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THEORY OF MATRIX

Homework

11

Problems

1. Problem 11

   对于下列矩阵 A, 求正交 (酉) 矩阵 P, 使 $P^{-1}AP$ 为对角矩阵:

   (a) $A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & j & 1 \\ -j & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   ### Answer

   (a) $\lambda I - A = \begin{vmatrix} \lambda-2 & -2 & 2 \\ -2 & \lambda-5 & 4 \\ 2 & 4 & \lambda-5 \end{vmatrix} = (\lambda-1)^2(\lambda-10) = 0.$

   所以 A 的特征值 $\lambda_1 = \lambda_2 = 1, \lambda_3 = 10$

   $\lambda_1 = \lambda_2 = 1$ 对应的特征向量为 $x_1 = (2,0,1)^T, x_2 = (-2,1,0)^T.\lambda_3$ 对应的特征值为 $x_3 = (1,2,-2)^T$

   因为有特征值具有 2 重根, 所以对象的向量不正交, 将其进行正交化后进行规范化.

   采用施密特正交化可得正交矩阵后进行单位化:

$$P = \begin{bmatrix} \frac{2}{3\sqrt{5}} & -\frac{2}{\sqrt{5}} & -\frac{1}{3} \\ \frac{4}{3\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{2}{3} \\ \frac{5}{3\sqrt{5}} & 0 & \frac{2}{3} \end{bmatrix}$$

   使得 $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

(b) $\lambda I-A=\begin{vmatrix}\lambda & -j & -1\\ j & \lambda & 0\\ -1 & 0 & \lambda\end{vmatrix}=\lambda(\lambda^2-2)=0.$

所以 A 的特征值 $\lambda_1=-\sqrt{2},\lambda_2=\sqrt{2},\lambda_3=0$

$\lambda_1=-\sqrt{2},x_1=(-\sqrt{2},-j,1)^T,\lambda_2=\sqrt{2},x_2=(\sqrt{2},-j,1).\lambda_3=0,x_3=(0,j,1)$

因为特征值互不相同, 所以特征向量相互正交, 只需将其规范化即可.

所以正交矩阵

$$P=\begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ -\frac{j}{2} & -\frac{j}{2} & \frac{j}{\sqrt{2}}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

使得 $P^{-1}AP=\begin{bmatrix}-\sqrt{2} & 0 & 0\\ 0 & \sqrt{2} & 0\\ 0 & 0 & 0\end{bmatrix}$