LECTURE 2

Lecture-2

$$C = \alpha C_f + (1-\alpha) C_b$$

$\alpha = 0$; $C = C_b$ (Transparent)

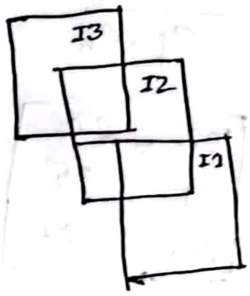
$\alpha = 1$; $C = C_f$ (Opaque)

$\alpha = 0.5$; $C = \frac{C_f + C_b}{2}$ (Partial Transparent)

Quiz- SetA

2

$$C_1 = \begin{bmatrix} 0.2 & 0.39 \\ 0 & 0.3 \\ 0.45 & 0.5 \end{bmatrix} \begin{bmatrix} 15 & 20 \\ 200 & 20 \\ 110 & 99 \end{bmatrix} + \begin{bmatrix} 1-0.2 & 1-0.39 \\ 1-0 & 1-0.3 \\ 1-0.45 & 1-0.5 \end{bmatrix} \begin{bmatrix} 130 & 20 \\ 50 & 85 \\ 230 & 9 \end{bmatrix}$$



$$= \begin{bmatrix} 3 & 7.8 \\ 0 & 6 \\ 49.5 & 49.5 \end{bmatrix} + \begin{bmatrix} 104 & 12.2 \\ 50 & 59.5 \\ 126.5 & 4.5 \end{bmatrix}$$

$$= \begin{bmatrix} 107 & 20 \\ 50 & 65.5 \\ 176 & 54 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.2 & 0.39 \\ 0 & 0.3 \\ 0.45 & 0.5 \end{bmatrix} \begin{bmatrix} 130 & 20 \\ 50 & 85 \\ 230 & 9 \end{bmatrix} + \begin{bmatrix} 1-0.2 & 1-0.39 \\ 1-0 & 1-0.3 \\ 1-0.45 & 1-0.5 \end{bmatrix} \begin{bmatrix} 107 & 20 \\ 50 & 65.5 \\ 176 & 54 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 7.8 \\ 0 & 25.5 \\ 103.5 & 4.5 \end{bmatrix} + \begin{bmatrix} 85.6 & 12.2 \\ 50 & 45.85 \\ 96.8 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 111.6 & 20 \\ 50 & 71.35 \\ 200.3 & 31.5 \end{bmatrix}$$

Ans.

Sol.-F

2.

We know,

$$c = \alpha c_f + (1-\alpha) c_b$$

$$c = \alpha c_f + c_b - \alpha c_b$$

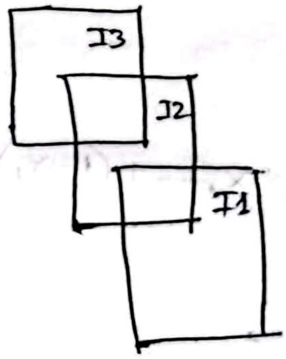
$$c - c_b = \alpha (c_f - c_b)$$

$$\therefore \alpha = \frac{c - c_b}{c_f - c_b}$$

$$\therefore \alpha = \begin{bmatrix} 102 & 20 \\ 74 & 85 \\ 201 & 27 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ 200 & 20 \\ 110 & 99 \end{bmatrix} \Big/ \begin{bmatrix} 130 & 20 \\ 50 & 85 \\ 230 & 9 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ 200 & 20 \\ 110 & 99 \end{bmatrix}$$

$=$

2



$C_1 : \alpha I_2 + (1-\alpha) I_3$

$C_2 : \alpha I_1 + (1-\alpha) C_1$

$\therefore C_2 = \alpha I_1 + (1-\alpha)\{\alpha I_2 + (1-\alpha) I_3\}$

$\therefore C_2 = \alpha I_1 + \alpha(1-\alpha) I_2 + (1-\alpha)^2 I_3$

$= \alpha I_1 + (\alpha - \alpha^2) I_2 + (1-\alpha)^2 I_3$

$= \alpha I_1 + \alpha I_2 - \alpha^2 I_2 + (1 - 2\alpha + \alpha^2) I_3$

$= \alpha I_1 + \alpha I_2 - \alpha^2 I_2 + I_3 - 2\alpha I_3 + \alpha^2 I_3$

$= \alpha(I_1 + I_2) - \alpha^2 (I_2 - I_3) - 2\alpha I_3 + I_3$

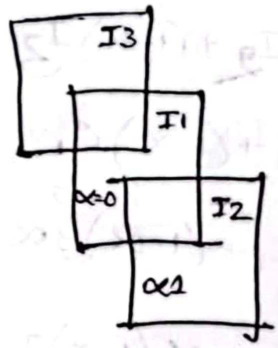
$\alpha^2 (I_3 - I_2) + \alpha(I_1 + I_2 - 2I_3) + (I_3 - C_2) = 0$

$\therefore ax^2 + bx + c = 0$

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Integer:

2(d)



$$C_{output} = \alpha_1 I_2 + (1-\alpha_1)\{\alpha_0 I_1 + (1-\alpha_0) I_3\}$$

$=$

Quiz- Sete

Image size = 6x7

∴ 42 pixels

1-byte : 8 bits

original image size = $(42 \times 8)/8$

$= 42$ byte

$= 0.041$ KB

R1 : 303112

R2 : 5021

R3 : 421021

R4 : 2253

R5 : 5321

R6 : 332220

Total pixel : 30

~~∴ compression ratio = $\frac{0.041 - 0.029}{0.041}$ = 29.26%~~

Compressed size = $(30 \times 8)/8$

$= 240/8$

$= 30$ byte$/1024$

$= 0.029$ KB

∴ compression ratio = $\frac{0.041}{0.029}$

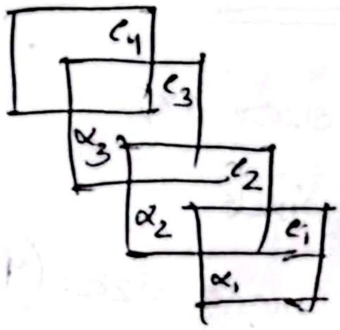
$= 1.41$

Set-D

α_1: Alpha compositing parameter to blend c_1 c_2

α_2 : " " " " " c_2 c_3

α_3 : " " " " " c_3 c_4



$$C_{output} = \alpha_1 C_1 + (1-\alpha_1) C_2$$

$$= \alpha_1 C_1 + (1-\alpha_1)\{\alpha_2 C_2 + (1-\alpha_2) C_3\}$$

$$= \alpha_1 C_1 + (1-\alpha_1)\{\alpha_2 C_2 + (1-\alpha_2)\{\alpha_3 C_3 + (1-\alpha_3) C_4\}\}$$

LECTURE 3

Lecture-3

Bezier Curves:

Degree, $d: N-1$; N: control points

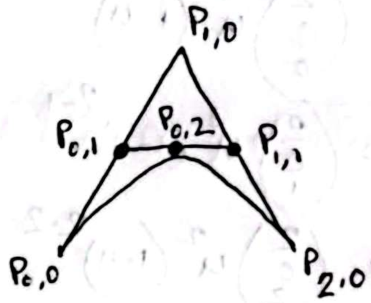
quadratic $(d=2)$, $N=3$.

For 2 control points P_0 and P_1 ;

$$Q_0 = P_0 + u\,(P_1 - P_0)$$

De casteljau's Algorithm .

$$P_{i,j} = (1-u)P_{i,j-1} + u\,P_{i+1,j-1}$$



For degree 2;

$$P_{0,2} = (1-u)P_{0,1} + uP_{1,1}$$

$$= (1-u)\left[(1-u)\underline{P_{0,1}} + uP_{1,0}\right] + u\left[(1-u)P_{1,0} + uP_{2,0}\right]$$

$$= (1-u)^2 P_{0,1} + u(1-u)P_{1,0} + u(1-u)P_{1,0} + u^2 P_{2,0}$$

$$= (1-u)^2 P_{0,1} + 2u(1-u)P_{1,0} + u^2 P_{2,0}$$

For degree 4;

$$P_{0,4} = (1-u)P_{0,3} + uP_{1,3}$$

$$= (1-u)\left[(1-u)P_{0,2} + uP_{1,2}\right] + u\left[(1-u)P_{1,2} + uP_{2,2}\right]$$

$$= (1-u)^2 P_{0,2} + 2u(1-u)P_{1,2} + u^2 P_{2,2}$$

$$= (1-u)^2\left[(1-u)P_{0,1} + uP_{1,1}\right] + 2u(1-u)\left[(1-u)P_{1,1} + uP_{2,1}\right] + u^2\left[(1-u)P_{2,1} + uP_{3,1}\right]$$

$$= (1-u)^3 P_{0,1} + u(1-u)^2 P_{1,1} + 2u(1-u)^2 P_{1,1} + 2u^2(1-u)P_{2,1} + u^2(1-u)P_{2,1} + u^3 P_{3,1}$$

$$= (1-u)^3 P_{0,1} + 3u(1-u)^2 P_{1,1} + 3u^2(1-u)P_{2,1} + u^3 P_{3,1}$$

$$= (1-u)^3\left[(1-u)P_{0,0} + uP_{1,0}\right] + 3u(1-u)^2\left[(1-u)P_{1,0} + uP_{2,0}\right] + 3u^2(1-u)\left[(1-u)P_{2,0} + uP_{3,0}\right]$$
$$+ u^3\left[(1-u)P_{3,0} + uP_{4,0}\right]$$

Using Polynomial:

$$Q(u) = \sum_{i=0}^{d} B_{i,d}(u)\, P_i \qquad 0 \le u \le 1$$

$$B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$$

For $d=2$;

$$B_{0,2}(u) = \binom{2}{0} u^0 (1-u)^{2-0} = (1-u)^2$$

$$B_{1,2}(u) = \binom{2}{1} u^1 (1-u)^{2-1} = 2u(1-u)$$

$$B_{2,2}(u) = \binom{2}{2} u^2 (1-u)^{2-2} = u^2$$

$$Q_2(u) = (1-u)^2 P_{0,1} + 2u(1-u) P_{1,0} + u^2 P_{2,0}$$

Set-B

3) $P_0 = (1,0)$

$P_1 = (1,4)$ $\qquad Q\left(\frac{1}{2}\right) = ?$

$P_2 = (3,4)$

$P_3 = (4,2)$ $\qquad$ Here, $N = 5$

$P_4 = (4,0)$ $\qquad$ degree $= 5-1 = 4$

For degree 4;

$$Q(u) = B_{0,4}(u) + B_{1,4}(u) + B_{2,4}(u) + B_{3,4}(u) + B_{4,4}(u)$$

$$= \binom{4}{0} u^0 (1-u)^{4-0} + \binom{4}{1} u^1 (1-u)^{4-1} + \binom{4}{2} u^2 (1-u)^{4-2} + \binom{4}{3} u^3 (1-u)^{4-3} + \binom{4}{4} u^4 (1-u)^{4-4}$$

$$= (1-u)^4 + 4u(1-u)^3 + 6u^2(1-u)^2 + 4u^3(1-u) + u^4$$

$$= (1-u)^4 P_0 + 4u(1-u)^3 P_1 + 6u^2(1-u)^2 P_2 + 4u^3(1-u) P_3 + u^4 P_4$$

$$Q\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 \begin{bmatrix}1\\0\end{bmatrix} + 2\cdot\left(\frac{1}{2}\right)^3 \begin{bmatrix}1\\4\end{bmatrix} + \frac{3}{2}\left(\frac{1}{2}\right)^2 \begin{bmatrix}3\\4\end{bmatrix} + 4\cdot\frac{1}{8}\cdot\left(\frac{1}{2}\right) \begin{bmatrix}4\\2\end{bmatrix} + \left(\frac{1}{2}\right)^4 \begin{bmatrix}4\\0\end{bmatrix}$$

$$= \begin{bmatrix}1/16\\0\end{bmatrix} + \begin{bmatrix}\frac{1}{4}\\1\end{bmatrix} + \begin{bmatrix}\frac{9}{8}\\\frac{3}{2}\end{bmatrix} + \begin{bmatrix}1\\\frac{1}{2}\end{bmatrix} + \begin{bmatrix}\frac{1}{4}\\0\end{bmatrix}$$

$$= \begin{bmatrix}2.6875\\3\end{bmatrix}$$

Set-D

(3)

1

1 1

d= 2 ; 1 2 1

d=3 ; 1 3 3 1

d=4 ; 1 4 6 4 1

d=5 ; 1 5 10 10 5 1

$Q(u) = (1-u)^5 P_0 + 5u(1-u)^4 P_1 + 10u^2(1-u)^3 P_2 + 10u^3(1-u)^2 P_3 + \underline{5u^4(1-u)P_4 + u^5 P_5}$

$Q(1) = 0+0+0+0+0+P_5$

$\therefore \begin{bmatrix} 5 \\ 1 \end{bmatrix} = P_5$

~~P_4:~~

$\cancel{Q(u) = 5u^4(1-u)P_4}$

$\cancel{\begin{bmatrix} 0.68 \\ 3.56 \end{bmatrix} = 5\left(\tfrac{1}{2}\right)^4\left(1-\tfrac{1}{2}\right)P_4}$

$Q(0.5) = (1-0.5)^5 \begin{bmatrix} -3 \\ 3 \end{bmatrix} + 5\times(0.5)\times(1-0.5)^4 \begin{bmatrix} -1 \\ 4 \end{bmatrix} + 10\times(0.5)^2(1-0.5)^3 \begin{bmatrix} 0 \\ 5 \end{bmatrix} +$

$10\times(0.5)^3(1-0.5)^2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + 5\times(0.5)^4(1-0.5)P_4 + (0.5)^5 \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -\frac{3}{32} \\ 3/32 \end{bmatrix} + \begin{bmatrix} -\frac{5}{32} \\ 5/8 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{25}{16} \end{bmatrix} + \begin{bmatrix} 5/16 \\ 15/16 \end{bmatrix} + 5/32 \cdot P_4 + \begin{bmatrix} 5/32 \\ 1/32 \end{bmatrix}$

$\begin{bmatrix} 0.68 \\ 3.56 \end{bmatrix} = \begin{bmatrix} 7/32 \\ 13/4 \end{bmatrix} + 5/32\, P_4$

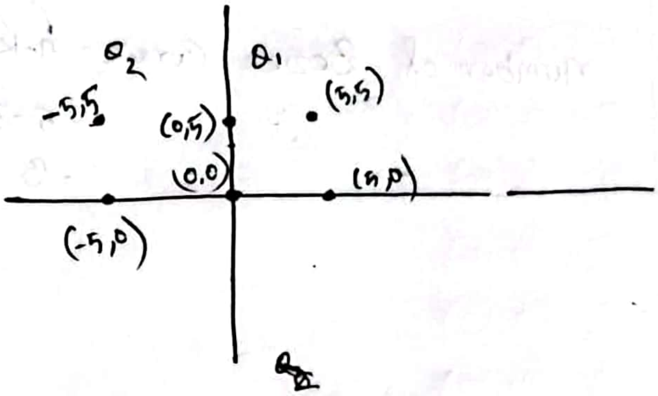
$5/32\, P_4 = \begin{bmatrix} 0.46125 \\ 0.31 \end{bmatrix} \quad \therefore P_4 = \begin{bmatrix} 2.952 \\ 1.984 \end{bmatrix}$

Dedpker

3(a)

For Q_1;

$P_0\ (0,0);\ P_1\ (0,5);\ P_2(5,5);\ P_3(5,0)$



$Q_1(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$

(0.6)

$=$

For Q_2;

$P_0\ (0,0);\ P_1\ (0,5);\ P_2\ (-5,5);\ P_3\ (-5,0)$

$Q_2(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$

(0.8)

$=$

B-spline Curve:

control point -1 → kind of curve (cubic)

Number of Bezier Curve $= n-k+1$

$= 5-3+1$

$= 3$ Bezier curve

Uniform Quadratic B-spline:

$$S_i(t) = \begin{pmatrix} P_i & P_{i+1} & P_{i+2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

Open Uniform:

$$S_0(t) = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 2 & -4 & 2 \\ -3 & 4 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$S_i(t) = \begin{pmatrix} P_i & P_{i+1} & P_{i+2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$S_{n-2}(t) = \begin{pmatrix} P_{n-2} & P_{n-1} & P_n \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -3 & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

Set-A

3

$P_0 = (-3,1)$

$P_1 = (-1,2)$

$P_2 = (1,3)$

$P_3 = (3,4)$

$P_4 = (4,5)$

$P_5 = (6,7)$

$P_6 = (7,8)$

# of curve $= n-k+1$

$= 6-2+1$

$= 5$

S_0, S_1, S_2, S_3, S_4

$$S_4(0.3) = \begin{pmatrix} P_4 & P_5 & P_6 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 6 & 7 \\ 5 & 7 & 8 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} (0.3)^2 \\ 0.3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 6 & 7 \\ 5 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1/2 & -1 & \frac{1}{2} \\ -1 & 1 & 1/2 \\ 1/2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.9 \\ 0.3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -0.5 & 2 & 5 \\ -0.5 & 2 & 6 \end{pmatrix} \begin{pmatrix} 0.9 \\ 0.3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5.15 \\ 6.15 \end{pmatrix}$$

$$S_3(0.3) = \begin{pmatrix} P_3 & P_4 & P_5 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$=$

Set-C

3

$P_0 = (1,10)$
$P_1 = (3,15)$
$P_2 = (5,20)$
$P_3 = (7,15)$
$P_4 = (9,13)$
$P_5 = (11,10)$

number of Bezier Curve $= n-k+1$
$= 5-2+1$
$= 4$

S_0, S_1, S_2, S_3

For uniform;

$$S_0(0.5) = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$=$

For open uniform;

$$S_0(0.5) = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 2 & -4 & 2 \\ -3 & 4 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$=$

Set, E

$t = 1$