6}{\sqrt{109}} \quad ; \quad c_z = \frac{3}{\sqrt{109}} \quad ; \quad d = \sqrt{c_x^2 + c_y^2} = 0.9578$$

$$\sin\alpha = \frac{c_x}{d} = \left(\frac{8}{\sqrt{109}}\right) / 0.9578 = 0.80002$$

$$\cos\alpha = \frac{c_y}{d} = 0.80002$$

$$\cos\beta = \frac{c_z}{u_e} = \frac{3}{\sqrt{109}}$$

$$\sin\beta = \frac{d}{u_e} = 0.9578$$

$$A'B' = \text{Rotate-x}(\beta) * \text{Rotate\_z}(\alpha) * \text{Translate}(-1,-1,1) * AB$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 9 \\ 1 & 7 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

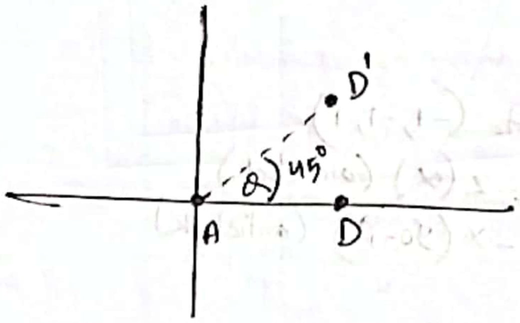
3(g)

Part-1:

① Translate $(0.5, -0.5)$

② Rotate $(45°)$

③ Translate $(-0.5, 0.5)$



$$AD'' = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \\ 1 & 1 \end{bmatrix}$$

For lid AD:

$\theta = 45°$ (anticlock)

=

Part-2:

① Tranlate $(-0.5, 0.5)$ (c point)

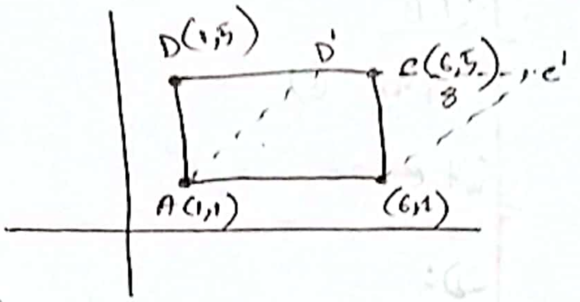
② Rotate$^{-1}(90)$

③ Translate $(0.5, -0.5)$

$$A'B'C'D'D'' = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90° & \sin 90° & 0 \\ -\sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 & D'' \\ 0.5 & -0.5 & -0.5 & 0.5 & D'' \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Quibits Quiz-2

Set-A



ⓐ
① Translate $(-1,-1)$
② Shear-x (2)
③ Translate $(1,1)$

Shear along x-axis by 8 units

$$\text{Shear factor} = \frac{8}{\Delta y} = \frac{8}{5-1} = \frac{8}{4} = 2$$

ⓑ Composite transformation matrix,

$$M_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ⓒ $A'B'C'D' = M_1 * V$

$$= \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

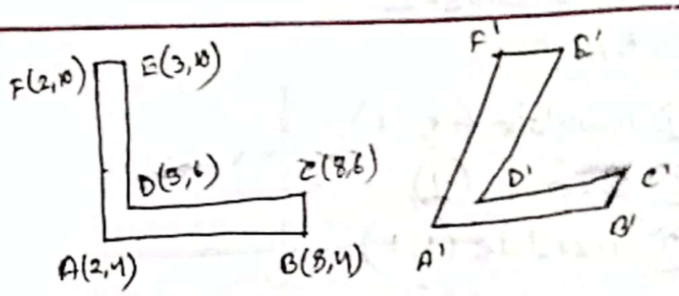
$$= \begin{bmatrix} 1 & 6 & 14 & 9 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(Ans)

Set-B

Decipher 8(c)

Set-e



CD:

Shearing along x-axis by 3 units

$\therefore$ Shear factor: $\frac{3}{6-4} = 1.5$

Steps:

① Translate (-2,4)

② Shear-x (1.5)

③ Translate (2,4)

$$A'B'C'D' = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 8 & 8 & 3 \\ 4 & 4 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

=

FE:

Shearing along x-axis by 3 units

$\therefore$ Shear factor = $\frac{3}{6} = 0.5$

Steps:

① Translate (-2,-4)

② Shear-x (0.5)

③ Translate (2,4)

$$A'D'E'F' = \text{Translate}(2,4) * \text{Shear-x}(0.5) * \text{Translate}(-2,-4) * \begin{bmatrix} 2 & 3 & 3 & 2 \\ 4 & 6 & 10 & 10 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Set-D

① $R(\theta_1)$ and $R^{-1}(\theta_2)$

$$m_1 = R(90^\circ) * R^{-1}(90^\circ)$$

$$= \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} \cos 90^\circ & \sin 90^\circ \\ -\sin 90^\circ & \cos 90^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$m_2 = R(90-90) = R(0)$$

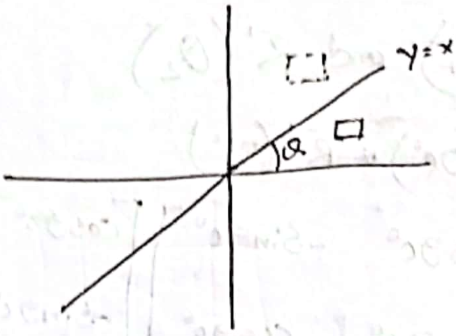
$$= \begin{bmatrix} \cos 0^\circ & -\sin 0^\circ \\ \sin 0^\circ & \cos 0^\circ \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore m_1 = m_2$

② Same integer (3b)

Set-E

① ① Rotate (45°)
② Reflect_y
③ Rotate^{-1} (45°)



$m = 1$
$\tan Q = 1$
$Q = 45^\circ$

$m_1 = \text{Rotate}(-45^\circ) * \text{Reflect\_y} * \text{Rotate}(45^\circ)$

$$= \begin{vmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{vmatrix} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{vmatrix}$$

$$= \begin{vmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{vmatrix} \begin{vmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

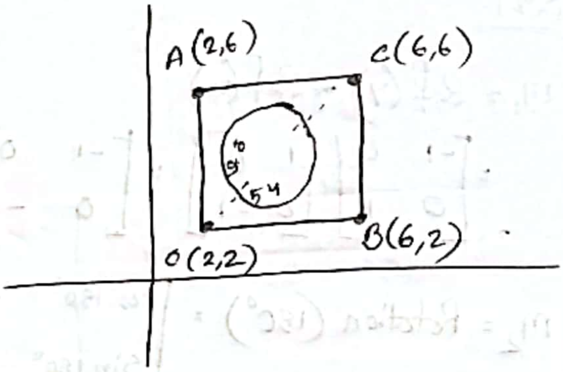
~~m_2 = Rotation (90°) * Reflection_y~~

$m_2 = \text{Reflection\_y} * \text{Rotation}(90^\circ)$

$$= \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$m_1 = m_2$.

2



Steps:

① Translate $(-2,-2)$

② Rotate (90°)

③ Scale $(1, 1.5+1)$

④ Rotate (-90°)

⑤ Translate $(2,2)$

Scale factor: 150% = 1.5

$$O'A'C'B' = \text{Translate}(2,2) * \text{Rotate}(-90^\circ) * \text{Scale}(1, 2.5) * \text{Rotate}(90^\circ) * \text{Translate}(-2,-2) * \begin{bmatrix} 2 & 2 & 6 & 6 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$=$

Set-F

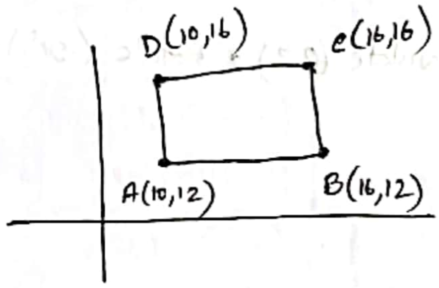
① $m_1 = Ref(y) * Ref(x)$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$m_2 = \text{Rotation } (180^\circ) = \begin{bmatrix} \cos 180^\circ & -\sin 180^\circ \\ \sin 180^\circ & \cos 180^\circ \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\therefore m_1 = m_2$

② Given line,



$2y - 6x + 2 = 0$

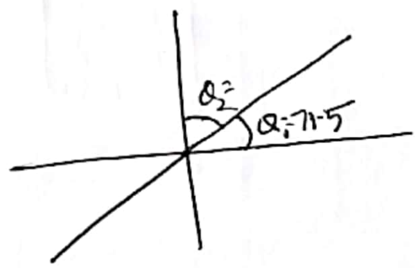
$y = 3x - \cancel{2}\,1$ ---①

$\therefore m = 3$

in ①; the line is 1 unit below y-axis.

Translating to the centre.

Translate (0,1)



$Q_1 = \tan^{-1}(3)$

$\therefore Q_1 = 71.5$

$Q_2 = 90 - 71.5 = 18.5^\circ$

Steps:

① Translate (0,1)

② Rotate (18.5°)

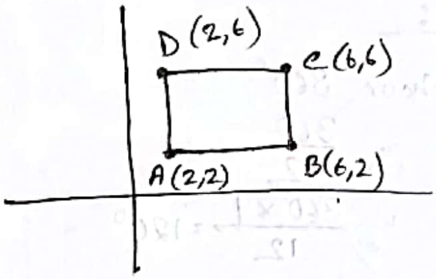
③ Reflect_y

④ Rotate (-18.5°)

⑤ Translate (0,-1)

Excel Previous Quiz | Set-B

2



For Scaling factor;

AD and BC edges are increased by 3unit.

upward scaling. So, $\Delta y = 6-2 = 4$

Total scaling $(4+3) = 7$ unit

$\therefore$ Scaling factor $= \frac{7}{4} = 1.75$

For Shear factor;

upward shearing. So, $\Delta x = |6-2| = 4$

$\therefore$ Shear factor $= \frac{6}{4} = 1.5$

Steps:

1. Translate $(-2,-2)$
2. Scale $(1, 1.75)$
3. Shear -y (1.5)
4. ~~Scale~~ Translate $(2,2)$.

Excel Previous Quiz/Set-D

② For OA:

12 hour 360°

1 " $\frac{360}{12}$

$\therefore$ 4 " $\frac{360 \times 4}{12} = 120°$

Steps:

① Translate (-8,-8)

② Rotate (-120°)

③ Translate (8,8)

$O'A' = m_1 * \begin{bmatrix} 8 & 8 \\ 8 & 14 \\ 1 & 1 \end{bmatrix}$

For OB:

60 minute 360°

1 " $\frac{360°}{60°}$

30 " $\frac{\cancel{360}^{180} \times \cancel{30}}{\cancel{60}_{2}} = 180°$

① Translate (-8,-8)

② Rotate (-180°)

③ Translate (8,8)

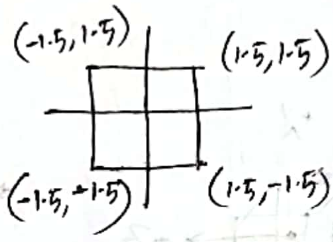
$O'B' = m_2 * \begin{bmatrix} 8 & 16 \\ 8 & 8 \\ 1 & 1 \end{bmatrix}$

Lecture-5

Quiz-2 Set-A

(1)

Steps:



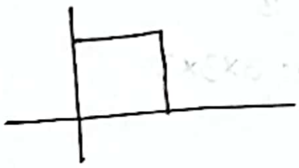
cannonical

(1) Translate $(1.5, 1.5)$

(2) Scaling $\left(\frac{nx}{3}, \frac{ny}{3}\right)$

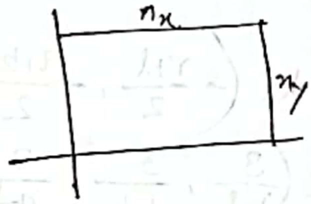
(3) Translate $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(1)



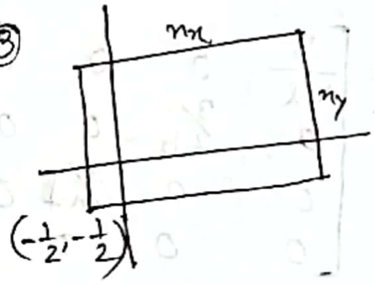
$$M_1 = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} nx/3 & 0 & 0 \\ 0 & ny/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & 1.5 \\ 0 & 0 & 1 \end{bmatrix}$$

(2)



$$= \begin{bmatrix} nx/3 & 0 & -\frac{1}{2} \\ 0 & ny/3 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & 1.5 \\ 0 & 0 & 1 \end{bmatrix}$$

(3)



; Screen / viewport

$$= \begin{bmatrix} nx/3 & 0 & \frac{nx-1}{2} \\ 0 & ny/3 & \frac{ny-1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

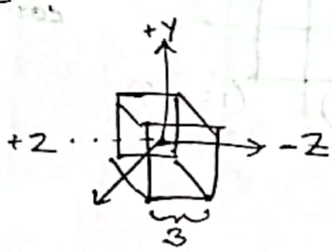
$$\begin{bmatrix} x_{screen} \\ y_{screen} \\ 1 \end{bmatrix} = M_1 * \begin{bmatrix} x_{cannonical} \\ y_{cannonical} \\ 1 \end{bmatrix}$$

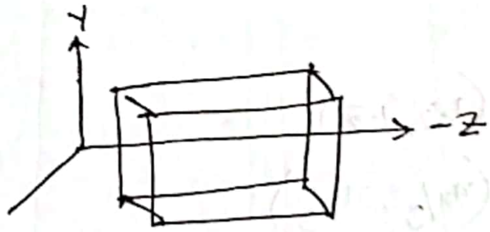
$$\frac{nx}{\cancel{3}} \times \frac{\cancel{3}}{2} - \frac{1}{2}$$

$$= \frac{n_x - 1}{2}$$

Set-e

①





Shape: 3×3×3

Shape: $|r-l| \times |t-b| \times |n-f|$

Steps:

① Translate $\left(-\frac{r+l}{2}, -\frac{t+b}{2}, -\frac{n+f}{2}\right)$

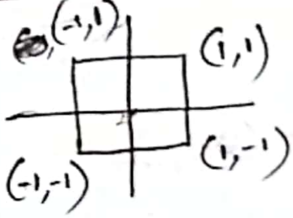
② Scale $\left(\frac{3}{r-l}, \frac{3}{t-b}, \frac{3}{n-f}\right)$

$$M_{orth} = \begin{bmatrix} \frac{3}{r-l} & 0 & 0 & 0 \\ 0 & \frac{3}{t-b} & 0 & 0 \\ 0 & 0 & \frac{3}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{r-l} & 0 & 0 & -\frac{3(r+l)}{2(r-l)} \\ 0 & \frac{3}{t-b} & 0 & -\frac{3(t+b)}{2(t-b)} \\ 0 & 0 & \frac{3}{n-f} & -\frac{3(n+f)}{2(n-f)} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Set-B

1. # Top to Bottom



① Translate$(1,-1)$

② Scaling$(n_x/2, n_y/2)$

③ Translate$(-\frac{1}{2}, \frac{1}{2})$

$$M_1 = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x/2 & 0 & 0 \\ 0 & n_y/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

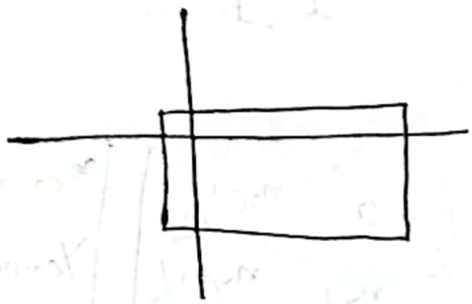
$$= \begin{bmatrix} n_x/2 & 0 & -\frac{1}{2} \\ 0 & n_y/2 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x/2 & 0 & \frac{n_x-1}{2} \\ 0 & n_y/2 & \frac{-n_y+1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $T(1,-1)$:

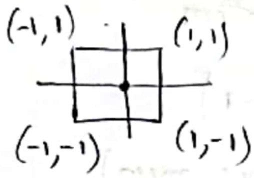
Apply $S(n_x/2, n_y/2)$:

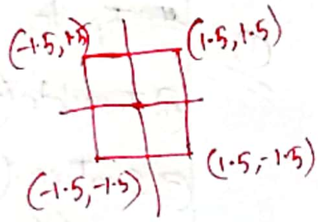
Apply $T(-\frac{1}{2}, \frac{1}{2})$



Viewport transformation

Bottom to top





① Translate $(1,1)$ $T(1.5, 1.5)$

② Scaling $(n_x/2, n_y/2)$ $(n_x/3, n_y/3)$

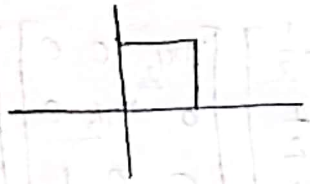
③ Translate $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

$$M_1 = \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x/2 & 0 & 0 \\ 0 & n_y/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x/2 & 0 & -\frac{1}{2} \\ 0 & n_y/2 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

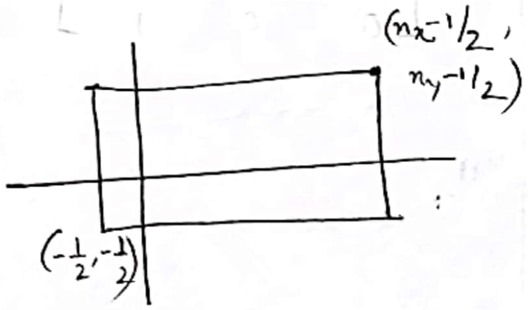
$$= \begin{bmatrix} n_x/2 & 0 & \frac{n_x-1}{2} \\ 0 & n_y/2 & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $T(1,1)$:

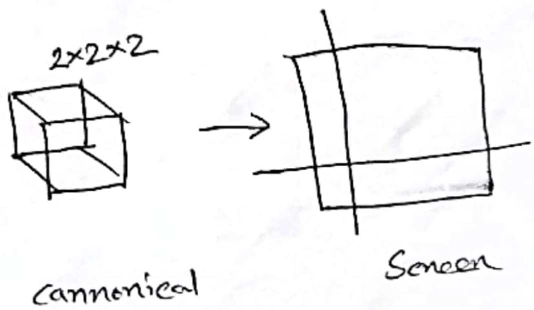


Apply $S(n_x/2, n_y/2)$:

Apply $\left(-\frac{1}{2}, -\frac{1}{2}\right)$:

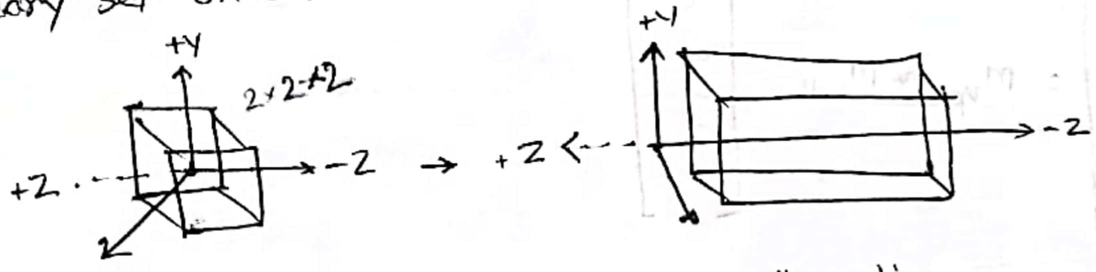


$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} n_x/2 & 0 & \frac{n_x-1}{2} \\ 0 & n_y/2 & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canno} \\ y_{canno} \\ 1 \end{bmatrix}$$



Orthographic Projection Transformation

In cannonical view, the space is limited $[-1,1]$. We can render a geometry in some other ~~or~~ region using orthographic Project, where we can have boundary set on our will.



Shape: $|r-l| \times |t-b| \times |n-f|$

cannonical

orthographic

① Translate $\left(-\frac{r+l}{2}, -\frac{t+b}{2}, -\frac{n+f}{2}\right)$

② Scaling $\left(\frac{2}{r-l}, \frac{2}{t-b}, \frac{2}{n-f}\right)$ → Because cannonical shape is $2\times2\times2$

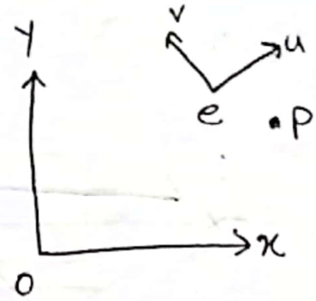
For ~~Scale~~ $3\times3\times3$: Scale $= \left(\frac{3}{r-l}, \frac{3}{t-b}, \frac{3}{n-f}\right)$

t = top plane
b = bottom plane
r = right plane
l = left plane
n = near plane
f = front plane

$$M_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Frame to cannonical transformation can be expressed as rotation followed by a translation.



Steps:

① Rotate (θ) of basis u and v

② Translate $(-x_e, -y_e)$

These steps needs to be followed for the Frame to cannonical transformation.

Basis Matrix

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_e \\ 0 & 1 & y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & x_v & 0 \\ y_u & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

Here,

$p(u_p, v_p) = e + u_p u + v_p v$

- u and v are the basis vectors, e is the origin of frame.

Here,

Rotation involving (u and v) followed by a translation (involving e). So, we can say that . . . . . .

Basis matrix:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{cc} \underline{u} & \underline{v} \\ \end{array}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$=$

For Cannonical to frame:

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

translation followed by a rotation.

Prev year Quiz Set-A

1(a)

Using top to Bottom:

1. Translate $(1,-1)$
2. Scaling $(nx/2, ny/2)$
3. Translate $(-\frac{1}{2}, \frac{1}{2})$

The final matrix will be

$$M_{vp} = \begin{bmatrix} nx/2 & 0 & nx^{-1}/2 \\ 0 & ny/2 & -ny+1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $nx = 256$

$ny = 128$

1(b)

$A = (-3,-4,-3)$; $B(2,4,-6)$

$l=-6$, $r=6$, $b=-7$, $t=7$, $n=-2$, $f=-8$

$$\begin{bmatrix} x_{Pixel} \\ y_{pixel} \\ z_{camo} \\ 1 \end{bmatrix} = M_{vp} * \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -3 & 2 \\ -4 & 4 \\ -3 & -6 \\ 1 & 1 \end{bmatrix}$$

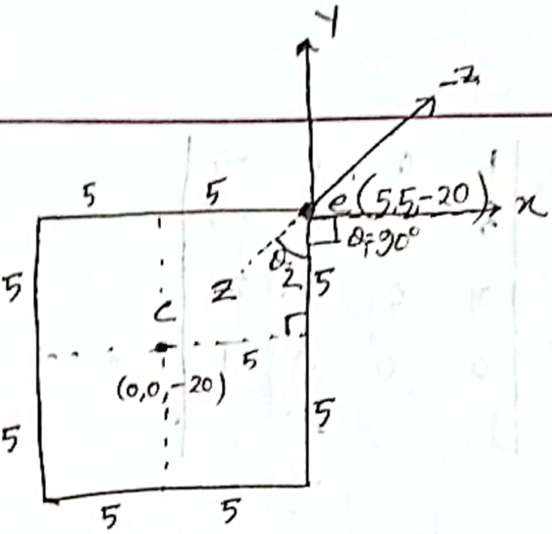
$=$

Quiz-3

(1)

Length each side: 10 units.

$\therefore e = (5,5,-20)$



$\tan\theta_2 = \frac{5}{5}$

$\therefore \theta_2 = 45^\circ$

Initial basis vectors: $U = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $V = \begin{bmatrix}0\\1\\0\end{bmatrix}$ $W = \begin{bmatrix}0\\0\\1\end{bmatrix}$

Rotating the Basis matrix:

Basis matrix = $Rotate_{-z}(-45^\circ) * Rotate_{-x}(-90^\circ) *$ Initial Basis

$$= \begin{bmatrix}\cos45^\circ & \sin45^\circ & 0 & 0\\ -\sin45^\circ & \cos45^\circ & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos90^\circ & \sin90^\circ & 0\\ 0 & -\sin90^\circ & \cos90^\circ & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$= \begin{vmatrix}1/\sqrt{2} & \frac{1}{\sqrt{2}} & 0 & 0\\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}\begin{vmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$$= \begin{vmatrix}1/\sqrt{2} & 0 & 1/\sqrt{2} & 0\\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 1\end{vmatrix} = \begin{vmatrix}1/\sqrt{2} & 0 & 1/\sqrt{2}\\ -1/\sqrt{2} & 0 & 1/\sqrt{2}\\ 0 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}$$

Eye matrix:- $\begin{vmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

Cannonical to frame:

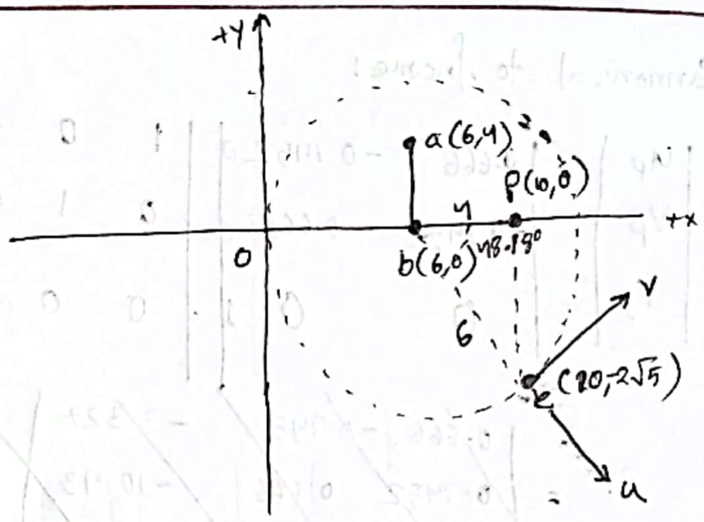
$$\begin{vmatrix} u_p \\ v_p \\ w_p \\ 1 \end{vmatrix} = \begin{vmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ -20 \\ 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ -20 \\ 1 \end{vmatrix}$$

Integer

3(g)

$be = 6$; $bp = 10$; $Pe = \sqrt{6^2 - 4^2}$
$= 2\sqrt{5}$



$\therefore e = (10, -2\sqrt{5})$

$\tan\theta = \dfrac{2\sqrt{5}}{4}$

$\therefore \theta = 48.18^\circ$

Basis vector; $U = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

After rotation;

Basis ~~vector~~ matrix = Rotate(-48.18) * Basis vector

$$= \begin{bmatrix} \cos(48.18) & \sin(48.18) & 0 \\ -\sin(48.18) & \cos(48.18) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

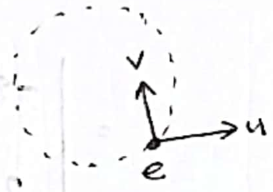
$$= \begin{bmatrix} 0.666 & 0.7452 & 0 \\ -0.7452 & 0.666 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.666 & 0.7452 \\ -0.7452 & 0.666 \\ 1 & 1 \end{bmatrix}$$

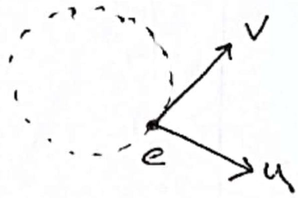
Eye matrix = $\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 2\sqrt{5} \\ 0 & 0 & 1 \end{bmatrix}$

Why clockwise.

① We first translate the frame co-ordinate to eye position normally.



② Now, observe which rotation you will be needing to get the frame of the question.



So, clock-wise rotation.

Cannonical to frame:

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} 0.666 & -0.7452 & 0 \\ 0.7452 & 0.666 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 2\sqrt{5} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 6 \\ 4 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 0.666 & -0.7452 & -3.327 \\ 0.7452 & 0.666 & -10.43 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 6 \\ 4 & 0 \\ 1 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 0.666 & -0.7452 & -9.99 \\ 0.7452 & 0.6666 & -4.47 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 6 \\ 4 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8.97 & -5.994 \\ 2.667 & 0 \end{bmatrix}$$

(Ans.)

Decipher

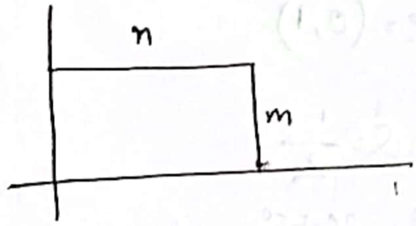
2(a)

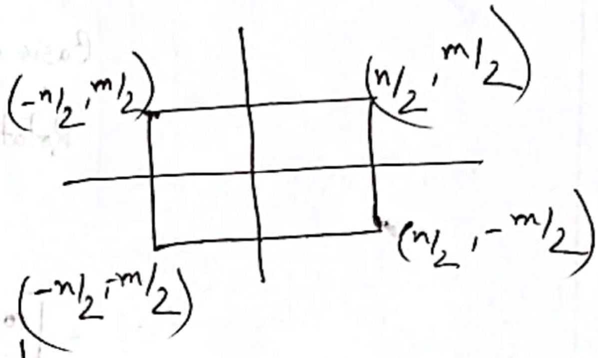
① Translate $(1,1)$

② Scaling $\left(\frac{n}{2}, \frac{m}{2}\right)$

③ Translate $\left(-\frac{n}{2}, -\frac{m}{2}\right)$

After scaling:





$$M_1 = \begin{vmatrix} 1 & 0 & -\frac{n}{2} \\ 0 & 1 & -\frac{m}{2} \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \frac{n}{2} & 0 & 0 \\ 0 & \frac{m}{2} & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n}{2} & 0 & -\frac{n}{2} \\ 0 & \frac{m}{2} & -\frac{m}{2} \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

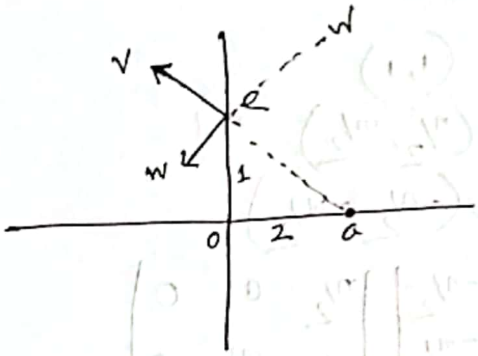
$$= \begin{vmatrix} \frac{n}{2} & 0 & 0 \\ 0 & \frac{m}{2} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Decipher

3(f)

$a = \cancel{(0,2)}\ (-2,0)$

$e = (0,1)$



$\tan\theta = \frac{1}{2}$

$\theta = 26.55^\circ$

Basis matrix;

Rotate (θ) * $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{vmatrix} \cos(26.55^\circ) & -\sin(26.55^\circ) & 0 \\ \sin(26.55^\circ) & \cos(26.55^\circ) & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{vmatrix}$$

P

$$= \begin{vmatrix} 0.8945 & -0.4469 & 0 \\ 0.4469 & 0.8945 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

P

Spring 2023

Quiz-3 (Set-A)

Ans

(1) viewport matrix is (See Decipher 2(a))

$$M_{vp} = \begin{bmatrix} n_x/2 & 0 & 0 & 0 \\ 0 & n_y/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$n_x = 256$

$n_y = 128$

(2) $A = (-2,-3,-4)$ $B(2,4,-6)$

$l = -6$, $r = 6$, $b = -7$, $t = 7$, $n = -2$, $f = -8$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 128 & 0 & 0 & 0 \\ 0 & 64 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{12} & 0 & 0 & -\frac{0}{12} \\ 0 & \frac{2}{14} & 0 & -\frac{0}{14} \\ 0 & 0 & \frac{2}{6} & -\frac{-10}{6} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ -3 & 4 \\ -4 & -6 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 128 & 0 & 0 & 0 \\ 0 & 64 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/6 & 0 & 0 & 0 \\ 0 & 1/7 & 0 & 0 \\ 0 & 0 & 1/3 & 5/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ -3 & 4 \\ -4 & -6 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21.33 \\ \\ \\ \end{bmatrix}$$

$=$