Slide-3 //

At; 90% confidence, $Z = \pm 1.65$
95% " , $Z = \pm 1.96$
99% " , $Z = \pm 2.58$

Example-1:

When σ is known, Z-values are used.

E = Margin Error is the maximum difference between the observed mean and the true value of the population mean.

$E = Z_{\alpha/2} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$; The interval, $\bar{X} - E < \mu < \bar{x} + E$

=>

Sample size, $n = 100$

Sample mean, $\bar{x} = 150\,g$

$\sigma = 40\,g$

$CI = 95\% = 0.95$

Now,

$CI/2 = 0.95/2 = 0.4750 = \alpha/2$

For 0.4750;

$Z = 1.9 + 0.06$
$= 1.96$

or You can just memorize the Z value directly

$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = (1.96) \cdot \left(\frac{40}{\sqrt{100}}\right)$
$= 7.84$

The interval is;

$\bar{X} - E < \mu < \bar{x} + E$

$150 - 7.84 < \mu < 150 + 7.84$

$142.16 < \mu < 157.84$

(Ans)

Example-2:

When σ ~~[struck]~~ is not known, we use t- distribution

Degree of freedom $= n-1$

Margin of error, $E = t\left(\frac{s}{\sqrt{n}}\right)$; s: Standard Deviation from the Sample

$\Rightarrow$

Given,

Sample Size, $n = 30$

$\bar{x} = 5.3$

D of F $= n-1 = 30-1 = 29$

Sample Standard Deviation, $s = 1.1$

Confidence ~~level~~, Interval $= 95\% = 0.95$

One tail, $\alpha = 1 - CI = 1 - 0.95$

$= 0.05$

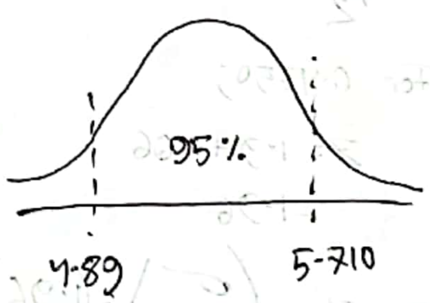
two tail, $\alpha/2 = (1-CI)/2 = (0.05)/2 = 0.025$

From table, (Table will be Given in exam)

$t = 2.045$

So, $E = 2.045 \times \left(\frac{1.1}{\sqrt{30}}\right)$

$= 0.410$



The interval is; $\bar{x} - E < \mu < \bar{x} + E$

$5.3 - 0.410 < \mu < 5.3 + 0.410$

$4.89 < \mu < 5.710$

Example-3:

Estimating population variance (σ^2) for $n < 30$; non normal distribution

Interval estimation;

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

; χ^2 = chi square

$1-\alpha$ = confidence coefficient

$\Rightarrow$ $n = 18$

d of F $= 18 - 1 = 17$

$\bar{x} = 9.38$

$s = 1.62$

$CI = 90\% = 0.90$

$\alpha = 1 - 0.90$

$= 0.1$

$\alpha/2 = 0.05$; $(1-\alpha/2) = 0.95$

The interval; $$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

$$\text{or, } \frac{17 \times (1.62)^2}{\chi^2_{0.05}} \leq \sigma^2 \leq \frac{17 \times (1.62)^2}{\chi^2_{0.95}}$$

$$\text{or, } \frac{44.6148}{27.587} \leq \sigma^2 \leq \frac{44.6148}{8.672}$$

$$\therefore 1.6172 \leq \sigma^2 \leq 5.14469$$

Example-4: (for $n \geq 30$) normal distribution

$$\left(\frac{s_1^2}{1+Z_{\alpha/2}\sqrt{2/(n-1)}} , \frac{s_1^2}{1-Z_{\alpha/2}\sqrt{2/(n-1)}} \right)$$

$\Rightarrow$

Given,

$n = 50$

$s_1 = 7.93$

$\bar{x} = 43.24$

$CI = 95\%$; $\alpha = 1-0.95$
$= 0.05$

$\therefore \alpha/2 = 0.025$

The interval;

$$\frac{(7.93)^2}{1+\left(1.96\sqrt{2/49}\right)} \leq \sigma^2 \leq \frac{(7.93)^2}{1-\left(1.96\sqrt{2/49}\right)}$$

or, $45.04 \leq \sigma^2 \leq 104.11$

Example-5:

$H_0 : \mu <= 74{,}914$

$H_1 : \mu > 74{,}914$

level of significance = 5% = 0.05

$\sigma = 14{,}530$

$n = 112 \geq 30$; normal distribution

$\bar{x} = 78{,}695$

~~For one tailed;~~

95% → For one tailed : 1.645
For two tailed : 1.96

99% → For one tailed : 2.33
For two tailed : 2.58

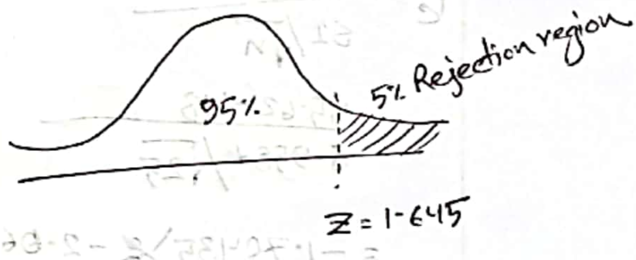
90% → For one tailed : 1.28
For two tailed : 1.645

Now,

$$Z_e = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{78695 - 74914}{14530 / \sqrt{112}}$$

$$= 2.7539 > Z$$



∴ The null hypothesis has been rejected.

Example-6:

$n = 25 < 30$; t distribution

$H_0 = \mu \geq 48$

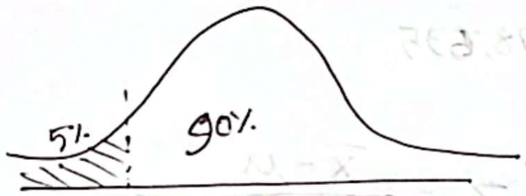
$H_1 = \mu < 48$

level of significance $= 5\% = 0.05$

$\overline{x} = 45.62$

$S_1 = 6.9587$

D of F $= 25 - 1 = 24$



$t = -2.064$

$$T_c = \frac{\overline{x} - \mu}{S_1/\sqrt{n}}$$

$$= \frac{45.62 - 48}{6.9587/\sqrt{25}}$$

$$= -1.70435 > -2.064$$

So, we will accept the null hypothesis.

Although the sample mean 45.62 which is less than 48 but we don't have enough evidence to reject the hypothesis.

Quiz-1 (Solve)

Set A:

$n = 400$

$\bar{X} = 67.47$

$\sigma = 1.30$

Level of Significance $= 5\%$

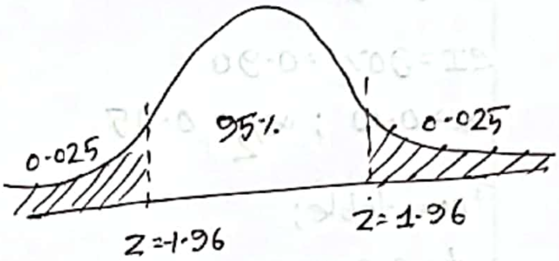
$C.I = 95\%$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$H_0 : \mu = 67.39$

$H_1 : \mu \neq 67.39$



Now, $Z_c = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{67.47 - 67.39}{1.30/\sqrt{400}} = 1.230$, falls inbetween the Interval.

$\therefore$ The null hypothesis is acceptable.

Set-B:

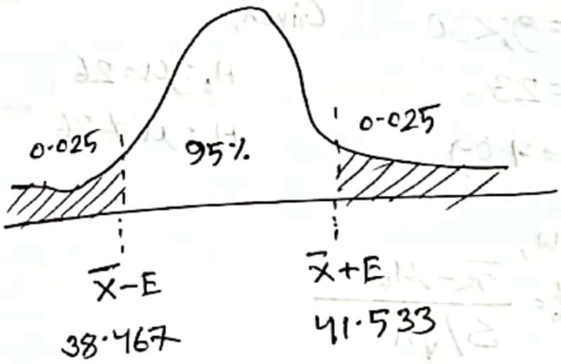
$n = 36$; d of F $= n-1 = 35$

$\bar{x} = 40$

$S = 4.5$

$CI = 95\%$

$\alpha = 0.05$; $\alpha/2 = 0.025$



From t-table:

$t = 2.045$

$E = t\left(\dfrac{S}{\sqrt{n}}\right)$

$= 2.045 \times \left(\dfrac{4.5}{\sqrt{36}}\right)$

$= 1.533$

The interval is;

$40 - 1.533 < \mu < 40 + 1.533$

$38.467 < \mu < 41.533$

Set-C:

$n = 4$; d of F $= 3$

$S = 2.8$

$\bar{X} = 36.8$

$CI = 90\% = 0.90$

$\alpha = 0.10$; $\alpha/2 = 0.05$

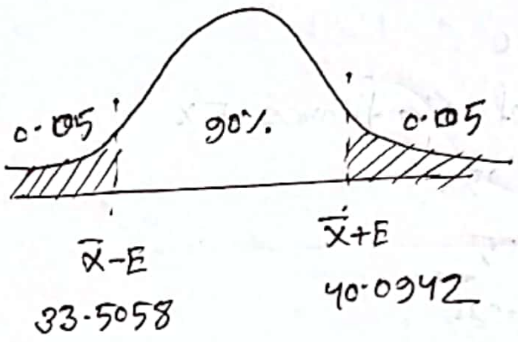
From table;

$t = 2.353$

$E = t\left(\frac{S}{\sqrt{n}}\right)$

$= 2.353 \times \left(\frac{2.8}{\sqrt{4}}\right)$

$= 3.2942$



The interval is;

$36.8 - 3.2942 < \mu < 38.8 + 3.2942$

$33.5058 < \mu < 40.0942$

Set-D:

$n = 9; < 30$

$\bar{x} = 23$

$S_1 = 4.09$

Given,

$H_0 : \mu = 26$

$H_1 : \mu \neq 26$

Now,

$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$

$= \frac{23-26}{4.09/\sqrt{9}}$

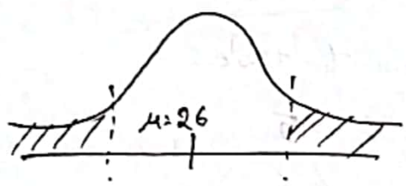
$= -2.2004$

Sample mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Sample SD:

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$



For 90% ;

if t value is greater than 1·86 or less than −1·86 , we reject H_0

For 95% ;

if t value is " " 2·306 or less than −2·306, " " H_0

For 99% ;

if t " " " " 2·896 or " " −2·896 " " H_0

Decision:

For 90% ;

$t = -2 \cdot 2004 < -1 \cdot 86$, So, reject H_0

For 95% ,

$t = -2 \cdot 2004 > -2 \cdot 306$, So, accept H_0

For 99% ,

$t = -2 \cdot 2004 > -2 \cdot 896$, So, accept H_0

So, the H_0 is rejected at 90% ~~and~~ confidence Interval but not in 95 and 99 .

Set E:

$n = 125; > 30$

$\bar{x} = 68695$

$\sigma = 10530$

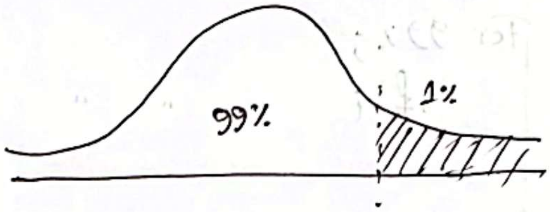
level of significance $= 1\% = 0.01$

$\alpha = 0.01$

$\alpha/2 = 0.005$

$H_0: \mu <= 60014$

$H_1: \mu > 60014$



$Z = 2.58$

Now,

$$Z_c = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{68695 - 60014}{10530/\sqrt{125}}$$

$= 9.21 > Z$, So, H_0 is rejected

The average salary has increased.

Set F:

$\bar{x} = 40.24$

$S_1 = 8.93$

$n = 25; < 30$

d of $F = 24$

$CI = 90\% = 0.90$

$\alpha = 0.10$

$\alpha/2 = 0.05$

$(1 - \alpha/2) = 1 - 0.05$
$= 0.95$

The interval;

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

$$\text{or, } \frac{24 \times (8.93)^2}{\chi^2_{0.05}} \le \sigma^2 \le \frac{24 \times (8.93)^2}{\chi^2_{0.95}}$$

$$\text{or, } \frac{24 \times (8.93)^2}{36.416} \le \sigma^2 \le \frac{24 \times (8.93)^2}{13.848}$$

$$\therefore 52.56 \le \sigma^2 \le 138.20$$