Lecture-6

The process of finding all the pixels in an image that are occupied by a geometric primitive is called rasterization.

Graphics Pipeline

The sequence of operation that is required, starting with objects and ending by updating pixels in the image, is known as Graphics pipeline.

Bresenham's Line drawing Algo

(*) Implicit equation of a Line.

$y = mx + B$

$\Rightarrow y = \frac{dy}{dx} * x + B$.

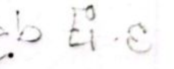
$\Rightarrow y\,dx = x\,dy + B$.

$\Rightarrow x\,dy - y\,dx + B = 0$.

$\Rightarrow ax + by + c = 0$ $\left[\begin{array}{l} \text{here} \\ a = dy \;;\; c = B \\ b = -dx \end{array}\right]$

$\Rightarrow \boxed{F(x,y) = ax + by + c = 0}$ —— (1)

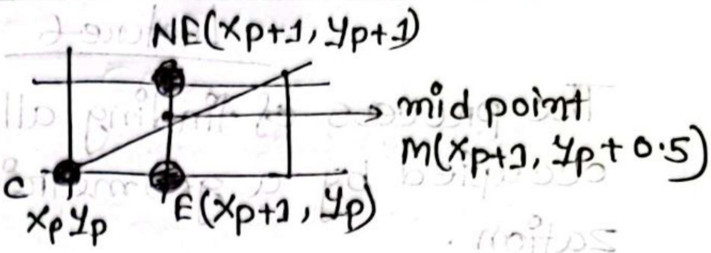
(*) Now if equation (1) equals to zero for any point (x,y) so it means that point (x,y) is on the line.



* If $F(x,y) > 0$, the point (x,y) is under the line

* if $F(x,y) < 0$, the point (x,y) is above the line

Midpoint criteria



So, decision Variable, d

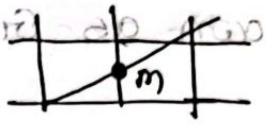
$d = F(M)$

$= F(x_p+1, y_p+0.5)$

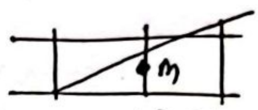
$= a(x_p+1) + b(y_p+0.5) + c \quad [ax + by + c = 0]$

Conditions:

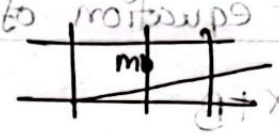
1. if $d=0$, Mid point on the line.



2. if $d>0$, Mid point below the line
$\boxed{\text{So NE is selected}}$



3. If $d<0$, Mid point above the line
$\boxed{\text{So E (East) is selected.}}$



Successive Updating for E:

$d_1 = F(M)$

$= F(x_p+1, y_p+0.5)$

$= a(x_p+1) + b(y_p+0.5) + c \quad [ax+by+c = F(x,y)]$

If $d_1 \leq 0$ select E $(x_p = x_p+1, y_p)$

$d_2 = F(M_2)$

$= F(x_p+2, y_p+0.5)$

$= a(x_p+2) + b(y_p+0.5) + c$

$= ax_p + 2a + by_p + 0.5b + c.$

$= ax_p + a + by_p + 0.5b + c + a$

$= [a(x_p+1) + b(y_p+0.5) + c] + a$

$= d_1 + a$

Every iteration after selecting E, updating decision variable is $d_{New} = d_{old} + a = d_{old} + dy$.

$$\boxed{d_{New} = d_{OLD} + dy} \rightarrow \text{Update E}$$

Successive Updating for NE:

$d_1 = f(M_1) = a(x_p+1) + b(y_p+0.5) + c$

If $d > 0$, select NE ($x_p = x_p+1$, $y_p = y_p+1$).

$d_2 = f(M_2)$

$= f(x_p+2, y_p+1.5)$

$= a(x_p+2) + b(y_p+1.5) + c$.

$= ax_p + 2a + by_p + 1.5b + c$

$= ax_p + a + by_p + 0.5b + c + a + b$.

$= [a(x_p+1) + b(y_p+0.5) + c] + a + b$

$= d_1 + a + b$.

Every iteration after selecting NE, updating decision variable is $d_{New} = d_{OLD} + a + b$

$$\boxed{d_{New} = d_{OLD} + dy - dx}$$

Finally: Midpoint ~~Updating~~ critera with successive Updating.

if $d \leq 0 \rightarrow M$ is above the line
$\rightarrow$ E closer to line $\rightarrow$ $d = d + \Delta E$ [$\Delta E = dy$]

if $d > 0 \rightarrow M$ is under the line
$\rightarrow$ NE closer to line $\rightarrow$ $d = d + \Delta NE$ [$\Delta NE = dy - dx$]

Bresenham's Mid point Algorithm:

(Loop)

1. while ($x <= x_1$)
2. if $d <= 0$ [close to E]
3. $d = d + \Delta E$;
4. else [close to NE]
5. ~~d = d + ΔNE~~; $y = y + 1$;
6. $d = d + \Delta NE$
7. End if
8. $x = x + 1$
9. PlotPoint (x, y)
10. end while

Given,
Start point (x_0, y_0)
End point (x_1, y_1)

Initialization:

$x = x_0 \quad y = y_0$

$dx = x_1 - x_0$; $dy = y_1 - y_0$

$\Delta E = 2dy$; $\Delta NE = 2(dy - dx)$

$d = 2dy - dx$;

Plot Point (x, y);

Initializing the Deceision Variable

$d_{init} = F(M)$

$= F(X_0 + 1, Y_0 + 0.5)$

$= a(X_0 + 1) + b(Y_0 + 0.5) + c$.

$= aX_0 + a + bY_0 + b0.5 + c$.

$= [aX_0 + bY_0 + c] + a + 0.5b$

$= F(X_0, Y_0) + a + 0.5b$

$= a + 0.5b$

$= dy - 0.5dx$ [floating point operation is slower than integer operation.]

$= 2dy - 2 \cdot \frac{1}{2} dx$ [multiply by 2]

updated.

$\boxed{d_{init} = 2dy - dx}$

& $\Delta E = 2dy$

$\Delta NE = 2(dy - dx)$

Quiz-4

CSE 4203

ID: 20200104038 Name: Talisha Tashim Pria Section: A

Ans to the Question NO:1

(a)

Here, $n = 38$ $p = (-1)^{38} \times 38 = 38$

Given,

Start Point $= (1, P) = (1, 38) \rightarrow (x_0, y_0)$

End Point $= (-3, p+9) = (-3, 38+9) = (-3, 47) \rightarrow (x_1, y_1)$

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{47-38}{-3-1} = \frac{9}{-4} = -2.25 < -1$$

3rd octant: $-1 > m > -\infty$ $y_0 = 38 < y_1 = 47$

Now,

$x = -x$

swap(x, y)

plot(-y, x)

$x_0 = -x_0 = -1$ $x_1 = -x_1 = 3$

swap(x_0, y_0) swap(x_1, y_1)

New $(x_0, y_0) = (38, -1)$ new $(x_1, y_1) = (47, 3)$

Here, $dy = y_1 - y_0 = 3 - (-1) = 4$; $dx = x_1 - x_0 = 47 - 38 = 9$

~~d = 2dy = 2×4~~

~~AE =~~

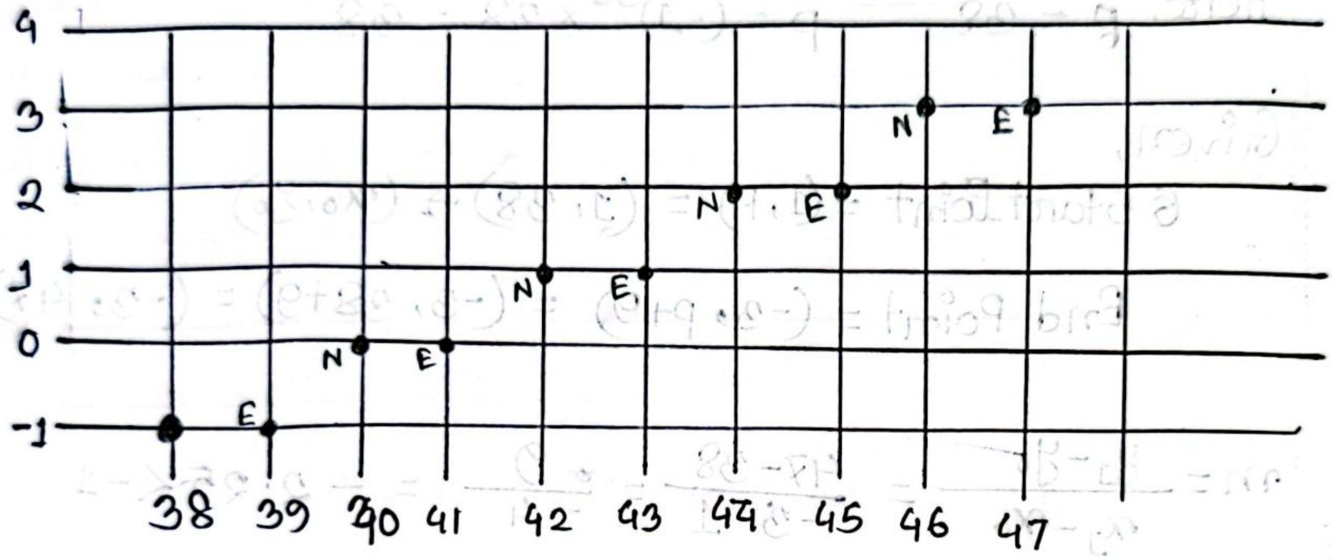
$d = 2dy - dx = 2 \times 4 - 9 = -1$

$\Delta E = 2dy = 2 \times 4 = 8$

$\Delta NE = 2(dy - dx) = 2(4-9) = -10$

~~Base~~ if $E \to (x_1+1, y_1)$ $NE \to (x_1+1, y_1+1)$

$d_{new} = d_{old} + \Delta E / \Delta NE$

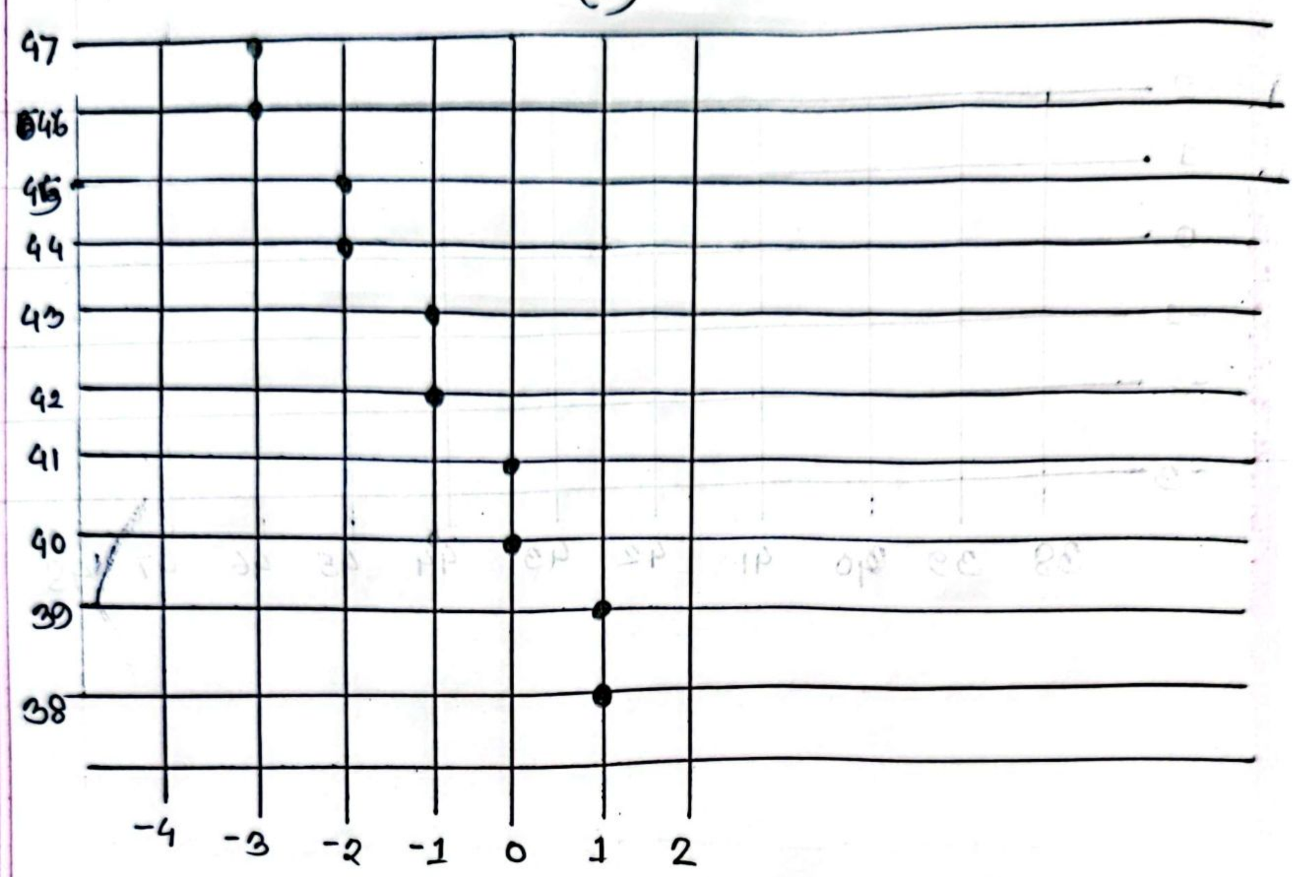


d	$d<=0$, E -1	$d=d+\Delta E$ $=-1+8$ $=7$ $d>0$, NE	$d=d+\Delta NE$ $=7-10$ $=-3$ $d<=0$, E	$d=d+\Delta E$ $=-3+8$ $=5$ $d>0$, NE	$d=d+\Delta NE$ $=5-10$ $=-5$ $d<=0$, E	$d=d+\Delta E$ $=-5+8$ $=3$ $d>0$, NE	$d=d+\Delta NE$ $=3-10$ $=-7$ $d<=0$, E	$d=d+\Delta E$ $=-7+8$ $=1$ $d>0$, NE	$d=d+\Delta NE$ $=1-10$ $=-9$ $d<=0$, E
(x,y)	E(39,-1)	NE(40,0)	E(41,0)	NE(42,1)	E(43,1)	NE(44,2)	E(45,2)	NE(46,3)	E(47,3)
$(-y,x)$	E(1,39)	NE(0,40)	E(0,41)	NE(-1,42)	E(-1,43)	NE(-2,44)	E(-2,45)	NE(-3,46)	E(-3,47)

$NE = (x+1, y+1)$

$E = (x+1, y)$

Final Plot
(b)



Line Drawing
Practice Problem

Perform the midpoint algorithm for a line with two points (5,8) and (-9,-11).

Ans:

Step1: Find the Octant: $(x_0, y_0) = (5,8)$ & $(x_1, y_1) = (-9,-11)$

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{-11-8}{-9-5} = 1.36$$

so match with 6^{th} octant $[y_1 < y_0 \quad 1 < m < \alpha]$

Step2: Convert it in 1^{st} octant:

$x = -x$	$x_0 = -x_0 = -5$	$x_1 = -x_1 = 9$
$y = -y$	$y_0 = -y_0 = -8$	$y_1 = -y_1 = 11$
swap (x, y)	swap $(x_0, y_0) = (-8, -5)$	swap $(x_1, y_1) = (11, 9)$
plot $(-y, -x)$		

Step3: Perform Line drawing Algo:

Start point $(\underset{x_0}{-8}, \underset{y_0}{-5})$ End Point $(\underset{x_1}{11}, \underset{y_1}{9})$

$dy = y_1 - y_0 = 9 - (-5) = 14$; $dx = x_1 - x_0 = 11 - (-8) = 19$

$\Delta E = 2dy = 2 \times 14 = \underline{28}$ $\quad \Delta NE = 2(dy - dx) = 2(14-19) = \underline{-10}$

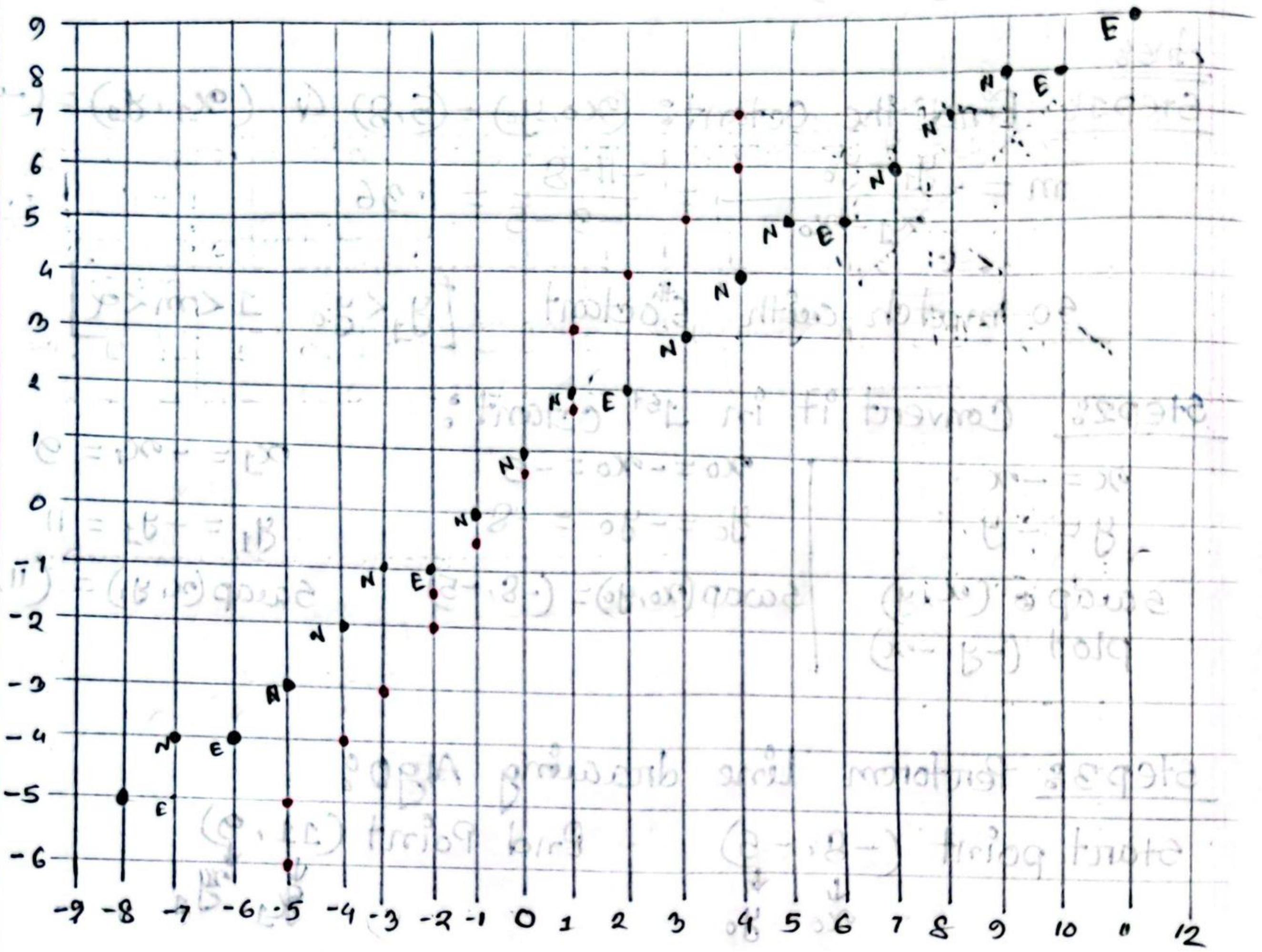
$d = 2dy - dx = 2 \times 14 - 19 = 9$

$$\boxed{d_{new} = d_{old} + \underset{d<=0}{\Delta E} / \underset{d>E}{\Delta NE}}$$

Step 4: Draw the grid:

in x-axis (-8 to 11) y-axis (-5 to 9)

start (-8, -5) End (11, 9)



Red point গুলো plot (-y, -x) র জন্য But ওটার জন্য new graph এঁকে plot করতে হবে।

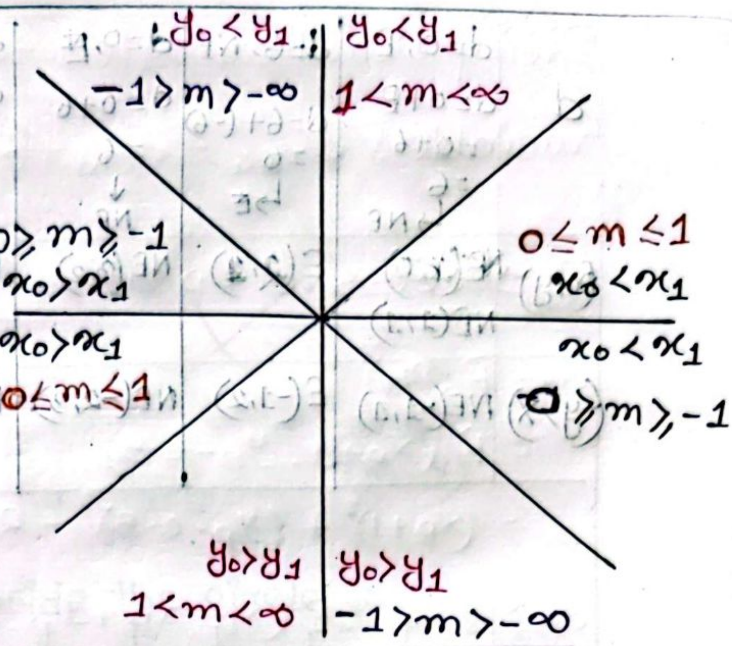
		d>0, NE	d≤0, ΔE	d>0, ΔNE	d>0 N	d>0, N	d<=0, E	d>0, NE	d>0, NE
	d	9	d=9+ΔNE =9-10=-1	d=-1+ΔE =-1+28=27	d=27+ΔN =24-10=17	d=17+NE =17-10=7	d=7+NE =7-10=-3	d=-3+ΔE =-3+28=25	d=25+ΔNE =25-10=15
Plot	(-y,-x)	NE(4,7)	E(4,6)	NE(3,5)	NE(2,4)	NE(1,3)	E(2,2)	NE(0,1)	NE(-1,0)
plot	(x,y)	(-7,-4)	E(-6,-4)	NE(-5,-3)	NE(-4,-2)	NE(-3,-1)	E(-2,-1)	NE(-1,0)	NE(0,1)

plot
↓
graph থেকে উল্টা আগে পাবো then (-y,-x) করব।

	d>0, NF	d<=0; E	d>0, Nf							
d	d=15-10 =5	d=5-10 =-5	d=-5+28 =23	d=23-10 =13	d=13-10 =3	d=3-10 =-7	d=-7+28 =21	d=21-10 =11	d=11-10 =1	d=1-10 =-9
(-y,-x)	NE(-2,-1)	E(-2,-2)	NE(-3,-3)	NE(-4,-4)	NE(-5,-5)	E(-5,-6)	NE(-6,-7)	(-9,-8)	-8,-9	-8,-10
(x,y)	NE(1,2)	E(2,2)	NF(3,3)	NF(4,4)	NF(5,5)	E(6,5)	NE(7,6)	E(8,9)	NE= 9,8	E=10,8

End

	d=-9+28 =19
	-9,-11
	N(11,9)



Dichipher 2(e)

$A(0,0)$ $B(-3,6)$

(x_0, y_0 under A; x_1, y_1 under B)

Step 1: find $m = \dfrac{y_1 - y_0}{x_1 - x_0} = \dfrac{6-0}{-3-0} = \dfrac{6}{-3} = -2$

Here $-1 > \underset{m}{-2} > -\infty$ $\quad y_0 = 0 < y_1 = 6$

So 3rd octant.

Step 2:

3rd octant	$(x_0, y_0) = (0,0)$	$(x_1, y_1) = (-3, 6)$
$x = -x$	$x_0 = -x_0 = 0$	$x_1 = -x_1 = 3$
swap(x, y)	swap(x_0, y_0)	swap(x_1, y_1)
plot $(-y, x)$	$x_0, y_0 = (0,0)$	$x_1, y_1 = (6, 3)$

Step 3: $(x_0, y_0) = (0,0)$ $\quad (x_1 y_1) = (6,3)$

$dy = y_1 - y_0 = 3 - 0 = 3$; $\quad dx = x_1 - x_0 = 6 - 0 = 6$.

$\boxed{dy = 3}$ $\boxed{dx = 6}$

$d_{init} = 2dy - dx = 6 - 6 = 0$

$\Delta E = 2dy = 6$

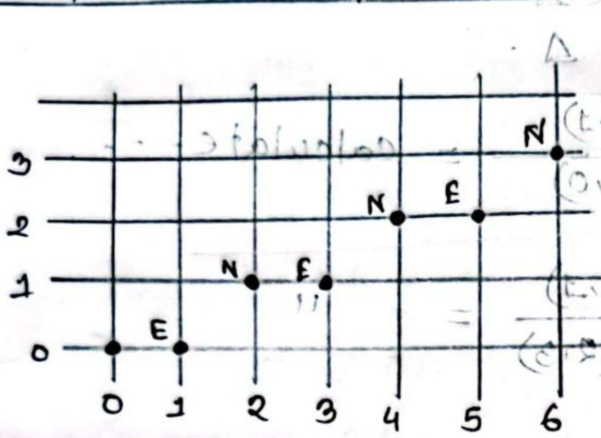
$\Delta NE = 2(dy - dx) = 2(3 - 6) = -6$

$d \leq 0$ ΔE	$d < 0 \rightarrow \Delta E$
$d > 0 \rightarrow \Delta NE$	$d \geq 0 \rightarrow SE$

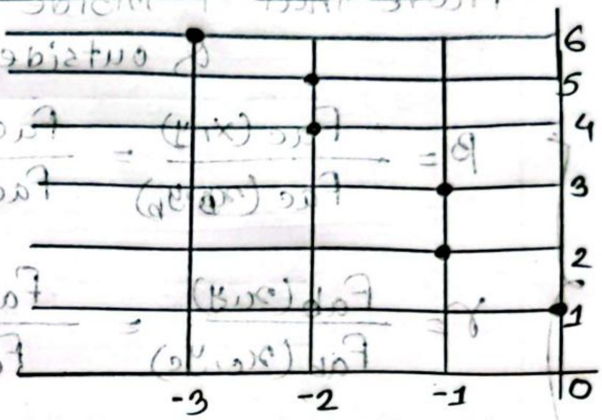
End (6,3)

$d=0 \quad \Delta E = 6 \quad \Delta NE = -6 \quad E = x+1, y \quad NE = x+1, y+1$

d	$d=0, E$	$d=0+\Delta E$ $=0+6$ $=6$ ↓	$d=6+\Delta NE$ $=6-6$ $=0$ ↓	$d=0+\Delta E$ $=0+6$ $=6$ ↓	$d=6+\Delta NE$ $=6-6$ $=0$ ↓	$d=0+\Delta E$ $=0+6$ $=6$
x, y	E(1,0)	NE(2,1)	E(3,1)	NE(4,2)	E(5,2)	NE(6,3)
(-y, x)	E(0,1)	NE(-1,2)	E(-1,3)	NE(-2,4)	E(-2,5)	NE(-3,6)



For table

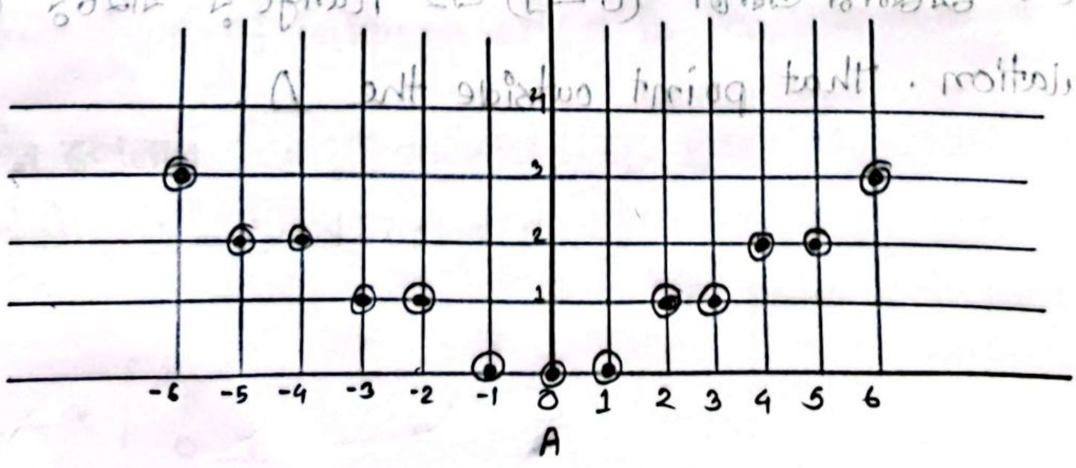


(-y,x) → plot

To draw $AB' = Ref\text{-}Y \times AB$

$$= \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 1 & 1 & 2 & 2 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -1 & -2 & -3 & -4 & -5 & -6 \\ 0 & 0 & 1 & 1 & 2 & 2 & 3 \end{vmatrix}$$

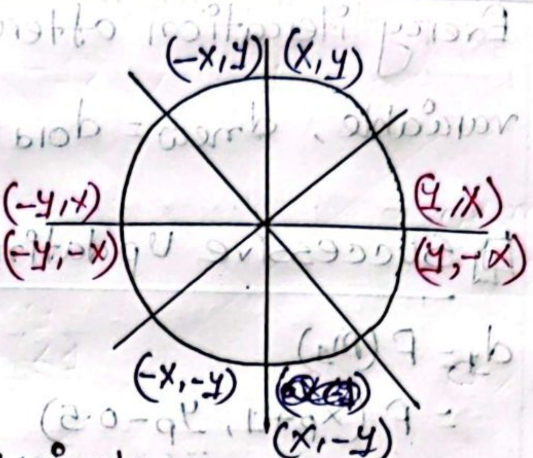


6-Part B

Equation of Circle:

$x^2 + y^2 = R^2$.

$F(x, y) = x^2 + y^2 - R^2 = 0$.



Conditions:

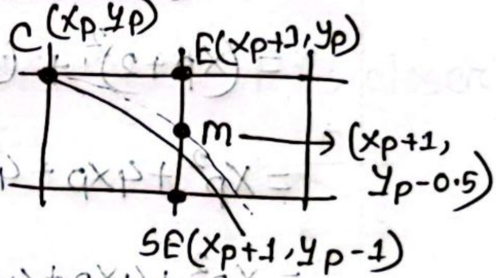
① If $F(x, y) = 0$, point (x, y) on the circle.

② If $F(x, y) > 0$, point (x, y) outside the circle

③ " $F(x, y) < 0$, point (x, y) inside " "

Mid Point Criteria

Successive Updating (For selecting E)



$d_1 = F(M_1)$

$= F(x_p+1, y_p-0.5)$

$= (x_p+1)^2 + (y_p-0.5)^2 - R^2$

$d_1 < 0 \rightarrow E \; (x_p = x_p+1, y_p)$

$d_2 = F(M_2)$

$= F(x_p+2, y_p-0.5)$

$= (x_p+2)^2 + (y_p-0.5)^2 - R^2$

$\cancel{= x_p^2 + 4x_p + 4 + y_p^2 - y_p + 0.25 - R^2}$

$= x_p^2 + 4x_p + 4 + (y_p-0.5)^2 - R^2$

$= [x_p^2 + 2x_p + 1 + (y_p-0.5)^2 - R^2] + 2x_p + 3$

$= [(x_p+1)^2 + (y_p-0.5)^2 - R^2] + 2x_p + 3$.

$= d_1 + (2x_p + 3)$

Every iteration after selecting E, update decision variable, $d_{new} = d_{old} + (2x_p + 3)$

Successive Updating (For selecting SE)

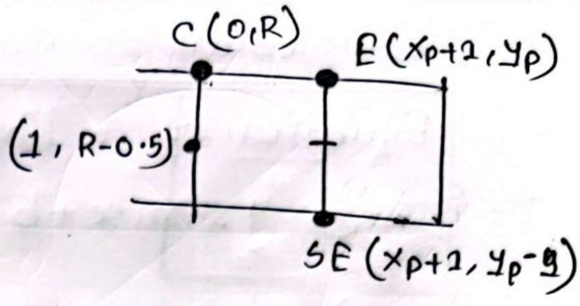
$d_1 = F(M_1)$
$= F(x_p+1, y_p - 0.5)$
$= (x_p+1)^2 + (y_p - 0.5)^2 - R^2$

$d_1 > 0 \rightarrow SE(x_p = x_p + 1, y_p = y_p - 1)$

$d_2 = F(M_2)$
$= F(x_p+2, y_p - 1.5)$
$= (x_p+2)^2 + (y_p - 1.5)^2 - R^2$
$= x_p^2 + 4x_p + 4 + y_p^2 - 2 \cdot y_p (1.5) + 2.25 - R^2$
$= x_p^2 + 4x_p + 4 + y_p^2 - 3y_p + 2.25 - R^2$ (2.25 - 0.25)
$= (x_p^2 + 2x_p + 1) + (y^2 - 2 \cdot y_p \cdot \frac{1}{2} + (0.25)) - R^2 + 2x_p - 2y_p + 3 + 2$
(4-1)
$= [(x_p+1)^2 + (y - 0.5)^2 - R^2] + 2x_p - 2y_p + 3 + 2$
$= d_1 + 2x_p - 2y_p + 5$

Every iteration after selecting SE, update decision variable, $d_{new} = d_{old} + 2(x_p - y_p) + 5$

D) Initialization:



$d_{init} = F(M_1)$

$= F(1, R-0.5)$

$= 1^2 + (R-0.5)^2 - R^2.$

$= 1^2 + R^2 - 2 \cdot R \frac{1}{2} + (\frac{1}{2})^2 - R^2.$

$= 1 - R + 0.25$

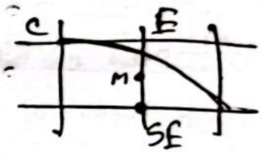
$= 1.25 - R$

$\approx 1 - R$

$\boxed{d_{init} = 1 - R}$

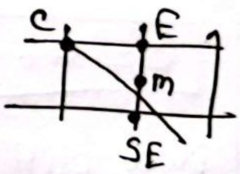
Mid Point Criteria : Successive Update:

① IF $d<0$ → mid point m is inside the circle, E is closer to the circumference. So E is selected.



$d = d + \Delta E \qquad \Delta E = 2X_p + 3.$

② IF $d \geq 0$ → mid point m is outside the circle, SE is closer to the circumference, so SE is selected.



$d = d + \Delta SE \quad \Delta SE = 2X_p - 2Y_p + 5$

$$\boxed{\begin{array}{ll} d_{init} = 1 - R & \\ d = d + \Delta E & d < 0 \\ d = d + \Delta SE & d \geq 0 \end{array}}$$

$\Delta E = 2X + 3$

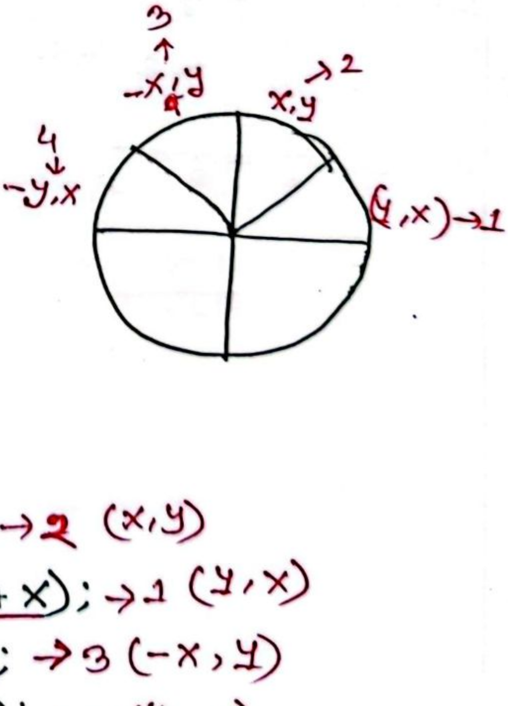
$\Delta SE = 2X - 2Y + 5$

Lecture-6

① ⊛ Write down the algorithm to create a half circle given the radius and center using Bresenham's circle drawing algorithm.

Ans:

```
Void Midpoint Half Circle (int radius)
{ int x=0;
  int y = radius;
  int d =1- radius;
  Half Circle Points (x,y);  <- function call.
  while (y>x)
  { if (d<0)
     { d= d + 2*x+3;} // move East
     else {
        d= d+ 2*(x-y) +5;   // move South East
        y = y-1;
     }
   x= x+1
   Half Circle Points (x,y);
  }
}
```




```
void Half Circle Points (int x, int y){
  Plot Point (centerX+x, centerY+y); -> 2 (x,y)
  Plot Point (centerX+y , centerY +x); -> 1 (y,x)
  Plot Point(centerX-x, centerY+y); -> 3 (-x,y)
  Plot Point (centerX-y, centerY+x); -> 4 (-y,x)
}
```

② Consider a line with a start and end point of (0,0) & (-1,-2) respectively. Apply the necessary transformation to increase the size of line by 100% and find the final vertices after the transformation.

Also determine the coordinates of each pixel along the transformed line segment using the midpoint line drawing algo.

Table given ....

<u>Ans:</u> To increase the size of the line by 100% means length of line should be double. So we need to apply scaling transformation & scaling factor = 2. (double)

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & -4 \\ 1 & 1 \end{bmatrix}$$

So after transformation new start = $\underset{x_0\ y_0}{(0,0)}$ end $\underset{x_1\ y_1}{(-2,-4)}$.

Now, $m = \dfrac{y_1 - y_0}{x_1 - x_0} = \dfrac{-4-0}{-2-0} = \dfrac{-4}{-2} = 2$

<u>6th octant:</u> $y_1 < y_0$; $1 < m < \infty \rightarrow \underset{y_1}{-4} < \underset{y_0}{0}$; $1 < 2 < \infty$

(rest of the math is same as line drawing)

***

m-এর range

diagonally same

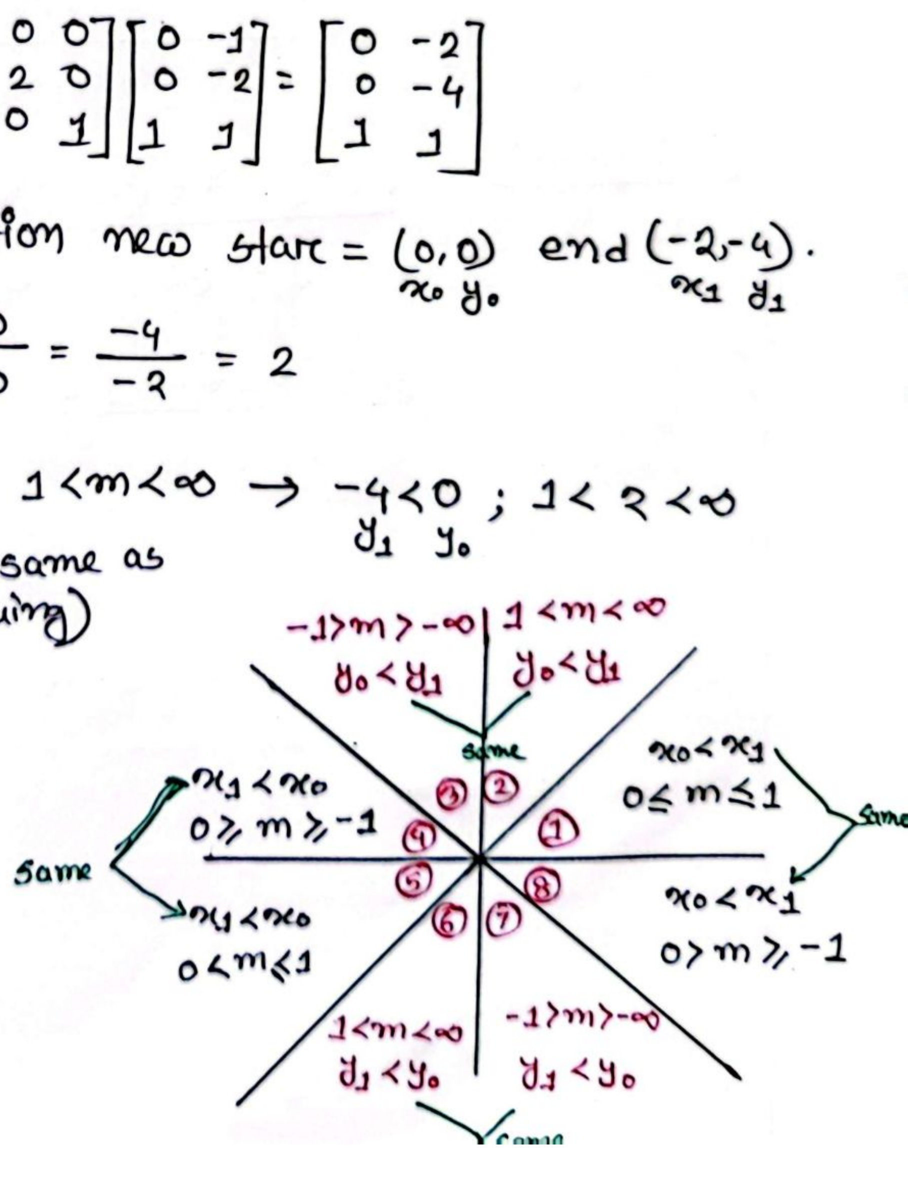
যেমন।

1, 5 → m → same

2, 6 → m → "

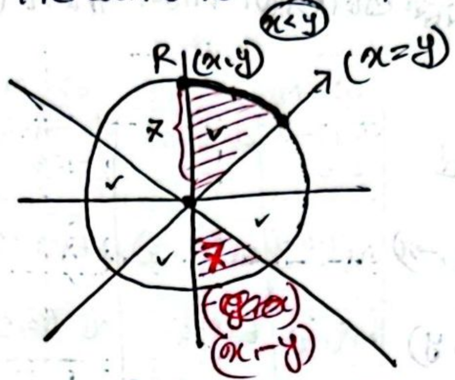
3, 7 → m → "

4, 8 → m → "



Circle drawing
Practice Problem

Perform midpoint algorithm to draw circle's portion at 7th octant which has center at (2, -3) and a radius of 7 pixels. Show each iterations and plot the points.



Ans:

Given, Radius, R = 7

$(x, y) = (0, 7)$

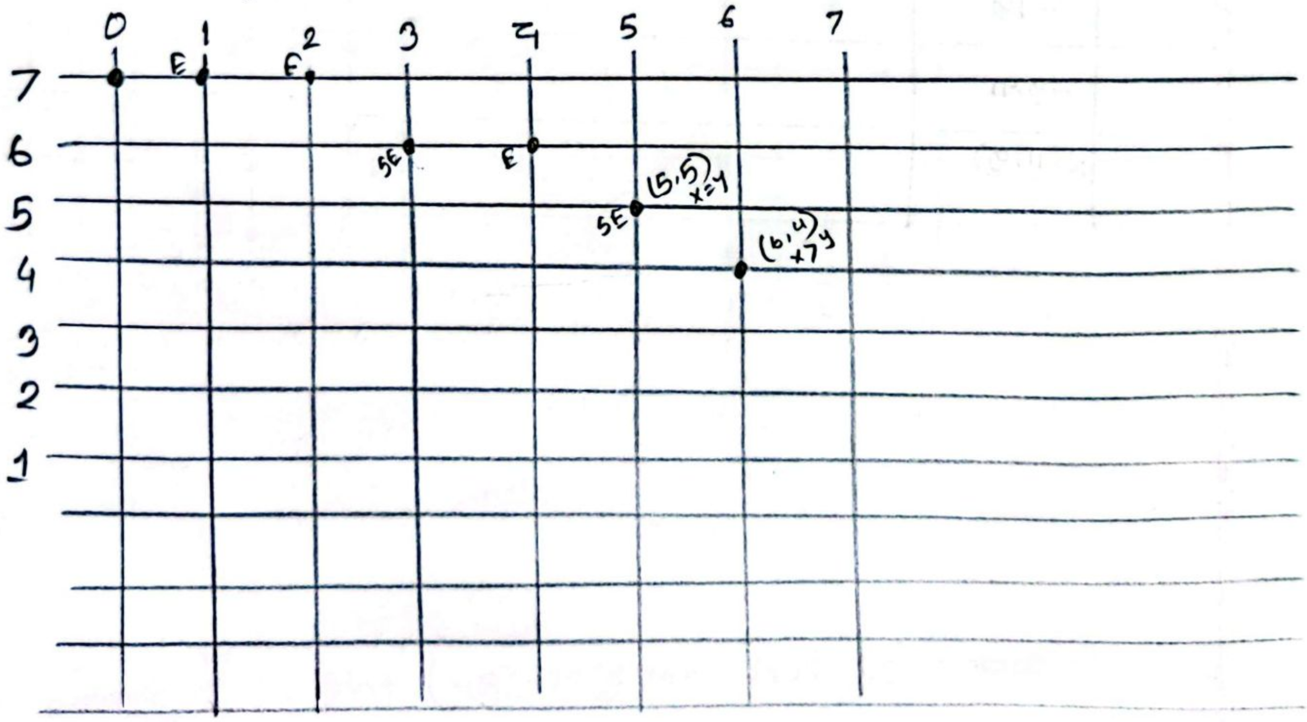
$d_{init} = h = 1 - R = 1 - 7 = -6$

$\Delta E = 2x + 3$

$\Delta SE = 2x - 2y + 5$

$= 2(x - y) + 5$

$$\boxed{d_{new} = h_{new} = h_{old} + \Delta E / \Delta SE \quad \Delta E: d<0 \quad \Delta SE: d>=0}$$



(x, y) = 0,7

K	1 (0,7)	2 (1,7)	3 (2,7)	4 (3,6)	5 (4,6)	6 5,5
2x	2×0 =0	2×1 = 2	4	6	8	10
2y	2×7 =14	2×7 = 14	14	12	12	10
h (then h update করো)	h = h + ΔE = −6 + 2×0 + 3 = −3	h + ΔE = −3 + 2×1 + 3 = 2	h + ΔSE = 2 + 4 − 14 + 5 = −3	h + ΔE = −3 + 6 + 3 = 6	h + SE = 6 + 2x − 2y + 5 = 6 + 8 − 12 + 5 = 7	7 + 10 − 10 + 5 = 12
(x, y) (আগে করো)	h = −6 (init) E(1,7)	E = (2,7)	SE = (3,6)	E(4,6)	SE(5,5) (x = y) End	SE(6,4) (x > y)
(x, −y)	E(1,−7)	E(2,−7)	SE = (3,−6)	E(4,−6)	SE(5,−5)	
(x+2, −y−3) new center	E(3,−10)	E(4,−10)	SE(5,−9)	E(6,−9)	SE(7,−8)	

$p = (-1)^n \times n = (-1)^{38} \times 38$

$n = 38$ $\quad P = 38$

$(-2, 38) \rightarrow$ center radius $7 \rightarrow$ for circle drawing.

$(-1, P)$ to $(4, p-6)$
$= (-1, 38)$ to $(4, 32)$ } line drawing.

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{32-38}{4+1} = \frac{-6}{5} = -1.2$$

7th octant

$-1 > m > -\alpha$

$32 = y_1 < y_0 = 38$

$y = -y$	$y_0 = -y_0 = -38$	$y_1 = -32$
swap(x, y)	swap(x_0, y_0)	swap(x_1, y_1)
plot$(y, -x)$	$(x_0, y_0) = (-38, -1)$	$(x_1, y_1) = (-32, 4)$

$dy = 4 + 1 = 5$

$dx = -32 + 38 = 6$

$d = 2dy - dx = 2 \times 5 - 6 = 10 - 6 = 4$

$\Delta E = 2dy = 2 \times 5 = 10$

$\Delta NE = 2(dy - dx) = -2$

$d_{new} = d_{old} + \Delta E / \Delta NE$

$\Delta E \downarrow d <= 0$ $\quad \Delta NE \downarrow d > 0$

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$n = 38$ $\quad P = (-1)^n \times n = (-1)^{38} \times 38 = 38$

center at $(-2, 38) \rightarrow (-2, P)$

radius, $R = 7$.

initial decision variable, $d_{init} = h = 1 - R = 1 - 7 = -6$

$\Delta E = 2x + 3$ $\quad \Delta SE = 2x - 2y + 5 = 2(x - y) + 5$

$h_{new} = h_{old} + \underset{\substack{\downarrow \\ d<0}}{\Delta E} / \underset{\substack{\downarrow \\ d>=0}}{\Delta SE}$

k	1 (0,7)	2 (1,7)	3 (2,7)	4 (3,6)	5 (4,6)	
2x	0	2	4	6	8	
2y	14	14	14	12	12	
h	h=h+ΔE =-6+2x+3 =-6+0+3 =-3	h=h+ΔE =-3+2+3 =2	h=h+ΔSE =2+2x-2y+5 =2+4-14+5 =-3	h=h+ΔE =-3+6+3 =6	h=h+ΔSE =6+8-12+5 =7	
Oct-2 (x, y)	h=-6<0 E=(x+1, y) E(1,7)	h=-3<0 E(x+1, y) E(2,7)	h=2>0 SE(x+1, y-1) SE(3,6)	h=-3<0 E(x+1, y) E(4,6)	h=6>0 SE(x+1, y-1) SE(5,5)	End (5,5) = (x, y)
(x, y)	E(-1, 45)	E(0, 45)	SE(1, 44)	E(2, 44)	SE(3, 43)	
Oct-4 (-y, x)	E(-7,1) E(-9,39)	E(-7,2) E(-9,40)	SE(-6,3) SE(-8,41)	E(-6,4) E(-8,42)	SE(-5,5) SE(-7,43)	
Oct-6 (-x, -y)	E(-1,-7) E(-3,31)	E(-2,-7) E(-4,31)	SE(-3,-6) SE(-5,32)	E(-4,-6) E(-6,32)	SE(-5,-5) SE(-7,33)	
Oct-8 (y, -x)	E(7,-1) E(5,37)	E(7,-2) E(5,36)	SE(6,-3) SE(4,35)	E(6,-4) E(4,34)	SE(5,-5) SE(3,33)	

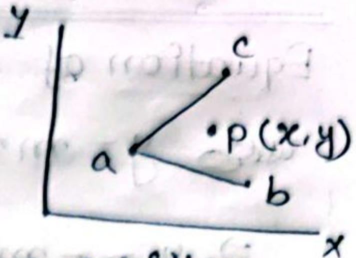
with center (-2,38)

Part-C

Barycentric Coordinate:

For cartesian coordinate,

$P(x,y) = O + xX + yY$



For Barycentric Coordinate eqn can be written,

$P(\beta,\gamma) = a + \beta(b-a) + \gamma(c-a)$

$= a + \beta b - \beta a + \gamma c - \gamma a$

$= a(1-\beta-\gamma) + \beta b + \gamma c$

$= a\alpha + \beta b + \gamma c$

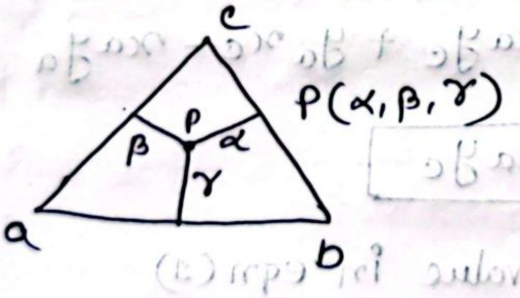
$\left.\begin{matrix} b-a \\ c-a \end{matrix}\right]$ basis.

triangle-এ respect-এ coordinate system কাজ করে।

$\alpha+\beta+\gamma = 1$

if point inside the triangle

***In a Barycentric coordinate system, Location of a point is specified by reference to a triangle for points in a plane.



For any point inside the triangle must satisfy the following criteria,

$\alpha+\beta+\gamma = 1$ $\quad 0<\alpha<1$

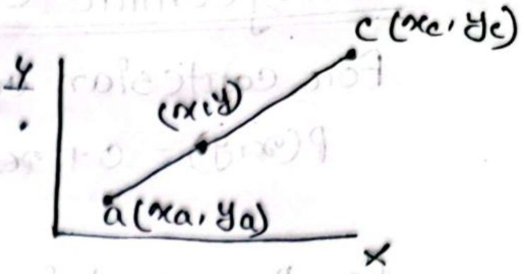
$0<\beta<1$

$0<\gamma<1$

Our goal is to convert cartesian coordinate to Barycentric coordinate for triangle rasterization.

$P(x,y) \rightarrow P(\alpha,\beta,\gamma)$

Cartesian to Barycentric



Equation of a line ac,

$y = mx + b$

$\Rightarrow y - mx - b = 0.$

$\Rightarrow y - \dfrac{y_c - y_a}{x_c - x_a} x - b = 0$

$\Rightarrow y(x_c - x_a) - x(y_c - y_a) - b(x_c - x_a) = 0.$

Let
$b(x_c - x_a) = c$
$y_c - y_a = -(y_a - y_c)$

$\Rightarrow (y_a - y_c)x + (x_c - x_a)y - c = 0$ —(1)

Now calculate the value of c,

$c = (y_a - y_c)x + (x_c - x_a)y \rightarrow$ Point (x, y)

$= (y_a - y_c)x_a + (x_c - x_a)y_a \rightarrow$ Point (x_a, y_a).

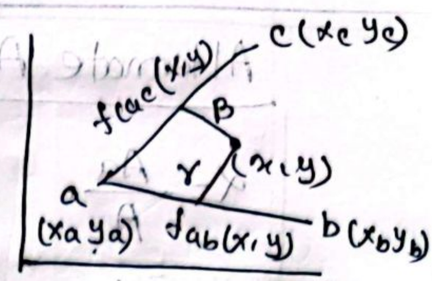
$= x_a y_a - x_a y_c + y_a x_c - x_a y_a$

$$\boxed{c = x_c y_a - x_a y_c}$$

Now putting c value in eqn (1)

$f_{ac} = (y_a - y_c)x + (x_c - x_a)y - x_c y_a + x_a y_c = 0.$

$$\beta = \frac{f_{ac}(x,y)}{f_{ac}(x_b,y_b)}$$



$$= \frac{(y_a - y_c)x + (x_c - x_a)y - x_a y_c + x_c y_a}{(y_a - y_c)x_b + (x_c - x_a)y_b - x_a y_c + x_c y_a}$$

$$\gamma = \frac{f_{ab}(x,y)}{f_{ab}(x_c,y_c)}$$

$$= \frac{(y_a - y_b)x + (x_b - x_a)y - x_a y_b + x_b y_a}{(y_a - y_b)x_c + (x_b - x_a)y_c - x_a y_b + x_b y_a}$$

$\alpha = 1 - \beta - \gamma$.

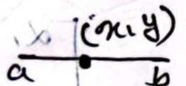
Q: ~~~~In Which case β become 1?

$\beta = \frac{\text{same}}{\text{same}} = 1$ b point এ then $\beta = 1$.

$(x,y) = (x_b, y_b)$

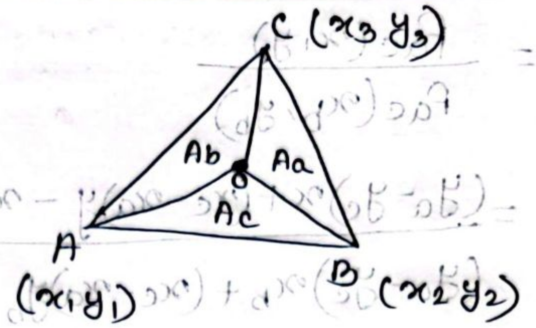
Q: What will happen when (x,y) lies on $f_{ab}(x,y)$.

$ab = 0$.



যখন point (x,y) line এর উপরে থাকবে তখন equation $= 0$.

Alternate Approach



$\alpha = \frac{A_a}{A}$

$\beta = \frac{A_b}{A}$

$\gamma = \frac{A_c}{A}$

$$\text{area } A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \rightarrow \text{Area of } \Delta ABC.$$

A_a = Area of Δ OCB

A_b = Area of Δ OAC

A_c = Area of Δ OAB

Interpolation

If the vertices have color c_0, c_1 & c_2, the color at a point in the triangle with Barycentric coordinate (α, β, γ) is

$$C = c_0\alpha + c_1\beta + c_2\gamma \quad [\alpha, \beta, \gamma \rightarrow \text{scalar value}]$$

This type of interpolation is called Gouraud Interpolat.

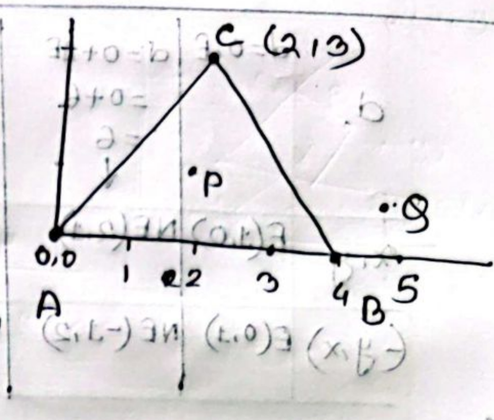
তিন কোনায় তিন রঙ mixed হয়ে পুরো triangle color হবে এটাই। এ mixed color ই C
→ এই point-এ interpolation হচ্ছে।

$$C = \alpha\begin{bmatrix} Red \\ 0 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ Green \\ 0 \end{bmatrix} + \gamma\begin{bmatrix} 0 \\ 0 \\ Blue \end{bmatrix}$$

$$= \begin{bmatrix} Red\,\alpha \\ Green\,\beta \\ Blue\,\gamma \end{bmatrix}$$

Practice Problem:

Let's $A(0,0)$ $B(4,0)$ $C(2,3)$



Let $P(2,1)$

$Q(5,1)$

Prove that P inside the Δ

Q outside " Δ.

P

$$\beta = \frac{F_{ac}(x,y)}{F_{ac}(x_b,y_b)} = \frac{F_{ac}(2,1)}{F_{ac}(4,0)} = \text{calculate}\ldots$$

$$\gamma = \frac{F_{ab}(x,y)}{F_{ab}(x_c,y_c)} = \frac{F_{ab}(2,1)}{F_{ab}(2,3)} =$$

$$\alpha = 1-\beta-\gamma$$

Q:

$$\beta = \frac{F_{ac}(5,1)}{F_{ac}(4,0)} \quad \gamma = \frac{F_{ab}(5,1)}{F_{ab}(2,3)} \quad \alpha = 1-\beta-\gamma.$$

it will not match $\boxed{\alpha+\beta+\gamma = 1}$

β or γ যেকোন একটা $(0-1)$ এই range র বাইরে থাকলে stop calculation. that point outside the Δ.

③ Consider a triangle with vertices A(1,1) B(5,1) & C(3,3) and color values of red (1,0,0) green (0,0.9,0) blue (0,0,0.8) at each vertex of the triangle. Find the color of the point P(3,2) inside the triangle using the concept of barycentric interpolation.

Ans: Given, $A(\overset{x_A}{1},\overset{y_A}{1})$ $B(\overset{x_B}{5},\overset{y_B}{1})$ $C(\overset{x_C}{3},\overset{y_C}{3})$ $P(\overset{x}{3},\overset{y}{2})$

$$\beta = \frac{f_{AC}(x,y)}{f_{AC}(x_B,y_B)} = \frac{(y_A-y_C)x+(x_C-x_A)y+x_Ay_C-x_Cy_A}{(y_A-y_C)x_B+(x_C-x_A)y_B+x_Ay_C-x_Cy_A}$$

$$= \frac{(1-3)3+(3-1)2+(3\times1)-(3\times1)}{(1-3)5+(3-1)1+(1\times3)-(3\times1)}$$

$$= \frac{(-2)\times3+(2\times2)+3-3}{(-2)\times5+(2\times1)+3-3} = 0.25$$

$$\gamma = \frac{f_{AB}(x,y)}{f_{AB}(x_C,y_C)} = \frac{(y_A-y_B)x+(x_B-x_A)y+x_Ay_B-x_By_A}{(y_A-y_B)x_C+(x_B-x_A)y_C+x_Ay_B-x_By_A}$$

$$= \frac{(1-1)\times3+(5-1)2+(1\times1)-(5\times1)}{(1-1)3+(5-1)3+(1\times1)-5\times1}$$

$$= 0.5$$

$\alpha = 1-\beta-\gamma = 1-0.25-0.5 = 0.25$

Hence for point P(3,2) → interpolation → α, β, γ

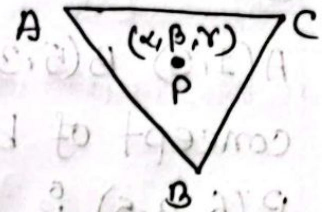
$$P(\alpha,\beta,\gamma) \rightarrow (0.25, 0.25, 0.5)$$

Given

Color Red, $P_0 (1,0,0)$

Green, $P_1 (0, 0.9, 0)$

Blue, $P_2 (0, 0, 0.8)$



we know,

$$P = \alpha P_0 + \beta P_1 + \gamma P_2 = \alpha \begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ G \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}$$

$$\Rightarrow P = 0.25 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0.25 \begin{bmatrix} 0 \\ 0.9 \\ 0 \end{bmatrix} + 0.5 \begin{bmatrix} 0 \\ 0 \\ 0.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.225 \\ 0 \end{bmatrix}}_{(0.25 \times 0.9)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.4 \end{bmatrix}}_{(0.5 \times 0.8)} = \begin{bmatrix} 0.25 \\ 0.225 \\ 0.4 \end{bmatrix} = \begin{bmatrix} Red \\ Green \\ Blue \end{bmatrix}$$

Ans:

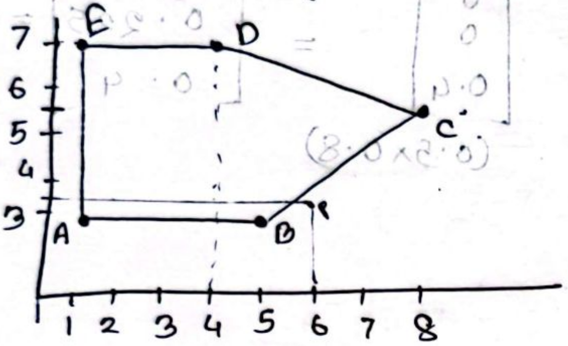
(4) Consider a ~~percentage~~ pentagon ABCDE with vertices A(1,3) B(5,3) ~~C(8,5.5)~~ C(8, 5.5) D(4,7) E(1,7). Using the concept of barycentric coordinates, determine if a point P(6, 3.5) is inside the pentagon or not. Describe your approach and show your calculation.

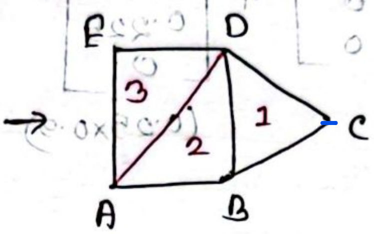
Ans:

(1) **Points:** $A(\overset{x_A}{1}, \overset{y_A}{3})$ $B(5,3)$ $C(8, 5.5)$ $D(4,7)$ $E(\overset{x_E}{1}, \overset{y_E}{7})$

(2) $P(\underset{x}{6}, \underset{y}{3.5})$

(3) Lets plot pentagon to find out exact triangles.





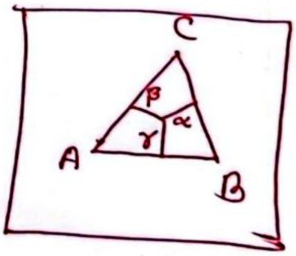
there are 3-triangles. Now for each triangle we have to find whether P inside that specific triangle or not.

(4) Triangle-1 (DBC) $D = \underset{x_D}{4}, \underset{y_D}{7}$ $B = \underset{x_B}{5}, \underset{y_B}{3}$ $C = \underset{x_C}{8}, \underset{y_C}{5.5}$

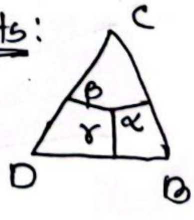
$$\beta = \frac{f_{CD}(x,y)}{f_{CD}(x_B, y_B)}$$

$$= \frac{(y_C - y_D)x + (x_D - x_C)y + x_C y_D - y_C x_D}{(y_C - y_D)x_B + (x_D - x_C)y_B + x_C y_D - y_C x_D}$$

$$= \frac{(5.5 - 7)6 + (4 -}{}$$



Lets:



$$\beta = \frac{F_{DC}(x,y)}{F_{DC}(x_B,y_B)} = \frac{(y_D-y_C)x + (x_C-x_D)y + x_D y_C - x_C y_D}{(y_D-y_C)x_B + (x_C-x_D)y_B + x_D y_C - x_C y_D}$$

$$= \frac{(7-5.5)6 + (8-4)3.5 + (4\times 5.5) - (8\times 7)}{(7-5.5)5 + (8-4)3 + (4\times 5.5) - (8\times 7)}$$

$$= 0.759$$

*** $\boxed{\beta = \frac{F_{CD}(x,y)}{F_{CD}(x_B,y_B)} = (y_C - y_D)x + (x_D - x_C)y \cdots}$

এভাবে করলেও same-ই আসবে ans:

$$\gamma = \frac{F_{DB}(x,y)}{F_{DB}(x_C,y_C)} = \frac{(y_D-y_B)x + (x_B-x_D)y + x_D y_B - x_B y_D}{(y_D-y_B)x_C + (x_B-x_D)y_C + x_D y_B - x_B y_D}$$

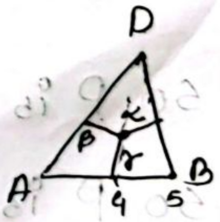
$$= \frac{(7-3)6 + (5-4)3.5 + (4\times 3) - (5\times 7)}{(7-3)8 + (5-4)5.5 + (4\times 3) - (5\times 7)}$$

$$= 0.31$$

$\alpha = 1 - \beta - \gamma = 1 - 0.759 - 0.31 = -0.069$

So P is not in ΔDBC. $\left[\begin{array}{l} 0<\alpha<1 \\ 0<\beta<1 \\ 0<\gamma<1 \end{array}\right\}$ only then P will be inside the Δ

Again check for Triangle-2 ΔABD



$$\beta = \frac{F_{AD}(x,y)}{F_{AD}(x_B,y_B)}$$

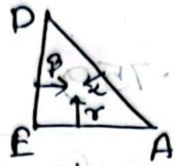
$$= \frac{(y_A-y_D)x + (x_D-x_A)y + x_A y_D - x_D y_A}{(y_A-y_D)x_B + (x_D-x_A)y_B + x_A y_D - x_D y_A}$$

$$= \frac{(3-7)6 + (4-1)3.5 + (1\times 7) - (4\times 3)}{(3-7)5 + (4-1)3 + (1\times 7) - (4\times 3)} = 1.15625 > 1$$

no need to do the further calculation. Because, β is not in range of $0 < \beta < 1$ so $\alpha+\beta+\gamma=1$ condition is also not satisfied.

So P is not inside ΔADB.

For triangle-3 ΔEAD



$$\beta = \frac{f_{DE}(x,y)}{f_{DE}(x_A, y_A)} = \frac{(y_D - y_E)x + (x_E - x_D)y + x_D y_E - x_E y_D}{(y_D - y_E)x_A + (x_E - x_D)y_A + x_D y_E - x_E y_D}$$

$$= \frac{(7-7)6 + (1-4)3.5 + (4\times 7) - (1\times 7)}{(7-7)1 + (1-4)3 + (4-7) - (1\times 7)}$$

$$= 0.875$$

$$\gamma = \frac{f_{AE}(x,y)}{f_{AE}(x_D, y_D)} = \frac{(y_A - y_E)x + (x_E - x_A)y + x_A y_E - x_E y_A}{(y_A - y_E)x_D + (x_E - x_A)y_D + x_A y_E - x_E y_A}$$

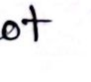
$$= \frac{(3-7)6 + (1-1)3.5 + (1\times 7) - (1\times 3)}{(3-7)4 + (1-1)7 + (1\times 7) - (1\times 3)}$$

$$= 1.67 > 1 \ (X)$$

So P is not even inside of ΔEAD

so P is not inside of pentagon ABCDE.

Ans:

[quadratic  হলে check for two triangles]

Lecture-8

Fractal

→ is a geometric shape generated using set of recursive rules.

→ Share a self-similarity — ~~to~~ resemble part of fractal ~~or~~ to make whole fractal

Rule of Cantor

→ A simplest example of a fractal is a cantor.

→ Starting with a straight line at $n=0$ we remove middle part of third piece of each line at each iteration.

→ As n gets larger, line segments get smaller.

Fractal Dimension

→ is a measure of the complexity of a fractal.

→ N_n = # of new element add ~~in~~ at ~~each~~ n iteration.

ε = Scaling factor of the size of new elements

$$\text{Fractal Dimension, } D = -\lim_{n \to \infty} \frac{\log(N_n)}{\log(\varepsilon_n)}$$

$\log(N_n)$ → n iteration-এ element কয়টা

$\log(\varepsilon_n)$ → n iteration-এ size কত?

for 2D Fractal $D \in [1,2]$

3D " $D \in [2,3]$

Dimension of cantor set

nth iteration → no of line segment $= 2^n = N_n$

nth " length of segment $= 3^{-n} = \varepsilon_n$

So,
$$D = -\lim_{n\to\infty} \frac{\log N_n}{\log \varepsilon_n}$$

$$= -\lim_{n\to\infty} \frac{\log 2^n}{\log 3^{-n}}$$

$$= -\lim_{n\to\infty} \frac{\log 2^n}{\log (3^{-1})^n}$$

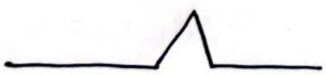
$$= -\lim_{n\to\infty} \frac{\cancel{n} \log 2}{\cancel{n} \log 1/3} = -\lim_{n\to\infty} \frac{\log 2}{\log 1 - \log 3}$$

$$= \lim_{n\to\infty} \frac{\log 2}{\log 3}$$

$$D = 0.6309 \ldots$$

The koch snowflake

→ is a shape with a finite area bounded by infinite circumference.



Stage	no of sides	length of sides
0	3	1
1	3×4	$1/3$
2	$3\times 4\times 4$	$1/3^2$
⋮		
n	3×4^n	$1/3^n$.

Dimension of koch Snowflake

$$D = -\lim_{n\to\infty} \frac{\log 3\times 4^n}{\log 1/3^n} = -\lim_{n\to\infty} \frac{\log 3 + n\log 4}{\log 3^{-n}}.$$

$$= \lim_{n\to\infty} \frac{\log 3 + n\log 4}{n\log 3}$$

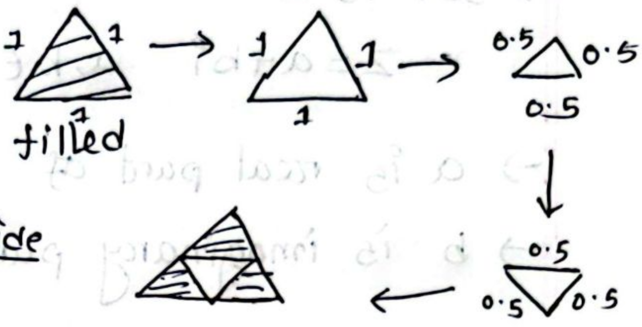
$$= \lim_{n\to\infty} \frac{\log 3}{n\log 3} + \frac{\cancel{n}\log 4}{\cancel{n}\log 3}$$

$$= \lim_{n\to\infty} \frac{1}{n} + \frac{\log 4}{\log 3}.$$

$$= \frac{\log 4}{\log 3} = 1.2619.$$

Sierpinski Triangle

→ For each filled triangle, make an unfilled copy.
→ Scale the copy by half and invert it.
→ Place the copy in the center of the filled triangle.



Stage	no of side	Length of side
0	3	1
1	3×3	$\frac{1}{2}$
2	$3\times3\times3$	$\frac{1}{2^2}$
⋮		
n	3^n	$\frac{1}{2^n}$

Dimension of Sierpinski

$$D = -\lim_{n\to\infty} \frac{\log 3^n}{\log \frac{1}{2^n}} = -\lim_{n\to\infty} \frac{\log 3^n}{\log 2^{-n}} = -\lim_{n\to\infty} \frac{n \log 3}{-n \log 2}$$

$$D = \lim_{n\to\infty} \frac{\log 3}{\log 2}$$

$$D = \frac{\log 3}{\log 2} = 1.585.$$

The Mandelbrot set

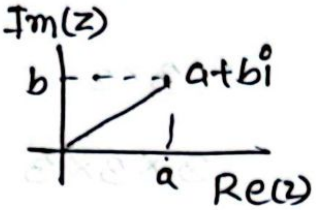
→ can be generated by a simple iterative formula, called quadratic recurrence relation.

→ set is defined using complex number.

$$z = a + bi \quad a, b \in R \quad i^2 = -1 \quad i = \sqrt{-1}.$$

→ a is real part of z → Re(z)

→ b is imaginary part of z → Im(z)



Modulas of a complex number $|a+bi|$ is the distance between the complex number & origin.

$$|a+bi| = \sqrt{a^2+b^2}$$

To Generate Mandelbrot set,

$$z_{n+1} = z_n^2 + \underbrace{c}_{\text{complex number}}$$

A point c is said to have escaped (therefore not member of Mandelbrot set) if z_n is larger than some escape radius (usually 2)

Given, $c = -0.771 - 0.326i \rightarrow$ do 10 steps.

$z_{n+1} = z_n^2 + c$.

$z_0 = 0 + c = (0 - 0.771 - 0.326i)^2 = 0.59 - 2$

$|z_0| = \sqrt{(-0.771)^2 + (-0.326)^2} = 0.837 \approx 0.8$

$z_1 = z_0^2 + c$

$= (-0.771 - 0.326i)^2 + (-0.771 - 0.326i)$

$= 0.59 - 0.5i + 0.1i^2 - 0.771 - 0.326i$

$= 0.6 - 0.5i - 0.1 = 0.5 - 0.5i -$ " "

$z_1 = -0.27 - 0.826i$

$|z_1| = \sqrt{(-0.27)^2 + (-0.826)^2} = 0.8690$

$z_2 = z_1^2 + c$

Lindenmayer-system

Structure

$G = (V, \omega, P)$.

* V (alphabet) → set of symbols containing both elements that can be replaced (variable) and those which cannot be replaced (constants or terminals).

* ω (start, axiom, or initiator) → starting symbol from V defining the initial state of the system.

* P → set of production rules.

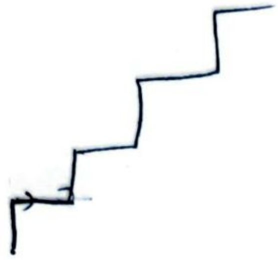
Example:

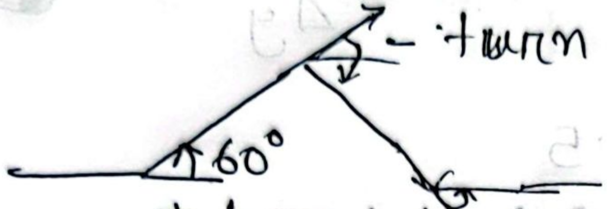
Variable X, F
constant: +, -
axiom: F+XF+F+XF
rules: X → XF-F+F-XF+F+XF-F+F-X
angle 90°

X → do nothing
F → draw a line forward
+ → rotate clockwise 90°
- → " counter " 90°

1. F + XF + F+ XF

2. F+ XF-F+F -XF + F+ XF-F +F-XF+ F+ XF-F+F-XF+F+
XF-F+F- XF

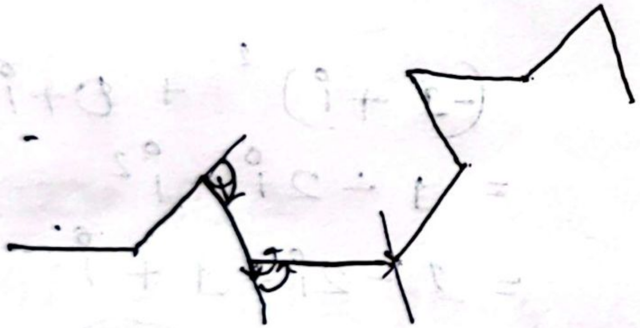




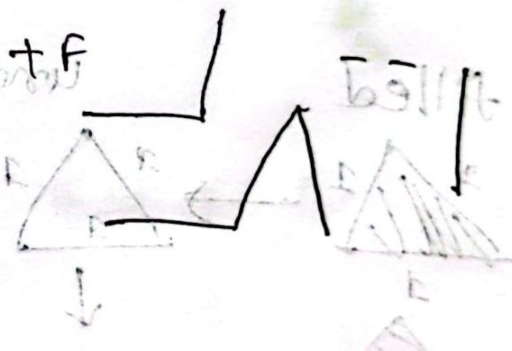
- turn 60° right.

+ turn left 60°.

F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F



F+F



F+F--F+F+F+F--