

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No ✓
2	No	Married	100K	No ✓
3	No	Single	70K	No ✓
4	Yes	Married	120K	No ✓
5	No	Divorced	95K	Yes ✓
6	No	Married	60K	No ✓
7	Yes	Divorced	220K	No ✓
8	No	Single	85K	Yes ✓
9	No	Married	75K	No ✓
10	No	Single	90K	Yes ✓

class - Refund
Marital status
Taxable income

$$P(\text{No}) = \frac{7}{10} \quad P(\text{Yes}) = \frac{3}{10}$$

for discrete Attribute:

$$P(\text{status} = \text{Married} | \text{No}) = \frac{4}{7} \checkmark \rightarrow \frac{4+1}{7+10}$$

$$P(\text{Refund} = \text{Yes} | \text{Yes}) = 0$$

$$P(\text{status} = \text{Married} | \text{Yes}) = 0 \checkmark \frac{0+1}{3+10}$$

$$P(\text{Refund} = \text{No} | \text{Yes}) = \frac{3}{3} = 1$$

$$P(\text{Refund} = \text{Yes} | \text{No}) = \frac{3}{7} \checkmark$$

$$P(\text{Refund} = \text{No} | \text{No}) = \frac{4}{7}$$

$$P(\text{status} = \text{Married} | \text{Yes}) = 0$$

$P(\text{status} = \text{Married}, \text{Refund} = \text{No}, \text{Income} = 120\text{K})$

$$P(X | \text{claim} = \text{No}) = P(\text{Status} = \text{Married} | \text{No}) \times$$

$$P(\text{Refund} = \text{No} | \text{No}) \times P(\text{Income} = 120\text{K} | \text{No}) \times P(\text{No})$$

$$= 9/7 \times 4/7 \times 7.19 \times 10^{-3} \times 7/10$$

$$= 0.0024$$

$$P(X | \text{claim} = \text{Yes}) = P(\text{Status} = \text{Married} | \text{Yes}) P(\text{Refund} = \text{No} | \text{Yes})$$

$$P(\text{Income} = 120\text{K} | \text{Yes}) \times P(\text{Yes})$$

$$= 0 \times 1 \times 1.2 \times 10^{-9} \times 3/10$$

$$= 0$$

So, No has the highest Probability, that's why

the resulted claim is no.

Taxable Income	Evade
125K	No ✓
100K	No ✓
70K	No ✓
120K	No ✓
95K	Yes .
60K	No ✓
220K	No ✓
85K	Yes .
75K	No ✓
90K	Yes .

$$P(\text{Taxable Income} = 120 \mid \text{No})$$

NO

$$\bar{x} = \frac{125 + 100 + 70 + 120 + 60 + 220 + 75}{7}$$

$$= 110 \text{ K} \quad \text{and } \checkmark$$

$$s^2 = \frac{(x - \bar{x})^2}{n-1}$$

$$= \frac{(125-110)^2 + (100-110)^2 + (70-110)^2 + (120-110)^2 + (60-110)^2 + (220-110)^2 + (75-110)^2}{7-1}$$

$$= 2975$$

$$P(\text{Taxable Income} = 120 \mid \text{No}) \approx$$

$$\frac{1}{\sqrt{2\pi}s} e^{-\frac{(x-\bar{x})^2}{2s^2}}$$

$$= \frac{1}{\sqrt{2\pi} \times \sqrt{2975}} e^{-\frac{(120-110)^2}{2 \times 2975}}$$

$$= 7.19 \times 10^{-3} \quad \checkmark$$

$$P(\text{Taxable Income} = 120 \mid \text{Yes}) = 1.2 \times 10^{-9}$$

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class	
human	yes ✓	no	no	yes	mammals	1
python	no	no	no	no	non-mammals	1
salmon	no	no	yes	no	non-mammals	2
whale	yes ✓	no	yes	no	mammals	2
frog	no	no	sometimes	yes	non-mammals	3
komodo	no	no	no	yes	non-mammals	4
bat	yes ✓	yes	no	yes	mammals	3
pigeon	no	yes	no	yes	non-mammals	5
cat	yes ✓	no	no	yes	mammals	4
leopard shark	yes	no	yes	no	non-mammals	6
turtle	no	no	sometimes	yes	non-mammals	7
penguin	no	no	sometimes	yes	non-mammals	8
porcupine	yes ✓	no	no	yes	mammals	5
eel	no	no	yes	no	non-mammals	7
salamander	no	no	sometimes	yes	non-mammals	10
gila monster	no	no	no	yes	non-mammals	11
platypus	no	no	no	yes	mammals	6
owl	no	yes	no	yes	non-mammals	12
dolphin	yes ✓	no	yes	no	mammals	7
eagle	no	yes	no	yes	non-mammals	13

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

$M \rightarrow \text{Mammals}$, $N = \text{Non mammals}$.

$$P(M) = 9/20 \quad P(N) = 13/20$$

$$P(B = \text{Yes} | M) = 6/7 \quad P(L = \text{Yes} | M) = 2/7$$

$$P(B = \text{Yes} | N) = 1/13 \quad P(L = \text{Yes} | N) = 3/13$$

$$P(C = \text{No} | M) = 6/7 \quad P(H = \text{No} | M) = 2/7$$

$$P(C = \text{No} | N) = 10/13 \quad P(H = \text{No} | N) = 9/13$$

$$\begin{aligned} P(A | M) &= P(B = \text{Yes} | M) \times P(C = \text{No} | M) \times P(L = \text{Yes} | M) \\ &\quad \times P(H = \text{No} | M) \times P(M) \\ &= 6/7 \times 6/7 \times 2/7 \times 2/7 \times 1/20 \end{aligned}$$

$$\begin{aligned}
 P(A|N)P(N) &= P(B = \text{Yes}|N)P(C = \text{No}|N)P(L = \text{Yes}|N) \\
 &\quad P(H = \text{No}|N) \times P(N) \\
 &= \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{9}{13}
 \end{aligned}$$

Doc	I	loved	this	Hotel	Hated	A	Great	Bad	Service	Good	Class
1	1	1	1	1							+
2	1			1	1	1					-
3					2		1	1		1	+
4								1	1		-
5	1	.	1	.	1	1	+

Doc	I	loved	this	Hotel	Hated	A	Great	Bad	Service	Good	Class
1	1	1	1	1							+
3					2		1	1		1	+
5					1		1	1		1	+

smoothed: $P(x_i = T | y = \text{spam})$

= frequent of T in spam + 1

total number of spam instances + V

$$P(w_k | t) = (n_{kt} + 1) / (n_t + V)$$

$$P(I|t) = (I+1) / (A+10)$$

$$P(\text{loved}|t) = (I+1) / (A+10)$$

$$P(\text{thin}|t) = (I+1) / (A+10)$$

$$P(\text{4-to-1} | +) = (4+1) / (14+10)$$

$$P(x_1) = 0.60 \quad P(y_1|x_1) = 0.40 \quad P(z_1|y_1) = 0.25 \quad P(w_1|z_1) = 0.45$$

$$P(y_1|x_0) = 0.30 \quad P(z_1|y_0) = 0.60 \quad P(w_1|z_0) = 0.30$$



Random Variables, x, y, z, w

$$P(x_1) = 0.60$$

$$P(x_1) = 0.40$$

$$P(y_1|x_1) = 0.40$$

$$P(y_1|x_0) = 0.30$$

$$P(z_1|y_1) = 0.25$$

$$P(z_1|y_0) = 0.60$$

$$P(w_1|z_1) = 0.45$$

$$P(w_1|z_0) = 0.30$$

$$P(y_1) = 0.36$$

$$P(y_0) = 0.64$$

$$P(z_1) = 0.474$$

$$P(z_0) = 0.526$$

$$P(w_1) = 0.37$$

$$P(w_0) = 0.63$$

i) Compute $P(y_1)$, y in dependent on x

$$P(y_1) = \sum P(y_1 | x) = P(y_1, x_1) + P(y_1, x_0) P(x_0)$$
$$= 0.4 \times 0.6 + 0.3 \times 0.4 = 0.36$$

$$P(z_1) = \sum P(z_1 | y) = P(z_1, y_1) + P(z_1, y_0) P(y_0)$$
$$= \frac{0.25}{0.36} + 0.6 \times 0.64$$
$$= 0.474$$

$$P(w_1) = \sum P(w_1 | z) = \frac{P(w_1, z_1) \times P(z_1) + P(w_1, z_0)}{P(z_0)}$$
$$= \frac{0.45 \times 0.474 + 0.30 \times 0.526}{0.36}$$
$$= 0.374$$

$P(x1)=0.60$	$P(y1 x1)=0.40$	$P(z1 y1)=0.25$	$P(w1 z1)=0.45$
x	y	z	w
$P(x0)=0.40$	$P(y0 x1)=0.60$	$P(z0 y1)=0.75$	$P(w0 z1)=0.30$
	$P(y0 x0)=0.70$	$P(z0 y0)=0.40$	$P(w0 z0)=0.70$
	$P(y1)=0.36$	$P(z1)=0.47$	$P(w1)=0.37$
	$P(y0)=0.64$	$P(z0)=0.53$	$P(w0)=0.63$

a) If x is measured to be $x=1$ (x_1), compute $P(z1|x1)$ and $P(w0|x1)$.

b) If w is measured to be $w=1$ (w_1) compute $P(z1|w1)$.

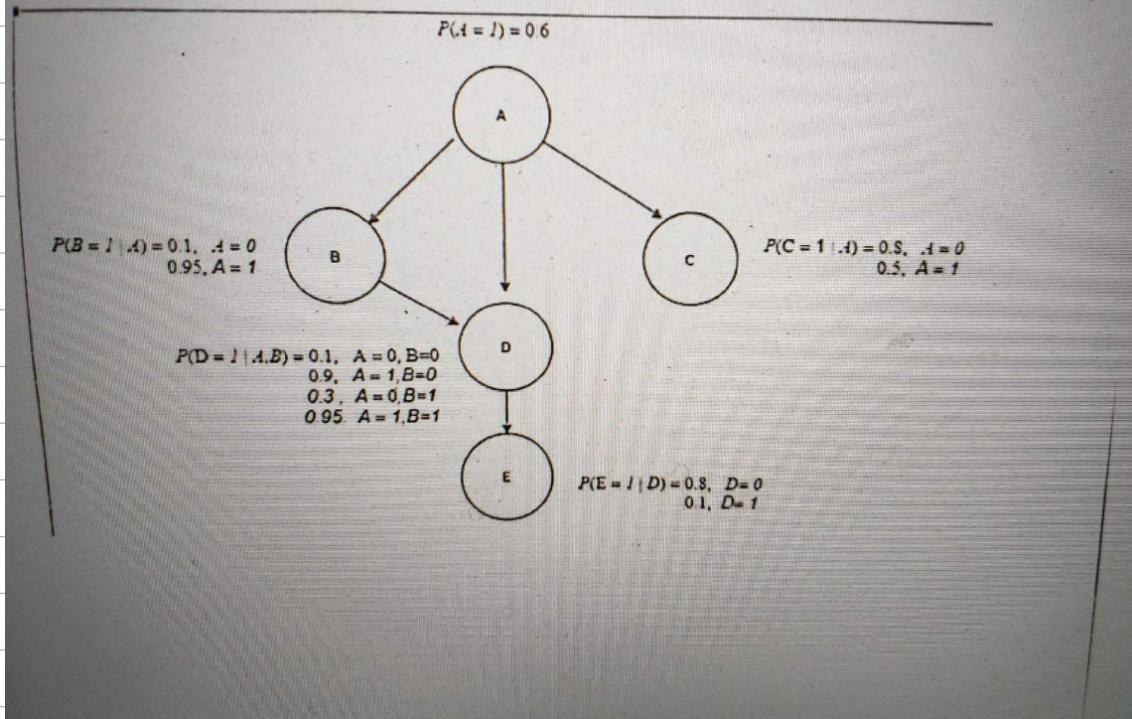
$$\begin{aligned} i) P(z_1|x_1) &= P(z_1|y_1, x_1) P(y_1|x_1) + P(z_1|y_0, x_1) \\ &\quad \times P(y_0|x_1) \\ &= P(z_1|y_1) P(y_1|x_1) + P(z_1|y_0) \times P(y_0|x_1) \\ &= 0.25 \times 0.40 + 0.60 \times 0.60 \\ &= \end{aligned}$$

$$\begin{aligned} P(w_0|x_1) &= P(w_0|z_1, x_1) P(z_1|x_1) + P(w_0|z_0, x_1) \\ &\quad P(z_0|x_1) \\ &= P(w_0|z_1) P(z_1|x_1) + P(w_0|z_0) \times P(z_0|x_1) \end{aligned}$$

$$P(z_1|w_1) = \frac{P(w_1|z_1) P(z_1)}{P(w_1)} =$$

Find the value of $P(A=1, B=1, C=1, D=1, E=1)$ from the Bayes network below.:

[3]



$$P(B_1 | A_0) = 0.1$$

$$P(B_1 | A_1) = 0.95$$

$$P(A_1) = 0.6$$

$$P(C_1 | A_0) = 0.5$$

$$P(C_1 | A_1) = 0.5$$

$$P(D_1 | A_0, B_0) = 0.1$$

$$P(D_1 | A_0, B_1) = 0.3$$

$$P(D_1 | A_1, B_0) = 0.9$$

$$P(D_1 | A_1, B_1) = 0.95$$

$$P(E_1 | D_0) = 0.8$$

$$P(E_1 | D_1) = 0.1$$

$$P(A_1, B_1, C_1, D_1, E_1)$$

$$= P(A_1) \times P(B_1 | A_1) \times P(C_1 | A_1) \times P(D_1 | B_1, A_1) \times P(E_1 | D_1)$$

$$= 0.6 \times 0.95 \times 0.5 \times 0.95 \times 0.1$$

=

1. A Bayesian network and corresponding conditional probability tables for are shown below. Compute the following probabilities. [6]

$P(E)$	
$+e$	0.4
$-e$	0.6
$P(S E, M)$	
$+e$	$+m \quad +s \quad 1.0$
$+e$	$+m \quad -s \quad 0.0$
$+e$	$-m \quad +s \quad 0.8$
$+e$	$-m \quad -s \quad 0.2$
$-e$	$+m \quad +s \quad 0.3$
$-e$	$+m \quad -s \quad 0.7$
$-e$	$-m \quad +s \quad 0.1$
$-e$	$-m \quad -s \quad 0.9$

$P(M)$	
$+m$	0.1
$-m$	0.9
$P(B M)$	
$+m$	$+b \quad 1.0$
$+m$	$-b \quad 0.0$
$-m$	$+b \quad 0.1$
$-m$	$-b \quad 0.9$

$$(a) P(-e, -s, -m, -b)$$

$$(b) P(+b)$$

$$(c) P(+m | +b)$$

2. What is Maximum a Posteriori (MAP) estimation in statistical inference and how does it differ from Maximum Likelihood Estimation (MLE)? [4]

$$1) P(-e, -s, -m, -b)$$

$$= P(-e) \times P(-s | -e, -m) \times P(-m) \times P(-m | -b)$$

$$= 0.6 \times 0.9 \times 0.9 \times 0.9$$

$$2) P(+b) = \sum P(+b | M)$$

$$= P(+b | +M) \times P(+M) + P(+b | -M) P(-M)$$

$$= 1 \times 0.1 + 0.9 \times 0.1$$

$$3) P(+m | +b) = \frac{P(+b | +M) P(+M)}{P(+b)}$$

$$= \frac{1 \times 0.1}{0.19} = 0.52$$