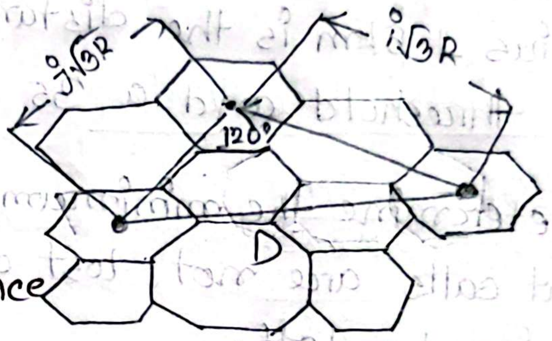
Saturday
26 Oct 2024

Maths
(Lecture - 4,5,6,7,8) (Finals)

1. Describe the frequency reuse concept in cellular system. Prove that for a hexagonal geometry, the co-channel reuse ratio is given by $Q = \sqrt{3N}$ where $N = i^2 + ij + j^2$.

Ans:

When size of cell is the same and Base station transmit the same power.



The co-channel interference ratio depends on

→ Radius of the cell (R)

→ The distance between centers of the nearest co-channel (D)

So, $D = \sqrt{(i\sqrt{3}R)^2 + (j\sqrt{3R})^2 - 2\, i\sqrt{3}R\, j\sqrt{3}R \cos 120^\circ}$ $(\sqrt{3}R)$

$= \sqrt{i^2 3R^2 + j^2 3R^2 - 2ij\, 3R^2 (-1/2)}$

$= \sqrt{3R^2(i^2 + j^2 + ij)}$

$= R\sqrt{3N}$

Hence co-channel ratio, $Q = \frac{D}{R} = \frac{R\sqrt{3N}}{R} = \sqrt{3N}$

Where,
D = Reuse distance
R = Radius
N = Cluster size / no of cell in cluster

*** Q is related to the cluster size (N) for a hexagonal geometry. A smaller value of Q indicates small N and it can reuse more so the capacity increases but generates higher co-channel Interference.

$Q\downarrow\ N\downarrow\ CCI\uparrow\ \text{Capacity}\uparrow$.

② Suppose that a mobile station is moving at a speed of 72 km/hr along a straight line between base station BS1 and BS2 with path loss 4. The received power at a reference distance 1km is equal to 15W. Let the cell radius 1.5km is the distance at which the power is at the threshold and a 3s handoff time.

(i) Determine the minimum required margin Δ to assure that calls are not lost due to weak signal conditions during handoff.

(ii) Describe the effect of the margin Δ on the performance of cellular system.

Ans: Given, distance covered, $s = vt = 72\ km/hr \times 3s$

$$= \frac{72 \times 1000}{3600} \times 3$$

Power threshold, $d = 1.5km = 1500m$; $= 60m$

Margin, $\Delta = P_r(\text{handoff}) - P_r(\text{min useable})$

$$= (P_0 - 10n\log(d-60)) - (P_0 - 10n\log d)$$

$$= P_0 - 10n\log(d-60) - P_0 + 10n\log d$$

$$= -10n\log(d-60) + 10n\log d$$

$$= 10n(\log d - \log(d-60))$$

$$= 10n\log\frac{d}{d-60}$$

$$= 10\times 4\log\frac{1500}{1500-60} = 0.709\ dB$$

Alternal

এই math টা $P_r = P_0 \left(\frac{d}{d_0}\right)^{-n}$ এ হবে।

Given-
$d_0 = 1km = 1000m$
$d = radius = 1500m.$ (1.5km)
$P_0 = 15W.$
$n = 4$

P_r (min usable) $= P_0 \left(\frac{d}{d_0}\right)^{-n}$

$= 15 \times \left(\frac{1500}{1000}\right)^{-4}$

$= 10 \log (\downarrow)$

$= 4.6717$ dB.

P_r (hand off) $= P_0 \left(\frac{d-60}{d_0}\right)^{-n} = 15 \times \left(\frac{1500-60}{1000}\right)^{-4} = 10 \log (\downarrow)$

$= 5.42$

$$\boxed{\Delta = P_r(h) - P_r(m) = 0.703 \text{ dB}}$$

(Ans:)

(ii) → Δ can't be too large; because unnecessary handoffs which burden MSC (mobile switching center) may occur.

→ Δ can't be too small; may be a insufficient time to complete a hand off before a call is lost due to weak signal conditions.

(3) Show that cell sectoring decreases co-channel radio trunking efficiency, increase SIR which turn decrease the cluster size and hence increase capacity.

Ans: Let's go for 120° sectoring.

1st without sectoring:

$$SIR = \frac{(\sqrt{3N})^n}{i_0}$$

Lets
$N = 7$
$n = 4$
$i_0 = 6$ (hexagonal)

$$= \frac{(\sqrt{3\times7})^4}{6}$$

$$= 73.5 = 10\log_{10} 73.5 = 18.66\,dB.$$

With 120° sectoring,

$$SIR = \frac{(\sqrt{3\times7})^4}{2}$$

Here, interferrer for 120° sectoring is 2 (2 টা cluster effect করে)
$i_0 = 2$

$$= 220.5 = 23.43\,dB.$$

Greater than without sectoring.

120° sector uses 3 antennas for this channels are seperated and SIR get increased that other co-channel can't hamper their signal so CCI is low.

In sectoring if we have to achieve 18.66 dB SIR N (cluster size must be be reduce. $N=4$ $(i=2, j=0)$

$$SIR = \frac{(\sqrt{3\times4})^4}{2} = 72 = 18.57 \text{ (much closer)}$$

so $N\downarrow$

Capacity $\propto \frac{1}{N}$ thus Capacity increase as small N can reuse frequency more times.

Now, Effect of Trunking Efficiency: (Assume a secnerio)

Let's assume, Hold time, $H = \frac{2}{60}$ hr = | total channel = 395

average $\lambda = 1$ h | $N = 7$

Channel per cell $= \frac{395}{7} = 57$

From chart, for 0.01 blocking and 57, $A = 44.2$

We know, $A = UA_U$

$\Rightarrow U = \frac{A}{A_U}$

$= \frac{44.2}{1/30}$

$= 1326$

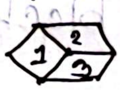
A_U = Traffic per user/

U = Total no of user supported per cell

A = Traffic supported per cell.

$A_U = \lambda H = \frac{1 \times 2}{60} = \frac{1}{30}$

Now with 120° sector:



So cell per sector $= \frac{57}{3} = 19$

0.01 & 19 channel $A = 11.2$ $U = \frac{11.2}{1/30} = 336$

So for 3 sectors $= 3 \times 336$ calls per hour.

$= 1008$

Decreased, $\left(\frac{1326 - 1008}{1326}\right) \times 100\% = 23.91 \approx 24\%$.

Thus sectoring decreaes the thrunking efficiency. while improving the SI for each user in system.

④ A city has a population ~~of~~ of 3 million people that are evenly distributed over an area of 1000 km². We know that a percentage of the population is subscribed to a cellular system. Assume that the cellular system is an Erlang B system with a total Band of 14 MHz, full duplex channel bandwidth of 40 KHz, covers the city using hexagonal cells with radius 2 km and a cluster size of 7 cells. Assume that each user makes 1 call each 2 hours with average call duration of 1 minute and the desired probability of call blocking is 0.005.

(a) The total no of cells in the system.

(b) The number of channels per cell

(c) The total no of channels in the system.

(d) Traffic intensity per cell.

(e) Maximum carried traffic for the whole system.

(f) The total no of users who can use the system.

(a) (cell জানতে total area, per cell area বের করতে হবে)

Given,

total area = $1000 \ km^2$.

Hexagonal Radius, R = 2 km = 2000 m.

" Cell Area, $= \frac{3\sqrt{3}}{2} R^2 = \frac{3\sqrt{3}}{2}(2km)^2 = 6\sqrt{3} \ km^2$

Total no of cells in the system, $= \dfrac{1000 \text{ (total system area)}}{\text{one cell area}}$

$= \dfrac{1000}{6\sqrt{3}}$

≈ 96.

Ans:

(b)

Given,

total Bandwidth $= 14\,MHz = 14 \times 10^3\,KHz$

full duplex " $= 40KHz$.

total ~~cell~~ chanel $= \dfrac{14 \times 10^3}{40} = 350$ channels in ~~system~~ bandwith.

cluster size; $N = 7$.

Channels per cell $= \dfrac{\text{total channels}}{\text{cluster size}} = \dfrac{350}{7} = 50$.

(Ans:)

(c)

Total no of channels in system = Total cells x (channels per cell)

$= 96 \times 50 = 4800$.

(Ans:)

(d)

From graph, for 0.005 blocking, channels per cell = 50

Traffic intensity per cell, $A = 37$ Erlang/cell.

↳ graph থেকে

(e)

Maximum carried traffic in the whole system,

= Totall cell in the system x Traffic intensity per cell

$= 96 \times 37$

$= 3552$.

(d)

$\lambda = 1$ call each 2hours $= \frac{1}{2} = 0.5$ call/hour.

Average call duration, $H = 1$ min $= \frac{1}{60}$ hour.

Traffic per user, $A_U = \lambda H = 0.5 \times \frac{1}{60} = \frac{1}{120}$

Total no of user supported per cell, $U = \frac{A}{A_U}$

$= \frac{37}{1/120}$

$= 4440$ users.

Total user in system = Total cell in system × per cell

$= 96 \times 4440$

$= 426240.$

(Ans:)

(5) Assum a ~~geographic~~ system of 128 cells with radius of 0.8km a total frequency bandwidth that supports 336 traffic channels and a reuse factor of N = 7

(i) What geographic area is covered by the system?

(ii) How many channels are there per cells?

(iii) What is the capacity of the system?

(i)

cell Radius, $R = 0.8km = 800m$.

cell Area $= \frac{3\sqrt{3}}{2} R^2 = \frac{3\sqrt{3}}{2}(0.8)^2 = 1.67 \text{ km}^2$

Geographic covered = total cell × Area of a cell

$= 128 \times 1.67$

$\approx 213.76 \approx 214$ (Ans:)

(ii)

$$\text{Channels per cell} = \frac{\text{total traffic channels}}{\text{cluster size}(N)} = \frac{336}{7} = 48.$$

(iii)

Capacity = (Total cells in system × channels per cell)

$= 128 \times 48 = 6144.$

<u>Extra</u> How many times the total frequency are reused?

$$\text{Total reused frequency} = \frac{\text{No of total cell}}{N}$$

$$= \frac{128}{7} = 18.25 \approx 18.$$

(6) Assume a geographical area of 212.48 km² is covered by a cellular system with a cell radius of 1.6 km. A total frequency bandwidth that supported 309 channels and a reuse factor of N = 9. If there are 0.96 MHz is dedicated to the control channel which uses 10 kHz for simplex channel.

(a) How many cells are there in geographical area?

(b) How many traffic channels are there per cell?

(c) What is the capacity of the system?

(d) How many times the total frequency are reused?

(a)

Area of cell $= \frac{3\sqrt{3}}{2} R^2 = \frac{3\sqrt{3}}{2} (1.61km)^2 = 6.651 \, km^2$

Total cells of area $= \dfrac{\text{Total area}}{\text{one cell arce}}$

$= \dfrac{212.48 \, km^2}{6.651 \, km^2}$

≈ 31.95

$\approx 32.$

(b)

Given, total bandwidth $= 0.96 MHz = 0.96 \times 10^3 KHz$

control channel's " $= 10 KHz \times 2 = 20 KHz.$

$\uparrow$ do duplex

Total control channel $= \dfrac{0.96 \times 10^3 KHz}{20 \, KHz} = 48$

total Traffic channels = Total channels − Control channels

$= 309 - 48 = 261$ channels.

(c)

Traffic channels per cell $= \dfrac{\text{Traffic channel}}{N}$

$= \dfrac{261}{9}$

$= 29$ channels per cell.

(c)

Capacity = no of total cell × no of channels per cell

$= 32 \times 29$

$= 928$

(d)

Total reused frequency = $\frac{\text{no of total cells}}{N}$

$= \frac{32}{9} \approx 4$ Ans.

* total channel কে N দিয়ে divide করলে per cell channel পাই

* total cell কে N দিয়ে divide করলে reused freq পাই। (কতবার use করা যাবে।)

Extra: What is the number of concurrent calls that can be handled?

total concurrent calls = total cell × channels per cell

$= 32 \times 29$

$= 928$

capacity

⑦ Discuss the microcell zone concept and show that microcell zone concept decreases co-channel ratio, increases SIR which in turn decreases the cluster size and hence increase the capacity.

Ans:

Microcell Zone Concept:

(*) A cell is conceptually divided into micro cells.

(*) Directional Antenna placed on cell edge, radiating power inward 

(*) Central BS connects to all zones, assigns channels as needed

(*) As BS has all channel's information, seemless zone transition with no handoff, simply switches channel to the next zone.

(*) Benefit → impoved coverage, fewer call drops, higher capacity in dense areas.

Let assum without microcell zone concept, SIR

$$SIR = \frac{(\sqrt{3N})^n}{6} \qquad n=4,\ N=7,\ i_0=6$$

$$= 18.66dB$$

with microcell ...

$$SIR = \frac{(\sqrt{3\times3})^4}{1} \qquad n=4,\ N=9 \to 3\text{ zone}$$

$= 20dB.$ $i=1$ → at least 1টা interferer পাওয়া যেতে পারে।

So as N↓ SIR↑ and CCI↑ Capacity↑ (reason description আগের গুলা)

(8) If a total 33 MHz of bandwidth is allocated to perticular FDD cellular system which uses two 25 kHz simplex channels to provide full duplex voice and control channels compute the number of channels available per cell if system uses 4 cell reuse. If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell.

Ans: Total channels = $\frac{\text{total Bandwidth}}{\text{one traffic channe bandthe}}$

$$= \frac{33\,MHz}{25 \times 2\,KHz} = \frac{33 \times 10^3\,KHz}{50\,KHz}$$

$$= 660 \text{ channels.}$$

no of channel availabe per cell = $\frac{\text{total channel}}{N}$

$$= \frac{660}{4} = 165 \text{ channel}$$

Now total control channel = $\frac{}{\text{one duplex control channel band width}}$

$$= \frac{\text{total Control channel bandwidth}}{\text{One duplex Control channel bandwidth}}$$

$$= \frac{1\,MHz}{2 \times 25\,KHz} = \frac{1000\,KHz}{50\,KHz}$$

$$= 20.$$

Total Control channels per cell = $\frac{20}{4} = 5$ channel

Total Voice channels per cell = $\frac{660 - 20}{4} = \frac{640}{4} = 160$ channel

or simple $(165 - \underset{\text{control}}{5}) = 160$ voice channel.

CamScanner

** However each cell only needs a ~~sig~~ single control channel. The control channels have a greater reuse distance than the voice channels. Thus one control and 160 voice channels would be assigned to each cell.

(3) If signal ratio (SIR) of 15dB is required for satisfactory forward link performance of a cellular system what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is $n=4$?
Assume that the mobile receiver is located at the $\boxed{\text{boundary}}$ of its omnidirectional operating cell.
Use suitable approximation.

Ans: Given SIR = 15dB. $n=4$.

Let's assume $N=7$

Co-channel Ration $Q=\sqrt{3N} = \sqrt{21}$.

At boundary,

$$SIR = \frac{1}{2(Q-1)^{-4}+2(Q+1)^{-4}+2Q^{-4}}$$

$$= \frac{1}{2(\sqrt{21}-1)^{-4}+2(\sqrt{21}+1)^{-4}+2\times(\sqrt{21})^{-4}}$$

$= 53.375$

$= 10\log_{10} 53.375 = 17.273\text{dB}$ which is greater than required so $N=7$ can be used.

(10) A total of 43.2 MHz of band width is allocated to a particular FDD cellular system that uses 15 KHz simplex channel. If each cell covers $5\,km^2$ and the cellular system support total area of $50\,km \times 50\,km$. Assume reuse factor of $N = 12$ path loss $n = 4$

(a) Identify the number of interferer co-channel cell for 60° sectoring. i- value → Pic থেকে দেখে বের করতে হবে।

(b) Calculate the SIR in dB for the omnidirectional antena and 60° sectoring.

(c) Find the number of user in each sector and each cell using 60° sectoring that can be supported at 0.5% blocking in an Erlang-B system, if each user average three calls per hour at an average call duration of 1 min.

(b)

$$SIR = \frac{(\sqrt{3N})^n}{i_0}$$

$n = 4$, $N = 12$, $i_0 = 6$

$$= \frac{(\sqrt{3 \times 12})^2}{6}$$

$$= 216$$

$$= 10 \log_{10} 216$$

$$= 23.34\,dB$$

60° sectoring: $N = 12$, $n = 4$, $i =$

ওটা দেখে দেখতে হবে কতটা interferer co-channel আছে then i value পাবো। impossible to do for me.

$$SIR_{60} = \frac{(\sqrt{3 \times 12})^n}{i}$$

$= ?$

normally figure থেকে value বের হবে তাছাড়া 60° র জন্য

$N = 4$ $i = 1$

$N = 3$ $i = 2$

[illegible]

(c)

$$\text{Total cell} = \frac{\text{Area}}{\text{each cell Area}} = \frac{50 \times 50 \ km^2}{5 km^2} = 500$$

$$\text{Total channels in system} = \frac{\text{System bandwidth}}{\text{each duplex bandwidth}}$$

$$= \frac{43.2 \ mHz}{15 \times 2 \ KHz}$$

$$= \frac{43.2 \times 10^3 \ KHz}{15 \times 2 \ KHz}$$

$$= 1440$$

Given, GOS, call blocking percent $= 0.5\% = 0.005$

3 calls per hour, $\lambda = \frac{3}{1} hr^{-1}$

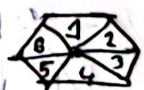
Average call duration, $H = 1 \text{ min} = \frac{1}{60}$ hr.

Traffic per user, $A_u = \lambda H = 3 \times \frac{1}{60} = \frac{1}{20}$

~~From graph, for 0.005 an~~

$N = 12$

$$\text{Channels per cell} = \frac{1440}{12} = 120$$

$$\text{Channels per sector } (60) = \frac{120}{6} = 60$$ 

with 0.005 blocking and 60 channel,

Traffic supported per sector $A_s = 13.18$ Erl → value is assumed (noco) [Graph থেকে বের করা লাগবে]

" " " cell $A = 6 \times 13.18 =$ ~~[illegible]~~ 79.08 Erl.

Total user supported per cell, $U = \frac{A}{A_u} = \frac{79.08}{0.005}$

$= 15816$

Total user supported per sector, $U = \frac{13.18}{0.005}$

$= 2636$

$\boxed{\text{or simply } (2636 \times 6) = 15816 \rightarrow \text{for cell user}}$ Ans

~~নোট~~ GoS % and channel দেখে Erl value বের করতে হবে for (A) → traffic intensity / supported per cell.

Lecture- 1, 2

① Describe with an example the relationship between bandwidth and data rate when considering 3 frequency components of a pulse:

Ans: Case 1:

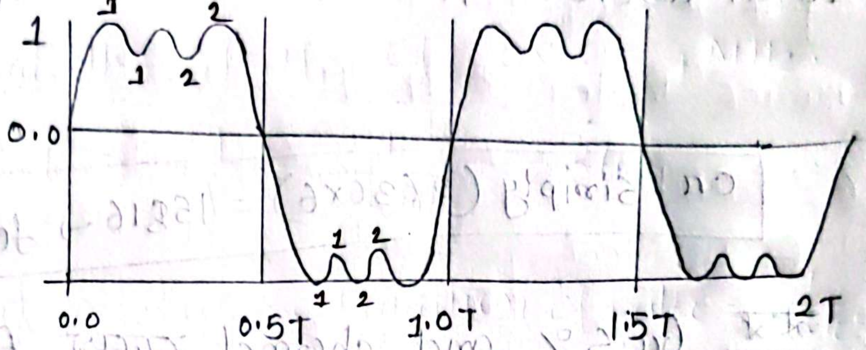


Figure: output

Lets, frequency, $f = 10^6$ cycle/sec $= 1MHz$.

frequency components $= 1f, 3f, 5f$

$$\text{Bandwidth} = 5f - 1f = 4f = 4 \times 1MHz = 4MHz$$

$$\text{Data rate} = 2f \text{ bps} \leftarrow$$

$$= 2 \times 1MHz \text{ bps}.$$

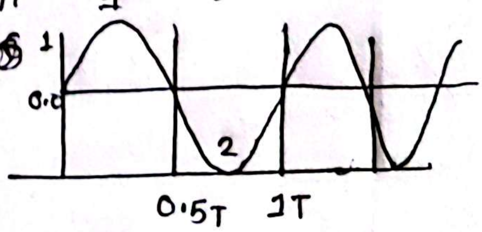
$$= 2mbps.$$

$$T = \frac{1}{f} = \frac{1}{10^6} = 10^{-6} = 1\mu s \left[\frac{10^6}{10^6}\right]$$

Now if we treat this wave form as bit string of 1s and 0s, 1 bit occurs at every $\frac{1\mu s}{2} = 0.5\mu s$

$\downarrow$ 2bit

So Duration of each pulse $= \frac{1}{2 \times 10^6}$



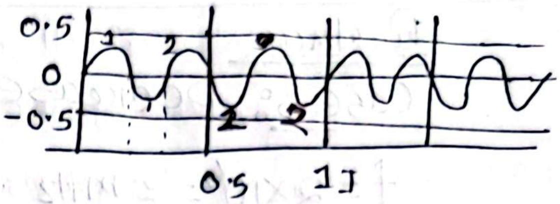
0.5µs-এ 1টা pulse

figure-input

Case 2: (increase frequency)

~~Exam~~

Let, $f = 2\times10^6 = 2MHz$



frequency components = 1f, 3f, 5f

Bandwidth = 5f - 1f = 4f = $4\times2\times10^6$ = 8 MHz

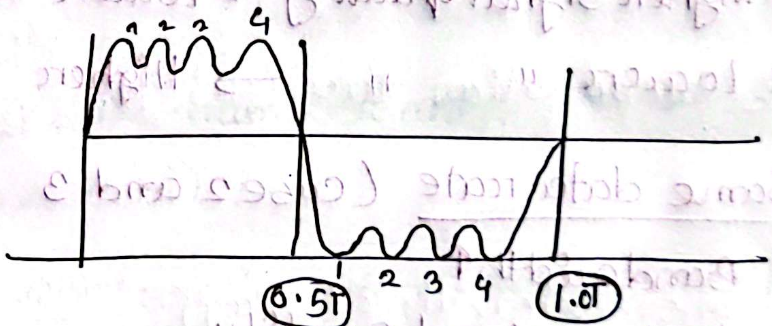
~~Data rate = 4*f~~

$$T = \frac{1}{f} = \frac{1}{2\times10^6} = 0.5\mu s.$$

Now from figure 1 bit occurs at every 0.25 μs

Duration of each pulse = $\frac{1}{4\times10^6}$ ↳ 1μs [1μs-এ কয়টা cycle হবে, $\frac{1}{0.25} = 4$]

Datarate = 4×10^6 = 4mbps. [1μs-এ কয়টা 1s হবে 0s]



[case-3 লাগবে এই ans-এ coz considering 3 component তাই]

So same signal Quality if Bandwidth and Datarate increase

Further

Case 3: Decrease frequency components.

$f = 2\times10^6 = 2\,\text{MHz}$.

frequency component $= 1f, 3f$

Bandwidth $= 3f - 1f = 2f = 2\times 2 = 4\,\text{MHz}$.

$$T = \frac{1}{f} = \frac{1}{2\,\text{MHz}} = 0.5\,\mu s.$$

1 bit occurs at $0.25\,\mu s$ so Duration of pulse $= \frac{1}{4\times10^6}$

Data rate $= 4\times10^6 s = 4\,\text{mbps}$.

Here Bandwidth 4 MHs Datarate = 4 mbps.

→ Same bandwidth (case 1 and 3 4 MHz)

* Higher signal quality → Lower data rate.
* Lower " " → Higher " "

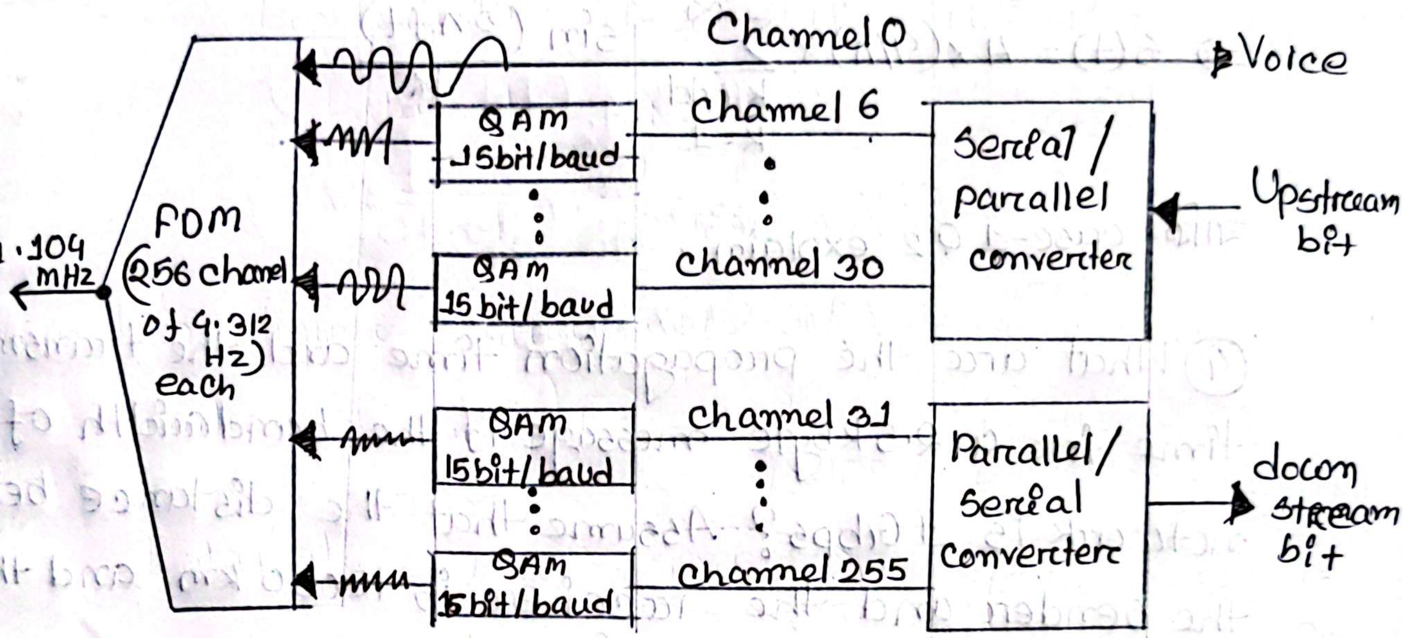
→ Same data rate (case 2 and 3 4 mbps)

* Bandwidth ↑
* Higher signal quality.

Now → If Bandwidth ↑ (increase) (1 & 2)

* Data rate ↑
* Same signal quality.

② Describe discrete multitone technique (DMT) used in ADSL modem with a diagram and how do we get 1.44 Mbps upstream and 13.4 Mbps downstream data rate for ADSL modem?



Fingure: ADSL modem.

~~Given~~ from, Each channel data transfer rate = 4.312 ≈ 4 Hz.

QAM speed = 15 bit /baud

Upstream channel = 25 channels = 24 traffic + 1 control channel
(6 to 30)

$= 24 \times 4\,kHz \times 15 = 1440\,kbps = 1.44\,Mbps.$

Downstream channel = 225 = 224 traffic + 1 control channel
(31 to 255)

$= 224 \times 4\,kHz \times 15 = 13500\,kbps = 13.5\,Mbps$

[Proved]

③ Show the mathmatical expression of the frequency components of the squared wave. What happen if we limit the bandwidth to just the first 3 ~~com~~ frequency components?

$$\Rightarrow s(t) = A \times (4/\pi) \times \sum_{\substack{k \text{ odd} \\ k=1}}^{\infty} \frac{\sin(2\pi f t)}{k}$$

বাকি case-1 & 2 explain.

④ What are the propagation time and the transmission time for a 2.5 kbyte message if the bandwidth of network is 1 Gbps? Assume that the distance between the sender and the receiver is 12000 km and the light travels at $2.4 \times 10^8 \, ms^{-1}$

Ans: Propagation time (delay) $= \frac{\text{Distance}}{\text{Speed}} = \frac{12000 \times 10^3 \, m}{2.4 \times 10^8 \, ms^{-1}}$

$= 0.05s$

Transmission time (delay) $= \frac{\text{Message size}}{\text{Data rate}}$

$= \frac{2.5 \, kB}{1 \, Gpbs}$

$= \frac{2.5 \, kB}{1 \times 10^6 \, kpbps}$

$= \frac{2.5 \times 1024 \times 8 \, bit}{10^9 \, bps}$

$= 2.0 \times 10^{-5} s.$

⑤ You are trying to design a cellular network that will cover an area of at least $2800 km^2$. There are $k=300$ available voice channels. Your design is required to support at least 100 concurrent calls in each cell. If the co-channel cell center distance is required to be 9km, How many base stations will you need in this network? Also find the signal to co-channel interference ratio with path-loss exponent is 4.

Ans:

Total area $= 2800 km^2$

$k = 300$ voice channel means each cluster can have 300 chanel.

Need to support 100 concurrent call in each cell,

mean $N = \frac{300}{100} = 3$

Co-channel center Distance, $D = 9km$.

$$D = R\sqrt{3N}$$

$$\Rightarrow 9km = R\sqrt{3\times3}$$

$$\Rightarrow R = \frac{9}{\sqrt{9}} = \frac{9}{3} = 3km$$

Each cell area, $= \frac{3\sqrt{3}}{2} R^2 = \frac{3\sqrt{3}}{2}(3)^2 = 23.38 km^2$

cell coz প্রত্যেক একটা cell base station থাকে।

← Required base station $= \frac{2800}{23.38} = 119.74 \simeq 120$.

$$SIR = \frac{(\sqrt{3N})^n}{6} = \frac{(\sqrt{3\times3})^4}{6}$$

$$= 13.5$$

$$= 11.303 dB.$$

⑥ What is the maximum capacity of a medium with a bandwidth of 750 kHz and a signal-to-noise ration of 30 dB?

$\Rightarrow$ $\text{Capacity} = B \log_2 (1 + SNR)$

$= 750 \times 10^3 \log_2 \left(1 + 10^{SNR/10}\right)$

$= 750 \times 10^3 \log_2 \left(1 + 10^{30/10}\right)$

$= 7475.4197$ kbps.

CamScanner

⑨

① If we consider the 6 trailing bits, stealing bits, guard bits and training bits in a GSM frame as overhead, and rest of the bits as data, then what is the percentage overhead in a GSM frame? What is the duration of a bit in GSM? If a user is allocated one time slot per frame, what is the delay between successive transmission in successive frames?

Ans: GSM frame has 8 timeslot. Each time slot is as follow

Trail	Voice	Stealing	Train	Stealing	Voice	trail	gaurd
3	57	1	26	1	57	3	8.25

1 Frame = 8 timeslot

Total bit = 156.25 bits.

So percentage of overhead $= \frac{156.25 - (57+57)}{156.25} \times 100 = 27.04\%$

We know, system sends 216.66 frame/sec.

Total frame width = 156.25 * 8 = 1250 bits/frame.

Transmission Rate = (216.66 × 156.25) bits/sec

= 270.33 kbps.

[1sec -এ যতগুলো frame
1 fram-এ " bit]

Duration of 1 bit $= \frac{1}{270.33} = 3.69 \mu s$

Time Duration of Each Frame $= \frac{\text{Frame width}}{\text{Transmission rate (1s যতbit)}}$

$= \frac{1250}{270.33 \text{kps}}$

$= 4.6239 \text{ ms}$. (Ans.)

270.33 bit যায় →			1 ms.
1	"	"	$\frac{1}{270.33}$ "
1250	"	"	$\frac{1250}{270.33}$ "

② GSM uses a frame structure where each frame consists of eight time slots and each time slot contains 156.25 bits and data is transmitted over a channel at 270.833 kbps.

(a) Time duration of a bit.

(b) Time duration of a TDMA frame

(c) Control data rate.

(d) traffic user rate.

(a)

Time duration of a bit $= \frac{1}{270.833} = 3.69\ \mu s$.

(b)

previous math এর নিয়মে করা যায়।

or.

Time duration of a time slot $= 156.25 \times 3.69$
$= 577\ \mu s$.

time duration of a TDMA frame $= \cancel{57}\ 577 \times 8 = \cancel{4616}\ \mu s$
$= \cancel{4.616\ ms}$
$= 4.61\ ms$

(c)

We know, system sends 216.66 frame/sec.

Control bit $= 156.25 - 144 = 42.25$ bits/frame.

Control bit data rate $= 216.66 \times 42.25 \times 8 = 73.66$ kbps.
$\hookrightarrow$ one fram

(d)

Traffic data rate $= 144 \times 216.66 \times 8 = 197.59$ Kbps.
$\downarrow$
$(57+57)$

২

Fall 2022

Ques 2: (a) → Done. সামনে লিখা আছে

(b) Distinguish between SNR and SIR. Identify the equation of SIR for mobile station located at the center of the cell and boundary of the cell.

⇒ SNR VS SIR

Aspect	SNR (Signal to noise Ratio)	SIR (Signal to interference Ratio)
Defination	Ratio of Signal Power and background noise power.	Ration of Signal Power & power of inteferring signals.
Focus	measure signal clearity in relation to random noise.	relative to interference from other signals.
Cause of degradation	Caused by thermal noise, environmen or device noise	Caused by interference from nearby cells or overlapping frequencies.
Typical Use	Communication channel and radio broadcasting	Key matric in Cellular network and wireless system.
Improvement methode	signal power ↑ or noise ↓	reduce interference.

SIR at center: with Radius R co-channel distance D reference d. path loss m.

$$SIR = \frac{\text{Signal Power}}{\text{Interference Power}}$$

$$= \frac{P_0 \left(R/d_0\right)^{-n}}{P_0 \left(\frac{D}{d_0}\right)^{-n}}$$

$$= (R/D)^{-n}$$

$$= \sum_{i=1}^{i_0} (R/D)^{-n}$$ [D = summation because there might be so many co-channel interfering]

$$= \frac{\left(\frac{D}{R}\right)^{n}}{6} \quad [i_0 = 6] \rightarrow \text{hexagonal.}$$

$$= \frac{(D/R)^n}{6} = \frac{(\sqrt{3N})^n}{i_0}$$

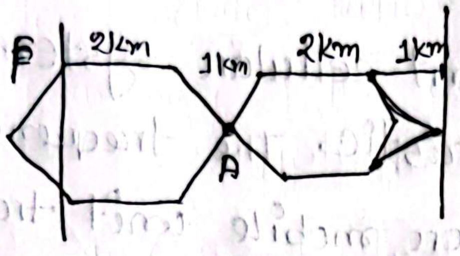
Now MS (mobile station) at the boundary cell.

$$SIR = \frac{S}{I} = \frac{R^{-4}}{2(D-R)^{-4} + 2(D+R)^{-4} + 2(D)^{-4}}$$

$$= \frac{1}{2(Q-1)^{-4} + 2(Q+1)^{-4} + 2Q^{-4}}$$

(c) → done.

(d) ওটা sir-র করে দেয়া slider মতই হবে যেহেতু original cell radius 2km and micro cell 1km.

So 

= 6km. [see the video]

~~[illegible]~~ just channel no is changed to 70.

→ BCDEF center A = (5) Base station.

(i) $5 \times 70 = 350$

(ii) $5+6 = 11 \times 70 = 770$ channel (original co-channe = 6)

(iii) $5+12 = 17 \times 70 = 1190$ channel.

[Ensure/ correct me if I'm wrong]

(iv) take, $n=4$

$P_r(\text{new small}) = P_r(\text{old}) \left(\frac{1}{2}\right)^n$

$$\Rightarrow \frac{P_r(\text{new small})}{P_r(\text{old})} = \left(\frac{1}{2}\right)^4 = \frac{1}{16} = 10 \log_{10}\left(\frac{1}{16}\right)$$

$= -12.04$.

So power will be reduced.

(e) done (Lecture 9, 10, 11) note.

(f) ↑

Ques 3: (a) × included.

(b) Consider two different cellular system that share the following characteristic. The frequency bands are 825 to 845 MHz for mobile unit transmission and 870 to 890 MHz for base station transmission. A duplex circuit consists of one 30 kHz channel in each direction. reduce factor 12, 19 respectively for the two system.

(i) Calculate the total forward link and reverse link bandwidth.

(ii) Suppose that in each of the system, the cluster of cells (12, 19) is duplicated 16 times. Find out the number of simultaneous communications that can be supported by each system.

(iii) Find the number of simultaneous communication that can be supported by a single cell in each system.

(iv) What is the area covered, in cells, by each system.

①

Channel = Revers + Forward

$= (845-825) + (890-870)$

$= 20\,MHz + 20\,MHz$

$= 40\,MHz$.

Total channels $= \frac{40 \times 10^3\,KHz}{2 \times 30\,KHz} = 666.67 \approx 667$ channels.

ⅱ & ⅲ → Confused

System-1 N=12,

Channels per cell $= \frac{667}{12} \approx 55$.

System-2 N=19

Channels per cell $= \frac{667}{19} = 35$.

Duplicate 16 times → $55 \times 16 = 880$
$35 \times 16 = 560$] area covered by each.

ⅳ

Area → 1 (System) $12 \times 16 = 192$ cells.

→ 2 " $19 \times 16 = 304$ cells.

(c) A hexagonal cell within a four-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the ~~total~~ load per user is 0.029 Erlangs and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call.

(i) How many users per square kilometer will this system support?

(ii) What is the probability that a delayed call will have to wait for more than 10 sec?

(iii) What is the probability that a call will be delayed for more than 10 second?

(i)

Cell Radius, $R = 1.387$ km.

Area covered by cell $= \frac{3\sqrt{3}}{2} R^2 = 5$ sq. km^2.

Number of cells per cluster $= 4$.

$$\text{Channels per cell} = \frac{60}{4} = 15 \text{ channels.}$$

traffic per user $A_u = 0.029$

From C chart 5% probability delay with $C=15$ traffic intensity $A=9$ Erlangs.

number of users $= \frac{A}{A_u} = \frac{9}{0.029} = 310$ users.

Users per sq km $\frac{310}{5} = 62$ user/sq.km.

(b)

$\lambda = 1$ $H = A_u/\lambda = 0.029/1 = 0.029$ hour $= \frac{0.029}{3600}$

$= 104.4$ sec.

The probability that a delayed call will have to wait for more than 10s is

channel per cell

$$Pr[\text{delay} > t(\text{delay})] = \exp(-(C-A)t/H)$$

$$= \exp(-(15-9)10/104.4) = 0.5629$$

$$= 56.29\%$$

(c)

$Pr(\text{delay} > 0) = 5\% = 0.05$

probability that a call is delayed more than 10s.

$$Pr(\text{delay} > 10) = Pr(\text{delay} > 0) \times Pr(\text{delay} > t(\text{delay}))$$

$$= 0.05 \times 0.5629$$

$$= 2.81\%$$

d→ slide math.

(e) The GSM fram structure is designated as hyperframe, superframe, multiframe and frame. The minimum unit being frame (or TDMA) is made 8 time slot.

(i) GMS data transmission Rate.

(ii) GMS user traffic rate.

(iii) The duration of superframe if GSM traffic channel is used.

(iv) Frame width of superframe and hyperframe.

(i)

Time slot width = 156.25 bit ~~frame~~.

System sends = 216.66 Frame/sec.

Frame width = 156.25 × 8 = 1250 bits/Fram.

Total Transmission Rate = 1250 × 216.66

= 270.8kps.

User (ii)

traffic rate = 144 × 216.66 × 8 = 197.59 kbps.

(*) If multilevel 26 = 24 (2 not included)

$$\frac{2 \times 57 \times 24}{120ms} = 22.8 kbps.$$

[26×4.61 = 120ms]

(iii)

system sends 216.66 frame/se.

Time duration of frame $= \frac{1}{216.66} = 4.615$ ms.

Time duration for superframe $= 4.615 \times 26 \times 51$ multifram → super-

$= 6112.86$ ms

$= 6.112$ s

(iv)

Time slot width = 156.25 bit

Frame width $= 8 \times 156.25$

~~mot~~ superframe $= 51 \times 26 \times 8 \times 156.25 =$ multi frame

Hyperframe $= 2048 \times 51 \times 26 \times 8 \times 156.25$

$=$

1 Hyperframe = 2048 superframe.

1 super " = 51 multiframe.

1 multi " = 26 frame

1 frame " = 8 timeslot.

(f) → বাকী আছে (9, 10, 11) → pdf

Spring 2023

Ques2 → All solved.

Ques3 → (a) → Confused

Given, total cell = 32

$R = 1.6 km$.

total Bandwidth = 33 MHz.

full duplex = $2 \times 25 kHz$

$N = 12$

(i)

What ~~is~~ geographycal area is covered?

⇒ Area of a cell = $\frac{3\sqrt{3}}{2}(1.6) = 6.651 km^2$

Total geographical area = total cell × a cell area

$= 32 \times 6.651$

$= 212.48 km^2$.

A.

(ii)

How many channels are there per cell?

Total Channels ~~per cell~~ $= \frac{33 MHz}{50 KHz} = \frac{33 \times 10^3 KHz}{50 KHz}$

$= 660$.

Channels per cell $= \frac{660}{12} = 55$

(III)

System's capacity = Total channel per cell × no of cell

$= 55 \times 32$

$= 1760$.

(IV)

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three system?

?

→ Control channel $= \frac{1\,MHz}{50\,KHz} = \frac{10^3 KHz}{50} = 20$ channels.

traffic channel $= 660 - 20 = 640$

Controll channel per cell $= \frac{20}{12} = 1.6 \approx 2$.

traffic " " " " $= \frac{640}{12} = 53.33 \approx 53$

?

(b) An urban area has a population of two million residents. Three competing trunked mobile networks (system A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, System B has 98 cells with 57 channels each and system C has 49 cells each with 100 channels.

Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of 3 min.

Assuming that all three trunked system are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Ans: System A

Given,

blocking = 2% = 0.02 $\quad \lambda = 2\ hr$

Channel per cell, $c = 19$ $\quad H = 3min = \frac{3}{60} = \frac{1}{20}$

From graph $A = 12$

$$A_U = \lambda H = 2 \times \frac{1}{20} = 0.1\ \text{Erlangs}.$$

User supported per cell $U = \frac{A}{A_U} = \frac{12}{0.1} = 120$

there are 394 cells. So total User supported by A

$$120 \times 394 = 47280$$

System B:

$0.02 \rightarrow C = 57 \rightarrow A = 45$.

User per cell $U = \frac{A}{A_u} = \frac{45}{0.1} = 450$.

there are 98 cells, so total users $= 450 \times 98 = 44100$

System C:

$0.02 \rightarrow C = 100 \rightarrow A = 88$.

User per cell, $U = \frac{A}{A_u} = \frac{88}{0.1} = 880$.

there are 49 cells, so total users $= 880 \times 49 = 43120$

Now total user ~~sup~~ suppoted by 3 system.

$(47280 + 44100 + 43120) = 134500$ users.

Again, there are $2m = 2000,000$ people. For system A percentage of market penetration, $\frac{47280}{2000000} = 2.36\%$

System B $= \frac{44100}{2000000} = 2.205\%$

System C $= \frac{43120}{2000,000} = 2.156\%$

total 3 system $= \frac{134500}{2000000} = 6.6725\%$

2.6

A certain city has an area of 1300 square mile and is covered by a cellular system using 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60kHz.

Assume a GOS of 2% for an Erlang B chart is specified. If the offered traffic per user is 0.03 Erlangs, compute

(a) Number of cells in the service area

(b) the # of channels per cell

(c) traffic intensity of each cell.

(d) the maximum carried traffic

(e) total # of user that can be served for 2% GOS

(f) # of mobile per channel

(g) the theoretical maximum # of users that could be served at one time by the system.

Ans: is the ~~no~~ # of channels in the system

95X31 = 2945 → 3.4%

(a)

Total area $= 1300$ mile^2.

cell radius $= 4$ miles.

Area of cell $= \frac{3\sqrt{3}}{2} 4^2 = 41.57$ mile^2

Total cell $= \frac{1300}{41.57} = 31$ cells.

(b)

Total channel $= \frac{40 \times 10^3 \text{ KHz}}{60 \text{ KHz}} = \cancel{85}\ 666$

Channel per cell $= \frac{666}{7} = 95$

(c)

$C = 95 \quad GOS = 0.02 \longrightarrow$ traffic intensity $A = 84$ Erlang/cell.

(d)

Maximum carried traffic $= \#$ of cell $\times$ traffic intensity

$= 31 \times 84 = 2604$ Erlangs.

(e)

$A_U = 0.03$.

$U = \frac{A}{A_U} = \frac{2604}{0.03} = 86,800$ users.

(f)

$\#$ of mobile per channel $= \frac{\text{number of users}}{\text{number of channels}}$.

$= \frac{86800}{666} = 130$.

A microwave transmitter has output of 0.1W at 2GHz. Assum that this transmitter is used in a microwave communication system where the transmitting and receiving antennas are parabolals each 1.2m in diameter.

(i) What is the gain in dB of each antenna in decible?

(ii) Taking into account antenna gain, what is the effective radiated power of the transmitted signal?

(iii) If the receiving antenna is located 24km from the transmitting antena over a free space path, find the available signal power out of the receiving antenna in dB units.

(i)

Gain of each antenna for parabola,

$G = \frac{7A}{\lambda^2}$ $\qquad D = 1.2m.$

$\qquad R = 0.6m.$

$= \frac{7 \times 1.13 \times f^2}{c^2}$ $\qquad A = \pi R^2 = 1.13$

$\qquad = 3.1416 \times 0.6^2$

$= \frac{7 \times 1.13 \times (2 \times 10^9)^2}{(3 \times 10^8)^2}$ $\qquad f = 2GHz = 2 \times 10^9 Hz$

$= 351.85$ $\qquad c = 3 \times 10^8 ms^{-1}$

$G_{dB} = 25.46 dB.$

(ii)

Effective radiated power of the transmitter.

signal P_s = $0.1 \times 351.85 = 35.185W$.

(iii)

- Free space loss:

$$\frac{P_t}{P_r} = \frac{(4\pi)^2 d^2}{G_r G_t \lambda^2}$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = 0.0225$$

$$= \frac{(4\times3.146\times24\times10^3)^2}{25.46\times25.46\times(0.0225)^2}$$

$$L_{dB} = 20\log(4\pi) + 20\log(d) + 20\log(f) - 10\log(G_r) - 10\log(G_t) - 20\log(c)$$

$$= 21.89 + 87.6 + 186.02 - 169.54 - 25.46 - 25.46$$

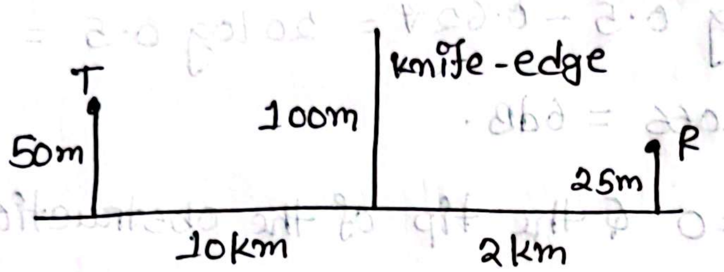
$$= 75.04\,dB.$$

Transmitter power $= 10\log(0.1\times10^3) = 20$.

receiver " $= 20 - 75.04 = -55.04\,dBm$.

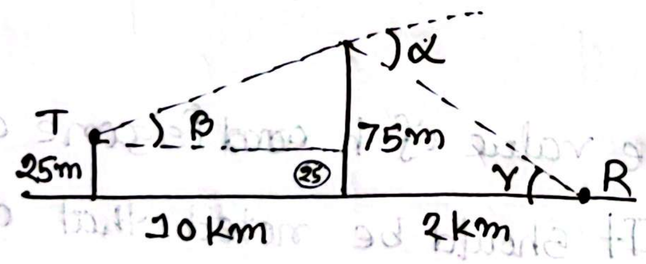
CamScanner

3.8 Given the follo wing geometry, determine (a) the Loss due to knife-edge diffraction and (b) the height of the obstacle required to induces 6dB diffraction Loss. Assume $f = 900\,MHz$.



(a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = \frac{1}{3}\,m$.

Now re-draw the geometry by subtracting the height of the smallest structure. Here R = smallest.



$\beta = \tan^{-1}\frac{75-25}{10{,}000} = 0.2865°$

$\gamma = \tan^{-1}\frac{75}{2000} = 2.15°$

$\alpha = \beta + \gamma = 0.2865 + 2.15 = 2.434° = \frac{2.434\times\pi}{180}$

$= 0.0424\ rad.$

CS CamScanner

The equation $v = \alpha \sqrt{\frac{2 \times d_1 \times d_2}{\lambda (d_1 + d_2)}}$

$$= 0.0424 \sqrt{\frac{2 \times 10{,}000 \times 2000}{1/3\,(10000 + 2000)}}$$

$$= 4.24 > 2.4$$

So $G_d(dB) = 20 \log \frac{0.225}{v} = 20 \log \frac{0.225}{4.24} = -25.50\,dB$

So the diffraction loss is 25.5 dB.

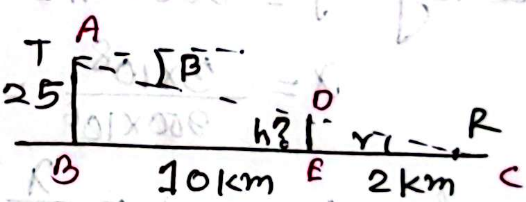
(b) For 6dB diffraction loss, $v = 0$.

$G_d = 20 \log 0.5 - 0.62 v$ or $20 \log 0.5 e^{-0.95 v}$

$= 20 \log 0.5 - 0.62 \times 0$ or $20 \log 0.5 e^{-0.95 \times 0}$

$= -6.0$ or -6.0.

So, $v = 0$.



ΔABC -

$\tan \gamma = \frac{AB}{BC} = \frac{25}{BE + EC} = \frac{25}{12000}$

ΔDCE - $\tan \gamma = \frac{h}{2000}$

$$\frac{h}{2000} = \frac{25}{12000}$$

$$\Rightarrow h = \frac{25 \times 2000}{12000} = 4.16\,m.$$

Example 3.6 A mobile is located 5km away from a base station and uses a verticle $\lambda/4$ monopole antenna with gain of 2.55 dB to receive cellular radio signals. The E-field at $\boxed{1\,km}$ from the transmitter is measured to be 10^{-3} V/m. The carrier frequency used for this system is 900 MHz.

(a) Find the ~~Gain~~ length and the gain of receiving antenna.

Ans: Given,

T-R seperation distance, $d = 5km = 5000m$.

E-field at a distance of 1 km $= 10^{-3}$ V/m.

& $d_0 = 1km = 1000$

Frequency $f = 900\,MHz$

$$\lambda = \frac{3\times10^8}{900\times10^6} = \frac{1}{3}$$

$$\text{Length of antenna, } L = \frac{\lambda}{4} = \frac{1/3}{4} = 0.0833\,m$$

$$= 8.33\,cm.$$

$$\text{Gain} = \frac{4\pi A_e}{\lambda^2}$$ যেখানে Ae value নাই।

$$\text{Gain} = 10^{2.55/10} = 1.8.$$

(b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and receiver antenna 1.5m above ground.

Ans: since $d \gg \sqrt{h_t h_r}$, the electric field,

$$E_R(d) = \frac{2E_0 d_0}{d} \cdot \frac{2\pi h_t h_r}{\lambda d}$$

$$= \frac{2\times10^{-3} \times 1000 \times 2\times3.1416\times50\times1.5}{5000 \times 0.333\times 5000}$$

$$= 113.1\times10^{-6}\,V/m.$$

The received power at a distance d can be obtained

$$P_r(d) = \frac{(113.1\times10^{-6})^2}{377}\left[\frac{1.8\,(0.33)^2}{4\pi}\right]$$

$$P_r(d) = P_d A_e$$

$$= \frac{|E(d)|^2}{120\pi} A_e$$

$$A_e = \frac{G\lambda^2}{4\pi}$$

$120\times3.1416 = 376.99$

$\simeq 377$

$$\Rightarrow P_r(5km) = \cancel{5.3}\ 5.4\times10^{-13}\,W$$

$$= 10 \log 5.4\times10^{-13}$$

$$= -122.68\ dBW$$

$$= 10\log_{10}\left(\frac{5.4\times10^{-13}}{10^{-3}}\right) dBm.$$

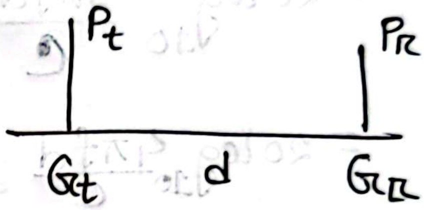
$$= -92.68\ dBm.$$

Friis Free Space Equation

দূরত্ব বাড়ার সাথে সাথে signal ছড়িয়ে যায়। অনেকগুলো signal·

The ~~equation~~ relation between the transmit and receiver power is given by Friis Free space equation

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$



Free space loss:

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{\lambda^2} = \frac{(4\pi f d)^2}{c^2} \quad \left[\lambda = \frac{c}{f}\right]$$

isotropic antenna gain = 1

$G_t = G_r = 1$

d & λ are in same unit (m)

$$L_{dB} = 10 \log_{10} \frac{P_t}{P_r}$$

$$= 10 \log_{10} \left(\frac{4\pi d}{\lambda}\right)^2$$

$$= 2 \times 10 \log_{10} \frac{4\pi d}{\lambda}$$

$$= 20 \log_{10} 4\pi d - 20 \log_{10} \lambda$$

$$= 20 \log_{10}(4\pi) + 20 \log_{10}(d) - 20 \log_{10}(\lambda)$$

$$= 21.98\,dB + 20 \log_{10}(d) - 20 \log_{10}(\lambda)$$

(Ans:)

Again,

$$L_{dB} = 10 \log_{10} \frac{P_t}{P_r}$$

$$= 10 \log_{10} \left(\frac{4\pi f d}{c}\right)^2$$

$$= 20 \log_{10} \frac{4\pi f d}{c}$$

$$= 20 \log_{10} \frac{4\pi f d}{c}$$

$$= 20 \log_{10} \frac{4\pi}{c} + 20 \log_{10} (f) + 20 \log_{10} (d)$$

$$= 20 \log_{10} \frac{4 \times 3.1416}{3 \times 10^8} + 20 \log_{10} (f) + 20 \log_{10} (d)$$

$$= -147.56 dB + 20 \log_{10} (f) + 20 \log_{10} (d).$$

(Ans:)

এগুলো prove করার আগে Pic দিয়ে $\frac{P_t}{P_r}$ figure explain করে then prove.

→ Free space loss accounting for Gain of other antenna

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{G_t G_r \lambda^2} \qquad G_t = \frac{4\pi A_t}{\lambda^2}$$

$$G_r = \frac{4\pi A_r}{\lambda^2}$$

$$= \frac{(4\pi)^2 d^2}{\lambda^2} \times \frac{\lambda^2 \times \lambda^2}{4\pi A_t \times A_r 4\pi}$$

$$= \frac{(4\pi)^2 d^2}{\cancel{\lambda^2}} \times \frac{\lambda^4}{A_t A_r (4\pi)^2}$$

$$= \frac{\lambda^2 d^2}{A_t A_r} = \frac{(\lambda d)^2}{A_t A_r} = \frac{(cd)^2}{A_t A_r f^2} \qquad \boxed{\lambda = \frac{c}{f}}$$

$$L_{dB} = 10 \log_{10} \frac{P_t}{P_r} = 10 \log_{10} \frac{(\lambda d)^2}{A_t A_r}$$

$$= 10 \log_{10} (\lambda d)^2 - 10 \log_{10} (A_t A_r)$$

$$= 20 \log_{10} \lambda d - 10 \log_{10} (A_t A_r)$$

$$= 20 \log_{10} \lambda + 20 \log_{10} d - 10 \log_{10} (A_t A_r)$$

$$L_{dB} = 10 \log_{10} \frac{(cd)^2}{A_t A_r f^2}$$

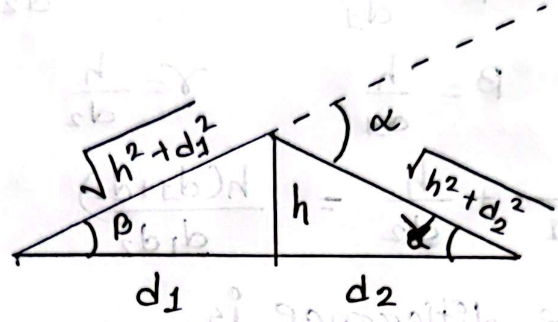
$$= 10 \log_{10} \left(\frac{cd}{f}\right)^2 - 10 \log_{10} A_t A_r$$

$$= 20 \log_{10} cd - 20 \log_{10} f - 10 \log_{10} (A_t A_r)$$

$$= 20 \log_{10} c + 20 \log_{10} d - 20 \log_{10} f - 10 \log_{10} A_t A_r$$

(11) lecture

knife-edge Diffraction model:



The electrical field due to the differacted path is,

$$E_d = E_0 \exp(-j\phi) \quad \text{——(1)}$$

The difference between the direct path and the diffracted path called the excess path length (Δ) can be obtained

$$\Delta = \sqrt{h^2+d_1^2} + \sqrt{h^2+d_2^2} - (d_1+d_2)$$

$$= d_1\sqrt{1+\frac{h^2}{d_1^2}} + d_2\sqrt{1+\frac{h^2}{d_2^2}} - d_1 - d_2$$

$$= d_1\left(1+\frac{h^2}{2d_1^2}\right) + d_2\left(1+\frac{h^2}{2d_2^2}\right) - d_1 - d_2 \quad \left[\begin{array}{l}\sqrt{1+x} = 1+\frac{x}{2} \\ x \ll 1\end{array}\right]$$

$$= d_1 + \frac{d_1h^2}{2d_1^2} + d_2 + \frac{d_2h^2}{2d_2^2} - d_1 - d_2$$

$$= \frac{h^2}{2}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)$$

$$= \frac{h^2(d_1+d_2)}{2d_1d_2}$$

The angle $\alpha = \beta + \gamma$ since $d_1, d_2 >> h$

$$\tan\beta = \frac{h}{d_1} \qquad \tan\gamma = \frac{h}{d_2}$$

$$\beta = \frac{h}{d_1} \qquad \gamma = \frac{h}{d_2} \qquad [\tan x \simeq x]$$

$$\alpha = \frac{h}{d_1} + \frac{h}{d_2} = \frac{h(d_1+d_2)}{d_1 d_2}$$

The phase difference is,

$$\phi = \omega T_d$$

$$= 2\pi f \frac{\Delta}{c}$$

$$= 2\pi f \frac{\Delta}{f\lambda}$$

$$= \frac{2\pi\Delta}{\lambda}$$

$$= \frac{\cancel{2}\pi}{\lambda} \frac{h^2(d_1+d_2)}{\cancel{2}d_1 d_2}$$

$$= \frac{\pi h^2 (d_1+d_2)}{\lambda d_1 d_2} \quad \text{if keep 2}$$

$$\phi = \frac{\pi}{2} \frac{h^2\, 2(d_1+d_2)}{\lambda d_1 d_2} \quad \text{——— (4)}$$

The phase difference is usually normalized using Fresnel kirchhoff parameter ν given by.

$$\nu = h\sqrt{\frac{2}{\lambda}\left(\frac{d_1+d_2}{d_1d_2}\right)} = \alpha\sqrt{\frac{2}{\lambda}\left(\frac{d_1d_2}{d_1+d_2}\right)}$$

$$\nu^2 = h^2\frac{2(d_1+d_2)}{\lambda\, d_1d_2}$$

From equa (11) →

$$\phi = \frac{\pi}{2}\nu^2$$

from equ^n (1) →

$$E_d = E_0 \exp\left(-j\frac{\pi}{2}\nu^2\right)$$

Now we include all other rays produced by the Hygen's sources. These are produced for all the Hygen's sources above the screen and hence we sum or integrate from ν to ∞

$$E_{TOT} = E_0\frac{1+j}{2}\int_{\nu}^{\infty}\exp\left(-j\frac{\pi}{2}t^2\right)dt.$$

$$\Rightarrow \frac{E_{TOT}}{E_0} = \frac{1+j}{2}\int_{\nu}^{\infty}\exp\left(-j\,\pi/2\, t^2\right)dt.$$

$\Rightarrow \frac{E_{TOT}}{E_0} = F(\nu)$ → Complex Fresnal Integral.

$$\Rightarrow \left|\frac{E_{TOT}}{E_0}\right|^2 = |F(\nu)|^2$$

$$\Rightarrow G_d\,(dB) = 20\log|F(\nu)|$$

G_d (dB) = Diffraction gain for positive value.

for negative value → Diffraction loss

1

Tuesday
09 July 24

Here,

E_{LOS} = Direct LOS component.

E_g = Ground Reflected component.

E_{TOT} = Total received feild.

h_t = Height of the Tx

h_r = " " " Rx

d = Distance betn Tx and Rx

Propagation delay $= \frac{\text{distance (d)}}{\text{speed (c)}}$

Let, E_0 = Free space E-field (V/m) at distance d_0

$$E(d,t) = \frac{E_0 d_0}{d} \cos\left(\omega(t - d/c)\right)$$ [~~see~~ d/c = propagation delay (t)

E-field for LOS

$$E_{LOS}(d',t) = \frac{E_0 d_0}{d'} \cos\left(\omega\left(t - \frac{d'}{c}\right)\right)$$ [d' = distance of LOS wave]

E-field for reflected wave,

$$E_g(d',t) = \frac{E_0 d_0}{d''} \cos\left(\omega\left(t - \frac{d''}{c}\right)\right)$$ [d' = distance of reflected wave]

$$E_{TOT} = E_{LOS} + E_g$$

$$\Rightarrow E_{TOT}(d,t) = \frac{E_0 d_0}{d'} \cos\left(\omega\left(t - \frac{d'}{c}\right)\right) + (-1)\frac{E_0 d_0}{d''} \cos\left(\omega\left(t - \frac{d''}{c}\right)\right)$$

[(-1) = Reflection co-efficient]

Evaluate E-field when reflected path arrives at receiver $\rightarrow$

$$E_{TOT}\left(d, t=\frac{d''}{c}\right)=\frac{E_0d_0}{d'}\cos\left(\omega\left(\frac{d''-d'}{c}\right)\right)-\frac{E_0d_0}{d''}\cos 0^\circ$$

$$=\frac{E_0d_0}{d'}\cos\theta_\Delta-\frac{E_0d_0}{d''}$$

If d becomes large then $d''=d'=d$

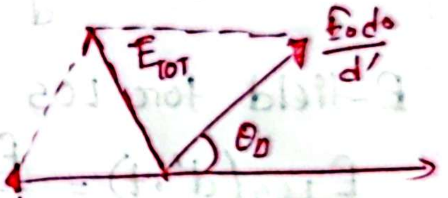
$$E_{TOT}(d)=\frac{E_0d_0}{d}\cos\theta_\Delta-\frac{E_0d_0}{d}=\frac{E_0d_0}{d}(\cos\theta_\Delta-1) \quad \text{--- (I)}$$

→ Electric Feild.

Here E = Power

$$E_{TOT}(d)=\sqrt{\left(\frac{E_0d_0}{d}\right)^2(\cos\theta_\Delta-1)^2+\left(\frac{E_0d_0}{d}\right)^2\sin^2\theta_\Delta} \quad \text{[Pythagorus]}$$

$$=\frac{E_0d_0}{d}\sqrt{2-2\cos\theta_\Delta}$$



phasor diagram

$$=2\frac{E_0d_0}{d}\sin\left(\frac{\theta_\Delta}{2}\right)$$

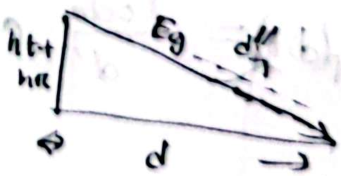
$$=2\frac{E_0d_0}{d}(\theta_\Delta/2) \quad \left[\text{Since } \sin\left(\frac{\theta_\Delta}{2}\right)=\frac{\theta_\Delta}{2}\right] \quad \text{--- (II)}$$

② Path difference,

Now, Path diff $\Delta = d''-d'$

$$=\sqrt{(h_t+h_r)^2+d^2}-\sqrt{(h_t-h_r)^2+d^2}$$

[Slide & diagram Xm-এ চিত্র proof করতে হবে]



$d'' =$ 

2

$$\Delta = d\sqrt{1+\left(\frac{h_t+h_r}{d}\right)^2} - d\sqrt{1+\left(\frac{h_t-h_r}{d}\right)^2}$$

$$= d\left(1+\frac{1}{2}\left(\frac{h_t+h_r}{d}\right)^2\right) - d\left(1+\frac{1}{2}\left(\frac{h_t-h_r}{d}\right)^2\right)$$

$\left[\sqrt{1-x} = 1+\frac{x}{2}\right.$, Tylor series.$\left.\right]$

$$= \frac{1}{2d}\left((h_t+h_r)^2 - (h_t-h_r)^2\right) \quad \left[(a+b)^2-(a-b)^2 = 4ab\right]$$

$$\boxed{\Delta = \cancel{2h_th_r}\ \frac{2h_th_r}{d}} \longleftarrow \text{Path difference.}$$

Phase diff^n, $\theta_\Delta = \omega \tau_d^{\to\text{time}} = 2\pi f\frac{\Delta}{c} = 2\pi f \frac{\Delta^{\to\text{distance}}}{\lambda f} = \frac{2\pi\Delta}{\lambda}$

Time delay. $\tau_d^{\to\text{time}} = \frac{\Delta}{c} = \frac{\theta_\Delta \lambda}{2\pi f\cancel{\lambda}} = \frac{\theta_\Delta}{2\pi f}$

From Equation (a)

$$E_{TOT}(d) = 2\frac{E_0 d_0}{d}\left(\theta_\Delta/2\right)$$

$$= 2\frac{E_0 d_0}{d}\left(\frac{2\pi\Delta}{2\lambda}\right)$$

$$= 2\frac{E_0 d_0}{d}\ \frac{2\pi\ 2h_th_r}{2\lambda d}$$

$$\boxed{E_{TOT}(d) = \frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2}}$$

$$|E_{TOT}(d)|^2 = \left| \frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2} \right|^2 \quad [P \propto E^2]$$

$$\cancel{d(d} \; P_r(d) = \frac{(4\pi)^2 E_0^2 d_0^2 h_t^2 h_r^2}{\lambda^2 d^4}$$

$$= \frac{(4\pi)^2 P_0 d_0^2 h_t^2 h_r^2}{\lambda^2 d^4} \quad \text{——(III)}$$

Free space loss for LOS,

$$\frac{P_t}{P_r} = \frac{(4\pi d)^2}{G_t G_r \lambda^2} \Rightarrow P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$$

Here, $P_0 = P_r$, $P_0 = P_t G_t G_r \frac{\lambda^2}{(4\pi d)^2}$

From (III),

$$P_r(d) = P_t G_t G_r \frac{\cancel{\lambda^2}}{(4\pi d_0)^2} \times \frac{(4\pi d_0)^2 h_t^2 d_r^2}{\lambda^2 d^4}$$

$$\boxed{P_r(d) = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}}$$

[Proved]

Free Space Loss for 2-ray model,

$$\frac{P_t}{P_r} = \frac{d^4}{G_t G_r h_t^2 h_r^2} \quad \text{(Proved)} \longrightarrow (2)$$

Free Space loss for 2-ray model in dB

$$L_{dB} = 40 \log(d) - 10 \log(G_t G_r) - 20 \log(h_t h_r)$$
(Proved) $-$ (3

(*) এই Formula use করে Rappaport বইয়ে math আছে করতে হবে।