Question Pattern

Set	Total Questions	Need to Answer	Marks
1	7	5	3x5 = 15
2	7	5	5x5 = 25
3	7	5	6x5 = 30
		Total:	70

Qubits45-Notes

All Quiz CG by Era-038.pdf
CG-Lecture-1,2,3,6,8 by Farliha-151.pdf ————————————
Lecture 2,3 by Paul-023.pdf —————————————
Lecture 4,5 by Paul-023.pdf
Lecture 6,8 by Era-038.pdf
5C-prove By Era-038.pdf
Eta vul chilo ektu. Sin r cos red color ra hbe

Quiz-2 set B

$\sin\alpha = \frac{c_y}{d} = \frac{6/\sqrt{109}}{0.96} = 0.5986 \doteq 0.6$

$\cos\alpha = \frac{c_x}{d} = \frac{8/\sqrt{109}}{0.96} \doteq 0.798 = 0.8$

$\sin\alpha = \frac{c_x}{d}$

$\cos\alpha = \frac{c_y}{d}$

$\cos\beta = d = 0.96 \quad \beta = \cos^{-1}(0.96) = 16.26$

16.699

$\sin\beta = \frac{c_z}{u_e} = \frac{3}{\sqrt{109}} = 0.287 \approx 0.3$

$(u_e = 1)$

$\beta = \sin^{-1}\frac{3}{\sqrt{109}} = \cos^{-1}\left(\sqrt{(8/\sqrt{109})^2 + (6/\sqrt{109})^2}\right) = 16.69$

$$Rot_z(\alpha) = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0.8 & -0.6 & 0 & 0 \\ 0.6 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$Rot_x(90-\beta) = Rot_x(90-16.7)$
$= Rot_x(73.3)$

$\cos\beta = \cos(73.3) = 0.287 \approx 0.3$

$\sin\beta = \sin(73.3) = 0.957 \approx 0.96$

No need to do $(90-\beta)$ just exchange value of $\sin\beta$ and $\cos\beta$.

$$Rot_x = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.3 & -0.96 & 0 \\ 0 & 0.96 & 0.3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

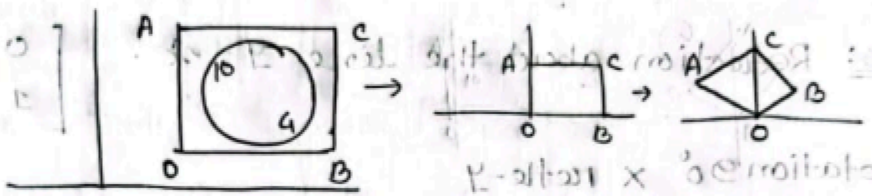
$C.M = Rot_x \; Rot_z \; T(-1,-1,1)$

$A' = CM \times A$

$B' = CM \times B$

$A'B' = CM \times [AB]$

② Stretch the clock OACB by 150% along one of its diagonals so that 10:00 through 4:00 move to the northeast and 9:00 through 5:00 move to the south keeping the center of the clock fixed. The four vertices of the clock are O(2,2) A(2,6) C(6,6) B(6,2). Perform all the transformations and find the final vertices.



Scale 150% $= \frac{150}{100} = 1.5$

~~Steps:~~

~~① Translate, T(-6, -2)~~

?

~~② Rot(-45°)~~

~~③ Scale S(1+1.5, 1) → S(2.5, 1)~~

~~④ Rot(45°)~~

~~⑤ T(6,2)~~

Steps:

① T(-2, -2)

② Rot(45°)

③ Scale (1, 2.5)

④ Rot(-45°)

⑤ T(2,2)

Ans:

O = 2,2

A′ = 5,9

B′ = 9,5

C′ = 12,12

$$M = T(2,2)\ Rot(-45^\circ)\ S(1, 2.5)\ Rot(45^\circ)\ T(-2,-2)$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.75 & 0.75 & 2 \\ 0.75 & 1.75 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 6 & 6 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{array}{c} \begin{matrix} O & A & C & B \\ \downarrow & \downarrow & \downarrow & \downarrow \end{matrix} \\ \begin{bmatrix} 2 & 5 & 12 & 9 \\ 2 & 8 & 12 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Important Notes

Important Suggestion for the Exam:

- Questions will be mostly **conceptual**, not straightforward. Try to have a deeper understanding of each topic rather than just memorizing.
- You will find **at least one question in each set from the quiz questions** with some modifications. I have shared all the quiz questions in the attachment for you to practice. However, I did not provide the solutions because I want you to try the solutions yourself for proper understanding.
- Manage your time properly. Be specific about the answer, don't need to exaggerate it where it's not necessary. In case of the matrix operation, showing detailed calculations is optional. You can use calculators to find results.
- For the theory questions, explaining the topic in your own words should be enough, but the answer needs to be correct.
- You must solve each and every problem given as a practice problem in the lecture slides.

- Try to see the last few semesters' questions. However, there is no guarantee that any direct questions from the previous semester will come. But it will help you a lot.
- For midpoint questions (if any), show detailed calculations. You should present them in a tabular format [(x, y), the decision variable, and related calculation]
- Drawing figures is not mandatory unless mentioned explicitly. But you can draw if you find it necessary.

Also, I have ordered the lectures according to the proportion of the marks they carry in the final exam. You can follow the following order while preparing for the final exam. However, I encourage you to go through each topic.

[top to bottom: most important to less important]
Lecture - 6
Lecture - 5
Lecture - 3
Lecture - 4
Lecture - 8
Lecture - 2
Lecture - 1

Good Luck!

Lecture 1

Decipher44

1) State the differences between hardware and software pipelines. 1 [3]

2) Explain why triangles are commonly used as the primary primitive in computer graphics. 1 [3]

Why we use Triangles?

*** Decipher 44 + (Quiz 3)

- It is the simplest universal surface element. → line, points এদের surface নয়।

- It is the Convex Hull, of 3 points.
  → কিছু points নিয়ে এমন একটি surface তৈরি করা যাতে points গুলো connect করলে কখনো একটি point আরেকটি point কে insect না করে।

Aspect	Software Graphics Pipeline	Hardware Graphics Pipeline
Processing Speed	Relies on the CPU, slower, less suited for real-time rendering.	Runs on the GPU, optimized for high-speed, real-time rendering.
Flexibility	Highly customizable, easy to modify each stage in software.	Less flexible; fixed stages, though programmable shaders add some customization.
Parallel Processing	Limited parallelism, as it depends on CPU threads.	Designed for massive parallelism on GPU cores, improving efficiency.
Power Efficiency	Consumes more power due to CPU's general-purpose processing.	More power-efficient for graphics tasks, as GPUs are specialized for them.
Use Cases	Ideal for prototyping and non-real-time rendering.	Best suited for high-performance, real-time rendering in games and 3D applications.

Origin42

1. Lecture -01

a) Explain the level-of-detail rendering. [3]

1. a. Solution:

<u>LOD:</u> (Level of Detail)

- Rendering Speed $\propto \frac{1}{\text{number of triangles}}$
- more triangle : more storage
- কাছের জিনিসে বেশি detail show করে। ক্রমান্বয়ে দূরের জিনিসে detailing কমতে থাকে।

1.

b) What is a vanishing point? Give an example scenario of multiple vanishing points. [3]

1. b. Solution: ***by 45***

In computer graphics and perspective drawing, a vanishing point is a point in the distance where parallel lines appear to converge or "vanish" when extended. A single vanishing point is often used in one-point perspective, where all parallel lines converge to a single point on the horizon line.Multiple vanishing points are used in two-point and three-point perspective to create a more realistic depiction of three-dimensional space.

Two-Point Perspective: For example, drawing a cityscape with tall buildings. The vertical edges of the buildings converge to one vanishing point on the left and another on the right.

Lecture 2

Decipher44

c) State the differences between lossless and lossy compression. 2		[3]
d) Describe how emissive display device produces colored images. 2		[3]

Difference between Lossless and Lossy :

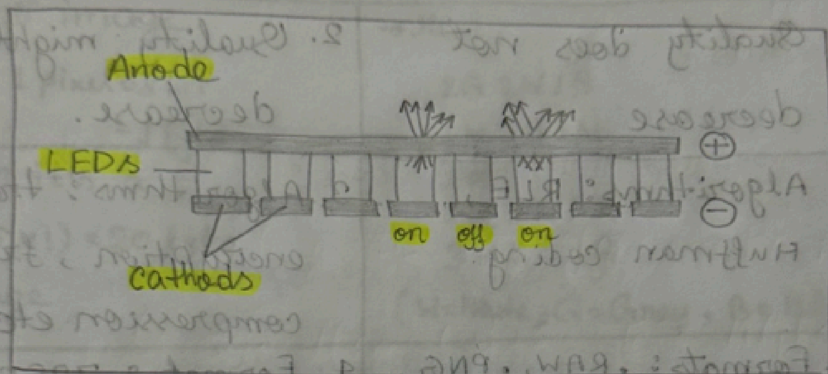
Quiz + Decipher44

Lossless Compression	Lossy Compression
1. get back original file.	1. can not get back origirlal file.
2. Quality does not decrease	2. Quality might decrease.
3. Algorithms: RLE, Huffman coding.	3. Algorithms: transform encryption, fractal compression etc.
4. Formats: .RAW, .PNG, .GIF	4. Formats: JPEG, AVC, MPEG
5. File size doesn't reduce significantly	5. File size reduce significantly.

(Quiz2 + decipher44)

⧉ Emissive Display: *Explain how Emissive Display works with an example?*

- has its own light source. (Each Pixel)
- Each pixel is composed of one or more LEDs (semiconductor devices)



- Sub-pixel: এখানে pixel গুলোকে 3 টি independently controlled sub-pixel -এ ভাগ করা হয়। These are: R,G,B.
- Example: LED (light-emitting diode)

Integer43

a)	Explain how a transmissive device works with an example.	[3]

Ques: Explain how transmissive display works with an example.

⟹ Example : LCD (light crystal display)

- Molecular structure of liquid crystal rotates the polarization of light that passes through it.

- have sub-pixels

- degree of rotation ⟷ applied voltage

Working Steps:

1) Display pannel এর পিছনে থাকে light source ব্যবহার হয়।
2) Liquid Crystal Layer কে allign করা হয় based on applied electric ~~source~~ currents.
3) Liquid Crystals adjust to block or allow specific amounts of light to pass through.
4) Colour filters sits above the liquid crystal
5) Liquid Crystal এর allignment control করে the display can adjust brightness and color for each pixel.
6) The screen is reliant on the backlight, so visibility can be reduced in bright environment. Such as Sunlight.

2	d) Consider 3 images img1, img2 and img3 (see the image below) overlapping each other where img2 is the foreground of img1 and img1 is the foreground of img3. Additionally, img2 has an alpha mask $\alpha 1$ given below and img1 is fully transparent. Find the pixel values for the output image.	[5]

img1

30	21	140
27	78	200
222	25	224

img2

50	22	152
55	85	20
230	19	100

img3

150	20	1
90	25	70
112	99	165

$\alpha 1$

0.2	0.39	1
0	0.5	0.82
0.45	0.5	0.7

Origin42

7. Lecture -02

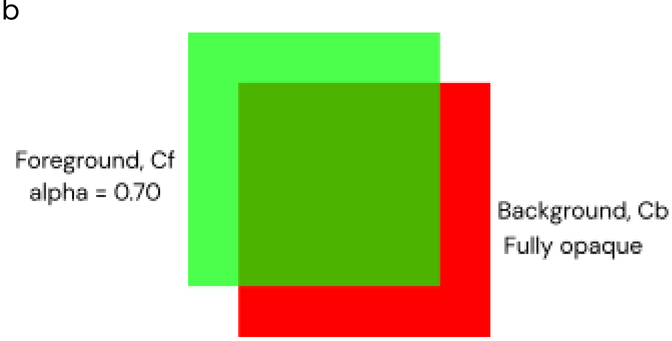
(b) Consider there are two objects overlapping each other, where C_1 is the color of the foreground object and C_2 is the color of the background object. Construct an alpha compositing formula if the foreground object has 30% transparency and the background object is fully opaque. [4]

7. b. Solution: 024

Foreground color = C1 = Cf
Background color = C2 = Cb

Foreground object has 30% transparency means the foreground is 70% opaque.
Background is fully opaque.(given).

Alpha compositing formula,
C = alpha x (Cf) + (1 – alpha) x Cb
= 0.70 x Cf + 0.30 X Cb



7.

(c) Explain how to determine whether a polygon is facing towards or away from the camera. [3]

7. c. Solution: 45

Camera Position:

- Determine the position of the camera or the viewpoint in 3D space.

Camera-to-Polygon Vector:

- Calculate a vector from the camera position to any point on the polygon.

Dot Product:

- Calculate the dot product of the camera-to-polygon vector and the normal vector of the polygon.

Determine Facing Direction:

- If the dot product is positive, the polygon is facing towards the camera

Enigma41

7. Lecture -02

(b) How does a transmissive device work? Explain with appropriate diagrams. [4]

7. b. Solution: 024
Liquid crystal displays (LCDs) are an example of the transmissive type. A liquid crystal is a material whose molecular structure enables it to rotate the polarization of light that passes through it, and the degree of rotation can be adjusted by an applied voltage. An LCD pixel has a layer of polarizing film behind it, so that it is illuminated by polarized light—let's assume it is polarized horizontally.
A second layer of polarizing film in front of the pixel is oriented to transmit only vertically polarized light. If the applied voltage is set so that the liquid crystal layer in between does not change the polarization, all light is blocked and the pixel is in the "off" (minimum intensity) state. If the voltage is set so that the liquid crystal rotates the polarization by 90 degrees, then all the light that entered through the back of the pixel will escape through the front, and the pixel is fully "on"—it has its maximum intensity. Intermediate voltages will partly rotate the polarization so that the front polarizer partly blocks the light, resulting in intensities between the minimum and maximum. Like color LED displays, color LCDs have red, green, and blue subpixels within each pixel, which are three independent pixels with red, green, and blue color filters over them.

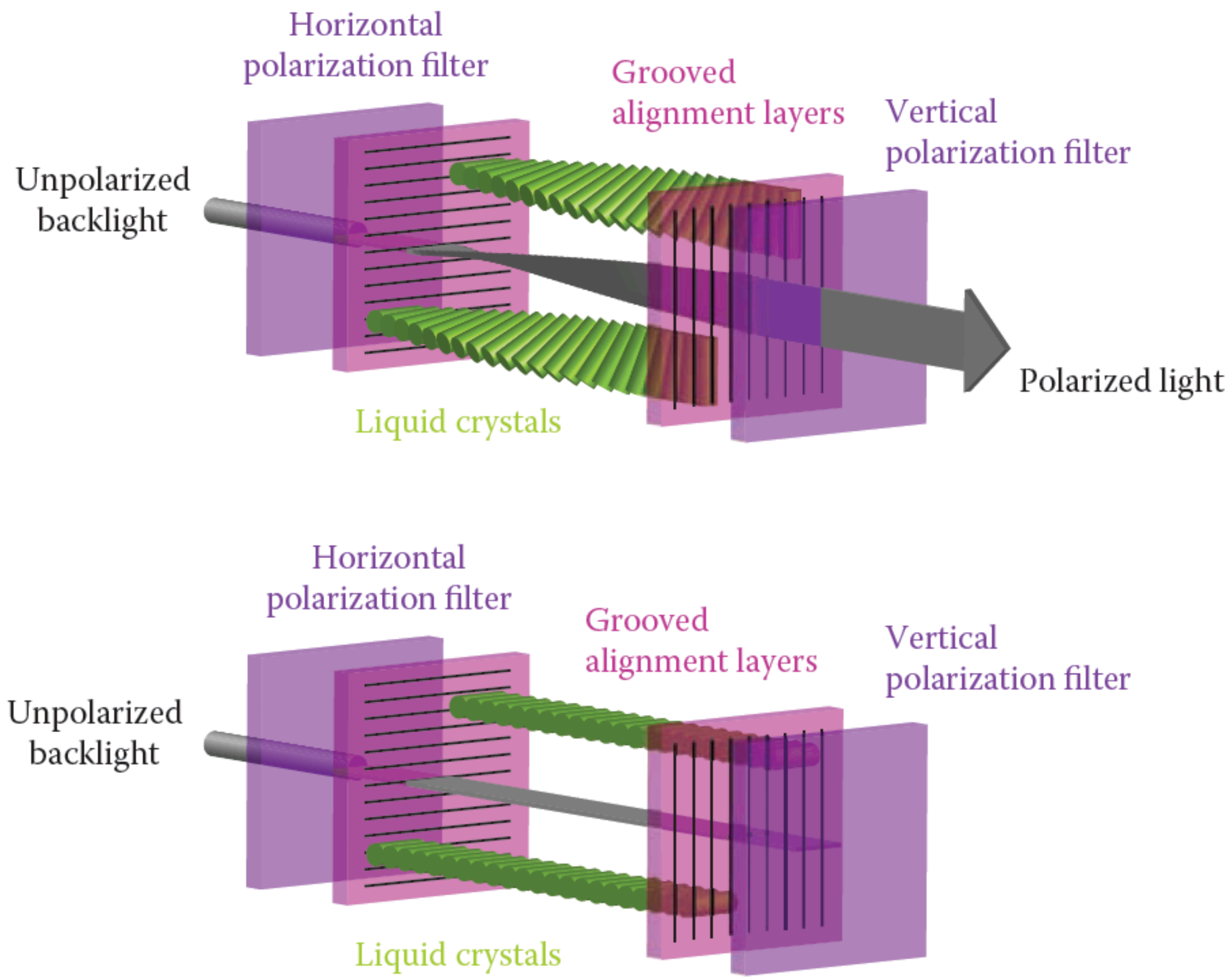
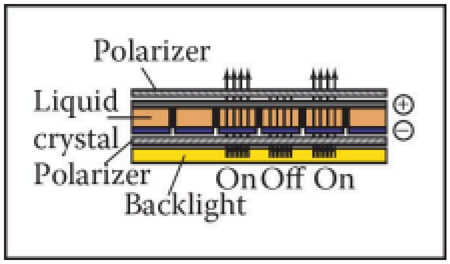


Figure: One pixel of a LCD display in ON and OFF state

Transmissive Displays (1/3)

- Transmissive Displays:
  - Example: light crystal display (LCD)
  - Molecular structure of liquid crystal rotates the polarization of light that passes through it
  - LCDs also have sub-pixels.



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Transmissive Displays (2/3)

- Degree of rotation ↔ applied voltage

Recursive40

1. lecture-2 b) Given that, $C_f = 1.0$, $C_b = 0.2$ and $C = 0.8$, where C_f, C_b and C are the foreground, background and composite intensities respectively. What is the alpha value to perform this composition? [3]
1. b. Solution: ***Solved by Younus-131***

$$C = \alpha C_f + (1-\alpha) C_b$$

$$\Rightarrow \alpha = \frac{C - C_b}{C_f - C_b} = \frac{0.8 - 0.2}{1 - 0.2}$$

$$= 0.75$$

Lecture 3

Decipher44

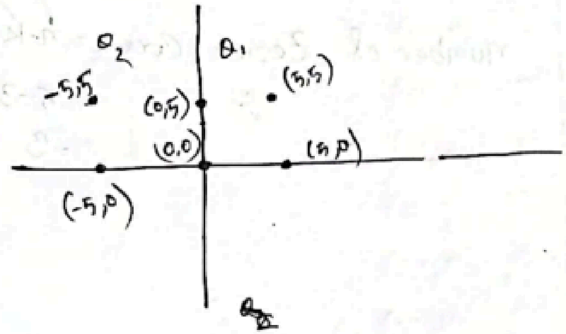
a) Suppose we have a circle created using two Bézier curves, Q1 and Q2. Q1 is defined by four control points: (0, 0), (0, 5), (5, 5), and (5, 0). The control points of Q2 are the reflections of those of Q1. Find the Bézier curve value of Q1(0.6) and Q2(0.8).

Dedpoker

3(a)

For Q_1;

$P_0(0,0)$; $P_1(0,5)$; $P_2(5,5)$; $P_3(5,0)$



$Q_1(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$

(0.6)

$=$

For Q_2;

$P_0(0,0)$; $P_1(0,5)$; $P_2(-5,5)$; $P_3(-5,0)$

$Q_2(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$

(0.8)

$=$

b) Explain why the degree of a B-spline curve remains unaffected by the number of control points used in the curve. 3 [3]

Integer43

a) Consider a Bezier curve Q, defined by 6 control points (-3, 3), (-1, 4), (0, 5), (1, 3), P_4 and P_5. Find the control points P_4 and P_5, if $Q(0.5) = [0.68, 3.56]^T$ and $Q(1) = [5, 1]^T$	[6]
d) State the problems associated with the higher degree Bezier Curve. Explain how this problem can be solved.	[3]
e) A uniform quadratic B-Spline curve S is defined by 7 control points $P_0(-3, -1)$, $P_1(-2, 0)$, $P_2(-1, 1)$, $P_3(0, 2)$, $P_4(1, 3)$, $P_5(2, 4)$ and $P_6(3, 5)$. Find the midpoint and endpoint of the first 2 curve segments of the quadratic B-Spline curve.	[5]

Origin42

2. Lecture -03

a) State the differences between raster and vector images. [4]

2. a. Solution: solved by- 146 (from chatgpt)

Raster image: convert (an image stored as an outline) into pixels that can be displayed on a screen or printed.

Vector image: store instructions for displaying the image rather than the pixels needed to display it.

Aspect	Raster Image	Vector Image
Basic Representation	Grid of pixels	Geometric shapes and paths
Resolution Dependency	**Resolution-dependent (DPI)**	**Resolution-independent**
Scaling	May result in quality degradation when scaled up or down :**Resizing can result quality degradation**	No loss of quality when scaled
File Size	**Larger file sizes**	**Smaller file size**
Editing Flexibility	Limited flexibility; editing may degrade quality	Highly editable without quality loss
Image Quality	**Suitable for photorealistic images**	**Limited photorealism**; best for line art, logos, and illustrations
Storage Format Examples	**JPEG, PNG, BMP**	**SVG, AI, PDF**

Printing Quality	Quality may vary based on resolution	Consistently high-quality printing
Ideal Use Cases	Photographs, detailed images	Logos, icons, illustrations

6. **Lecture -03** ~~(a)~~ Suppose we have a cubic Bézier curve defined by the control points $P_0 = (0, 0)$, $P_1 = (2, 5)$, $P_2 = (5, 5)$, and $P_3 = (8, 0)$. Find the mid-point and end-point of the cubic curve. [7]
6. a. Solution: ***by 102***

$P_0(0,0)$, $P_1(2,5)$, $P_2(5,5)$, $P_3(8,0)$ · Find mid point, end point

Control points, $N = 4$; $d = N - 1 = 4 - 1 = 3$

We know, $B_{i,d}(u) = \frac{d!}{i!(d-1)!} u^i (1-u)^{(d-i)}$

For, $B(\frac{1}{2})$

$$\therefore B_{0,3}(\tfrac{1}{2}) = \frac{3!}{0!(3-0)!} (\tfrac{1}{2})^0 (1 - \tfrac{1}{2})^{3-0}$$

$$= \frac{(3 \times 2)}{1(3 \times 2)} \times 1 \times \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$$

$$\therefore B_{\cancel{0}1,3}\left(\frac{1}{2}\right) = \frac{3!}{1!(3-1)!} \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^{3-1}$$

$$= \frac{6}{2} \times \frac{1}{2} \times \frac{1}{4}$$

$$= 0.375$$

$$\therefore B_{2,3}\left(\frac{1}{2}\right) = \frac{3!}{2!(3-3)!} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{3-2}$$

$$= \frac{6}{2 \times 1} \times \frac{1}{4} \times \frac{1}{2}$$

$$= 0.375$$

$$B_{3,3}\left(\frac{1}{2}\right) = \frac{3!}{3!(3-3)!} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^{3-3}$$

$$\therefore Q_3\left(\tfrac{1}{2}\right) = 0.125 \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.375 \times \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 0.375 \times \begin{bmatrix} 5 \\ 5 \end{bmatrix} + 0.125 \times \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 \\ 1.875 \end{bmatrix} + \begin{bmatrix} 1.875 \\ 1.875 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3.625 \\ 3.75 \end{bmatrix}$$

For $Q_3(1)$, $\quad B_{i,d}(u) = \dfrac{d!}{i!(d-i)!} \times (u)^i \times (1-u)^{d-i}$

$$\therefore B_{0,3}(1) = \frac{3!}{0!\,(3-0)!} \times 1^0 \times (1-1)^{3-0}$$

$$= \underline{1}$$

$$\therefore B_{1,3}(1) = \frac{3!}{1!\,(3-1)!} \times (1)^1 \times \underbrace{(1-1)}_{0}{}^{3-1}$$

$$= \frac{3 \times \cancel{2}}{\cancel{2}} \times 0 = 0$$

$$\therefore B_{2,3}(1) = \frac{3!}{2!\;1!}(1)^2 \times \underbrace{(1-1)}_{0}{}^{3-2}$$

$$= \frac{3 \times 2}{2} \times 0 = 0$$

$$\therefore B_{3,3}(1) = 1$$

Enigma41

1. Lecture -03

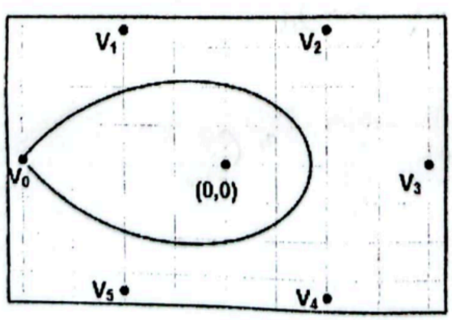
b) State the differences between raster and vector images. **[4]**

1. b. Solution: 024

Raster Graphics	Vector Graphics
They are composed of pixels.	They are composed of paths.
In Raster Graphics, refresh process is independent of the complexity of the image.	Vector displays flicker when the number of primitives in the image become too large.
Graphic primitives are specified in terms of end points and must be scan converted into corresponding pixels.	Scan conversion is not required.
Raster graphics can draw mathematical curves, polygons and boundaries of curved primitives only by pixel approximation.	Vector graphics draw continuous and smooth lines.
Raster graphics cost less.	Vector graphics cost more as compared to raster graphics.
They occupy more space which depends on image quality.	They occupy less space.
File extensions: .BMP, .TIF, .GIF, .JPG	File Extensions: .SVG, .EPS, .PDF, .AI, .DXF

6. Lecture -03

(b) A 2D Bezier curve Q is situated inside a regular hexagon $V_0V_1V_2V_3V_4V_5$ (see the following figure). The control points are chosen from the vertices of the hexagon. If Q has the same starting and ending point V_0, what is the value of $Q(\frac{1}{5})$? Given that, the vertices V_0 and V_1 are $(-1,0)$ and $(-\frac{1}{2},\frac{\sqrt{3}}{2})$ respectively. Show your calculations. (Hint: a regular hexagon has symmetric property) **[7]**



6. b. Solution: Rabab 039
We know, rectangular hexagons have symmetric properties. From the figure,
$V0 = (-1,0)$
$V1 = (-\frac{1}{2}, \sqrt{3}/2)$
V3 is at the opposite x-axis end of V0. So, $V3 = (1,0)$
V5 is at the opposite y-axis end of V1. So, $V5 = (-\frac{1}{2}, -\sqrt{3}/2)$
V2 is at the opposite side of the origin from V1. So, $V2 = (\frac{1}{2}, \sqrt{3}/2)$
V4 is at the opposite y-axis end of V2. So, $V4 = (\frac{1}{2}, -\sqrt{3}/2)$

So, the number of control points, N = 7 (As V0 is both the starting and ending point)
So, dimensions, d = N - 1 = 7 - 1 = 6

Pro tip: Too tired to calculate the coefficients? Use Pascal's triangle! (jodi time na thake)

d = 0	1
d = 1	1 1
d = 2	1 2 1
d = 3	1 3 3 1
d = 4	1 4 6 4 1
d = 5	1 5 10 10 5 1
d = 6	1 6 15 20 15 6 1

Using Bezier curve equation,
$Q(u) = (1-u)^6V0 + 6(1-u)^5uV1 + 15(1-u)^4u^2V2 + 20(1-u)^3u^3V3 + 15(1-u)^2u^4V4 + 6(1-u)u^5V5 + u^6V0$
So,
$Q(0.2) = (0.8)^6V0 + 6(0.8)^5(0.2)V1 + 15(0.8)^4(0.2)^2V2 + 20(0.8)^3(0.2)^3V3 + 15(0.8)^2(0.2)^4V4 + 6(0.8)(0.2)^5V5 + (0.2)^6V0$
Calculate the rest…

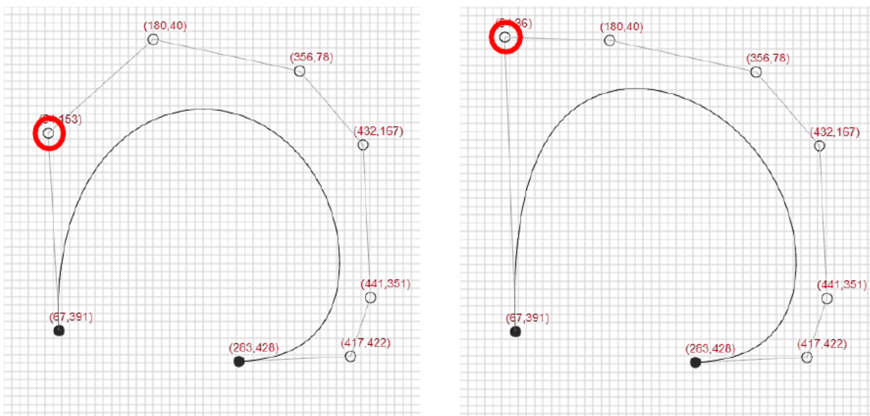
Recursive40

2. Lecture - 03
b) Discuss the limitations of Bezier curve. [2]

2. b. Solution: Rabab 039

Disadvantages

- A change to any of the control point alters the entire curve.
- Having a large number of control points requires high polynomials to be evaluated. This is expensive to compute.



Credit: CPSC 589/689 Course Notes, University of Calgary, Faramarz Samavati

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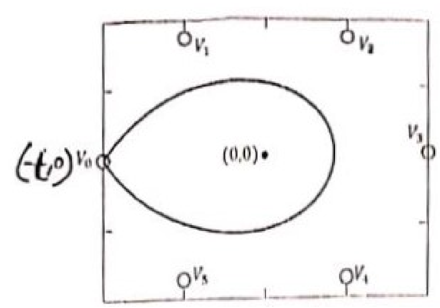
5. lecture-3

(a) A 2D Bezier curve Q is situated inside a regular hexagon $V_0V_1V_2V_3V_4V_5$ (see the following figure). The control points are chosen from the vertices of the hexagon. If Q has the same starting and ending point V_0, what is the Euclidean distance between $Q\left(\frac{1}{2}\right)$ and $Q\left(\frac{1}{6}\right)$? Given that, the vertices V_0 and V_1 are $(-1,0)$ and $(-1,\frac{\sqrt{3}}{2})$ respectively. Show your calculations. **[10]**



5. a. Solution:

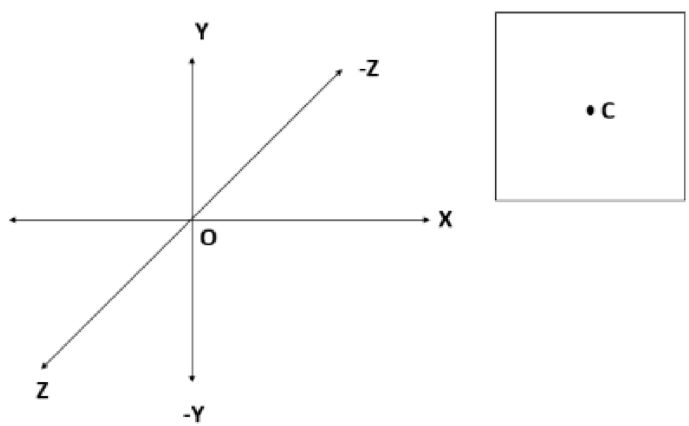
Same as Enigma41 6(b)

Lecture 4

Qubits45 - Quiz

1) Consider a square in a 3D canonical coordinate system (see the following figure). The square has a side of 10 units with a center at C(0, 0, -20). Also, consider a camera coordinate with origin e and basis {u, v, w}. The eye of the camera frame is placed in the upper right corner of the square. The goal is to point the camera viewing direction at point **C** and capture it.

   a) **[10 Marks]** Determine the basis and eye matrix
   b) **[10 Marks]** Determine the position of point **C** w.r.t the camera coordinate.



Decipher44

~~a)~~ Briefly explain different stages of viewing transformation. 4 [3]

~~a)~~ Consider a screen of size n x m in which the origin coordinate is in the center of the screen and pixel coordinates can have negative values. Apply the necessary transformation to construct the viewport matrix for this screen. [5] 2

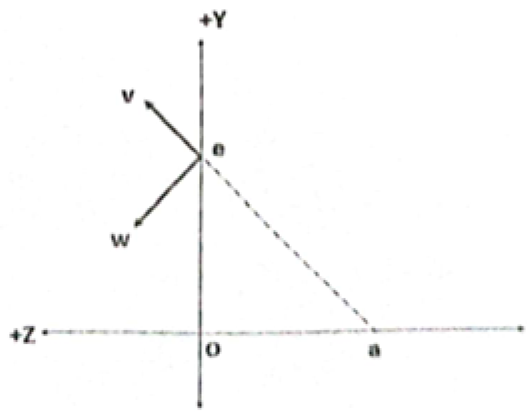
c) Show that canonical-to-frame transformation is a translation followed by a rotation. [5]

c) Consider a line AB in a 3D space, where point A and B are (1, 1, -1) and (9, 7, 2) respectively. Apply appropriate transformations to align the line AB to the z-axis so that point A stays at the origin. Calculate and determine the new point A' and B' after the transformation. [6]

3

f) Origin O and basis vectors {z, y} construct a 2D canonical coordinate system where −z is the viewing direction and y is the up vector. Consider a frame coordinate with origin e and basis {w, v}. Here e is located on the y-axis and edge oe and oa of the triangle *oea* has a length of 1 and 2 unit respectively. Determine the position of the point a w.r.t the frame coordinate.

4



Integer43

d) Assume, ABCD is a 2D rectangle and the vertices are A(2, 2), B(7, 2), C(7, 7) and D(2,7). Apply appropriate transformation on ABCD to obtain A'B'C'D' such that A'D' and B'C' both create 45 degrees with X-axis after the transformation. Determine the composite transformation matrix to perform this task and plot A'B'C'D'. [6]

c) Explain with appropriate example that the frame-to-canonical transformation can be expressed as a rotation followed by a translation. [3]

a) Apply appropriate transformations to construct the orthographic transformation matrix. [5]

g) Describe why perspective projection is considered a non-affine transformation.
b) Consider a line AB in a 3D space, where point A and B are (5, -2, 3) and (10, 3, 2) respectively. Apply appropriate transformations to align the line AB to y-axis so that point A stays at origin. Calculate and determine the new point A' and B' after the transformation. [6]
d) Assume, ABCD is a 2D rectangle and the vertices are A(2, 2), B(7, 2), C(7, 7) and D(2,7). Apply appropriate transformation on ABCD to obtain A'B'C'D' such that A'D' and B'C' both create 45 degrees with X-axis after the transformation. Determine the composite transformation matrix to perform this task and plot A'B'C'D'.

Origin42

3. Lecture - 04

a) Consider a rectangle with vertices A(1, 1), B(6, 1), C(6, 5) and D(1, 5). Apply appropriate transformation to the rectangle to obtain a parallelogram in such a way that point C and D move 4 units to its right from the original position and point A and B remain unchanged. [8]

3. a. Solution: 024

Steps:

1. Translate by (-1, -1)
2. Shear along X-axis by 1
3. Translate by (1, 1)

// Explanation:
For shear factor along X-axis we have to divide the shear value with Δy
For shear factor along Y-axis we have to divide the shear value with Δx

For this question, C and D move to the right means shear along the X-axis by 4 points. As shear by X-axis we need to calculate Δy. Here, Y_value_difference = Δy = |5 - 1| = 4
So, shear value for X-axis, Sh_x = 4/Δy = 4/4 = 1
//

M = T(1, 1) x Shear_X(1) x T(-1, -1) =

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}.\begin{pmatrix}1 & 1 & -2\\0 & 1 & -1\\0 & 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 1 & -1\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$$

*// here signs between matrices are **x** (not dot) and multiplications are **cross product.***

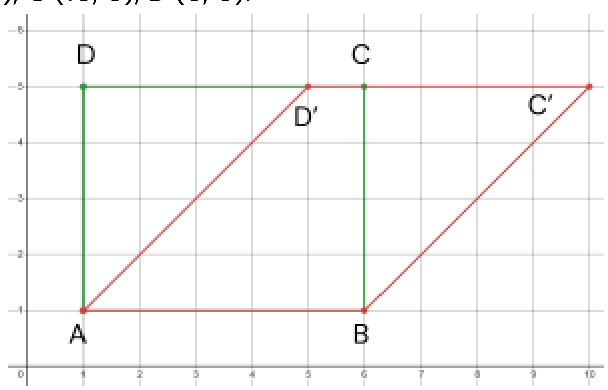
So, M x V =

$$\begin{pmatrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}.\begin{pmatrix}1 & 1 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}.\begin{pmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 1\end{pmatrix}.\begin{pmatrix}1 & 6 & 6 & 1\\1 & 1 & 5 & 5\\1 & 1 & 1 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 1 & -1\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}.\begin{pmatrix}1 & 6 & 6 & 1\\1 & 1 & 5 & 5\\1 & 1 & 1 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 6 & 10 & 5\\1 & 1 & 5 & 5\\1 & 1 & 1 & 1\end{pmatrix}$$

So, points are A(1, 1), B(6, 1), C′(10, 5), D′(5, 5).



2. Lecture -04

b) What are the properties of affine transformation? Mention an example of non-affine transformation operation. **[4]**

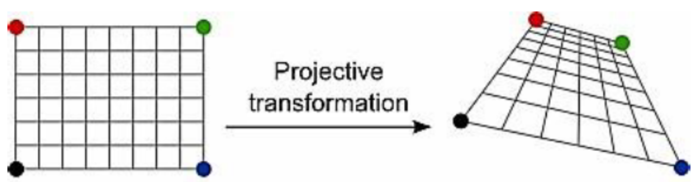
2. b. Solution:

Affine:

Affine transformation (1/2)

- Maps points to points, lines to lines, planes to planes.
- Preserves the ratio of lengths of parallel line segments.
- Sets of parallel lines remain parallel.
- Does not necessarily preserve angles between lines or distances between points.

Non-Affine:



5. Lecture -04

(b) Consider a square OACB with vertices O(3, 2), A(3, 6), C(7, 6) and B(7, 2). Reflect the square along a line x = -1 using 2D transformation. Determine the composite transformation matrix and find the final vertices. [7]

5. b. Solution: 024

Steps:

1. Translate by (1, 0) // to align x=-1 line with Y-axis we have to move it to right by 1 point
2. Reflect by Y-axis // given x=-1 is parallel to Y-axis. So, reflect by Y-axis
3. Translate by (-1, 0) // undo step 1

M = T(-1, 0) x Reflect_Y x T(1, 0) =

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

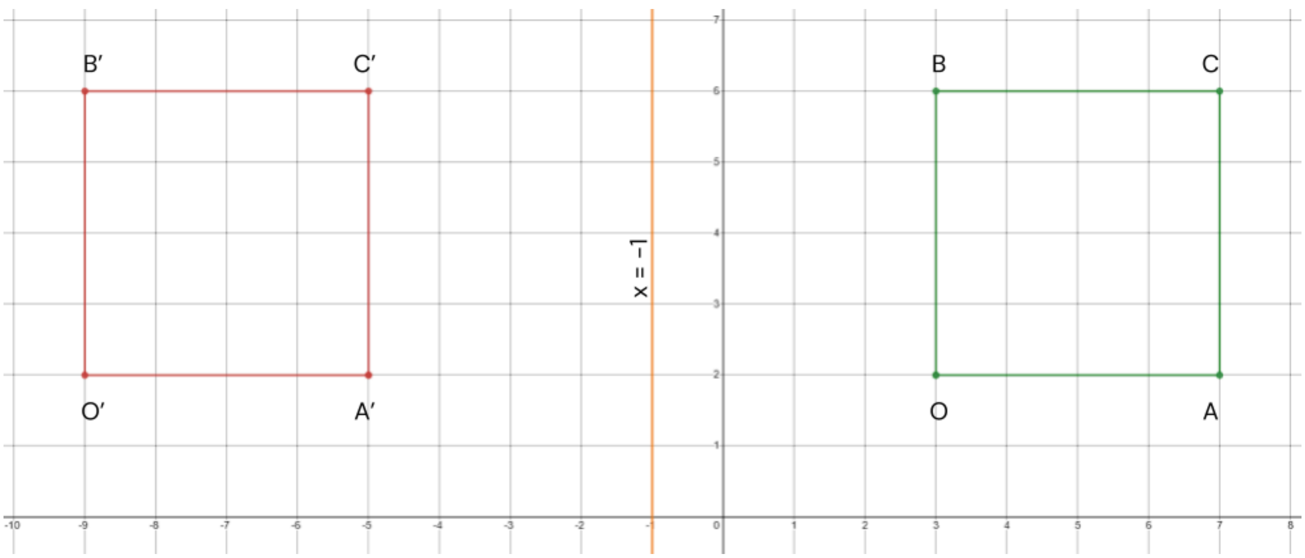
*// here signs between matrices are **x** (not dot) and multiplications are **cross product.***

So, M x V =

$$\begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 3 & 3 & 7 & 7 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -5 & -9 & -9 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, points are A′(−5, 2), B′(−5, 6), C′(−9, 6), D′(−9, 2).



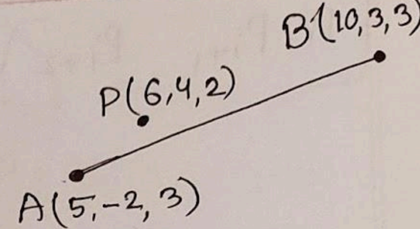
Enigma41

2. **Lecture -04**

a) AB is a line and P is a point in 3D space; where the points A, B and P are (5,-2,3), (10,3,3) and (6,4,2) respectively. We want to rotate P along AB by -90°. Determine the composite transformation matrix to do the task and calculate the rotated point P'. **[10]**	
2. a. Solution: 024, Sohom bolse "Thikase"	

Steps

1. Translate by $(-5, 2, -3)$
2. Rotate along Z
3. Rotate along X
4. Rotate along Y
5. Rotate along X
6. Rotate along Z
7. Translate by $(5, -2, 3)$



Step-1

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~Ue =~~

Step-2

$$u_e = \frac{B-A}{|B-A|} = c_x, c_y, c_z$$

$$c_x = \frac{10-5}{\sqrt{(10-5)^2 + (3+2)^2 + (3-3)^2}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$c_y = \frac{3-(-2)}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$c_z = \frac{3-3}{5\sqrt{2}} = 0$$

Step 5 (undo 3)

$$R\_x(\beta^{-1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6 (undo 2)

$$R\_z(\alpha^{-1}) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 7 (undo 1)

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ M = Step 7 matrix X ..... X Step 1 matrix

=

Beta value ta negative hobe cause clockwise rotation

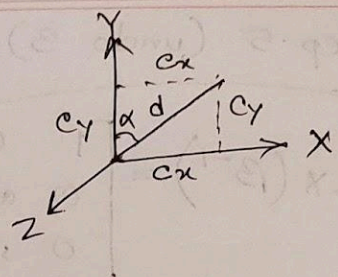
Calculation incomplete

2. Lecture -04

$$d = \sqrt{Cx^2 + Cy^2} = 1$$

$$\cos\alpha = \frac{Cy}{d} = \frac{1}{\sqrt{2}}$$

$$\sin\alpha = \frac{Cx}{d} = \frac{1}{\sqrt{2}}$$

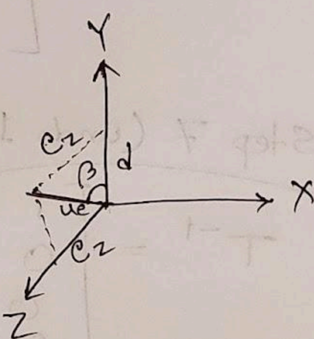


$$R\_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3

$$\cos\beta = \frac{d}{ue} = \frac{1}{1} = 1$$

$$\sin\beta = \frac{Cz}{ue} = 0$$



$$R\_x(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 4 //it was asked to do

$$R\_Y = \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) What are the properties of affine transformation? Mention an example of non-affine transformation operation. **[4]**

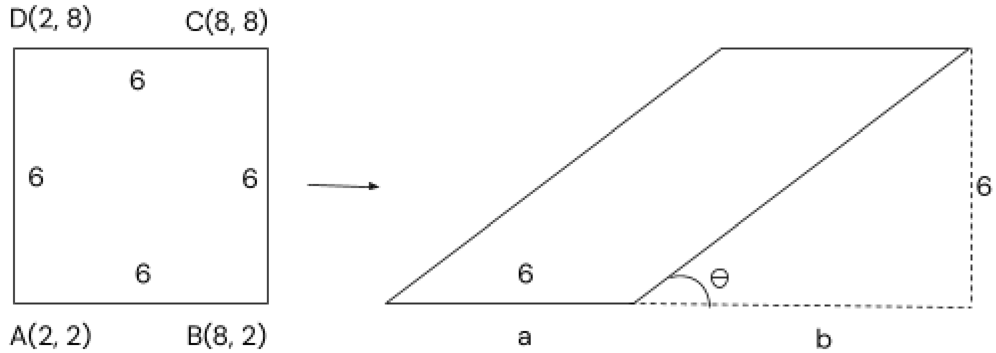
2. b. Solution: ***same as Origin 2(b)***

7. Lecture -04 **(a)** Consider a rectangle with vertices A(1,1), B(6,1), C(6,5) and D(1,5). Reflect the rectangle along the line $y = \frac{1}{\sqrt{3}}x - 3$ using 2D transformation. Determine the composite transformation matrix and find the final vertices. **[8]**
7. a. Solution: almost same as set-c (2b)

Recursive40

2. a) AB is a line and P is a point in 3D space; where the points A, B and P are $(1,1,1)$, $(3,3,3)$ and $(2,2,4)$ respectively. We want to rotate a point P with respect to AB by 90^o. Determine the composite transformation matrix to perform the task. [12]
2. a. Solution: ch4

3. (b) Assume, $ABCD$ is a 2D rectangle and the vertices are $A(2,2), B(8,2), C(8,8)$, and $D(2,8)$. Apply shear to obtain $A'B'C'D'$ such that $A'D'$ and $B'C'$ both create 30 degree with X-axis after the transformation. Design the steps to perform the task and determine the composite transformation matrix. Plot $A'B'C'D'$. [8]
3. b. Solution: 024 (shear-x er khetre angle with Y axis consider korte hoy i think, as per book, Please Check) ***r****eply: book page no please? as said "30 degree with X-axis" so I did that.*



Here, theta = 30 degree (given)
tan(30) = 6 / b => b = 10.392

So, to shear by X-axis for 10.392 point,
Shear factor = 10.392 / 6 = 1.732

Steps for transformation:

1. Translate by (-2, -2)
2. Shear along X-axis by 1.732
3. Translate by (2, 2)

M = T(2, 2) x Shear_X(1.732) x T(-2, -2) =

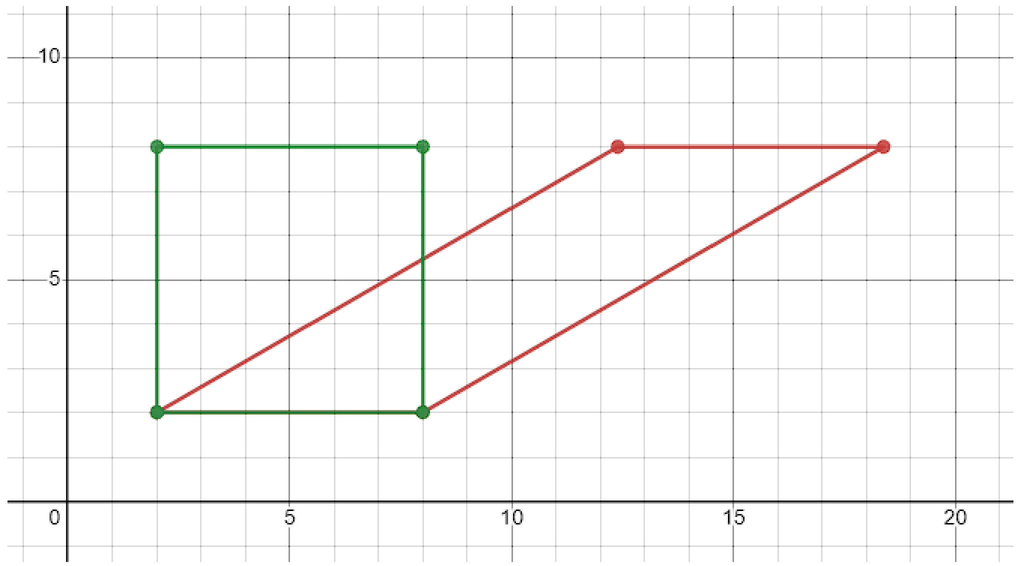
$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 1.732 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So, M x V =

$$\begin{pmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 2 & 8 & 8 & 2 \\ 2 & 2 & 8 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2. & 8 & 18.392 & 12.392 \\ 2 & 2 & 8 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



4.

(b) Suppose we want to reflect a 2D point $P(4,5)$ against a line that goes through $(-1,-3)$ and $(3,2)$. Determine the composite transformation to perform this task. What is the final position of P? [6]

4. b. Solution: 024, Correction: 019, 076

For (-1, -3) and (3, 2) line equation:

=> $(x-x1) / (x1-x2) = (y-y1) / (y1-y2)$

=> $(x+1) / (-1-3) = (y+3) / (-3-2)$

=> $5x - 4y - 7 = 0$

=> $y = (5/4)x + (-7/4)$ // y=mx+c

Steps for transformation:

1. Translate by (0, +7/4) // as the equation cuts Y-axis through (-7/4)
2. Rotate by 38.66 // m = tan(theta) = 5/4 => theta = 51.34 with X-axis. So with Y: (90-51.34)
3. Reflect by Y-axis // after rotation the line became parallel to Y-axis
4. Rotate by -38.66 // undo step 2
5. Translate by (0, -7/4) // undo step 1

M =

M x V =

Calculations incomplete

Lecture 5

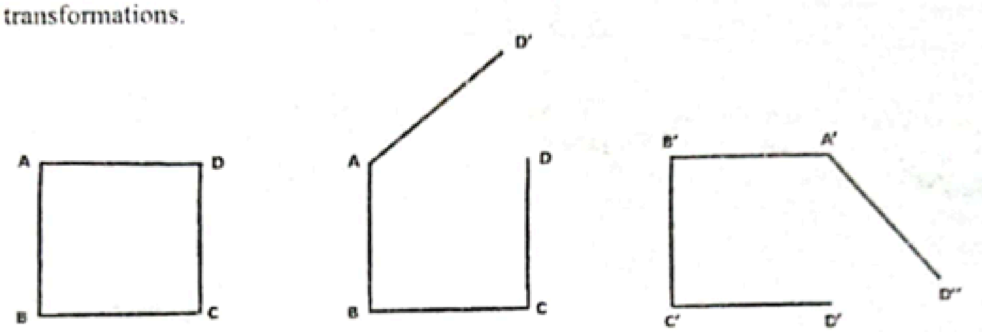
Decipher44

g) Suppose we have a 2D box ABCD where the vertices are A(-0.5, 0.5), B(-0.5, -0.5), C(0.5, -0.5) and D(0.5, 0.5). The box consists of two parts: the body ABCD and the lid AD. You need to perform appropriate transformations so that the lid AD opens by creating a +45 degree angle with respect to the point A and its final position becomes AD'. Additionally, rotate both the body and lid clockwise by 90 degrees with respect to the origin. Find the final position of the body A'B'C'D' and the lid A'D" after the transformations. [6]

5

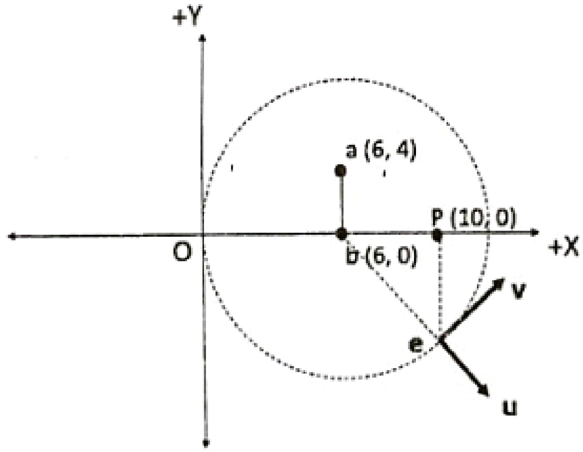
3

(-3



(i) The box (ii) Output after opening the lid (iii) Output rotating the body and the lid

Integer43

g) Here (in the figure), origin O and basis {x,y} construct a 2D canonical coordinate system. Within this, line ab is our model (P_{xy}). Now, we want to view it from a new 2D camera with eye e and basis {u,v}; which is rotated by θ degrees from its' default orientation. Assume that, u is the viewing direction and b is the center of the circle. 	[6]
e) State the differences between image-order and object-order rendering.	[3]
b) Show that the transformation matrix for the reflection about the line y = -x is equivalent to a reflection relative to the y-axis followed by a counter-clockwise rotation of 90 degrees.	[5]
g) Consider the following parameters for an orthographic ray-tracing: Camera frame: $E = [2, 6, 10]^T$, $U = [1, 0, 0]^T$, $V = [0, 0.6, -0.6]^T$, $W = [0, 0.6, 0.6]^T$ Image plane: $l = -12$, $r = 12$, $t = 12$, $b = -12$ Raster image resolution: 12×10 A ray (with length = 20) is generated from the lower left corner pixel of the raster image. Find the position of the ray start and end point on the image plane.	[5]

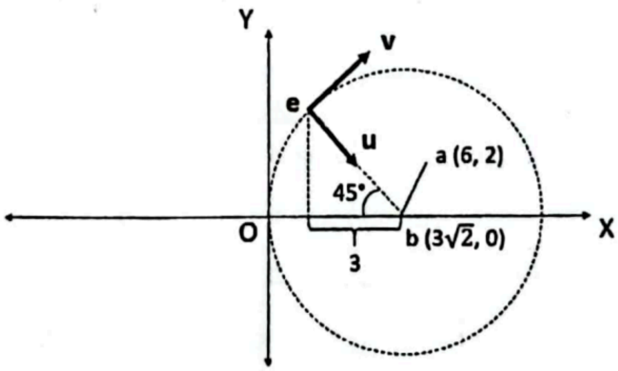
Origin42

2. Lecture -05 b) Explain the problems associated with it if homogeneous coordinates were not used in matrix transformation. [2]
2. b. Solution: 024 In general, Translation process is a summation process. But other transformation processes are matrix multiplication. So, if homogeneous coordinates were not used, the translation process

could not be combined with other transformations.
45
Homogeneous coordinates are especially important in 3D graphics, where transformations involve both rotations and translations in three dimensions. Without homogeneous coordinates, the representation and manipulation of 3D transformations would be significantly more complex and error-prone.
perspective projection, are essential for creating realistic 3D scenes. Homogeneous coordinates are crucial for representing and efficiently applying projective transformations. Without them, handling perspective projection would be extremely complex and inefficient

2. **Lecture - 05**

c) Here (in the figure), origin O and basis {x,y} construct a 2D canonical coordinate system. Within this, line ab is our model (Pxy). Now, we want to view it from a new 2D camera with eye e and basis {u,v}; which is rotated by −45 degrees around b. Determine the position of a and b w.r.t camera coordinate. [8]
Assume that, u is the viewing direction and b is the center of the circle.



2. c. Solution:

4.

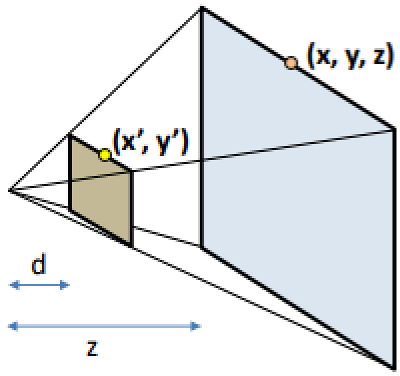
(b) Derive 2D perspective projection matrix. [4]

4. b. Solution: 45

Perspective Projection (8/17)

For 2D:

$$y' = dy/z$$
$$x' = dx/z$$



$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} dx \\ dy \\ z \end{bmatrix} \sim \begin{bmatrix} dx/z \\ dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

7. Lecture -05*

(a) Transform a 3D line AB from an orthographic view volume to the viewport of size 256 x 128. Consider the vertices of the line are A(-2, -4, -1), B(1, 5, -5) and the orthographic view volume has the following setup: [7]

l = - 6, r = 6, b = - 7, t = 7, n = - 2, f = - 8

7. a. Solution: 024

Given,

nx = 256, ny = 128

l = -6, r = 6, t = 7, b = -7, n = -2, f = -8

For Orthographic -> Canonical -> Viewport:

M = Mvp x Morth =

$$\begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \frac{256}{2} & 0 & 0 & \frac{256-1}{2} \\ 0 & \frac{128}{2} & 0 & \frac{128-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \frac{2}{6--6} & 0 & 0 & -\frac{6-6}{6--6} \\ 0 & \frac{2}{7--7} & 0 & -\frac{7-7}{7--7} \\ 0 & 0 & \frac{2}{-2--8} & -\frac{-2-8}{-2--8} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{64}{3} & 0 & 0 & \frac{255}{2} \\ 0 & \frac{64}{7} & 0 & \frac{127}{2} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Also given, A(-2, -4, -1) and B(1, 5, -5).
We know,

$$\begin{bmatrix} x_{\text{pixel}} \\ y_{\text{pixel}} \\ z_{\text{canonical}} \\ 1 \end{bmatrix} = (\mathbf{M}_{\text{vp}}\mathbf{M}_{\text{orth}}) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

// here signs between matrices are ***x*** *(not dot) and multiplications are* ***cross product.***

A' = MA

$$\begin{pmatrix} \frac{64}{3} & 0 & 0 & \frac{255}{2} \\ 0 & \frac{64}{7} & 0 & \frac{127}{2} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} -2 \\ -4 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{509}{6} \\ \frac{377}{14} \\ \frac{4}{3} \\ 1 \end{pmatrix}$$

B′ = MB

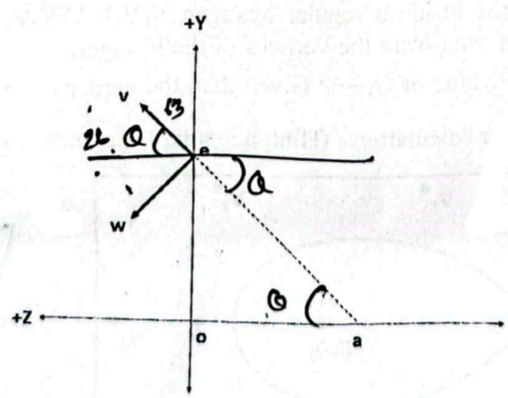
$$\begin{pmatrix} \frac{64}{3} & 0 & 0 & \frac{255}{2} \\ 0 & \frac{64}{7} & 0 & \frac{127}{2} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ -5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{893}{6} \\ \frac{1529}{14} \\ 0 \\ 1 \end{pmatrix}$$

Enigma41

3. Lecture -05

a) Origin **O** and basis vectors {z, y} construct a 2D canonical coordinate system where –z is the viewing direction and y is the up vector. Consider a frame coordinate with origin **e** and basis **{w, v}**. Here **e** is located on the y-axis and edge **oe** and **oa** of the triangle *oea* has a length of 1 and 2 unit respectively. Determine the position of the point **a** w.r.t the frame coordinate. **[8]**

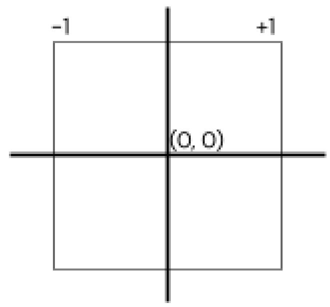


3. a. Solution:

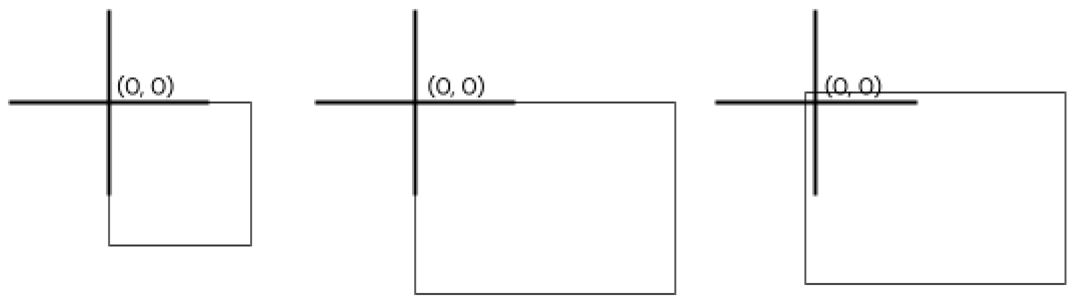
3. Lecture -05

b) Construct the viewport matrix required for a system in which pixel coordinates count down from the top of the image, rather than up from the bottom. **[6]**

3. b. Solution: 024
This is a general unit image rectangle. We have to convert it to screen pixels.



Given, pixel coordinates count from top to down.
So, transformations are as follows:



So viewport matrix, Mvp = T(-1/2, 1/2) x S(nx/2, ny/2) x T(1, -1)

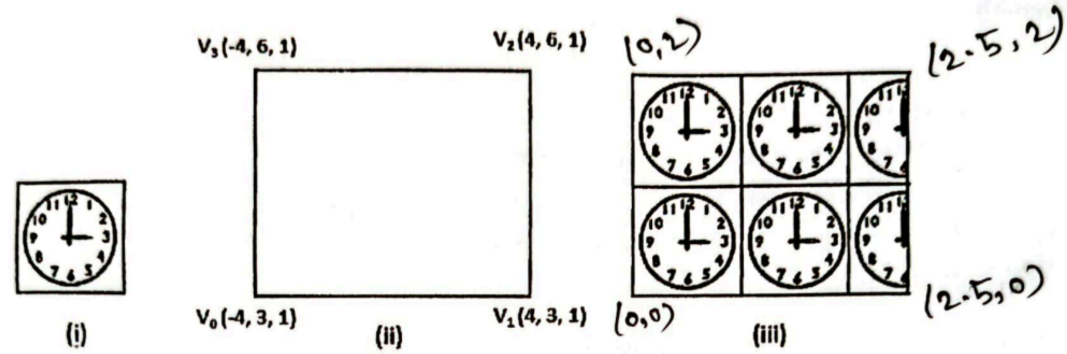
$$\begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{nx}{2} & 0 & 0 \\ 0 & \frac{ny}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{nx}{2} & 0 & \frac{nx}{2} \\ 0 & \frac{ny}{2} & -\frac{ny}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{nx}{2} & 0 & \frac{nx}{2} - \frac{1}{2} \\ 0 & \frac{ny}{2} & \frac{1}{2} - \frac{ny}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

4.

(b) In the following figure, (i) is a texture, (ii) is a rectangular face $V_0V_1V_2V_3$ to be mapped with the texture, and (iii) is the output after texture mapping. List the texture coordinates for corresponding xyz-coordinates to perform texture lookup. (assume any data if necessary) [5]



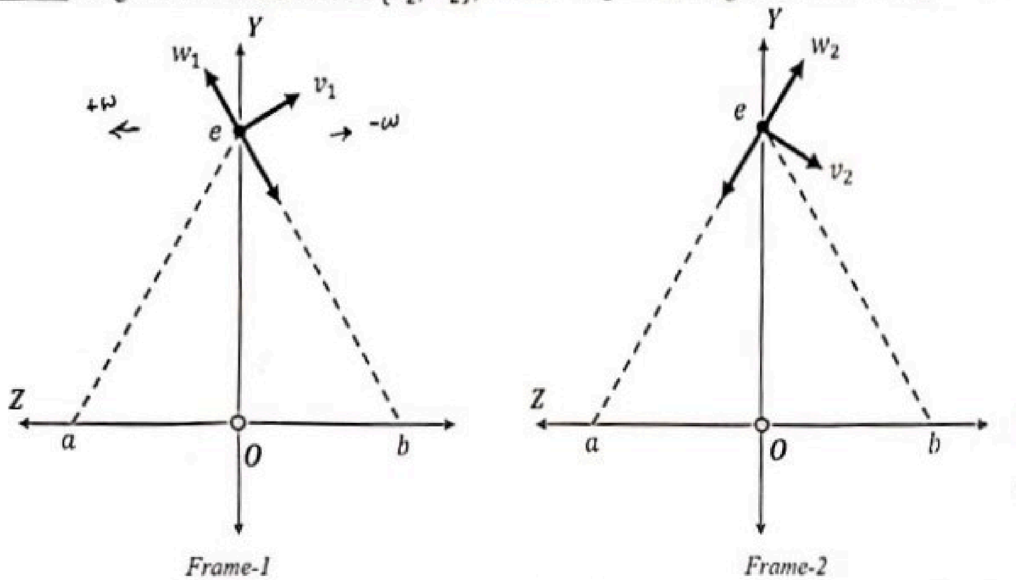
4. b. Solution:

5. Lecture -05

(c) Differentiate between orthographic and oblique projections. [3]

5. c. Solution:

6.

a) Consider a 2D canonical camera coordinate system with origin O and basis vectors $\{y, z\}$; where $-z$ is the viewing direction and y is the up vector. Also consider 2 frame coordinate system inside the canonical (see the following figures), which are – • Frame-1: Origin e and basis vectors $\{v_1, w_1\}$; where $-w_1$ = viewing direction, and v_1 = up • Frame-2: Origin e and basis vectors $\{v_2, w_2\}$; where $-w_2$ = viewing direction, and v_2 = up  Here e is located on y axis and is a vertex of an equilateral Δeab, where each edge has a length of *one* unit for both the frames. Determine the positions of the O w.r.t *Frame-1* and *Frame-2*.	[11]
6. a. Solution:	

7. b) Consider a 3D line AB that needs to be transformed from an orthographic view volume to a viewport with 64×64 resolution. Vertices of the line are $A(-1, -3, -6)$ and $B(2, 4, -7)$. The orthographic view volume has the following setup: $l = -5, \quad r = 5, \quad b = -5, \quad t = 5, \quad n = -3, \quad f = -10$ Determine the matrix M to transform the vertices of the line to viewport. Determine the transformed vertices.	[6]
7. b. Solution: ch5	

Recursive40

5.

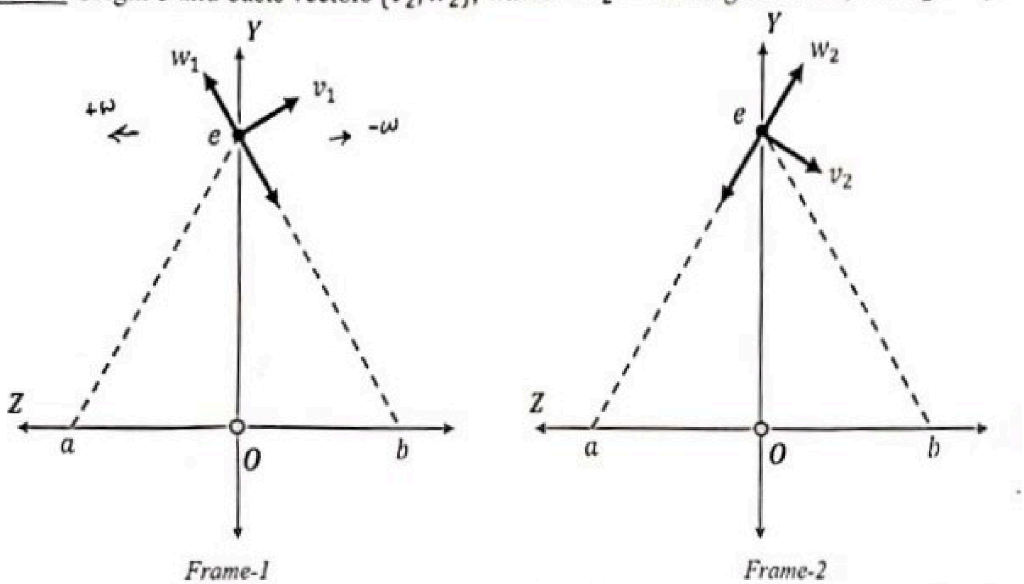
b) With an example, explain the shading computation for a simple *ray-tracing* algorithm. [4]

5. b. Solution:

6.

a) Consider a 2D canonical camera coordinate system with origin O and basis vectors $\{y, z\}$; where $-z$ is the viewing direction and y is the up vector. Also consider 2 frame coordinate system inside the canonical (see the following figures), which are – [11]

- Frame-1: Origin e and basis vectors $\{v_1, w_1\}$; where $-w_1$ = viewing direction, and v_1 = up
- Frame-2: Origin e and basis vectors $\{v_2, w_2\}$; where $-w_2$ = viewing direction, and v_2 = up



Here e is located on y axis and is a vertex of an equilateral Δeab, where each edge has a length of *one* unit for both the frames. Determine the positions of the O w.r.t *Frame-1* and *Frame-2*.

6. a. Solution: ch5

7.

b) Consider a 3D line AB that needs to be transformed from an orthographic view volume to a viewport with 64×64 resolution. Vertices of the line are $A(-1,-3,-6)$ and $B(2,4,-7)$. The orthographic view volume has the following setup: $$l=-5, \quad r=5, \quad b=-5, \quad t=5, \quad n=-3, \quad f=-10$$ Determine the matrix M to transform the vertices of the line to viewport. Determine the transformed vertices.	[6]
7. b. Solution: ch5	

Lecture 6

Qubits45 - Quiz

1. Apply the midpoint line drawing algorithm to draw a line from (1, p) to (-3, p + 9) and plot the points.

Here, $p = (-1)^n \times n$
[n = last 2 digits of your ID]

Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=−x; swap(x, y); plot(−y, x)	(4) x=−x; plot(−x, y)
(5) x=−x; y=−y; plot(−x, −y)	(6) x=−x; y=−y; swap(x, y); plot(−y, −x)	(7) y=−y; swap(x, y); plot(y, −x)	(8) y=−y; plot(x, −y)

a) **[15 marks]** Show the values of the decision variables and the points for each step (in a tabular format).
b) **[5 marks]** Plot the final points

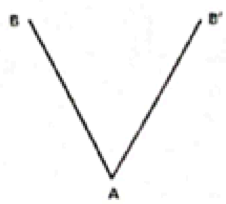
Decipher44

6 b) Consider a clipping rectangle which has width and height of 10 and 8 units respectively. Its lower left corner is located at (2, 2). Also, consider a line which has starting and ending points of (1, 1) and (15, 15) respectively. Perform the line-edge intersecting points with respect to all four edges of the clipping rectangle using Cyrus-Beck algorithm and determine the true clipping points. Show the steps and calculations for your solution (assume any data if necessary). [6]

6 e) Construct an algorithm to create a half circle given the radius and the center using the Bresenham's Circle drawing algorithm. [8]

c) Consider an English letter V created by joining 2 lines. The pixel coordinates on the first line AB is obtained using the midpoint line drawing algorithm along the points A(0, 0) and B(-3, 6). Second line AB' is obtained by applying reflection on the obtained coordinates of the first line. Apply necessary transformations and determine the points on the letter V. [6]

4



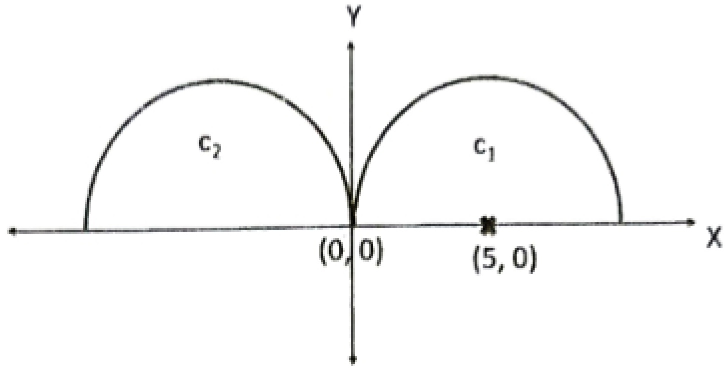
Necessary adjustments of the original line drawing algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=−x; swap(x, y); plot(−y, x)	(4) x=−x; plot(−x, y)
(5) x=−x; y=−y; plot(−x, −y)	(6) x=−x; y=−y; swap(x, y); plot(−y, −x)	(7) y=−y; swap(x, y); plot(y, −x)	(8) y=−y; plot(x, −y)

6 f) Consider a triangle with vertices A(1, 1), B(9, 1) and C(5, 5) and color values of (1, 0, 0), (1, 1, 0) and (0, 1, 0) at each respective vertex. Find the color of the point P(2, 2) inside the triangle using the concept of barycentric interpolation. [5]

$\begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix}$

Integer43

e) In the following figure, there are 2 half circles C_1 and C_2 where C_2 is the reflection of C_1. Here C_1 has a center at (5, 0) and both share the same point (0, 0) in their circumference. Using affine transformation and Bresenham's circle drawing algorithm, find the points in the circumference for C_2. 6 
c) Consider a rectangle with vertices A(1, 1), B(5, 1), C(5, 5) and D(1, 5) and color values of (1, 0, 0), (0, 1, 0), (1, 1, 0) and (0, 0, 1) at each respective vertex. Find the color of the point P(4, 2) inside the rectangle using the concept of barycentric interpolation. [6]
f) Show that, in case of the midpoint line drawing algorithm, we can successively update the decision variable by adding $(y_1 - x_1) - (y_0 - x_0)$ for each selection of a northeast pixel. Here, (x_0, y_0) and (x_1, y_1) are two endpoints of the line. [5]

Origin42

3. Lecture - 06 b) Write down the algorithm to create a half circle given the radius and the center using Bresenham's Circle drawing algorithm. [6]
3. b. Solution:

Algorithm

```
void MidpointCircle(int radius)
{
    int x = 0;
    int y = radius ;
    int d = 1 - radius ;
    CirclePoints(x, y);
    while (y > x)
    {
        if (d < 0)   /* Select E*/
                d = d + 2 * x + 3;
        else
        {     /* Select SE*/
            d = d + 2 * ( x - y ) + 5;
            y = y - 1;
        }
     x = x + 1;
     CirclePoints(x, y);
    }
}
```

```
CirclePoints (x,y)
    Plotpoint(x,y) ;
    Plotpoint (x,-y);
    Plotpoint(-x,y) ;
    Plotpoint(-x, -y);
    Plotpoint(y,x) ;
    Plotpoint(y, -x);
    Plotpoint(-y, x);
    Plotpoint( -y, -x);
end
```

4. Lecture -06

(a) Consider a line with a start and end point of (0, 0) and (-1, -2) respectively. Apply the necessary transformation to increase the size of the line by 100% and find the final vertices after the transformation. Also, determine the coordinates of each pixel along the transformed line segment using the midpoint line drawing algorithm. [10]

Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=−x; swap(x, y); plot(−y, x)	(4) x=−x; plot(−x, y)
(5) x=−x; y=−y; plot(−x, −y)	(6) x=−x; y=−y; swap(x, y); plot(−y, −x)	(7) y=−y; swap(x, y); plot(y, −x)	(8) y=−y; plot(x, −y)

4. a. Solution: Rabab 039

#Origin42 4(a)

$\Delta x = (-1-0) = -1$, $\Delta y = (-2-0) = -2$

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{-1} = 2$$

$1 < m < \infty$ and $\Delta x, \Delta y$ (-)ve $\therefore$ It's in the 6th octant.

$\therefore (x_0, y_0) = (0,0)$ and $(x_s, y_s) = (2,1)$

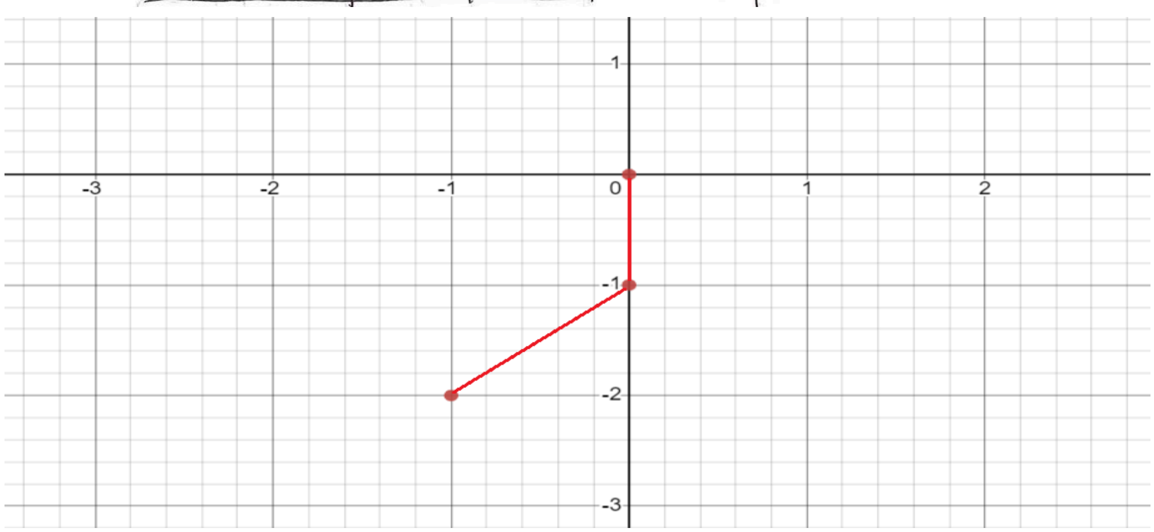
Now, $dy = (1-0) = 1$, $dx = (2-0) = 2$

$\therefore d_{init} = 2dy - dx = (2\times1) - 2 = 0$

$\therefore \Delta E = 2dy = 2\times1 = 2$

$\therefore \Delta NE = 2(dy - dx) = 2(1-2) = -2$

x, y	0, 0	1, 0	2, 1
d	$0 \le 0$ $\therefore$ E	$2 > 0$ $\therefore$ NE	$0 \le 0$ $\therefore$ E
d_{next}	$0+2 = 2$	$2-2=0$	–
$-y, -x$	0, 0	0, -1	-1, -2



5. Lecture -06

(a) Consider a triangle with vertices A(1, 1), B(5, 1), and C(3, 3) and color values of red(1, 0, 0), green(0, 0.9, 0), and blue(0, 0, 0.8) at each vertex of the triangle. Find the color of the point P(3, 2) inside the triangle using the concept of barycentric interpolation.	[7]
5. a. Solution: 024	

Barycentric Interpolation.

Given, $A(1,1)$, $B(5,1)$, $C(3,3)$; $P(3,2)$

$$\beta = \frac{f_{AC}(x,y)}{f_{AC}(x_B,y_B)}$$

$$= \frac{(y_A-y_c)x + (x_c-x_a)y + x_a y_c - x_c y_a}{(y_a-y_c)x_B + (x_c-x_a)y_B + x_a y_c - x_c y_a}$$

$$= \frac{(1-3)\,3 + (3-1)\,2 + 1\times 3 - 3\times 1}{(1-3)\,5 + (3-1)\,1 + 1\times 3 - 3\times 1}$$

$$= 0.25$$

$$\gamma = \frac{f_{AB}(x,y)}{f_{AB}(x_c,y_c)}$$

$$= \frac{(y_A-y_B)\,x + (x_B-x_a)\,y + x_a y_B - x_B y_a}{(y_a-y_B)\,x_c + (x_B-x_a)\,y_c + x_a y_B - x_B y_a}$$

$$= \frac{(1-1)\,3 + (5-1)\,2 + 1\times 1 - 5\times 1}{(1-1)\,3 + (5-1)\,3 + 1\times 1 - 5\times 1}$$

$$= 0.5$$

$\therefore \alpha+\beta+\gamma = 1 \Rightarrow \alpha = 1 - 0.25 - 0.5 = 0.25$

So, for $P(3, 2) \Rightarrow P(0.25, 0.25, 0.50)$

$P(x, y) \Rightarrow P(\alpha, \beta, \gamma)$

For color,

Given, red $(1, 0, 0)$, green $(0, 0.9, 0)$,

blue $(0, 0, 0.8)$

we know,

$$C = \alpha C_0 + \beta C_1 + \gamma C_2$$

$$\Rightarrow C = 0.25 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0.25 \begin{bmatrix} 0 \\ 0.9 \\ 0 \end{bmatrix} + 0.50 \begin{bmatrix} 0 \\ 0 \\ 0.8 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.225 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \text{red} \\ \text{green} \\ \text{blue} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.225 \\ 0.4 \end{bmatrix}$$ Ans.

Enigma41

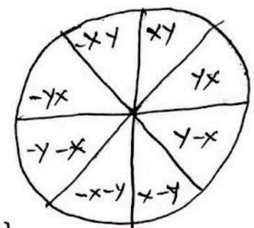
4. Lecture -06

(a) Apply the midpoint algorithm to draw a circle's portions of circumference centered at (-5,-1) on the 5^{th}, 6^{th}, 7^{th} and 8^{th} octant with radius 6. Plot the obtained points. For each step, show the values of the decision variables and the points (in a tabular format). **[9]**

4. a. Solution: Rabab 039

Enigma41 4(a)

$h = 1 - R = 1 - 6 = -5$

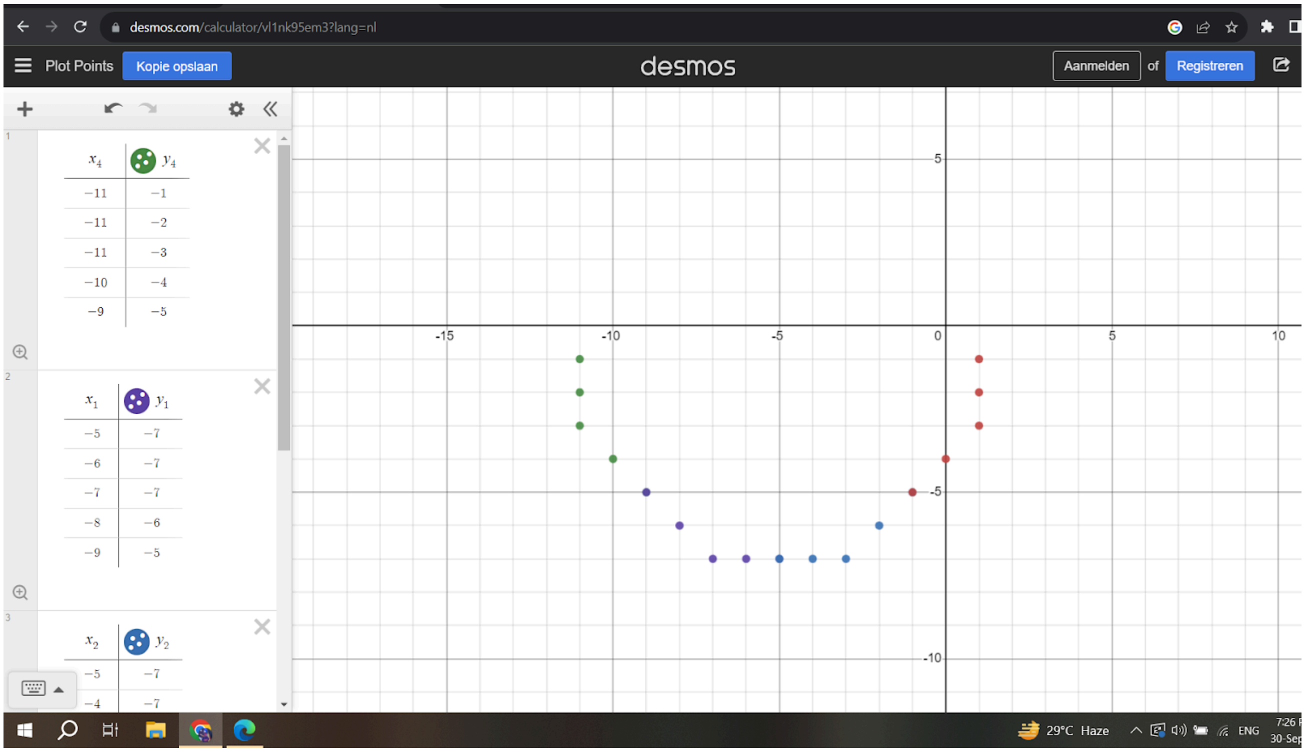


For 2nd octant,

#	x, y	h	h_{next}
0	0, 6	$-5 < 0$ ∴ E	$-5 + 2x + 3 = -5 + (2\times0) + 3 = -2$
1	1, 6	$-2 < 0$ ∴ E	$-2 + (2\times1) + 3 = 3$
2	2, 6	$3 > 0$ ∴ SE	$3 + (2\times2) + 5 - (2\times6) = 0$
3	3, 5	$0 \geq 0$ ∴ SE	$0 + (2\times3) - (2\times5) + 5 = 1$
4	4, 4		

Now, for other octants, and shifting by (-5, -1),

5th		6th		7th		8th	
-y, -x	-y', -x'	-x, -y	-x', -y'	x, -y	x', -y'	y, -x	y', -x'
-6, 0	-11, -1	0, -6	-5, -7	0, -6	-5, -7	6, 0	1, -1
-6, -1	-11, -2	-1, -6	-6, -7	1, -6	-4, -7	6, -1	1, -2
-6, -2	-11, -3	-2, -6	-7, -7	2, -6	-3, -7	6, -2	1, -3
-5, -3	-10, -4	-3, -5	-8, -6	3, -5	-2, -6	5, -3	0, -4
-4, -4	-9, -5	-4, -4	-9, -5	4, -4	-1, -5	4, -4	-1, -5



5. Lecture -06

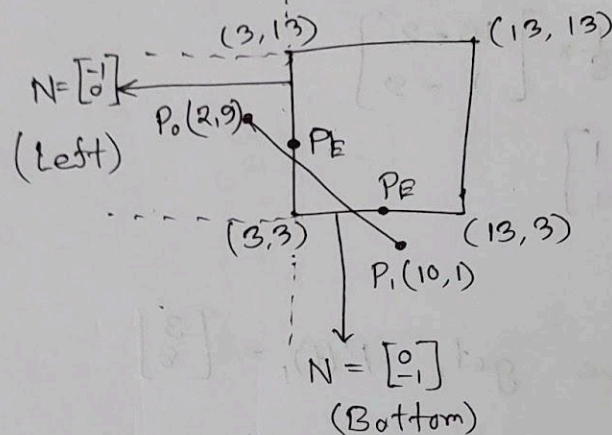
Question 5. [Marks: 14]

(a) Consider a clipping rectangle defined by the vertices (3,3), (13,3),(13,13) and (3,13). Also, consider a line which has starting and ending points of (10,1) and (2,9) respectively. Find the line-edge intersecting points with respect to all four edges of the clipping rectangle using the Cyrus-Beck clipping algorithm and determine the true clipping points. Show the steps and calculations for your solution. **[8]**

5. a. Solution: 024, Correction: 018

Start point = P0(10, 1) and end point = P1(2, 9)

Cyrus - Beck Line Algorithm



For Left:

$N_L = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$P_E = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$;(Let) and $P_0 = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$, $P_1 = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$

we know,

$$t_1 = \frac{N \cdot [P_0 - P_E]}{-N \cdot [P_1 - P_0]}$$

$$= \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2-3 \\ 9-7 \end{bmatrix}}{-\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}}{-\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix}} = \frac{1}{8}$$

Again we know,

$$P(t) = P_0 + t(P_1 - P_0)$$

$$\Rightarrow \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \frac{1}{8} \times \begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

So, for Left we get, $P(t)_1 = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

For bottom:

$N_b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$P_E = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ (let) and $P_0 = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$, $P_1 = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$

We know,

$$t_2 = \frac{N \cdot [P_0 - P_E]}{-N \cdot [P_1 - P_0]}$$

$$= \frac{\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2-7 \\ 9-3 \end{bmatrix}}{-\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 6 \end{bmatrix}}{-\begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix}} = \frac{-6}{-8} = \frac{3}{4}$$

D = P1 – P0 *N.D* **< 0 →** *Pe* **•** *N.D* **> 0 →** *P*

So,

$P(t) = P_0 + t(P_1 - P_0)$

$$\Rightarrow \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \frac{3}{4}\begin{bmatrix} 10-2 \\ 1-9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 9 \end{bmatrix} + \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

So, for bottom we get $P(t)_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

$\therefore$ Line-edge intersecting points are $P(t)_1 = (3, 8)$ and $P(t)_2 = (8, 3)$.

<u>For true clipping ~~mask~~: points:</u>

$N_l \cdot D = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix} = -8 < 0 \rightarrow$ Potentially entering

$N_b \cdot D = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -8 \end{bmatrix} = 8 > 0 \longrightarrow$ Potentially leaving

$\therefore$

P_e	P_l
$t_1 = 0.125$	$t_2 = 0.75$

As $\max(P_e) < \min(P_l)$; So we can say that $t_1 = 0.125$ and $t_2 = 0.75$ are true intersecting points.

6. Lecture -06

(a)	Consider a pentagon ABCDE with vertices A(1,3), B(5,3), C(8,5.5), D(4,7) and E(1,7). Using the concept of barycentric coordinate, determine if a point P(6, 3.5) is inside the pentagon or not. Describe your approach and show your calculations.	**[7]**

6. a. Solution: 109

021 - Need to check all triangles (or until a triangle is found with P inside it)

nigma 6(a) A(1,3) B(5,3), C(8, 5·5), ~~E(8·5, 5)~~ D(4,7) E(1,7)

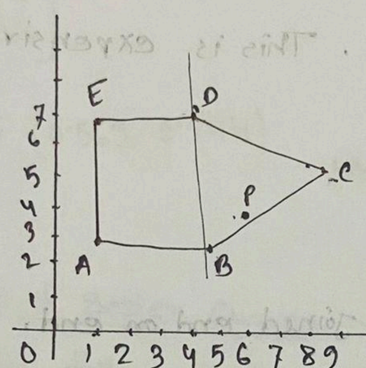
P(6, 3·5)

$B(x_b, y_b) = (5,3)$

$C(x_c, y_c) = (8, 5.5)$

$D(x_d, y_d) = (4,7)$

$P(x, y) = (6, 3.5)$



$$\beta = \frac{f_{cd}(x, y)}{f_{cd}(x_b, y_b)}$$

$$= \frac{(y_c - y_d)x + (x_d - x_c)y + x_c y_d - x_d y_c}{(y_c - y_d)x_b + (x_d - x_c)y_b + x_c y_d - x_d y_c}$$

$$= \frac{(5.5 - 7) \times 6 + (4 - 8) \times 3.5 + (8 \times 7) - (4 \times 5.5)}{(5.5 - 7) \times 5 + (4 - 8) \times 3 + 56 - 22}$$

$$= \frac{-9 - 14 + 56 - 22}{-7.5 - 12 + 56 - 22} = 0.759$$

$$\gamma = \frac{f_{bc}(x, y)}{f_{bc}(x_d, y_d)}$$

$$= \frac{(y_b - y_c)x + (x_c - x_b)y + x_b y_c - x_c y_b}{(y_b - y_c) \times x_d + (x_c - x_b) y_d + x_b y_c - x_c y_b}$$

$$= \frac{(3 - 5.5) \times 6 + (8 - 5) 3.5 + (5 \times 5.5) - (8 \times 3)}{(3 - 5.5) \times \cancel{6}4 + (8 - 5) \times 7 + (5 \times 5.5) - (8 \times 3)}$$

$$= \frac{-15 + 10.5 + 27.5 - 24}{-10 + 21 + 27.5 - 24} = 0.276$$

$$\alpha = 1 - \beta - \gamma$$

$$= (1 - 0.759 - \cancel{2.76}\ 0.276) = -0.035$$

Not inside.

Recursive40

1.

(3) Apply the midpoint algorithm to draw a line from $(2, 1)$ to $(-8, -6)$ and plot the obtained points. [11]
Show step-wise values of the decision variables and the points (in a tabular format).

1. a. Solution: ch6

3. lecture-6

(a) Suppose we have a 2D quad $OABC$ with the vertices $O(0,0)$, $A(1,0.5)$, $B(2,1.5)$ and $C(0.75,3)$. Using the concept of barycentric coordinate, determine if a point $P(1.5,2.5)$ is inside the quad. Describe your approach and show your calculations. [6]

3. a. Solution: Rabab 039

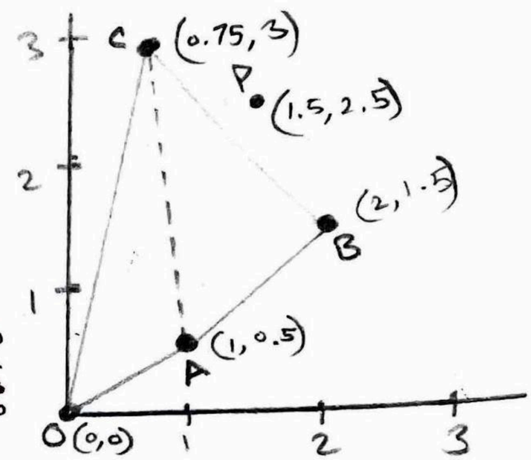
021 - Need to check all triangles (or until a triangle is found with P inside it)

Recursive40, 3(a)

$$\alpha = \frac{f_{bc}(x,y)}{f_{bc}(x_a,y_a)}$$

$$= \frac{(y_b-y_c)x + (x_c-x_b)y + x_b y_c - x_c y_b}{(y_b-y_c)x_a + (x_c-x_b)y_a + x_b y_c - x_c y_b}$$

$$= \frac{(1.5-3)1.5 + (.75-2)2.5 + (2\times3) - (.75\times1.5)}{(1.5-3)1 + (.75-2)0.5 + (2\times3) - (.75\times1.5)} = -0.181$$



Shurutei khela shesh. There's a negative value so, outside

4. . lecture-6

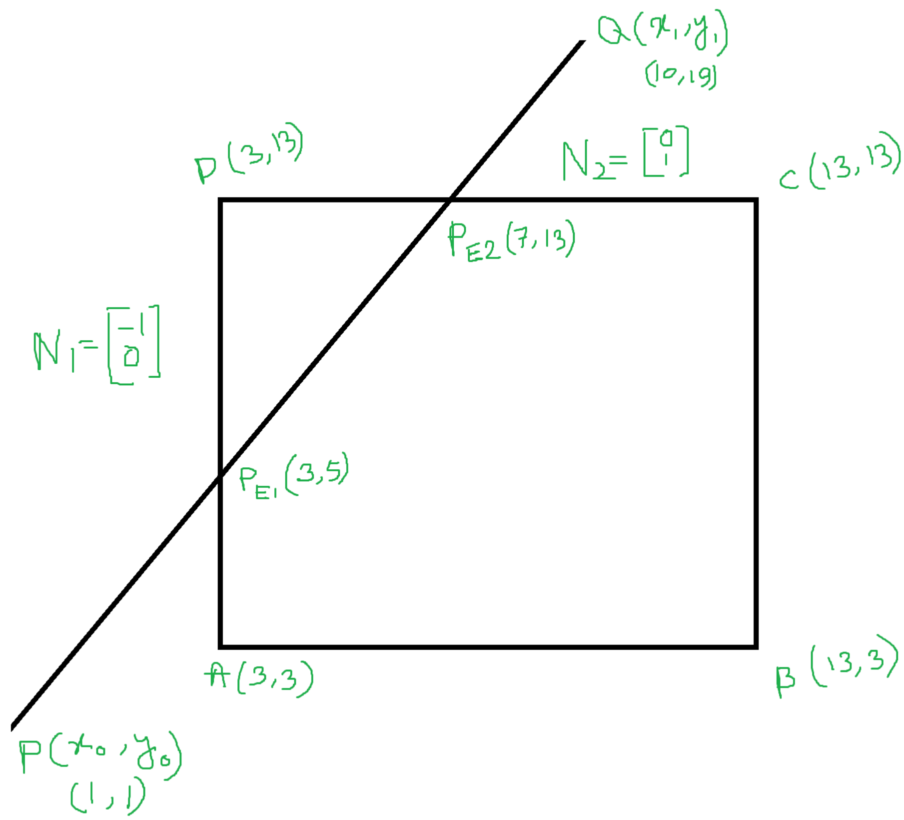
a) Apply the midpoint algorithm to draw a circle's portions of circumference centered at $(2,0)$ on the 3^{rd}, 4^{th}, 5^{th} and 6^{th} octant with radius 7. Plot the obtained points. For each step, show values of the decision variables and the points (in a tabular format). [8]

4. a. Solution:

7. lecture-6

a)	Consider a clipping rectangle which has width and height of 10 units. Its lower left corner is located at (3, 3). Also consider a line which has a starting point at (1, 1), length = 20 units, and *slope* = 2. Perform the line-edge intersecting points with respect to *all four edges* of the clipping rectangle using Cyrus-Beck algorithm and determine the true clipping points. Show your steps and calculations for your solution (assume any data if necessary).	[8]

7. a. Solution: Younus-131



Given, $m = 2$

$$\Rightarrow \frac{y_1 - y_0}{x_1 - x_0} = 2$$

$$\Rightarrow \frac{y_1 - 1}{x_1 - 1} = 2$$

$$\Rightarrow 2x_1 - 2 - y_1 + 1 = 0$$

$$\Rightarrow y_1 = 2x_1 - 1 \quad \text{---} \quad (1)$$

$\therefore$ Length of PQ = 20

$$\Rightarrow \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} = 20$$

$$\Rightarrow (1 - x_1)^2 + (1 - y_1)^2 = 400$$

$$\Rightarrow 1^2 - 2x_1 + x_1^2 + 1^2 - 2y_1 + y_1^2 = 400$$

$$\Rightarrow 1 - 2x_1 + x_1^2 + 1 - 2(2x_1 - 1) + (2x_1 - 1)^2 = 400$$

$$\Rightarrow 5x_1^2 - 10x_1 - 395 = 0 \quad [\text{from } (1)]$$

$x_1 = 9.94 \approx 10$ & $x_1 = -7.94 \approx -8$

From (1) $\Rightarrow$ $y_1 = 2x_1 - 1$

$\therefore y_1 = 19$ & -17

$(-8, -17)$ is invalid because PQ line intersects the rectangle. So $Q \equiv (10, 19)$.

The PQ line intersects the AD line at P_{E1} point.

Eq^n of AD line is $x = 3$ [parallel line of y axis]

From (1) $\Rightarrow$ $y_1 = 2 \times 3 - 1 = 5$

$\therefore P_{E1} \equiv (3, 5)$

PQ line intersects the CD line at P_{E2} point.

Eq^n of CD line is $y = 13$ [parallel line of x axis]

From (1) $\Rightarrow$ $13 = 2x - 1$

$\Rightarrow x = 7$

$\therefore P_{E2} \equiv (7, 13)$

For left half Plane:

$$t_1 = \frac{N_1 [P_0 - P_{E1}]}{N_1 [P_1 - P_0]} = \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}}{-\begin{bmatrix} -1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}}$$

$$= \frac{\begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \end{bmatrix}}{-\begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} 9 \\ 18 \end{bmatrix}} = \frac{2}{9} = 0.222$$

For top half Plane:

$$t_2 = \frac{N_2 [P_0 - P_{E2}]}{-N_2 [P_1 - P_0]} = \frac{\begin{bmatrix} 0 \\ 1 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 13 \end{bmatrix} \right\}}{-\begin{bmatrix} 0 \\ 1 \end{bmatrix} \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}}$$

$$= 0.666$$

Intersection points are:

$$P(t_1) = P_0 + t_1 (P_1 - P_0)$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.222 \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 2.98 \\ 4.99 \end{bmatrix} \approx \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$P(t_2) = P_0 + t_2 (P_1 - P_0)$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.66 \left\{ \begin{bmatrix} 10 \\ 19 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 6.94 \\ 12.88 \end{bmatrix} \approx \begin{bmatrix} 7 \\ 13 \end{bmatrix} \quad \text{Answer}$$

Lecture 8

Decipher44

b) Find the dimension of a fractal generated by the following steps below: [5]
Step 1: Start with an unfilled equilateral triangle at stage 0
Step 2: For each unfilled triangle, make a filled copy
Step 3: Scale the copy by half and invert it
Step 4: Place the copy in the center of the unfilled triangle

8 3

d) A curve is characterized by the following rules: [5]
variables: X, F
constants: +, −

8

axiom: F+XF+F+XF
rules: X -> XF-F+F-XF+F+XF-F+F-X
Angle: 90 degrees

Here, X means "do nothing", F means "draw a line forward", + means "rotate clockwise by 90 degrees" and − means "rotate counter-clockwise by 90 degrees". Apply the concept of L-systems to draw the curve for the second iteration.

d) Using the iterative scheme of the Mandelbrot Set for a maximum iteration up to 10 steps, determine whether the complex number c = -0.771 - 0.326i is a member of the Mandelbrot set or not. Also determine the color of the point using the following criteria:

8

Number of iterations to divergence	Color
less than 5	Red
5 to 8	Yellow
more than 8	Blue

Integer43

c) A curve is characterized by the following rules: [5]

variables: F
constants: +, −
axiom: F
rules: (F → F+F−F−F+F)
Angle: 90 degrees

Here, F means "draw a line forward", + means "turn left 90 degrees", and − means "turn right 90 degrees". Apply the concept of L-systems to draw the curve for the second iteration.

f) Using the iterative scheme of the Mandelbrot Set for a maximum iteration up to 10 steps, determine whether the complex number $c = -0.5 + 0.5i$ is a member of the Mandelbrot set or not. [6]

Origin42

Solution:

Enigma41

Solution:

Recursive40

Solution:

Some Important Derivations

Chapter-2

2. Propose an alpha compositing formula for blending the colors of three objects C1, C2 and C3. Where C1 is the foreground of C2 and C2 is the foreground of C3 [6]

2. Solution: ***Added from "radia-all-mathcg.pdf"***

Ans: Let,

α = Alpha compositing parameter to blend C_1 and C_2

β = " " " " " C_2 " C_3

We know,

$$C = \alpha C_f + (1-\alpha) C_b$$

Here,
C_f - Foreground color
C_b - background color
C - mixed/final color

$$\therefore C = \alpha C_1 + (1-\alpha) C_2$$

$$= \alpha C_1 + (1-\alpha) \boxed{\{\beta C_2 + (1-\beta) C_3\}}$$

$$= \alpha C_1 + \beta C_2 - \alpha\beta C_2 + (1-\beta) C_3 - \alpha(1-\beta) C_3$$

$$= \alpha C_1 + (1-\alpha)\beta C_2 + (1-\alpha)(1-\beta) C_3$$

$\therefore$ This is the required alpha compositing formula.

Chapter-3

2. Derive the equation of a Bezier curve of degree 4 using de Casteljau's Algorithm. [6]
2. Solution: de Casteljau's Algorithm: $P_{i,j} = (1-u)\,P_{i,j-1} + u\,P_{i+1,j-1}$ $P_{0,4} = (1-u)\,P_{0,3} + u\,P_{1,3}$ $= (1-u)\,((1-u)\,P_{0,2} + u\,P_{1,2}) + u\,((1-u)\,P_{1,2} + u\,P_{2,2})$ $= (1-u)\,((1-u)\,((1-u)\,P_{0,1} + u\,P_{2,1}) + u\,((1-u)\,P_{1,1} + u\,P_{2,1})) + u\,((1-u)\,((1-u)\,P_{1,1} + u\,P_{2,1}) + u\,((1-u)\,P_{2,1} + u\,P_{3,1}))$ $= (1-u)\,((1-u)\,((1-u)\,((1-u)\,P_{0,0} + u\,P_{1,0})+ u\,((1-u)\,P_{2,0} + u\,P_{3,0})) + u\,((1-u)\,((1-u)\,P_{1,0} + u\,P_{2,0})) + u\,((1-u)\,P_{2,0} + u\,P_{3,0})))) + u\,((1-u)\,((1-u)\,((1-u)\,P_{1,0} + u\,P_{2,0})) + u\,((1-u)\,P_{2,0} + u\,P_{3,0}))) + u\,((1-u)\,((1-u)\,P_{2,0} + u\,P_{3,0})) + u\,((1-u)\,P_{3,0} + u\,P_{4,0})))\,))$

Chapter-4

1. Show that two successive reflections about either of the principle axis is equivalent to a single rotation about the coordinate origin. [8]
1. a. Solution: 1. Transformation matrix for two successive reflections about either of the principle axis, M1 = Ref-Y * Ref-X Single rotation matrix M2 = R(180) You need to show that, M1 == M2

1. Show that, transformation for a reflection about the line y = x, is equivalent to a reflection relative to the x-axis followed by counterclockwise rotations of 90 degree. [8]
1. a. Solution: 1. Transformation matrix for reflection about the line y = x, $M1 = R^{-1}(45) * Ref * R(45)$ Transformation matrix for Reflection relative to the x-axis followed by counterclockwise rotations of 90 degrees, M2 = R(90) * Ref-X You need to show that, M1 == M2 Solution: 053

$$M1 = R^{-1}(45) * Ref * R(45)$$

$$= \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$M2 = R(90) * Ref\text{-}X$$

$$= \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

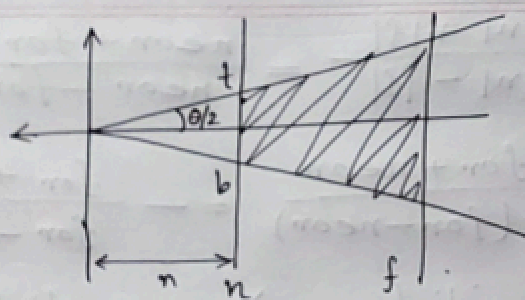
$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

So m1 == m2

Chapter-5

- Show that, the M_{OpenGL} can be written as follows –

$$\mathrm{M}_{\mathrm{OpenGL}} = \begin{bmatrix} \frac{2|n|}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2|n|}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{|n|+|f|}{|n|-|f|} & \frac{2|f||n|}{|n|-|f|} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{aspect * \tan(\frac{fov}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{fov}{2})} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2*far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



n: near
f : far
t : top
b : bottom

$$M(1,1) = \frac{2|n|}{r-l} = \frac{1}{\frac{r-l}{2|n|}} = \frac{1}{\frac{r-l}{t-b} \times \frac{t-b}{2|n|}}$$

$$= \frac{1}{aspect * \frac{\cancel{2}t}{\cancel{2}|n|}} \quad // \; b = -t$$

$$= \frac{1}{\underset{(ratio)}{aspect} * \tan\left(\frac{fov}{2}\right)}$$

$l = -r$
$b = -t$
; Field-of-view, fov = θ
$\tan(\theta/2) = \frac{t}{|n|} = \tan\left(\frac{fov}{2}\right)$

$$M(1,3) = \frac{r+l}{r-l} = \frac{r-r}{r-l} = 0$$

$$M(2,2) = \frac{2|n|}{t-b} = \frac{1}{\frac{t-b}{2|n|}} = \frac{1}{\frac{2t}{2|n|}} = \frac{1}{\tan\left(\frac{fov}{2}\right)}$$

$$M(2,3) = \frac{t+b}{t-b} = \frac{t-t}{t-b} = 0$$

Correction: M(2,2) er sheshe 1/tan(fov/2) hobe. corrected

$$M(3,3) = \frac{|n| + |f|}{|n| - |f|} = \frac{near + far}{near - far}$$

$$= \frac{far + near}{-(far - near)} = -\frac{far + near}{far - near}$$

// far distance > near distance
so putting far first.

$$M(3,4) = \frac{2|f||n|}{|n| - |f|} = \frac{2 * far * near}{-(far - near)}$$

$$= -\frac{2 * far * near}{far - near}$$