Photoacoustic Tomography | Spring 2020

# Time Reversal Related Numerical Experiments

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## 1 Forward Modeling

Throughout the whole experimental process, I used the finite difference scheme to simulate wave propagation in unbounded domain. The scheme follows the paper: '**The perfectly matched layer for acoustic waves in absorptive media**' by Qing-Huo Liu and Jianping Tao. To be more specific, the original second order wave equation is decomposed into 2 first order equations:

$$\begin{cases} \partial_t \mathbf{v}(\mathbf{x},t) = -\nabla p(\mathbf{x},t), \\ \partial_t p(\mathbf{x},t) = -c^2(\mathbf{x}) \nabla \cdot \mathbf{v}(\mathbf{x},t). \end{cases}$$

Here $p(\mathbf{x},t)$ denotes pressure field, and vector $\mathbf{v}(\mathbf{x},t)$ denotes particle velocity fields.

By complex coordinate stretching, the above formula in 2D can be modified as

$$\begin{cases} \partial_t \mathbf{v}_j(\mathbf{x},t) + \omega_j(\mathbf{x})\mathbf{v}_j(\mathbf{x},t) = -\partial_j p(\mathbf{x},t), \quad \text{for } j = 1,2, \\ \partial_t p_j(\mathbf{x},t) + \omega_j(\mathbf{x}) p_j(\mathbf{x},t) = -c^2(\mathbf{x}) \partial_j \mathbf{v}_j(\mathbf{x},t), \quad \text{for } j = 1,2, \end{cases}$$

where $\mathbf{x} = (x_1, x_2)$, $\mathbf{v} = (\mathbf{v}_1, \mathbf{v}_2)$, $\partial_j$ denotes partial derivative along $x_j$, and $p = p_1 + p_2$. Also, $\omega_j$ denotes the absorption coefficient along axis $x_j$ in computational domain. According to kWave package, the absorption coefficient is chosen to be a fourth order polynomial:

$$\omega_j(\mathbf{x}) = \begin{cases} \text{const} * \left( \frac{\text{dist}(\mathbf{x}, \text{PML})}{\text{length of PML}} \right)^4 \\ 0, \qquad \text{otherwise} \end{cases}$$

And the forward modeling follows standard-staggered finite difference scheme. We compute the pressure field at uniform standard grids $p(x_j, y_k, t_n)$, and particle velocity field at staggered grid $v_x(x_{j+1/2}, y_k, t_{n+1/2})$, $v_y(x_j, y_{k+1/2}, t_{n+1/2})$. The second order finite difference scheme becomes