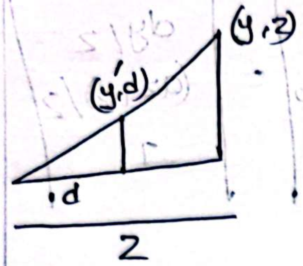
Perspective Projection



For 1D

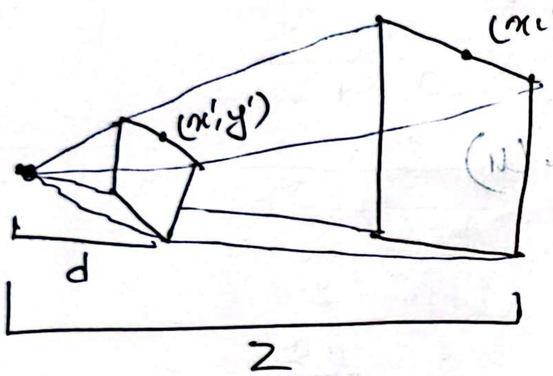
$$\frac{y'}{d} = \frac{y}{z}$$

$$\Rightarrow y' = \frac{dy}{z}$$

$$\begin{bmatrix} d & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} y' \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} dy \\ z \end{bmatrix} = \begin{bmatrix} dy/z \\ z/z \end{bmatrix} = \begin{bmatrix} dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} y' \\ 1 \end{bmatrix}$$

For 2D



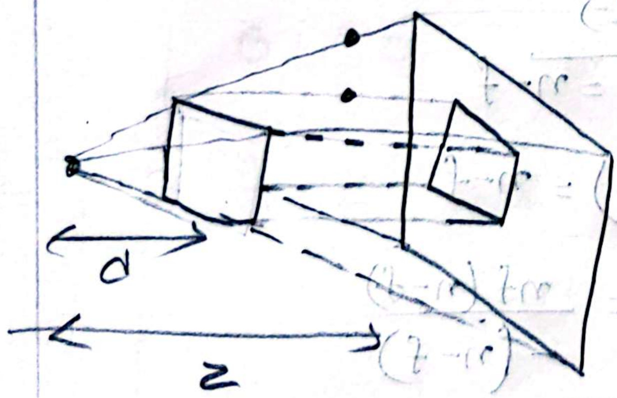
$$y' = \frac{dy}{z}$$

$$x' = \frac{dx}{z}$$

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} dx \\ dy \\ z \end{bmatrix} = \begin{bmatrix} dx/z \\ dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

For 3D



$$y' = \frac{dy}{z}$$

$$x' = \frac{dx}{z}$$

$$z' = z$$

There will be always division by z so $z' = z$ not possible.

If $z' = z$ it would imply no perspective effect ~~remaining~~ objects at different depths would appear the same size as if viewed orthographically. To keep depth info in 2D compressed z is important. so find new $z = az+b$

$y' = dy/z$

$x' = dx/z$

$z' = \dfrac{az+b}{z}$

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ az+b \\ z \end{bmatrix} = \begin{bmatrix} dx/z \\ dy/z \\ (az+b)/z \\ 1 \end{bmatrix}$$

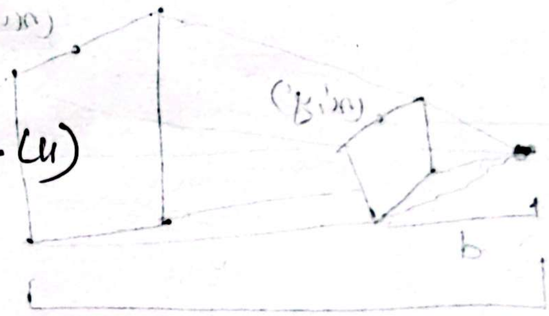
Now determine (a, b)

$z = n$
$x = 0$
$y = 0$

$$\begin{bmatrix} 0 \\ 0 \\ (an+b)/n \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ n \\ 1 \end{bmatrix} \text{ —— (i)}$$

$z = f$
$x = 0$
$y = 0$

$$\begin{bmatrix} 0 \\ 0 \\ (fa+b)/f \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ f \\ 1 \end{bmatrix} \text{ —— (ii)}$$



(i) − (ii)

$\dfrac{an+b}{n} = n$

$\dfrac{af+b}{f} = f$

$\rightarrow$

$a + \dfrac{b}{n} = n$

$a + \dfrac{b}{f} = f$

$b\left(\dfrac{1}{n} - \dfrac{1}{f}\right) = n - f$

$\Rightarrow b\left(\dfrac{f-n}{nf}\right) = n - f$

$\Rightarrow b = \dfrac{nf(n-f)}{-(n-f)}$

$\boxed{b = -nf}$

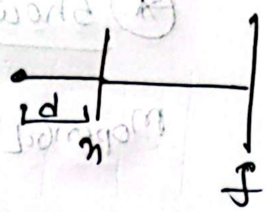
Put $b = -nf$ (i)

$a + \dfrac{-nf}{n} = n$

$\Rightarrow a - f = n$.

$\Rightarrow \boxed{a = n+f}$

Perspective Matrix: $\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$



$M = M_{vp} * M_{orth} * P * M_{cam}$

$M_{per} = M_{orth} * P$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{-r+l} & \cancel{0} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & \cancel{0} & 0 \\ 0 & 0 & \frac{2(n+f)}{n-f} - \frac{n+f}{n-f} & & \frac{2}{n-f}(-nf) \\ 0 & 0 & \cancel{0}\ 1 & & \cancel{0} \end{bmatrix}$$

$$\frac{2nf}{f-n}$$

$$\cancel{0}\ \frac{n+f(2-1)}{n-f} = \frac{n+f}{n-f}$$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{n+f}{n-f} & \frac{2nf}{f-n} \\ 0 & 0 & \cancel{0}\ 1 & \cancel{0} \end{bmatrix} = \underline{M_{per}}$$

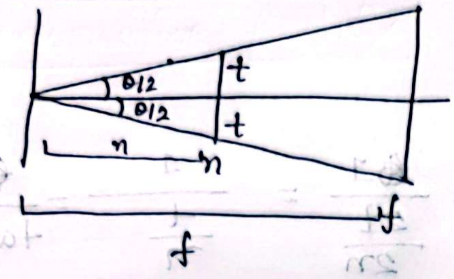
(*) Show that, the MopenGL can be written as follow:

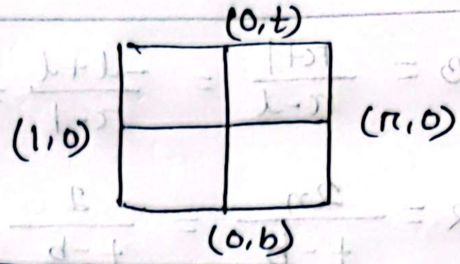
$$M_{openGL} = \begin{bmatrix} \overset{(m_{11})}{\frac{2|n|}{r-l}} & 0 & \overset{(m_{13})}{\frac{r+l}{r-l}} & 0 \\ 0 & \overset{(m_{22})}{\frac{2|n|}{t-b}} & \overset{m(23)}{\frac{t+b}{t-b}} & 0 \\ 0 & 0 & \overset{m(33)}{\frac{|n|+|f|}{|n|-|f|}} & \overset{m(34)}{\frac{2|f||n|}{|n|-|f|}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \overset{(m_{11})}{\frac{1}{aspect * \tan\left(\frac{fov}{2}\right)}} & 0 & \overset{(m_{13})}{0} & 0 \\ 0 & \overset{(m_{22})}{\frac{1}{\tan\left(\frac{fov}{2}\right)}} & \overset{m(2,3)}{0} & 0 \\ 0 & 0 & \overset{m(33)}{-\frac{far+near}{far-near}} & \overset{m(3,4)}{-\frac{2*far*near}{far-near}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Start 8

$\theta/2 = fov$ [field of view]





$\boxed{r = -l \, ; \, t = -b}$

From M openGL,

$$m_{11} = \frac{2n}{r-l}$$

$$= \frac{1}{\frac{r-l}{2n}}$$

$$= \frac{1}{\frac{r-l}{2n} \times \frac{t-b}{t-b}}$$ cal·রজন্য → নিয়ে আসছি

$$= \frac{1}{\frac{r-l}{t-b} \times \frac{t-b}{2n}}$$

aspect $= \frac{width}{heigth}$

$$= \frac{1}{aspect \times \frac{t-(-t)}{2n}}$$

$$= \frac{1}{aspect \times \frac{t+t}{2n}}$$

$$= \frac{1}{aspect \times \frac{2t}{2n}}$$

$$= \frac{1}{aspect \times \frac{t}{n}}$$

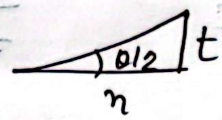
$$= \frac{1}{aspect \tan\left(\frac{fov}{2}\right)}$$

$t - b = height$

$\Rightarrow t - (-t) = $ "

$\Rightarrow 2t = height.$

$$aspect = \frac{r-l}{t-b} = \frac{width}{height}$$



$$\tan\theta/2 = \frac{t}{n}$$

$$\theta/2 = \frac{fov}{2} = \tan^{-1}(t/n)$$

$$m_{13} = \frac{r+l}{r-l} = \frac{-l+l}{r-l} = 0.$$

$$m_{22} = \frac{2n}{t-b} = \frac{1}{\frac{t-b}{2n}} = \frac{1}{\frac{t-(-t)}{2n}} = \frac{1}{\frac{2t}{2n}} = \frac{1}{\frac{t}{n}} = \frac{1}{\tan\left(\frac{fov}{2}\right)}$$

$$m_{23} = \frac{t+b}{t-b} = \frac{t+(-t)}{t-b} = \frac{t-t}{t-b} = 0.$$

$$m_{33} = \frac{n+f}{n-f} = -\frac{n+f}{f-n} = -\frac{far+near}{far-near}$$

$$m_{34} = \frac{2fn}{n-f} = -\frac{2fn}{f-n} = -\frac{2*far*near}{far-near}.$$

[Proved]