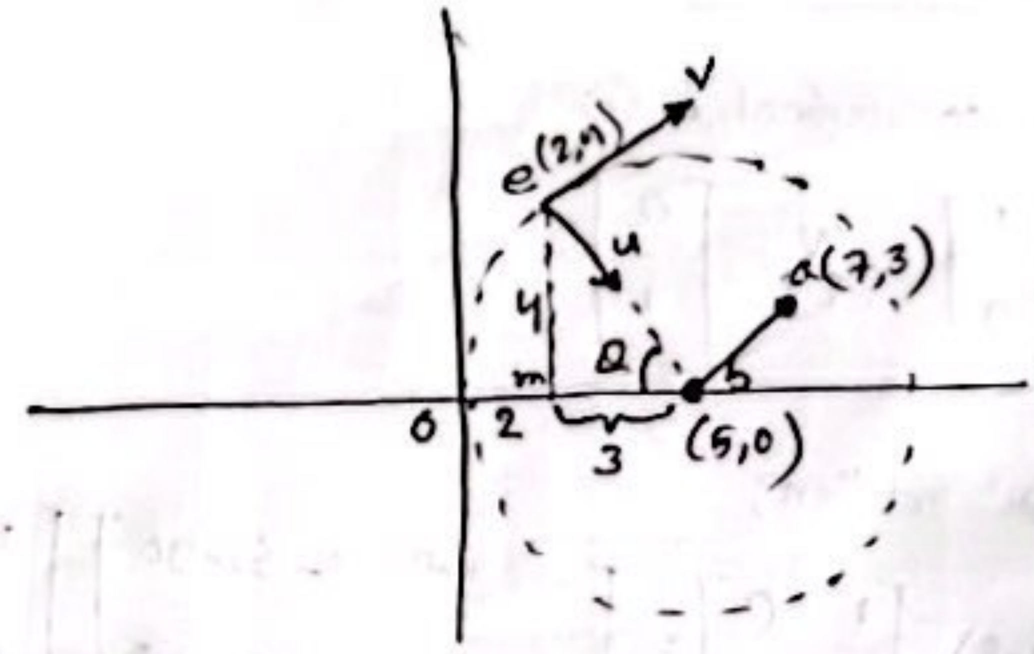
CHAP 5A

Prev year Quiz set-B

ab



ob=5, mb=3, om = 5-3 = 2

ob = eb = 5

For Δemb ;

$eb^2 = em^2 + mb^2$

$5^2 = em^2 + 3^2$

$\therefore em = 4$

$\therefore$ em is (2,4)

$\tan Q = \frac{4}{3}$

$\therefore Q = 53.13$

Basis vectors are ; $U = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

After rotation:

: Rotate (-53.13) * Basis matrix

$$= \begin{bmatrix} \cos(-53.13) & \sin(53.13) & 0 \\ -\sin(53.13) & \cos(53.13) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

:

is our required basis matrix.

Practice Problem-1 (Lecture5-A)

(1)

$$M = M_{vp} * M_{ortho}$$

$$= \begin{bmatrix} n_x/2 & 0 & 0 & \frac{n_x-1}{2} \\ 0 & n_y/2 & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$=$

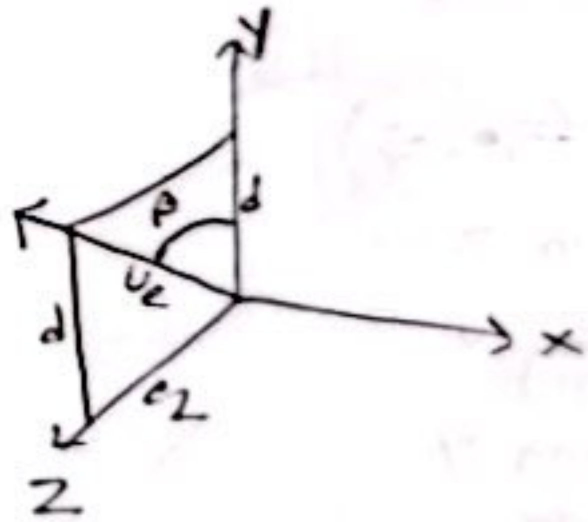
(2)

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = M * \begin{bmatrix} -1 & 2 \\ -3 & 4 \\ -5 & -6 \\ 1 & 1 \end{bmatrix}$$

$$\cos\beta = \frac{d}{v_e}$$

$$= d = \frac{5\sqrt{102}}{51}$$

$$\sin\beta = \frac{e_z}{v_e} = -\frac{1}{\sqrt{51}}$$



$$\text{Rotate\_z}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$AB = \cancel{\begin{bmatrix} 5 & -2 & 3 & 1 \\ 10 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

$$\text{Rotate\_x}(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*** For Question ans would be:

$\cancel{M_1 = T^{-1}(5,-2,3)}$

$M_1 = \text{Rotate\_x}(\beta) * \text{Rotate\_z}(\alpha) * \text{Translate}(5,-2,3) * AB$

$$\text{Rotate\_y}(\theta = -90^\circ) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$; AB = \begin{bmatrix} 5 & 10 \\ -2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$M_1 = \text{Translate}^{-1}(5,-2,3) * \text{Rotate\_z}(-\alpha) * \text{Rotate\_x}(-\beta) * \text{Rotate\_y}(\theta) *$
$\text{Rotate\_x}(\beta) * \text{Rotate\_z}(\alpha) * \text{Translate}(5,-2,3) * P$

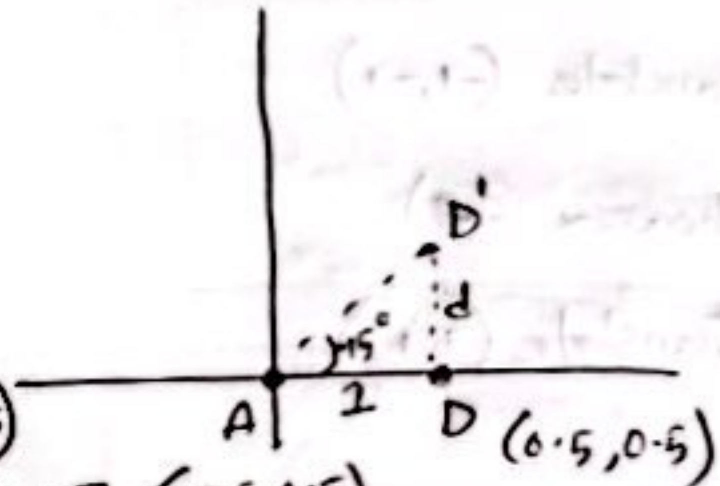
:

3(g)

① Translate (-0.5, 0.5)

② Shear along y-axis by 1

③ Translate (-0.5, -0.5)



$m_1 = T(-0.5, 0.5) * \text{Shear-y}(1) * T(0.5, -0.5)$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate (45°)

$=$

$\theta = 45°$

$\tan\theta = \frac{d}{1}$

$\therefore d = 1$

$\therefore$ to shear by y axis for 1 points

shear factor $\frac{1}{1} = 1$

$$m_1 \times V = m_1 \times \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \\ 1 & 1 \end{bmatrix}$$

$=$

Again;

① Translate (-0.5, 0.5)

② Rotate (-90°)

③ Translate (0.5, -0.5)

$m_2 = T(0.5, -0.5) * \text{Rotate}(-90°) * \text{Translate}(-0.5, 0.5)$

$=$

$$m_2 \times V = m_2 \times \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 & \cdot \\ 0.5 & -0.5 & -0.5 & 0.5 & \cdot \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Integer

3(f)

$c = a+bi$

$z_{n+1} = z_n^2 + c \quad ; c = -0.5+0.5i$

$z_0 = 0 + (-0.5+0.5i)$ $\qquad |z_0| = 0.707 < 2$

$z_1 = (-0.5+0.5i)^2 + (-0.5+0.5i)$

:

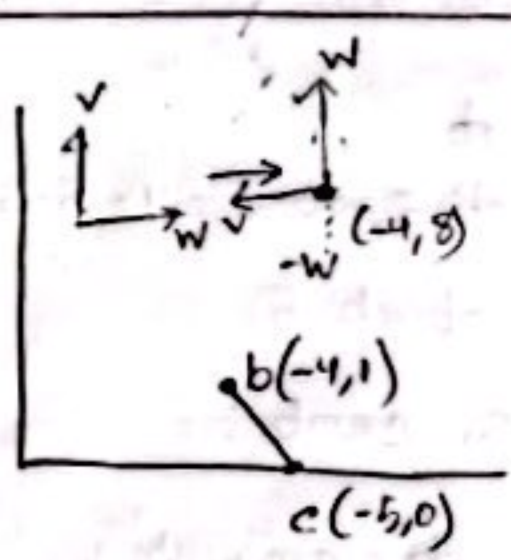
⋮

if z_{10} stays inside 2, then it is a member of mandlebrot set.

Chap 5B

1-5 (B)

Practice Problem- 1



Initially cannonical basis:

$$W = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

After 90° rotation;

$$R(90^\circ)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Canonical to frame matrix:

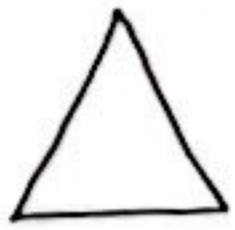
$$\begin{bmatrix} U_p \\ V_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -(-4) \\ 0 & 1 & -(8) \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -4 & -5 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

=

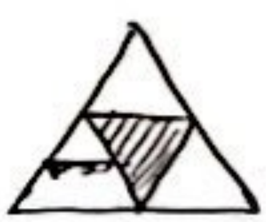
Decipher

2(b)

①  stage 0

②  stage 1

③  stage 2

④ 

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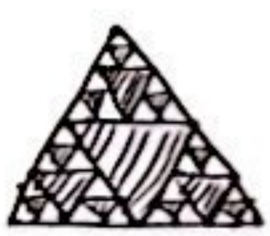

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Stage	no. of ~~sides~~ triangles	Length of ~~sides~~ triangles
0	0	0
1	1	$\frac{1}{2}$
2	4	$\frac{1}{4}$
3	13	$\frac{1}{8}$
4	40	$\frac{1}{16}$
	$(3^{n-1})+(n-1)$	$\frac{1}{2^n}$

Here, $\epsilon_n = \frac{1}{2^n}$; $N_n = (3^{n-1}) + (n-1)$

$$D = -\lim_{n\to\infty} \frac{\log\left((3^{n-1})+(n-1)\right)}{\log\left(\frac{1}{2^n}\right)} = -\lim_{n\to\infty} \frac{\log(3^{n-1})+\log(n-1)}{n\log(1/2)}$$

$$= -\lim_{n\to\infty} \frac{\cancel{\log}(n-1)\log 3 + \log(n-1)}{-n\log(2)}$$

$$= -\lim_{n\to\infty} \frac{n\log 3 - \log 3 + \log(n-1)}{-n\log 2}$$

$$= -\lim_{n\to\infty} -\frac{n\log 3}{n\log 2} = \frac{\log 3}{\log 2} = 1.5849$$

Decipher

3(f)

$e = (0,1)$

~~a = (-2,0)~~

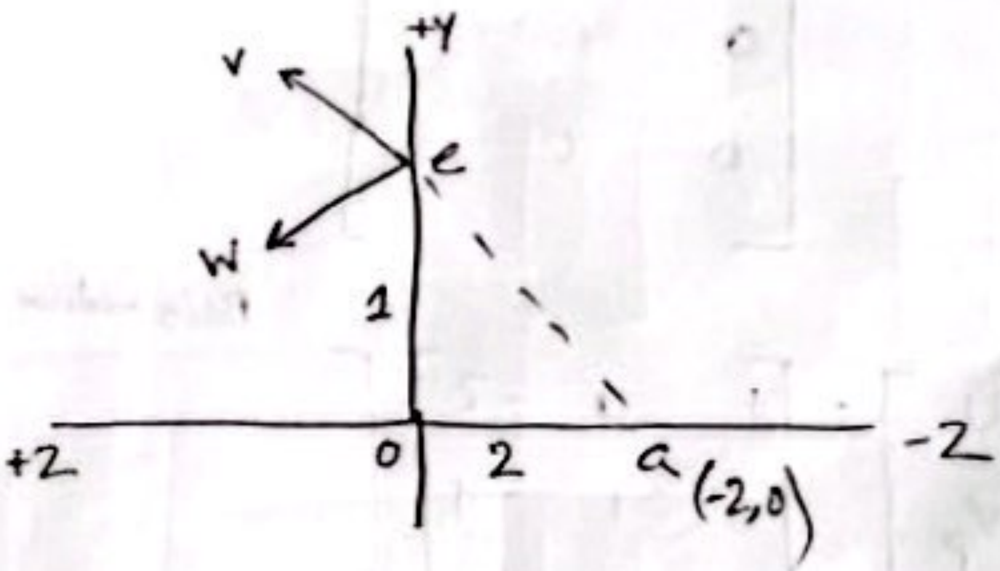
$a = (-2,0)$

$ae = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$\tan\theta = \frac{1}{2}$

$\theta = 26.56°$



Basis matrix $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$=$

$$P_{uv} = \begin{bmatrix} U_p \\ V_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$=$

3(d) Decipher

$c = a + bi$

$z_{n+1} = z_n^2 + c$; converge if $z_n > 2$

complex number : $c = -0.771 - 0.326i$

$z_0 = 0 + (-0.771 - 0.326i)$ $|z_0| = 0.8370$

$= -0.771 - 0.326i$

$z_1 = (-0.771 - 0.326i)^2 + (-0.771 - 0.326i)$

$= (-0.771)^2 + 2 \times 0.771 \times 0.326 + (0.326i)^2 - 0.771 - 0.326i$

$= 0.5944 + 0.5026 - 0.1062 - 0.771 - 0.326i$

$= 0.2198 - 0.326i$ $|z_1| = \sqrt{(0.2198)^2 + (-0.326)^2}$

$= 0.3931$

$z_2 = (0.2198 - 0.326i)^2 + (-0.771 - 0.326i)$

$=$

$\vdots$

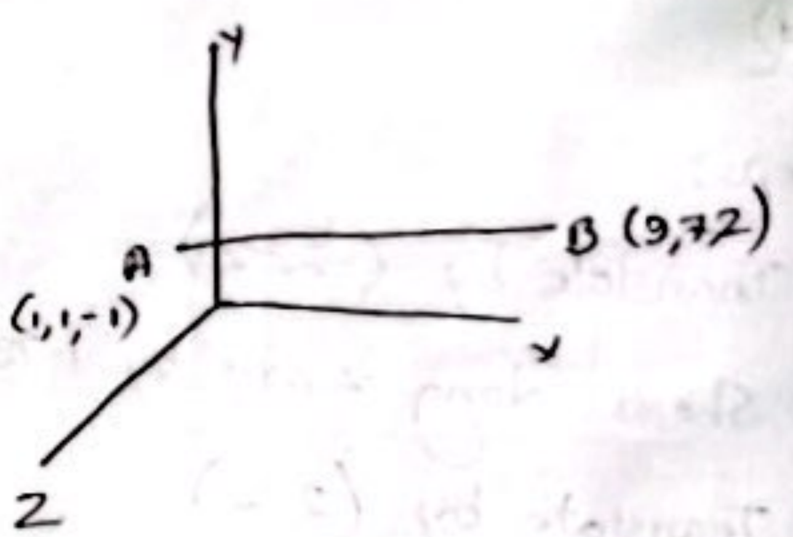
upto z_{10}

it will converge after z_8 (GPT bolche...)

So, The colour of the points are Blue.

Decipher

3(c)



① Translation $(-1,-1,1)$

~~② Rotate along y (anticlockwise)~~

~~③ Rotate along~~

② Rotate along z

③ Rotate along x (anticlock)

Translate $(-1,-1,1)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find unit vectors:

$$U_e = \frac{B-A}{|B-A|} = c_x, c_y, c_z$$

$$c_x = \frac{9-1}{\sqrt{(9-1)^2+(7-1)^2+(2+1)^2}} = \frac{8}{\sqrt{109}}$$

$$c_y = \frac{7-1}{\sqrt{109}} = \frac{6}{\sqrt{109}}$$

$$c_z = \frac{2+1}{\sqrt{109}} = \frac{3}{\sqrt{109}}$$

$$; d = \sqrt{c_x^2 + c_y^2} = \sqrt{\left(\frac{8}{\sqrt{109}}\right)^2 + \left(\frac{6}{\sqrt{109}}\right)^2} = 0.9578$$

$$\sin\alpha = \frac{c_y}{d} = \frac{\frac{6}{\sqrt{109}}}{0.9578}$$

$$\cos\alpha = \frac{c_x}{d} = \frac{\frac{8}{\sqrt{109}}}{0.9578}$$

$$\cos\beta = \frac{d}{u_e} = 0.9578 \; ; \; \sin\beta = \frac{c_z}{u_e} = \frac{3}{\sqrt{109}}$$

$$m_1 = Rot\_x(\beta) * Rot\_z(\alpha) * T(-1,-1,1)$$

$$=$$

$$m_1 \times v = m_1 \times \begin{bmatrix} 1 & 9 \\ 1 & 7 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

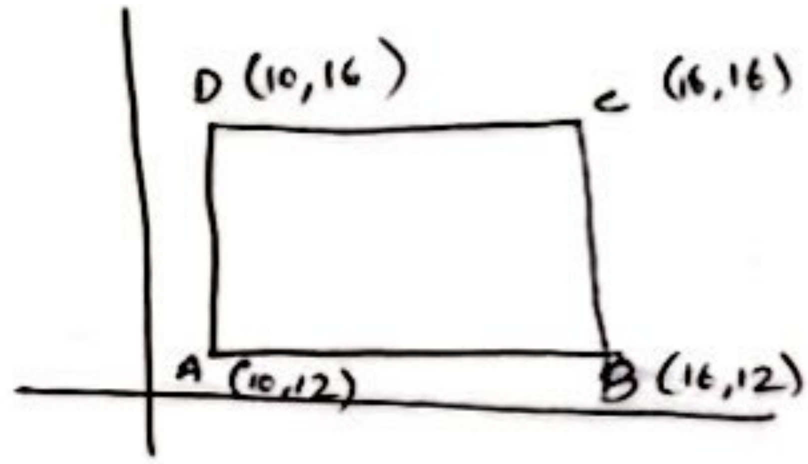
(Ans.)

Quiz-2 Set-F

line: $2y - 6x + 2 = 0$

$$y - 3x + 1 = 0$$

$$\therefore y = 3x - 1$$



The line is 1 unit below origin on y-axis.

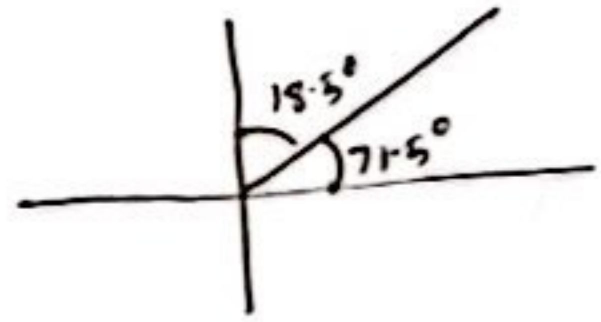
by translating (0,1).

$$y = 3x$$

here, $m = 3$

$$\tan\theta = 3$$

$$\theta = 71.5$$



(1) Translate (0,1)

(2) Rotate (18.5°)

(3) Reflect-Y

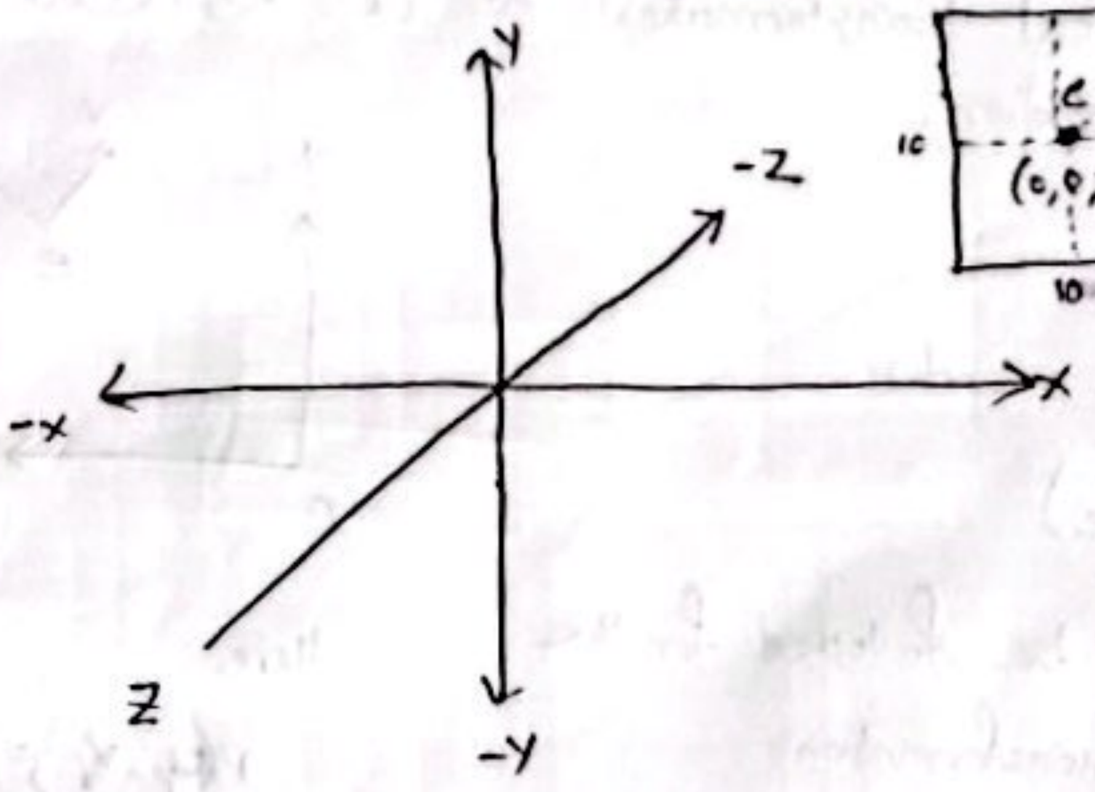
(4) Rotate (-18.5°)

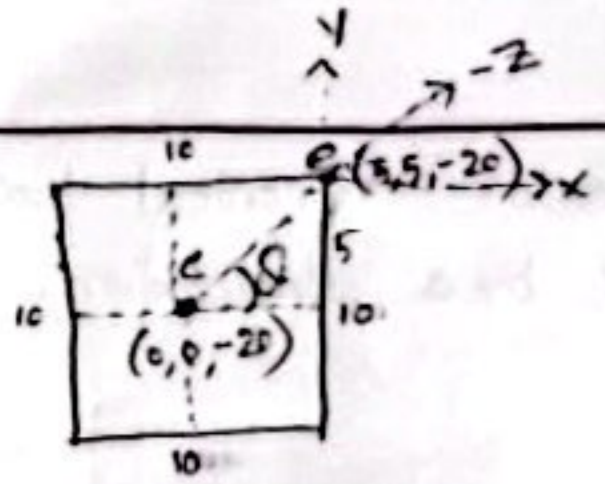
(5) Translate (0,-1)

Scanned with CamScanner

Quiz

① a)





$\tan Q = \frac{5}{5}$

$Q = \tan^{-1}(1)$

$\therefore Q = 45^\circ$

Basis vectors: $U = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $V = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $W = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

After rotation:

Basis matrix = Rotate (-45°) * Basis vectors

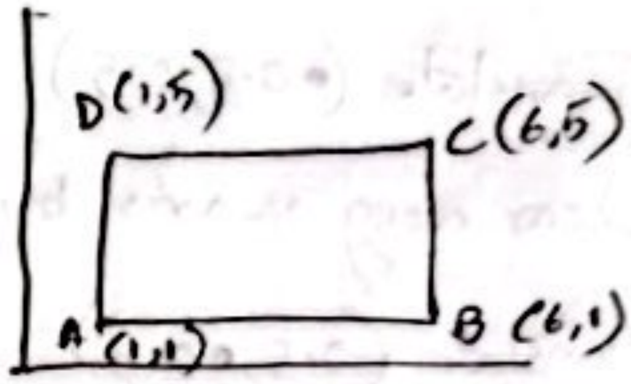
$$= \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Quiz- Set-A

① Translate $(-1,-1)$

② Shear_x (2)

③ Translate $(1,1)$



$M_1 = T(-1,-1) * Shear\_x(2) * T(-1,-1)$

$\Delta y = |5-1| = 4$

$\therefore$ Shear factor $= \frac{8}{4}$

$= 2$

$=$

$$M_1 \times V = M_1 \times \begin{bmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Show that two succesive reflections about eith of the principal axis is equivalent to a single rotation about the coordinate origin.

$\Rightarrow M_1 = Ref\_y * Ref\_x$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$=$

$M_2 = \text{Rotation}(180^\circ)$

$=$

Lectur-8 (A)

For Cantor set:

\# ϵ_n: Size of new element at n iteration $=$ length $= \frac{1}{3^n}$

N_n: the number of new elements at " $n = 2^n$

Fractal dimension, $D: -\lim_{n\to\infty} \frac{\log(N_n)}{\log(\epsilon_n)}$

$\rightarrow$ Defines the complexity of fractal.

Dimension of Cantor Set

$$D = -\lim_{n\to\infty} \frac{\log(N_n)}{\log(\epsilon_n)} = -\lim_{n\to\infty} \frac{\log(2^n)}{\log(1/3^n)}$$

$$= -\lim_{n\to\infty} \frac{n\log 2}{n\log 1/3}$$

$$= \lim_{n\to\infty} \frac{\log 2}{\log 3}$$

$$= 0.6309$$

For Koch Snowflake:

$\epsilon_n = \frac{1}{3^n}$

$N_n = 3\times 4^n$

$$D = -\lim_{n\to\infty} \frac{\log[3(4^n)]}{\log[1/3^n]} = -\lim_{n\to\infty} \frac{\log(3) + n\log(4)}{-n\log(3)}$$

$$= \lim_{n\to\infty} \frac{\cancel{\log(3)}}{n\log(3)} + \frac{\cancel{n}\log 4}{\cancel{n}\log 3}$$

$$= \lim_{n\to\infty} \frac{1}{n} + \frac{\log 4}{\log 3} = \log 4 / \log 3 = 1.2619$$

四 Sierpinski Triangle:

$$\epsilon_n = \frac{1}{2^n}$$

$$N_n = 3^n$$

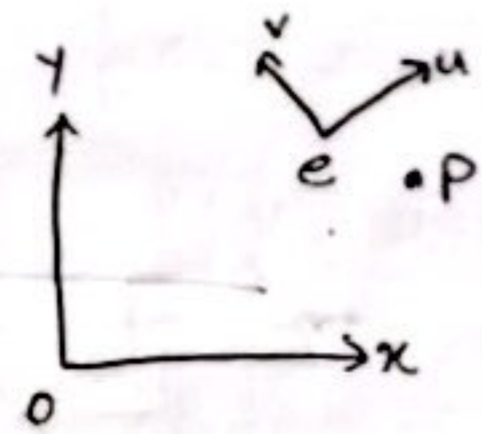
$$D = -\lim_{n \to \infty} \frac{\log(3^n)}{\log(1/2^n)}$$

$$= \lim_{n \to \infty} \frac{\log(3)}{\log(2)}$$

$$= 1.58$$

五 The ~~mad~~ mandlebnot set

Frame to cannonical transformation can be expressed as rotation followed by a translation.



Steps:

(1) Rotate (Q) of basis u and v

(2) Translate $(-x_e, -y_e)$

These steps needs to be followed for the Frame to cannonical transformation.

Here,

$$p(u_p, v_p) = e + u_p u + v_p v$$

- u and v are the basis vectors, e is the origin of frame.

Basis Matrix

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_e \\ 0 & 1 & y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & x_v & 0 \\ y_u & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

Here,

Rotation involving (u and v) followed by a translation (involving e). So, we can say that .....

Basis matrix:

$$\begin{bmatrix} \cos Q & -\sin Q & 0 \\ \sin Q & \cos Q & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{cc} \underline{u} & \underline{v} \\ \multicolumn{2}{c}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}} \end{array}$$

$=$

For Cannonical to frame:

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

translation followed by a rotation.

eye matrix = $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

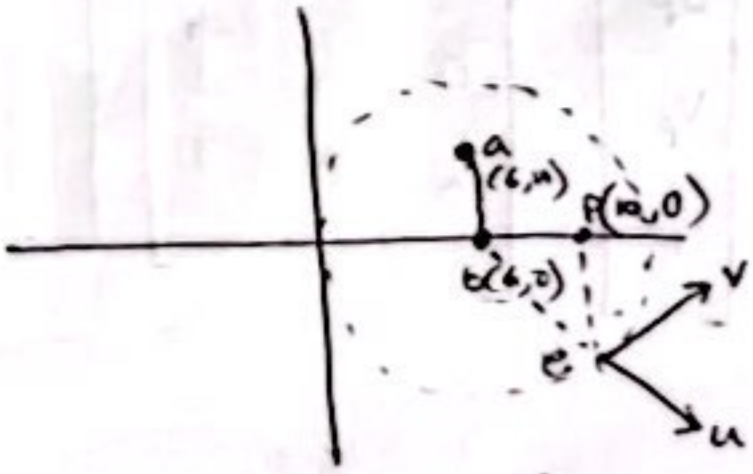
Basis matrix

$P_{uv} = \begin{bmatrix} U_P \\ V_P \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 3 & 0 \\ 1 & 1 \end{bmatrix}$

$=$

Integer

3(g)



$be = 6$; $bp = 4$

$pe = \sqrt{6^2 - 4^2}$

$= 2\sqrt{5}$

$\therefore e = (10, 2\sqrt{5})$

$\tan Q = \frac{2\sqrt{5}}{4}$

$\therefore Q = 48.18$

Basis vector: $U = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Eye = $\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -2\sqrt{5} \\ 0 & 0 & 1 \end{bmatrix}$

After rotation:

$P_{uv} =$

Rotate(48.18) * Basis vector

$= \begin{bmatrix} \cos(48.18) & -\sin(48.18) & 0 \\ \sin(48.18) & \cos(48.18) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} =$

Prev sem quiz-Set D

For OA;

12 hour 360°

1 " 30°

∴ 4 hour 120°

① Translate (-8,-8)

② Rotate (-120°)

③ Translate (8,8)

For OB;

60 minute : 360°

1 " : 6°

30 " : 180°

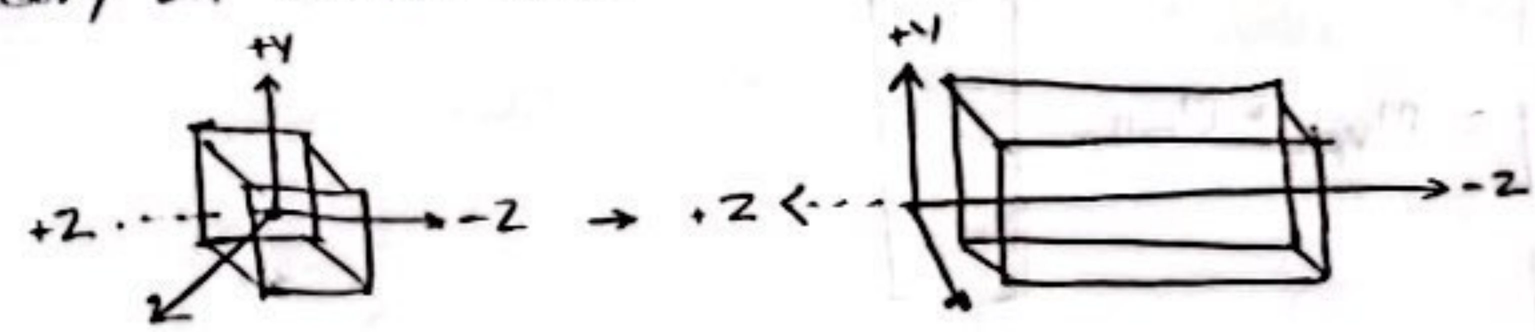
① Translate (-8,-8)

② Rotate (-180°)

③ Translate (8,8)

Orthographic Projection Transformation

In connonical view, the space is limited [-1, 1]. We can render a geometry in some other ~~or~~ region using orthographic Project, where we can have boundary set on our will.



cannonical   orthographic

Shape: $|r-l| \times |t-b| \times |n-f|$

① Translate $\left(-\frac{r+l}{2}, -\frac{t+b}{2}, -\frac{n+f}{2}\right)$

② Scaling $\left(\frac{2}{r-l}, \frac{2}{t-b}, \frac{2}{n-f}\right)$ → Because connonical shape is $2 \times 2 \times 2$

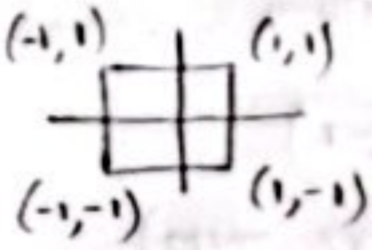
t = top plane
b = bottom plane
r = right plane
l = left plane
n = near plane
f = front plane

$$M_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Viewport transformation

Bottom to top



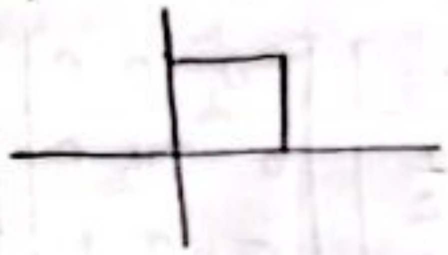
1. Translate $(1,1)$
2. Scaling $(n_x/2, n_y/2)$
3. Translate $(-\frac{1}{2}, -\frac{1}{2})$

$$M_1 = \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x/2 & 0 & 0 \\ 0 & n_y/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

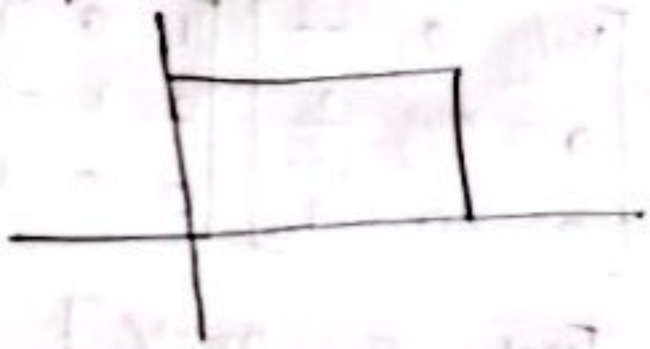
$$= \begin{bmatrix} n_x/2 & 0 & -\frac{1}{2} \\ 0 & n_y/2 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x/2 & 0 & n_x-1/2 \\ 0 & n_y/2 & n_y-1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

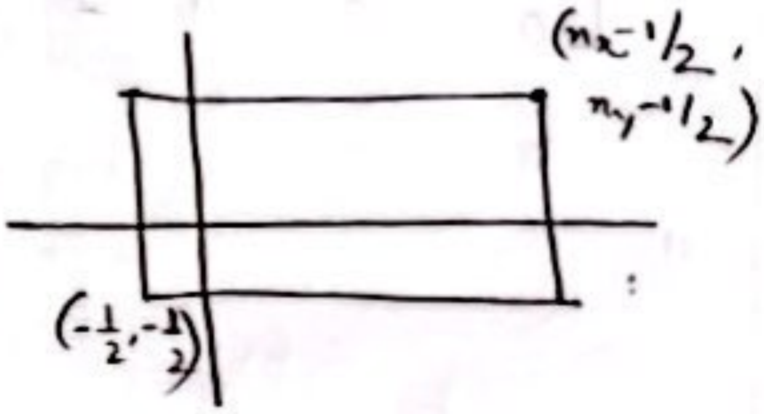
Apply $T(1,1)$:



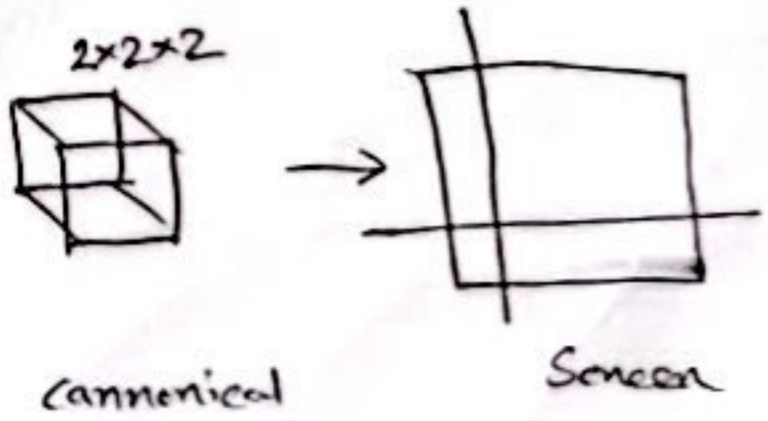
Apply $S(n_x/2, n_y/2)$:



Apply $(-\frac{1}{2}, -\frac{1}{2})$:



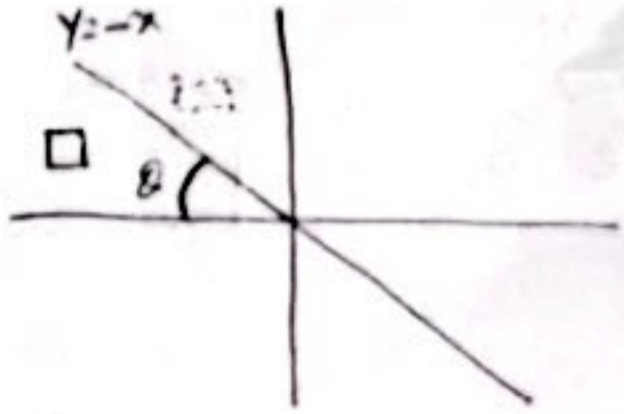
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} n_x/2 & 0 & n_x-1/2 \\ 0 & n_y/2 & n_y-1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canno} \\ y_{canno} \\ 1 \end{bmatrix}$$



Integer

2(b)

Transformation matrix for the reflection about the line. $y=-x$:



$$m_1 = Rot(45^\circ) * Ref\text{-}y * Rot(-45^\circ)$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

Here, $m=-1$

$\tan\theta = -1$; $\theta = \tan^{-1}(-1)$

$\theta = -45^\circ$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Reflection related to y-axis followed by a counter-clockwise rotation of 90°

$$m_2 = Rot(90^\circ) * Ref\text{-}y$$

$$= \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$\therefore m_1 = m_2$

2(d)

V: X, F

C: +, −

axiom: F + XF + F + XF

Rules: X → XF − F + F − XF + F + XF − F + F − X

Angle: 90°

X = do nothing

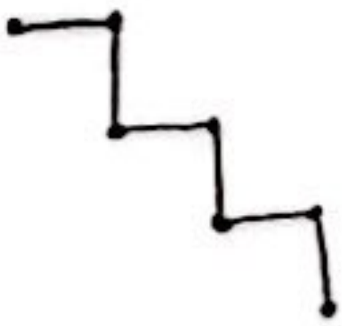
F: draw a line forward

+ : rotate clockwise by 90°

− : " counterclock by 90°

n=0: F + XF + F + XF

n=1: F + XF − F + F − XF + F + XF − F + F − XF + F + XF − F + F − XF + F + XF − F + F − XF



Ki kortesi aishob?

Eye matrix: $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

b) Frame / Camera to cannonical

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -20 \\ 1 \end{bmatrix}$$

$=$

Prev year Quiz Set A

1(a)

Using top to Bottom:

① Translate $(1,-1)$

② Scaling $(n_x/2, n_y/2)$

③ Translate $(-\frac{1}{2}, \frac{1}{2})$

The final matrix will be

$$M_{vp} = \begin{bmatrix} n_x/2 & 0 & n_x^{-1}/2 \\ 0 & n_y/2 & -n_y+1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $n_x = 256$

$n_y = 128$

1(b)

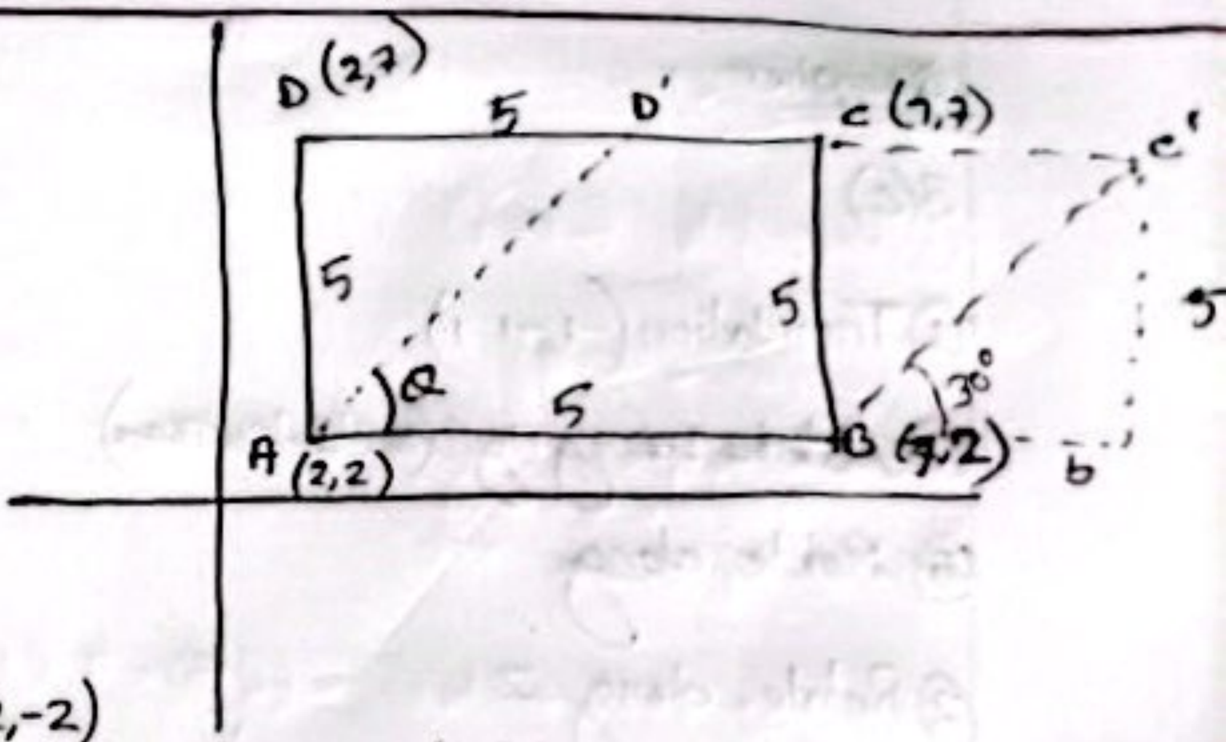
$A = (-3,-4,-3)$; $B(2,4,-6)$

$l = -6$, $r = 6$, $b = -7$, $t = 7$, $n = -2$, $f = -8$

$$\begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{canon} \\ 1 \end{bmatrix} = M_{vp} * \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -3 & 2 \\ -4 & 4 \\ -3 & -6 \\ 1 & 1 \end{bmatrix}$$

=

3(d)



Steps:

① Translate by $(-2,-2)$

② Shear along x-axis by 1.732

③ Translate by $(2,2)$

$M_1 = T(2,2) * Shear\_x(1.732) * T(-2,-2)$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1.732 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S, M_1 \times V = M_1 \times \begin{bmatrix} 2 & 7 & 7 & 2 \\ 2 & 2 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$=$

$Q = 30$

$\tan Q = \frac{5}{b}$

$b = 5\sqrt{3}$

$= 8.6602$

∴ to shear by x-axis for 8.6602 points

Shear factor $= \frac{8.6602}{5} \rightarrow \Delta y$

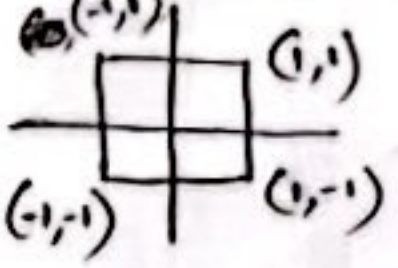
$= 1.732$ $\quad (7-2) = 5$

For shear factor

along x-axis; divide by Δy

along y-axis; divide by Δx

Top to Bottom



① Translate$(1,-1)$

② Scaling$(nx/2, ny/2)$

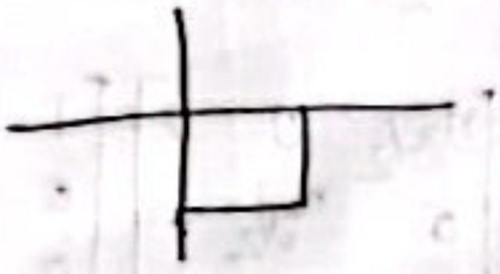
③ Translate$\left(-\frac{1}{2}, \frac{1}{2}\right)$

$$M_1 = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} nx/2 & 0 & 0 \\ 0 & ny/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

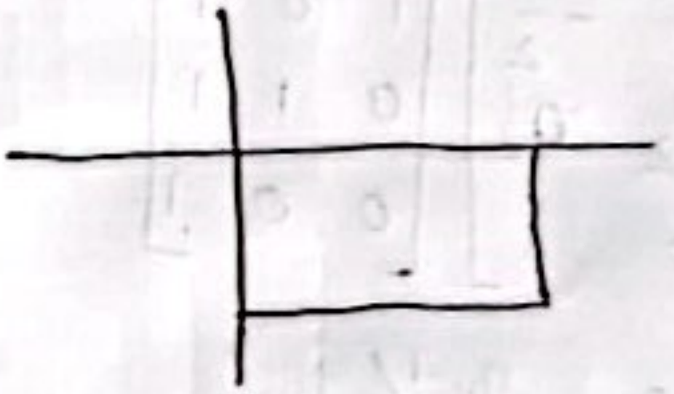
$$= \begin{bmatrix} nx/2 & 0 & -\frac{1}{2} \\ 0 & ny/2 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} nx/2 & 0 & nx-1/2 \\ 0 & ny/2 & \frac{-ny+1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

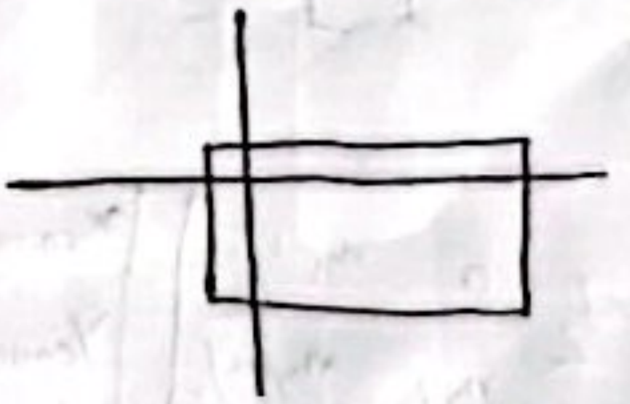
Apply $T(1,-1)$:



Apply $S(nx/2, ny/2)$:



Apply $T\left(-\frac{1}{2}, \frac{1}{2}\right)$



$M_1 = R(45^\circ) \cdot R^{-1}(45^\circ)$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$=$

$M_2 = R(45^\circ - 45^\circ)$

$= R(0^\circ)$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3(b)

① Translate $(-5,2,-3)$

② Rotate along Z

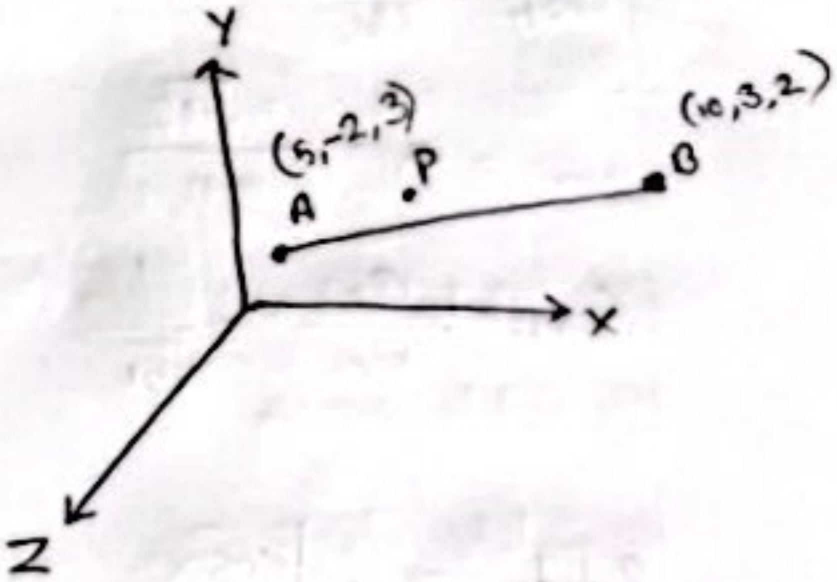
③ Rotate along X

④ Rotate along Y

⑤ Rotate along X

⑥ Rotate along Z

⑦ Translate $(5,-2,3)$



\# Translate $(5,-2,3)$:

$$T = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

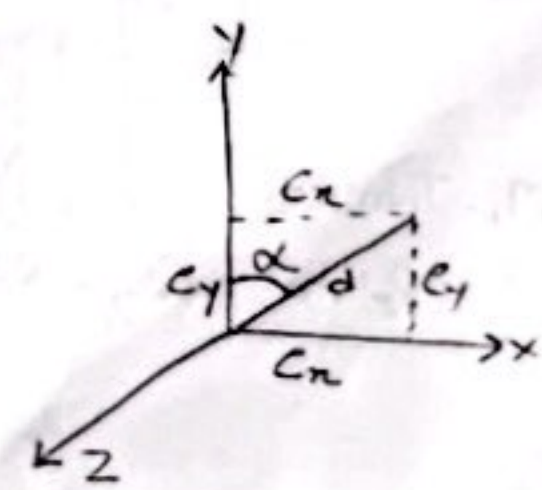
\# Find unit vectors:

$$U_e = \frac{B-A}{|B-A|_{x,y,z}} = C_x, C_y, C_z$$

$$C_x = \frac{10-5}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$C_y = \frac{(3+2)}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$C_z = \frac{2-3}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{-1}{\sqrt{51}}$$



$$d = \sqrt{C_x^2+C_y^2} = \sqrt{\left(\frac{5}{\sqrt{51}}\right)^2+\left(\frac{5}{\sqrt{51}}\right)^2} = \frac{5\sqrt{102}}{51}$$

$$\cos\alpha = \frac{C_y}{d} = \frac{\frac{5}{\sqrt{51}}}{\frac{5\sqrt{102}}{51}} = \frac{1}{\sqrt{2}}$$

$$\sin\alpha = \frac{C_x}{d} = \frac{1}{\sqrt{2}}$$