

3/1 隨堂練習

(1) Cobb-Douglas 生產函數:  $Q = f(K, L) = L^\alpha K^\beta$

① 產出彈性:

$$AR = \frac{Q}{L} = \frac{L^\alpha K^\beta}{L} = L^{\alpha-1} K^\beta$$

$$MR_L = \frac{dQ}{dL} = \alpha L^{\alpha-1} K^\beta$$

$$AP_K = \frac{Q}{K} = \frac{L^\alpha K^\beta}{K} = L^\alpha K^{\beta-1}$$

$$MR_K = \frac{dQ}{dK} = \beta L^\alpha K^{\beta-1}$$

勞動產出彈性:

$$\epsilon^L = \frac{MR_L}{AR} = \frac{\alpha L^{\alpha-1} K^\beta}{L^{\alpha-1} K^\beta} = \alpha$$

資本產出彈性:

$$\epsilon^K = \frac{MR_K}{AP_K} = \frac{\beta L^\alpha K^{\beta-1}}{L^\alpha K^{\beta-1}} = \beta$$

生產力彈性:

$$\epsilon^R = \epsilon^L + \epsilon^K = \alpha + \beta$$

替代彈性:

$$MRTS = \frac{MR_L}{MR_K} = \frac{\alpha L^{\alpha-1} K^\beta}{\beta L^\alpha K^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{K}{L}$$

$$\epsilon^K = \frac{d \ln(\frac{K}{L})}{d \ln(MRTS)} = \frac{d \ln(\frac{K}{L})}{d \ln(\frac{\alpha}{\beta}) + d \ln(\frac{K}{L})} = 1$$

(2)  $Q = 3K + 2L$  (K: 資本, L: 勞力, Q: 產出)

假設 K 和 L 同時增加  $\alpha$  倍, 為  $\alpha K, \alpha L$

$$F(\alpha K, \alpha L) = 3(\alpha K) + 2(\alpha L) = \alpha(3K + 2L) = \alpha Q$$

① K, L 增加  $\alpha$  倍, Q 也增加  $\alpha$  倍, 故生產函數為固定規模報酬

$$\textcircled{2} \quad MR_L = \frac{\partial Q}{\partial L} = 2, \quad MR_K = \frac{\partial Q}{\partial K} = 3$$

$MR_L, MR_K$  固定, 沒有邊際產量遞減的現象

$$\textcircled{3} \quad MRTS_{LK} = \frac{MR_L}{MR_K} = \frac{2}{3}, \text{ 邊際技術替代率為一固定值。}$$