Lectur-8 (A)

For Cantor set :

\# ϵ_n : Size of new element at n iteration = length = $\frac{1}{3^n}$

N_n : the number of new elements at " $n = 2^n$

Fractal dimension, $D = -\lim_{n\to\infty} \frac{\log(N_n)}{\log(\epsilon_n)}$

→ Defines the complexity of fractal.

Dimension of Cantor Set

$$D = -\lim_{n\to\infty} \frac{\log(N_n)}{\log(\epsilon_n)} = -\lim_{n\to\infty} \frac{\log(2^n)}{\log(\frac{1}{3^n})}$$

$$= -\lim_{n\to\infty} \frac{n\log 2}{n\log \frac{1}{3}}$$

$$= \lim_{n\to\infty} \frac{\log 2}{\log 3}$$

$$= 0.6309$$

For Koch Snowflake:

$\epsilon_n = \frac{1}{3^n}$

$N_n = 3\times4^n$

$$D = -\lim_{n\to\infty} \frac{\log[3(4^n)]}{\log[\frac{1}{3^n}]} = -\lim_{n\to\infty} \frac{\log(3) + n\log(4)}{-n\log(3)}$$

$$= \lim_{n\to\infty} \frac{\log(3)}{n\log(3)} + \frac{n\log 4}{n\log 3}$$

$$= \lim_{n\to\infty} \frac{1}{n} + \frac{\log 4}{\log 3} = \log 4 / \log 3 = 1.2619$$

Sierpinski Triangle:

$$\epsilon_n = \frac{1}{2^n}$$

$$N_n = 3^n$$

$$D = -\lim_{n \to \infty} \frac{\log(3^n)}{\log(1/2^n)}$$

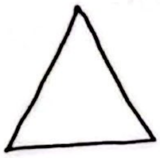
$$= \lim_{n \to \infty} \frac{\log(3)}{\log(2)}$$

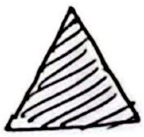
$$= 1.58$$

The ~~mad~~ mandlebrot set

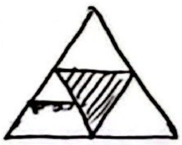
Decipher

2(b)

①  stage 0

②  Stage 1

③  stage 2

④ 

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Stage	no. of ~~sides~~ triangles	Length of ~~sides~~ triangles
0	0	0
1	1	$\frac{1}{2}$
2	4	$\frac{1}{4}$
3	13	$\frac{1}{8}$
4	40	$\frac{1}{16}$
	$(3^{n-1}) + (n-1)$	$\frac{1}{2^n}$

Here, $\epsilon_n = \frac{1}{2^n}$; $N_n = (3^{n-1}) + (n-1)$

$$D = -\lim_{n\to\infty} \frac{\log\left((3^{n-1}) + (n-1)\right)}{\log\left(\frac{1}{2^n}\right)} = -\lim_{n\to\infty} \frac{\log(3^{n-1}) + \log(n-1)}{n\log(\frac{1}{2})}$$

$$= -\lim_{n\to\infty} \frac{\cancel{\log}(n-1)\log 3 + \log(n-1)}{-n\log(2)}$$

$$= -\lim_{n\to\infty} \frac{n\log 3 - \log 3 + \log(n-1)}{-n\log 2}$$

$$= -\lim_{n\to\infty} -\frac{n\log 3}{n\log 2} = \frac{\log 3}{\log 2} = 1.5849$$

2(d)

V: X, F

C: +, −

axiom: F+XF+F+XF

Rules: X → XF−F+F−XF+F+XF−F+F−X

Angle: 90°

X = do nothing

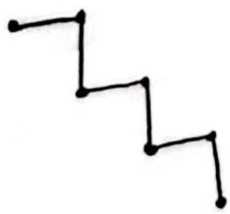
F = draw a line forward

+ : rotate clockwise by 90°

− : " counterclock by 90°

n=0: F+XF+F+XF

n=1: F+XF−F+F−XF+F+XF−F+F−XF+F+XF−F+F−XF+F+XF−F+F−XF



Ki kortesi aishob?

3(d) Decipher

$c = a + bi$

$Z_{n+1} = Z_n^2 + c$; converge if $Z_n > 2$

complex number : $c = -0.771 - 0.326i$

$Z_0 = 0 + (-0.771 - 0.326i)$ $\quad |Z_0| = 0.8370$

$= -0.771 - 0.326i$

$Z_1 = (-0.771 - 0.326i)^2 + (-0.771 - 0.326i)$

$= (-0.771)^2 + 2 \times 0.771 \times 0.326 + (0.326i)^2 - 0.771 - 0.326i$

$= 0.5944 + 0.5026 - 0.1062 - 0.771 - 0.326i$

$= 0.2198 - 0.326i$ $\quad |Z_1| = \sqrt{(0.2198)^2 + (-0.326)^2}$

$= 0.3931$

$Z_2 = (0.2198 - 0.326i)^2 + (-0.771 - 0.326i)$

$=$

$\vdots$

upto Z_{10}

it will converge after Z_8 (GPT bolche...)

So, The colour of the points are Blue.

Integer

3(f)

$c = a + bi$

$z_{n+1} = z_n^2 + c \quad ; c = -0.5 + 0.5i$

$|z_0| = \cancel{\ } 0.707 < 2$

$z_0 = 0 + (-0.5 + 0.5i)$

$z_1 = (-0.5 + 0.5i)^2 + (-0.5 + 0.5i)$

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⋮

if z_{10} stays inside 2, then it is a member of mandlebrot set.