LECTURE 4

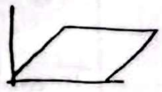
Lecture-4

Scaling:

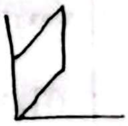
$$Scale(S_x, S_y) = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

Shearing:

$$shear\text{-}x(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$



$$Shear\text{-}y(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$



Reflection:

$$Reflect\text{-}y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Reflect\text{-}x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Translation:

For 2D:

$$\begin{bmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{bmatrix}$$

For 3D:

$$\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling:

For 2D:

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 3D:

$$\begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation:

For 2D:

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 3D:

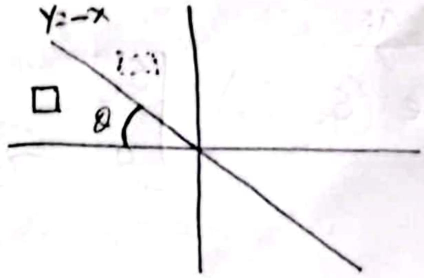
$$Rot\text{-}x: \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$Rot\text{-}z: \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Rot\text{-}y: \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

Integer

2(b)



Transformation matrix for the reflection about the line $y=-x$:

$m_1 = Rot(45°) * Ref\text{-}y * Rot(-45°)$

$$= \begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{bmatrix}$$

Here, $m=-1$

$\tan\theta = -1$; $\theta = \tan^{-1}(-1)$

$\theta = -45°$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Reflection related to y-axis followed by a counter-clockwise rotation of 90°

$m_2 = Rot(90°) * Ref\text{-}Y$

$$= \begin{bmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

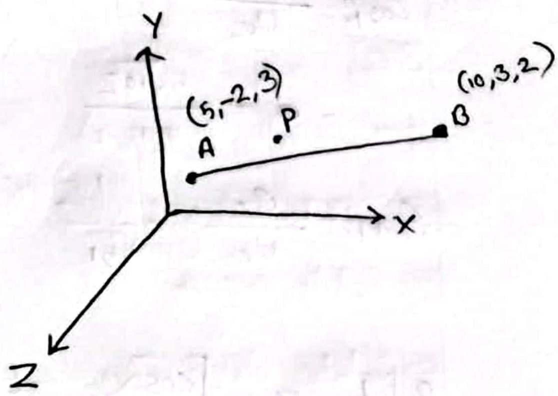
$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$\therefore m_1 = m_2$

3(b)

1. Translate $(-5,-2,-3)$
2. Rotate along Z
3. Rotate along X
4. Rotate along Y
5. Rotate along X
6. Rotate along Z
7. Translate $(5,-2,3)$



Translate $(5,-2,3)$:

$$T = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

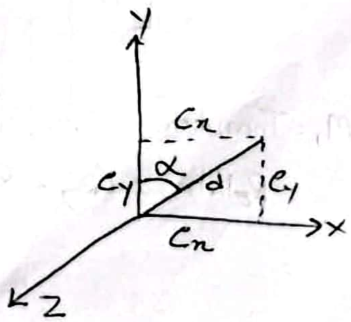
Find unit vectors:

$$U_e = \frac{B-A}{|B-A|_{x,y,z}} = C_x, C_y, C_z$$

$$C_x = \frac{10-5}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$C_y = \frac{(3+2)}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{5}{\sqrt{51}}$$

$$C_z = \frac{2-3}{\sqrt{(10-5)^2+(3+2)^2+(2-3)^2}} = \frac{-1}{\sqrt{51}}$$



$$d = \sqrt{C_x^2+C_y^2} = \sqrt{\left(\frac{5}{\sqrt{51}}\right)^2+\left(\frac{5}{\sqrt{51}}\right)^2} = \frac{5\sqrt{102}}{51}$$

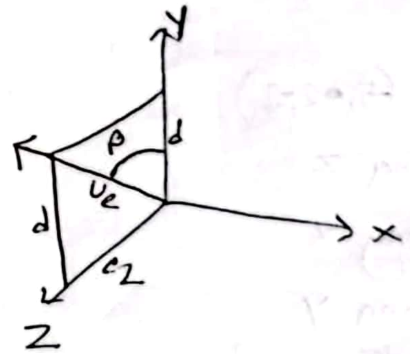
$$\cos\alpha = \frac{C_y}{d} = \frac{\frac{5}{\sqrt{51}}}{\frac{5\sqrt{102}}{51}} = \frac{1}{\sqrt{2}}$$

$$\sin\alpha = \frac{C_x}{d} = \frac{1}{\sqrt{2}}$$

$$\cos\beta = \frac{d}{v_e}$$

$$= d = \frac{5\sqrt{102}}{51}$$

$$\sin\beta = \frac{c_z}{v_e} = -\frac{1}{\sqrt{51}}$$



$$\text{Rotate\_Z}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{AB = \begin{bmatrix} 5 & -2 & 3 & 1 \\ 10 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

$$\text{Rotate\_X}(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*** For Question ans would be:

$\cancel{M_1 = T^{-1}(5,-2,3)}$

$M_1 = \text{Rotate\_X}(\beta) * \text{Rotate\_Z}(\alpha) * \text{Translate}(5,-2,3) * AB$

$$\text{Rotate\_Y}(Q = -90^\circ) = \begin{bmatrix} \cos Q & 0 & \sin Q & 0 \\ 0 & 1 & 0 & 0 \\ -\sin Q & 0 & \cos Q & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$; AB = \begin{bmatrix} 5 & 10 \\ -2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$M_1 = \text{Translate}^{-1}(5,-2,3) * \text{Rotate\_Z}(-\alpha) * \text{Rotate\_X}(-\beta) * \text{Rotate\_Y}(Q) * \text{Rotate\_X}(\beta) * \text{Rotate\_Z}(\alpha) * \text{Translate}(5,-2,3) * P$$

$=$

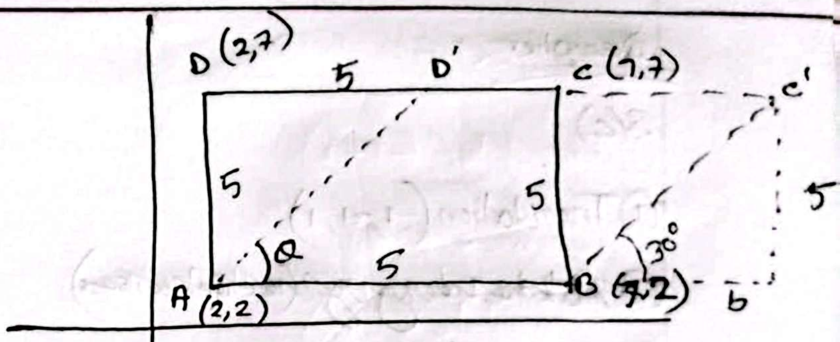
3(d)

Steps:

① Translate by $(-2,-2)$

② Shear along x-axis by 1.732

③ Translate by $(2,2)$



$M_1 = T(2,2) * Shear\_x(1.732) * T(-2,-2)$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1.732 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1.732 & -3.464 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $M_1 \times V = M_1 \times \begin{bmatrix} 2 & 7 & 7 & 2 \\ 2 & 2 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

=

$Q = 30$

$\tan Q = \frac{5}{b}$

$b = 5\sqrt{3}$

$= 8.6602$

$\therefore$ to shear by x-axis for 8.6602 points

Shear factor $= \frac{8.6602}{5} \xrightarrow{} \Delta y$

$= 1.732$ $(7-2) = 5$

For shear factor

along x-axis; divide by Δy

along y-axis; divide by Δx

Decipher

3(c)

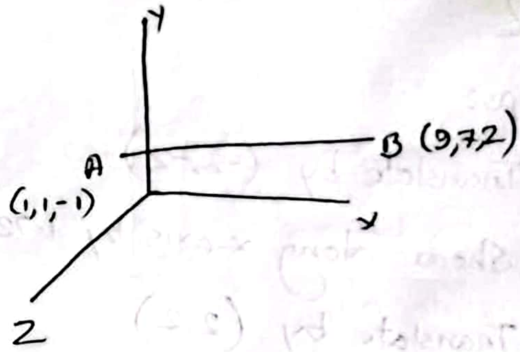
① Translation $(-1,-1,1)$

~~Rotate along y (anticlockwise)~~

~~Rotate along~~

② Rotate along z

③ Rotate along x (anticlock)



Translate $(-1,-1,1)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find unit vectors:

$$U_e = \frac{B-A}{|B-A|} = c_x, c_y, c_z$$

$$c_x = \frac{9-1}{\sqrt{(9-1)^2 + (7-1)^2 + (2+1)^2}} = \frac{8}{\sqrt{109}}$$

$$c_y = \frac{7-1}{\sqrt{109}} = \frac{6}{\sqrt{109}}$$

$$c_z = \frac{2+1}{\sqrt{109}} = \frac{3}{\sqrt{109}}$$

$$; \; d = \sqrt{c_x^2 + c_y^2} = \sqrt{\left(\frac{8}{\sqrt{109}}\right)^2 + \left(\frac{6}{\sqrt{109}}\right)^2} = 0.9578$$

$$\sin\alpha = \frac{c_y}{d} = \frac{\frac{6}{\sqrt{109}}}{0.9578}$$

$$\cos\alpha = \frac{c_x}{d} = \frac{\frac{8}{\sqrt{109}}}{0.9578}$$

$$\cos\beta = \frac{d}{u_e} = 0.9578 \; ; \; \sin\beta = \frac{c_z}{u_e} = \frac{3}{\sqrt{109}}$$

$$M_1 = Rot\_x(\beta) * Rot\_z(\alpha) * T(-1,-1,1)$$

$$=$$

$$M_1 \times V = M_1 \times \begin{bmatrix} 1 & 9 \\ 1 & 7 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

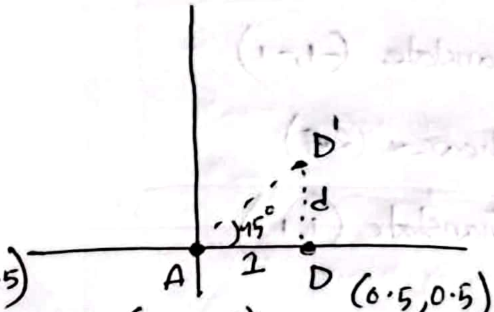
(Ans.)

3(g)

(1) Translate (0.5, -0.5)

(2) Shear along y-axis by 1

(3) Translate (-0.5, 0.5)



$M_1 : T(-0.5, 0.5) * \text{Shear\_y}(1) * T(0.5, -0.5)$ (-0.5, 0.5)

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate (45°)

$\theta = 45°$

$\tan\theta = \frac{d}{1}$

$\therefore d = 1$

$\therefore$ to shear by y axis for 1 points

shear factor $\frac{1}{1} = 1$

=

$$M_1 \times V = M_1 \times \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \\ 1 & 1 \end{bmatrix}$$

=

Again;

(1) Translate (-0.5, 0.5)

(2) Rotate (-90°)

(3) Translate (0.5, -0.5)

$M_2 = T(0.5, -0.5) * \text{Rotate}(-90°) * \text{Translate}(-0.5, 0.5)$

=

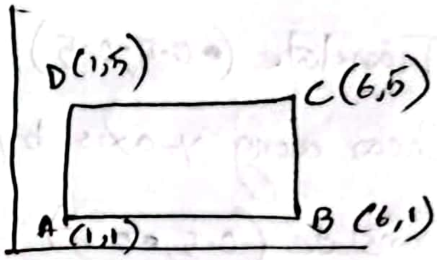
$$M_2 \times V = M_2 \times \begin{bmatrix} -0.5 & -0.5 & 0.5 & 0.5 & . \\ 0.5 & -0.5 & -0.5 & 0.5 & . \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Quiz- Set-A

(1) Translate (-1,-1)

(2) Shear_x (2)

(3) Translate (1,1)



$m_1 = T(-1,-1) * Shear\_x(2) * T(-1,-1)$

$\Delta y = |5-1| = 4$

$\text{Shear factor} = \frac{8}{4}$

$= 2$

$=$

$$m_1 \times V = m_1 \times \begin{bmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Show that two succesive reflections about eith of the principal axis is equivalent to a single rotation about the coordinate origin.

$\Rightarrow m_1 = Ref\_y * Ref\_x$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$=$

$m_2 = Rotation(180^\circ)$

$=$

$M_1 = R(45°) * R^{-1}(45°)$

$$= \begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix} \begin{bmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$=$

$M_2 = R(45° - 45°)$

$= R(0°)$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Prev sem quiz-Set D

For OA;

12 hour 360°

1 " 30°

$\therefore$ 4 hour 120°

① Translate $(-8,-8)$

② Rotate (-120°)

③ Translate $(8,8)$

For OB;

60 minute $= 360^\circ$

1 " $= 6^\circ$

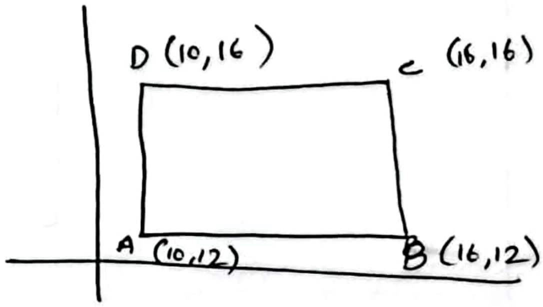
30 " $= 180^\circ$

① Translate $(-8,-8)$

② Rotate (-180°)

③ Translate $(8,8)$

Quiz-2 Set-F



line: $2y - 6x + 2 = 0$

$y - 3x + 1 = 0$

$\therefore y = 3x - 1$

The line is 1 unit below origin on y-axis.

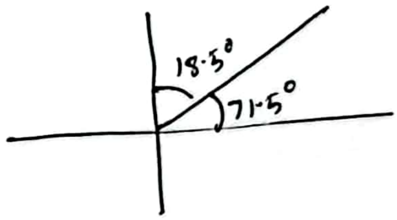
by translating (0,1),

$y = 3x$

here, $m = 3$

$\tan\theta = 3$

$\theta = 71.5$



① Translate (0,1)

② Rotate (18.5°)

③ Reflect-Y

④ Rotate (-18.5°)

⑤ Translate (0,-1)