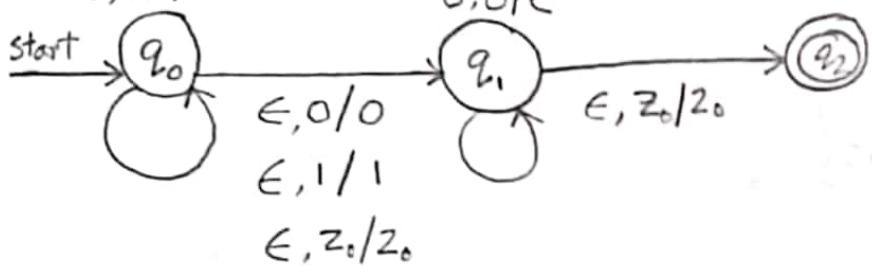
Final

Pushdown Automata (PDA)

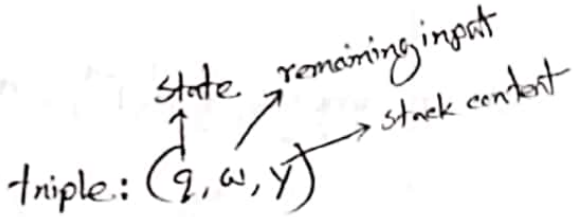
Language L_{wwr} over $\{0,1\}$, even length palindrome over alphabet $\{0,1\}$

1,1/11
1,0/10
0,0/0,0
0,1/01
1,Z_0/1Z_0
0,Z_0/0Z_0

1,1/ϵ
0,0/ϵ



0 0 1 1



Verifying using ID: (1111)

$(q_0, 1111, Z_0)$

$(q_0, 111, 1Z_0)$ $\quad (q_1, 1111, Z_0) \rightarrow (q_2, 1111, Z_0)$

$(q_0, 11, 11Z_0)$ $\quad (q_1, 111, 1Z_0) \rightarrow (q_1, 11, Z_0) \rightarrow (q_2, 11, Z_0)$

$(q_0, 1, 111Z_0)$ $\quad (q_1, 11, 11Z_0) \rightarrow (q_1, 1, 1Z_0) \rightarrow (q_1, \epsilon, \epsilon Z_0)$

$(q_0, \epsilon, 1111Z_0)$ $\quad (q_1, 1, 111Z_0) \rightarrow (q_1, \epsilon, 11Z_0)$

$(q_1, \epsilon, 1111Z_0)$

$(q_2, \epsilon, \epsilon Z_0)$

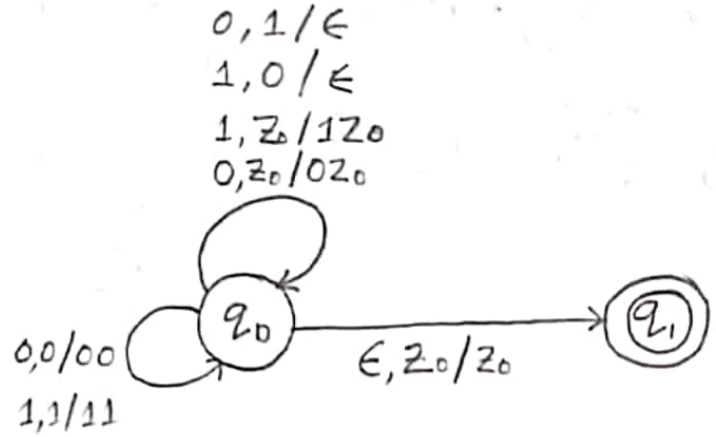
input verified

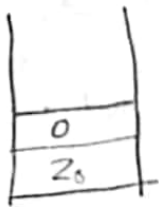
To be accepted:
- remaining part input: ϵ
- state content: Z_0

Language $\{\omega \in (0,1)^* \mid n_0(\omega) = n_1(\omega)\}$

110 001

010011





Language $\{0^n 1^{2n}, n>0\}$

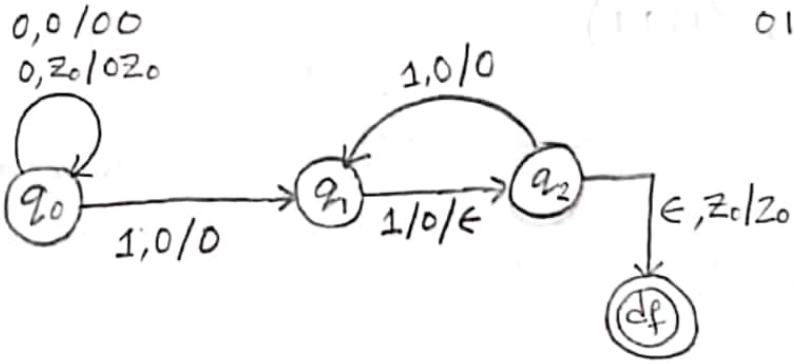
011

00 1111

00 111111 ×

10 ×

01 ×



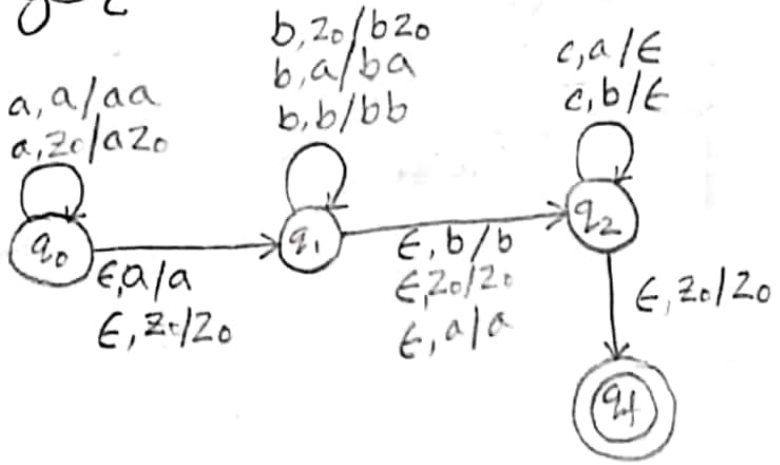
Language $\{a^x b^y c^z \mid x,y,z \geq 0 \text{ and } x+y=z\}$

bc

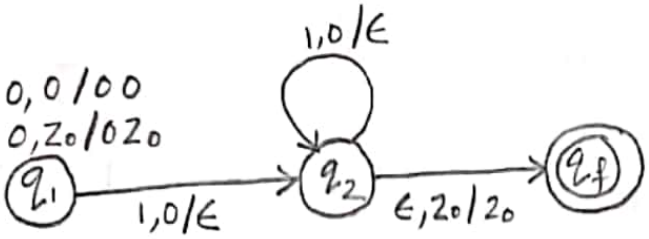
ac

aabccc

aaabbccccc



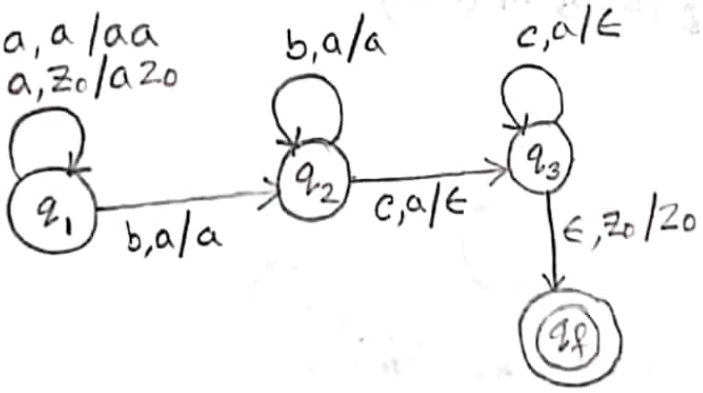
Language = $\{0^n 1^{2n} \mid n \geq 1\}$



0 1

0 0 1 1

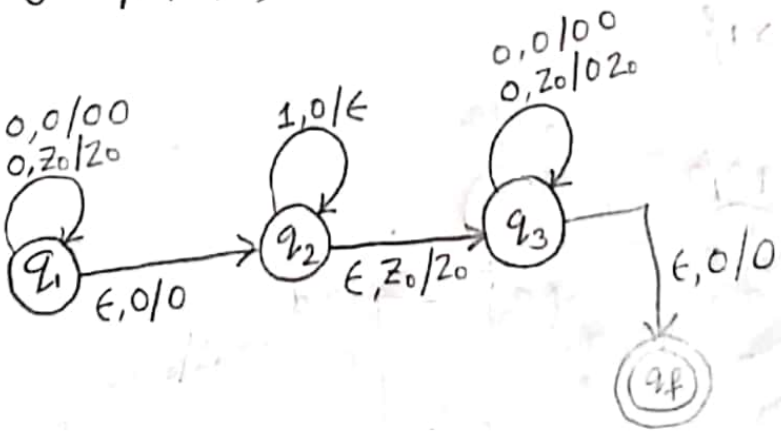
Language $L = \{a^n b^m c^n \mid n \geq 1 \; m \geq 1\}$



aabbbcc

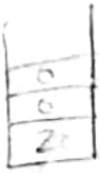
a bbbbc

$L = \{0^n 1^n 0^m \mid m,n \geq 1\}$

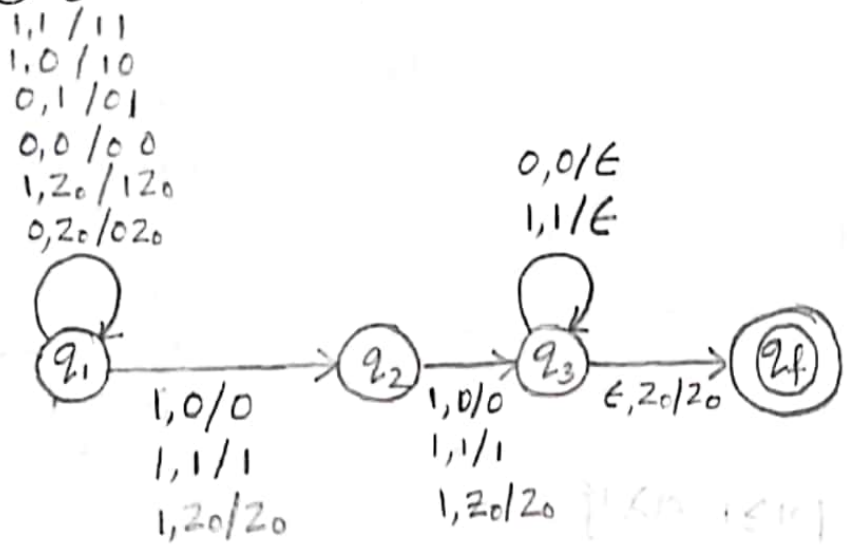


0011000

0001100



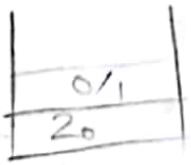
Language = $\{ w\,11\,w^R \mid w \text{ is in } (0+1)^* \}$ over alphabet $\{0,1\}$



ϵ ∣ ϵ

0 1 1 0

0 1 1 1 1 0

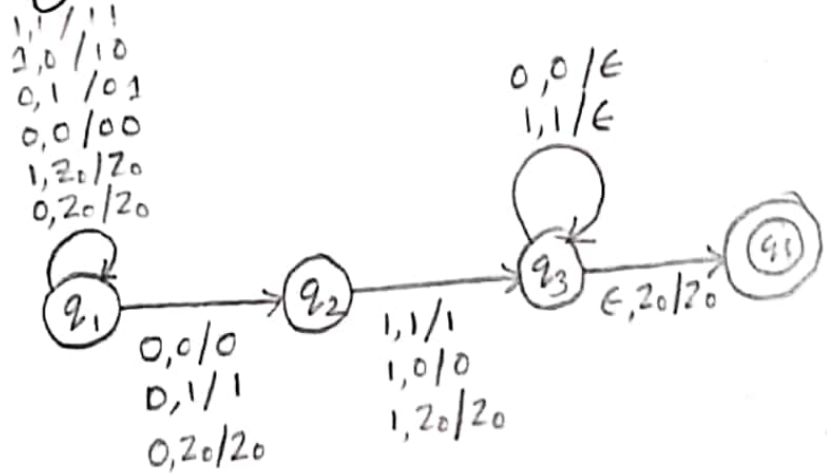


Language = $\{ w\,01\,w^R \mid w \text{ is in } (0+1)^* \}$

1 0 1 1

0 0 1 0

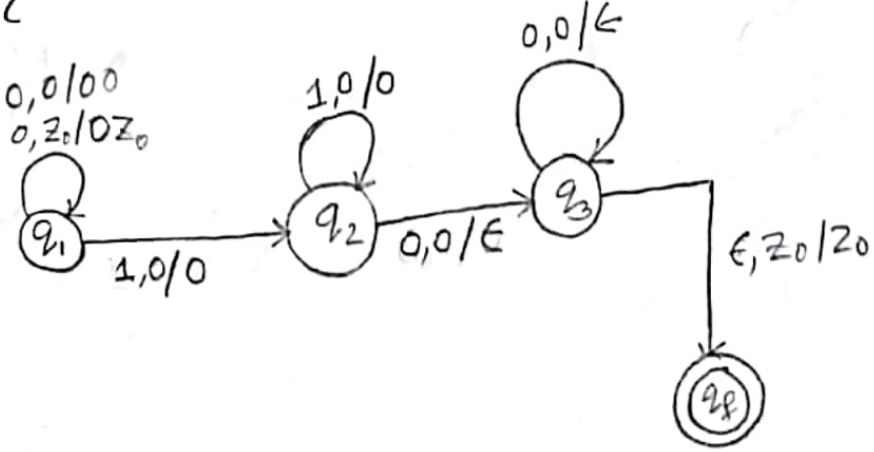
0 1 0 1 1 0



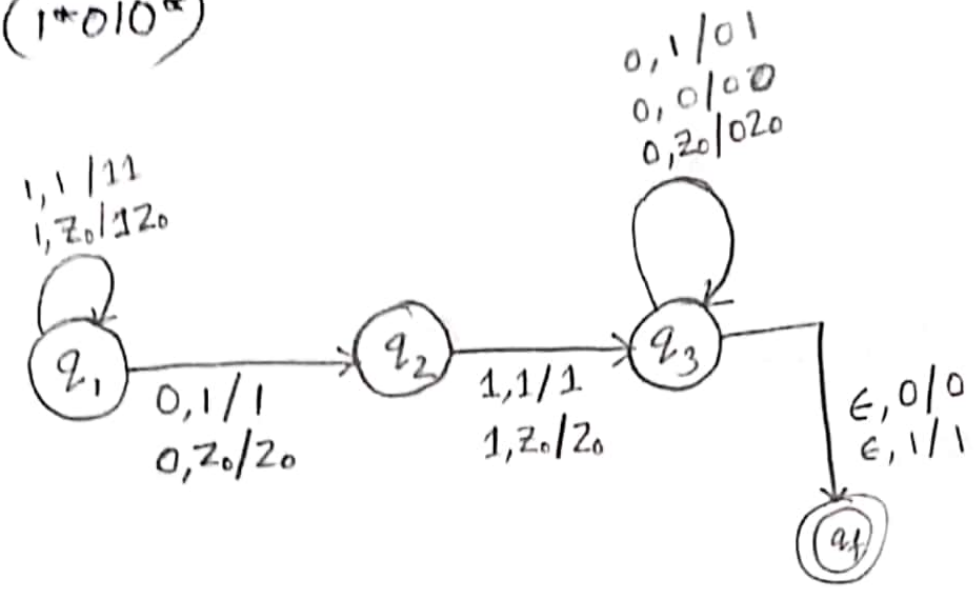
L = $\{ 0^n 1^m 0^n \mid m,n \geq 1 \}$

0 1 0

0 0 1 0 0



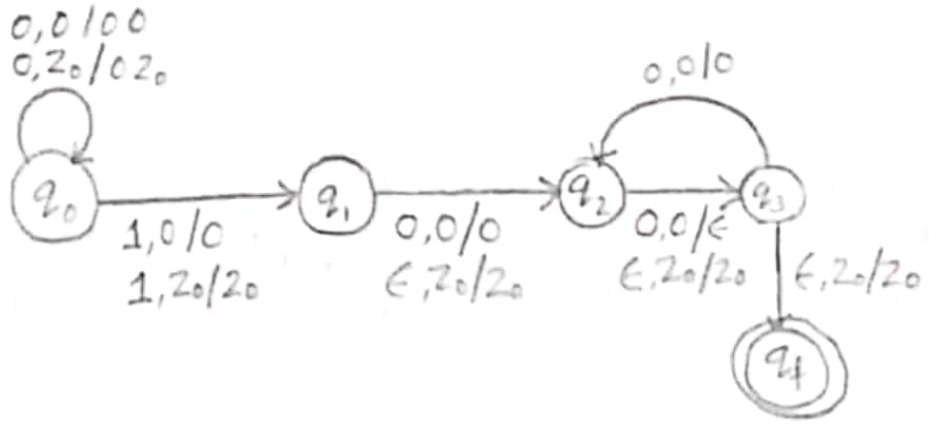
Q1) $L(1^*010^*)$


1,1/11
1,Z0/1Z0
0,1/01
0,0/00
0,Z0/0Z0
q1
q2
q3
qf
0,1/1
0,Z0/Z0
1,1/1
1,Z0/Z0
ϵ,0/0
ϵ,1/1

Quiz-02

$L = \{0^n 1 0^{2n}\}$ $n \geq 0$

1
0100
001000



IDA: 0010000

$(q_0, 0010000, Z_0)$

↓

$(q_0, 010000, 0Z_0)$

↓

$(q_0, 10000, 00Z_0)$

↓

$(q_1, 0000, 00Z_0)$

↓

$(q_2, 000, 00Z_0)$

↓

$(q_3, 00, 0Z_0)$ → $(q_f, 00, 0Z_0)$

↓

$(q_2, 0, Z_0)$

↓

(q_3, ϵ, Z_0) → (q_f, ϵ, Z_0)

Three Address Code

- At most 3 address
- only one operator

- $x = y + z$
- $x = -y$
- $x = y$
- goto L (unconditional)
- if x goto L (conditional)
  if False x goto L (")
- if $x > y$ goto L

Examples:

① $t_1 = b * c$
$t_2 = t_1 + d$
$t_3 = t_2 + a$
$x = t_3$

③ if a < b goto L
b = 2
goto END
L: a = 3
END:

6.5

L1: i = i + 1
$t_1 = i * 8$
$t_2 = a[t_1]$
if $t_2 < V$ goto L1
END:

④ i = 2
j = 1
L2: if i < 5 goto L1:
goto END
L1: j = j + 1
i = i + 1
goto L2
END:

Integer:

```
a=4
b=2
if a<5 goto L1
goto L4
```

```
L1: if a==b goto L2
    goto L3
```

```
L2: b=a+b
    param a
    param b
    call func1, 2
```

```
L3: a=a-1
    if True a!=0 goto L1
    goto L4
```

```
L4: switch b
    case 1: goto L5
    case 2: goto L7
    default: goto L6
```

```
L5: a=a-1
    goto L6
```

```
L6: a=0
```

```
END:
```

```
L7:  t1 = uminus a
     t2 = t1 * 3
     t3 = t2 + 5
     b = t3
     goto END
```

Decipher:

```
b=3
t1=b*3
a=1+t1
if a>b goto L0
goto L4

L0 : a=a-2
     goto L1

L1: if a>b goto L2
    goto L4

L2: b=a+b
    param b
    call func,1,1
    goto L3

L3: a=a-1
    goto L1

END:

L4: if b==1 goto L5
    if b==2 goto L6
    goto L7

L5: a=a-1
    goto END

L6: t1=uminus a
    t2=t1*3
    b=5+t2
    goto L7

L7: a=0
```

origin

```
i=1
a=0
goto L1:

L1: if i<5 goto L2
    goto L7

L2: j=i
    goto L3

L3: if j<4 goto L4
    goto L6

L4: a=a+1
    goto L5

L5: j=j+1
    goto L3

L6: if i==3 goto L7
    goto L8

L8: i=i+1
    goto L1

L7: if i==0: goto L9
    if i==1: goto L10
    if i==2: goto L11

L9: a=a-1
    goto END

L10: j=0

L11: a=0

END:
```

CFG, Simplification of CFG

Decipher

4(c)

$S \rightarrow ERE$

$E \rightarrow E+T \mid E-T \mid T$

$T \rightarrow T+F \mid T-F \mid F$

$F \rightarrow (E) \mid a \mid b \mid c$

$R \rightarrow < \mid > \mid <= \mid >=$

S → Start Symbol
E → Expressions
T → Terms (arithmetic)
F → Bracket / individual
R → logical operators

Example-1:

① $S \rightarrow ERE$
$\rightarrow TRE$
$\rightarrow FRE$
$\rightarrow aRE$
$\rightarrow a<T$
$\rightarrow a<F$
$\rightarrow a<b$

② $S \rightarrow ERE$
$\rightarrow (E) R (E)$ $[E \rightarrow T] [T \rightarrow F] [F \rightarrow (E)]$
$\rightarrow (E+T) R (E+T)$ $[E \rightarrow E+T]$
$\rightarrow (E+T-F) R (E+T-F)$ $[T \rightarrow T-F]$
$\rightarrow (F+F-F) R (F+F-F)$ $[E \rightarrow T] [T \rightarrow F]$
$\rightarrow (a+b-c) <= (b+c-a)$ $[F \rightarrow a \mid b \mid c] [R \rightarrow <=]$

Examples:

① $a<b$
② $b-c$
③ $(a+b)<c$
④ $(a+b-c) <= (b+c-a)$
⑤ $(a<b)+(c>=b)$

Integer

5(c)

S → Start Symbol

$S \rightarrow RI(P);$

$R \rightarrow void \mid float \mid int$

$I \rightarrow id \mid \epsilon$

$P \rightarrow void \mid LIT \mid \epsilon \mid$

$T \rightarrow ,LIT \mid \epsilon$

$L \rightarrow int \mid float \mid \epsilon$

Examples:

- void id (void);
- void id (int id);
- ③ int id (int id, float id);
- float id (float id, float id, int i

Example:

$S \rightarrow RI(P);$

$\rightarrow int\ I\ (P);$

$\rightarrow int\ id\ (P);$

$\rightarrow int\ id\ (LIT);$

$\rightarrow int\ id\ (int\ id\ T);$

$\rightarrow int\ id\ (int\ id\ ,LIT);$

$\rightarrow int\ id\ (int\ id\ ,float\ id);$

Simplification of CFGs

Step-1 : Eliminate useless symbols
2 : " ϵ productions
3 : " unit productions

Elimination of Useless symbols :

- Generating Symbols : if any symbol generates terminals.
- Reachable Symbols : Symbols that can be obtained from head.

$S \to AB / AC$
$A \to aAb / bAa / a$
$B \to bbA / aaB / AB$
$C \to abCA / aDb$? ← Loop
$D \to bD / a\epsilon$

- Removing non-generating:

$S \to AB$
$A \to aAb / bAa / a$
$B \to bbA / aaB / AB$

- Removing non reachable:

$S \to AB$
$A \to aAb \mid bAa \mid a$
$B \to bbA \mid aaB \mid AB$

$S \to aZ / SY / XA$
$X \to bSZa$
$Y \to aSY / bYZ$
$Z \to aYZ / ad$
$A \to ab \mid aA$

- non-generating:

$S \to aZ \mid XA$
$X \to bSZa$
$Z \to ad$
$A \to ab \mid aA$

- non-reachable

$S \to aZ \mid XA$
$X \to bSZa$
$Z \to ad$
$A \to ab \mid aA$

⊞ $A \to xyz \mid Xyzz$
$X \to Xz \mid xYz$
$Y \to yYy \mid Xz$
$Z \to Zy \mid z$

• Generating:
$A \to xyz$
$Z \to Zy \mid z$

• Reachable
$A \to xyz$

⊞ $S \to abS \mid abA \mid abB$
$A \to cd$
$B \to aB$
$C \to dc$

• Generating:
$S \to abS \mid abA$
$A \to cd$
$C \to dc$

• Reachable
$S \to abS \mid abA$
$A \to cd$

Class-Test (2)

Set-A

$S \to AB \mid AC$
$A \to aAb \mid bAa \mid a$
$B \to bbA \mid aaB \mid AB$
$C \to abCA \mid aDb$
$D \to bD \mid aC$
$Z \to bX$
$X \to \epsilon$ (ϵ is not terminal)

• Generating
$S \to AB$
$A \to aAb \mid bAa \mid a$
$B \to bbA \mid aaB \mid AB$
$Z \to b$

• Reachable
$S \to AB$
$A \to aAb \mid bAa \mid a$
$B \to bbA \mid aaB \mid AB$

Set-B

$A \to xyz \mid Xyzz \mid B$
$X \to Xz \mid xYz$
$Y \to yYy \mid Xz$
$Z \to Zy \mid z$
$B \to b \mid \epsilon$
$U \to bV$
$V \to \epsilon$

• Generating
$A \to xyz \mid B$
$Z \to Zy \mid z$
$B \to b$
$U \to b$

• Reachable
$A \to xyz \mid B$
$B \to b$

Eliminating ϵ-production:

Nullable variable: Variables from which we can go to ϵ.

$S \to AB$
$A \to aAA \mid \epsilon$
$B \to bBB \mid \epsilon$

$S \to AB \mid A \mid B$
$A \to aAA \mid aA \mid aA \mid a$
$B \to bBB \mid bB \mid bB \mid b$

Spring-22

1(b)

$S \to cAdB$
$A \to Ab \mid \epsilon$
$B \to Aa \mid \epsilon$

$\to$

$S \to cAdB \mid cdB \mid cAd \mid cd$
$A \to Ab \mid b$
$B \to Aa \mid a$

Derive cdbba:

$S \to cdB$
$\to cdAa \quad [B \to Aa]$
$\to cdAba \quad [A \to Ab]$
$\to cdbba \quad [A \to b]$

Eliminating Unit Production

E → T | E+T

T → F | T*F

F → I | (E)

I → a|b|Ia|Ib|I0|I1

A → B ✓

A → a ✗

unit pairs : (E,E) (T,T) (F,F) (I,I)

① (E,E) and the production E→T gives us unit pair ~~(E→E+T) (E→T*F)~~ (E,T)

② (E,T) " " " T→F " " " " (E,F)

③ (E,F) " " " F→I " " " " (E,I)

④ (E,I) no production

⑤ (T,T) and the production T→F gives us unit pair (T,F)

⑥ (T,F) " " " F→I " " " " (T,I)

⑦ (T,I) no production " " " (F→I)

⑧ (F,F) " " " F→I " " " "

⑨ (F,I) no production

⑩ (I,I) no production

Pair	Productions
(E,E)	E→E+T
(E,T)	E→T*F
(E,F)	E→(E)
(E,I)	E→a|b|Ia|Ib|I0|I1
(T,T)	T→T*F
(T,F)	T→(E)
(T,I)	T→a|b|Ia|Ib|I0|I1
(F,F)	F→(E)
(F,I)	F→a|b|Ia|Ib|I0|I1
(I,I)	I→a|b|Ia|Ib|I0|I1

The final Grammar:

E → E+T | T*F | (E) | a|b|Ia|Ib|I0|I1

T → T*F | (E) | a|b|Ia|Ib|I0|I1

F → (E) | a|b|Ia|Ib|I0|I1

I → a|b|Ia|Ib|I0|I1

Spring-22

3(b)

$S \rightarrow XY$

$X \rightarrow a$

$Y \rightarrow Z \mid b$

$Z \rightarrow M$

$M \rightarrow N$

$N \rightarrow a$

unit pairs: (S,S) (X,X) (Y,Y) (Z,Z) (M,M) (N,N)

① (S,S) and the production is nothing

② (X,X) and the production is nothing

③ (Y,Y) and the production $Y \rightarrow Z$ gives us unit pair (Y,Z)

④ (Y,Z) and " " $Z \rightarrow M$ " " " " (Y,M)

⑤ (Y,M) " " " $M \rightarrow N$ " " " " (Y,N)

⑥ (Y,N) and the production is nothing

⑦ (Z,Z) and the production $(Z \rightarrow M)$ " " " " (Z,M)

⑧ (Z,M) " " " $(M \rightarrow N)$ " " " (Z,N)

⑨ (Z,N) and the production is nothing

⑩ (M,M) and the production $(M \rightarrow N)$ " " " " (M,N)

⑪ (M,N) and the production is nothing

⑫ (N,N) and the production " "

Pair	Productions
(S,S)	S → XY
(X,X)	X → a
(Y,Y)	Y → b
(Y,Z)	
(Y,M)	
(Y,N)	Y → a
(Z,Z)	
(Z,M)	
(Z,N)	Z → a
(M,M)	
(M,N)	M → a
(N,N)	N → a

The final Grammar:

$S \rightarrow XY$

$X \rightarrow a$

$Y \rightarrow b \mid a$

$Z \rightarrow a$
$M \rightarrow a$
$N \rightarrow a$
} won't be included as they are not reachable.

Chomsky Normal form:

order formation → ① Eliminate ϵ-productions
② Eliminate unit productions
③ Eliminate useless productions

- $A \to BC$
- $A \to \alpha$
- For more than length 2, make it 2.

$E \to E+T \mid T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$
$T \to T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$
$F \to (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$
$I \to a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

we introduce:

$P \to +$ $\quad M \to *$ $\quad L \to ($ $\quad R \to)$
$A \to a$ $\quad Z \to 0$ $\quad 0 \to 1$ $\quad B \to b$

$E \to EPT \mid TMF \mid LER \mid a \mid b \mid IA \mid IB \mid I2 \mid I0$
$T \to TMF \mid LER \mid a \mid b \mid IA \mid IB \mid I2 \mid I0$
$F \to LER \mid a \mid b \mid IA \mid IB \mid I2 \mid I0$
$I \to a \mid b \mid IA \mid IB \mid I2 \mid I0$

$C_1 = PT$
$C_2 = MF$
$C_3 = ER$

So final Grammer:

$E \to EC_1 \mid TC_2 \mid LC_3 \mid a \mid b \mid IA \mid IB \mid I2 \mid I0$
$T \to TC_2 \mid LC_3 \mid a \mid b \mid IA \mid IB \mid I2 \mid I0$
$F \to LC_3 \mid a \mid b \mid IA \mid IB \mid I2 \mid I0$
$I \to a \mid b \mid IA \mid IB \mid I2 \mid I0$
$C_1 \to PT$ $\quad C_2 \to MF$ $\quad C_3 = ER$
$A \to +$ $\quad M \to *$ $\quad L \to ($ $\quad R \to)$
$Z \to 0$ $\quad 0 \to 1$ $\quad B \to b$

} has either terminal or two variables.

Fall-2022

2(c)

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \epsilon$

Eliminate ϵ production:

$S \rightarrow ASA \mid AS \mid SA \mid S \mid aB \mid a$

$A \rightarrow B \mid S$

$B \rightarrow b$

Eliminate unit production:

unit pairs: (S,S) (A,A) (B,B)

① (S,S) and the production (S→S), gives us unit pair (S,S)

② (A,A) " " " (A→B) " " " " (A,B)

③ (A,B) " " " nothing

④ (B,B) " " " nothing

Pair	Productions
(S,S)	$S \rightarrow ASA \mid AS \mid SA \mid aB \mid a$
(A,A)	$A \rightarrow S$
(A,B)	$A \rightarrow b$
(B,B)	$B \rightarrow b$

The grammar will be:

$S \rightarrow ASA \mid AS \mid SA \mid aB \mid a$

$A \rightarrow S \mid b$

$B \rightarrow b$

- Eliminate useless production

$S \to ASA \mid AS \mid SA \mid aB \mid a$

$A \to S \mid b$

$B \to b$

Generating Symbols:

$S \to ASA \mid AS \mid SA \mid aB \mid a$

$A \to S \mid b$

$B \to b$

Reachable Symbols:

$S \to ASA \mid AS \mid SA \mid aB \mid a$

$A \to S \mid b$

$B \to b$

The Grammer is

$S \to ASA \mid AS \mid SA \mid aB \mid a$

$A \to S \mid b$

$B \to b$

Converting to chomsky :

we introduce →

$C_1 \to SA$ $\quad$ $X \to a$

The final Grammer in CNF :

$S \to AC_1 \mid AS \mid SA \mid XB \mid a$

$A \to S \mid b$

$B \to b$

$C_1 \to SA$

$X \to a$

} either terminal or two variables.

Fall-2022

2(c) $S \rightarrow AcBd$

$A \rightarrow bA \mid \epsilon$

$B \rightarrow cBd \mid a$

(i) First (a) = {a} Follow(S) = {$}

(b) = {b} (A) = {c}

(c) = {c} (B) = {d}

(d) = {d}

~~(S) = {}~~

(B) = {c, a}

(A) = {b, ε}

(S) = {b, c}

(ii)

LL(1) table:

NT-Symbols	Terminals a	b	c	d	$
S		S→AcBd	S→AcBd		
A		A→bA	A→ε		
B	B→a		B→cBd		

(iii) ccad

Matched	STACK	INPUT	ACTION
	S $	ccad $	S→AcBd
	AcBd $	ccad $	A→ε
	cBd $	ccad $	matched c
c	Bd $	cad $	B→cBd
c	cBdd $	cad $	matched c
cc	Bdd $	ad $	B→a
cc	add $	ad $	matched a
cca	dd $	d $	matched d
ccad	d $	$	
ccad	d $	$	error

3(c)

$S \rightarrow (L) \mid a$

$L \rightarrow L,S \mid S$

$L \rightarrow L,S \mid S$

$A \rightarrow A\alpha \mid \beta \rightarrow A \rightarrow \beta A'$

$A' \rightarrow \epsilon \mid \alpha$

After left-recursion:

$L \rightarrow SL'$

$L' \rightarrow \epsilon \mid ,S$

The grammer will be:

$S \rightarrow (L) \mid a$

$L \rightarrow SL'$

$L' \rightarrow \epsilon \mid ,SL'$

First ( ) = { (}

( ) = {) }

(a) = { a }

(,) = { , }

(L') = { , , ϵ }

(L) = { (, a }

(S) = { (, a }

Follow (S) = { $, , }

(L) = {) }

(L') = {) }

No need