# LECTURE 2

## Lecture-2

$$C = \alpha C_f + (1-\alpha) C_b$$

$\alpha = 0$ ; $C = C_b$ (Transparent)

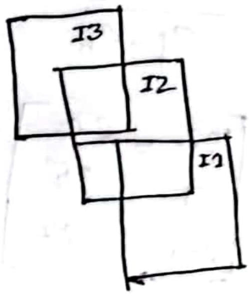
$\alpha = 1$ ; $C = C_f$ (Opaque)

$\alpha = 0.5$ ; $C = \dfrac{C_f + C_b}{2}$ (Partial Transparent)

### Quiz- Set A

2

$$C_1 = \begin{bmatrix} 0.2 & 0.39 \\ 0 & 0.3 \\ 0.45 & 0.5 \end{bmatrix} \begin{bmatrix} 15 & 20 \\ 200 & 20 \\ 110 & 99 \end{bmatrix} + \begin{bmatrix} 1-0.2 & 1-0.39 \\ 1-0 & 1-0.3 \\ 1-0.45 & 1-0.5 \end{bmatrix} \begin{bmatrix} 130 & 20 \\ 50 & 85 \\ 230 & 9 \end{bmatrix}$$




$$= \begin{bmatrix} 3 & 7.8 \\ 0 & 6 \\ 49.5 & 49.5 \end{bmatrix} + \begin{bmatrix} 104 & 12.2 \\ 50 & 59.5 \\ 126.5 & 4.5 \end{bmatrix}$$

$$= \begin{bmatrix} 107 & 20 \\ 50 & 65.5 \\ 176 & 54 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.2 & 0.39 \\ 0 & 0.3 \\ 0.45 & 0.5 \end{bmatrix} \begin{bmatrix} 130 & 20 \\ 50 & 85 \\ 230 & 9 \end{bmatrix} + \begin{bmatrix} 1-0.2 & 1-0.39 \\ 1-0 & 1-0.3 \\ 1-0.45 & 1-0.5 \end{bmatrix} \begin{bmatrix} 107 & 20 \\ 50 & 65.5 \\ 176 & 54 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 7.8 \\ 0 & 25.5 \\ 103.5 & 4.5 \end{bmatrix} + \begin{bmatrix} 85.6 & 12.2 \\ 50 & 45.85 \\ 96.8 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 111.6 & 20 \\ 50 & 71.35 \\ 200.3 & 31.5 \end{bmatrix}$$

Ans.

Sol.-F

2.

We know,

$$c = \alpha c_f + (1-\alpha) c_b$$

$$c = \alpha c_f + c_b - \alpha c_b$$

$$c - c_b = \alpha (c_f - c_b)$$

$$\therefore \alpha = \frac{c - c_b}{c_f - c_b}$$

$$\therefore \alpha = \begin{bmatrix} 102 & 20 \\ 74 & 85 \\ 201 & 27 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ 200 & 20 \\ 110 & 99 \end{bmatrix} \Big/ \begin{bmatrix} 130 & 20 \\ 50 & 85 \\ 230 & 9 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ 200 & 20 \\ 110 & 99 \end{bmatrix}$$

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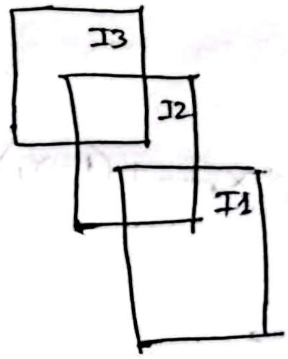
2




$C_1 : \alpha I_2 + (1-\alpha) I_3$

$C_2 : \alpha I_1 + (1-\alpha) C_1$.

$\therefore C_2 = \alpha I_1 + (1-\alpha)\{\alpha I_2 + (1-\alpha) I_3\}$

$\therefore C_2 = \alpha I_1 + \alpha(1-\alpha) I_2 + (1-\alpha)^2 I_3$

$= \alpha I_1 + (\alpha - \alpha^2) I_2 + (1-\alpha)^2 I_3$

$= \alpha I_1 + \alpha I_2 - \alpha^2 I_2 + (1 - 2\alpha + \alpha^2) I_3$

$= \alpha I_1 + \alpha I_2 - \alpha^2 I_2 + I_3 - 2\alpha I_3 + \alpha^2 I_3$

$= \alpha(I_1 + I_2) - \alpha^2 (I_2 - I_3) - 2\alpha I_3 + I_3$

$\alpha^2 (I_3 - I_2) + \alpha (I_1 + I_2 - 2I_3) + (I_3 - C_2) = 0$

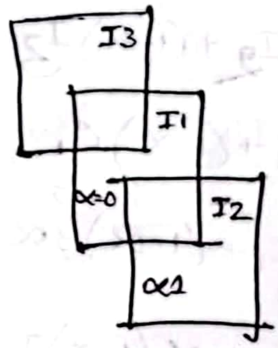
$\therefore ax^2 + bx + c = 0$

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

## Integer:

2(d)

$$C_{output} = \alpha_1 I_2 + (1-\alpha_1)\{\alpha_0 I_1 + (1-\alpha_0) I_3\}$$

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Quiz- Sete

1-byte = 8 bits

Image size = 6x7

$\therefore$ 42 pixels

original image size = $(42 \times 8)/8$

= 42 byte

= 0.041 KB

R1 : 303112

R2 : 5021

R3 : 421021

R4 : 2253

R5 : 5321

R6 : 332220

Total pixel : 30

Compressed size = $(30 \times 8)/8$

= 240/8

= 30 byte /1024

= 0.029 KB

~~$\therefore$ compression ratio = $\frac{0.041 - 0.029}{0.041}$ = 29.26%~~

$\therefore$ compression ratio = $\frac{0.041}{0.029}$

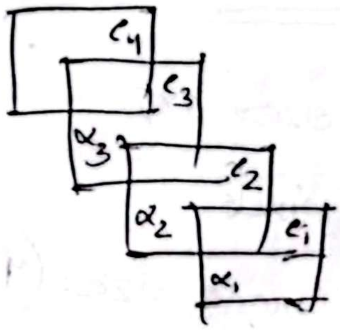
= 1.41

**Set-D**

$\alpha_1$ : Alpha compositing parameter to blend $c_1$ $c_2$

$\alpha_2$ : " " " " " $c_2$ $c_3$

$\alpha_3$ : " " " " " $c_3$ $c_4$




$$C_{output} = \alpha_1 c_1 + (1-\alpha_1) c_2$$

$$= \alpha_1 c_1 + (1-\alpha_1)\{\alpha_2 c_2 + (1-\alpha_2) c_3\}$$

$$= \alpha_1 c_1 + (1-\alpha_1)\{\alpha_2 c_2 + (1-\alpha_2)\{\alpha_3 c_3 + (1-\alpha_3) c_4\}\}$$

# LECTURE 3

Lecture-3

## Bezier Curves:

Degree, d: N-1 ; N: control points

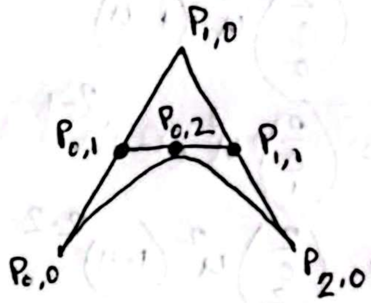
quadratic (d=2), N=3.

For 2 control points $P_0$ and $P_1$;

$$Q_0 = P_0 + u(P_1 - P_0)$$

## # De casteljau's Algorithm.

$$P_{i,j} = (1-u)P_{i,j-1} + u P_{i+1,j-1}$$




For degree 2;

$$P_{0,2} = (1-u)P_{0,1} + uP_{1,1}$$

$$= (1-u)\left[(1-u)P_{0,1} + uP_{1,0}\right] + u\left[(1-u)P_{1,0} + uP_{2,0}\right]$$

$$= (1-u)^2 P_{0,1} + u(1-u)P_{1,0} + u(1-u)P_{1,0} + u^2 P_{2,0}$$

$$= (1-u)^2 P_{0,1} + 2u(1-u)P_{1,0} + u^2 P_{2,0}$$

For degree 4;

$$P_{0,4} = (1-u)P_{0,3} + uP_{1,3}$$

$$= (1-u)\left[(1-u)P_{0,2} + uP_{1,2}\right] + u\left[(1-u)P_{1,2} + uP_{2,2}\right]$$

$$= (1-u)^2 P_{0,2} + 2u(1-u)P_{1,2} + u^2 P_{2,2}$$

$$= (1-u)^2\left[(1-u)P_{0,1} + uP_{1,1}\right] + 2u(1-u)\left[(1-u)P_{1,1} + uP_{2,1}\right] + u^2\left[(1-u)P_{2,1} + uP_{3,1}\right]$$

$$= (1-u)^3 P_{0,1} + u(1-u)^2 P_{1,1} + 2u(1-u)^2 P_{1,1} + 2u^2(1-u)P_{2,1} + u^2(1-u)P_{2,1} + u^3 P_{3,1}$$

$$= (1-u)^3 P_{0,1} + 3u(1-u)^2 P_{1,1} + 3u^2(1-u)P_{2,1} + u^3 P_{3,1}$$

$$= (1-u)^3\left[(1-u)P_{0,0} + uP_{1,0}\right] + 3u(1-u)^2\left[(1-u)P_{1,0} + uP_{2,0}\right] + 3u^2(1-u)\left[(1-u)^2 P_{2,0} + uP\right.$$

$$\left. + u^3\left[(1-u)P_{3,0} + uP_{4,0}\right]\right.$$

Using Polynomial:

$$Q(u) = \sum_{i=0}^{d} B_{i,d}(u)\, P_i \qquad 0 \le u \le 1$$

$$B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$$

For $d = 2$;

$$B_{0,2}(u) = \binom{2}{0} u^0 (1-u)^{2-0} = (1-u)^2$$

$$B_{1,2}(u) = \binom{2}{1} u^1 (1-u)^{2-1} = 2u(1-u)$$

$$B_{2,2}(u) = \binom{2}{2} u^2 (1-u)^{2-2} = u^2$$

$$Q_2(u) = (1-u)^2 P_{0,1} + 2u(1-u) P_{1,0} + u^2 P_{2,0}$$

Set-B

3) $P_0 = (1,0)$

$P_1 = (1,4)$

$P_2 = (3,4)$

$P_3 = (4,2)$

$P_4 = (4,0)$

$Q\left(\frac{1}{2}\right) = ?$

Here, $N = 5$

degree $= 5-1 = 4$

For degree 4;

$$Q(u) = B_{0,4}(u) + B_{1,4}(u) + B_{2,4}(u) + B_{3,4}(u) + B_{4,4}(u)$$

$$= \binom{4}{0} u^0 (1-u)^{4-0} + \binom{4}{1} u^1 (1-u)^{4-1} + \binom{4}{2} u^2 (1-u)^{4-2} + \binom{4}{3} u^3 (1-u)^{4-3} + \binom{4}{4} u^4 (1-u)^{4-4}$$

$$= (1-u)^4 + 4u(1-u)^3 + 6u^2(1-u)^2 + 4u^3(1-u) + u^4$$

$$= (1-u)^4 P_0 + 4u(1-u)^3 P_1 + 6u^2(1-u)^2 P_2 + 4u^3(1-u) P_3 + u^4 P_4$$

$$Q\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\cdot\left(\frac{1}{2}\right)^3 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \frac{3}{2}\left(\frac{1}{2}\right)^2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + 4\cdot\frac{1}{8}\cdot\left(\frac{1}{2}\right) \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \left(\frac{1}{2}\right)^4 \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/16 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{9}{8} \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2.6875 \\ 3 \end{bmatrix}$$

Set-D

③

$$\begin{array}{lccccccccccc}
 & & & & & & 1 & & & & & \\
 & & & & & 1 & & 1 & & & & \\
d=2; & & & & 1 & & 2 & & 1 & & & \\
d=3; & & & 1 & & 3 & & 3 & & 1 & & \\
d=4; & & 1 & & 4 & & 6 & & 4 & & 1 & \\
d=5; & 1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

$$Q(u)=(1-u)^5P_0+5u(1-u)^4P_1+10u^2(1-u)^3P_2+10u^3(1-u)^2P_3+\underline{5u^4(1-u)P_4}+\underline{u^5P_5}$$

$Q(1)=0+0+0+0+0+P_5$

$\therefore \begin{bmatrix}5\\1\end{bmatrix}=P_5$

$$\cancel{Q(u)=5u^4(1-u)P_4}$$

$$\cancel{\begin{bmatrix}0.68\\3.56\end{bmatrix}=5\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)P_4}$$

$$Q(0.5)=(1-0.5)^5\begin{bmatrix}-3\\3\end{bmatrix}+5\times(0.5)\times(1-0.5)^4\begin{bmatrix}-1\\4\end{bmatrix}+10\times(0.5)^2(1-0.5)^3\begin{bmatrix}0\\5\end{bmatrix}+$$

$$10\times(0.5)^3(1-0.5)^2\begin{bmatrix}1\\3\end{bmatrix}+5\times(0.5)^4(1-0.5)P_4+(0.5)^5\begin{bmatrix}5\\1\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{3}{32}\\ \frac{3}{32}\end{bmatrix}+\begin{bmatrix}-\frac{5}{32}\\ \frac{5}{8}\end{bmatrix}+\begin{bmatrix}0\\ \frac{25}{16}\end{bmatrix}+\begin{bmatrix}\frac{5}{16}\\ \frac{15}{16}\end{bmatrix}+\frac{5}{32}\cdot P_4+\begin{bmatrix}\frac{5}{32}\\ \frac{1}{32}\end{bmatrix}$$

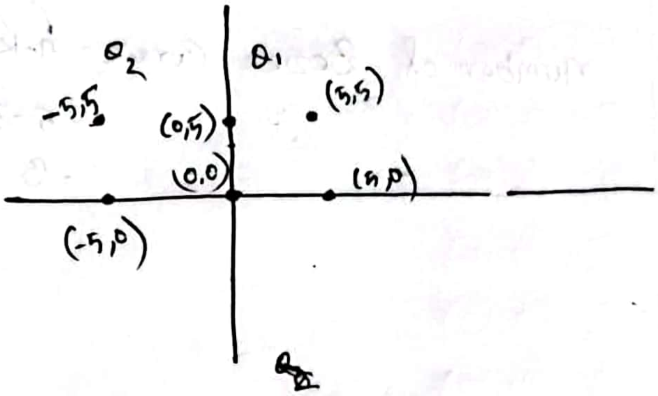
$$\begin{bmatrix}0.68\\3.56\end{bmatrix}=\begin{bmatrix}\frac{7}{32}\\ \frac{13}{4}\end{bmatrix}+\frac{5}{32}P_4$$

$$\frac{5}{32}P_4=\begin{bmatrix}0.46125\\0.31\end{bmatrix}\qquad \therefore P_4=\begin{bmatrix}2.952\\1.984\end{bmatrix}$$

Deqoker

3(a)

For $Q_1$;




$P_0\ (0,0);\ P_1\ (0,5);\ P_2(5,5);\ P_3(5,0)$

$Q_1(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2$
$\quad + u^3 P_3$

$(0.6)$

$=$

For $Q_2$;

$P_0\ (0,0);\ P_1\ (0,5);\ P_2\ (-5,5);\ P_3\ (-5,0)$

$Q_2(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$

$(0.8)$

$=$

B-spline Curve:

$$\text{number of Bezier Curve} = \overset{\text{control point} - 1}{n} - \overset{\text{kind of curve (cubic)}}{K} + 1$$

$$= 5 - 3 + 1$$

$$= 3 \text{ Bezier curve}$$

# Uniform Quadratic B-spline:

$$S_i(t) = \begin{pmatrix} P_i & P_{i+1} & P_{i+2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

# Open Uniform:

$$S_0(t) = \begin{pmatrix} P_0 & P_{1} & P_{2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 2 & -4 & 2 \\ -3 & 4 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$S_i(t) = \begin{pmatrix} P_i & P_{i+1} & P_{i+2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$S_{n-2}(t) = \begin{pmatrix} P_{n-2} & P_{n-1} & P_n \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -3 & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

Set-A

3

$P_0 = (-3,1)$

$P_1 = (-1,2)$

$P_2 = (1,3)$

$P_3 = (3,4)$

$P_4 = (4,5)$

$P_5 = (6,7)$

$P_6 = (7,8)$

# of curve $= n-k+1$

$= 6-2+1$

$= 5$

$S_0, S_1, S_2, S_3, S_4$

$$S_4(0.3) = \begin{pmatrix} P_4 & P_5 & P_6 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 6 & 7 \\ 5 & 7 & 8 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} (0.3)^2 \\ 0.3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 6 & 7 \\ 5 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1/2 & -1 & \frac{1}{2} \\ -1 & 1 & 1/2 \\ 1/2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.9 \\ 0.3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -0.5 & 2 & 5 \\ -0.5 & 2 & 6 \end{pmatrix} \begin{pmatrix} 0.9 \\ 0.3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5.15 \\ 6.15 \end{pmatrix}$$

$$S_3(0.3) = \begin{pmatrix} P_3 & P_4 & P_5 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$=$

Set-C

3

$P_0 = (1,10)$
$P_1 = (3,15)$
$P_2 = (5,20)$
$P_3 = (7,15)$
$P_4 = (9,13)$
$P_5 = (11,10)$

# number of Bezier Curve = $n-k+1$
$= 5-2+1$
$= 4$

$S_0, S_1, S_2, S_3$

For uniform;

$$S_0(0.5) = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

=

For open uniform;

$$S_0(0.5) = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 2 & -4 & 2 \\ -3 & 4 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

=

Set, E

$t = 1$

# LECTURE 4

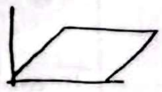
Lecture-4

## Scaling:

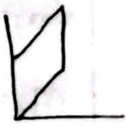
$$\text{Scale}(S_x, S_y) = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

## Shearing:

$$\text{Shear-}x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$



$$\text{Shear-}y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$



## Reflection:

$$\text{Reflect-}y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Reflect-}x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

## Translation:

For 2D:

$$\begin{bmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{bmatrix}$$

For 3D:

$$\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Scaling:

For 2D:

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 3D:

$$\begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Rotation:

For 2D:

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 3D:

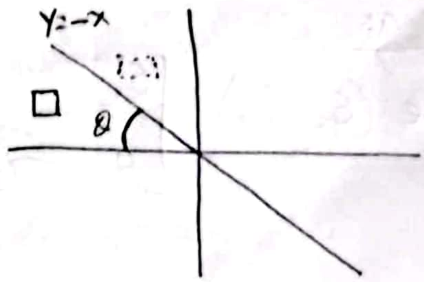
$$\text{Rot-}x: \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\text{Rot-}z: \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot-}y: \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

Integer

2(b)




Transformation matrix for the reflection about the line $y=-x$ :

$$m_1 = Rot(45^\circ) * Ref\text{-}y * Rot(-45^\circ)$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

Here, $m = -1$

$\tan\theta = -1$; $\theta = \tan^{-1}(-1)$

$\theta = -45^\circ$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Reflection related to y-axis followed by a counter-clockwise rotation of 90°

$$m_2 = Rot(90^\circ) * Ref\text{-}Y$$

$$= \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$\therefore m_1 = m_2$

3(b)

① Translate (-5,-2,-3)

② Rotate along Z

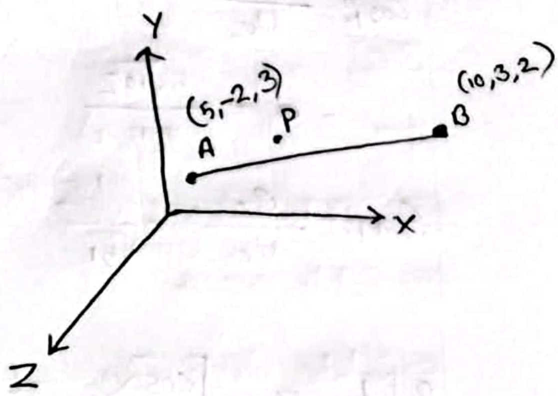
③ Rotate along X

④ Rotate along Y

⑤ Rotate along X

⑥ Rotate along Z

⑦ Translate (5,-2,3)




# Translate (5,-2,3):

$$T = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

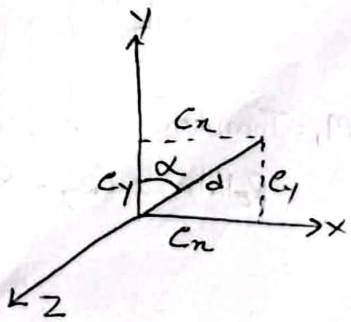
# Find unit vectors:

$$U_e = \frac{B-A}{|B-A|_{x,y,z}} = C_x, C_y, C_z$$

$$C_x = \frac{10-5}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$C_y = \frac{(3+2)}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$C_z = \frac{2-3}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{-1}{\sqrt{51}}$$




$$d = \sqrt{C_x^2 + C_y^2} = \sqrt{\left(\frac{5}{\sqrt{51}}\right)^2 + \left(\frac{5}{\sqrt{51}}\right)^2} = \frac{5\sqrt{102}}{51}$$

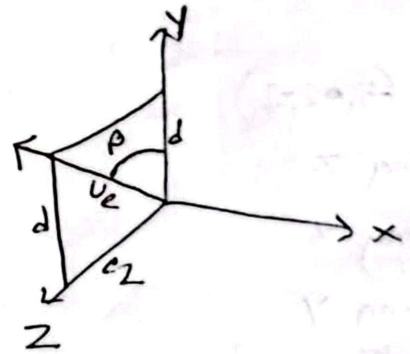
$$\cos\alpha = \frac{C_y}{d} = \frac{\frac{5}{\sqrt{51}}}{\frac{5\sqrt{102}}{51}} = \frac{1}{\sqrt{2}}$$

$$\sin\alpha = \frac{C_x}{d} = \frac{1}{\sqrt{2}}$$

$$\cos\beta = \frac{d}{U_e}$$

$$= d = \frac{5\sqrt{102}}{51}$$

$$\sin\beta = \frac{c_z}{U_e} = -\frac{1}{\sqrt{51}}$$




$$\text{Rotate\_Z}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & -2 & 3 & 1 \\ 10 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\text{Rotate\_X}(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*** For Question ans would be:

$\cancel{M_1 = T^{-1}(5,-2,3)}$

$M_1 = \text{Rotate\_X}(\beta) * \text{Rotate\_Z}(\alpha) * \text{Translate}(5,-2,3) * AB$

$$\text{Rotate\_Y}(Q = -90^\circ) = \begin{bmatrix} \cos Q & 0 & \sin Q & 0 \\ 0 & 1 & 0 & 0 \\ -\sin Q & 0 & \cos Q & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$; AB = \begin{bmatrix} 5 & 10 \\ -2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$M_1 = \text{Translate}^{-1}(5,-2,3) * \text{Rotate\_Z}(-\alpha) * \text{Rotate\_X}(-\beta) * \text{Rotate\_Y}(Q) *$
$\text{Rotate\_X}(\beta) * \text{Rotate\_Z}(\alpha) * \text{Translate}(5,-2,3) * P$

$=$

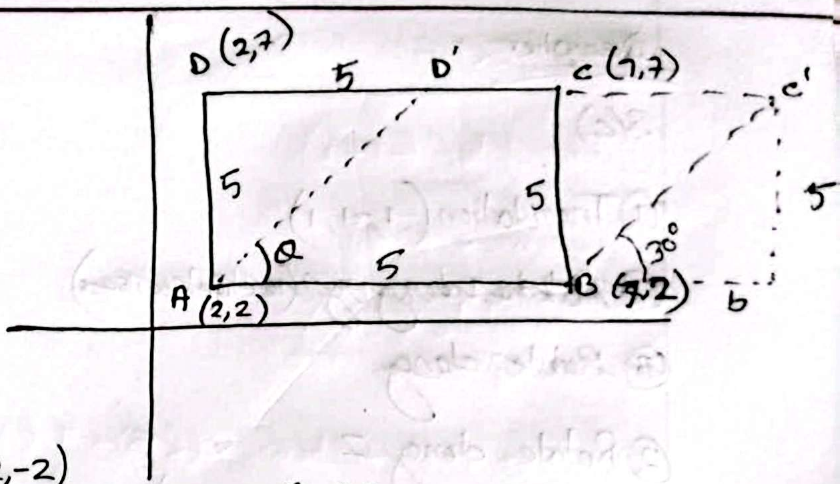
3(d)

Steps:

① Translate by $(-2,-2)$

② Shear along x-axis by 1.732

③ Translate by $(2,2)$




$M_1 = T(2,2) * Shear\_x(1.732) * T(-2,-2)$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1.732 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $M_1 \times V = M_1 \times \begin{bmatrix} 2 & 7 & 7 & 2 \\ 2 & 2 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$=$

$Q = 30$

$\tan Q = \frac{5}{b}$

$b = 5\sqrt{3}$

$= 8.6602$

$\therefore$ to shear by x-axis for 8.6602 points

Shear factor $= \frac{8.6602}{5} \rightarrow \Delta y$

$= 1.732$ $\quad (7-2) = 5$

For shear factor
along x-axis; divide by $\Delta y$
along y-axis; divide by $\Delta x$

Decipher

3(c)

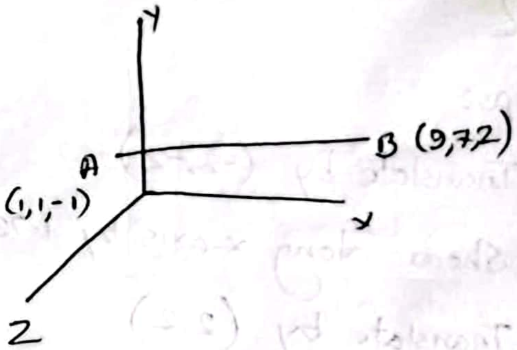
① Translation $(-1,-1,1)$

~~② Rotate along y (anticlockwise)~~

~~③ Rotate along~~

② Rotate along z

③ Rotate along x (anticlock)




Translate $(-1,-1,1)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find unit vectors:

$$U_e = \frac{B-A}{|B-A|} = c_x, c_y, c_z$$

$$c_x = \frac{9-1}{\sqrt{(9-1)^2+(7-1)^2+(2+1)^2}} = \frac{8}{\sqrt{109}}$$

$$c_y = \frac{7-1}{\sqrt{109}} = \frac{6}{\sqrt{109}}$$

$$c_z = \frac{2+1}{\sqrt{109}} = \frac{3}{\sqrt{109}}$$

$$; d = \sqrt{c_x^2 + c_y^2} = \sqrt{\left(\frac{8}{\sqrt{109}}\right)^2 + \left(\frac{6}{\sqrt{109}}\right)^2} = 0.9578$$

$$\sin\alpha = \frac{c_y}{d} = \frac{\frac{6}{\sqrt{109}}}{0.9578}$$

$$\cos\alpha = \frac{c_x}{d} = \frac{\frac{8}{\sqrt{109}}}{0.9578}$$

$$\cos\beta = \frac{d}{u_e} = 0.9578 \; ; \; \sin\beta = \frac{c_z}{u_e} = \frac{3}{\sqrt{109}}$$

$$M_1 = Rot\_x(\beta) * Rot\_z(\alpha) * T(-1,-1,1)$$

$$=$$

$$M_1 \times V = M_1 \times \begin{bmatrix} 1 & 9 \\ 1 & 7 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

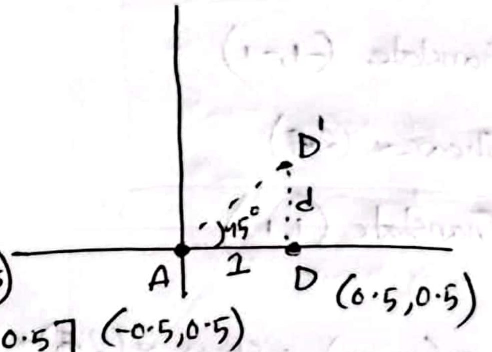
(Ans.)

3(g)

① Translate (0.5, 0.5)

② Shear along y-axis by 1

③ Translate (-0.5, 0.5)




$m_1 : T(-0.5, 0.5) * \text{Shear\_y}(1) * T(0.5, -0.5)$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate (45°)

$Q = 45°$

$\tan Q = \frac{d}{1}$

$\therefore d = 1$

$\therefore$ to shear by y axis for 1 points

shear factor $\frac{1}{1} = 1$

=

$$m_1 \times V = m_1 \times \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \\ 1 & 1 \end{bmatrix}$$

=

Again;

① Translate (-0.5, 0.5)

② Rotate (-90°)

③ Translate (0.5, -0.5)

$m_2 = T(0.5, -0.5) * \text{Rotate}(-90°) * \text{Translate}(-0.5, 0.5)$

=

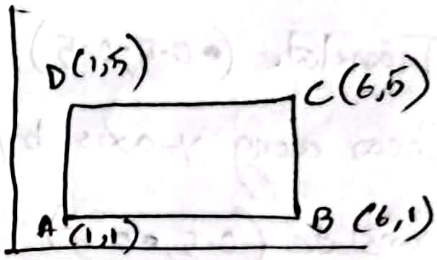
$$m_2 \times V = m_2 \times \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 & . \\ 0.5 & -0.5 & -0.5 & 0.5 & . \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Quiz- Set-A

① Translate $(-1,-1)$

② Shear_x $(2)$

③ Translate $(1,1)$




$M_1 = T(-1,-1) * Shear\_x(2) * T(-1,-1)$

$\Delta y = |5-1| = 4$

Shear factor $= \frac{8}{4}$

$= 2$

=

$$M_1 \times V = M_1 \times \begin{bmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

# Show that two succesive reflections about eith of the principal axis is equivalent to a single rotation about the coordinate origin.

=> $M_1 = Ref\_y * Ref\_x$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

=

$M_2 = Rotation(180^\circ)$

=

# $M_1 = R(45^\circ) * R^{-1}(45^\circ)$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$=$

$M_2 = R(45^\circ - 45^\circ)$

$$= R(0^\circ)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Prev sem quiz-Set D

For OA;

12 hour 360°

1 " 30°

∴ 4 hour 120°

① Translate (-8, -8)

② Rotate (-120°)

③ Translate (8, 8)

For OB;

60 minute = 360°

1 " = 6°

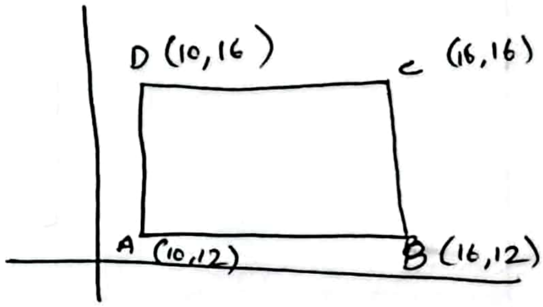
30 " = 180°

① Translate (-8, -8)

② Rotate (-180°)

③ Translate (8, 8)

Quiz-2 Set-F




line: $2y - 6x + 2 = 0$

$y - 3x + 1 = 0$

$\therefore y = 3x - 1$

The line is 1 unit below origin on y-axis.

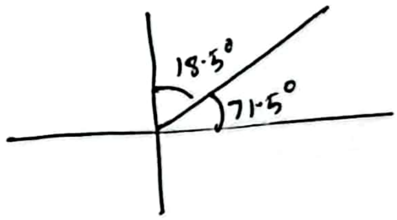
by translating (0,1),

$y = 3x$

here, $m = 3$

$\tan\theta = 3$

$\theta = 71.5$




① Translate (0,1)

② Rotate (18.5°)

③ Reflect-Y

④ Rotate (-18.5°)

⑤ Translate (0,-1)

# CHAP 5A

Prev year Quiz set-B

~~ab~~

$ob = 5, mb = 3, om = 5-3 = 2$

$ob = eb = 5$

For $\Delta emb$;

$eb^2 = em^2 + mb^2$

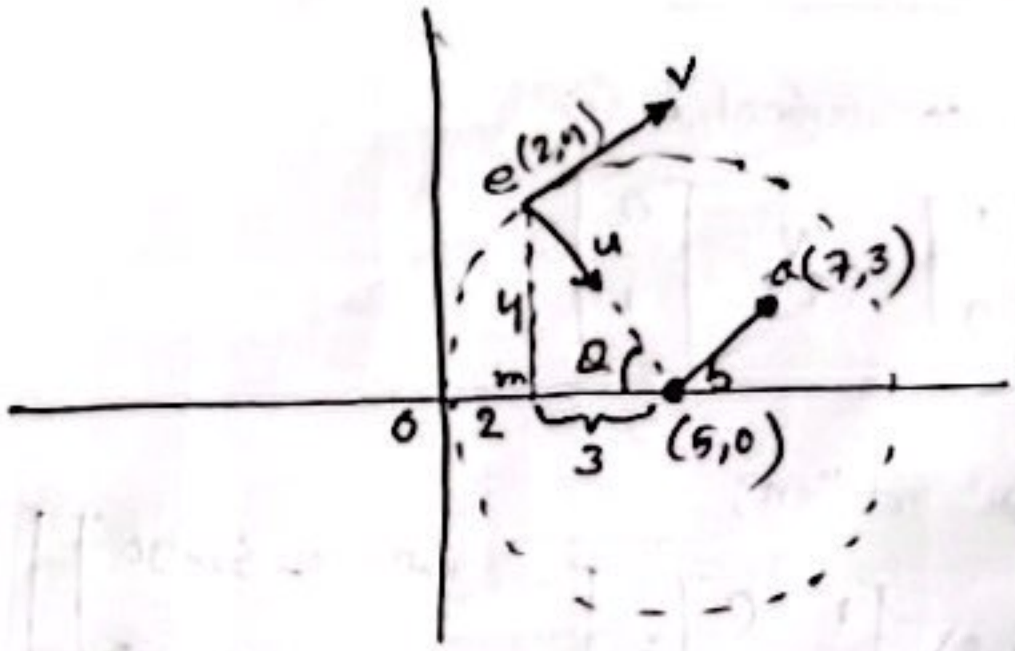
$5^2 = em^2 + 3^2$

$\therefore em = 4$

$\therefore$ em is $(2,4)$

$\tan Q = \frac{4}{3}$

$\therefore Q = 53.13$




Basis vectors are ; $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

After rotation:

: Rotate $(-53.13)$ * Basis matrix

$$= \begin{bmatrix} \cos(-53.13) & \sin(53.13) & 0 \\ -\sin(53.13) & \cos(53.13) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

:

is our required basis matrix .

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Practice Problem-1 (Lecture5-A)

(1)

$$M = M_{vp} * M_{ortho}$$

$$= \begin{bmatrix} n_x/2 & 0 & 0 & \frac{n_x-1}{2} \\ 0 & n_y/2 & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

:

(2)

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = M * \begin{bmatrix} -1 & 2 \\ -3 & 4 \\ -5 & -6 \\ 1 & 1 \end{bmatrix}$$

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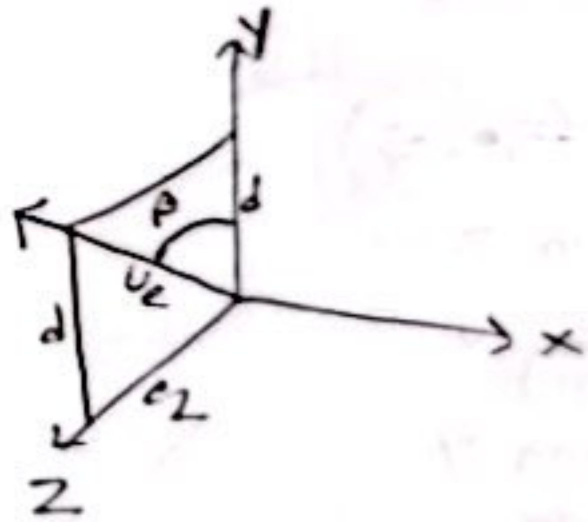
$$\cos\beta = \frac{d}{U_e}$$

$$= d = \frac{5\sqrt{102}}{51}$$

$$\sin\beta = \frac{e_z}{U_e} = -\frac{1}{\sqrt{51}}$$




$$\text{Rotate\_z}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{AB = \begin{bmatrix} 5 & -2 & 3 & 1 \\ 10 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

$$\text{Rotate\_x}(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*** For Question ans would be:

~~$M_1 = T^{-1}(5,-2,3)$~~

$M_1 = \text{Rotate\_x}(\beta) * \text{Rotate\_z}(\alpha) * \text{Translate}(5,-2,3) * AB$

$$\text{Rotate\_Y}(\theta = -90^\circ) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$; AB = \begin{bmatrix} 5 & 10 \\ -2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$M_1 = \text{Translate}^{-1}(5,-2,3) * \text{Rotate\_z}(-\alpha) * \text{Rotate\_x}(-\beta) * \text{Rotate\_Y}(\theta) * \text{Rotate\_x}(\beta) * \text{Rotate\_z}(\alpha) * \text{Translate}(5,-2,3) * P$

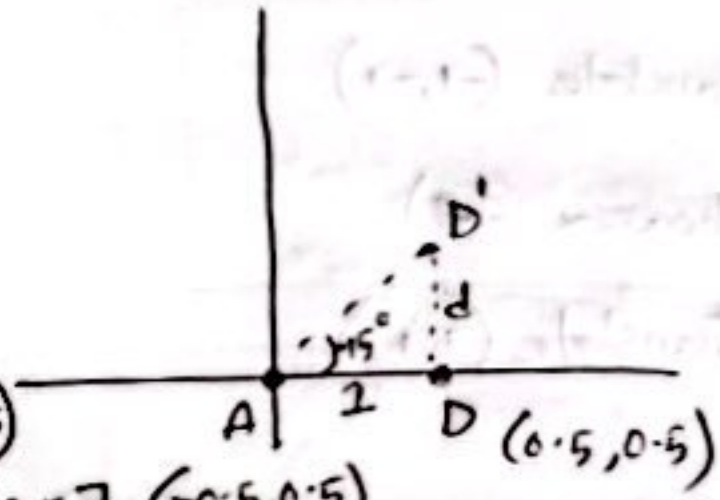
$=$

3(g)

① Translate $(0.5, 0.5)$

② Shear along y-axis by 1

③ Translate $(-0.5, 0.5)$




$m_1 : T(-0.5, 0.5) * \text{Shear\_y}(1) * T(0.5, -0.5)$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate (45°)

$=$

$\theta = 45^\circ$

$\tan\theta = \dfrac{d}{1}$

$\therefore d = 1$

$\therefore$ to shear by y axis for 1 points

shear factor $\dfrac{1}{1} = 1$

$$m_1 \times V = m_1 \times \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \\ 1 & 1 \end{bmatrix}$$

$=$

Again;

① Translate $(-0.5, 0.5)$

② Rotate $(-90^\circ)$

③ Translate $(0.5, -0.5)$

$m_2 : T(0.5, -0.5) * \text{Rotate}(-90^\circ) * \text{Translate}(-0.5, 0.5)$

$:$

$$m_2 \times V = m_2 \times \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 & \cdot \\ 0.5 & -0.5 & -0.5 & 0.5 & \cdot \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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Integer

3(f)

$c = a+bi$

$z_{n+1} = z_n^2 + c$ ; $c = -0.5+0.5i$

$z_0 = 0 + (-0.5+0.5i)$ $|z_0| = 0.707 < 2$

$z_1 = (-0.5+0.5i)^2 + (-0.5+0.5i)$

$\vdots$

if $z_{10}$ stays inside 2, then it is a member of mandlebrot set.

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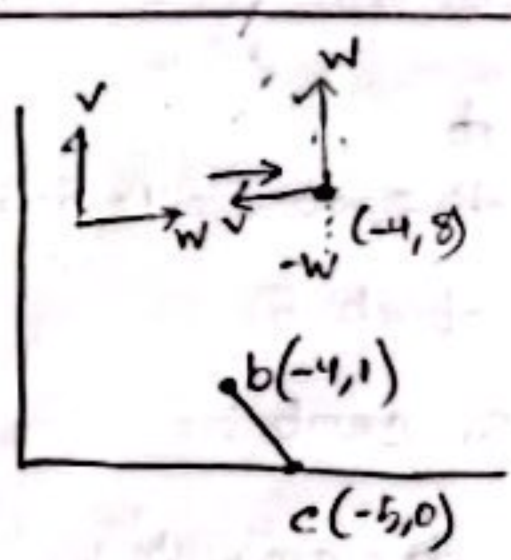
L-5 (B)

## Practice Problem-1




Initially cannonical basis:

$$W = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

After 90° rotation;

$$R(90^\circ) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Canonical to frame matrix:

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -(-4) \\ 0 & 1 & -(8) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -5 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

=

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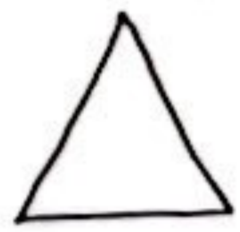
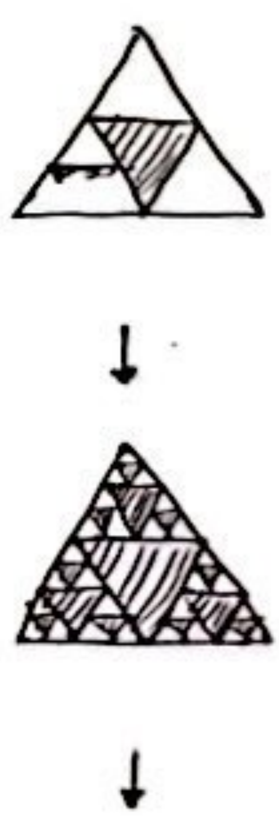
## Decipher

### 2(b)

①  stage 0

②  stage 1

③  stage 2

④ 

| Stage | no. of ~~sides~~ triangles | Length of ~~sides~~ triangles |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | $\frac{1}{2}$ |
| 2 | 4 | $\frac{1}{4}$ |
| 3 | 13 | $\frac{1}{8}$ |
| 4 | ~~39~~ 40 | $\frac{1}{16}$ |
| | $(3^{n-1}) + (n-1)$ | $\frac{1}{2^n}$ |

Here, $\epsilon_n = \frac{1}{2^n}$ ; $N_n = (3^{n-1}) + (n-1)$

$$D = -\lim_{n\to\infty} \frac{\log\left((3^{n-1}) + (n-1)\right)}{\log\left(\frac{1}{2^n}\right)} = -\lim_{n\to\infty} \frac{\log(3^{n-1}) + \log(n-1)}{n\log(1/2)}$$

$$= -\lim_{n\to\infty} \frac{\cancel{\log}(n-1)\log 3 + \log(n-1)}{-n\log(2)}$$

$$= -\lim_{n\to\infty} \frac{n\log 3 - \log 3 + \log(n-1)}{-n\log 2}$$

$$= -\lim_{n\to\infty} -\frac{n\log 3}{n\log 2} = \frac{\log 3}{\log 2} = 1.5849$$

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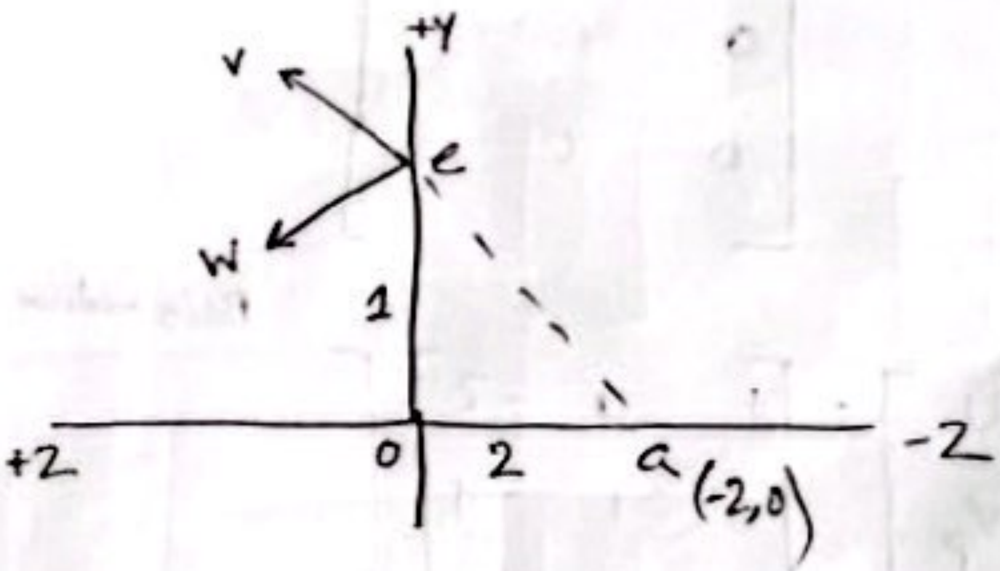
Decipher

3(f)

$e = (0,1)$

~~a = (2,0)~~

$a = (-2,0)$




$ae = \sqrt{1^2+2^2}$

$= \sqrt{5}$

$\tan\theta = \frac{1}{2}$

$\theta = 26.56°$

Basis matrix $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$=$

$P_{uv} = \begin{bmatrix} U_p \\ V_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

$=$

3(d) Decipher

$c = a + bi$

$Z_{n+1} = Z_n^2 + c$ ; converge if $Z_n > 2$

complex number : $c = -0.771 - 0.326i$

$Z_0 = 0 + (-0.771 - 0.326i)$ $\quad |Z_0| = 0.8370$

$= -0.771 - 0.326i$

$Z_1 = (-0.771 - 0.326i)^2 + (-0.771 - 0.326i)$

$= (-0.771)^2 + 2 \times 0.771 \times 0.326 + (0.326i)^2 - 0.771 - 0.326i$

$= 0.5944 + 0.5026 - 0.1062 - 0.771 - 0.326i$

$= 0.2198 - 0.326i$ $\quad |Z_1| = \sqrt{(0.2198)^2 + (-0.326)^2}$

$= 0.3931$

$Z_2 = (0.2198 - 0.326i)^2 + (-0.771 - 0.326i)$

$=$

$\vdots$

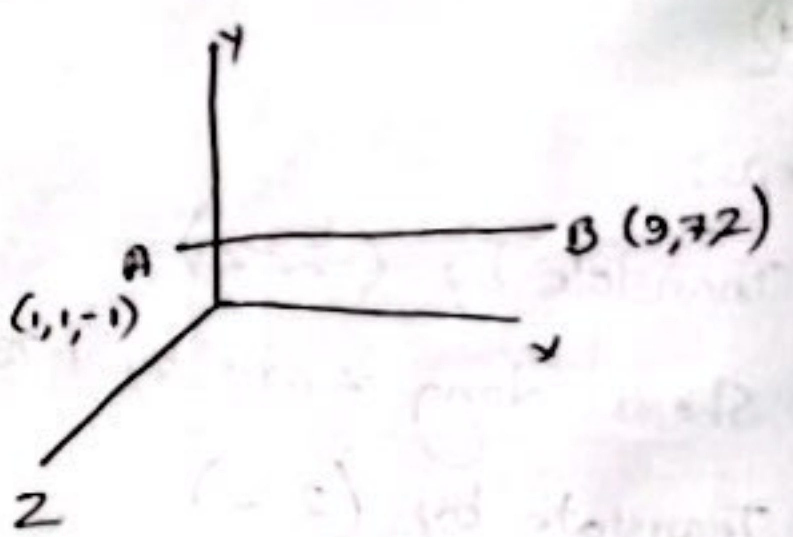
upto $Z_{10}$

it will converge after $Z_8$ (GPT bolehe...)

So, The colour of the points are Blue.

<u>Decipher</u>

3(c)




① Translation $(-1,-1,1)$

~~② Rotate along y/z (anticlockwise)~~

~~③ Rotate along~~

② Rotate along z

③ Rotate along x (anticlock)

Translate $(-1,-1,1)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find unit vectors:

$$U_e = \frac{B-A}{|B-A|} = c_x, c_y, c_z$$

$$c_x = \frac{9-1}{\sqrt{(9-1)^2+(7-1)^2+(2+1)^2}} = \frac{8}{\sqrt{109}}$$

$$c_y = \frac{7-1}{\sqrt{109}} = \frac{6}{\sqrt{109}}$$

$$c_z = \frac{2+1}{\sqrt{109}} = \frac{3}{\sqrt{109}}$$

$$; d = \sqrt{c_x^2 + c_y^2} = \sqrt{\left(\frac{8}{\sqrt{109}}\right)^2 + \left(\frac{6}{\sqrt{109}}\right)^2} = 0.9578$$

$$\sin\alpha = \frac{c_y}{d} = \frac{\frac{6}{\sqrt{109}}}{0.9578}$$

$$\cos\alpha = \frac{c_x}{d} = \frac{\frac{8}{\sqrt{109}}}{0.9578}$$

$$\cos\beta = \frac{d}{u_e} = 0.9578 \; ; \; \sin\beta = \frac{c_z}{u_e} = \frac{3}{\sqrt{109}}$$

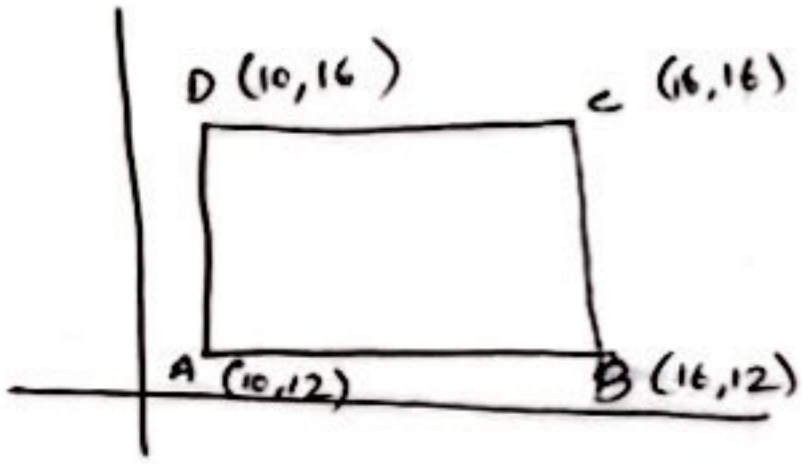
$$m_1 = Rot\_x(\beta) * Rot\_z(\alpha) * T(-1,-1,1)$$

$$=$$

$$m_1 \times V = m_1 \times \begin{bmatrix} 1 & 9 \\ 1 & 7 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

(Ans.)

Quiz-2 Set-F




line: $2y - 6x + 2 = 0$

$y - 3x + 1 = 0$

$\therefore y = 3x - 1$

The line is 1 unit below origin on y-axis.

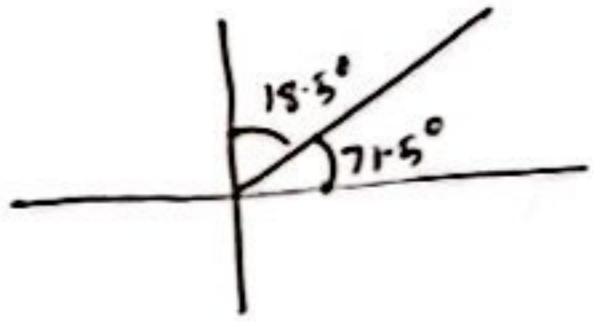
by translating (0, 1),

$y = 3x$

here, $m = 3$

$\tan\theta = 3$

$\theta = 71.5$




① Translate (0, 1)

② Rotate (18.5°)

③ Reflect-Y

④ Rotate (-18.5°)

⑤ Translate (0, -1)

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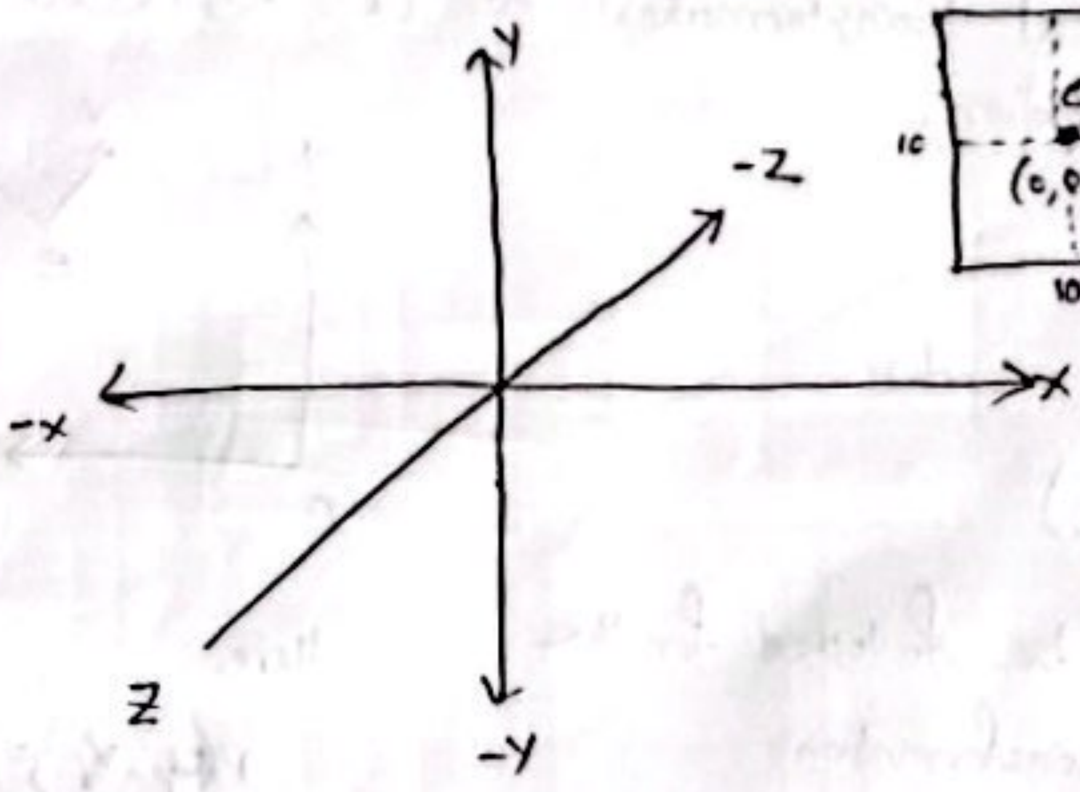
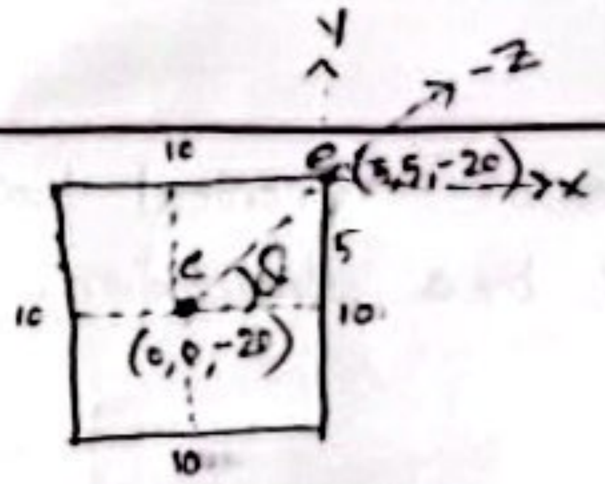
Quiz

① a)




$\tan\theta = \frac{5}{5}$

$\theta = \tan^{-1}(1)$

$\therefore \theta = 45^\circ$

Basis vectors: $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

After rotation.

Basis matrix = Rotate $(-45^\circ)$ * Basis vectors

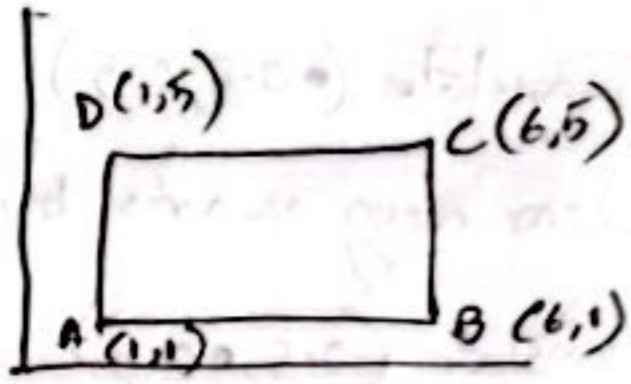
$$= \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Quiz- Set-A

① Translate $(-1,-1)$

② Shear_x $(2)$

③ Translate $(1,1)$




$m_1 = T(-1,-1) * \text{Shear\_x}(2) * T(-1,-1)$

$\Delta y = |5-1| = 4$

$\therefore$ Shear factor $= \frac{8}{4}$

$= 2$

$=$

$$m_1 \times V = m_1 \times \begin{bmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

# Show that two succesive reflections about eith of the principal axis is equivalent to a single rotation about the coordinate origin.

$\Rightarrow m_1 = \text{Ref\_y} * \text{Ref\_x}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$=$

$m_2 = \text{Rotation}(180^\circ)$

$=$

Lectur-8 (A)

For Cantor set:

\# $\epsilon_n$ : Size of new element at n iteration = length = $\frac{1}{3^n}$

$N_n$ : the number of new elements at " n = $2^n$

Fractal dimension, $D = -\lim_{n\to\infty} \frac{\log(N_n)}{\log(\epsilon_n)}$

→ Defines the complexity of fractal.

Dimension of Cantor Set

$$D = -\lim_{n\to\infty} \frac{\log(N_n)}{\log(\epsilon_n)} = -\lim_{n\to\infty} \frac{\log(2^n)}{\log(1/3^n)}$$

$$= -\lim_{n\to\infty} \frac{n\log 2}{n\log 1/3}$$

$$= \lim_{n\to\infty} \frac{\log 2}{\log 3}$$

$$= 0.6309$$

For Koch Snowflake:

$\epsilon_n = \frac{1}{3^n}$

$N_n = 3\times 4^n$

$$D = -\lim_{n\to\infty} \frac{\log[3(4^n)]}{\log[1/3^n]} = -\lim_{n\to\infty} \frac{\log(3) + n\log(4)}{-n\log(3)}$$

$$= \lim_{n\to\infty} \frac{\cancel{\log(3)}}{n\log(3)} + \frac{\cancel{n}\log 4}{\cancel{n}\log 3}$$

$$= \lim_{n\to\infty} \frac{1}{n} + \frac{\log 4}{\log 3} = \log 4/\log 3 = 1.2619$$

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四 Sierpinski Triangle:

$$\epsilon_n = \frac{1}{2^n}$$

$$N_n = 3^n$$

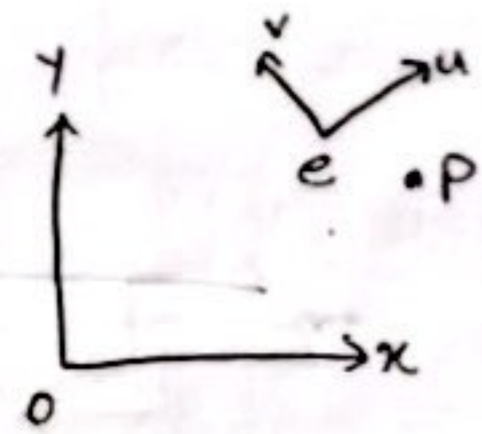
$$D = -\lim_{n\to\infty} \frac{\log(3^n)}{\log(1/2^n)}$$

$$= \lim_{n\to\infty} \frac{\log(3)}{\log(2)}$$

$$= 1.58$$

五 The ~~mad~~ mandelbrot set

# Frame to cannonical transformation can be expressed as rotation followed by a translation.




Steps:

1. Rotate $(\theta)$ of basis u and v
2. Translate $(-x_e, -y_e)$

These steps needs to be followed for the Frame to cannonical transformation.

Here,

$$p(u_p, v_p) = e + u_p u + v_p v$$

- U and V are the basis vectors, e is the origin of frame.

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_e \\ 0 & 1 & y_e \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} x_u & x_v & 0 \\ y_u & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Basis Matrix}} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

Here,

Rotation involving (u and v) followed by a translation (involving e). So, we can say that . . . . .

Basis matrix:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{cc} \underline{u} & \underline{v} \\ \multicolumn{2}{c}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}} \end{array}$$

$=$

# For Cannonical to frame:

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

translation followed by a rotation.

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eye matrix $= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

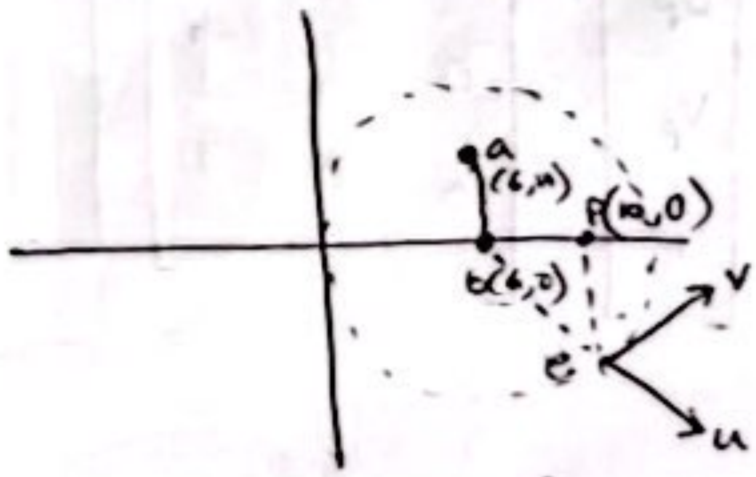
Basis matrix

$$P_{uv} = \begin{bmatrix} U_p \\ V_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 3 & 0 \\ 1 & 1 \end{bmatrix}$$

$=$

Integer

3(a)




$be = 6$ ; $bp = 4$

$pe = \sqrt{6^2 - 4^2}$

$= 2\sqrt{5}$

$\therefore e = (10, 2\sqrt{5})$

$\tan Q = \dfrac{2\sqrt{5}}{4}$

$\therefore Q = 48.18$

Eye $= \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -2\sqrt{5} \\ 0 & 0 & 1 \end{bmatrix}$

Basis vector: $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ; $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

After rotation:

$P_{uv} =$

Rotate(48.18) * Basis vector

$$= \begin{bmatrix} \cos(48.18) & -\sin(48.18) & 0 \\ \sin(48.18) & \cos(48.18) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} =$$

Prev sem quiz - Set D

For OA;

12 hour 360°

1 " 30°

∴ 4 hour 120°

For OB;

60 minute : 360°

1 " : 6°

30 " : 180°

① Translate (-8, -8)

② Rotate (-120°)

③ Translate (8, 8)

① Translate (-8, -8)

② Rotate (-180°)

③ Translate (8, 8)

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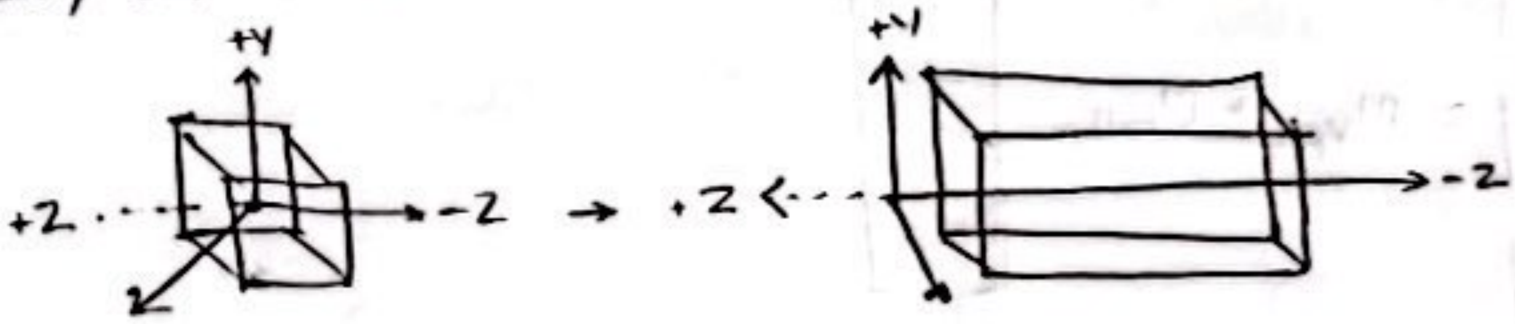
# Orthographic Projection Transformation

In cannonical view, the space is limited $[-1,1]$. We can render a geometry in some other ~~er~~ region using orthographic Project, where we can have boundary set on our will.




cannonical → orthographic

Shape: $|r-l| \times |t-b| \times |n-f|$

① Translate $\left(-\frac{r+l}{2}, -\frac{t+b}{2}, -\frac{n+f}{2}\right)$

② Scaling $\left(\frac{2}{r-l}, \frac{2}{t-b}, \frac{2}{n-f}\right)$ → Because cannonical shape is $2 \times 2 \times 2$

$t$ = top plane
$b$ = bottom plane
$r$ = right plane
$l$ = left plane
$n$ = near plane
$f$ = front plane

$$M_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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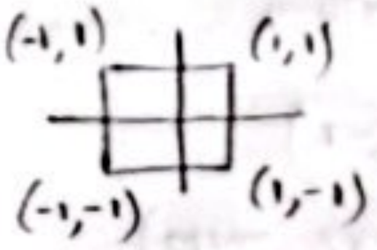
## Viewport transformation

# Bottom to top




1. Translate $(1,1)$
2. Scaling $(n_x/2, n_y/2)$
3. Translate $(-\frac{1}{2}, -\frac{1}{2})$

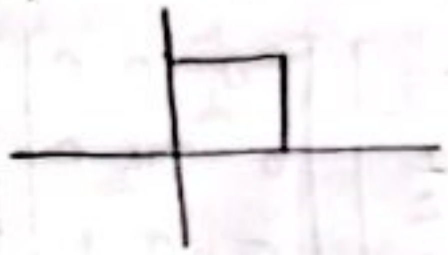
$$M_1 = \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x/2 & 0 & 0 \\ 0 & n_y/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x/2 & 0 & -\frac{1}{2} \\ 0 & n_y/2 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

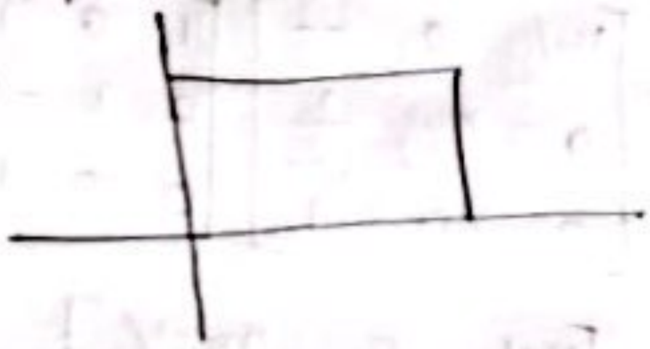
$$= \begin{bmatrix} n_x/2 & 0 & n_x - 1/2 \\ 0 & n_y/2 & n_y - 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} n_x/2 & 0 & n_x - 1/2 \\ 0 & n_y/2 & n_y - 1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canno} \\ y_{canno} \\ 1 \end{bmatrix}$$

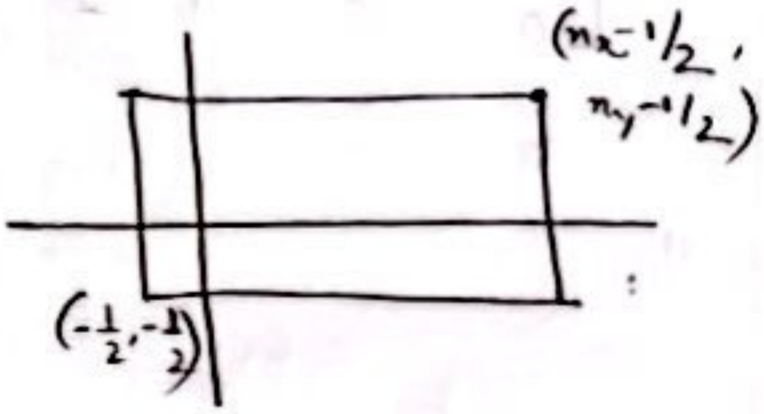
Apply $T(1,1)$:

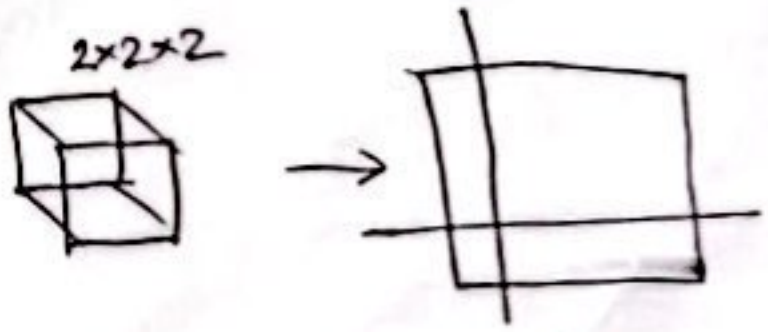


Apply $S(n_x/2, n_y/2)$:



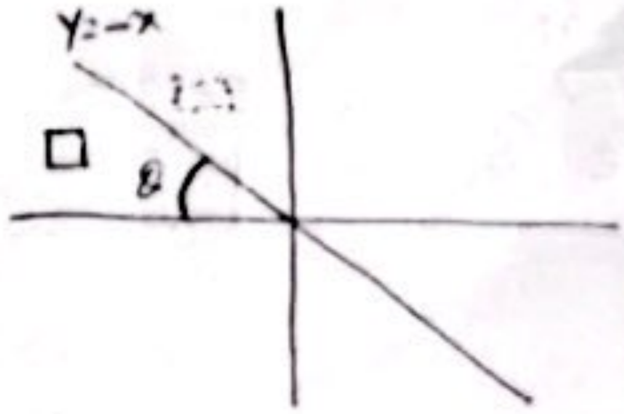
Apply $(-\frac{1}{2}, -\frac{1}{2})$:



Integer

2(b)




Transformation matrix for the reflection about the line $y=-x$ :

$m_1 = Rot(45°) * Ref\text{-}y * Rot(-45°)$

$$= \begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{bmatrix}$$

Here, $m = -1$

$\tan\theta = -1$; $\theta = \tan^{-1}(-1)$

$\theta = -45°$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Reflection related to y-axis followed by a counter-clockwise rotation of 90°

$m_2 = Rot(90°) * Ref\text{-}y$

$$= \begin{bmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$\therefore m_1 = m_2$

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2(d)

V: X, F

C: +, −

axiom: F + XF + F + XF

Rules: X → XF − F + F − XF + F + XF − F + F − X

Angle: 90°

X = do nothing

F: draw a line forward

+ : rotate clockwise by 90°

− : " counterclock by 90°

n=0: F + XF + F + XF

n=1: F + XF − F + F − XF + F + XF − F + F − XF + F + XF − F + F − XF + F + XF − F + F − XF

Ki korlesi aishob?

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Eye matrix: $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

b) Frame / Camera to cannonical

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -20 \\ 1 \end{bmatrix}$$

$=$

Prev year Quiz Set A

## 1(a)

Using top to Bottom:

1. Translate $(1,-1)$
2. Scaling $(n_x/2, n_y/2)$
3. Translate $(-\frac{1}{2}, \frac{1}{2})$

The final matrix will be

$$M_{vp} = \begin{bmatrix} n_x/2 & 0 & n_x - 1/2 \\ 0 & n_y/2 & -n_y+1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $n_x = 256$
$n_y = 128$

## 1(b)

$A = (-3,-4,-3)$ ; $B\,(2,4,-6)$

$l = -6$, $r = 6$, $b = -7$, $t = 7$, $n = -2$, $f = -8$

$$\begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{canno} \\ 1 \end{bmatrix} = M_{vp} * \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -3 & 2 \\ -4 & 4 \\ -3 & -6 \\ 1 & 1 \end{bmatrix}$$

$=$

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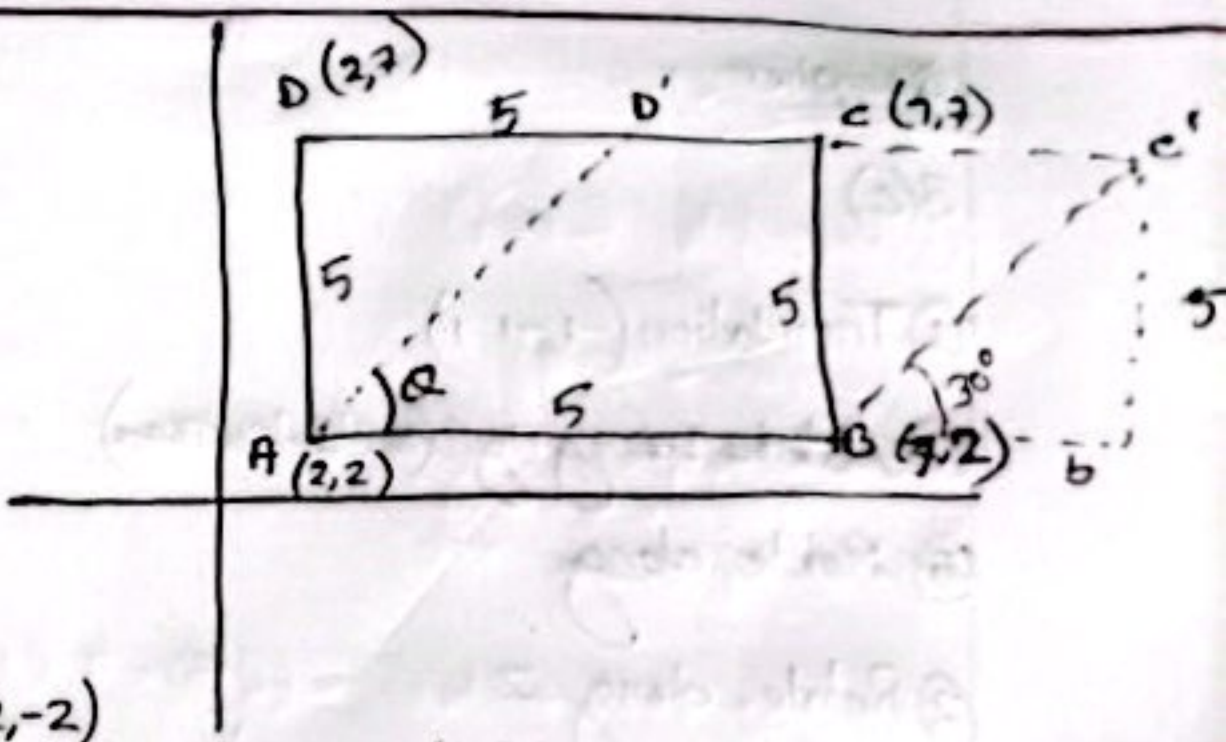
3(d)

Steps:

① Translate by $(-2,-2)$

② Shear along x-axis by 1.732

③ Translate by $(2,2)$




$M_1 = T(2,2) * Shear\_x(1.732) * T(-2,-2)$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1.732 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_1, M_3 \times V = M_1 \times \begin{bmatrix} 2 & 7 & 7 & 2 \\ 2 & 2 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$=$

$Q = 30$

$\tan Q = \frac{5}{b}$

$b = 5\sqrt{3}$

$= 8.6602$

$\therefore$ to shear by x-axis for 8.6602 points

Shear factor $= \frac{8.6602}{5} \rightarrow \Delta y$

$= 1.732$ $\quad (7-2) = 5$

For shear factor

along x-axis; divide by $\Delta y$

along y-axis; divide by $\Delta x$

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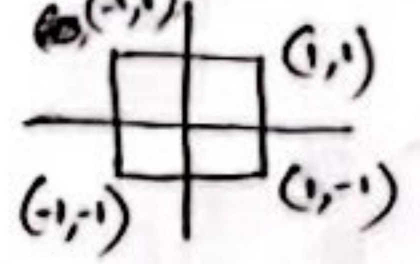
# Top to Bottom

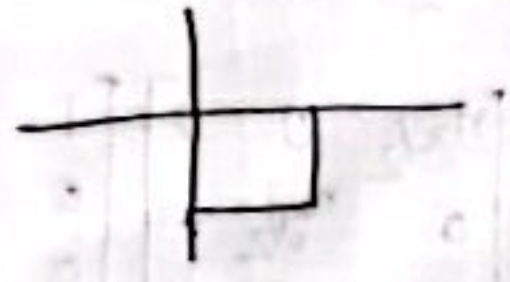



① Translate$(1,-1)$

② Scaling$(nx/2, ny/2)$

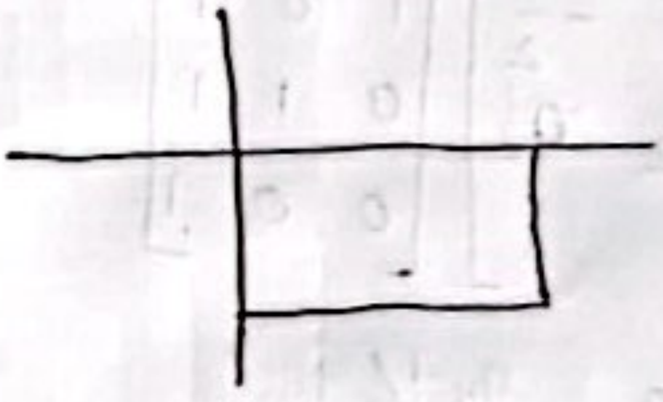
③ Translate$(-\frac{1}{2}, \frac{1}{2})$

Apply $T(1,-1)$:



$$M_1 = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} nx/2 & 0 & 0 \\ 0 & ny/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

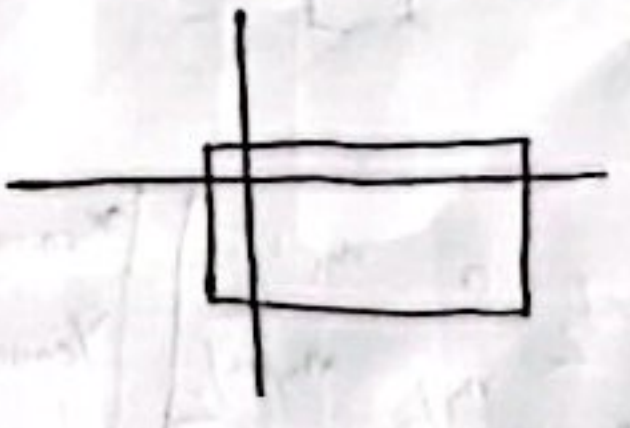
Apply $S(nx/2, ny/2)$:



$$= \begin{bmatrix} nx/2 & 0 & -\frac{1}{2} \\ 0 & ny/2 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} nx/2 & 0 & nx-1/2 \\ 0 & ny/2 & \frac{-ny+1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $T(-\frac{1}{2}, \frac{1}{2})$



# $m_1 = R(45^\circ) \cdot R^{-1}(45^\circ)$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$=$

$m_2 = R(45^\circ - 45^\circ)$

$= R(0^\circ)$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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3(b)

① Translate (-5, -2, -3)

② Rotate along Z

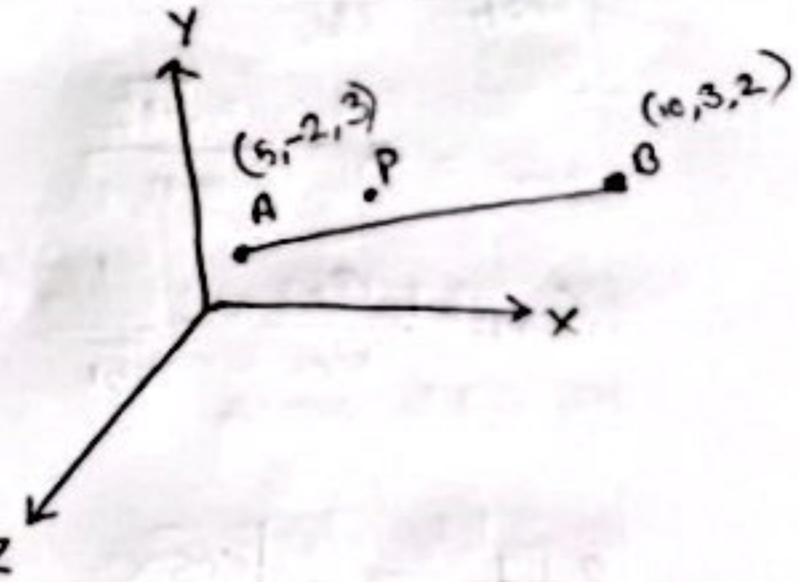
③ Rotate along X

④ Rotate along Y

⑤ Rotate along X

⑥ Rotate along Z

⑦ Translate (5, -2, 3)




# Translate (5, -2, 3):

$$T: \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

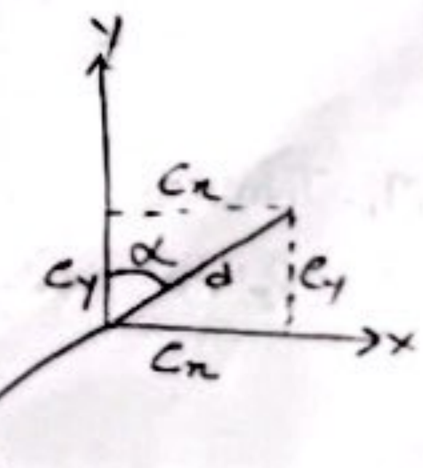
# Find unit vectors:

$$U_e: \frac{B-A}{|B-A|_{x,y,z}} : c_x, c_y, c_z$$

$$c_x: \frac{10-5}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} : \frac{5}{\sqrt{51}}$$

$$c_y: \frac{(3+2)}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} : \frac{5}{\sqrt{51}}$$

$$c_z: \frac{2-3}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} : \frac{-1}{\sqrt{51}}$$




$$d: \sqrt{c_x^2+c_y^2} = \sqrt{\left(\frac{5}{\sqrt{51}}\right)^2+\left(\frac{5}{\sqrt{51}}\right)^2} = \frac{5\sqrt{102}}{51}$$

$$\cos\alpha: \frac{c_y}{d} : \frac{\frac{5}{\sqrt{51}}}{\frac{5\sqrt{102}}{51}} : \frac{1}{\sqrt{2}}$$

$$\sin\alpha: \frac{c_x}{d} : \frac{1}{\sqrt{2}}$$

Scanned with CamScanner