Backward Propagation:

Here L = Loss function

a = Activation function.

$z = \omega^T x + b$.

$$L(a,y) = \frac{\partial L}{\partial a} \frac{\partial a}{\partial z} \frac{\partial z}{\partial \omega_1}$$

$$\frac{\partial L}{\partial a} = -\left[y \log_2 a + (1-y)\log_2(1-a)\right]$$

$$= -\left[y\frac{1}{a} + (1-y)\frac{1}{1-a}(-1)\right] \quad \left[\because \frac{\partial}{\partial a}(1-a) = -1\right]$$

$$= -\left[\frac{y}{a} - \frac{1-y}{1-a}\right] = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{\partial a}{\partial z} = \frac{\partial}{\partial z}\left[\frac{1}{1+e^{-z}}\right]$$

$$\frac{\partial}{\partial z}(1+e^{-z})^{-1}$$

$$= (-1)(1+e^{-z})^{-2}\frac{\partial}{\partial z}(1+e^{-z})$$

$$= \frac{-1}{(1+e^{-z})^2} e^{-z}(-1)$$

$$= \frac{1}{(1+e^{-z})^2} e^{-z}$$

$$= \frac{1}{(1+e^{-z})} \times \frac{1+e^{-z}-1}{1+e^{-z}}$$

$$= \frac{1}{1+e^{-z}} \times \left(\frac{1+e^{-z}}{1+e^{-z}} - \frac{1}{1+e^{-z}}\right)$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= a(1-a)$$

$(1+e^{-z})^{-1}$

$(-1)(1+e^{-z})^{-2} \cdot e^{-z}(-1)$

$\frac{e^{-z}}{(1+e^{-z})^2}$

$\frac{1+e^{-z}-1}{1+e^{-z}}$

$$\frac{\partial z}{\partial w_1} = w_1 x_1 + w_2 x_2 + b$$

$$= x_1$$

$$dz = \frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial a} \frac{\partial a}{\partial z} \frac{\partial z}{\partial w_1}$$

$$= \left(\frac{-y}{a} + \frac{1-y}{1-a}\right)\left(a(1-a)\right) x_1$$

$$= (a-y) x_1$$

If Activation function ~~sig~~ tanh.

$$\frac{\partial a}{\partial z} = \frac{\partial}{\partial z}\left[\frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}\right]$$

$$= \frac{(e^{z}+e^{-z})(e^{z}+e^{-z}) - (e^{z}-e^{-z})(e^{z}-e^{-z})}{(e^{z}+e^{-z})^2}$$

$$= \frac{(e^{z}+e^{-z})^2 - (e^{z}-e^{-z})^2}{(e^{z}+e^{-z})^2}$$

$$= \left(\frac{e^{z}+e^{-z}}{e^{z}+e^{-z}}\right)^2 - \left(\frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}\right)^2$$

$$= 1 - a^2$$

Step function:

$$\frac{\partial a}{\partial z} = 0$$

Linear

$$y = x$$

$$\Rightarrow \frac{dy}{dx} = 1 .$$

Leaky ReLU

$x \geq 0 \quad y = x$

$$\Rightarrow \frac{dy}{dx} = 1$$

$x < 0 \quad y = 0.001x .$

$$\Rightarrow \frac{dy}{dx} = 0.01 .$$

ReLU:

$x \geq 0 \quad y = x$

$$\Rightarrow \frac{dy}{dx} = 1 .$$

$x < 0 \quad y = 0$

$$\frac{dy}{dx} = 0 .$$