# CSCI 104
# Graph & Tree Search and Traversals Algorithms

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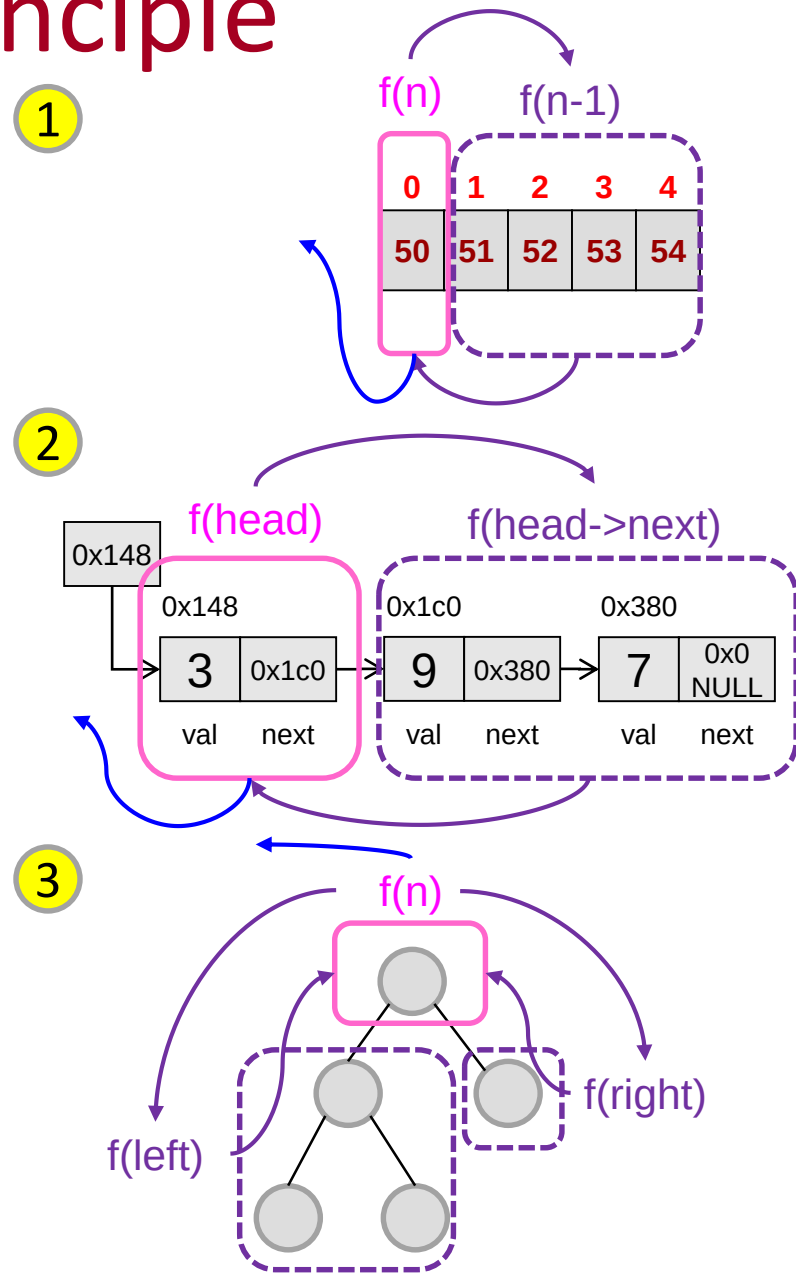
# RECURSIVE TREE TRAVERSALS

# Guiding Recursive Principle

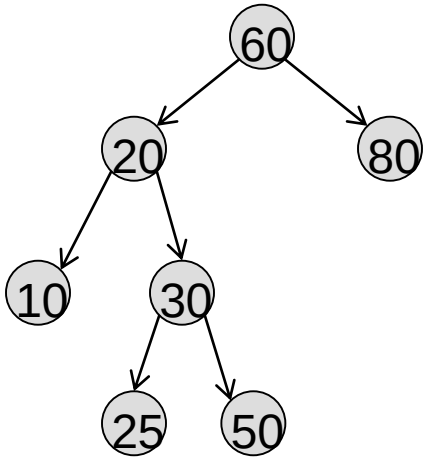
- A useful principle when trying to develop recursive solutions is that the recursive code should handle only 1 element, which might be:
  1. An element in an array
  2. A node a linked list
  3. A node in a tree
  4. One choice in a sequence of choices
- Then use recursion to handle the remaining elements
- And finally combine the solution(s) to the recursive call(s) with the one element being handled



# Recursive Tree Traversals

- A traversal iterates over all nodes of the tree
  - Usually using a depth-first, recursive approach
- Three general traversal orderings
  - Pre-order [Process root then visit subtrees]
  - In-order [Visit left subtree, process root, visit right subtree]
  - Post-order [Visit left subtree, visit right subtree, process root]

```
// Node definition
struct TNode
{
  int val;
  TNode *left, *right;
};
```




```
Preorder(TNode* t)
{  if t == NULL return
   process(t) // print val.
   Preorder(t->left)
   Preorder(t->right)
}
```

60 20 10 30 25 50 80

```
Inorder(TNode* t)
{  if t == NULL return
   Inorder(t->left)
   process(t) // print val.
   Inorder(t->right)
}
```

10 20 25 30 50 60 80

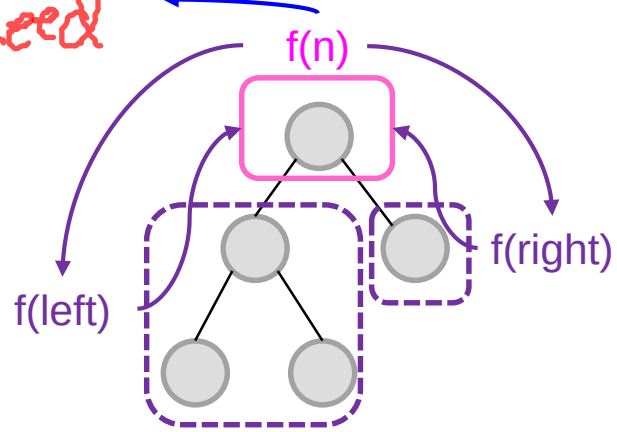
```
Postorder(TNode* t)
{  if t == NULL return
   Postorder(t->left)
   Postorder(t->right)
   process(t) // print val.
}
```

10 25 50 30 20 80 60

# Example 1: Count Nodes

- Write a recursive function to **count how many nodes** are in the binary tree
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node
  - For in- or post-order traversals, determine how to use/combine results from recursion on children

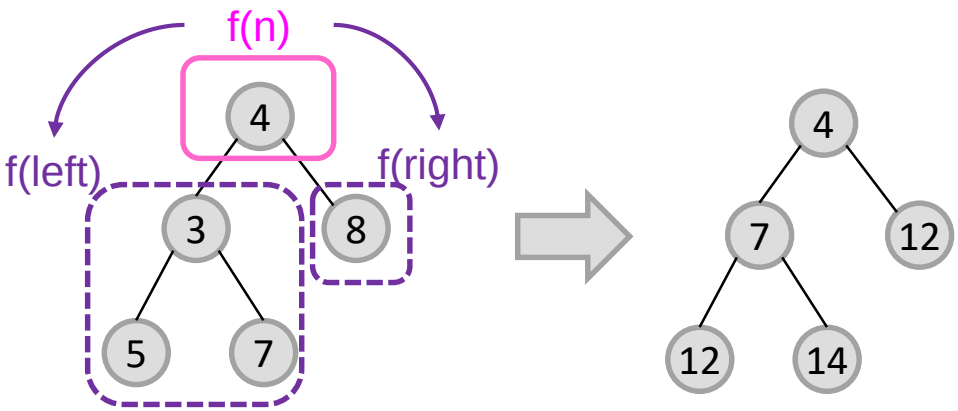
Pre: process on the way down
Post: need the results from kid




```
// Node definition
struct Tnode {
  int val;
  TNode *left, *right;
};

int count(TNode* root)
{
  if( root == NULL ) _____________;
  else {




} }
```

# Example 2: Prefix Sums

- Write a recursive function to **have each node store the sum of the values on the path from the root to each node.**
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node



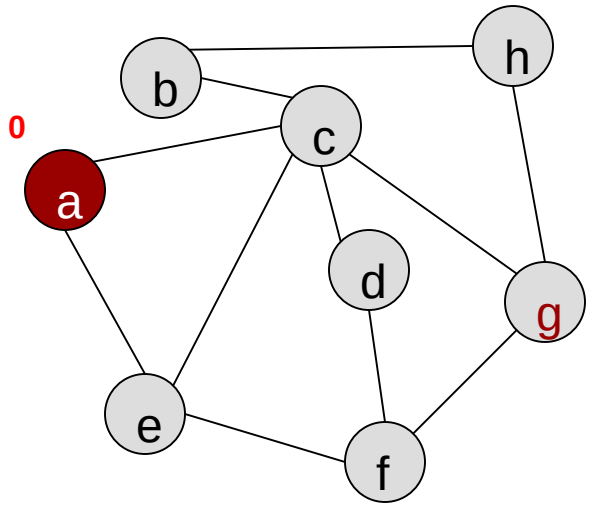

```
void prefixH(TNode* root, int psum)

void prefix(TNode* root)
{
  prefixH(root, 0);
}

void prefixH(TNode* root, int psum)
{
  if( root == NULL ) _____________;
  else {









  }
}
```

# GENERAL GRAPH TRAVERSALS

# BREADTH-FIRST SEARCH

# Breadth-First Search

- Given a graph with vertices, V, and edges, E, and a starting vertex that we'll call u
- BFS starts at u (‘a’ in the diagram to the left) and fans-out along the edges to nearest neighbors, then to their neighbors and so on
- Goal:  Find shortest paths (a.k.a. minimum number of hops or depth) from the start vertex to every other vertex




**Depth 0: a**

# Breadth-First Search

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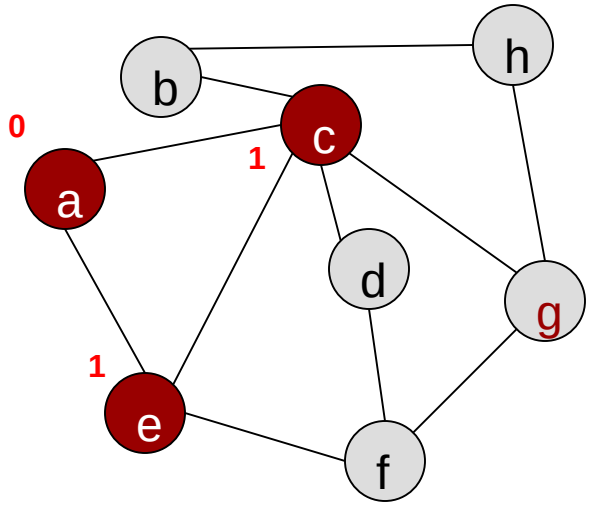


**Depth 0: a**
**Depth 1: c,e**

# Breadth-First Search

- Given a graph with vertices, V, and edges, E, and a starting vertex, u
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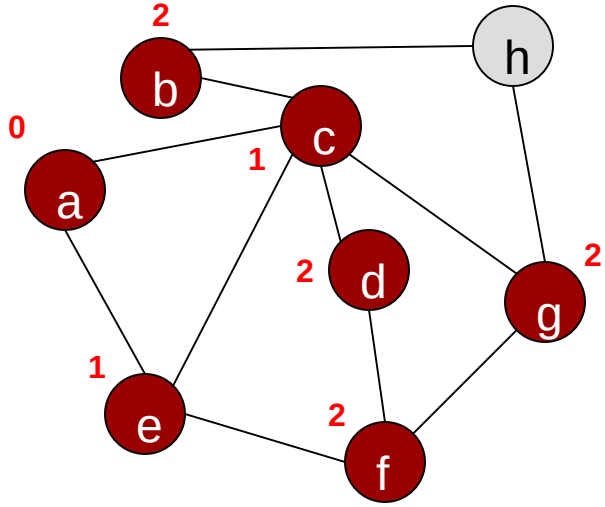


**Depth 0: a**
**Depth 1: c,e**
**Depth 2: b,d,f,g**

# Breadth-First Search

- Given a graph with vertices, V, and edges, E, and a starting vertex, u
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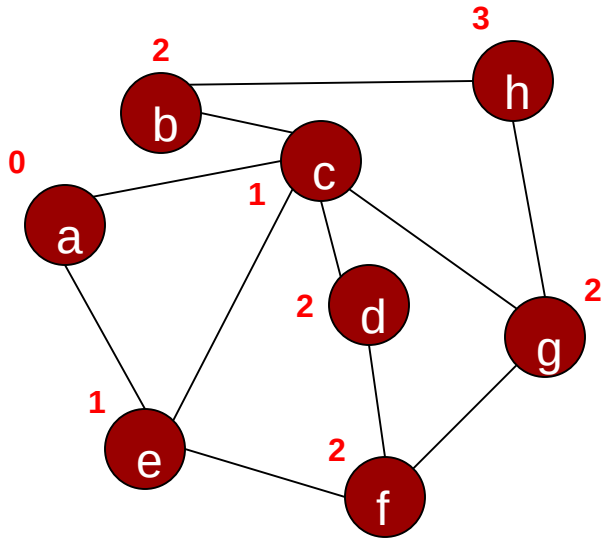
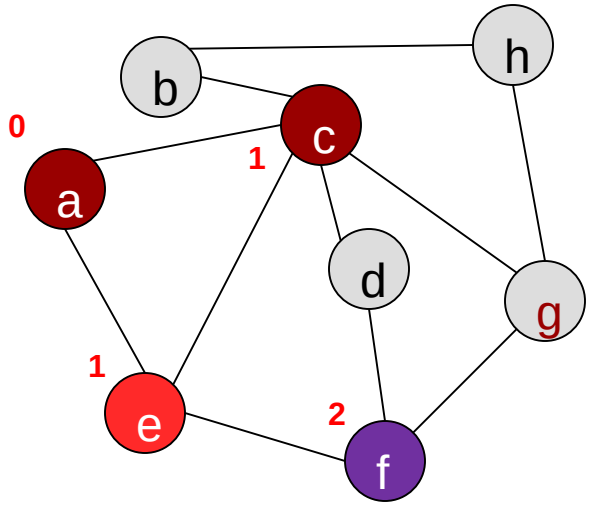



**Depth 0: a**
**Depth 1: c,e**
**Depth 2: b,d,f,g**
**Depth 3: h**

# Developing the Algorithm

- Key idea: Must explore all nearer neighbors before exploring further-away neighbors
- From ‘a’ we find ‘e’ and ‘c’
  - If we explore 'e' next and find 'f' who should we choose to explore from next: 'c' or 'f'?
- Must explore all vertices at depth i before any vertices at depth i+1
  - Essentially, the first vertices we find should be the first ones we explore from
  - What data structure may help us?




**Depth 0: a**
**Depth 1: c,e**
**Depth 2: b,d,f,g**
**Depth 3: h**

# Developing the Algorithm

- Exploring all vertices in the order they are found implies we will explore all vertices at shallower depth before greater depth
  - Keep a first-in / first-out queue (FIFO) of neighbors found
- Put newly found vertices in the back and pull out a vertex from the front to explore next
- We don't want to put a vertex in the queue more than once…
  - 'mark' a vertex the first time we encounter it
  - only allow unmarked vertices to be put in the queue
- May also keep a 'predecessor' structure that indicates how each vertex got discovered (i.e. which vertex caused this one to be found)
  - Allows us to find a shortest-path back to the start vertex

# Breadth-First Search

Algorithm:

```
BFS(G,u)
1  for each vertex v
2      pred[v] = nil, d[v] = inf.
3  Q = new Queue
4  Q.enqueue(u),  d[u]=0
5  while Q is not empty
6     v = Q.front(); Q.dequeue()
7     foreach neighbor, w, of v:
8        if pred[w] == nil // w not found
9           Q.enqueue(w)
10          pred[w] = v,  d[w] = d[v] + 1
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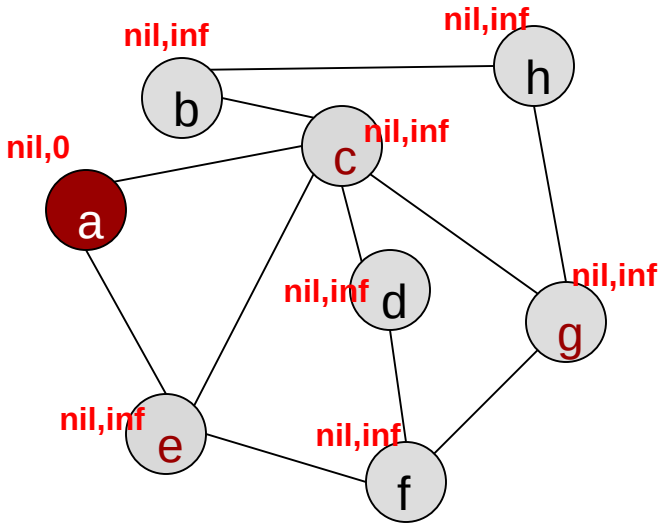


Q:

a

# Breadth-First Search

Algorithm:

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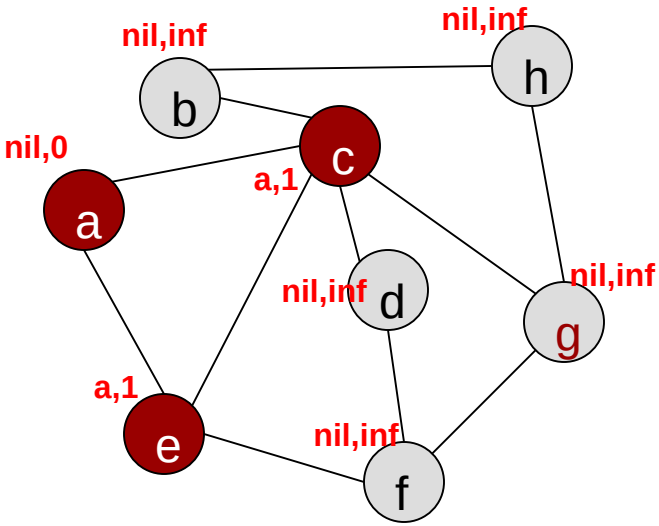


v = a

Q:

| e | c |
|---|---|

# Breadth-First Search

Algorithm:

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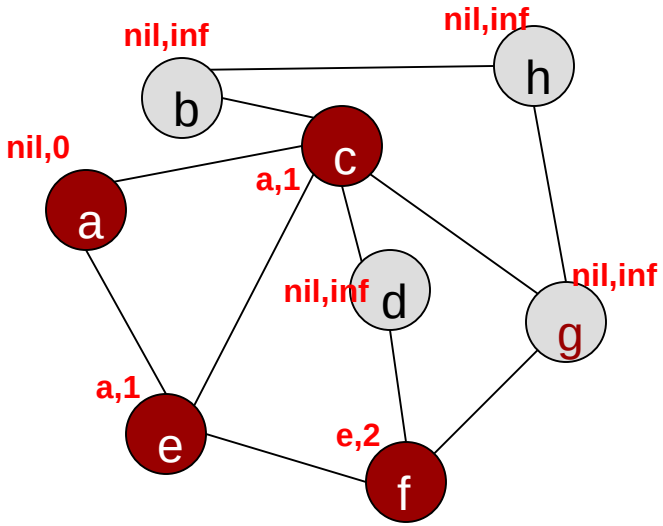


v = e

Q:

c | f

# Breadth-First Search

Algorithm:

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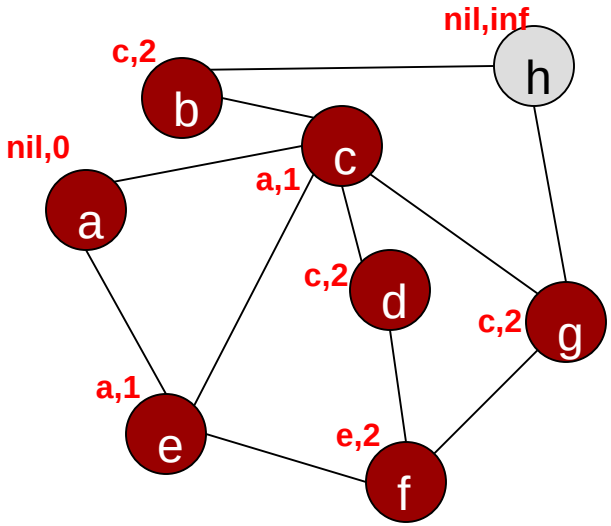


v = c

Q: f b d g

# Breadth-First Search

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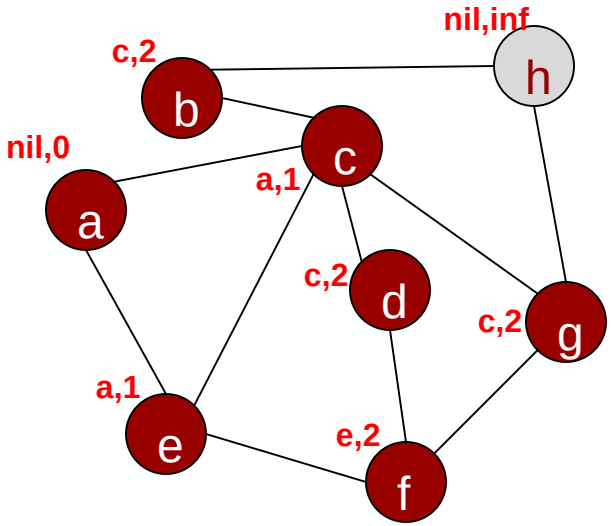


v = f

Q:

| b | d | g |
|---|---|---|

# Breadth-First Search

Algorithm:

```
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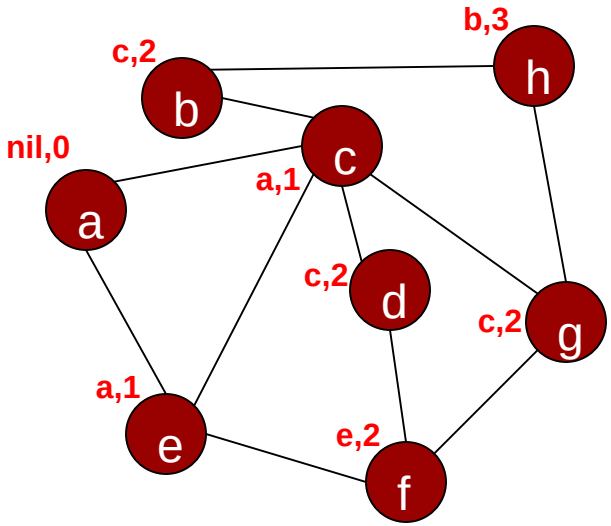


v = b

Q:

| d | g | h |
|---|---|---|

# Breadth-First Search

Algorithm:

```
BFS(G,u)
1  for each vertex v
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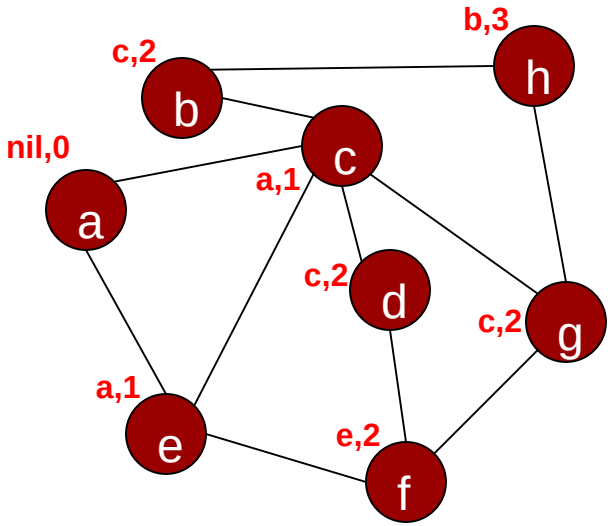


v = d

Q:

| g | h |
|---|---|

# Breadth-First Search

Algorithm:

```
BFS(G,u)
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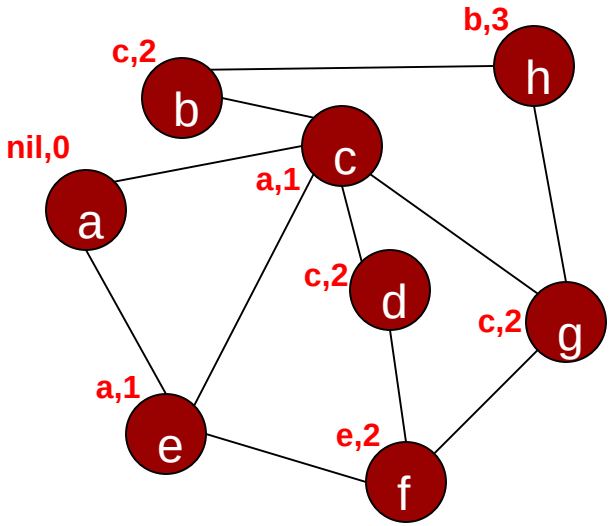


v = g

Q:

h

# Breadth-First Search

Algorithm:

```
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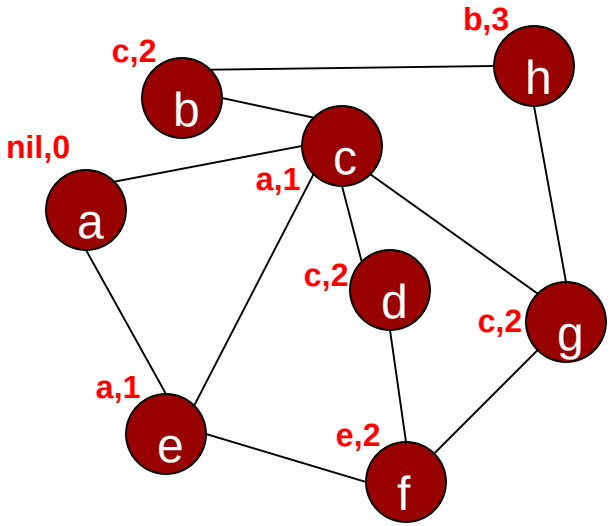
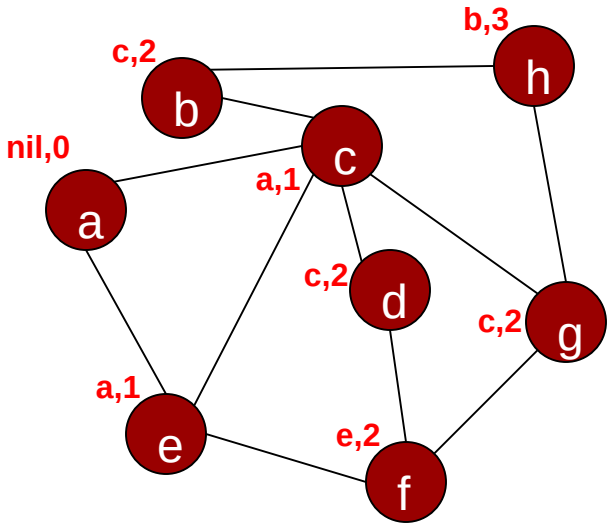



v = h

Q:

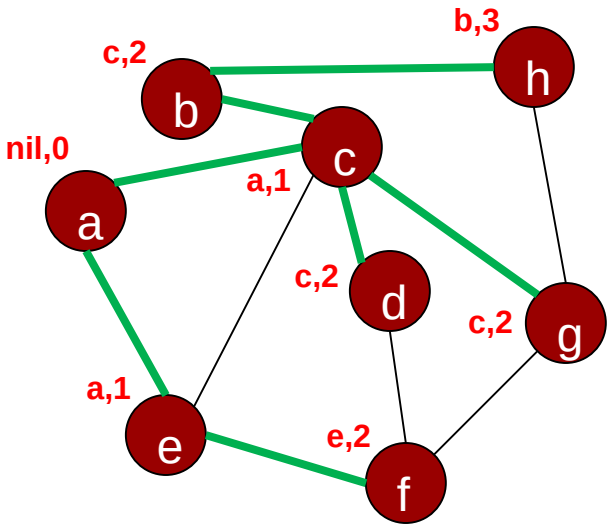
# Breadth-First Search

- Shortest paths can be found be walking predecessor value from any node backward
- Example:
  - Shortest path from a to h
  - Start at h
  - Pred[h] = b (so walk back to b)
  - Pred[b] = c (so walk back to c)
  - Pred[c] = a (so walk back to a)
  - Pred[a] = nil … no predecessor, Done!!

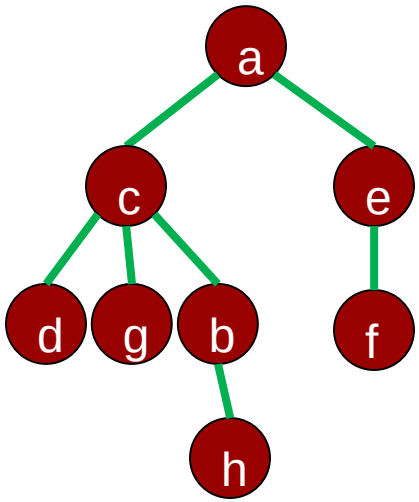


# Breadth-First Search Trees

- BFS (and later DFS) will induce a tree subgraph (i.e. acyclic, one parent each) from the original graph
  - BFS is tree of shortest paths from the source to all other vertices (in connected component)



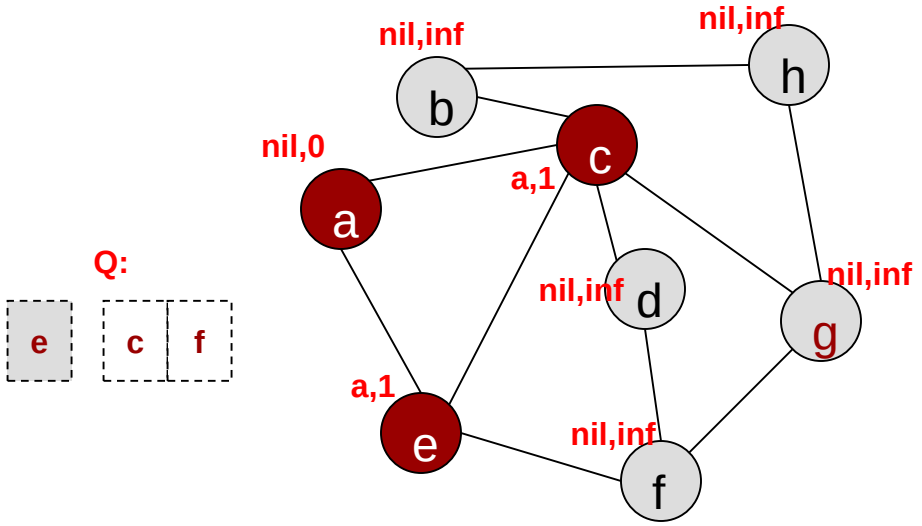

Original graph, G




BFS Induced Tree

# Correctness

- Define
  - dist(s,v) = correct shortest distance
  - d[v] = BFS computed distance
  - p[v] = predecessor of v
- Loop invariant
  - What can we say about the nodes in the queue, their d[v] values, relationship between d[v] and dist[v], etc.?

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```
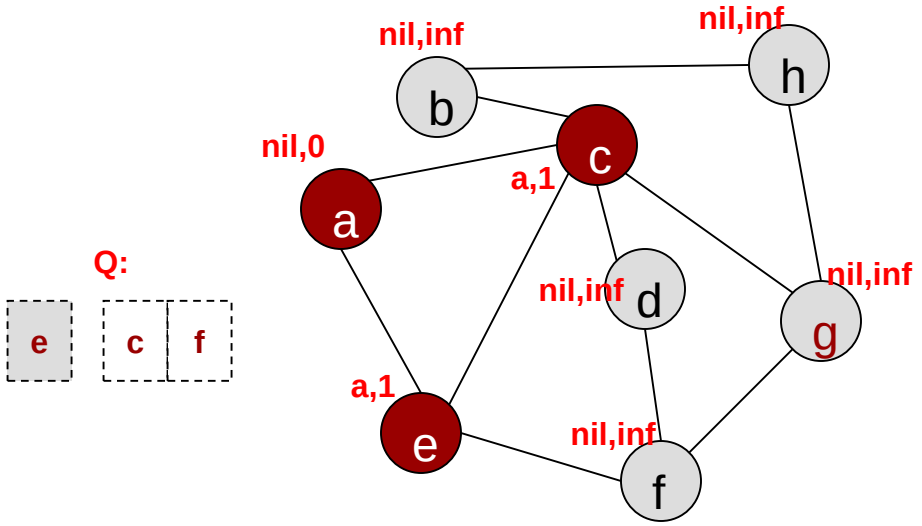

# Correctness

- Define
  - dist(s,v) = correct shortest distance
  - d[v] = BFS computed distance
  - p[v] = predecessor of v
- Loop invariant
  - All vertices with p[v] != nil (i.e. already in the queue or popped from queue) have d[v] = dist(s,v)
  - The distance of the nodes in the queue are sorted
    - If Q = {$v_1$, $v_2$, …, $v_r$} then d[$v_1$] <= d[$v_2$] <= … <= d[$v_r$]
  - The nodes in the queue are from 2 adjacent layers/levels
    - i.e. d[$v_k$] <= d[$v_1$] + 1
    - Suppose there is a node from a 3rd level (d[$v_1$] + 2), it must have been found by some, vi, where d[$v_i$] = d[$v_1$]+1

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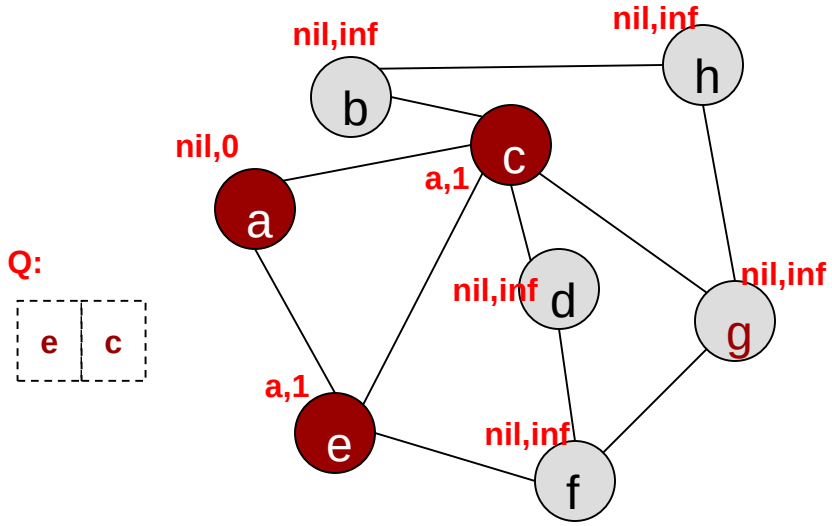
```

# Breadth-First Search

- Analyze the run time of BFS for a graph with n vertices and m edges
  - Find T(n,m)
- How many times does loop on line 5 iterate?
- How many times loop on line 7 iterate?

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6     v = Q.front(); Q.dequeue()
7     foreach neighbor, w, of v:
8        if pred[w] == nil // w not found
9           Q.enqueue(w)
10          pred[w] = v,  d[w] = d[v] + 1
```
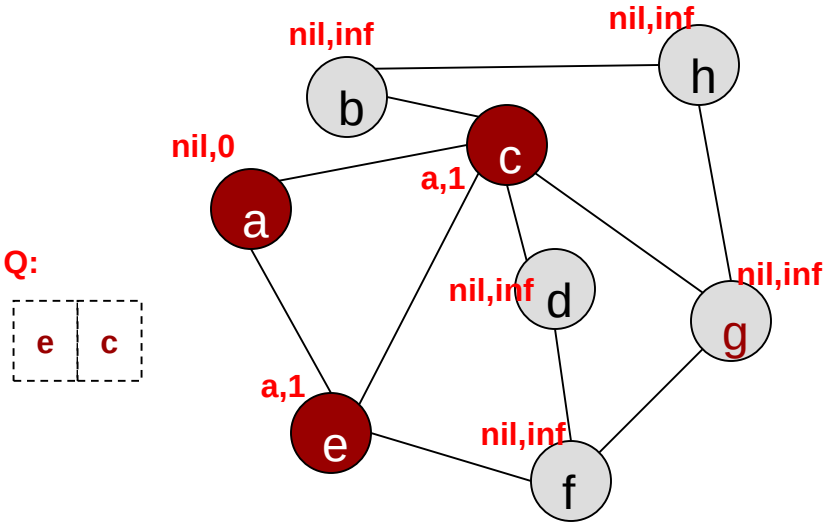

# Breadth-First Search

- Analyze the run time of BFS for a graph with n vertices and m edges
  - Find T(n)
- How many times does loop on line 5 iterate?
  - N times (one iteration per vertex)
- How many times loop on line 7 iterate?
  - For each vertex, v, the loop executes deg(v) times
  - $= \sum_{v \in V} \theta[1 + \deg(v)]$
  - $= \theta(\sum_v 1) + \theta(\sum_v \deg(v))$
  - = Θ (n) + Θ (m)
- Total = Θ(n+m)



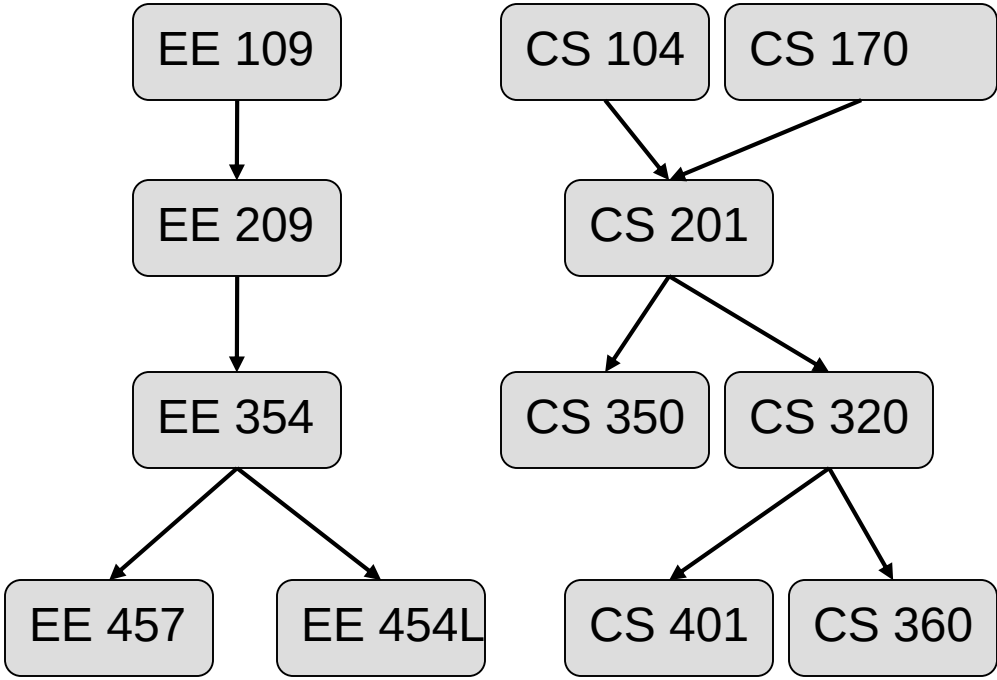

```
BFS(G,u)
1  for each vertex v
2      pred[v] = nil, d[v] = inf.
3  Q = new Queue
4  Q.enqueue(u),  d[u]=0
5  while Q is not empty
6     v = Q.front(); Q.dequeue()
7     foreach neighbor, w, of v:
8        if pred[w] == nil // w not found
9           Q.enqueue(w)
10          pred[w] = v,  d[w] = d[v] + 1
```

Topological Search

# DEPTH FIRST SEARCH MOTIVATING EXAMPLE

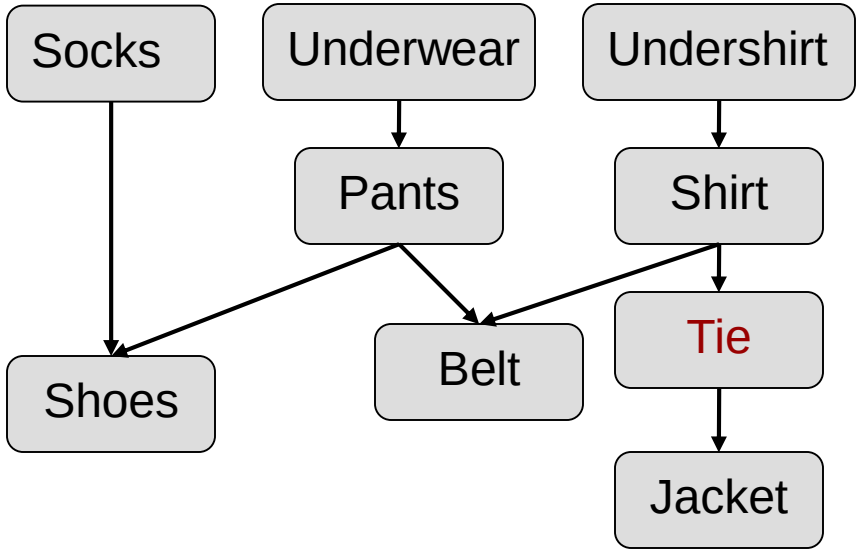
# DFS Application: Topological Sort

- Breadth-first search doesn't solve all our problems.
- Given a graph of dependencies (tasks, prerequisities, etc.) **topological** sort creates a consistent ordering of tasks (vertices) where no dependencies are violated
- Many possible valid topological orderings exist
  - EE 109, EE 209, EE 354, EE 454, EE 457, CS104, PHYS 152, CS 201,…
  - CS 104, EE 109, CS 170, EE 209,…



# Topological Sort

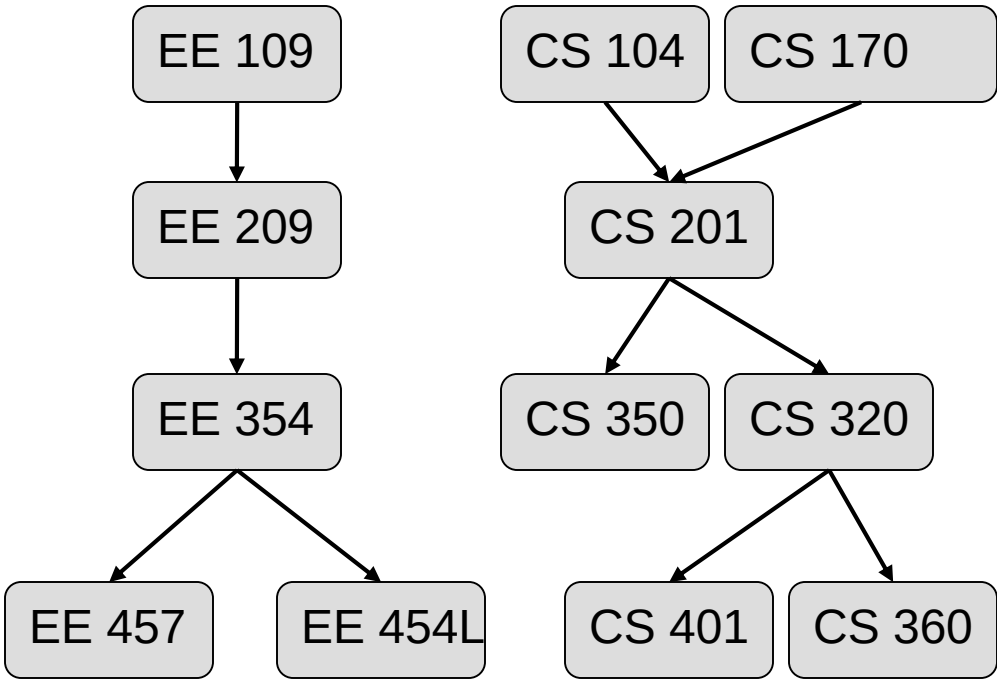
- Another example
  - Getting dressed
- More Examples:
  - Project management scheduling
  - Build order in a Makefile or other compile project
  - Cooking using a recipe
  - Instruction execution on an out-of-order pipelined CPU
  - Production of output values in a simulation of a combinational gate network




http://www.personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/GraphAlgor/topoSort.htm

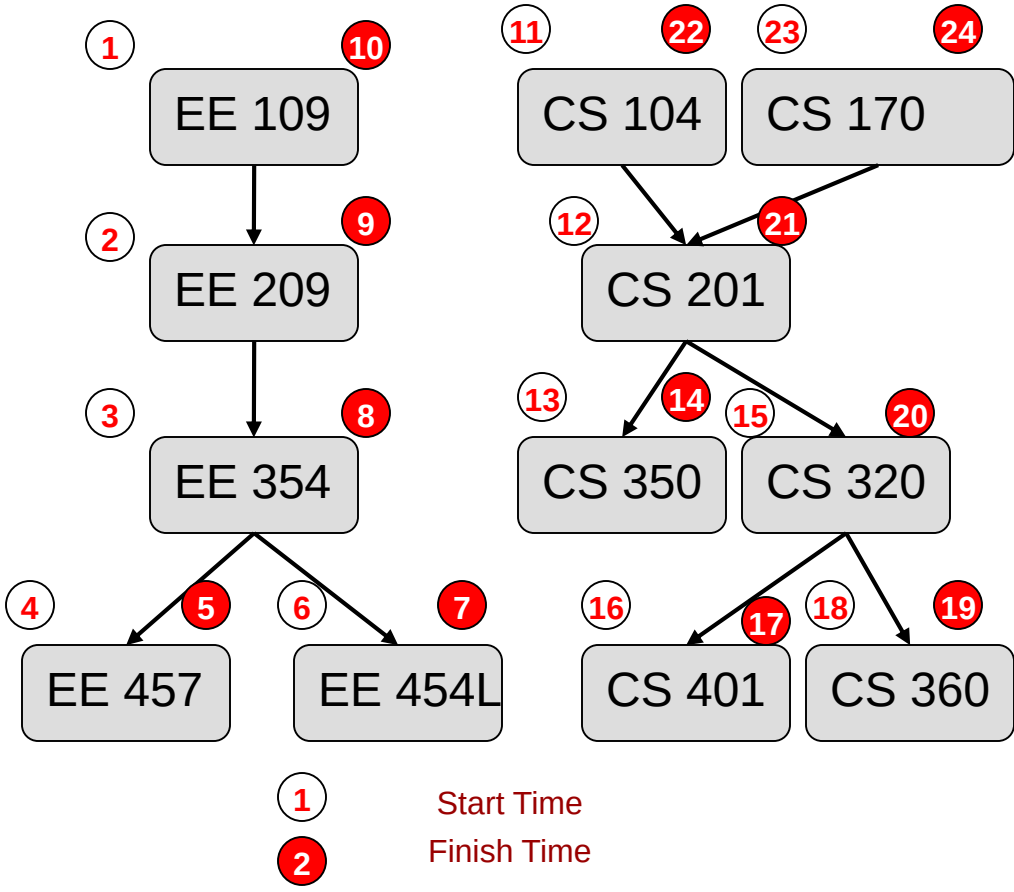
# Topological Sort

- Does breadth-first search work?
  - No. What if we started at CS 170…
  - We'd go to CS 201L before CS 104
- All parent nodes need to be completed before any child node
- BFS only guarantees *some* parent has completed before child
- Turns out a Depth-First Search will be part of our solution



# Depth First Search

- Explores ALL children before completing a parent
  - Note: BFS completes a parent before ANY children
- For DFS let us assign:
  - A start time when the node is first found
  - A finish time when a node is completed
- If we look at our nodes in reverse order of finish time (i.e. last one to finish back to first one to finish) we arrive at a…
  - Topological ordering!!!




**Reverse Finish Time Order**

**CS 170, CS 104, CS 201, CS 320, CS 360, CS 477, CS 350, EE 109, EE 209L, EE 354, EE 454L, EE 457**

# DFS Algorithm

- DFS visits and completes all children before completing (and going on to a sibling)
- Process:
  - Visit a node
  - Mark as visited (started)
  - For each visited neighbor, visit it and perform DFS on all of their children
  - Only then, mark as finished
- Let's trace recursive DFS!!
- If cycles in the graph, ensure we don't get caught visiting neighbors endlessly
  - Use some status (textbooks use "colors" but really just some integer)
  - White = unvisited,
  - Gray = visited but not finished
  - Black = finished

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

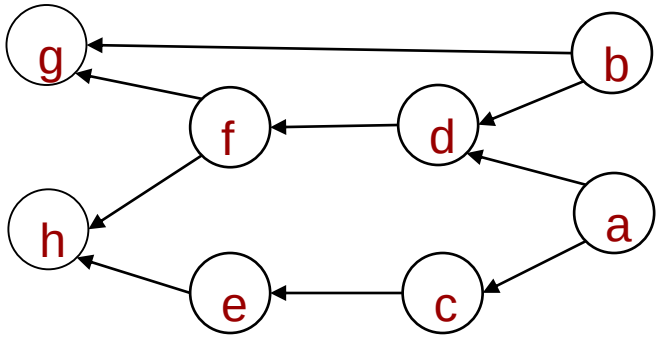
```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4           DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

Source: "Introduction to Algorithms", Cormer, Leiserson, Rivest

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

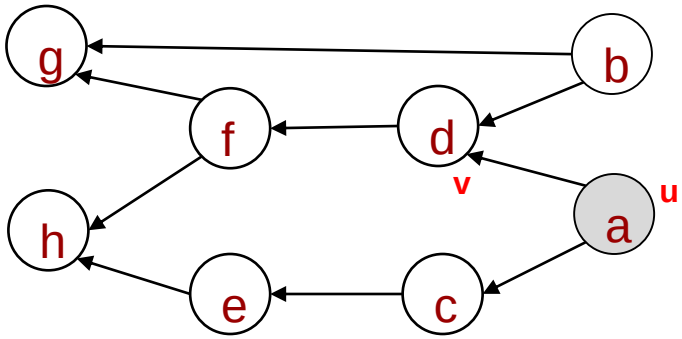
```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```



# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u,l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```

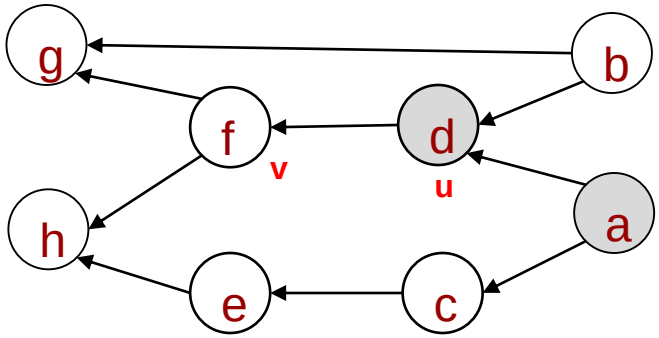



**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4           DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```

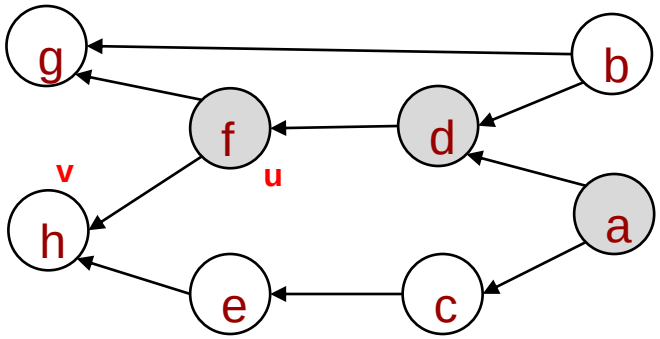



| DFS-Visit(G,d,l): |
| --- |
| DFS-Visit(G,a,l): |

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4           DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




| DFS-Visit(G,f,l): |
| --- |
| DFS-Visit(G,d,l): |
| DFS-Visit(G,a,l): |

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
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7  return finish_list
```
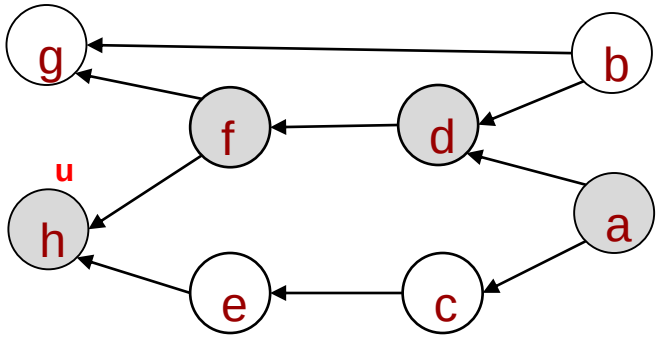

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4           DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




| DFS-Visit(G,h,l): |
| --- |
| DFS-Visit(G,f,l): |
| DFS-Visit(G,d,l): |
| DFS-Visit(G,a,l): |

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
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4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```
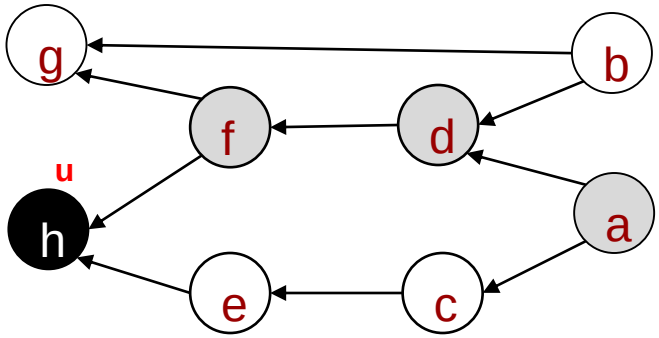

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

**h**

**DFS-Visit(G,h,l):**

**DFS-Visit(G,f,l):**

**DFS-Visit(G,d,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
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5    u.color = BLACK
6    l.append(u)
```
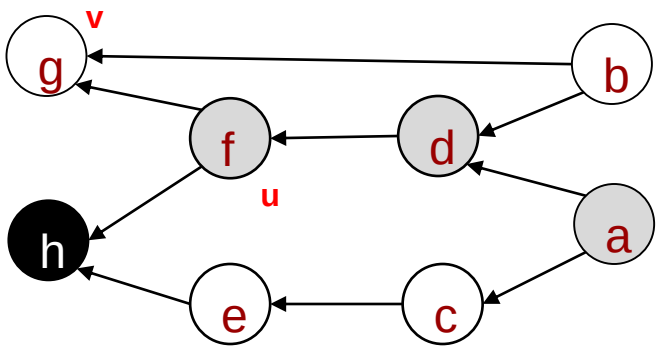
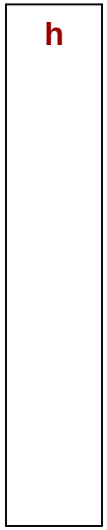




**Finish_list:**




**DFS-Visit(G,f,l):**

**DFS-Visit(G,d,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
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5    u.color = BLACK
6    l.append(u)
```
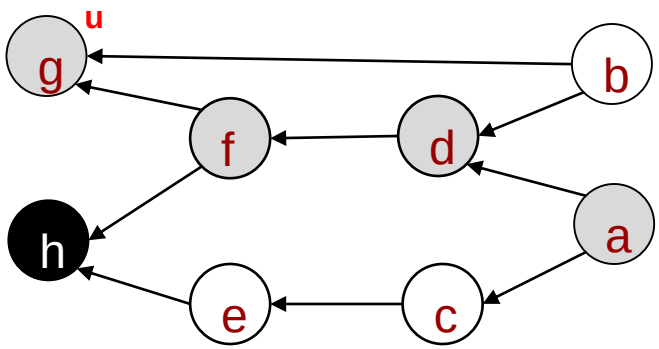
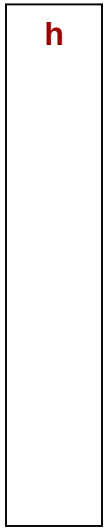




**Finish_list:**




**DFS-Visit(G,g,l):**

**DFS-Visit(G,f,l):**

**DFS-Visit(G,d,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

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DFS-All (G)
1  for each vertex u
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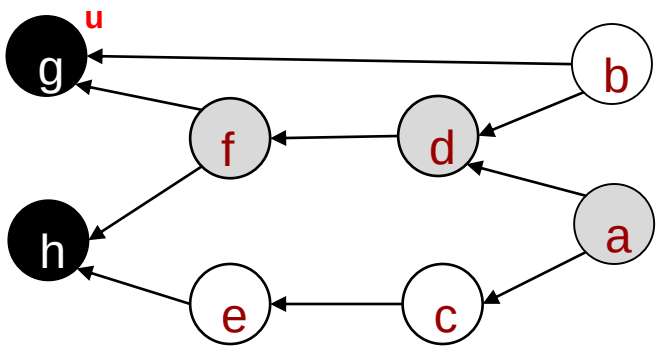
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4           DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

**h,**
**g**

**DFS-Visit(G,g,l):**

**DFS-Visit(G,f,l):**

**DFS-Visit(G,d,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
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5    u.color = BLACK
6    l.append(u)
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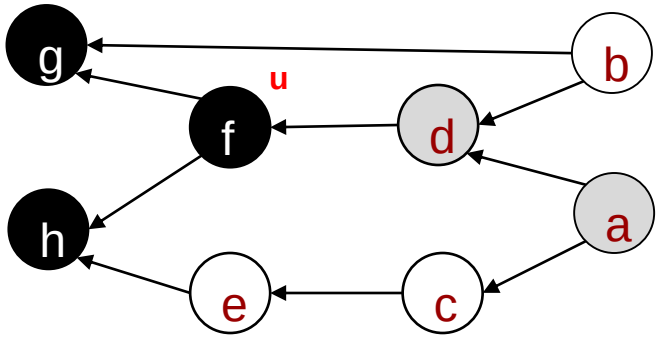


**Finish_list:**

**h,**
**g,**
**f**

**DFS-Visit(G,f,l):**

**DFS-Visit(G,d,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
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4  for each vertex u do
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5    u.color = BLACK
6    l.append(u)
```

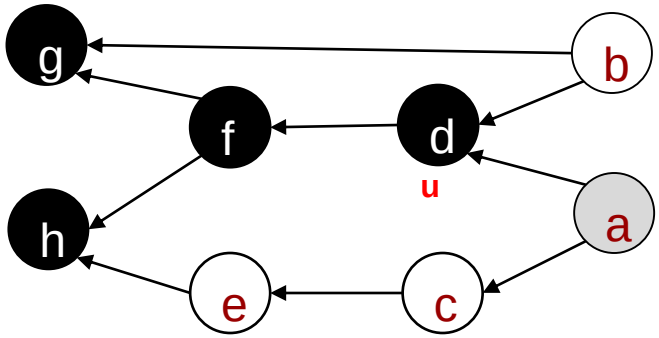


**Finish_list:**

**h,**
**g,**
**f,**
**d**

**DFS-Visit(G,d,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
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4  for each vertex u do
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```

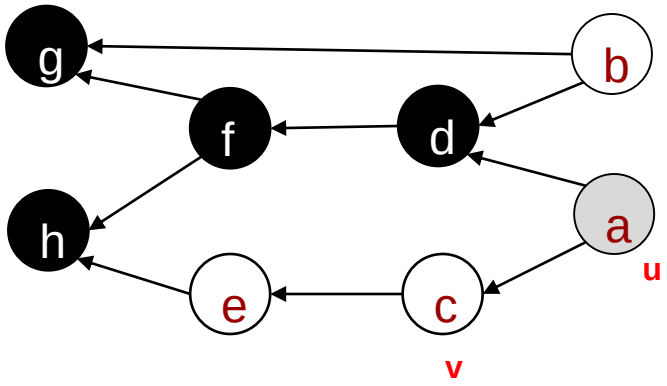


**Finish_list:**

**h,**
**g,**
**f,**
**d**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
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4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```
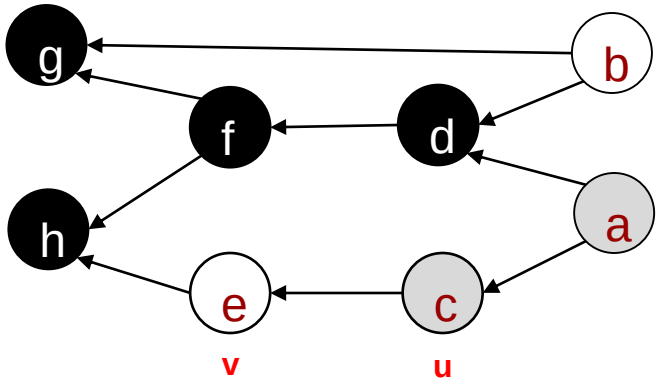




**Finish_list:**

**h,**
**g,**
**f,**
**d**

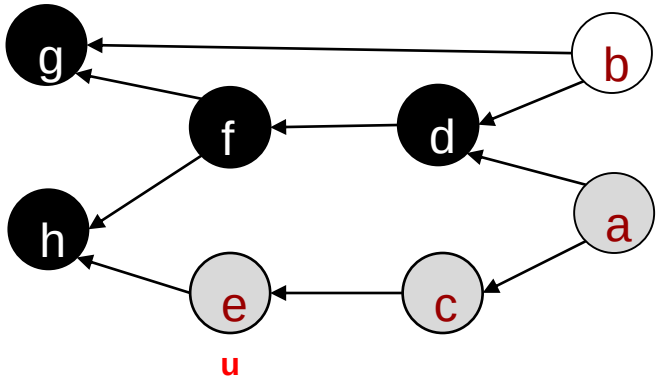
**DFS-Visit(G,c,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

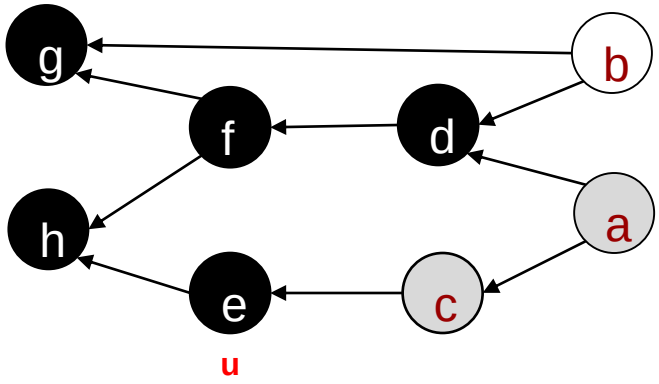
| |
|---|
| **h,** |
| **g,** |
| **f,** |
| **d** |

| |
|---|
| **DFS-Visit(G,e,l):** |
| **DFS-Visit(G,c,l):** |
| **DFS-Visit(G,a,l):** |

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

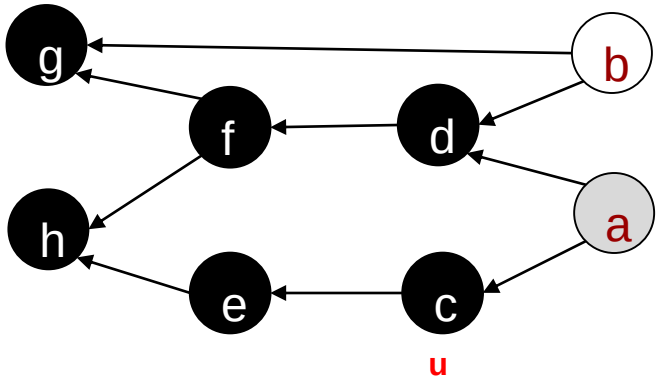
| |
|---|
| **h,** |
| **g,** |
| **f,** |
| **d.** |
| **e** |

| |
|---|
| **DFS-Visit(G,e,l):** |
| **DFS-Visit(G,c,l):** |
| **DFS-Visit(G,a,l):** |

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

**h,**
**g,**
**f,**
**d.**
**e,**
**c**

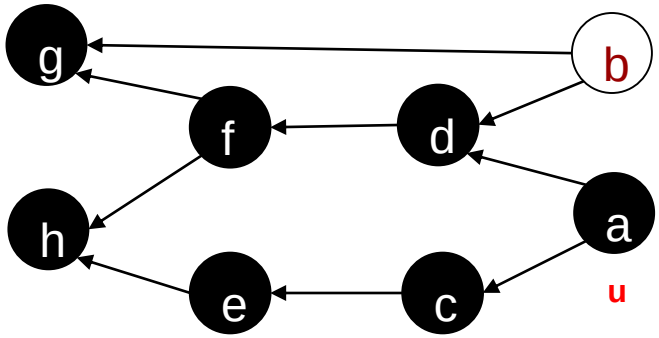
**DFS-Visit(G,c,l):**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

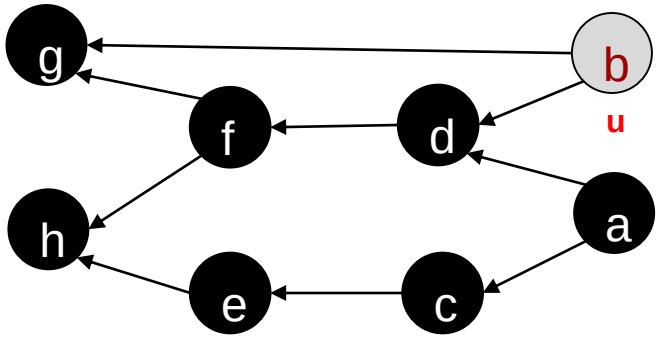
**h,**
**g,**
**f,**
**d.**
**e,**
**c,**
**a**

**DFS-Visit(G,a,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

May iterate through many complete vertices before finding b to launch a new search from




```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4           DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```

**Finish_list:**

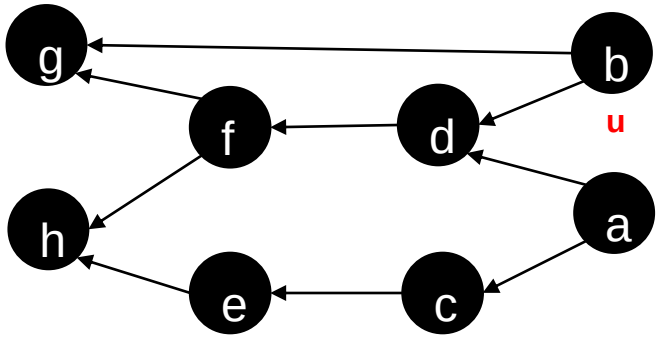
**h,**
**g,**
**f,**
**d.**
**e,**
**c,**
**a**

**DFS-Visit(G,b,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

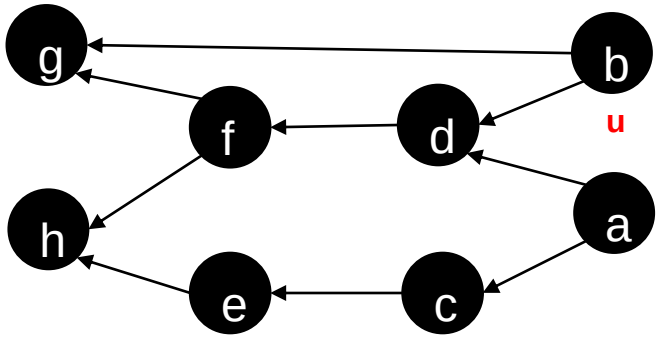
**h,**
**g,**
**f,**
**d.**
**e,**
**c,**
**a,**
**b**

**DFS-Visit(G,b,l):**

# Depth First-Search

```
DFS-All (G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
7  return finish_list
```

```
DFS-Visit (G, u, l)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4            DFS-Visit (G, v)
5    u.color = BLACK
6    l.append(u)
```




**Finish_list:**

**h,**
**g,**
**f,**
**d.**
**e,**
**c,**
**a,**
**b**

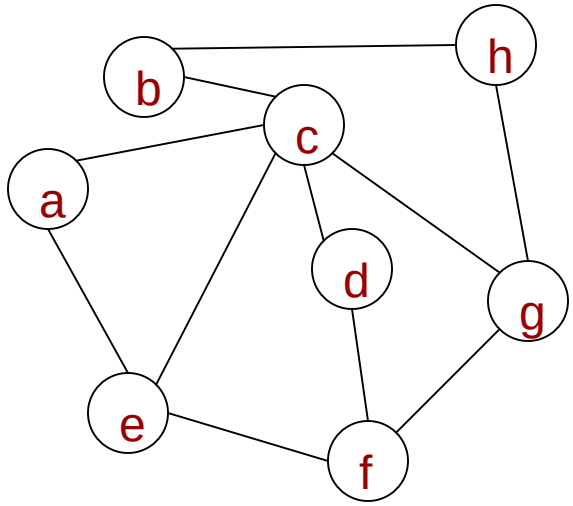
With Cycles in the graph

# ANOTHER EXAMPLE (IF TIME)

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

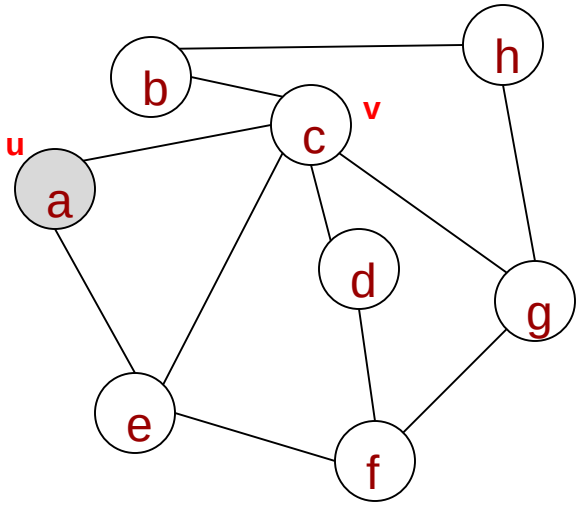
```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```



# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

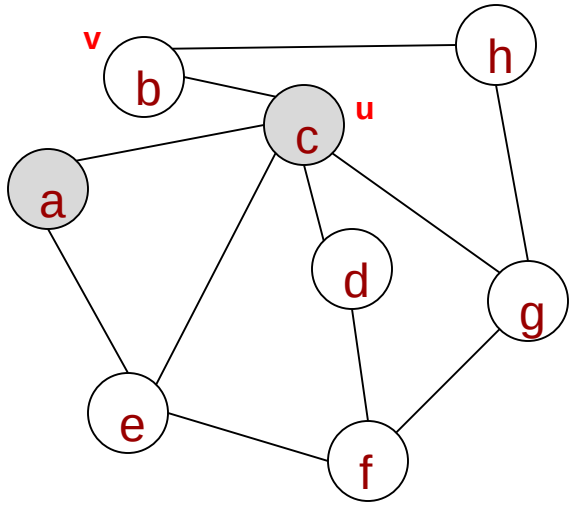



**DFS-Visit(G,a):**

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

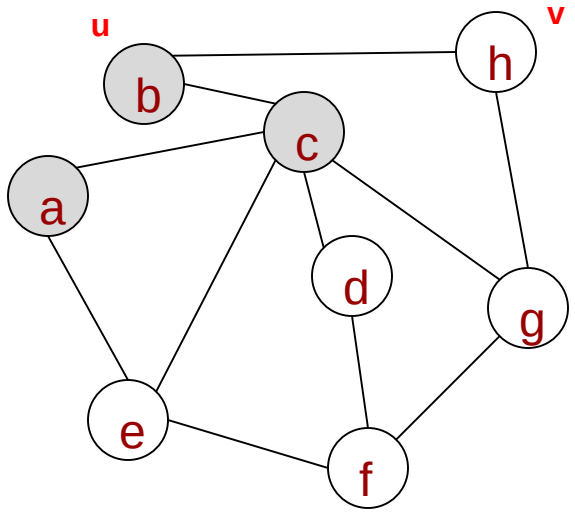



| DFS-Visit(G,c): |
| --- |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

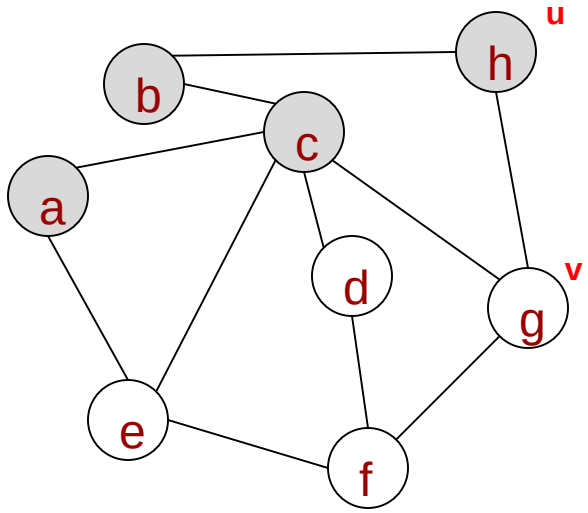



| DFS-Visit(G,b): |
| --- |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

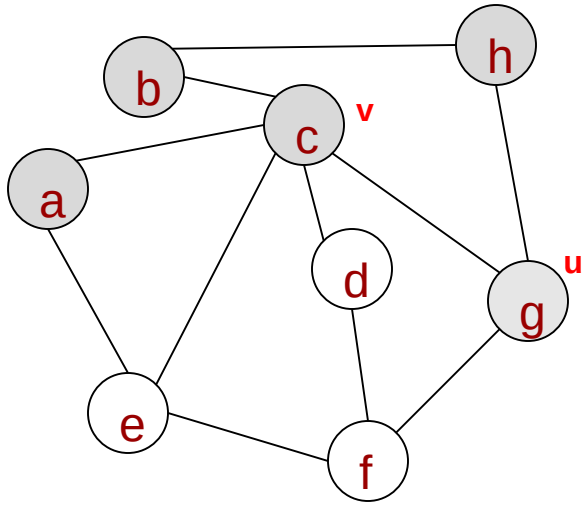



| DFS-Visit(G,h): |
| --- |
| DFS-Visit(G,b): |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

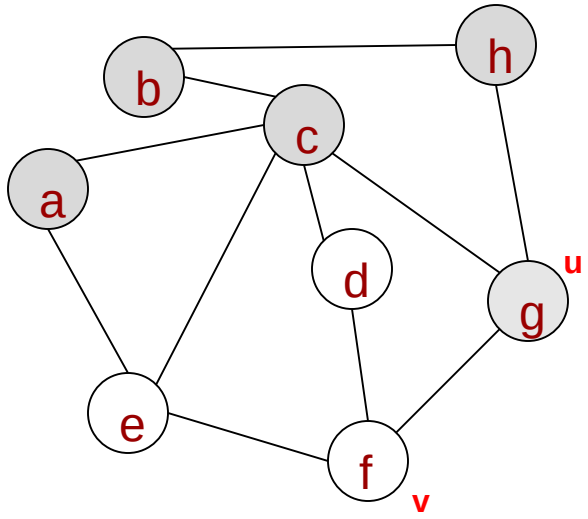



| DFS-Visit(G,g): |
| --- |
| DFS-Visit(G,h): |
| DFS-Visit(G,b): |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

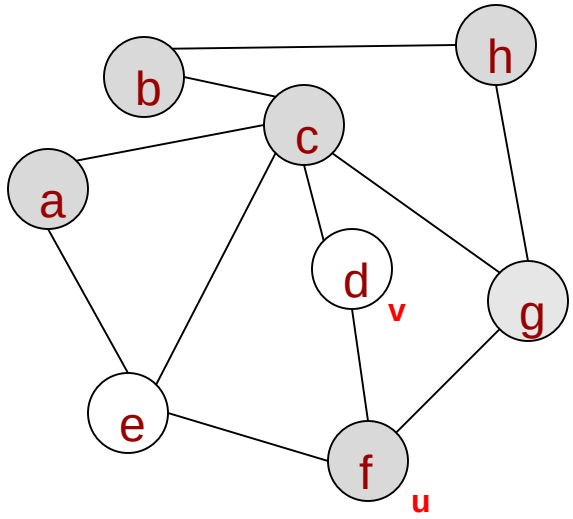



| DFS-Visit(G,g): |
| --- |
| DFS-Visit(G,h): |
| DFS-Visit(G,b): |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

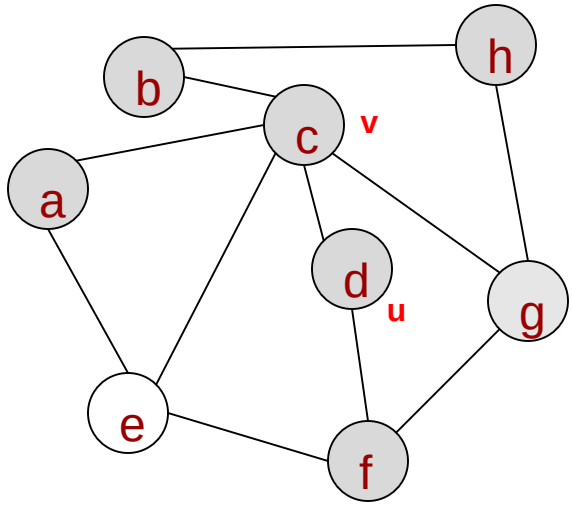



| DFS-Visit(G,f): |
| --- |
| DFS-Visit(G,g): |
| DFS-Visit(G,h): |
| DFS-Visit(G,b): |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3       if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

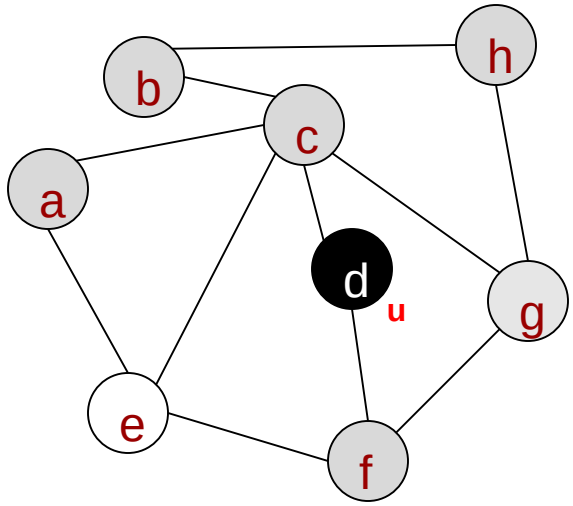



| DFS-Visit(G,d): |
| --- |
| DFS-Visit(G,f): |
| DFS-Visit(G,g): |
| DFS-Visit(G,h): |
| DFS-Visit(G,b): |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

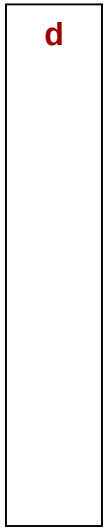
# Depth First-Search

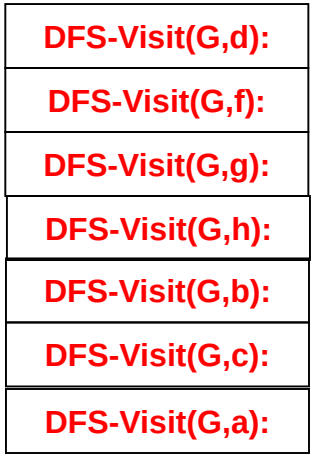
```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```




**DFSQ:**

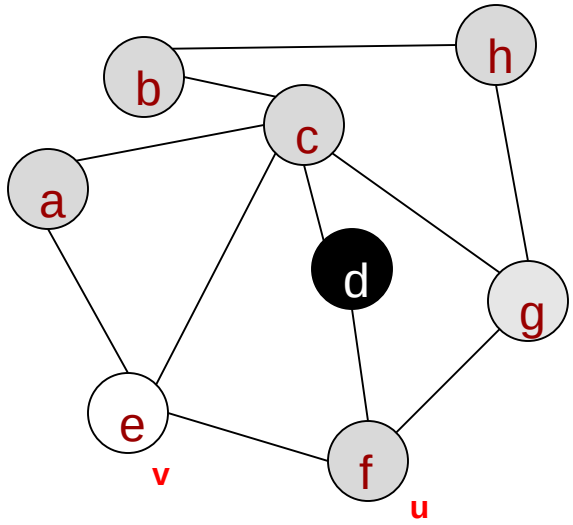


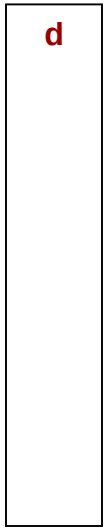
# Depth First-Search

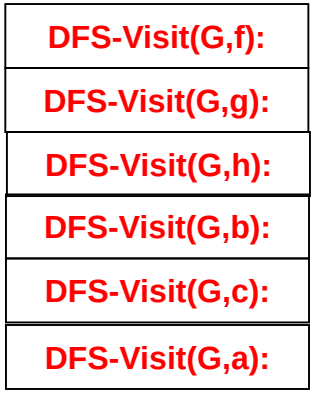
```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```




**DFSQ:**

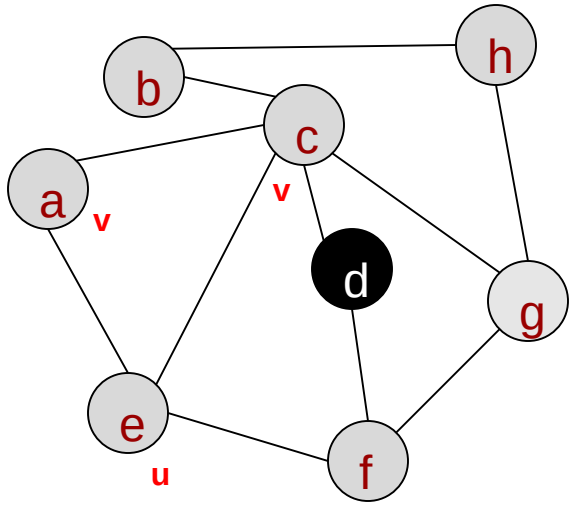


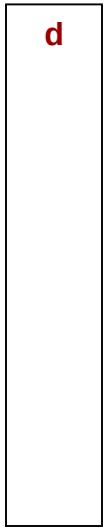
# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```




**DFSQ:**

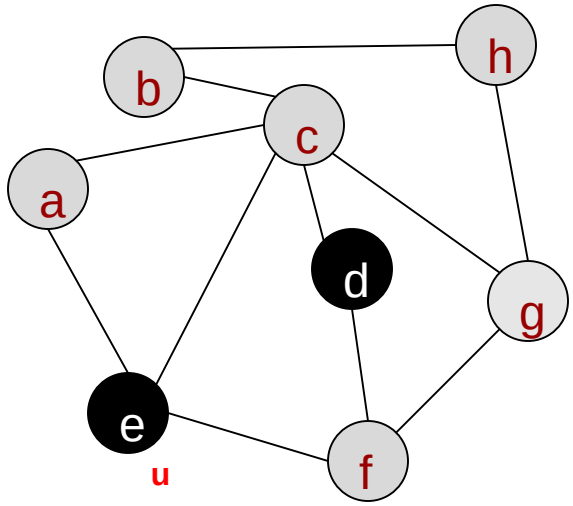



**DFS-Visit(G,e):**
**DFS-Visit(G,f):**
**DFS-Visit(G,g):**
**DFS-Visit(G,h):**
**DFS-Visit(G,b):**
**DFS-Visit(G,c):**
**DFS-Visit(G,a):**

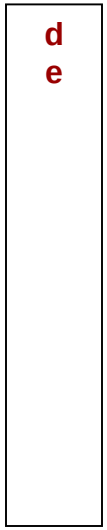
# Depth First-Search

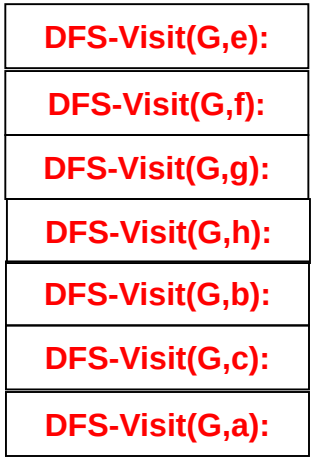
```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```




**DFSQ:**

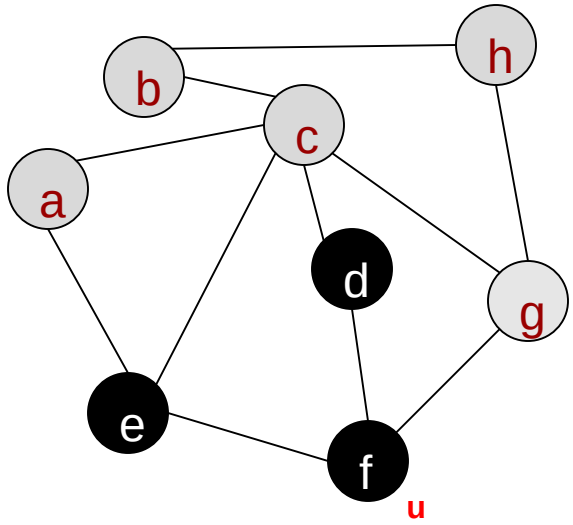


# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```




**DFSQ:**

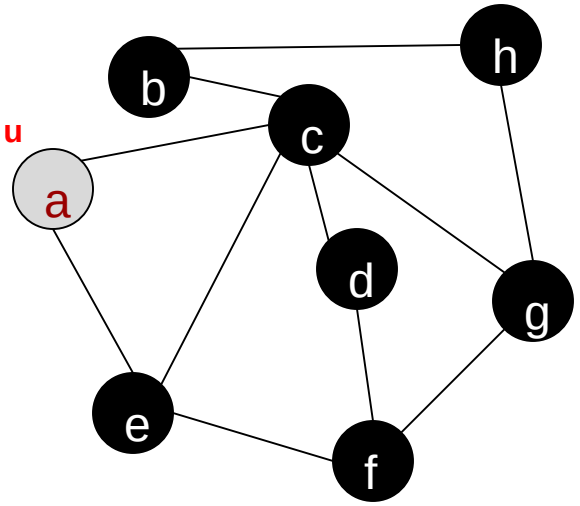
| |
|---|
| d |
| e |
| f |

| |
|---|
| DFS-Visit(G,f): |
| DFS-Visit(G,g): |
| DFS-Visit(G,h): |
| DFS-Visit(G,b): |
| DFS-Visit(G,c): |
| DFS-Visit(G,a): |

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```




**DFSQ:**

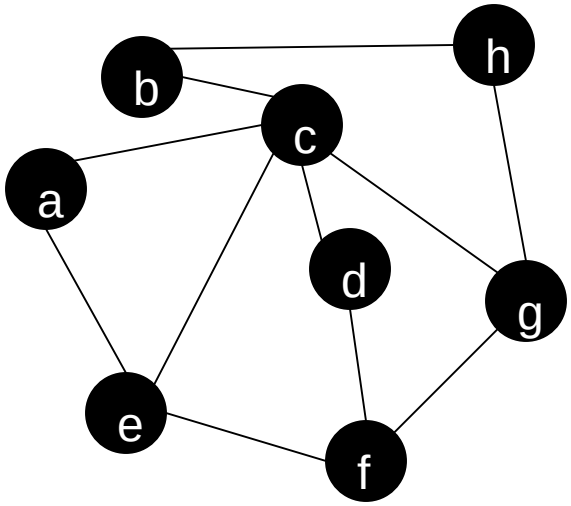
**d**
**e**
**f**
**g**
**h**
**b**
**c**

**DFS-Visit(G,a):**

# Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4          DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```



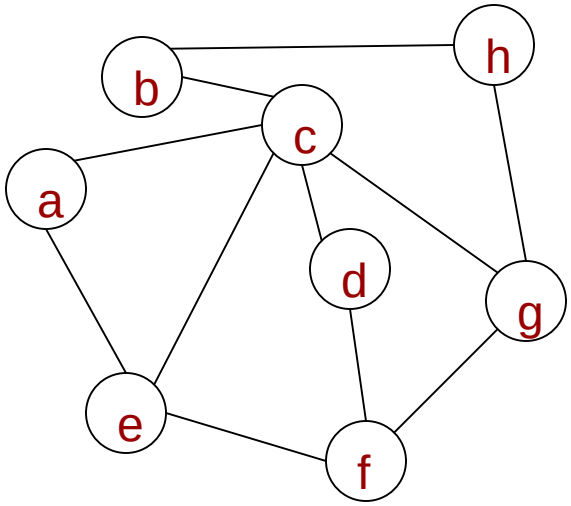

**DFSQ:**

**d**
**e**
**f**
**g**
**h**
**b**
**c**
**a**

# ITERATIVE VERSION

# Depth First-Search

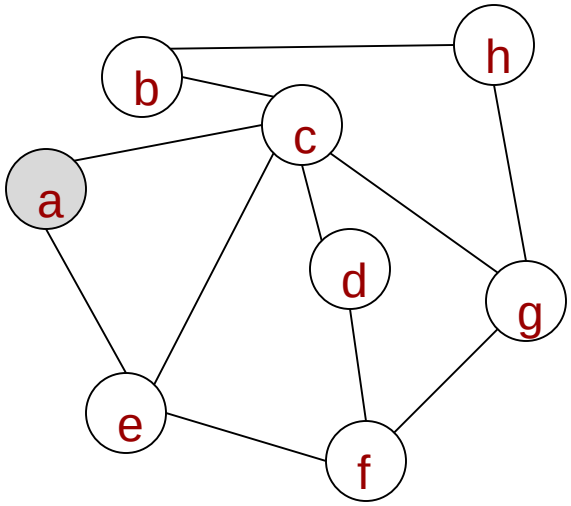
```
DFS (G,s)
1   for each vertex u
2     u.color = WHITE
3   st = new Stack
4   st.push_back(s)
5   while st not empty
6     u = st.back()
7     if u.color == WHITE then
8       u.color = GRAY
9       foreach vertex v in Adj(u) do
10        if v.color == WHITE
11          st.push_back(v)
12    else if u.color != WHITE
13      u.color = BLACK
14      st.pop_back()
```




st: a

# Depth First-Search

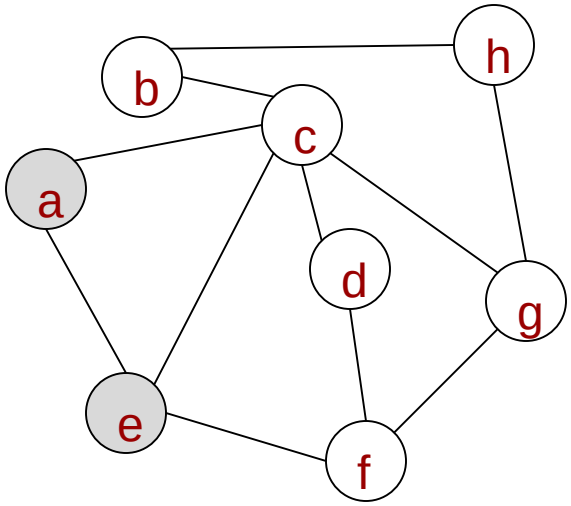
```
DFS (G,s)
1    for each vertex u
2      u.color = WHITE
3    st = new Stack
4    st.push_back(s)
5    while st not empty
6      u = st.back()
7      if u.color == WHITE then
8        u.color = GRAY
9        foreach vertex v in Adj(u) do
10         if v.color == WHITE
11           st.push_back(v)
12     else if u.color != WHITE
13       u.color = BLACK
14       st.pop_back()
```




st: | a | c | e |

# Depth First-Search

```
DFS (G,s)
1    for each vertex u
2      u.color = WHITE
3    st = new Stack
4    st.push_back(s)
5    while st not empty
6      u = st.back()
7      if u.color == WHITE then
8        u.color = GRAY
9        foreach vertex v in Adj(u) do
10         if v.color == WHITE
11           st.push_back(v)
12     else if u.color != WHITE
13       u.color = BLACK
14       st.pop_back()
```




st:

| a | c | e | c | f |
|---|---|---|---|---|

# Depth First-Search

```
DFS (G,s)
1    for each vertex u
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10            if v.color == WHITE
11               st.push_back(v)
12    else if u.color != WHITE
13       u.color = BLACK
14       st.pop_back()
```

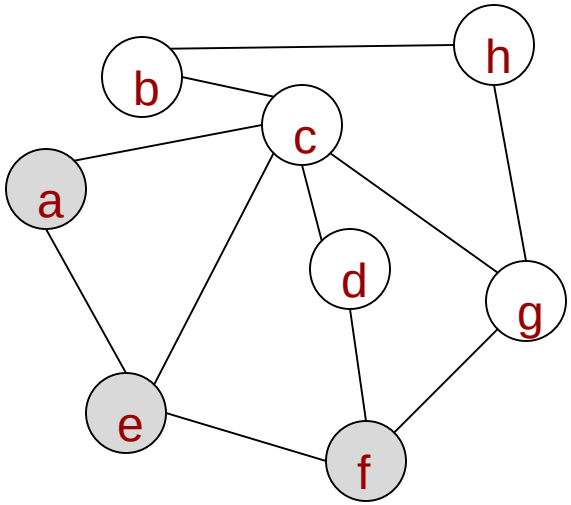


st:

| a | c | e | c | f | d | g |
|---|---|---|---|---|---|---|

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DFS (G,s)
1    for each vertex u
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10            if v.color == WHITE
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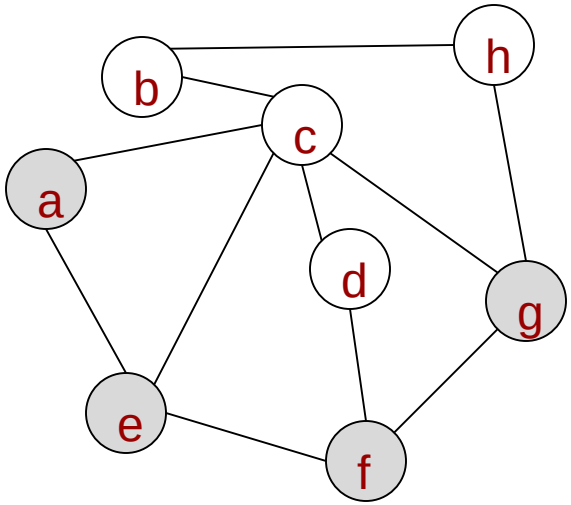
12    else if u.color != WHITE
13       u.color = BLACK
14       st.pop_back()
```




st:

| a | c | e | c | f | d | g | c | h |
|---|---|---|---|---|---|---|---|---|

# Depth First-Search

```
DFS (G,s)
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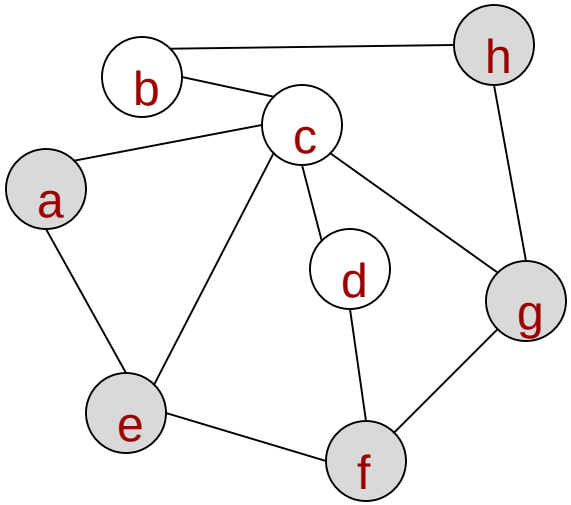


st:

| a | c | e | c | f | d | g | c | h | b |
|---|---|---|---|---|---|---|---|---|---|

# Depth First-Search

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10         if v.color == WHITE
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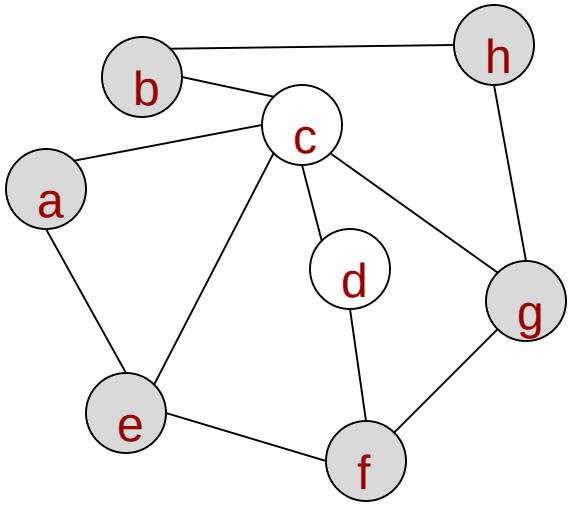
```




st:

| a | c | e | c | f | d | g | c | h | b | c |
|---|---|---|---|---|---|---|---|---|---|---|

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```

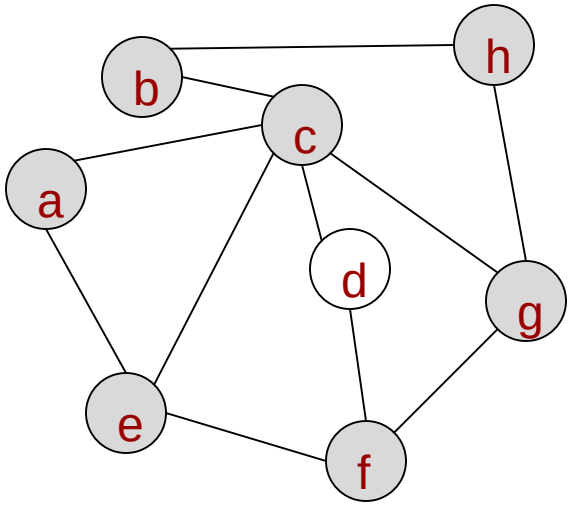


st:

| a | c | e | c | f | d | g | c | h | b | c | d |
|---|---|---|---|---|---|---|---|---|---|---|---|

# Depth First-Search

```
DFS (G,s)
1    for each vertex u
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```
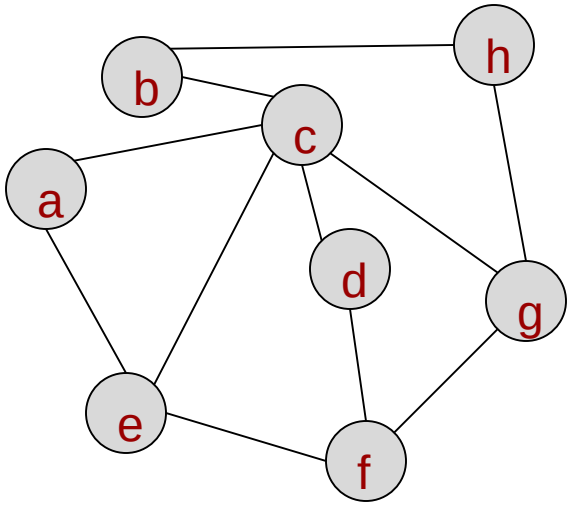
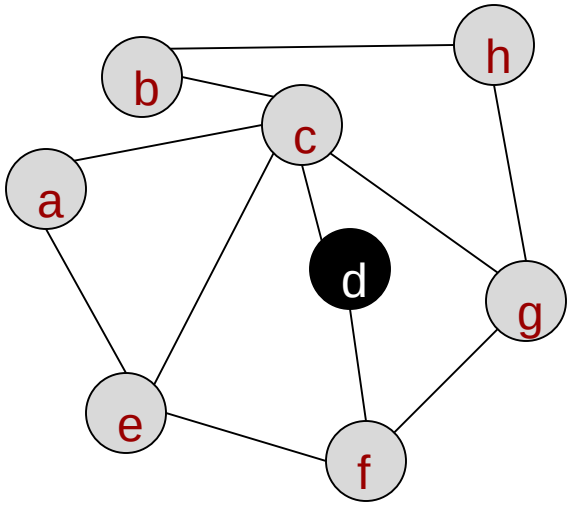




st:

| a | c | e | c | f | d | g | c | h | b | c | d |
|---|---|---|---|---|---|---|---|---|---|---|---|

# Depth First-Search

```
DFS (G,s)
1    for each vertex u
2      u.color = WHITE
3    st = new Stack
4    st.push_back(s)
5    while st not empty
6      u = st.back()
7      if u.color == WHITE then
8        u.color = GRAY
9        foreach vertex v in Adj(u) do
10         if v.color == WHITE
11           st.push_back(v)
12     else if u.color != WHITE
13       u.color = BLACK
14       st.pop_back()
```

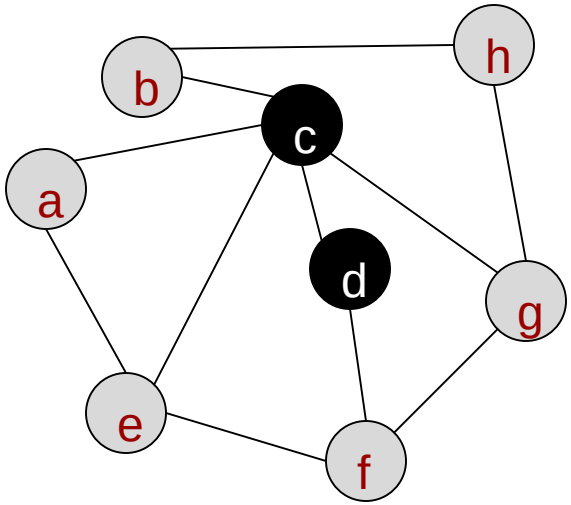



st:

| a | c | e | c | f | d | g | c | h | b | c | d |
|---|---|---|---|---|---|---|---|---|---|---|---|

# Depth First-Search

```
DFS (G,s)
1   for each vertex u
2     u.color = WHITE
3   st = new Stack
4   st.push_back(s)
5   while st not empty
6     u = st.back()
7     if u.color == WHITE then
8       u.color = GRAY
9       foreach vertex v in Adj(u) do
10        if v.color == WHITE
11          st.push_back(v)
12    else if u.color != WHITE
13      u.color = BLACK
14      st.pop_back()
```

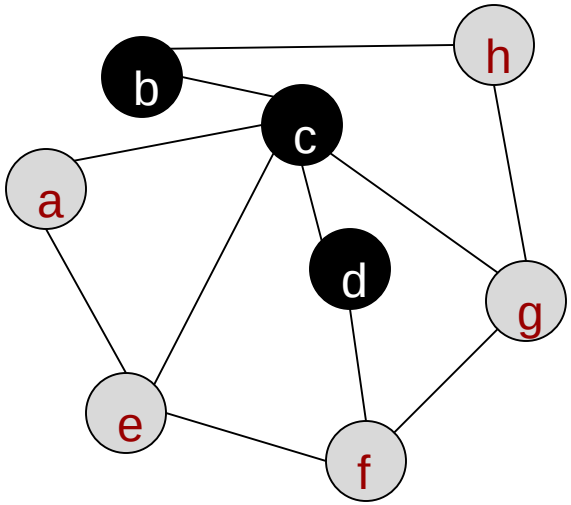



st:

| a | c | e | c | f | d | g | c | h | b | c |
|---|---|---|---|---|---|---|---|---|---|---|

# Depth First-Search

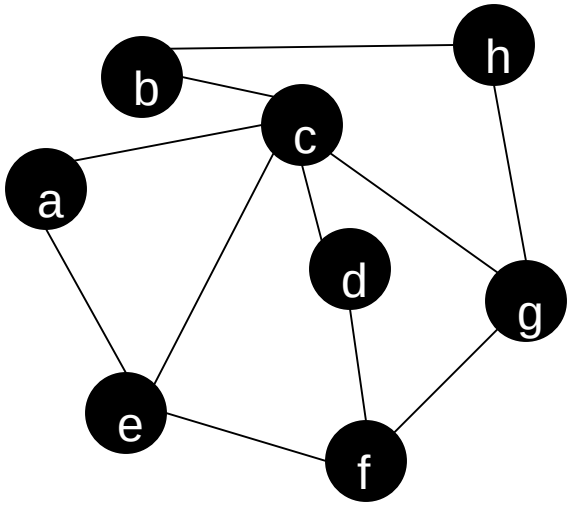
```
DFS (G,s)
1    for each vertex u
2      u.color = WHITE
3    st = new Stack
4    st.push_back(s)
5    while st not empty
6      u = st.back()
7      if u.color == WHITE then
8        u.color = GRAY
9        foreach vertex v in Adj(u) do
10         if v.color == WHITE
11           st.push_back(v)
12     else if u.color != WHITE
13       u.color = BLACK
14       st.pop_back()
```




st: a c e c f d g c h b

# Depth First-Search

```
DFS (G,s)
1   for each vertex u
2     u.color = WHITE
3   st = new Stack
4   st.push_back(s)
5   while st not empty
6     u = st.back()
7     if u.color == WHITE then
8       u.color = GRAY
9       foreach vertex v in Adj(u) do
10        if v.color == WHITE
11          st.push_back(v)
12    else if u.color != WHITE
13      u.color = BLACK
14      st.pop_back()
```




st: a

# BFS vs. DFS Algorithm

- BFS and DFS are more similar than you think
  - Do we use a FIFO/Queue (BFS) or LIFO/Stack (DFS) to store vertices as we find them

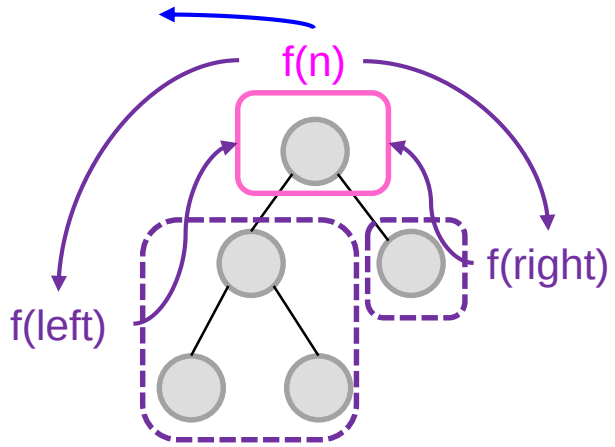
```
BFS-Visit (G, start_node)
1    for each vertex u
2      u.color = WHITE
3      u.pred = nil
4    bfsq = new Queue
5    bfsq.push_back(start_node)
6    while bfsq not empty
7       u = bfsq.pop_front()
8       if u.color == WHITE
9         u.color = GRAY
10        foreach vertex v in Adj(u) do
11          bfsq.push_back(v)
```

```
DFS-Visit (G, start_node)
1    for each vertex u
2      u.color = WHITE
3      u.pred = nil
4    st = new Stack
5    st.push_back(start_node)
6    while st not empty
7       u = st.top(); st.pop()
8       if u.color == WHITE
9         u.color = GRAY
10        foreach vertex v in Adj(u) do
11          st.push_back(v)
```

# SOLUTIONS

# Example 1: Count Nodes

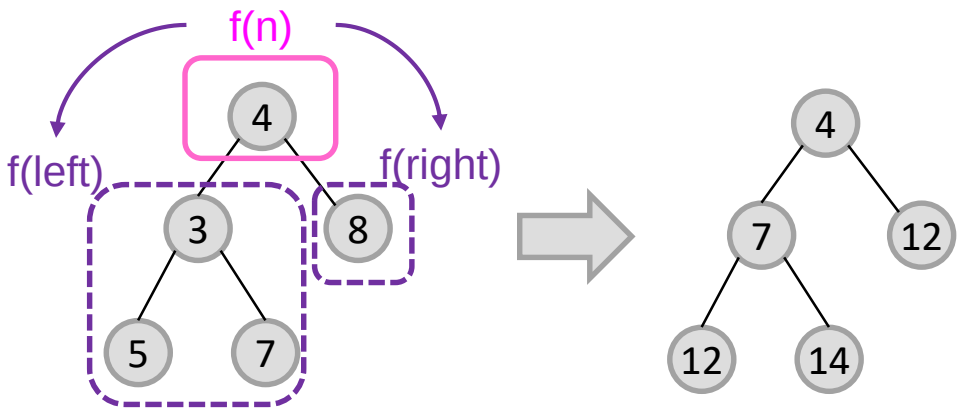
- Write a recursive function to **count how many nodes** are in the binary tree
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node
  - For in- or post-order traversals, determine how to use/combine results from recursion on children




```
// Node definition
struct Tnode {
  int val;
  TNode *left, *right;
};

int count(TNode* root)
{
  if( root == NULL ) return 0;
  else {
    return 1 + count(root->left) +
               count(root->right);
  }
}
```

# Example 2: Prefix Sums

- Write a recursive function to **have each node store the sum of the values on the path from the root to each node.**
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node




```
void prefixH(TNode* root, int psum)

void prefix(TNode* root)
{
  prefixH(root, 0);
}

void prefixH(TNode* root, int psum)
{
  if( root == NULL ) return;
  else {
    root->val += psum;
    prefixH(root->left,  root->val);
    prefixH(root->right, root->val);
  }
}
```