CSE4227 Digital Image Processing

Lecture 05 – Chapter 03 – **Histogram Processing**

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Today's Contents

- **What is an image Histogram?**
- **How it is created from an image?**
- **The importance of image histogram in image processing.**
- **Some basic histogram operations:**
  - Normalized Histogram
  - Histogram Stretching
  - Histogram Equalization
  - Histogram Specification / Matching

- Chapter 3 from R.C. Gonzalez and R.E. Woods, Digital Image Processing (3rd Edition), Prentice Hall, 2008 [**Section 3.3**] [**Exercise Problems 3.1, 3.4, 3.5, 3.6, 3.7**]

Perceived Image Quality

Image A

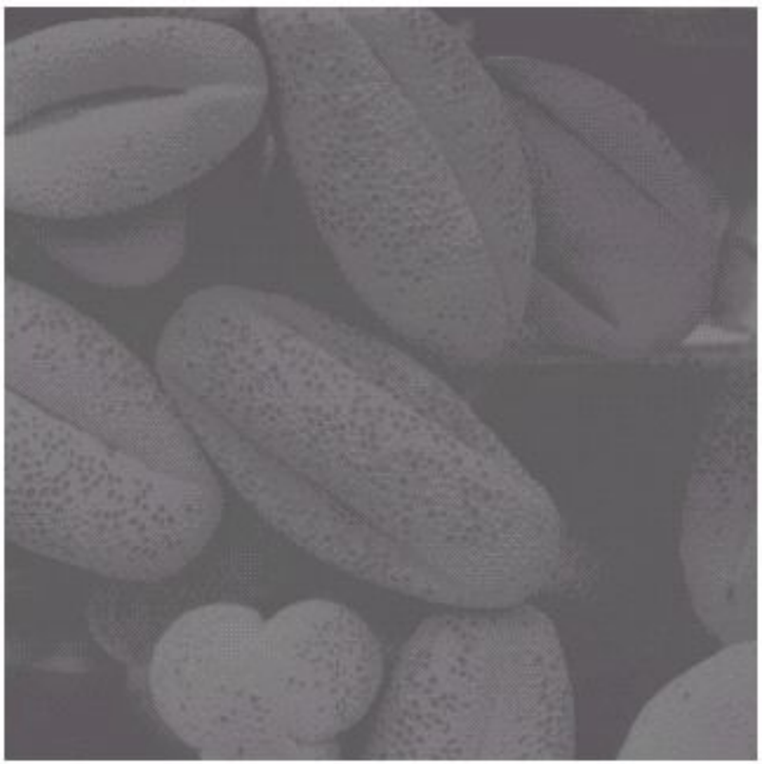


Image B

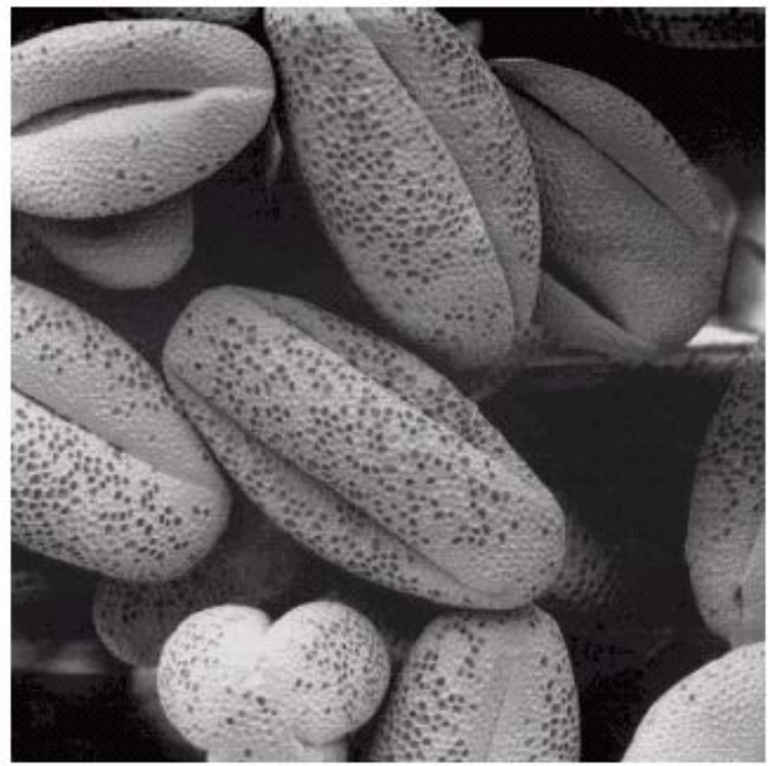


Image histogram - definitions

- Histogram
  - The histogram function is defined over all possible intensity levels.
  - For each intensity level, its value is equal to the number of the pixels with that intensity.
- Applications
  - Image enhancement, compression, segmentation
  - Very popular tool for realtime image processing

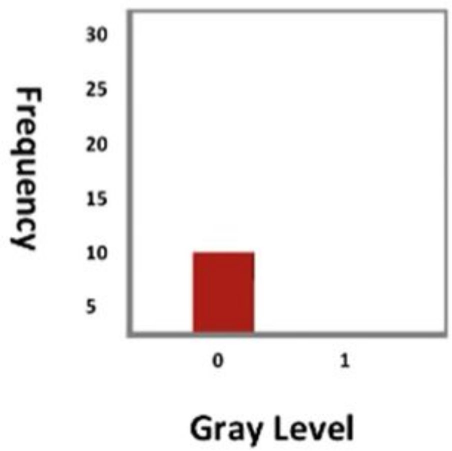
Generating an Image Histogram

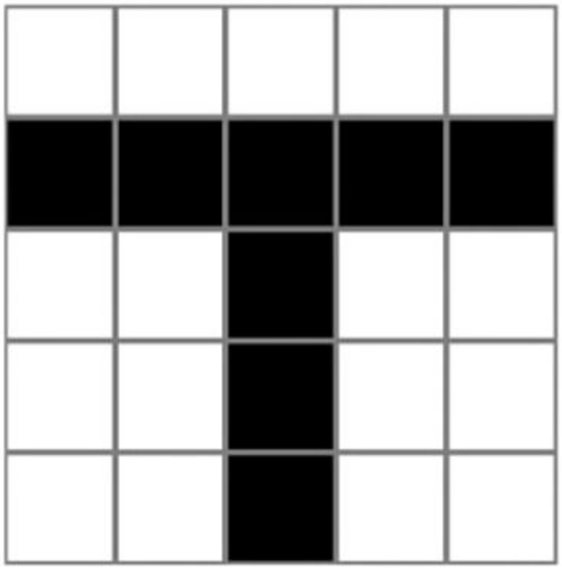
- The histogram generation process is a **systematic counting** of the number of pixels of every gray level
- The process is implemented by:
  - Iterating through all possible gray level values
  - For each gray level value, counting the number of pixels in the image that have that value.
  - Storing the results in a table or displaying it in a chart

Example (cont.)

- Step 1: count the number pixel with the gray level 0

Gray Level	Frequency
0	8
1	

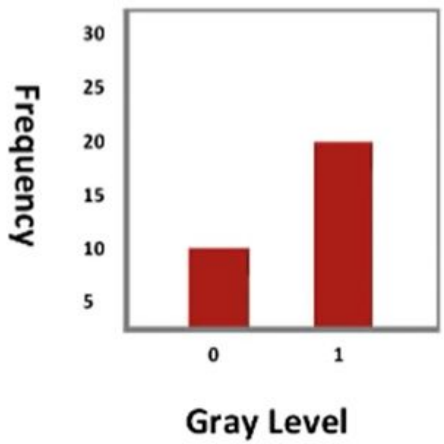


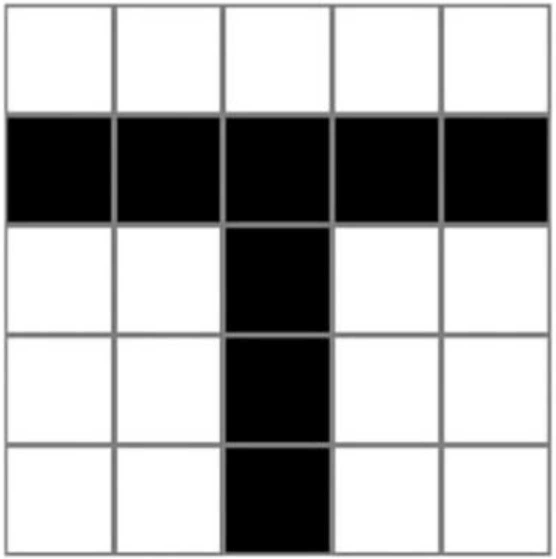


Example (cont.)

- Step 2: count the number pixel with the gray level 1

Gray Level	Frequency
0	8
1	17
Total	**25**





Introduction to Histogram

Example

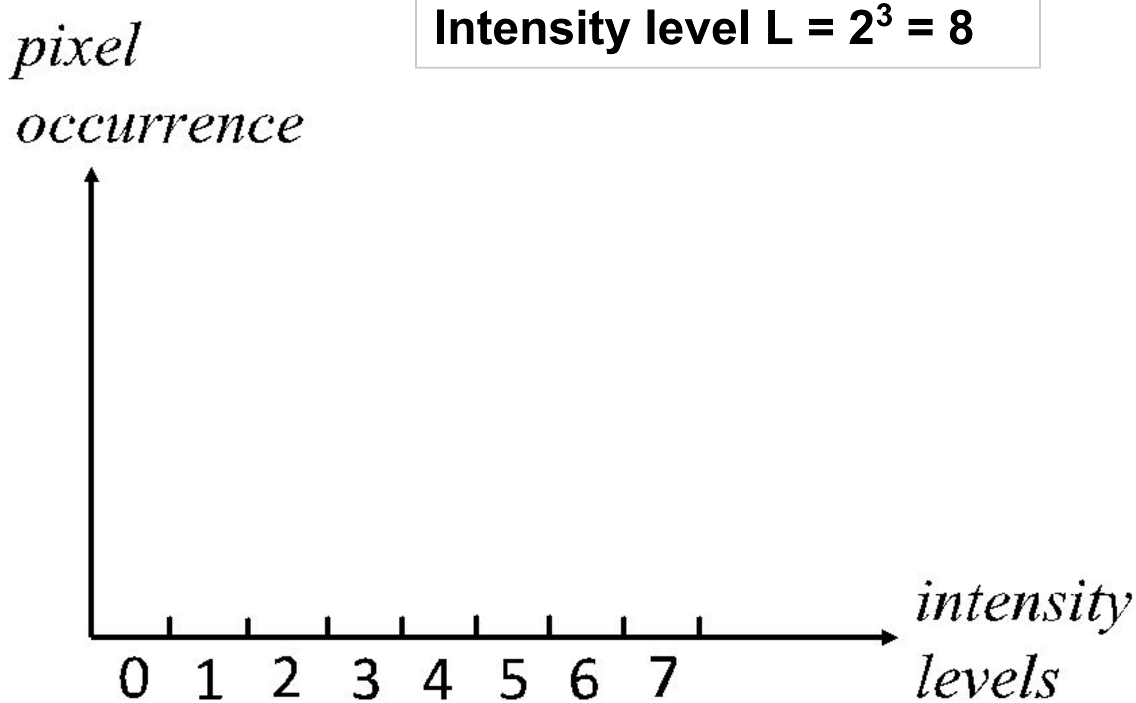
f is a 3-bit 4×4 image

Bin pos:	0	1	2	3	4	5	6	7
h:								

Intensity level L = 2^3 = 8

5	5	3	3
4	3	3	3
0	6	7	2
1	7	2	2

An image



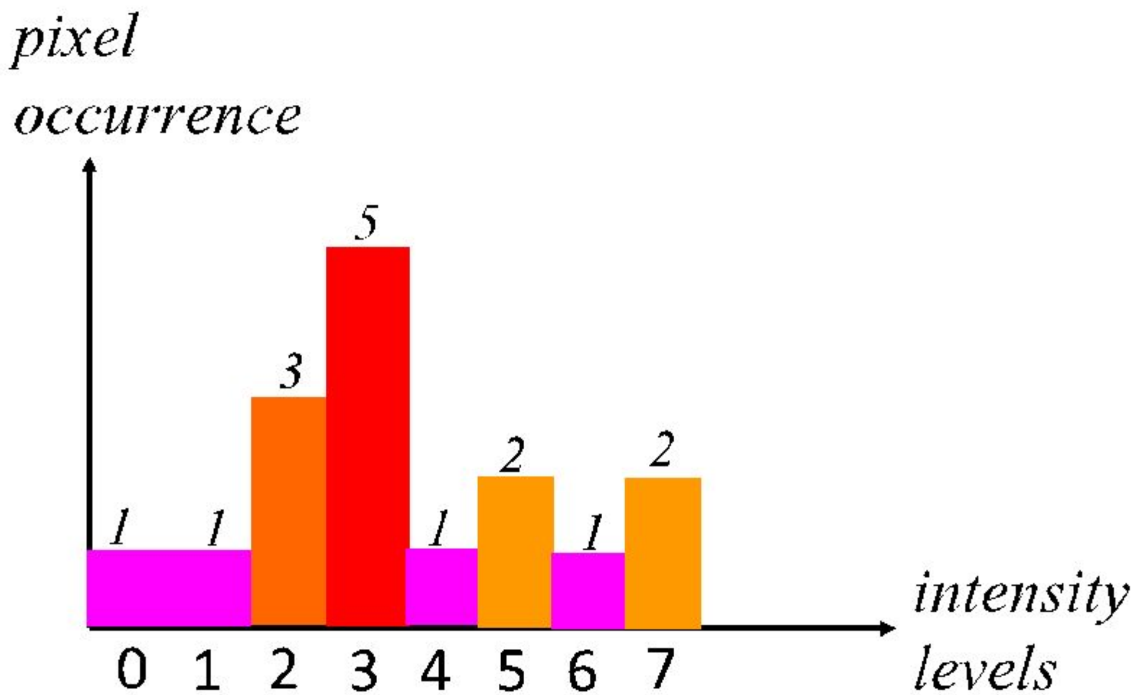
Introduction to Histogram

Example

f is a 3-bit 4×4 image

Bin pos:	0	1	2	3	4	5	6	7
h:	1	1	3	5	1	2	1	2

5	5	3	3
4	3	3	3
0	6	7	2
1	7	2	2



$$\sum_{x=0}^{2^b-1} h(x) = \textit{area of the image}$$

Image Histogram

- The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function:

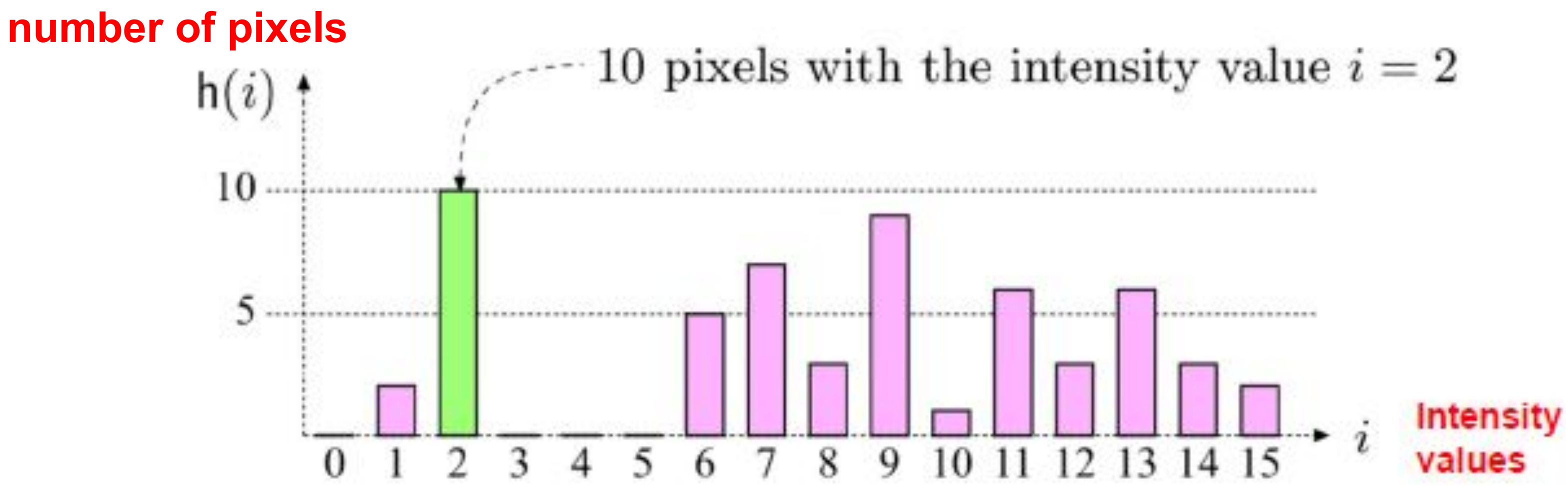
$$h(r_k) = n_k$$

Where:

r_k : kth gray level

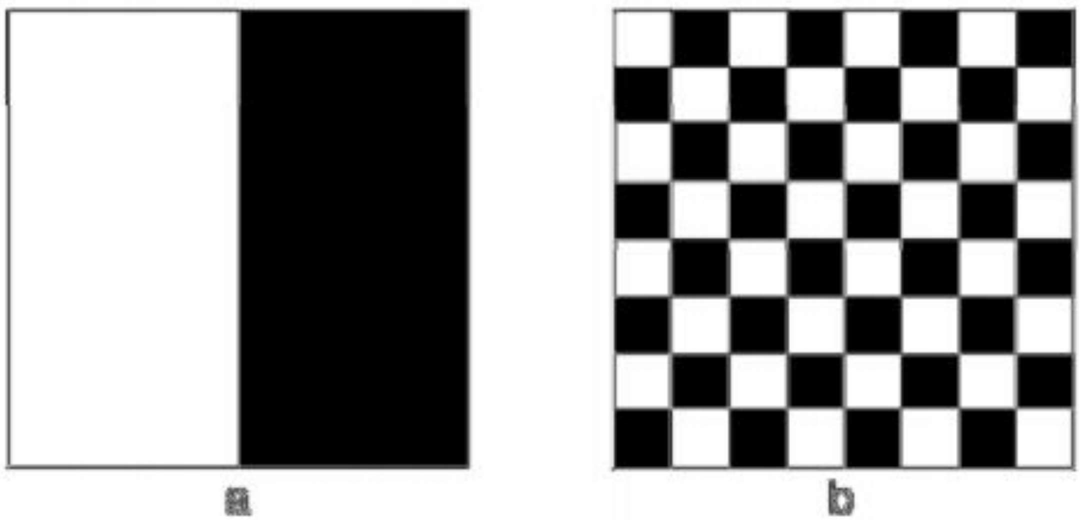
n_k : # of pixels with having gray level r_k

Histograms



h(i)	0	2	10	0	0	0	5	7	3	9	1	6	3	6	3	2
i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- E.g. L = 16, 10 pixels have intensity value = 2
- Histograms: only statistical information
- No indication of **location** of pixels



- Suppose that both images have size 20×20, with black (0) and white (1) pixels.

- **Their histograms are identical???** **YES**

Histograms

- Different images can have **same** histogram
- 3 images below have same histogram

- Half of pixels are gray, half are white
  - Same histogram = same statisics
  - Distribution of intensities could be different
- Can we reconstruct image from histogram?

Histograms

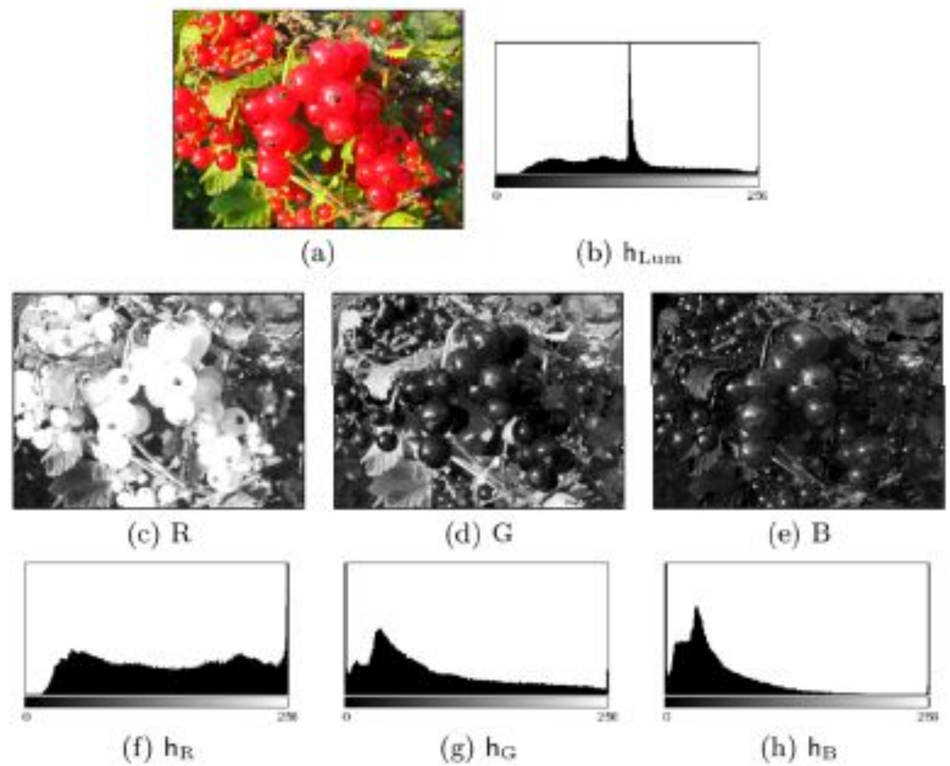
- Different images can have **same** histogram
- 3 images below have same histogram

- Half of pixels are gray, half are white
  - Same histogram = same statisics
  - Distribution of intensities could be different
- Can we reconstruct image from histogram? No!

Color Image Histograms

Two types:

1. **Intensity histogram:**
   - Convert color image to gray scale
   - Display histogram of gray scale
2. **Individual Color Channel Histograms:** 3 histograms (R,G,B)



Histogram: Example

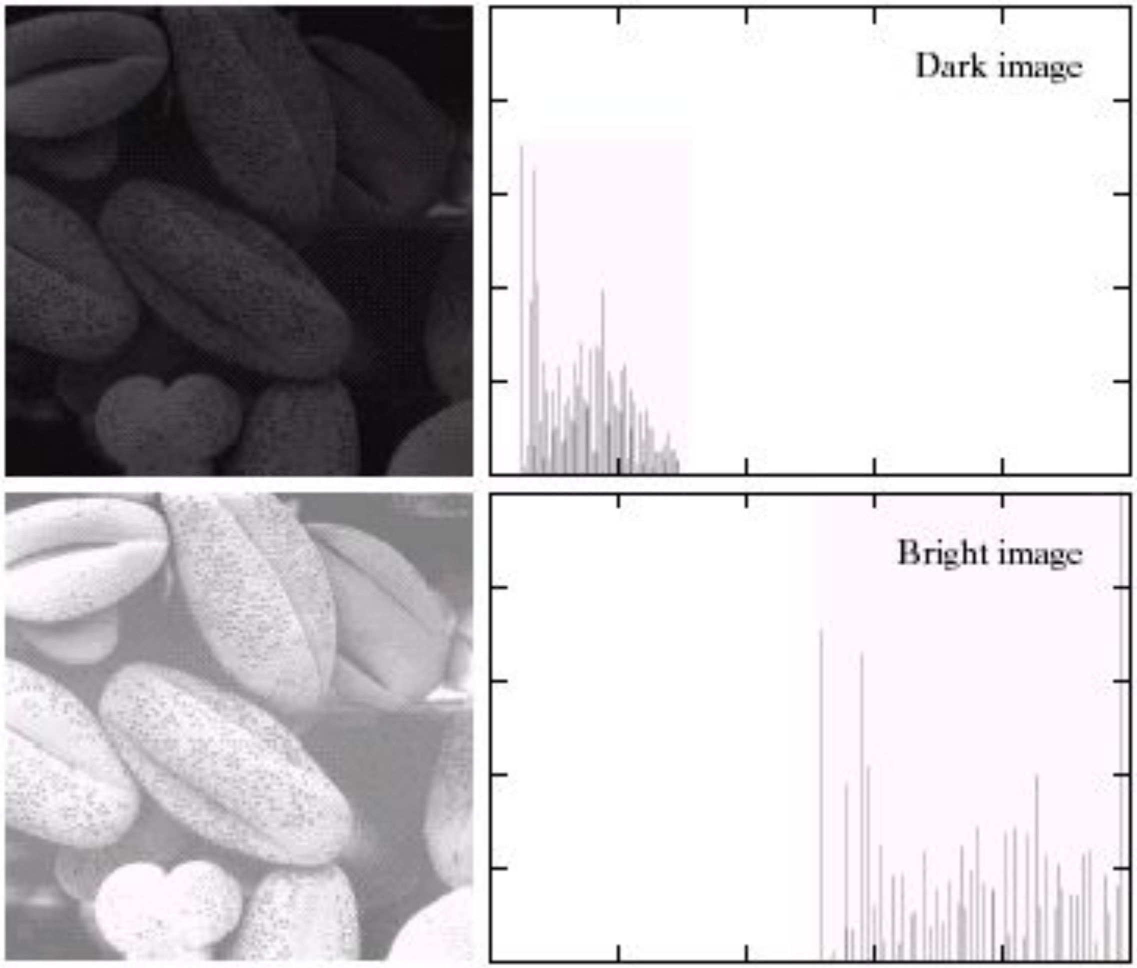
Four basic types of image and their corresponding histograms:

Dark Image

How would the histograms of these images look like?

Bright Image

Histogram: Example



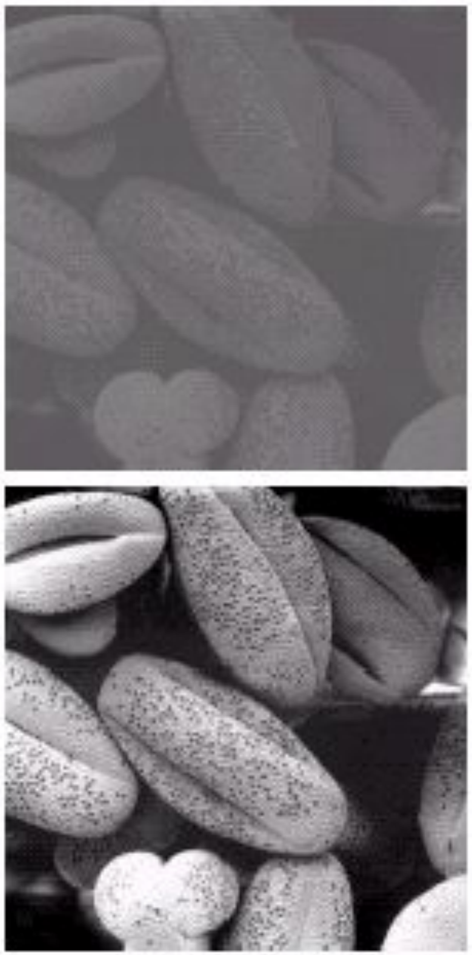
Dark image

Components of histogram are concentrated on the low side of the gray scale

Bright image

Components of histogram are concentrated on the high side of the gray scale

Histogram: Example

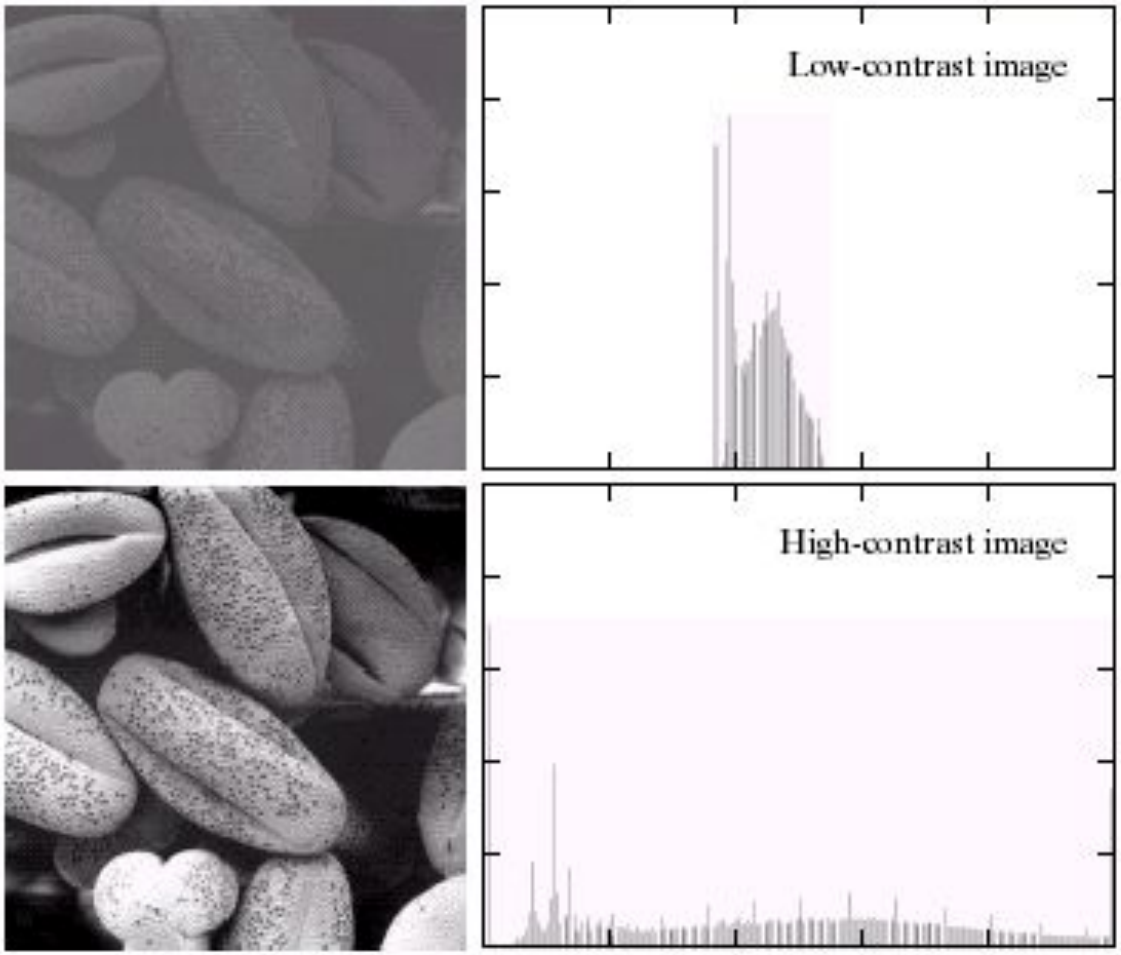


Low Contrast Image

How would the histograms of these images look like?

High Contrast Image

Histogram: Example



Low contrast image

Histogram is narrow and centered toward the middle of the gray scale

High contrast image

Histogram covers broad range of the gray scale and the distribution of pixels is not too far from uniform with very few vertical lines being much higher than the others

Why is the image histogram important?

- The shape of the histogram provides key information on
  - The overall quality of the image
  - The possibility for enhancement
- Visual inspection of the histogram is a fast way to analyze the overall image quality
- By processing the histogram an image can be quickly and automatically improved

Use of Histogram

- Histogram can tell us ***whether image was scanned properly or not***.
- It gives us ***idea about tonal distribution*** in the image.
- Histogram equalization can be applied ***to improve appearance*** of the image.
- Histogram also ***tells us about objects*** in the image.
- Object in an image have similar gray levels so histogram helps us ***to select threshold value for object detection***.
- Histogram can be used for ***image segmentation***.

Histogram Processing: Normalization

- Dividing each of histogram at gray level r_k by the total number of pixels in the image, n

$$p(r_k) = n_k / n$$

- $p(r_k)$ gives an estimation of the probability of occurrence of gray level r_k
- The sum of all components of a normalized histogram is equal to 1

Normalized Image Histogram(Cont.)

$$p(r_k) = n_k / n$$

where, $n = \sum_k n_k$

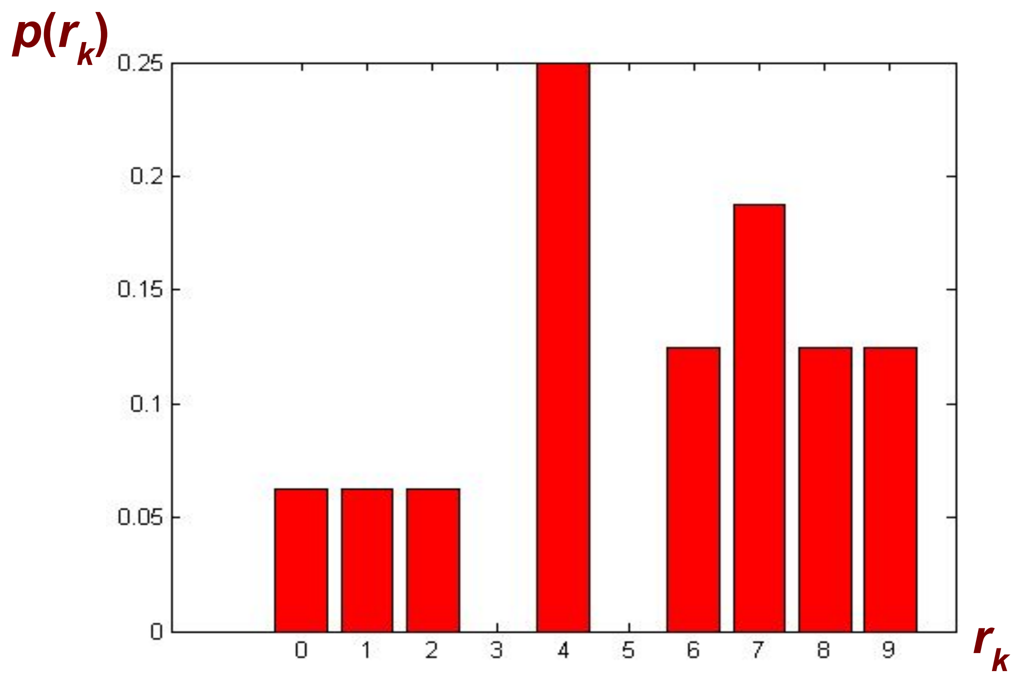
- Normalized histogram is more like probabilities
- $p(r_k)$: how likely a pixel will have gray level r_k

Normalized Image Histogram(Cont.)

Bin pos:	0	1	2	3	4	5	6	7	8	9
$h(r_k)$:	1	1	1	0	4	0	2	3	2	2
$p(r_k)$:	1/16	1/16	1/16	0	1/4	0	1/8	3/16	1/8	1/8

9	8	9	8
2	7	4	7
6	4	6	1
4	0	7	4

An image (cropped)



HISTOGRAM PROCESSING TECHNIQUES

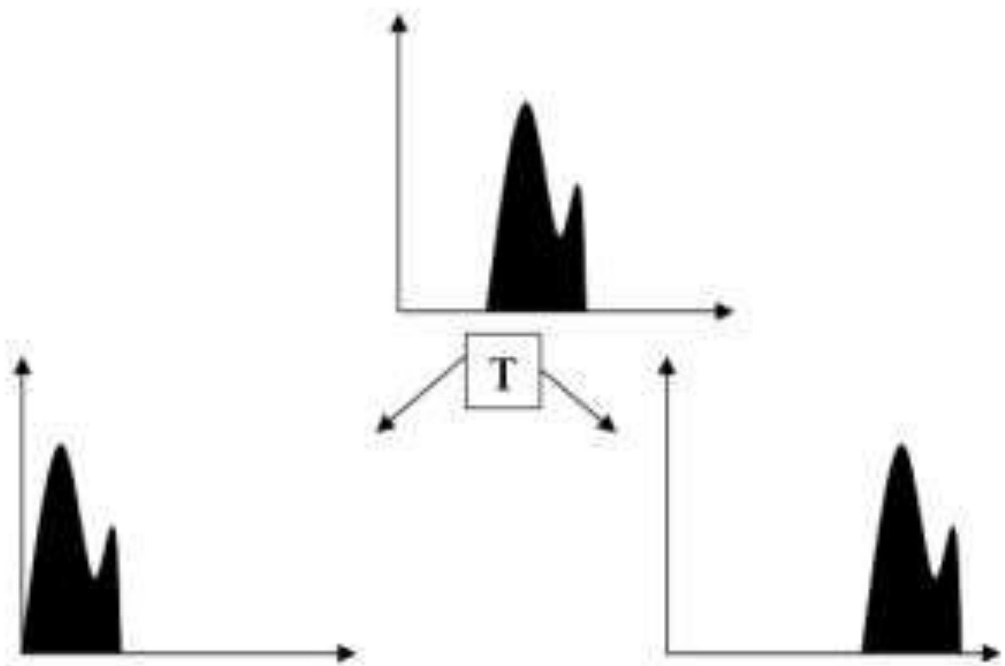
- The transformation function (processing technique) T is applied to an input image f(x, y) which gives the processed output image g(x, y).

$$g(x, y) = T\,(f(x, y))$$

i. **Histogram Sliding**
ii. **Histogram Stretching**
iii. **Histogram Equalization**

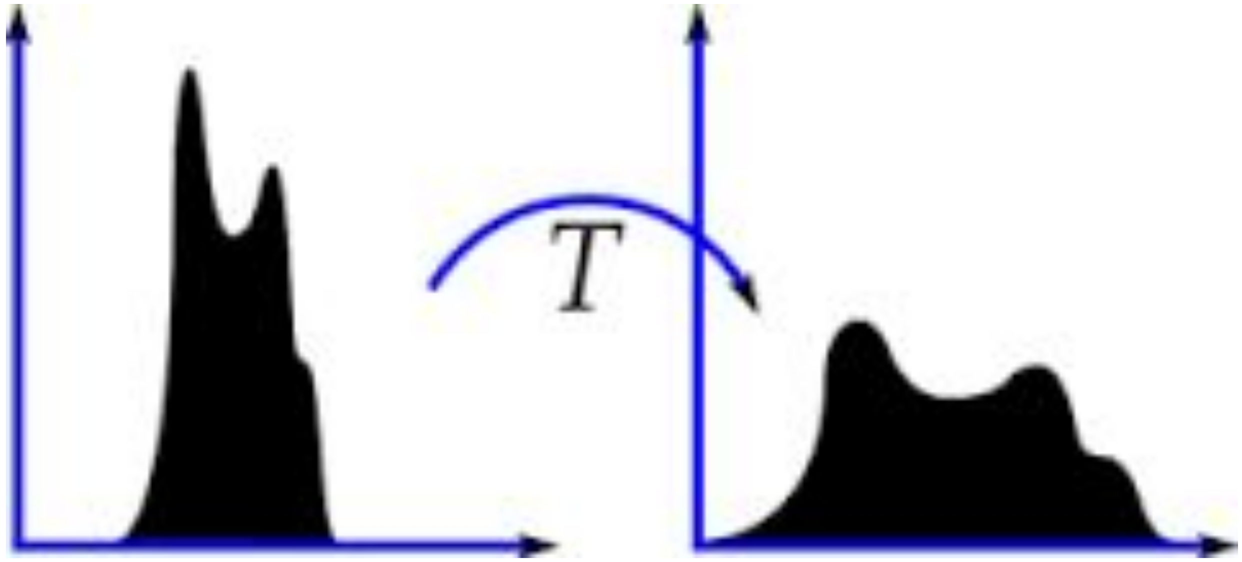
Histogram Sliding

- the complete histogram is simply shifted towards rightwards or leftwards.
- a clear change will seen in the brightness of image.

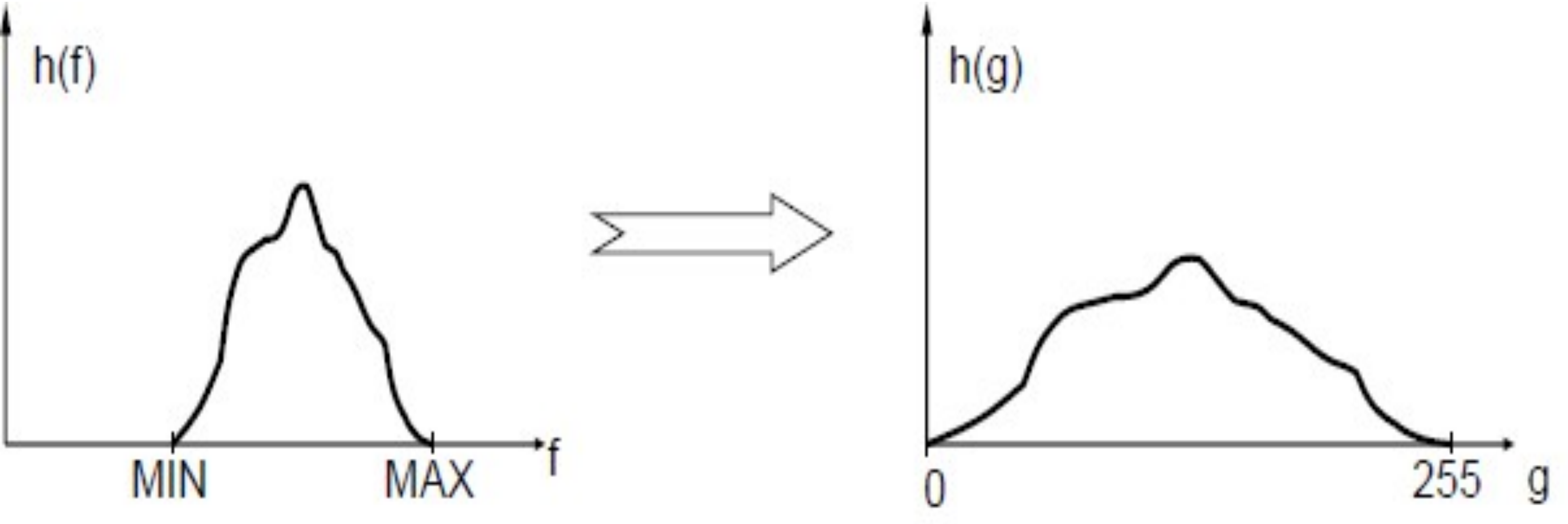


Histogram Stretching

- Histogram Stretching is process of increasing the contrast of an image.
- Contrast is defined as the difference between maximum and minimum pixel intensity values in an image.



Histogram Stretching



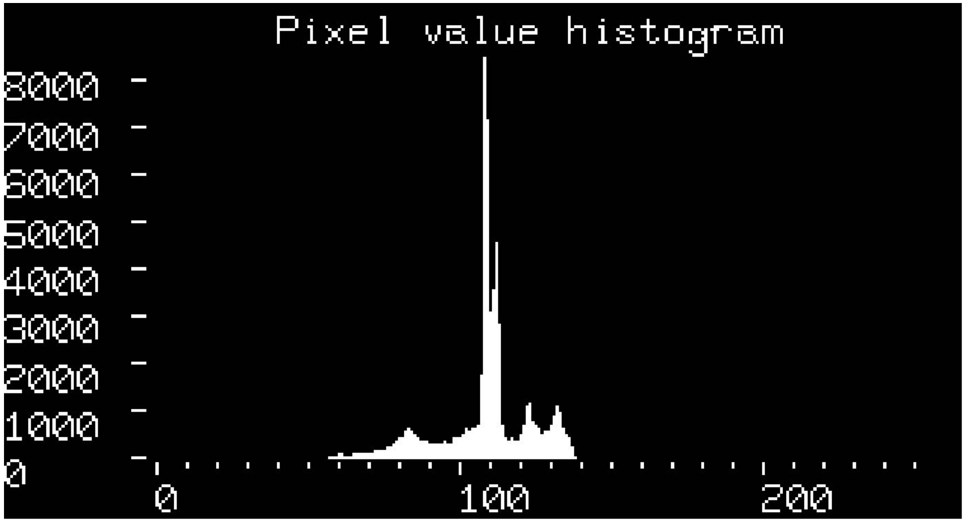
3) Rescale to [0,255] range

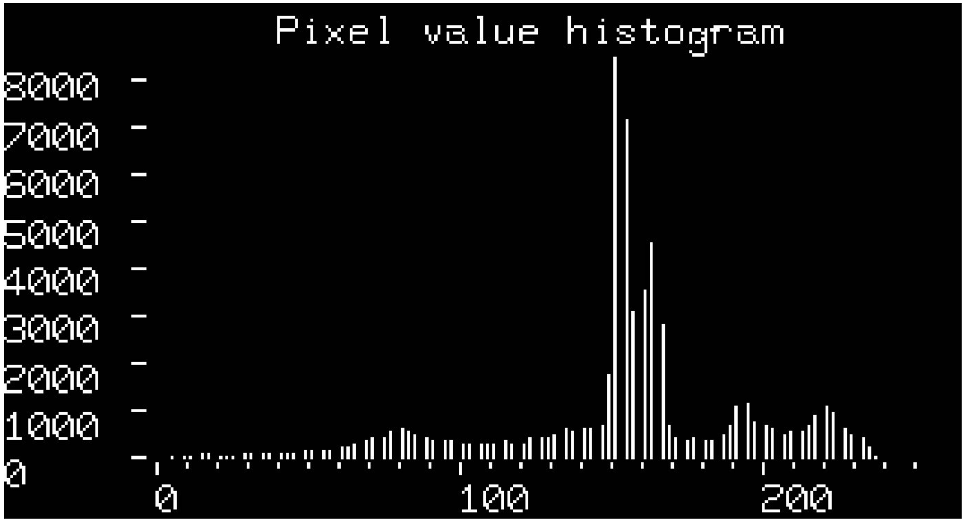
1) Slide histogram down to 0

$$g = \frac{255(f - MIN)}{MAX - MIN}$$

2) Normalize histogram to [0,1] range

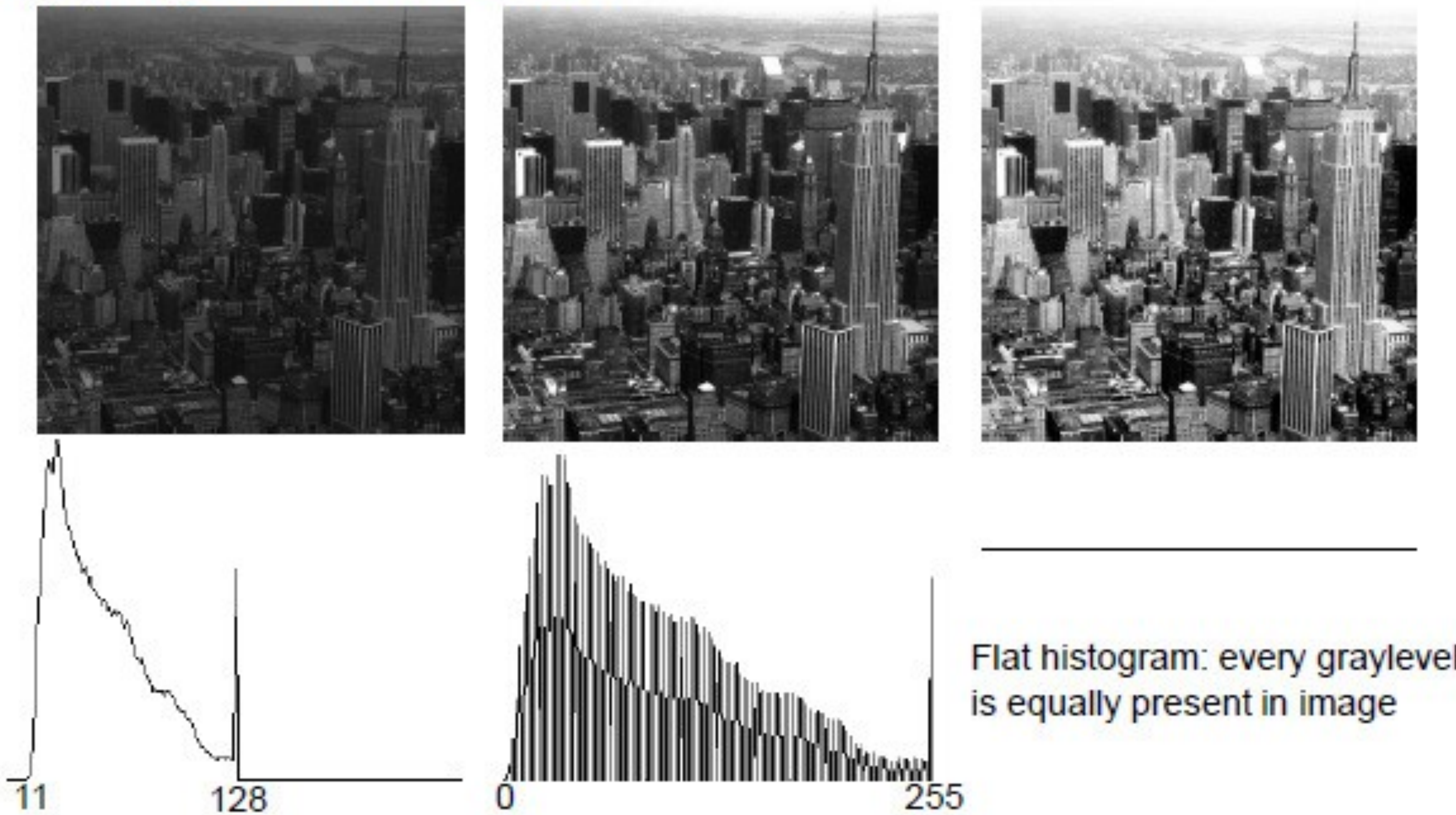
Histogram Stretching – Example 1





Example 2

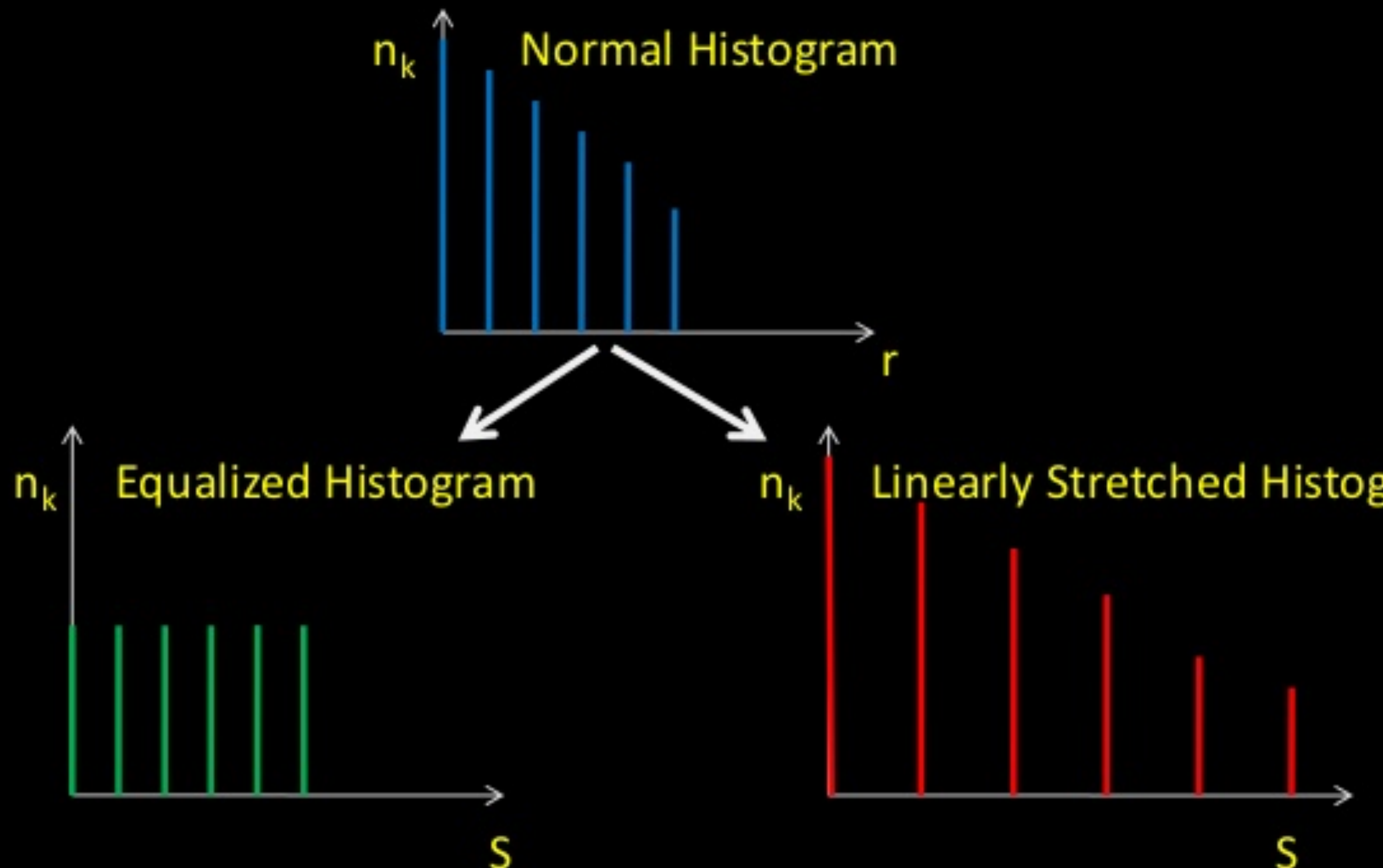
- Improve effectiveness of histogram stretching by clipping intensities first



Histogram Equalization

- Linear stretching is a good technique but not perfect since the shape remains the same.
- Most of the time we need a flat histogram.
- It can't be achieved by Histogram Stretching.
- Thus, new technique of Histogram Equalization came into use.
- Perfect image is one where all gray levels have equal number of pixels.
- Here, our objective is not only to spread the dynamic range but also to have equal pixels at all gray levels.

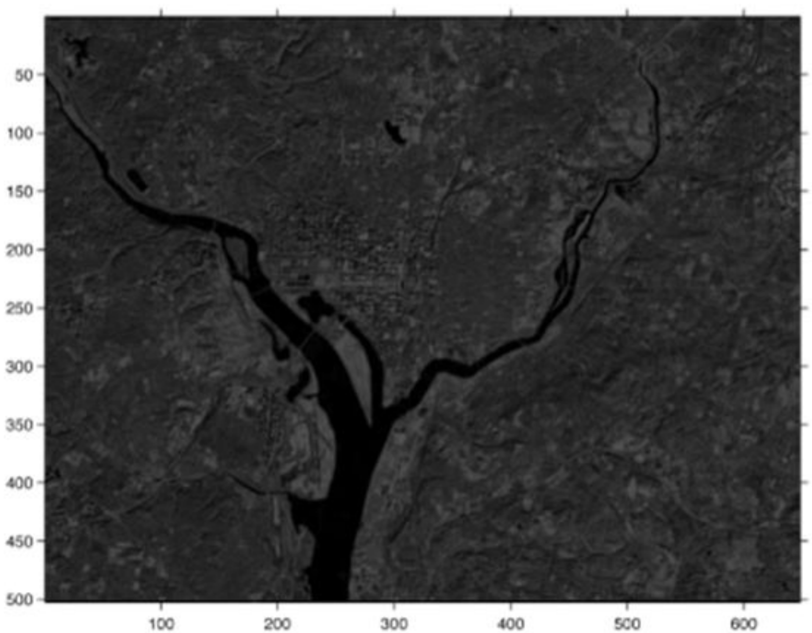
Histogram Equalization



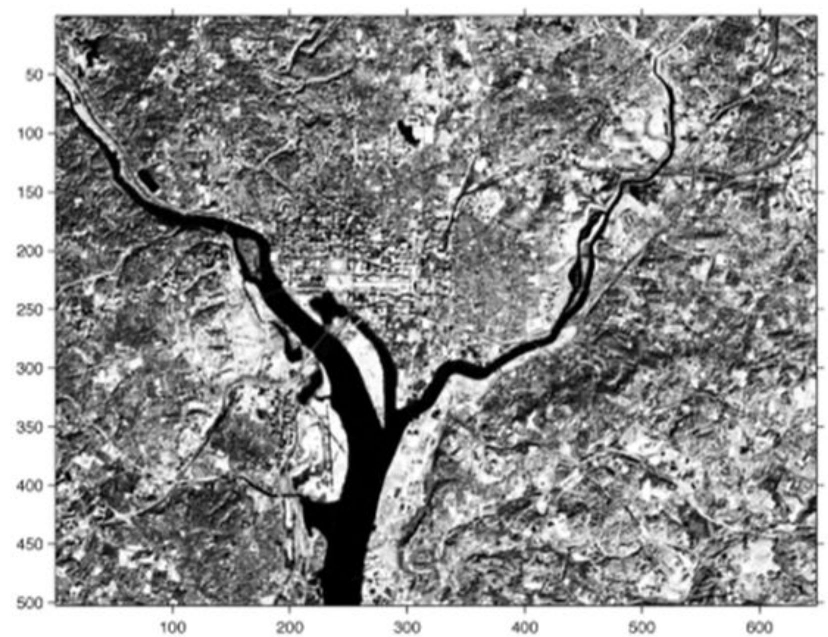
Histogram Equalization stretches the histogram to fill the dynamic range and at the same time tries to keep the histogram uniform.

Histogram Equalization: Example – Case A

Case A

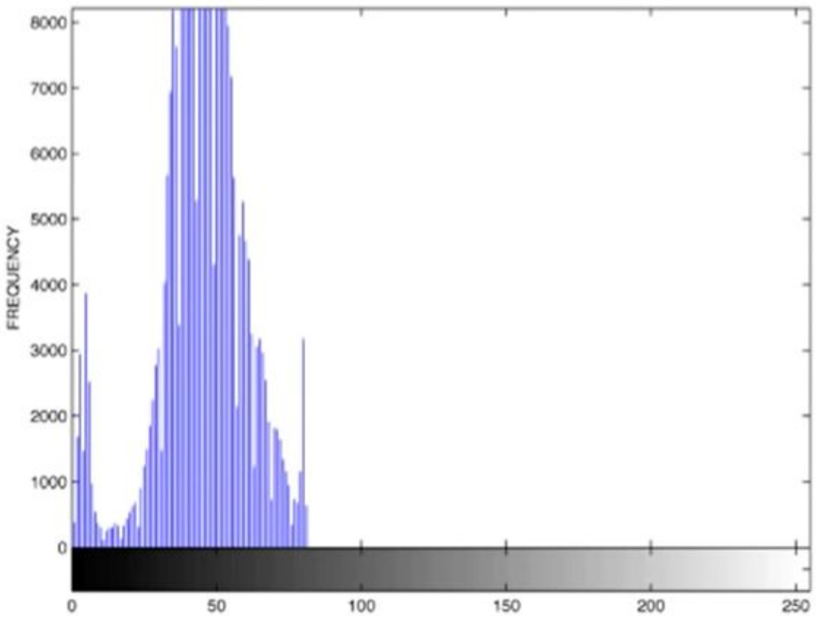


Case A **after** equalization

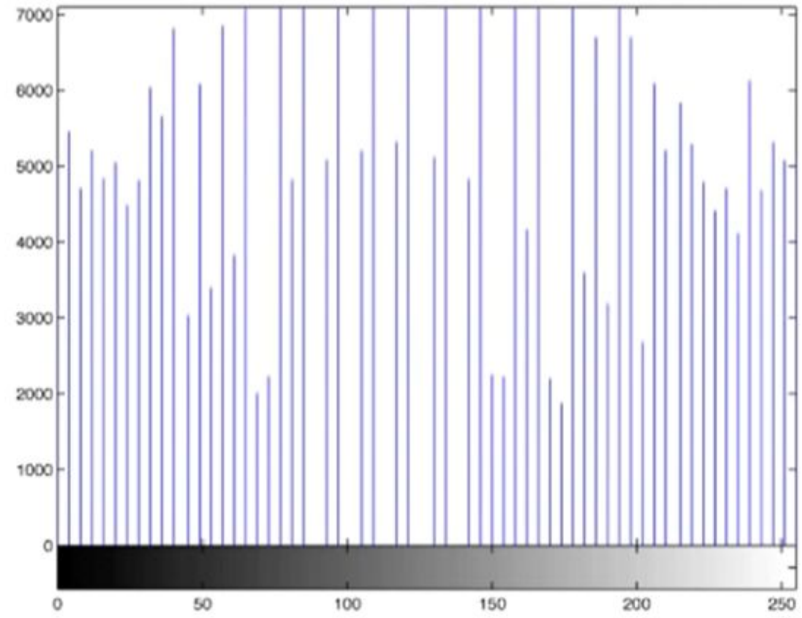


Histogram Equalization: Exploring The Histogram of Case A

The Histogram of Case A



The histogram of Case A **after** equalization



The resultant image have an appearance of high contrast and exhibits a large variety of grey tones. **But the histogram is not the perfectly flat.**

Histogram Equalization

- Histogram equalization is used to enhance contrast of an image.
- Mostly it will not be able to perfectly equalize the histogram of an image.
- This is only possible if we can assume continuous intensity values in the histogram.
- But in reality, intensity values are discrete thus perfectly flat histograms are rare in practical applications of the histogram equalization.

Histogram Processing: Equalization

- Histogram equalization modifies the values of pixels so that **the number of pixels at each gray level will be approximately the same.**
- Generally, this operation tends to enhance the image contrast.
- Can be performed automatically

Histogram Equalization - Algorithm

1. Find the frequency of each value represented on the horizontal axis of the histogram i.e. intensity in the case of an image.
2. Calculate the probability density function (PDF) for each intensity value.
3. After finding the PDF, calculate the cumulative density function (CDF) for each intensity's frequency.
4. The CDF value is in the range 0-1, so we multiply all CDF values by the largest value of intensity i.e. (L-1) or 255 for 8 bit image.
5. Round off the final values to integer values.

Histogram Equalization - Algorithm

- **Input**

A discrete image ***f*** of L levels of intensity

- **Algrithm**

Step 1: Compute the histogram $\mathbf{h(r_k)}$ of image ***f*** or normalized histogram $\boldsymbol{p}\mathbf{(r_k)}$ of image ***f*** i.e. PMF/PDF

Step 2: Compute the **CDF** of $\mathbf{h(r_k)}$ or $\boldsymbol{p}\mathbf{(r_k)}$, denoted $\mathbf{CDF}_{\boldsymbol{hf}}$

Step 3: Find a transformation T: [0. L-1] -> [0. L-1] such that

$$S = T(r) = (L-1)\sum_{q=0}^{r} h(q) = (L-1)CDF_{hf}(r)$$

If CDF is normalized.

Step 4: Apply the transformation on each pixel of the input image ***f***

**PMF = probability mass function* **CDF = Cumulative distribution function*

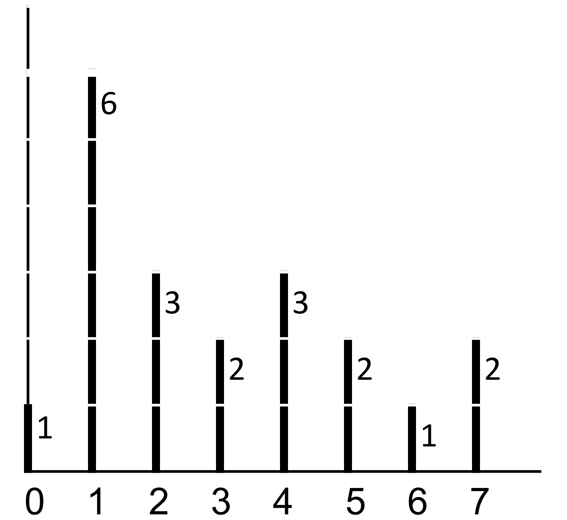
Histogram Equalization: Example

The Tabular form of the calculation for histogram equalization is given here:

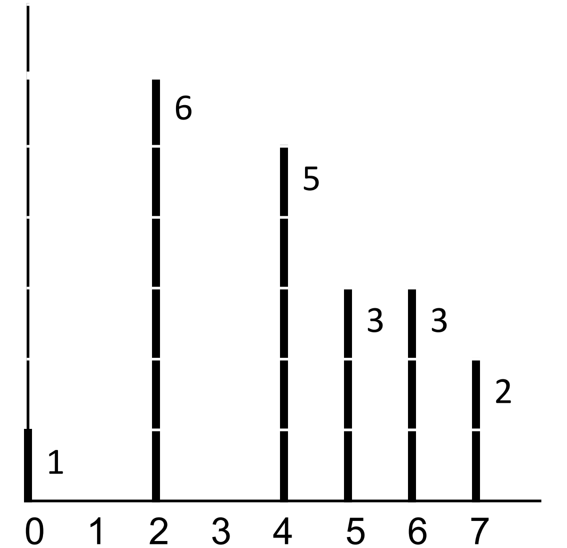
Intensity (r_k)	**Frequency $h(r_k) = n_k$**	**PDF $p(h(r_k)$**	**CDF**	**(L-1)*CDF= 7*CDF**	**Round-off (s_k)**
0	1	0.0500	0.0500	0.3500	0
1	6	0.3000	0.3500	2.4500	2
2	3	0.1500	0.5000	3.5000	4
3	2	0.1000	0.6000	4.2000	4
4	3	0.1500	0.7500	5.2500	5
5	2	0.1000	0.8500	5.9500	6
6	1	0.0500	0.9000	6.3000	6
7	2	0.1000	1.0000	7.0000	7

Mapping the new gray level values into number of pixels.

Gray level r_k	New Gray level s_k	no. of pixel n_k
0	0	1
1	2	6
2	4	3
3	4	2
4	5	3
5	6	2
6	6	1
7	7	2

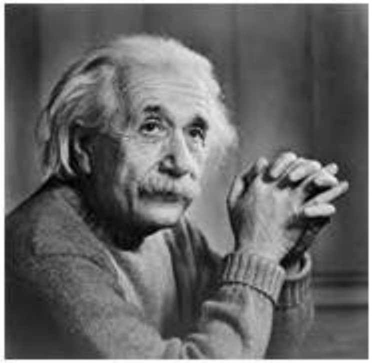


Original Histogram



Equalized Histogram

Another Example



$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

An 8x8 image
i.e. total number of pixels is 64.

Histogram Equalization: Example

Step 1

Compute the histogram $h(r_k)$

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52		64		72		85		113	
55		65		73		87		122	
58		66		75		88		126	
59		67		76		90		144	
60		68		77		94		[illegible]	
61		69		78		104			
62		70		79		106			
63		71		83		109			

Image Histogram (Non-zero values)

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

Histogram Equalization: Example

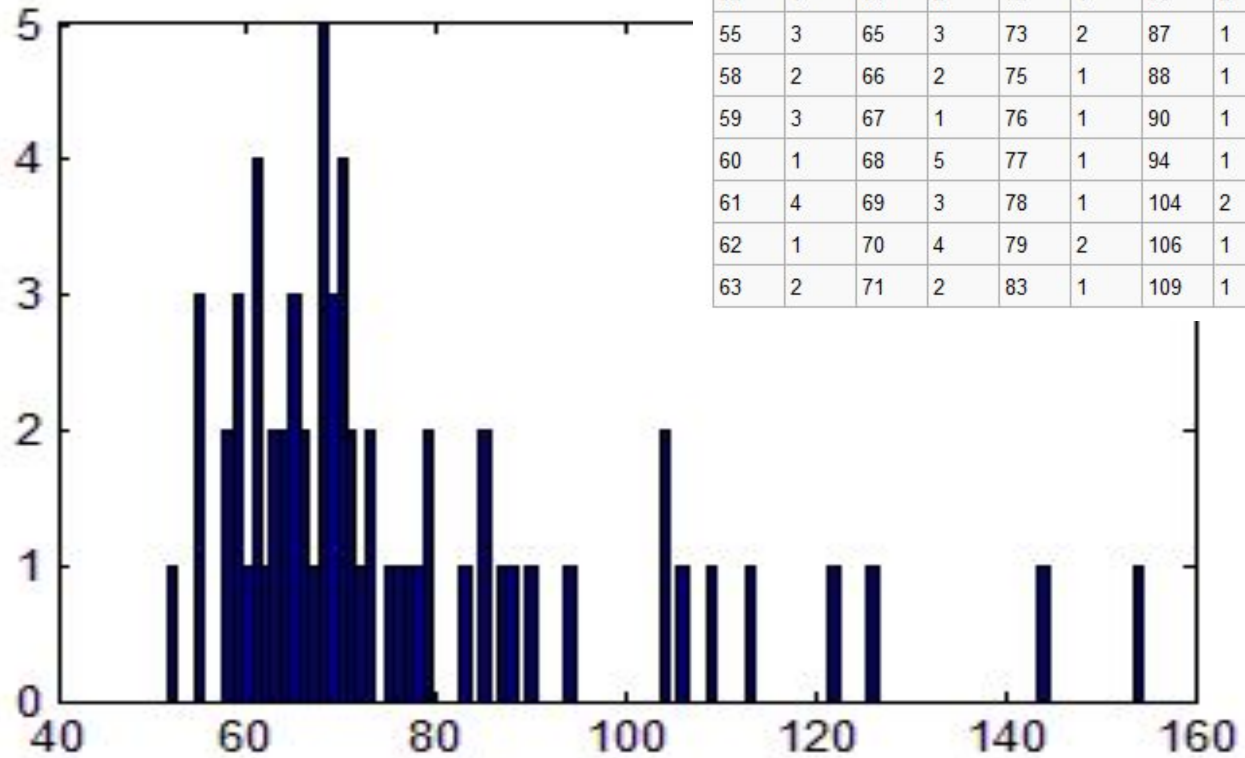
Image Histogram $\mathbf{h(r_k)}$ (Non-zero values shown)

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Step 1
But skipped normalization

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

Histogram Equalization: Example



Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Image Histogram

Histogram Equalization: Example

Step 2

Compute the Cumulative Distribution Function (CDF)

But skipped normalization

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52		64		72		85		113	
55		65		73		87		122	
58		66		75		88		126	
59		67		76		90		144	
60		68		77		94		154	
61		69		78		104			
62		70		79		106			
63		71		83		109			

Histogram Equalization: Example

Cumulative Distribution Function (CDF)

Step 2

CDF is NOT normalized

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52	1	64	19	72	40	85	51	113	60
55	4	65	22	73	42	87	52	122	61
58	6	66	24	75	43	88	53	126	62
59	9	67	25	76	44	90	54	144	63
60	10	68	30	77	45	94	55	154	64
61	14	69	33	78	46	104	57		
62	15	70	37	79	48	106	58		
63	17	71	39	83	49	109	59		

If CDF is normalized ***s*** = $T(r) = (L-1)\sum_{q=0}^{r} h(q) = (L-1)CDF_{hf}(r)$

Histogram Equalization: Example

If CDF is normalized

$$s = round(255.cdf(r))$$

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52	1	64	19	72	40	85	51	113	60
55	4	65	22	73	42	87	52	122	61
58	6	66	24	75	43	88	53	126	62
59	9	67	25	76	44	90	54	144	63
60	10	68	30	77	45	94	55	154	64
61	14	69	33	78	46	104	57		
62	15	70	37	79	48	106	58		
63	17	71	39	83	49	109	59		

$$s = round(255.\frac{cdf(r)}{M \times N})$$

$$s = round(255.(46/64))$$

$$s = 183$$

Step 3

If CDF is NOT normalized

Step 4

Repeat for each pixels

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

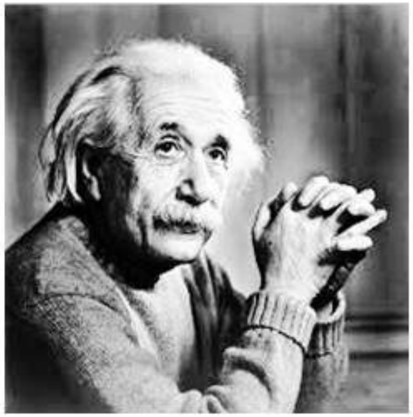
Original Image

4	**16**	**56**	**96**	**147**			
						183	

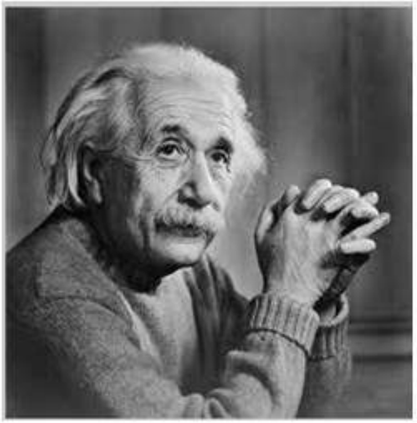
Map the new gray levels into the number of pixels

Before and After Histogram Equalization:

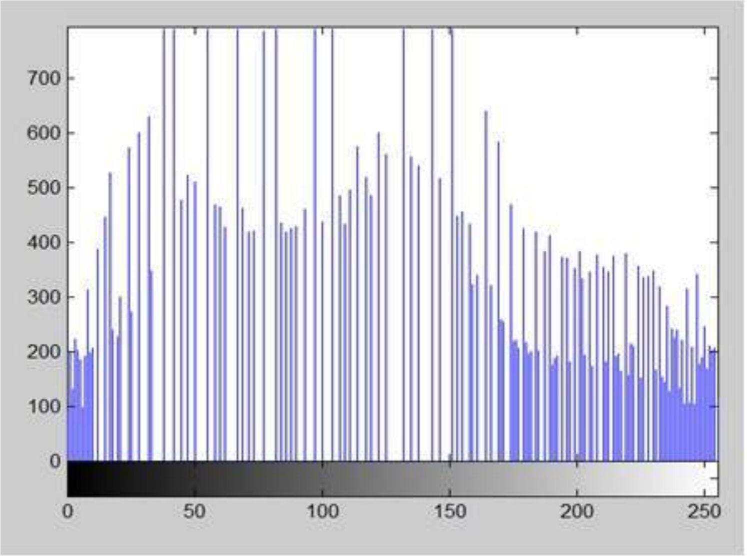
New Image



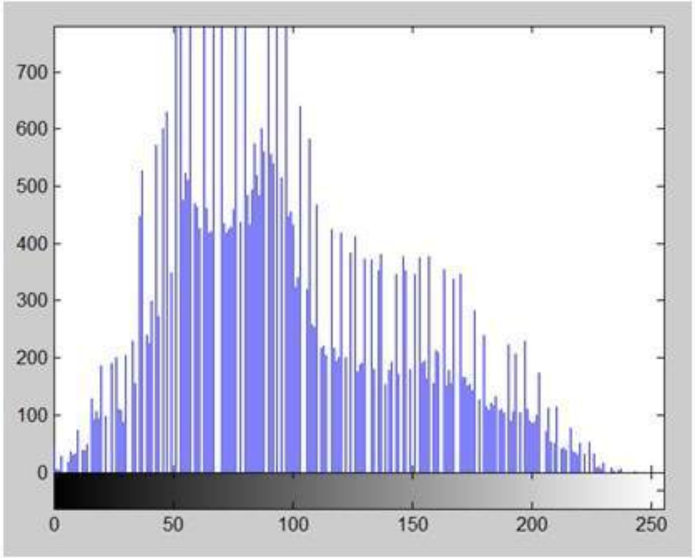
Old image



New Histogram



Old Histogram



Problem: Equalize the given histogram of image below:.

Grey level (r)	0	1	2	3	4	5	6	7
Number of pixels	790	1023	850	656	329	245	122	81

Grey level r_k	Histogram $h(r_k)$ (Original image)	(CDF)	S_k = ((L-1) /MxN) x CDF	Rounding off S_k	No. of Pixels n_k (Output image)
0	**790**	**790**	**1.35**	**1**	**790**
1	**1023**	**1813**	**3.09**	**3**	**1023**
2	**850**	**2663**	**4.55**	**5**	**850**
3	**656**	**3319**	**5.67**	**6**	(656+329 =) **985**
4	**329**	**3648**	**6.23**	**6**	
5	**245**	**3893**	**6.65**	**7**	(245+122+81=) **448**
6	**122**	**4015**	**6.86**	**7**	
7	**81**	**4096**	**7**	**7**	

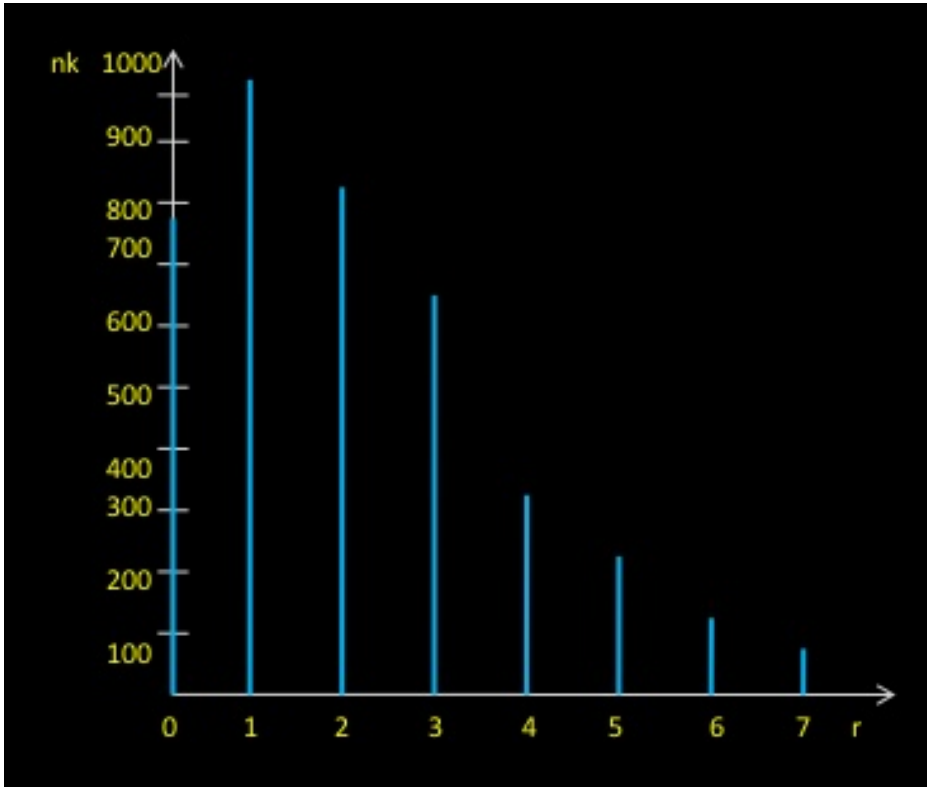
Histogram Equalization

Gray Levels (r_k)	No. of Pixels n_k	(PDF) Pr(rk) = nk/n	(CDF) Sk = $\sum$ Pk(rk)	(L-1) Sk = 7 x Sk	Rounding off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7
	n = 4096	**1**			

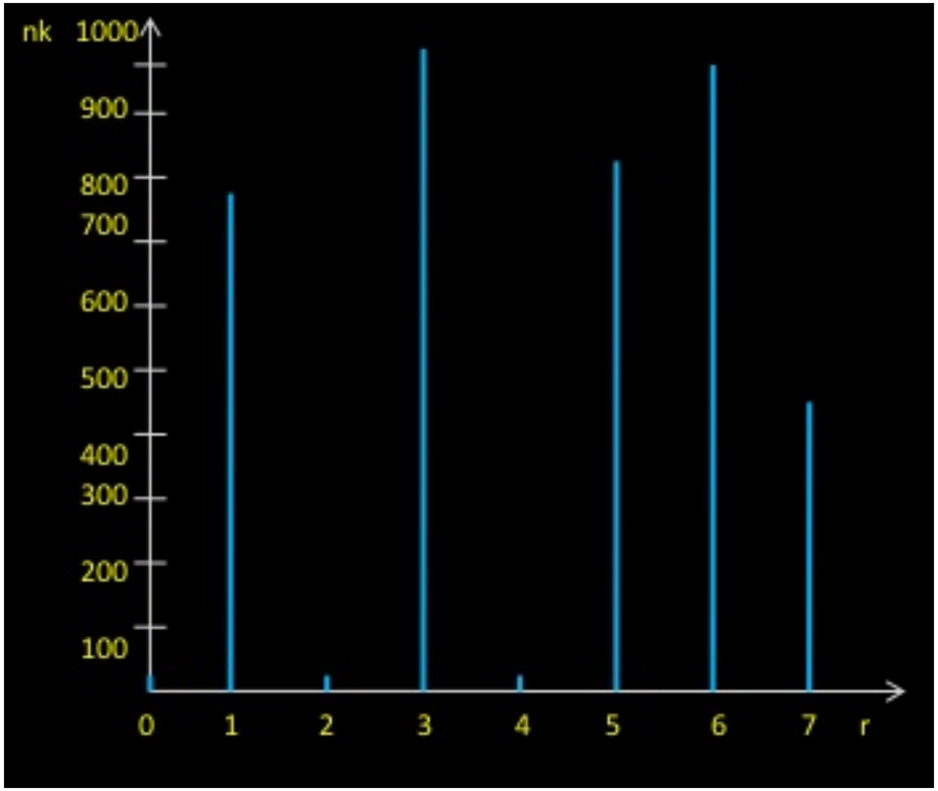
<u>Equating Gray Levels to No. of Pixels:</u>

0 -> 0	4 -> 0		
1 -> 790	5 -> 850		
2 -> 0	6 -> 985		
3 -> 1023	7 -> 448	Total : 4096	Hence Verified!!!

Before & after Histogram Equalization



Before the equalization



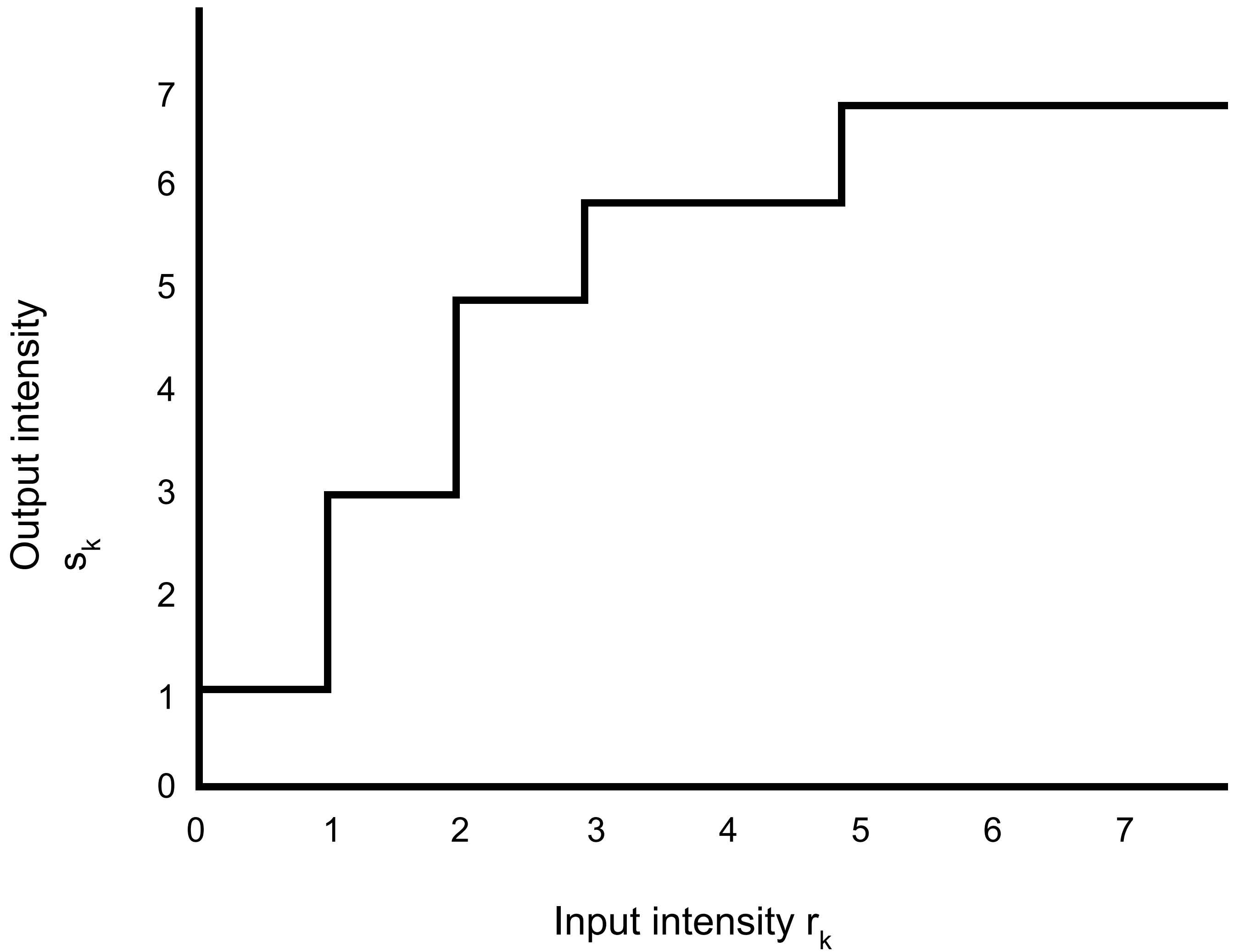
After the equalization

Image Histogram

Mapping the new gray level values into number of pixels.

Gray level r_k	Gray level s_k	no. of pixel n_k
0	1	790
1	3	1023
2	5	850
3	6	656
4	6	329
5	7	245
6	7	122
7	7	81

Histogram Equalization Transformation function



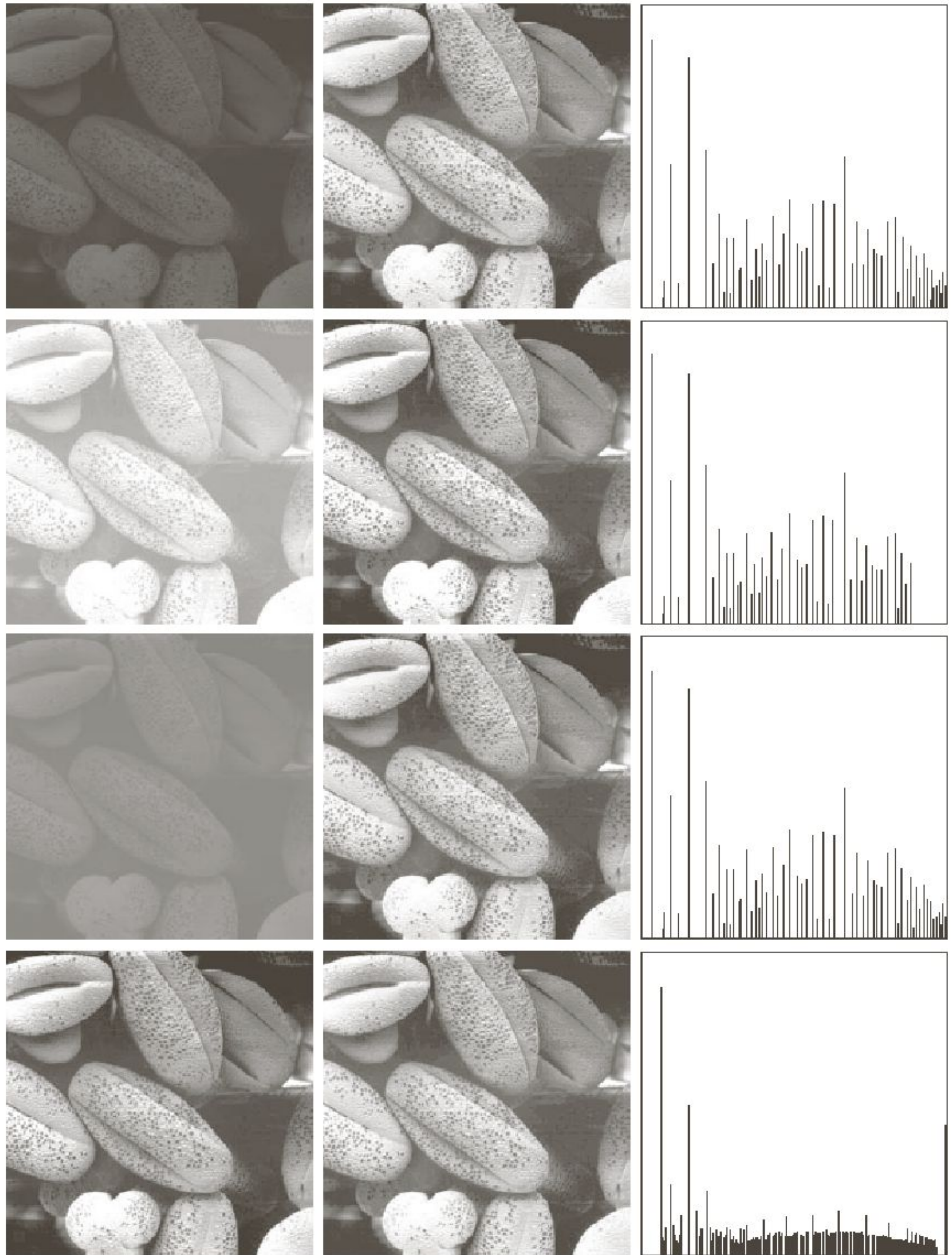


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Equalization Transformation Function

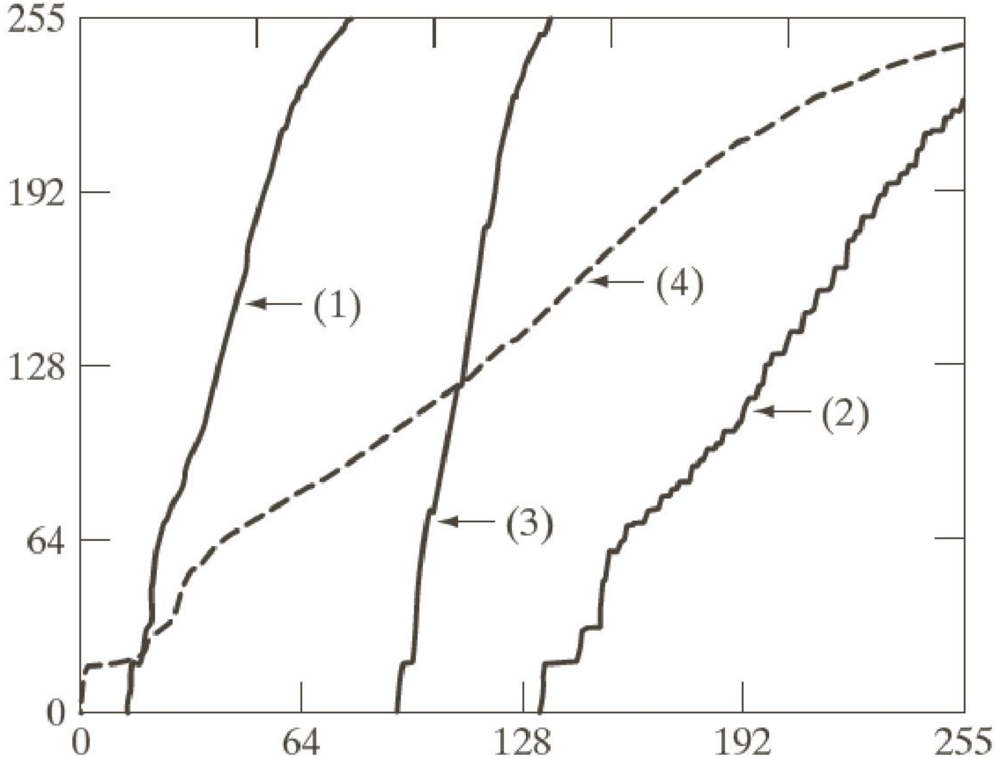
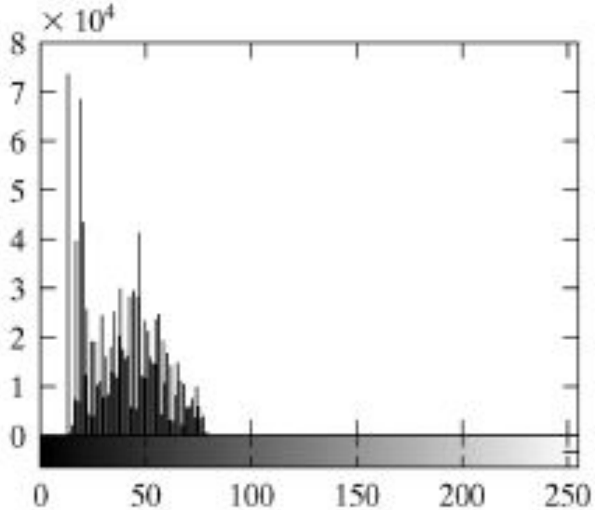
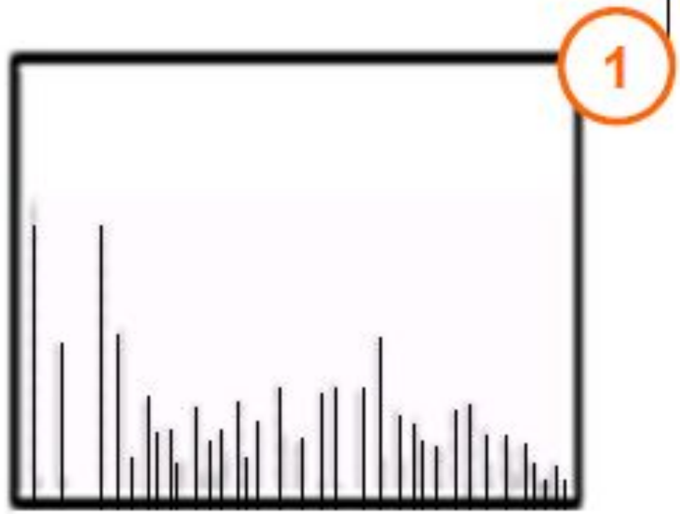
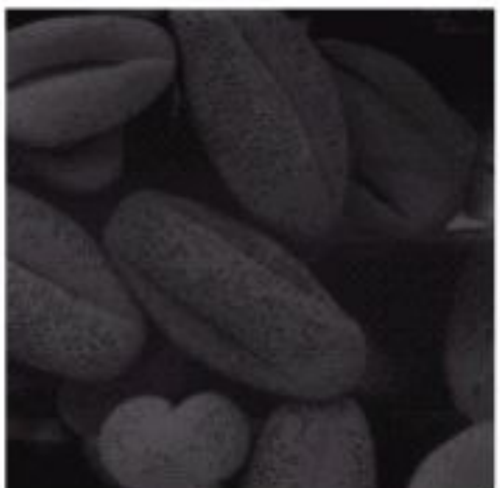


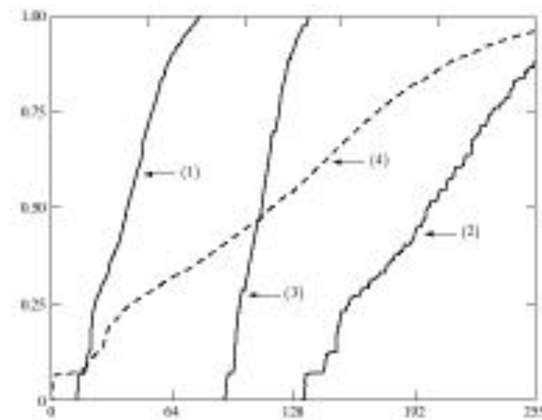
FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

Equalization Examples

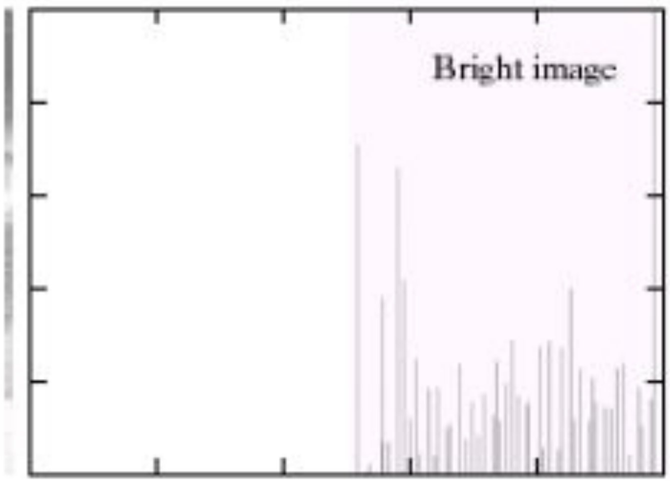




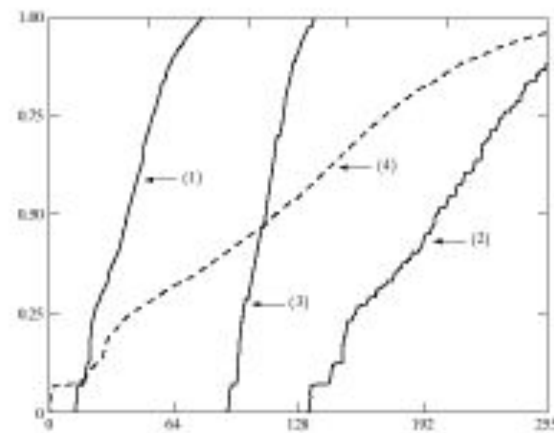




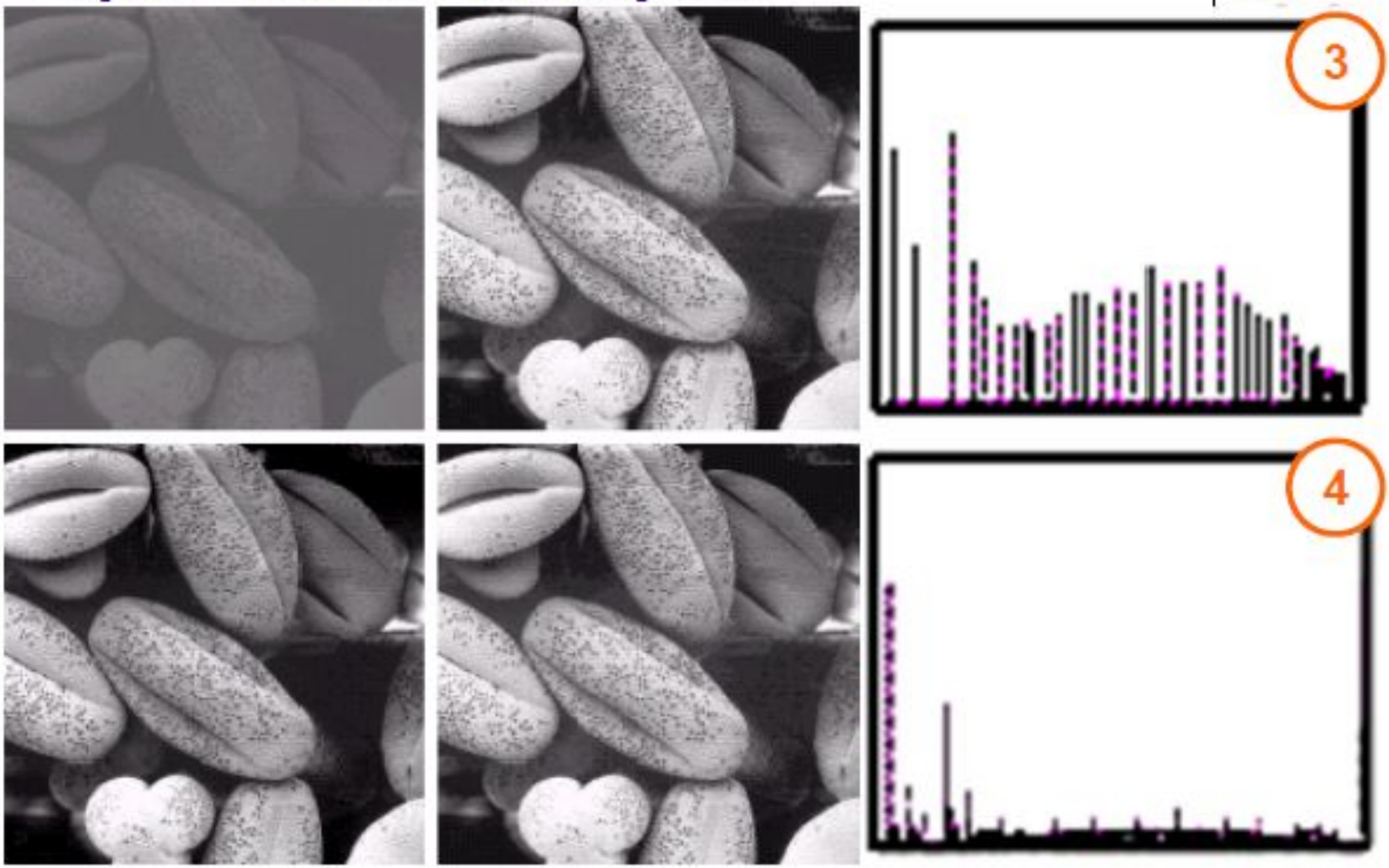
Equalization Examples



2



Equalization Examples



Note

- Histogram equalization has a disadvantage:
- It can generate only one type of output image.
- With **histogram specification** we can specify the shape of the histogram that we wish the output image to have.
- It doesn't have to be a uniform histogram.
- Histogram specification is a trial-and-error process.
- There are no rules for specifying histograms, and one must resort to analysis on a case-by-case basis for any given enhancement task.

HISTOGRAM SPECIFICATION/MATCHING

- Equalize the levels of the original image.
- **Histogram matching** is the transformation of an image.
- The process of Histogram Matching takes in an input image and produces an output image that is based upon a specified histogram.
- The well-known **histogram** equalization method is a special case in which the specified **histogram** is uniformly distributed.

Histogram Specification

Here we want to convert the image so that it has a particular histogram that can be arbitrarily specified. Such a mapping function can be found in three steps:

Algorithm:

1. Equalize the histogram of the input image
2. Equalize the specified histogram
3. Relate the two equalized histograms

EXAMPLE: HISTOGRAM MATCHING

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Step-1:
Equalize the original histogram of the image

Grey level r_k	No. of Pixels n_k	(PDF)r_k $P_r(r_k)=n_k/N$	(CDF)	(L-1)CDF = 7xCDF	(equalized gray level) Rounding off H_k
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1.0	7	7

Step-2:
Equalize the specified histogram of the image

Grey level r_k	No. of Pixels n_k	(PDF)r_k $P_r(r_k)=n_k/N$	(CDF)	(L-1)CDF = 7xCDF	(equalized gray level) Rounding off S_k
0	0	0.0	0.0	0.0	0
1	0	0.0	0.0	0.0	0
2	0	0.0	0.0	0.0	0
3	?	0.15	0.15	1.05	1
4	?	0.20	0.35	2.45	2
5	?	0.30	0.65	4.55	5
6	?	0.20	0.85	5.95	6
7	?	0.15	1.0	7	7

Step-3:
Map the original histogram to specified histogram

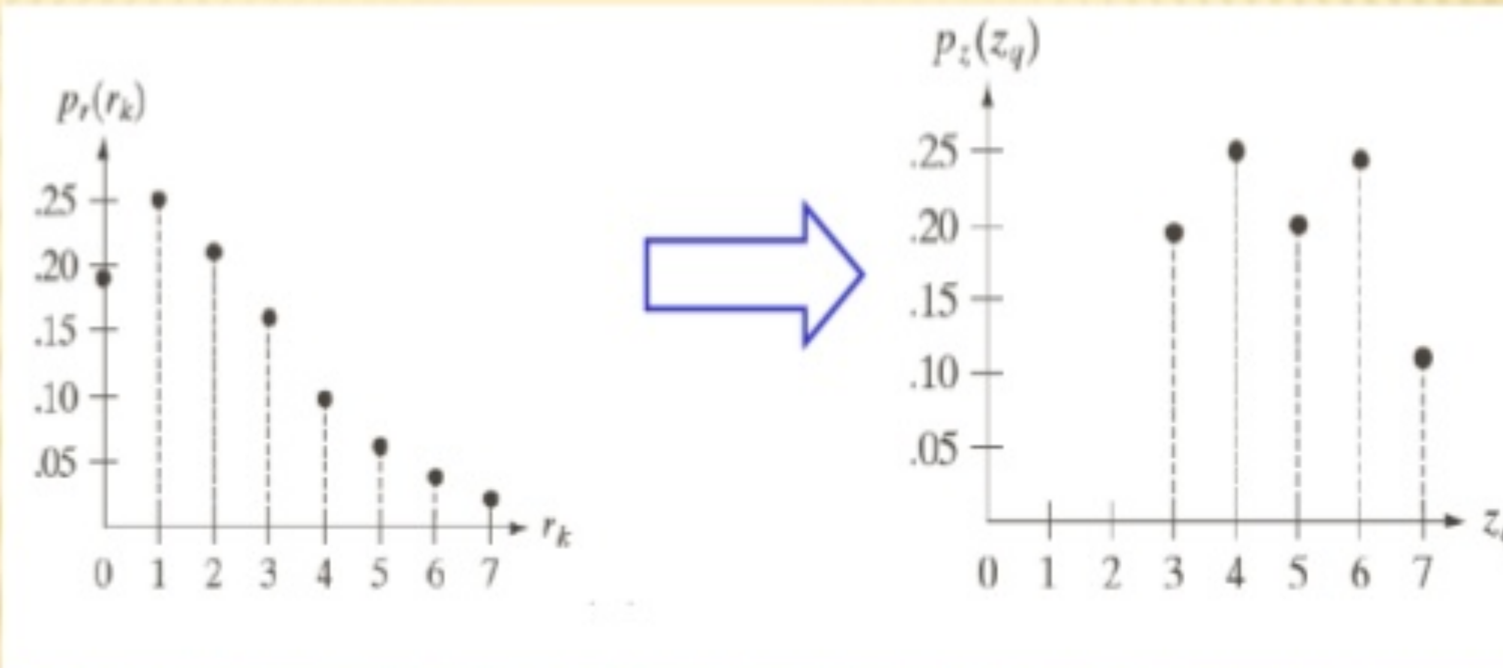
Grey level of input image R_k	(equalized gray level) (Original image) H_k	(equalized gray level) (Specified image) S_k	Map Gray level of Output image S_k
0	1	0	3
1	3	0	4
2	5	0	5
3	6	1	6
4	6	2	6
5	7	5	7
6	7	6	7
7	7	7	7

Histogram Matching

Mapping the new gray level values into number of pixels.

Grey level of input image r_k	Mapping		New grey level of desired output image R_k	No. of Pixels n_k (output image)
	No. of Pixels input image n_k	Gray level of Equalized image S_k		
0	790	3	0	0
1	1023	4	1	0
2	850	5	2	0
3	656	6	3	790
4	329	6	4	1023
5	245	7	5	850
6	122	7	6	985
7	81	7	7	448

EXAMPLE: HISTOGRAM MATCHING



z_q	P(s)
3	0.19
4	0.25
5	0.21
6	0.24
7	0.11

(a) Original image histogram

(b) Specified histogram

Histogram Matching / Specification Example

Original Image histogram

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	8	10	10	2	12	16	4	2

Desired image histogram

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	0	20	20	16	8

Histogram Matching / Specification Example

continuation….

Original Image histogram equalization

Gray level (**r_k**)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
No. of Pixels (n_k)	8	10	10	2	12	16	4	2	N=64
PDF $P_r(r_k)=n_k/N$	**0.13**	**0.16**	**0.16**	**0.03**	**0.18**	**0.25**	**0.06**	**0.03**	**1**
CDF	**0.13**	**0.29**	**0.45**	**0.48**	**0.66**	**0.91**	**0.97**	**1.0**	
(L-1)*CDF	**0.91**	**2.03**	**3.15**	**3.36**	**4.62**	**6.37**	**6.79**	**7**	
H_k	**1**	**2**	**3**	**3**	**5**	**6**	**7**	**7**	

Histogram Matching / Specification Example

continuation….

Desired Image histogram equalization

Gray level (**r_k**)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
No. of Pixels (n_k)	0	0	0	0	20	20	16	8	N=64
PDF $P_r(r_k)=n_k/N$	**0**	**0**	**0**	**0**	**0.31**	**0.31**	**0.25**	**0.13**	**1**
CDF	**0**	**0**	**0**	**0**	**0.31**	**0.62**	**0.87**	**1.0**	
(L-1)*CDF	**0**	**0**	**0**	**0**	**2.17**	**4.34**	**6.09**	**7**	
S_k	**0**	**0**	**0**	**0**	**2**	**4**	**6**	**7**	

Histogram Matching / Specification Example

continuation….

Histogram Mapping

Gray level	0	1	2	3	4	5	6	7
H_k	**1**	**2**	**3**	**3**	**5**	**6**	**7**	**7**
S_k	**0**	**0**	**0**	**0**	**2**	**4**	**6**	**7**
Map Gray level of Output image	**4**	**4**	**5**	**5**	**6**	**6**	**7**	**7**

Original Image histogram

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	8	10	**10**	**2**	**12**	**16**	**4**	**2**

N=64

Desired Image Histogram

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	0	18	12	**28**	**6**

N=64

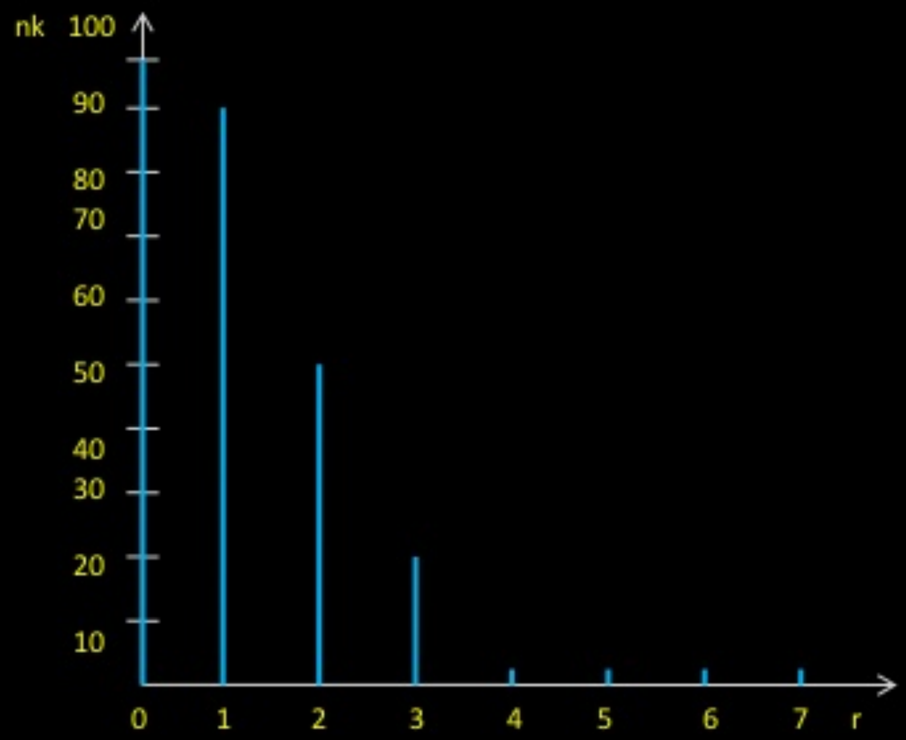
HOME WORK

Suppose that a digital image is subjected to histogram equalization . Show that a second pass of histogram equalization will produce exactly the same result as the first pass.

Class Work

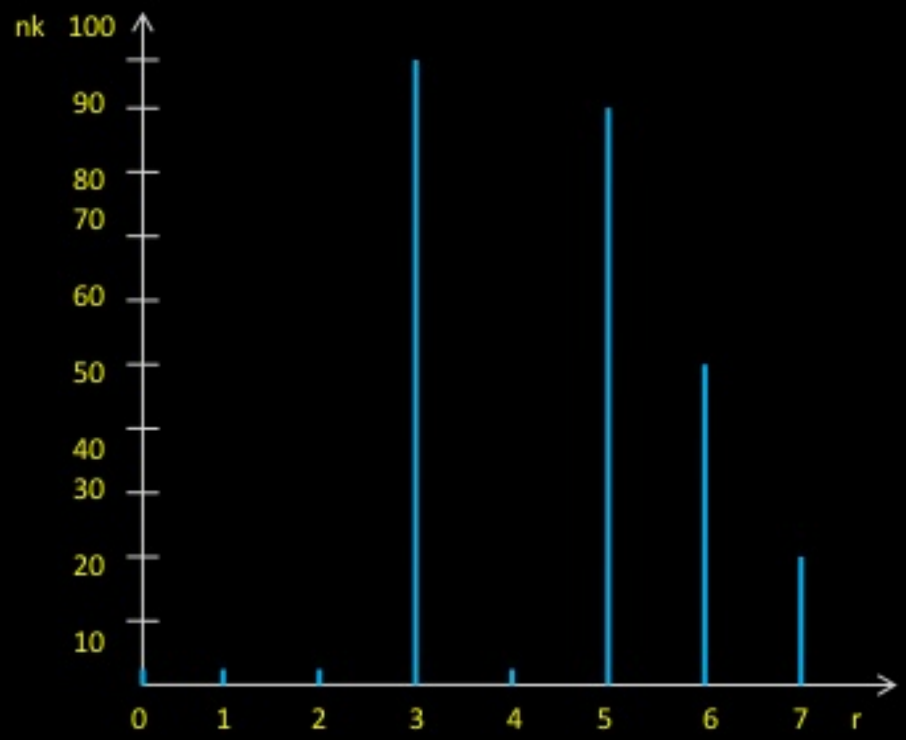
Prob. 2) Equalize the given histogram

Gray Levels (r)	0	1	2	3	4	5	6	7
No. of Pixels	100	90	50	20	0	0	0	0



Class Work: Answer

Gray Levels (r)	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	100	0	90	50	20



More Example of Histogram Equalization

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in following table.
Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Example: Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.19 = 1.33 \qquad \rightarrow 1$$

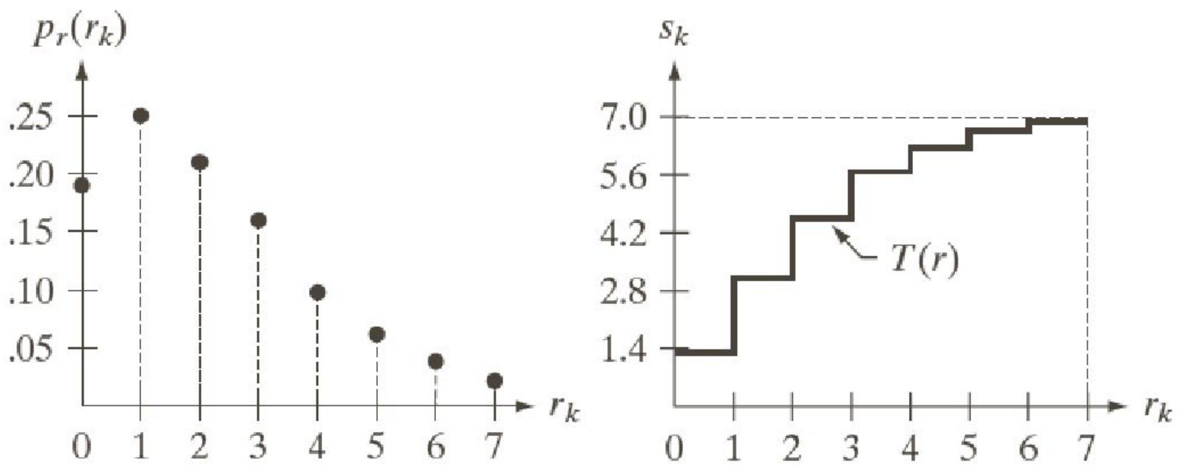
$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \qquad \rightarrow 3$$

$$s_2 = 4.55 \quad \rightarrow 5 \qquad s_3 = 5.67 \quad \rightarrow 6$$

$$s_4 = 6.23 \quad \rightarrow 6 \qquad s_5 = 6.65 \quad \rightarrow 7$$

$$s_6 = 6.86 \quad \rightarrow 7 \qquad s_7 = 7.00 \quad \rightarrow 7$$

Example: Histogram Equalization



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

❑**Consider "*cat.tif*" is a 64X64 image with 4 bit gray values. The histogram "*cat.tif*" is given below:**

Gray Values	No. of Pixels
2	90
4	123
5	1550
8	965
9	245
10	122
12	101

i. Sketch the normalize histogram of "***cat.tif***".

ii. Make a equalize histogram of "***cat.tif***".

iii. Sketch the equalize histogram transformation function of ***cat.tif***".

Prob. 1) What effect would setting to zero the lower order bit plane have on the histogram of an image shown?

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Prob. 1) What effect would setting to zero the lower order bit plane have on the histogram of an image shown?

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

➤

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

Prob. 1) What effect would setting to zero the lower order bit plane have on the histogram of an image shown?

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

0000	0000	0000	0000
0100	0100	0100	0100
1000	1000	1000	1000
1100	1100	1100	1100

Setting lower order bit plane to zero.

Prob. 1) What effect would setting to zero the lower order bit plane have on the histogram of an image shown?

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

0000	0000	0000	0000
0100	0100	0100	0100
1000	1000	1000	1000
1100	1100	1100	1100

0	0	0	0
4	4	4	4
8	8	8	8
12	12	12	12

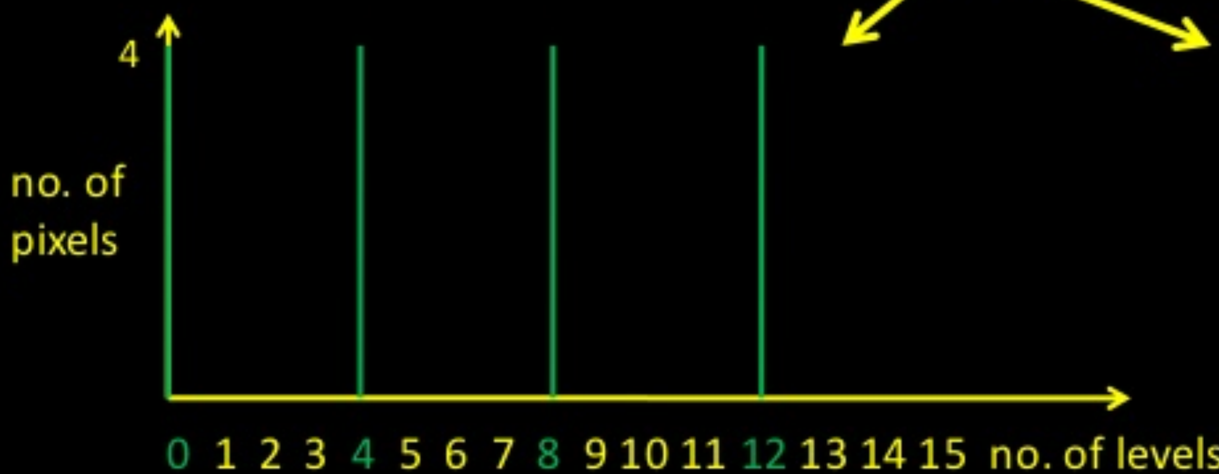
Setting lower order bit plane to zero.

Prob. 1) What effect would setting to zero the lower order bit plane have on the histogram of an image shown?

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

0	0	0	0
4	4	4	4
8	8	8	8
12	12	12	12

0000	0000	0000	0000
0100	0100	0100	0100
1000	1000	1000	1000
1100	1100	1100	1100



Prob. 2) What effect would setting to zero the higher order bit plane have on the histogram of an image shown? Comment on the image.

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

- Review: Consider sample.bmp is a 4x4 image with 8-bit gray values.

20	210	150	60
60	150	60	150
150	60	210	150
150	20	150	20

Sample.bmp

- Calculate the **normalized histogram (PDF)** of Sample.bmp.