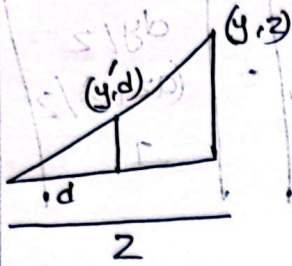


Perspective Projection



For 1D

$$\frac{y'}{d} = \frac{y}{z}$$

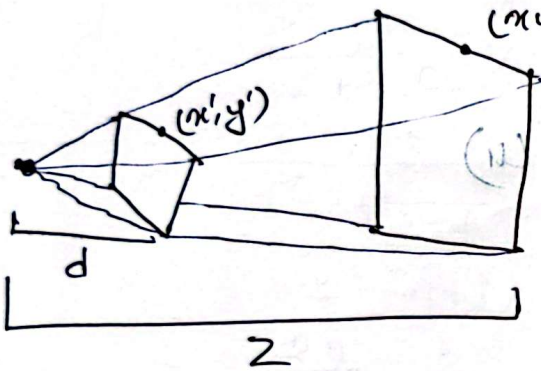
$$\Rightarrow y' = \frac{dy}{z}$$

$$\begin{bmatrix} d & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} y' \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} dy \\ z \end{bmatrix}$$

$$= \begin{bmatrix} dy/z \\ z/z \end{bmatrix} = \begin{bmatrix} dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} y' \\ 1 \end{bmatrix}$$

For 2D



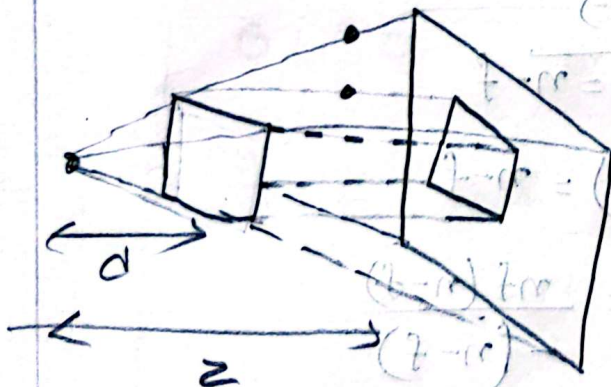
$$y' = \frac{dy}{z}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ z \end{bmatrix}$$

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} dx \\ dy \\ z \end{bmatrix} = \begin{bmatrix} dx/z \\ dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

For 3D



There will be always division by z
so $z' = z$ not possible.

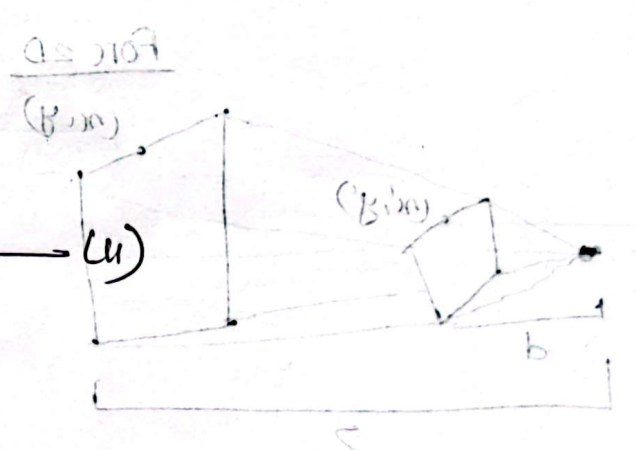
If $z' = z$ it would imply no perspective effect remaining objects at different depths would appear the same size as if viewed orthographically. To keep depth z into in 2D compressed z is important. so find new $z = az + b$

$$\begin{aligned} y' &= dy/z \\ x' &= dx/z \\ z' &= \frac{az+b}{z} \end{aligned} \quad \begin{vmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix} = \begin{vmatrix} dx \\ dy \\ az+b \\ z \end{vmatrix} = \begin{vmatrix} dx/z \\ dy/z \\ (az+b)/z \\ 1 \end{vmatrix}$$

Now determine (a, b)

$$\begin{aligned} z &= n \\ x &= 0 \\ y &= 0 \end{aligned} \quad \begin{bmatrix} 0 \\ 0 \\ (an+b)/n \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ n \\ 1 \end{bmatrix} \quad \text{--- (u)}$$

$$\begin{aligned} z &= f \\ x &= 0 \\ y &= 0 \end{aligned} \quad \begin{bmatrix} 0 \\ 0 \\ (fa+b)/f \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ f \\ 1 \end{bmatrix} \quad \text{--- (u)}$$



(i) $\bar{u}(u)$

$$\begin{aligned} \frac{an+b}{n} &= n \\ \frac{fa+b}{f} &= f \end{aligned} \quad \begin{aligned} a + \frac{b}{n} &= n \\ a + \frac{b}{f} &= f \end{aligned} \quad \begin{aligned} \frac{a+b}{f} &= \frac{n-f}{f} \\ b\left(\frac{1}{n} - \frac{1}{f}\right) &= n-f \\ \Rightarrow b\left(\frac{f-n}{nf}\right) &= n-f \end{aligned}$$

Put $b = -nf(i)$ $\Rightarrow b = \frac{nf(n-f)}{-(n-f)}$

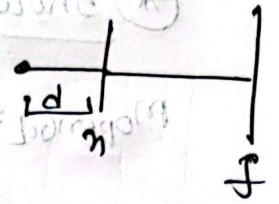
$$a + \frac{-nf}{n} = n \quad \boxed{b = -nf}$$

$$\Rightarrow a - f = n$$

$$\Rightarrow \boxed{a = n+f}$$

Perspective matrix:

$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$M = M_{vp} * M_{orth} * P * M_{cam}$$

$$M_{per} = M_{orth} * P$$

$$= \begin{bmatrix} \frac{2}{n-1} & 0 & 0 & -\frac{n+1}{n-1} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2n}{n-1} & 0 & -\frac{n+1}{n-1} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{2(n+f)}{n-f} - \frac{n+f}{n-f} & \frac{2}{n-f}(-fn) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} & \frac{2nf}{f-n} \\ & \frac{n+f(n-1)}{n-f} \\ & = \frac{nf}{f-n} \end{aligned}$$

$$= \begin{bmatrix} \frac{2n}{n-1} & 0 & \frac{n+1}{1-n} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{n+f}{n-f} & \frac{2nf}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{per}$$

* Show that, the MopenGL can be written as follow:

$$M_{openGL} = \begin{bmatrix} \frac{2n}{n-1} & 0 & \frac{n+1}{n-1} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{|n|+|f|}{|n|-|f|} & \frac{2|f||n|}{|n|-|f|} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

where $m_{11} = \frac{2n}{n-1}$, $m_{13} = \frac{n+1}{n-1}$, $m_{22} = \frac{2n}{t-b}$, $m_{23} = \frac{t+b}{t-b}$, $m_{33} = \frac{|n|+|f|}{|n|-|f|}$, $m_{34} = \frac{2|f||n|}{|n|-|f|}$.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\text{aspect} * \tan(\frac{fov}{2})} & 0 & 0 \\ 0 & 0 & \frac{1}{\tan(\frac{fov}{2})} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1+n}{1-n} & 0 & \frac{1-n}{1-n} & 0 \\ 0 & \frac{t+b}{t-b} & \frac{t-b}{t-b} & 0 \\ 0 & 0 & \frac{t+n}{t-b} & \frac{2*far*near}{far-near} \\ 0 & 0 & \frac{far+near}{far-near} & \frac{2*far*near}{far-near} \end{bmatrix}$$

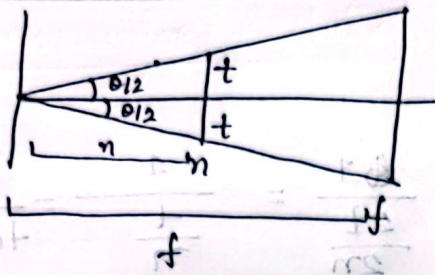
where $m_{11} = 1$, $m_{13} = \frac{1+n}{1-n}$, $m_{22} = \frac{1}{\tan(\frac{fov}{2})}$, $m_{23} = \frac{t+b}{t-b}$, $m_{33} = \frac{t+n}{t-b}$, $m_{34} = \frac{2*far*near}{far-near}$.

$$= \begin{bmatrix} 0 & \frac{1+n}{n-1} & 0 & \frac{1-n}{1-n} \\ 0 & \frac{t+b}{t-b} & \frac{t-b}{t-b} & 0 \\ \frac{t+n}{t-b} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $m_{11} = 0$, $m_{13} = \frac{1+n}{n-1}$, $m_{22} = \frac{t+b}{t-b}$, $m_{23} = \frac{t-b}{t-b}$, $m_{33} = \frac{t+n}{t-b}$, $m_{34} = 0$.

Start 8

$\theta/2 = \text{fov}$ [field of view]



From Mopen GL,

$$m_{11} = \frac{2n}{r-1}$$

$$= \frac{1}{\frac{r-1}{2n}}$$

$$= \frac{1}{\frac{r-1}{2n} \times \frac{t-b}{t-b}} \xrightarrow{\text{cal. বুজানো}} \text{নিম্নে আসছি}$$

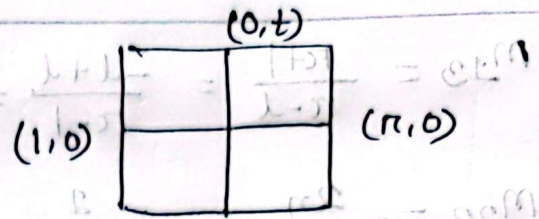
$$= \frac{1}{\frac{r-1}{t-b} \times \frac{2t-b}{2n}}$$

$$= \frac{1}{\text{aspect} \times \frac{t+t}{2n}}$$

$$= \frac{1}{\text{aspect} \times \frac{2t}{2n}}$$

$$= \frac{1}{\text{aspect} \times \frac{t}{n}}$$

$$= \frac{1}{\text{aspect} \tan\left(\frac{\text{fov}}{2}\right)}$$



$$r = -t; t = -b$$

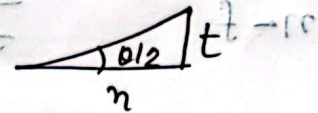
$$t-b = \text{height}$$

$$\Rightarrow t - (-t) = \frac{d-t}{d-t} = \text{height}$$

$$\Rightarrow 2t = \text{height}$$

$$\text{aspect} = \frac{r-1}{t-b} = \frac{\text{width}}{\text{height}}$$

$$\frac{r-1}{t-b} = \frac{2t-b}{2t-b} = \text{height}$$



$$\tan\left(\frac{\theta}{2}\right) = \frac{t}{n}$$

$$\theta/2 = \text{fov} = \tan^{-1}\left(\frac{t}{n}\right)$$

$$\text{aspect} = \frac{\text{width}}{\text{height}}$$

[view to b/obj] vob = obj

$$m_{13} = \frac{n+1}{n-1} = \frac{-1+1}{-1-1} = 0$$

$$m_{22} = \frac{2\eta}{t-b} = \frac{1}{\frac{t-b}{2\eta}} = \frac{1}{\frac{t-(t)}{2\eta}} = \frac{1}{\frac{0}{2\eta}} = \frac{1}{0} = \frac{1}{\tan(\frac{\text{fov}}{2})}$$

$$m_{23} = \frac{t+b}{t-b} = \frac{t+(t)}{t-b} = \frac{2t}{t-b} = 0$$

$$m_{33} = \frac{n+f}{n-f} = -\frac{n+f}{f-n} = -\frac{\text{far} + \text{near}}{\text{far} - \text{near}}$$

$$m_{34} = \frac{2fn}{n-f} = -\frac{2fn}{f-n}$$

From object

$$\frac{1}{1-n} = \text{obj}$$

$$\frac{1}{1-n} = \text{obj}$$

$$\frac{2 * \text{far} * \text{near}}{\text{far} - \text{near}}$$

$$\frac{d-f}{n} \times \frac{1-n}{f-d}$$

[Proved]

$$\frac{(f-d) \times \text{obj}}{n}$$

$$\frac{f+f \times \text{obj}}{n}$$

$$\frac{f}{n} \times \text{obj}$$

$$\frac{f}{n} \times \text{obj}$$

$$\frac{1}{\tan(\frac{\text{fov}}{2})} \times \text{obj}$$

(m) f - mat = vob = obj