CSCI 104
Searching and Sorted Lists

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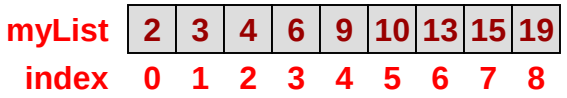
SEARCH

Linear Search

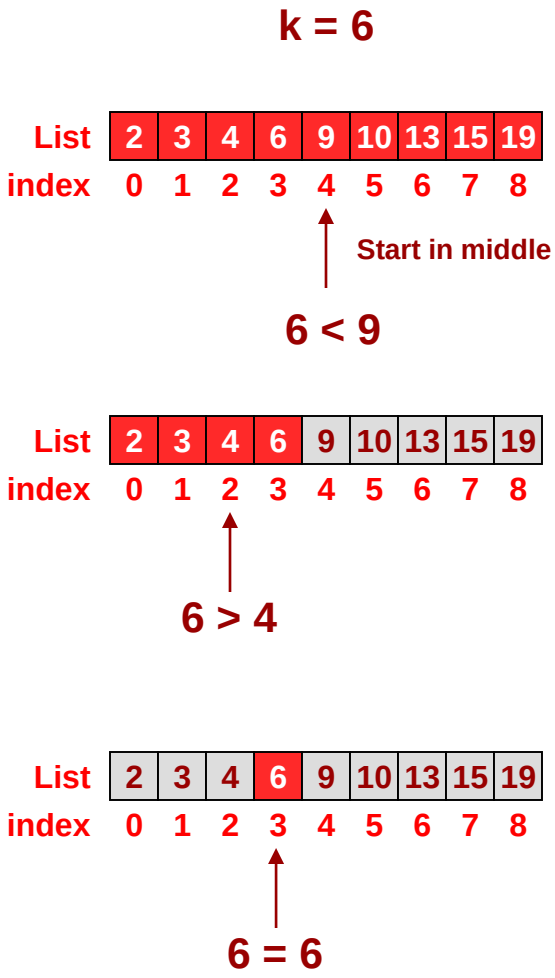
- Search a list (array) for a specific value, k, and return the location
- Sequential Search
  - Start at first item, check if it is equal to k, repeat for second, third, fourth item, etc.
- O(___)
- O(n)

```
int search(vector<int> mylist, int k)
{
  int i;
  for(i=0; i < mylist.size(); i++){
    if(mylist[i] == k)
       return i;
  }
  return -1;
}
```



Binary Search

- Sequential search does not take advantage of the ordered (a.k.a. sorted) nature of the list
  - Would work the same (equally well) on an ordered or unordered list
- Binary Search
  - Take advantage of ordered list by comparing k with middle element and based on the result, rule out all numbers greater or smaller, repeat with middle element of remaining list, etc.



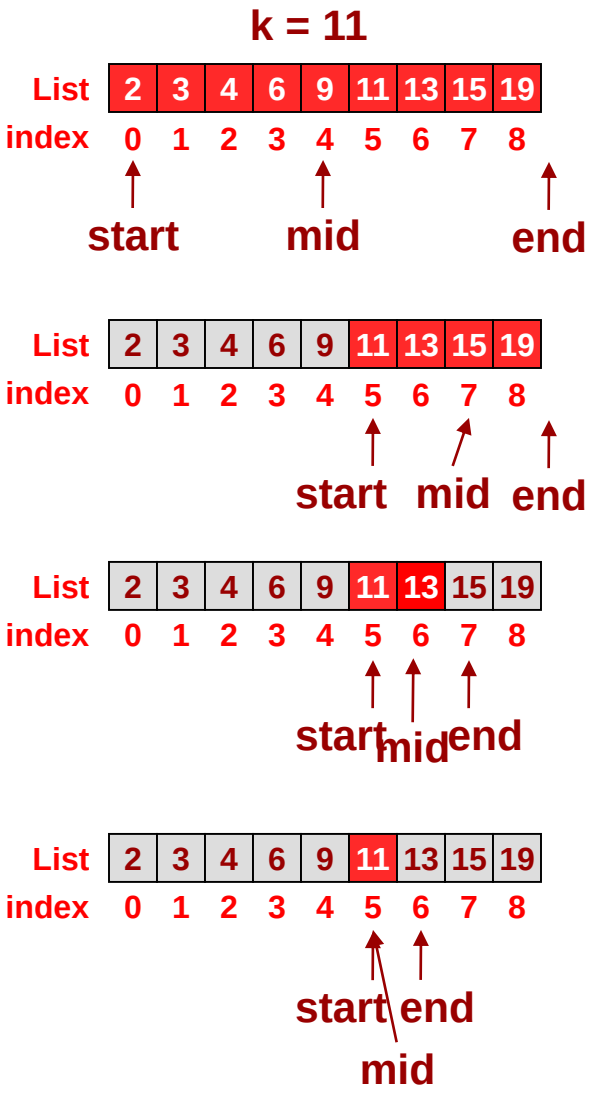
Binary Search

- Search an ordered list (array) for a specific value, k, and return the location
- Binary Search
  - Compare k with middle element of list and if not equal, rule out ½ of the list and repeat on the other half
  - "Range" Implementations in most languages are [start, end)
  - Start is inclusive, end is non-inclusive (i.e. end will always point to 1 beyond true ending index to make arithmetic work out correctly)

```
int bsearch(vector<int> mylist,
            int k,
            int start, int end)
{
  // range is empty when start == end
  while(start < end){
    int mid = (start + end)/2;
    if(k == mylist[mid])
      return mid;
    else if(k < mylist[mid])
      end = mid;
    else
      start = mid+1;
  }
 return -1;
}
```

myList	2	3	4	6	9	10	13	15	19
index	0	1	2	3	4	5	6	7	8

Binary Search




```
int bsearch(vector<int> mylist,
            int k,
            int start, int end)
{
  // range is empty when start == end
  while(start < end){
    int mid = (start + end)/2;
    if(k == mylist[mid])
      return mid;
    else if(k < mylist[mid])
      end = mid;
    else
      start = mid+1;
  }
 return -1;
}
```

Prove Time Complexity

- T(n) =

Search Comparison

- Linear search = O(_____)
- Precondition: None
- Works on (ArrayList / LinkedList)

```
int search(vector<int> mylist,int k)
{
  int i;
  for(i=0; i < mylist.size(); i++){
    if(mylist[i] == k)
       return i;
  }
  return -1;
}
```

- Binary Search = O(_____)
- Precondition: List is sorted
- Works on (ArrayList / LinkedList)

```
int bsearch(vector<int> mylist,
            int k,
            int start, int end)
{
  int i;
  // range is empty when start == end
  while(start < end){
    int mid = (start + end)/2;
    if(k == mylist[mid])
      return mid;
    else if(k < mylist[mid])
      end = mid;
    else {
      start = mid+1;
    }
  }
 return -1;
}
```

Search Comparison

- Linear search = O(n)
- Precondition: None
- Works on ArrayList or LinkedList

```
int search(vector<int> mylist,int k)
{
  int i;
  for(i=0; i < mylist.size(); i++){
    if(mylist[i] == k)
       return i;
  }
  return -1;
}
```

- Binary Search = O(log(n))
- Precondition: List is sorted
- Works on ArrrayList only

```
int bsearch(vector<int> mylist,
            int k,
            int start, int end)
{
  int i;
  // range is empty when start == end
  while(start < end){
    int mid = (start + end)/2;
    if(k == mylist[mid])
      return mid;
    else if(k < mylist[mid])
      end = mid;
    else {
      start = mid+1;
    }
  }
 return -1;
}
```

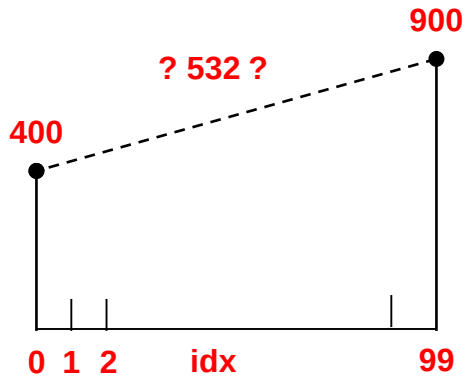
Introduction to Interpolation Search

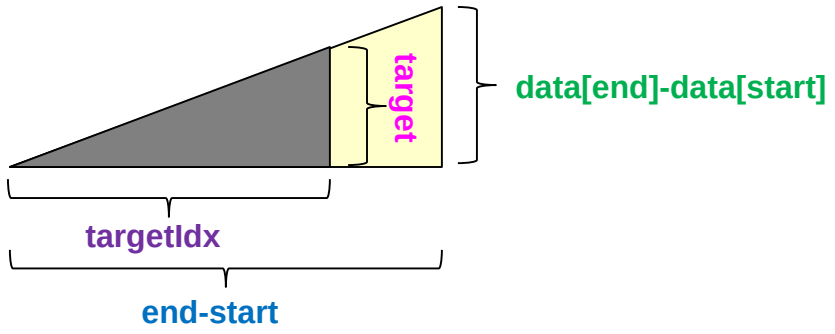
- Given a dictionary, if I say look for the word 'bag' would you really do a binary search and start in the middle of the dictionary?
- Assume a uniform distribution of 100 random numbers between [0 and 999]
  - [679 372 554 …]
- Now sort them
  - [002 009 015 …]
- At what index would you start looking for key=130

myList	002	009	015	024	039		981
index	00	01	02	03	04		99

Linear Interpolation

- If I have a range of 100 numbers where the first is 400 and the last is 900, at what index would I expect 532 (my target) to be?





$$\left(\frac{end - start + 1}{data[end] - data[start]}\right) = \left(\frac{targetIdx - startIdx}{target - data[start]}\right)$$

$$(target - data[start])\left(\frac{end - start + 1}{data[end] - data[start]}\right) + startIdx = targetIdx$$

$$(532 - 400)\left(\frac{100}{500}\right) + 0 = targetIdx$$

$$132 * 0.2 = targetIdx$$

$$26.4 = targetIdx$$

$$\lfloor 26.4 \rfloor = 26 = targetIdx$$

Interpolation Search

- Similar to binary search but rather than taking the middle value we compute the interpolated index

```
int bin_search(vector<int> mylist,
               int k,
               int start, int end)
{
  // range is empty when start == end
  while(start < end){
    int mid = (start + end)/2;

    if(k == mylist[mid])
      return mid;
    else if(k < mylist[mid])
      end = mid;
    else
      start = mid+1;
  }
 return -1;
}
```

```
int interp_search(vector<int> mylist,
                  int k,
                  int start, int end)
{
  // range is empty when start > end
  while(start <= end){
    int loc =
        interp(mylist, start, end, k);
    if(k == mylist[loc])
      return loc;
    else if(k < mylist[loc])
      end = loc;
    else
      start = loc+1;
  }
 return -1;
}
```

Another Example

- Suppose we have 1000 doubles in the range 0-1
- Do we have .7?
- Use interpolation search
- Key insight: Make sure the ratio of index range to the value range equals the ratio of the target index range to target value range, i.e.

$$\frac{\text{(Index Range)}}{\text{(Value Range)}} = \frac{\text{(Target Index – Start Index)}}{\text{(Target Value – Start Value)}}$$

- In contrast in binary search, what is this ratio?
- Interpolation search for .7
  - First find correct target index:
  - (0.7-0) * (1000/1)+0 = 700 = Target Index
  - Check List[700]

Another Example

- Key insight:

$$\frac{\text{(Index Range)}}{\text{(Value Range)}} = \frac{\text{(Target Index – Start Index)}}{\text{(Target Value – Start Value)}}$$

- If List[700] = 0.68: interpolation search again for 0.7 in a list of 300 items starting at value 0.68 and with max value of 1
- (0.7-0.68)/(1-0.68)*(Index Range) + Start Index = Target Index
  - Floor(0.0675*300 + 700) = 720
  - If List[720] = 0.71, search between 700 and 720
- Interpolate search again
- (Target Value Range/Value Range) = (0.7-0.68)/(0.71-0.68) = 0.6667
  - Interpolated index = floor(0.6667*20+700) = 713
  - Finally List[713] = .7

Example from "Y. Perl, A. Itai., and H. Avni, Interpolation Search – A Log Log N Search, Communications of the ACM, Vol. 21, No. 7, July 1978"

Another Example

- Suppose we have 1000 doubles in the range 0-1
- Find if 0.7 exists in the list and where
- Use interpolation search
  - First look at location: 0.7 * 1000 = 700
  - But when you pick up List[700] you find 0.68
  - We know 0.7 would have to be between location 700 and 1000 so we narrow our search to those 300
- Interpolate again to find where 0.7 would be in a list of 300 items that start with 0.68 and max value of 1
  - (0.7-0.68)/(1-0.68) = 0.0675
  - Interpolated index = floor(700 + 300*0.0675) = 720
  - You find List[720] = 0.71 so you narrow your search to 700-720
- Interpolate again
  - (0.7-0.68)/(0.71-0.68) = 0.6667
  - Interpolated index = floor(700 + 20*0.6667) = 713

Example from "Y. Perl, A. Itai., and H. Avni, Interpolation Search – A Log Log N Search, Communications of the ACM, Vol. 21, No. 7, July 1978"

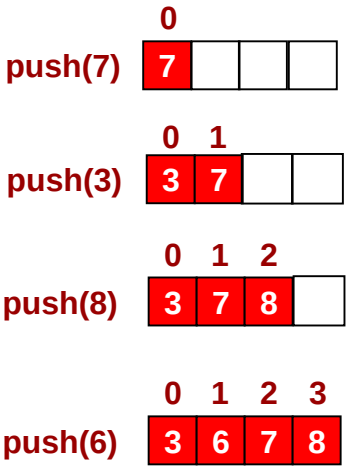
Interpolation Search Summary

- Requires a sorted list
  - An array list not a linked list (in most cases)
- Binary search = O(log(n))
- Interpolation search = O(log(log(n))
  - If n = 1000, O(log(n)) = 10, O(log(log(n)) = 3.332
  - If n = 256,000, O(log(n)) = 18, O(log(log(n)) = 4.097
- Makes an assumption that data is uniformly (linearly) distributed
  - If data is "poorly" distributed (e.g. exponentially, etc.), interpolation search will break down to O(log(n)) or even O(n)
  - Notice interpolation search uses actual values (target, startVal, endVal) to determine search index
  - Binary search only uses indices (i.e. is data agnostic)
- Assumes some 'distance' metric exists for the data type
  - If we store Webpage what's the distance between two webpages?

SORTED LISTS

Overview

- If we need to support fast searching we need sorted data
- Two Options:
  - Sort the unordered list (and keep sorting when we modify it)
  - Keep the list ordered as we modify it
- Now when we insert a value into the list, we'll insert it into the required location to keep the data sorted.
- See example



Sorted Input Class

- insert() puts the value into its correct ordered location
  - Backed by array: O( )
  - Backed by LinkedList: O( )
- find() returns the index of the given value
  - Backed by array: O( )
  - Backed by LinkedList: O( )

```
class SortedIntList
{
 public:
  bool empty() const;
  int  size() const;
  void insert(const int& new_val);
  void remove(int loc);

  // can use binary or interp. search
  int find(int val);

  int& get(int i);
  int const & get(int i) const;
 private:
  ???
};
```

Sorted Input Class

- insert() puts the value into its correct ordered location
  - Backed by array: O(n)
  - Backed by LinkedList: O(n)
- find() returns the index of the given value
  - Backed by array: O(log n)
  - Backed by LinkedList: O(n)

```
class SortedIntList
{
 public:
  bool empty() const;
  int  size() const;
  void insert(const int& new_val);
  void remove(int loc);

  // can use binary or interp. search
  int find(int val);

  int& get(int i);
  int const & get(int i) const;
 private:
  ???
};
```

Sorted Input Class

- Assume an array based approach, implement insert()

```
class SortedIntList
{
 public:

 private:
  int* data; int size; int cap;
};


void SortedIntList::insert(const int& new_val)
{









}
```