

2

a) Here, positive pulse: 0
negative pulse: 1

Data rate = $2f$ bps
Duration of each pulse = $1/2f$

Case-1:

Let, $f = 10^6$ cycle/sec = 1 MHz

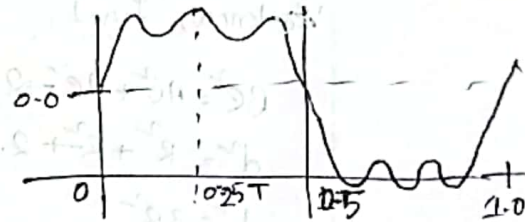
Frequency components = $1f, 3f, 5f$

Bandwidth = $5f - f = 4f = 4 \times 1 \text{ MHz} = 4 \text{ MHz}$

Time period, $T = \frac{1}{10^6} = 10^{-6} = 1 \mu\text{s}$

Duration of each pulse = $\frac{1}{2 \times 10^6}$ [2 because 1 bit occurs at every $0.5 \mu\text{s}$]

Data Rate = $2 \times 1 = 2 \text{ Mbps}$



Case-2: (Frequency increased)

Let, $f = 2 \times 10^6$ cycle/sec = 2 MHz

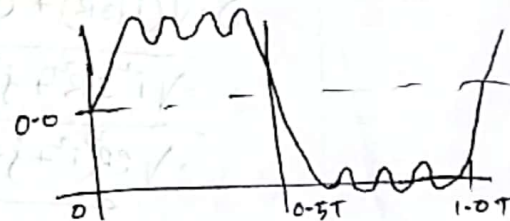
Frequency component = $1f, 3f, 5f$

Bandwidth = $5f - f = 4f = 4 \times 2 = 8 \text{ MHz}$

Time period, $T = \frac{1}{2 \times 10^6} = 0.5 \mu\text{s}$

Duration of each pulse = $\frac{1}{2 \times 2 \times 10^6}$ [4 because 1 bit occurs at every $0.25 \mu\text{s}$]

Data Rate = $2f = 2 \times 2 = 4 \text{ Mbps}$



Case-3: (Frequency component decreased)

Let, $f = 2 \times 10^6$ cycle/sec = 2 MHz

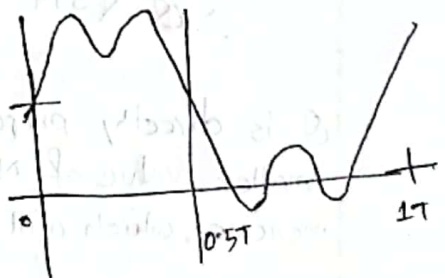
Frequency component = $1f, 3f$

Bandwidth = $3f - f = 2f = 2 \times 2 = 4 \text{ MHz}$

Time period, $T = \frac{1}{f} = \frac{1}{2 \times 10^6} = 0.5 \mu\text{s}$

Duration of each pulse = $\frac{1}{2 \times 2 \times 10^6}$

Data Rate = $2f = 2 \times 2 \times 10^6 = 4 \text{ Mbps}$



From Case 1 and 2:

- Bandwidth increases
- data rate increases
- same signal quality

From 1 and 3:

- Same bandwidth
- data rate decreases
- signal quality increases

From 2 and 3:

- Same data rate
- Bandwidth increase
- higher signal quality.

b)

For same size of cells and same power transmitted from Base station

We know, fig-1

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(120^\circ)$$

$$d^2 = R^2 + R^2 - 2 \cdot R \cdot R \cdot (-\frac{1}{2})$$

$$d^2 = 3R^2$$

$$d = \sqrt{3}R$$

Now, fig-2

$$D = \sqrt{(i\sqrt{3}R)^2 + (j\sqrt{3}R)^2 - 2 \cdot (i\sqrt{3}R) \cdot (j\sqrt{3}R) \cdot \cos(120^\circ)}$$

$$= \sqrt{i^2 3R^2 + j^2 3R^2 - 2 \cdot i\sqrt{3}R \cdot j\sqrt{3}R \cdot (-\frac{1}{2})}$$

$$= \sqrt{3R^2(i^2 + j + ij)}$$

$$= R\sqrt{3N}$$

$$\therefore D = R\sqrt{3N}$$

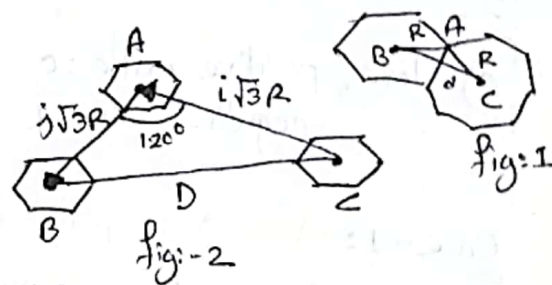
We know,

$$Q = \frac{D}{R} = \frac{R\sqrt{3N}}{R}$$

$$\therefore Q = \sqrt{3N}$$

Q is directly proportional to \sqrt{N} . A smaller value of Q means a smaller value of N , means reuse will be more and the capacity will increase, which will also generate higher co-channel interference.

$$Q \downarrow \quad N \downarrow \quad \text{capacity} \uparrow \quad \text{CCI} \uparrow$$



D = minimum distance between the centre of co-channel cells.

R = Radius of the cell

d = distance between centers of adjacent cells.

N = number of cells, where $N = i^2 + j + j^2$

$Q = \frac{D}{R}$; co-channel reuse ratio

c) Without Sectoring

We know,

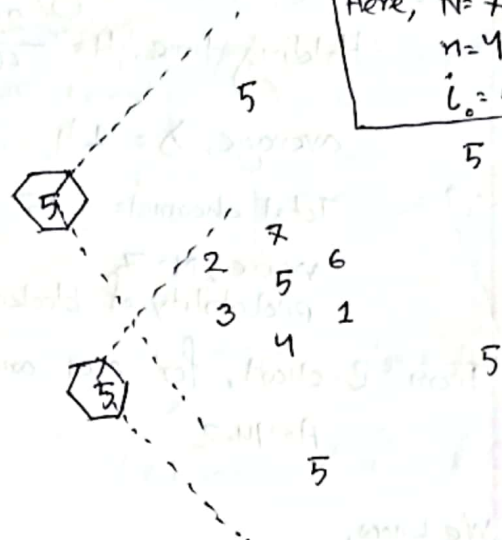
$$SIR = \frac{(\sqrt{3}N)^n}{i_0} = \frac{(\sqrt{3 \times 7})^4}{6}$$

$$= 73.5$$

$$= 10 \log(73.5)$$

$$= 18.66 \text{ dB}$$

Here, $N=7$
 $n=4$
 $i_0=6$ (hexa)
 5



With sectoring

$$SIR = \frac{(\sqrt{3}N)^n}{i_0} = \frac{(\sqrt{3 \times 7})^4}{2}$$

$$= 220.5$$

$$= 10 \log(220.5)$$

$$= 23.41 \text{ dB}$$

For 120° ;

- 3 sectors.
- number of affected clusters $= 2 = i_0$

\therefore SIR increases with sectoring.

For 120° sectoring 3 antennas are used, as SIR is increased, other co-channel cannot affect their signal.

\therefore CCI is low.

Our accepted SIR value is 18.66 dB , so, with sectoring we need to decrease the N to achieve the SIR value.

For $N=4$ ($i=2, j=0$)

$$SIR = \frac{(\sqrt{3 \times 4})^4}{2} = 72 = 18.57 \text{ dB}$$

So, $N \downarrow$

\therefore capacity increases.

\therefore Φ or reuse ratio decreases

Assume, (without sectoring)

$$\text{Holding time, } H = \frac{2 \text{ min}}{60} = \frac{1}{30} \text{ hour}$$

$$\text{average, } \lambda = 1 \text{ h}$$

$$\text{Total channel} = 395$$

$$\therefore \text{channel per cell} = \frac{395}{7} \approx 57$$

$$\text{reuse, } N = 7$$

$$\text{probability of blocking} = 0.01$$

From B-chart, for 0.01 and 57;

$$A = 44.2$$

We know,

$$A = U A_u$$

$$U = \frac{A}{A_u} = \frac{44.2}{0.03333}$$

$$\times 1326$$

$$\text{Here, } A_u = \lambda H$$

$$= 1 \times \frac{1}{30}$$

$$= 0.03333$$

With sectoring

For 120° sector;

3 sectors.

$$\therefore \text{number of channel per sector} = \frac{57}{3} = 19$$

For B-chart for 0.01 and 19;

$$A = 11.2$$

$$\therefore U = \frac{11.2}{0.03333}$$

$$= 336$$

$$\therefore \text{for 3 sectors} = (336 \times 3) = 1008$$

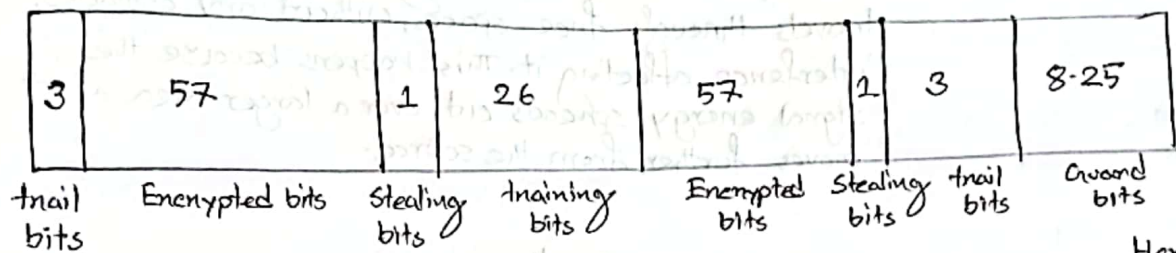
$$\text{Efficiency} = \left(\frac{1326 - 1008}{1326} \right) \times 100 = 24\%$$

\therefore sectoring decreases trunking efficiency.

A_u = Traffic intensity per user
 U = Total no. of user supported per cell
 A = Traffic intensity per cell

d)

GSM Frame structure for transmission



Here 1 slot
8 fields

The system sends = 216.66 frame/sec

Frame duration = $\frac{1}{216.66} = 4.61 \text{ ms}$

Each slot duration = $\frac{4.61}{8} = 0.57625 \text{ ms}$

Each slot contain 8 fields

Total traffic bits = $57 + 57 = 114 \text{ bits}$ (65 digitized voice)
49 bits for error detection

Total control bits = $3 + 1 + 26 + 1 + 3 + 8.25 = 42.25 \text{ bits}$

Slot width = $114 + 42.25 = 156.25 \text{ bits}$

Frame width = $156.25 \times 8 = 1250 \text{ bits/frame}$

Total transmission rate = $1250 \text{ bits/frame} \times 216.66 \text{ frame/sec} = 270825 \text{ bps}$
 $= 270.825 \text{ kbps}$

(Ans)

GSM transmission: each voice channel is digitized and compressed to 13 kbps
each slot carries 156.25 bits. Each slots here share a frame (TDMA)
Each 270.8 kbps digital channel modulates a carrier using GMSK
and results 200 kHz analog signal. Finally 124 analog channel of
200 kHz are combined using FDMA and results in 25 MHz band.

e) Free Space loss: refers to naturally weakening of a signal as it travels through free space, without any obstacles, interference affecting it. This happens because the signal energy spreads out over a larger area as it moves further from the source.

According to Friis Free Space Equation;

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{\lambda^2 G_t G_r} \quad \dots \text{--- ①}$$

According Antenna Gain Formula:

$$G = \frac{4\pi A_e}{\lambda^2} \quad \dots \text{--- ②}$$

Putting ② in ①;

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{\lambda^2 \cdot \frac{4\pi A_t}{\lambda^2} \cdot \frac{4\pi A_r}{\lambda^2}}$$

$$\frac{P_t}{P_r} = \frac{\lambda^2 d^2}{A_t A_r}$$

Free space loss: $L_{DB} = 10 \log \left(\frac{P_t}{P_r} \right)$

$$\therefore L_{DB} = 10 \log \left(\frac{\lambda^2 d^2}{A_t A_r} \right)$$

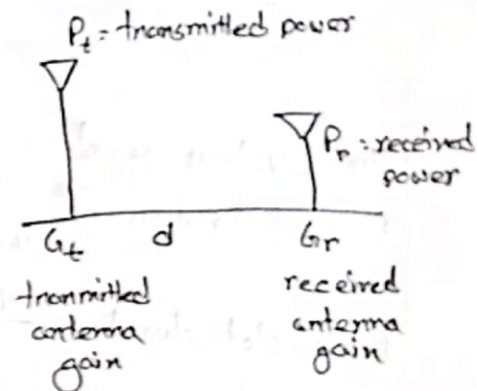
$$= 10 \log (\lambda^2 d^2) - 10 \log (A_t A_r)$$

$$= 10 \log (\lambda^2) + 10 \log (d^2) - 10 \log (A_t) - 10 \log (A_r)$$

$$= 10 \log \left(\frac{c}{f} \right)^2 + 10 \log (d^2) - 10 \log (A_t) - 10 \log (A_r)$$

$$= 10 \log (3 \times 10^8)^2 - 10 \log (f)^2 + 10 \log (d^2) - 10 \log (A_t) - 10 \log (A_r)$$

$$= 167.54 \text{ dB} - 20 \log (f) + 20 \log (d) - 10 \log (A_t A_r) \quad (\text{Ans})$$



Lecture-11

Prove:

① Compute E_{TOT} :

For Geometric calculation, we take Electric field (E-field)

Let, E_0 = Free space E-field (V/m)
at a distance d_0 , propagating
Free space E-field at distance
 $d > d_0$ is given by;

$$E(d, t) = \frac{E_0 d_0}{d} \cos\left(\omega \left(t - \frac{d}{c}\right)\right) \text{--- (i)}$$

With the help of (i);

$$E_{LOS}(d', t) = \frac{E_0 d_0}{d'} \cos\left(\omega \left(t - \frac{d'}{c}\right)\right) \text{--- (ii)}$$

$$E_g(d'', t) = \frac{E_0 d_0}{d''} \cos\left(\omega \left(t - \frac{d''}{c}\right)\right) \text{--- (iii)}$$

Here,

E_{TOT} = Total Received E-field
 E_{LOS} = Direct LOS component
 E_g = Ground reflected component
 h_t = Height of the Transmitter (T_x)
 h_r = " " " Receiver (R_x)
 d = distance between T_x and R_x
 d' = distance of LOS
 d'' = " " reflected wave
Propagation delay = $\frac{\text{distance}}{\text{speed}} = \frac{d}{c}$

(-1) = Reflection coefficient

Now,

We know,

$$E_{TOT} = E_{LOS} + E_g$$

$$\therefore E_{TOT}(d, t) = \frac{E_0 d_0}{d'} \cos\left(\omega \left(t - \frac{d'}{c}\right)\right) + (-1) \frac{E_0 d_0}{d''} \cos\left(\omega \left(t - \frac{d''}{c}\right)\right)$$

Assume, for $t = \frac{d''}{c}$

$$\begin{aligned} E_{TOT}\left(d, t = \frac{d''}{c}\right) &= \frac{E_0 d_0}{d'} \cos\left(\omega \left(\frac{d'' - d'}{c}\right)\right) - \frac{E_0 d_0}{d''} \cos 0 \\ &= \frac{E_0 d_0}{d'} \cos \theta_\Delta - \frac{E_0 d_0}{d''} \end{aligned}$$

If d becomes very large, $d'' = d' = d$

$$E_{TOT}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d} \cos \theta_\Delta - \frac{E_0 d_0}{d} = \frac{E_0 d_0}{d} (\cos \theta_\Delta - 1)$$

Using phasor diagram;

$$E_{TOT}(d) = \sqrt{\left(\frac{E_0 d_0}{d}\right)^2 (\cos \theta_D - 1)^2 + \left(\frac{E_0 d_0}{d}\right)^2 \sin^2 \theta_D}$$

$$= \frac{E_0 d_0}{d} \sqrt{2 - 2 \cos \theta_D}$$

$$= 2 \cdot \frac{E_0 d_0}{d} \sin\left(\frac{\theta_D}{2}\right)$$

$$= 2 \frac{E_0 d_0}{d} \left(\frac{\theta_D}{2}\right) \dots \dots (iv)$$

② Compute Path difference, Phase difference and Time delay

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$= d \sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} - d \sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2}$$

$$= d \left(1 + \frac{1}{2} \left(\frac{h_t + h_r}{d}\right)^2\right) - d \left(1 + \frac{1}{2} \left(\frac{h_t - h_r}{d}\right)^2\right)$$

$$= \frac{1}{2d} \left((h_t + h_r)^2 - (h_t - h_r)^2 \right)$$

$$\text{Path} = \frac{2 h_t h_r}{d}$$

$$\text{Phase, } \theta_D = \omega \mathcal{L} = 2\pi f \frac{\Delta}{c} = 2\pi f \frac{\Delta}{\lambda f} = \frac{2\pi \Delta}{\lambda}$$

$$\text{Time delay, } \mathcal{L} = \frac{\Delta}{c} = \frac{\theta_D \lambda}{2\pi \cdot f \lambda} = \frac{\theta_D}{2\pi f}$$

③ Compute received Power

From (iv)

$$E_{TOT}(d) = 2 \frac{E_0 d_0}{d} \left(\frac{\theta_0}{2} \right) = \frac{2 E_0 d_0}{d} \times \frac{2\pi 2 h_t h_r}{2\pi d}$$

$$= \frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2}$$

$$|E_{TOT}(d)|^2 = \left| \frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2} \right|^2 [P \propto E^2]$$

$$P_r(d) = \frac{(4\pi)^2 E_0^2 d_0^2 h_t^2 h_r^2}{\lambda^2 d^4} = \frac{(4\pi)^2 P_0 d_0^2 h_t^2 h_r^2}{\lambda^2 d^4} [P_0 \propto E_0^2] \quad \text{--- (v)}$$

Free Space Loss for LOS;

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{G_t G_r \lambda^2} \Rightarrow P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$

$$P_0 = P_r; P_0 = P_t G_t G_r \frac{\lambda^2}{(4\pi d_0)^2}$$

From (v),

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d_0)^2} \cdot \frac{(4\pi d_0)^2 h_t^2 h_r^2}{\lambda^2 d^4}$$

$$= P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

$$P_r \propto \frac{1}{d^4}$$

$$\frac{P_t}{P_r} = \frac{d^4}{G_t G_r h_t^2 h_r^2}$$

$$dB = 40 \log d - 10 \log (G_t G_r) - 20 \log (h_t h_r)$$

3) a)

$$\begin{aligned} \text{i) Area} &= \frac{\sqrt{3} \cdot 3}{2} \times R^2 \times N \\ &= \frac{3\sqrt{3}}{2} \times (1.6)^2 \times 32 \\ &= 212.83 \text{ km}^2 \end{aligned}$$

ii) Total bandwidth = 33 MHz

$$\begin{aligned} \text{Channel bandwidth} &= 25 \times 2 \text{ KHz} \\ &= 50 \text{ KHz} \end{aligned}$$

$$\text{Total available channel} = \frac{33000}{50} = 660 \text{ channel}$$

$$\therefore S = 660$$

$$S = KN ; N = 12$$

$$K = \frac{S}{N} = \frac{660}{12} = 55$$

iii) $S = KN$

$$= 55 \times 32$$

$$= 1760 \text{ channels}$$

iv) 1 MHz for control channel.

$$1 \text{ MHz} = 1000 \text{ KHz}$$

$$\therefore \text{Available Control channel} = \frac{1000}{50} = 20$$

$$\therefore \text{Available Traffic channel} = 660 - 20 = 640$$

$$\text{So, Control channel per cell} = \frac{20}{12} \approx 2$$

$$\text{Traffic channel per cell} = \frac{640}{12} \approx 54$$

b) System-A

Probability of blocking = 2% = 0.02

$$\text{Holding time} = \frac{3}{60} = \frac{1}{20} \text{ hr}$$

$$\text{Average, } \lambda = 2$$

channel per cell = 19

Total cell = 394

From Graph,

$$A = 12.5$$

We know,

$$A_0 = \lambda H = 2 \times \frac{1}{20} = \frac{1}{10} = 0.1 \text{ Erlangs}$$

$$U = \frac{A}{A_0} = \frac{12.5}{0.1} = 125 \text{ user per cell}$$

$$\therefore \text{total user supported by System A} = (125 \times 394) \\ = 49250$$

System-B

channel per cell = ~~98~~ 57

total cell = 98

From Graph, $A = 47$

$$U = \frac{47}{0.1} = 470 \text{ user per cell}$$

$$\therefore \text{total user supported by System B} = (470 \times 98) \\ = 46060$$

System-C

channel per cell = 100

total cell = 49

From Graph, $A = 88$

$$U = \frac{88}{0.1} = 880$$

$$\therefore \text{total user by system C} = (880 \times 49) \\ = 43120$$

total user supported by 3 systems.

$$= (49250 + 46060 + 43120) = 138430$$

Market penetration for system A;

$$\frac{49250}{2000000} = 0.024 = 2.46\%$$

System B;

$$\frac{46060}{2000000} = 2.303\%$$

System C;

$$\frac{43120}{2000000} = 2.156\%$$

Combined 3 system;

$$\frac{138430}{2000000} = 6.92\%$$

(Ans.)

3(c) use the process of 2(c)

~~4~~

e) ① Gain for parabolic antenna, $G_r = \frac{7A}{\lambda^2}$

$$\begin{aligned} &= \frac{7A f^2}{c^2} \\ &= \frac{7 \times \pi \times (0.6)^2 \times (2 \times 10^9)^2}{(3 \times 10^8)^2} \\ &= 351.85 \end{aligned}$$

$$G_{\text{ain dB}} = 25.46 \text{ dB}$$

$$\begin{aligned} \text{① Effective area} &= 0.56 A \\ &= 0.56 (\pi \times (0.6)^2) \\ &= 0.633 \text{ m}^2 \end{aligned}$$

$$\text{①① } \frac{P_t}{P_r} = \frac{(4\pi)^2 (d)^2}{G_r G_t \lambda^2} \quad ; d = 24 \text{ km} = 24 \times 10^3 \text{ m}$$

$$\begin{aligned} &= 20 \log \log(4\pi) + 20 \log(d) + 20 \log(f) - 20 \log(c) - 10 \log(G_r) - 10 \log(G_t) \\ &= 21.48 + 87.60 + 186.02 - 169.54 - 25.46 - 25.46 \\ &= 75.14 \text{ dB} \end{aligned}$$

$$\text{The transmitted power} = 10 \log(0.1) = -10$$

$$\begin{aligned} \text{The available received signal power} &= 75.14 - 10 \\ &= 65.14 \end{aligned}$$

f) a)

$$f = 900 \text{ MHz}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3} \text{ m}$$

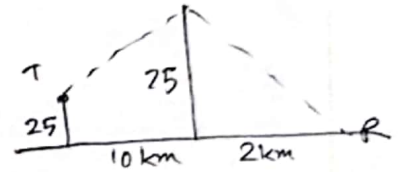
$$\beta = \tan^{-1} \left(\frac{75 \text{ m} - 25}{10000} \right) = \cancel{0.0025} = 0.2864^\circ$$

$$\gamma = \tan^{-1} \left(\frac{75}{2000} \right) = 2.1475^\circ$$

$$\alpha = \beta + \gamma = 2.43^\circ = 0.0424 \text{ rad}$$

$$v = \eta \sqrt{\frac{2 \times (10000 \times 2000)}{\frac{1}{3} \times (10000 + 2000)}} = 4.24$$

$$\text{diffraction loss} = 20 \log \left(\frac{0.225}{4.24} \right) = -25.50 \text{ dB}$$



b) For 6 dB ; $v = 0$

$$\frac{h}{2000} = \frac{25}{10000}$$

$$h = 4.16 \text{ m}$$

