

# Are Convertible Bonds Efficiently Priced in the Chinese Market? Insights from a Simulation-Based Pricing Model

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## Abstract

This study investigates the pricing efficiency of Chinese convertible bonds and presents evidence of systematic mispricing. To support this analysis, we develop a pricing framework based on the Least Squares Monte Carlo (LSM) method, tailored to reflect contractual features unique to the Chinese market. Using this model, we simulate fair values over the full lifespan of 154 convertible bonds issued between 2015 and 2019 and compare them to observed market prices. The model-predicted price curves generally align well with observed price patterns, demonstrating the robustness and practical value of our approach. However, we also find that trading prices occasionally deviate from model-implied values by more than 10%, with these deviations exhibiting consistent patterns rather than random fluctuations. Furthermore, we demonstrate that simple trading strategies—both at the individual bond level and at the portfolio level—can exploit these discrepancies to generate substantial excess returns. These findings suggest that the Chinese convertible bond market is only partially efficient and highlight persistent arbitrage opportunities, underscoring the importance of market-specific valuation models in emerging financial markets.

**Keywords:** Convertible Bonds; Monte Carlo Simulation; Least Squares Regression; Embedded Options; Chinese Market; Pricing Model; Stock Volatility; Trading Strategy

# 1 Introduction

## 1.1 Convertible bonds and their pricing

Convertible bonds are hybrid financial instruments that blend debt and equity characteristics, offering issuers flexible financing options and providing investors with a unique risk-return profile. Unlike traditional bonds, they give holders the option to convert their bonds into the issuer's stock under specific conditions, enabling participation in potential stock price gains. Convertible bonds also include features like call and put options, with the addition of a unique reset clause in the Chinese market. These embedded options increase the bond's flexibility, making convertible bonds a balanced solution that combines the income stability of bonds with the growth potential of equities, ultimately meeting the strategic needs of both issuers and investors.

Convertible bonds, with their combination of debt and equity features, are complex to price due to embedded options and varied contractual terms. Pricing convertible bonds requires a comprehensive approach that considers both the bond and option components and their interactions. Different convertible bonds may contain unique features, necessitating distinct parameters or even entirely different pricing methods. Generally, pricing methods can be divided into two main categories: component-based and holistic approaches.

The component-based approach separates the convertible bond into bond and option parts. While the bond component is relatively straightforward to price, the option component can be valued using traditional pricing theories, such as the Black-Scholes model [1], binomial trees, and Monte Carlo simulation. Although the Black-Scholes model is foundational, its assumptions of frictionless markets and absence of default risk limit its applicability to convertible bonds. Ingersoll (1977) [2] incorporated default risk into the model, while Lewis (1991) [3] extended it to more complex capital structures. Although classical models are simple to compute, they

overlook interactions among convertible bond clauses, making them less effective for pricing bonds with complex embedded options.

The holistic approach treats the convertible bond as a single entity for pricing. Since closed-form solutions are typically unattainable, numerical methods like binomial trees, finite difference methods, and Monte Carlo simulations are often used. The binomial model introduced by Cox, Ross, and Rubinstein [4] and the finite difference method by Brennan and Schwartz [5] address option pricing in discrete time. Monte Carlo methods were first applied by Boyle (1977) for valuing complex financial instruments. The Least Squares Monte Carlo (LSM) method by Longstaff and Schwartz [8] optimizes early exercise decisions for American options, making it suitable for convertible bonds with call and conversion features. The barrier Monte Carlo method by Cheuk and Vorst [9] and the credit risk model by Tsiveriotis and Fernandes [7] further enhance convertible bond pricing for instruments with complex clauses and default risk.

## 1.2 Chinese convertible bond market

### *Clauses of Chinese convertible bonds*

In the Chinese market, convertible bonds typically include four main clauses: the conversion clause, the redemption clause, the put provision, and the reset clause. Among these, the reset clause is unique to China. Next, we briefly introduce them with examples illustrating their features and implications.

The conversion clause defines the period when investors can convert bonds into the issuer's stock at a predetermined price, starting a few months after issuance and lasting until maturity. For example, the Everbright Convertible Bond (113011.SH), listed on May 5, 2017, allowed investors to begin converting their bonds into stock at a conversion price of 4.36 yuan per share starting from September 18, 2017. This means each bond with a face value of 100 yuan could be converted into approximately

139  $100/4.36 \approx 22.94$  shares. Investors do not always convert when the stock price exceeds  
140 the conversion price, as further stock appreciation is possible. Additionally, the bond's  
141 market price may hold a premium due to remaining bond value and embedded options,  
142 leading investors to sell in the secondary market instead.  
143

144 The put clause provides investors the right to sell bonds back to the issuer at  
145 a preset price if adverse conditions occur, such as a substantial stock price decline,  
146 thus reducing downside risk. For example, the Aviation Information Convertible Bond  
147 (110031.SH) stipulates that if the company's stock closes below 70% of the current  
148 conversion price for any 30 consecutive trading days during the last two interest-  
149 bearing years, bondholders have the right to sell all or part of their bonds back to the  
150 issuer at the face value plus accrued interest. Issuers generally prefer to avoid early  
151 redemption under the put provision, as it requires repaying the debt early, conflicting  
152 with the goal of converting debt into equity without cash outflows. The reset clause  
153 addresses this concern.  
154

155 The reset clause allows issuers to lower the conversion price if the stock falls below  
156 a certain level, making conversion more appealing and potentially avoiding the put  
157 option. For instance, the Aviation Information Convertible Bond (110031.SH) includes  
158 a reset provision that permits the company's board of directors to propose a downward  
159 adjustment of the conversion price if the stock's closing price is below 90% of the  
160 current conversion price for 10 out of any 20 consecutive trading days during the bond's  
161 life. The adjustment requires approval at a general shareholders meeting, and it is at  
162 the issuer's discretion. Although reset conditions are often triggered, the adjustment  
163 may not always be implemented. Modeling this clause requires adding a variable to  
164 account for the issuer's discretion.  
165

166 Table 1 summarizes the key terms of these clauses. Together, the conversion,  
167 redemption, put, and reset clauses embed options into the convertible bond, adding  
168 complexity to its valuation. Since terms vary across bonds, pricing models must  
169

| Clause            | Main Content   |     |
|-------------------|--|-----|
| Conversion Clause | Specifies the period during which investors can convert their convertible bonds into shares of the issuing company's stock at a predetermined conversion price.  | 185 |
| Redemption Clause | Allows the issuer to redeem the convertible bonds before maturity under certain conditions, typically when the stock price has appreciated significantly, by paying bondholders the face value plus accrued interest.  | 186 |
| Put Provision     | Grants investors the right to sell the bonds back to the issuer at a predetermined price under specific adverse conditions, such as a significant decline in the stock price, providing downside protection.   | 187 |
| Reset Clause      | Permits the issuer to adjust the conversion price downward under certain conditions, usually when the stock price has fallen below a specified threshold, to make conversion more attractive and discourage investors from exercising the put provision; unique to the Chinese market. | 188 |

**Table 1** Key clauses commonly found in Chinese convertible bonds, including the distinctive reset clause that permits issuers to lower the conversion price following sustained stock price declines, thereby reducing the value of the conversion option for investors.

adapt to each bond's unique features, necessitating a flexible approach to capture the interactions between options and cash flows.

### *Pricing Chinese convertible bonds*

Since the issuance of the first convertible corporate bond (hereinafter referred to as "convertible bond") in 1992, the Chinese convertible bond market has experienced more than 20 years of development. From 2019 to 2022, the financing scale of listed companies through convertible bonds exceeded 200 billion yuan annually ([18]), and convertible bonds have now replaced additional equity issuance as the primary financing option for listed companies. The pricing of Chinese convertible bonds has its particularities. Compared to capital markets such as the United States, Chinese convertible bonds generally have reset clauses, which means that pricing models used in other countries cannot be directly applied to Chinese convertible bonds.

To address these challenges, Wang Yintian and Wen Zhiying (2018) [17] investigated the influence of reset clauses on pricing accuracy, revealing that incorporating these clauses into models significantly reduces pricing errors and mitigates premium phenomena. Further advancing this research, Li (2023) [19] applied the Black-Scholes model to a sample of 20 listed Chinese convertible bonds, performing a regression analysis between theoretical and market prices. The findings demonstrated that the

231 model's theoretical prices align closely with actual market prices, suggesting that this  
232 model offers valuable explanatory power for Chinese convertible bond prices. Similarly,  
233 Xie Dejie (2016) [16], leveraging the Least-Squares Monte Carlo (LSM) simulation by  
234 Longstaff and Schwartz (2001) [8], adapted the model by incorporating clauses unique  
235 to the Chinese market to validate its applicability. Collectively, these studies under-  
236 score the substantial impact of reset clauses on the pricing of Chinese convertible  
237 bonds. However, they also reveal limitations in current research, which often relies  
238 on small samples and short time frames. For instance, Xie Dejie's (2016) [16] study  
239 was restricted to 13 convertible bonds over just 16 trading days. Additionally, these  
240 studies lack a standardized, objective benchmark for assessing pricing model accuracy.  
241 Therefore, current research still leaves one question unanswered: Is the pricing of con-  
242 vertible bonds in the Chinese market truly accurate? Or, put differently, does price  
243 volatility in convertible bonds create ongoing profit opportunities?

244 To address this issue, this paper employs a modified Least-Squares Monte Carlo  
245 (LSM) model to value 154 convertible bonds over their entire listing periods. By  
246 constructing trading strategies based on the model's valuations and actual trading  
247 prices, we investigate the accuracy of convertible bond pricing in the Chinese market.

### 259 1.3 Structure and Contribution of this Paper

260 The structure of this paper is as follows: Chapter One introduces the concept of con-  
261 vertible bonds and the common methods used for their pricing as well as an overview  
262 of the convertible bond market in China, with a focus on its unique features. Chap-  
263 ter Two explains the construction of the pricing model based on the Monte Carlo  
264 method. Chapter Three presents the empirical analysis and the corresponding results.  
265 Lastly, Chapter FOUR concludes with a summary of findings and suggestions for  
266 future research.

This paper makes the following contributions: First, it proposes a novel calculation approach that incorporates the unique characteristics of the Chinese convertible bond market. Specifically, Formula 2 and Algorithm 2 are designed to account for the distinct contractual terms and pricing dynamics in this market. Second, it conducts a comprehensive computational experiment using a large dataset of real Chinese market data. The results reveal systematic mispricing in Chinese convertible bonds, highlighting arbitrage opportunities. This finding underscores market inefficiencies and provides empirical evidence that deviations from theoretical values can be exploited for excess returns.

## 2 LSM Monte Carlo Simulation for Convertible Bonds Pricing

We begin by discussing the pricing of traditional bonds, which lays the foundation for extending the discussion to the pricing of convertible bonds. Traditional bonds are financial instruments that represent a loan from the investor to the issuer, typically offering fixed periodic payments (coupons) and a repayment of the principal amount at maturity. The pricing of a bond reflects the present value of these future cash flows, discounted at the appropriate rate to account for the time value of money and the credit risk associated with the bond. The price of a bond ( $V_{\text{bond}}$ ) is calculated using the following formula:

$$V_{\text{bond}} = \sum_{t=1}^T C_t e^{-r_t t} + N e^{-r_T T}, \quad (1)$$

where:

- $V_{\text{bond}}$ : The price of the bond, which is the present value of its expected future cash flows.

- 323 •  $C_t$ : The coupon payment at time  $t$ , typically a fixed percentage of the bond's face  
324 value.  
325
- 326 •  $r_t$ : The discount rate at time  $t$ , incorporating the bond's credit risk and the time  
327 value of money.  
328
- 329 •  $e^{-r_t t}$ : The discount factor for the cash flow at time  $t$ .  
330
- 331 •  $T$ : The maturity of the bond, representing the final time period when the principal  
332 is repaid.  
333
- 334 •  $N$ : The principal (or face value) of the bond, repaid by the issuer at the bond's  
335 maturity.  
336
- 337 •  $e^{-r_T T}$ : The discount factor for the principal repayment at maturity ( $T$ ).  
338

339  
340 Convertible bonds are traditional bonds with embedded options that allow the  
341 bondholder or issuer to modify the cash flow structure by holding the bond, converting  
342 it to equity, or exercising other options. These options create uncertainty in valuation  
343 by affecting both the bond's maturity ( $T$ ) and future cash flows. Before exercising  
344 these options, convertible bonds behave like traditional bonds with periodic coupon  
345 payments and principal repayment. Once exercised, the cash flow structure adjusts to  
346 reflect the specific payoff, adding complexity to their valuation.  
347  
348  
349  
350

351 The future cash flows of a convertible bond are uncertain. To determine its value,  
352 we calculate the expected present value of its cash flows under different scenarios,  
353 weighted by their probabilities. Thus, based on the traditional bond pricing formula  
354 (1), the pricing formula for a convertible bond at time  $t$  ( $V_t$ ) can be written as follows:  
355  
356  
357  
358  
359  
360

$$361 \quad V_t = E \left[ \sum_{i=1}^{t^*(t)} C_i e^{-r_i i} + \text{Payoff}(t^*(t), S_{t^*(t)}, \text{Conv}_{t^*(t)}, V'_{t^*(t)}, \text{Action}_{t^*(t)}) e^{-r_{t^*(t)} t^*(t)} \right],$$

362  
363  
364  
365  
366  
367  
368 (2)



where, in addition to the variables already defined in Equation (1), the remaining variables are defined as follows, and the whole process of calculation of  $Payoff(t^*(t), S_{t^*(t)}, Conv_{t^*(t)}, V'_{t^*(t)}, Action_{t^*(t)})$  is concluded in Algorithm 1:

- $t^*(t)$ : The optimal stopping time, determined by the exercise of embedded options.

We will discuss the determination of the optimal stopping time in subsection 2.1.

- $S_{t^*(t)}$ : The stock price at the optimal stopping time  $t^*(t)$ .
- $Conv_{t^*(t)}$ : The conversion price at  $t^*(t)$ , which is the price per share that must be paid when converting a convertible bond into company stock. It determines the cost at which investors acquire the shares, and we will discuss how the conversion price determined in section 2.3.2.
- $E[\cdot]$ : Under different stock paths  $\{S_i\}_{i=t+1, \dots, T}$ , which will be discussed in 2.3.1, there will be varying payoffs and stopping times ( $t^*(t)$ ), resulting in different present values of cash flows. This expectation value represents the average of these present values across all possible stock price paths. Note that  $t^*(t)$ ,  $Conv_{t^*(t)}$ ,  $Action_{t^*(t)}$ ,  $V'_{t^*(t)}$ , and  $Action_{t^*(t)}$  are all determined by the stock price paths  $\{S_i\}_{i=t+1, \dots, T}$ .
- $Action_{t^*(t)}$ : A variable indicating whether any embedded options—call, put, or convert—have been exercised from the bond's issuance up to time  $t^*(t)$ . If any option is exercised, the bond terminates immediately, and  $Action_{t^*(t)} = 1$ ; otherwise,  $Action_{t^*(t)} = 0$ . We will discuss how to determine  $Action_{t^*(t)}$  in section 2.2.
- $Payoff(t^*(t), S_{t^*(t)}, Conv_{t^*(t)}, V'_{t^*(t)}, Action_{t^*(t)})$ : The cash flow at the optimal stopping time  $t^*(t)$ . We will later discuss the calculation methods for payoffs under different scenarios in subsection 2.2.
- $V'_{t^*(t)}$ : Continuation value, the present value of the expected future cash flows from holding the bond. We will discuss this variable in detail in subsection 2.3.

Thus, the pricing formula for convertible bonds can be seen as a modification of the traditional bond pricing formula, incorporating the impact of embedded options and their associated uncertainties. The remainder of this section details a simulation-based

415 calculation of the quantities involved in Formula (2) and demonstrates its application  
416 in pricing a convertible bond.  
417

418

419

## 420 2.1 Determination of the Optimal Stopping Time $t^*(t)$

421

422 As shown in Table 1, the embedded options in a convertible bond may be exercised  
423 under specific conditions when the stock price  $S_{t^*(t)}$  and the conversion price  $\text{Conv}_{t^*(t)}$   
424 meet certain criteria. These options include the call option, conversion, and put option.  
425 The exercise of these options implies that the convertible bond will not be held until  
426 maturity ( $t^*(t) < T$ ). Therefore, the determination of  $t^*(t)$  depends on the time at  
427 which one of these options is exercised. In a Monte Carlo simulation, we identify  
428 the exercise times for all embedded options and select the earliest of these times as  
429  $t^*(t)$ . The payoff of a convertible bond is fundamentally influenced by the timing and  
430 execution of its embedded options by both investors and issuers.  
431

432 Investors aim to maximize their returns. During the holding period of a convertible  
433 bond, if the conditions for exercising an option, such as conversion or put, are met,  
434 they must decide whether to exercise the option or continue holding the bond. To  
435 make this decision, we define the continuation value ( $V_t'$ ) at time  $t$  as the present value  
436 of the expected future cash flows from holding the bond.  
437

438 Similarly, for issuers, their objective is to minimize the cash flows they are required  
439 to pay. To make this decision, the issuer will decide to exercise an option (such as a  
440 call) at time  $t$  when the continuation value ( $V_t'$ ) exceeds the cash flow resulting from  
441 exercising the option at  $t$ , provided the conditions for exercising the option are met.  
442

443 The optimal stopping time  $t^*(t)$  is determined by both the investor's and the  
444 issuer's decisions. Specifically,  $t^*(t)$  is the earliest time when either the investor chooses  
445 to exercise their option (e.g., conversion or put) or the issuer exercises their option  
446 (e.g., call), provided the respective conditions for exercising the option are met.  
447 Mathematically, this can be expressed as:  
448

449

450

$$t^*(t) = \min \left\{ \arg \max_{t_i \in \mathcal{T}_{\text{investor}}} \left\{ t_i \mid \text{Payoff}(t_i, S_{t_i}, \text{Conv}_{t_i}, V'_{t_i}, \text{Action}_{t_i}) \geq V'_{t_i} \right\}, \right. \\ \left. \arg \min_{t_c \in \mathcal{T}_{\text{call}}} \left\{ t_c \mid \text{Payoff}(t_c, S_{t_c}, \text{Conv}_{t_c}, V'_{t_c}, \text{Action}_{t_c}) \leq V'_{t_c} \right\} \right\}, \quad (3)$$

where:

- The set  $\mathcal{T}_{\text{investor}} = \mathcal{T}_{\text{put}} \cup \mathcal{T}_{\text{conv}}$  indicates when the investor can exercise their options.  $\mathcal{T}_{\text{put}}$  is for exercising the put option to sell the bond back, while  $\mathcal{T}_{\text{conv}}$  is for converting the bond into equity.
- $\mathcal{T}_{\text{call}}$  is for exercising the call option to redempt the bond back from investor.

## 2.2 Definition of the Payoff function

We will now define the Payoff function, which can take on one of seven possible values based on four potential outcomes for a convertible bond: redemption (call), repurchase (put), conversion, or holding the bond until maturity, along with their corresponding payoffs. Table 2 shows the detailed definition of the Payoff function, which involves the following notations:

- Call Price: Redemption price, representing the amount the issuer pays to redeem the bond.
- Put Price: Put price, representing the amount investors receive when selling the bond back to the issuer.
- $\text{Conv}_t$ : Conversion price at time  $t$ , determining the number of shares investors receive upon conversion. In the Chinese convertible bond market,  $\text{Conv}_t$  is a time-series variable because of the presence of reset clauses. When these clauses are triggered—typically due to stock price declines—the conversion price may be adjusted

| Outcome  | Condition   | Payoff( $t, S_t, \text{Conv}_t, V'_t, \text{Action}_t$ ) |
|--|---|--|
| <b>Redemption (Call) or Voluntary Conversion</b> | 1. $t \in \mathcal{T}_{\text{conv}} \cap \mathcal{T}_{\text{call}}$<br>2. $V'_t > \text{Call}$<br>3. $\text{Action}_t = 0$<br>4. Redemption clauses satisfied           | $\max\{n_t S_t, \text{Call Price}\}$                     |
| <b>Redemption (Call)</b>                         | 1. $t \in \mathcal{T}_{\text{call}}$ and $t \notin \mathcal{T}_{\text{conv}}$<br>2. $V'_t > \text{Call}$<br>3. $\text{Action}_t = 0$<br>4. Redemption clauses satisfied | Call Price   |
| <b>Repurchase (Put)</b>                          | 1. $t \in \mathcal{T}_{\text{put}}$<br>2. $V'_t \leq \text{Put}$<br>3. $\text{Action}_t = 0$<br>4. Repurchase clauses satisfied   | Put Price  |
| <b>Voluntary Conversion</b>                      | 1. $t \in \mathcal{T}_{\text{conv}}$<br>2. $n_t S_t > V'_t$<br>3. $\text{Action}_t = 0$<br>4. Redemption clauses not satisfied  | $n_t S_t = \frac{100}{\text{Conv}_t} S_t$                |
| <b>Holding to Maturity</b>                       | 1. $t = T$<br>2. $\text{Action}_t = 0$  | $\sum_{t=1}^T C_t e^{-r_t t} + \alpha N e^{-r_T T}$      |
| <b>Continue Holding</b>                          | 1. $t < T$<br>2. $\text{Action}_t = 0$<br>3. Redemption and repurchase clauses not satisfied  | 0  |
| <b>Option Exercised, Bond Terminated</b>         | 1. $\text{Action}_t = 1$  | 0  |

**Table 2** Payoff Calculation Formulas and Trigger Conditions for Convertible Bond Outcomes. Each row corresponds to a possible outcome for a convertible bond, with associated trigger conditions and the formula used to calculate the resulting payoff.

downward, making  $\text{Conv}_t$  time-dependent. The  $\text{Conv}_t$  is determined by stock path

$\{S_i\}_{i=t+1, \dots, T}$  which will be discussed in section 2.3.2

- $n_t = \frac{100}{\text{Conv}_t}$ : Conversion quantity at time  $t$ , representing the number of shares obtained for each bond.

In convertible bonds, the exercise of any embedded option—call, put, or convert—results in the immediate termination of the bond. Therefore, when determining the payoffs for these outcomes, it is crucial to ensure that no option has been exercised prior to the occurrence of a specific outcome. If an option has already been exercised, the bond ceases to be active, and the corresponding payoff is set to zero.

Next, we provide a detailed explanation of above equation.

|   |     |
|---|-----|
| <b>Redemption (Call)</b>  | 553 |
|   | 554 |
| Redemption occurs when the issuer exercises the call option during the call period                                    | 555 |
| $(\mathcal{T}_{\text{call}})$ . Before announcing the redemption decision, the issuer evaluates the continua-         | 556 |
| tion value $V'_t$ and the redemption price Call. If the continuation value $V'_t$ exceeds the                         | 557 |
| redemption price Call, the issuer announces its intention to redeem the convertible                                   | 558 |
| bonds at Call. Otherwise, the issuer will choose not to redeem the bonds.   | 559 |
|   | 560 |
|   | 561 |
|   | 562 |
| If the issuer announces its decision to redeem the bonds and the current time $t$                                     | 563 |
| also falls within the investor's conversion period ( $t \in \mathcal{T}_{\text{conv}}$ ), investors face two choices: | 564 |
| either wait for the issuer to redeem the bonds at the announced redemption price                                      | 565 |
| Call or actively convert the bonds into stock at the current conversion price $\text{Conv}_t$ ,                       | 566 |
| obtaining $n_t = \frac{100}{\text{Conv}_t}$ shares. The decision depends on the relationship between the              | 567 |
| value of conversion $n_t S_t$ , where $S_t$ is the stock price at time $t$ , and the redemption                       | 568 |
| price Call. If $n_t S_t > \text{Call}$ , investors will choose to convert the bonds into stock, as the                | 569 |
| value of conversion exceeds the redemption price. Otherwise, they will opt to wait for                                | 570 |
| the issuer to redeem the bonds at Call.   | 571 |
|   | 572 |
|   | 573 |
|   | 574 |
|   | 575 |
|   | 576 |
|   | 577 |
|   | 578 |
| <b>Resell (Put)</b>   | 579 |
|   | 580 |
| Resell occurs when investors exercise the put option during the put period $(\mathcal{T}_{\text{put}})$ . The         | 581 |
| decision to exercise the put option depends on the relationship between the put price                                 | 582 |
| $\text{Put}_t$ , the continuation value $V'_t$ , and the immediate conversion value $n_t S_t$ . However,              | 583 |
| the trigger condition for the put option is often that the current stock price remains                                | 584 |
| below a certain percentage of the conversion price for several consecutive trading days.                              | 585 |
| Therefore, in this context, the possibility of investors converting the bonds into stock                              | 586 |
| can be disregarded. So if $V'_t \leq \text{Put}$ , investors will exercise the put option.                            | 587 |
|   | 588 |
|   | 589 |
|   | 590 |
|   | 591 |
|   | 592 |
| However, it should be noted that when the stock price remains consistently below                                      | 593 |
| the conversion price, in addition to the put option potentially being triggered, the                                  | 594 |
| issuer's option to reset the conversion price may also be activated. In this case, the                                | 595 |
|   | 596 |
|   | 597 |
|   | 598 |

conversion price  $\text{Conv}_t$  is no longer constant, which could prevent the put option from being triggered.

### ***Voluntary Conversion by Investors***

Similarly, voluntary conversion occurs when investors decide to convert their bonds into shares during the conversion period ( $\mathcal{T}_{\text{conv}}$ ). The decision to convert is based on the comparison between the immediate conversion value  $n_t S_t$  and the continuation value  $V'_t$ .

### ***Holding to Maturity***

If none of the aforementioned scenarios occur during the life of the bond, the bond will be held to maturity. At maturity, investors will receive the maturity value, which consists of the principal amount  $N$  and any compensatory payments represented by  $\alpha N$ , where  $\alpha$  is a multiplier greater than one. In this case,  $\alpha N$  will replace the  $N$  in Equation (1).

This outcome reflects a situation where neither the issuer nor the investors find it optimal to exercise any embedded options, and the bondholder receives the contractual payoff at maturity.

We elaborate on the detailed calculation process of the Payoff function in Algorithm 2.

## **2.3 Calculation of Continuation Value $V'_t$ Based on LSM**

The continuation value  $V'_t$  represents the expected present value of future cash flows if the convertible bond is held beyond a given time  $t$ . It serves as a benchmark for determining whether to exercise an embedded option or to continue holding the bond. So  $V'_t$  is calculated as:

$$V'_t = \mathbb{E} \left[ e^{-r_f \Delta t} V_{t+\Delta t} \mid S_t, \text{Conv}_t \right], \quad (4)$$

where:

- $r_f$  is the risk-free interest rate, observed data,
- $\Delta t$  is the time step,
- $V_{t+\Delta t}$  is the value of the convertible bond at the next time step,
- $S_t$  is the stock price at time  $t$ ,
- $Conv_t$  is the convert price at time  $t$ ,
- $\mathbb{E}[\cdot]$ : This represents the conditional expectation of the future discounted payoff  $V_{t+\Delta t}$ , averaged across all possible paths of simulated stock prices and convert price.

Building upon the framework introduced by Longstaff and Schwartz (2001) [8], we employ an enhanced version of the Least Squares Monte Carlo (LSM) method. This approach leverages Monte Carlo simulations to generate multiple stock price paths and utilizes regression analysis to approximate the conditional expectation  $\mathbb{E}[\cdot]$ .

### 2.3.1 Stock Price Paths Generation

The stock price  $S_t$  is modeled using geometric Brownian motion (GBM), governed by the stochastic differential equation (SDE):

$$dS_t = r_f S_t dt + \sigma S_t dW_t, \quad (5)$$

where:

- $\sigma$  is the volatility of the stock,
- $W_t$  is a Wiener process, with  $W_t \sim \mathcal{N}(0, t)$ , representing random fluctuations.

The solution to this SDE provides the stock price at a future time  $t$ :

$$S_t = S_0 \exp \left( \left( r_f - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \right), \quad (6)$$

where:

- $S_0$  is the initial stock price at the start of the simulation,

- $\sigma$  is the volatility of the stock's return. In this paper,  $\sigma$  is estimated using the historical volatility of stock returns.

indicating that  $S_t$  follows a log-normal distribution. To capture the stochastic behavior of the stock price, multiple stock price paths  $\{S_i\}_{(i=t+1,\dots,T),(j=1,2,\dots)}$  are simulated over discrete time steps  $\Delta t$ . These simulated stock price paths represent potential future scenarios and form the foundation for subsequent calculations.

### 2.3.2 Conversion Price Sequence Generation

The conversion price sequence  $\text{Conv}_{t,j}$  is calculated based on these simulated stock price paths. At the time of issuance, the company specifies an initial conversion price  $\text{Conv}_0$ . If the stock price  $S_{t,j}$  continues to decline and meets certain reset conditions specified in the bond's contractual terms, a reset clause may be triggered. When this occurs, the conversion price may be adjusted downward with a probability  $p^1$ . The conversion price at time  $t$  is updated using the following formula:

$$\text{Conv}_t = \text{Conv}_{t-1} \cdot [1 - \delta \cdot \mathbb{I}(\text{Reset conditions are met}) \cdot \mathbb{B}(p)], \quad (7)$$

where:

- $\delta$  is the reset adjustment factor which is embedded in bond clauses,
- $\mathbb{I}(\cdot)$  is an indicator function that equals 1 if the reset conditions are met and 0 otherwise. The determination of whether the reset conditions are met is based on the stock price path  $\{S_i\}_{i=t+1,\dots,T}$ . Each convertible bond has its own specific rules that define the stock price conditions under which the reset clause will be triggered.
- $\mathbb{B}(p)$  is a Bernoulli random variable with success probability  $p$ , representing whether the reset occurs when conditions are met.

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<sup>1</sup>In this paper,  $p$  is set to 0.4 based on historical data, which indicates the proportion of cases where companies chose to reset the conversion price after the reset clause was triggered. Readers may adopt different values or models for  $p$  depending on changes in market conditions.



### 2.3.3 Backward Deduction for $V'_t$ Calculation

The estimation of the continuation value  $V'_t$  follows a backward deduction approach, which involves iterating over all simulated stock price paths in reverse order, starting from the maturity date ( $t = T$ ) and moving backward to the initial time ( $t = 0$ ).

First, we need to define the intrinsic value of a convertible bond: the immediate realizable value based on its conversion or other options. So, the intrinsic value is:

$$IV_{t,j} = \begin{cases} \max(n_{T,j}S_{T,j}, N), & t = T, \\ \max(n_{t,j}S_{t,j}, \text{Put Price}, \text{Call Price}), & t < T. \end{cases} \quad (8)$$

The bond value  $V_{t,j}$  is the maximum of the intrinsic value and the continuation value, representing the greater value between immediately realizing the bond's worth or the value of continue holding it. The formula for calculating the bond value is:

$$V_{t,j} = \max(IV_{t,j}, V'_{t,j}), t = 0, \dots, T \quad (9)$$

Finally, based on the above definitions, the value  $V'_t$  at each time point  $t$  can be calculated through iterative steps.

#### **STEP 1 Initialization at Maturity ( $t = T$ )**

At maturity, the bondholder does not have the option to hold the bond further. The bond value  $V_T$  is therefore equal to the intrinsic value  $IV_T$ . So, the initial continue value at time  $T$  is defined as:

$$V'_{T,j} = \max(n_{T,j}S_{T,j}, N)$$

Then, substituting  $V'_{T,j}$  into equations (8) and (9), we obtain the initial values at time  $T$ :

$$V_{T,j} = IV_{T,j} = \max(n_{T,j}S_{T,j}, N), \quad (10)$$

**STEP 2 Iteration for Previous Time Steps (Using data at time  $t + \Delta t$  to calculate values at  $t$ )**

Given  $V_{t+\Delta t,j}$ ,  $S_{t,j}$ , and  $\text{Conv}_{t,j}$  for all  $j$ , we iteratively calculate  $V'_{t,j}$ ,  $IV_{t,j}$ , and  $V_{t,j}$  for each  $t$ . At each time point  $t = 1, 2, 3, \dots$ , we estimate  $V'_t$  using the regression model:

$$V'_t = \alpha_t + \beta_{1,t}S_t + \beta_{2,t}\text{Conv}_t + \epsilon_t, \quad (11)$$

The purpose is to obtain  $\alpha_t$ ,  $\beta_{1,t}$ , and  $\beta_{2,t}$ , which can then be used to estimate  $V'_T$  for future data.

1. **Calculation of Continuation Value for a Single Path:** For any given simulated stock price path  $j$ , the continuation value at time  $t$ , denoted as  $V'_{t,j}$ , is calculated by discounting the convertible bond's value at the next time step  $t + \Delta t$ , denoted as  $V_{t+\Delta t,j}$ , back to the present. This approach incorporates the time value of money and is expressed as:

$$V'_{t,j} = e^{-r_f \Delta t} V_{t+\Delta t,j},$$

where:

- $V_{t+\Delta t,j}$  represents the convertible bond's value at  $t + \Delta t$  for the  $j$ th stock path, obtained through backward deduction,
- $\Delta t$  is the length of the time step.

Discounting ensures that all future cash flows are valued in terms of their present value at time  $t$ . This creates a consistent basis for comparing the continuation value and intrinsic value.

2. **Regression-Based Continuation Value Estimation Model:** To estimate  $V'_t$ , we leverage the simulated stock price paths  $S_{t,j}$  and corresponding conversion

prices  $\text{Conv}_{t,j}$  for each path  $j = 1, 2, \dots$ , generating a comprehensive dataset. In this dataset,  $V'_{t,j}$  serves as the dependent variable, while  $S_{t,j}$  and  $\text{Conv}_{t,j}$  act as independent variables. A regression model is then constructed to approximate the continuation value at time  $t$ . The model is expressed in equations (11).

3. **Update Bond Value:** After obtaining the regression model, the bond value at time  $t$ , denoted as  $V_{t,j}$ , is updated by comparing the continuation value  $V'_{t,j}$  with the intrinsic value  $IV_{t,j}$ . The intrinsic value reflects the immediate payoff of exercising the convertible bond options and is calculated as:

$$IV_{t,j} = \max(n_{t,j}S_{t,j}, \text{Put Price}, \text{Call Price}),$$

where:

- $n_t S_{t,j}$  is the value obtained from converting the bond into shares,
- Put Price and Call Price are the bond's put and call values as defined in the contract.

And the continuation value  $V'_{t,j}$  is calculated by fit the regression model using the  $j$ -th path data  $S_{t,j}, \text{Conv}_{t,j}$  at time  $t$ :

$$V'_{t,j} = \hat{\alpha} + \hat{\beta}_1 S_{t,j} + \hat{\beta}_2 \text{Conv}_{t,j}.$$

Where  $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$  are calculated by equations (11). The bond value is then updated as:

$$V_{t,j} = \max(IV_{t,j}, V'_{t,j}),$$

ensuring that the bondholder always selects the option yielding the highest value.

### ***STEP 3 Repeat Until All Time Steps $t$ Are Covered***

This process is repeated for all time steps, working backward from  $t = T$  to  $t = 0$ . At each time step  $t$ , the continuation value  $V'_t$  is estimated as a regression model based

875 on the simulated stock price  $S_t$  and conversion price  $\text{Conv}_t$ . After completing this  
876 backward induction process, we obtain a set of regression models for  $V_t'$  across all time  
877 steps  $t$ . These models allow us to calculate the continuation value at any time  $t$  based  
878 on the current stock price  $S_t$  and conversion price  $\text{Conv}_t$ .  
879

881       These continuation value models are essential for determining the optimal decision  
882 at any time  $t$ . With real-time stock price  $S_t$  and conversion price  $\text{Conv}_t$ , we can  
883 directly compute an accurate estimate of the continuation value and make informed  
884 decisions regarding whether to hold or exercise embedded options. The whole process  
885 is concluded in algorithm 1.  
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| <b>Algorithm 1</b> Calculation of Continuation Value $V'_t$ Using LSM  | 921 |
| <b>Require:</b> Simulated $M$ stock price paths $\{S_{t,j}\}$ and conversion prices $\{\text{Conv}_{t,j}\}$ for  | 922 |
| $j = 1, 2, \dots, M$ , time steps $t = 0, \Delta t, 2\Delta t, \dots, T$ , risk-free rate $r_f$ .                | 923 |
| <b>Ensure:</b> Regression models $V'_t$ for all time steps $t$ .   | 924 |
| 1: <b>Initialize at maturity</b> $t = T$ :   | 925 |
| 2: <b>for</b> each path $j = 1, 2, \dots, M$ <b>do</b>   | 926 |
| 3:     Compute intrinsic value at $t = T$ :  | 927 |
| $V_{T,j} = \max(n_T S_{T,j}, N),$  | 928 |
| 4: <b>end for</b>  | 929 |
| 5: <b>Backward deduction with step size</b> $\Delta t$ :   | 930 |
| 6: <b>for</b> each time step $t = T - \Delta t, T - 2\Delta t, \dots, 0$ <b>do</b>                               | 931 |
| 7: <b>for</b> each path $j = 1, 2, \dots, M$ <b>do</b>   | 932 |
| 8:         Discount the bond value at the next time step $t + \Delta t$ :  | 933 |
| $V'_{t,j} = e^{-r_f \Delta t} V_{t+\Delta t,j}.$   | 934 |
| 9: <b>end for</b>  | 935 |
| 10:     Construct a regression dataset:  | 936 |
| $\{(V'_{t,j}, S_{t,j}, \text{Conv}_{t,j}) : j = 1, 2, \dots, M\}.$   | 937 |
| 11:     Fit a regression model to estimate $V'_t$ :  | 938 |
| $V'_t = \alpha + \beta_1 S_t + \beta_2 \text{Conv}_t + \epsilon_t.$  | 939 |
| 12: <b>for</b> each path $j = 1, 2, \dots, M$ <b>do</b>  | 940 |
| 13:         Compute the intrinsic value at $t$ :   | 941 |
| $IV_{t,j} = \max(n_t S_{t,j}, \text{Put Price}, \text{Call Price}).$   | 942 |
| 14:         Update $V'_{t,j}$ using regression model which share information across all stock paths:             | 943 |
| $V'_{t,j} = \hat{\alpha} + \hat{\beta}_1 S_{t,j} + \hat{\beta}_2 \text{Conv}_{t,j}.$                             | 944 |
| where $(\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2)$ are fitted coefficients from regression model from Step 11. | 945 |
| 15:         Update the bond value at $t$ , which will be used in step 8 in next iteration:                       | 946 |
| $V_{t,j} = \max(IV_{t,j}, V'_{t,j})$   |     |

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967 Algorithm 2 Convertible Bond Valuation at time  $t$ , i.e. Calculating  $V_t$ 
968 Require:  $\{S_u\}_{u<t}$ ,  $S_t$ ,  $\text{Conv}_t$ ,  $r_f$ , bond terms ( $C_t$ , call/put prices, conversion rules,
969 reset clauses, maturity value  $N$ , reset factor  $\delta$ , simulation number  $M$ , and mul-
970 tiplier  $\alpha$ . Where  $\{S_u\}_{u<t}$  represents the stock prices over a past period before  $t$ .
971 The length of this period can be adjusted based on the model's requirements.
972
973 Ensure:  $V_t$ 
974
975 1: Step 1: Simulate Stock and Conversion Price Paths
976
977 2: for each path  $j = 1, \dots, M$  do
978
979 3:   Generate  $\{S_{i,j}\}_{i=t+1,\dots,T}$  based on  $\{S_u\}_{u<t}$ ,  $S_t$ ,  $r_f$  and equation (6)
980
981 4:   Compute  $\{\text{Conv}_{t,j}\}_{i=t+1,\dots,T}$  using equation (7) for each simulated stock price
982 path
983
984 5: end for
985
986 6: Step 2: Get  $V'$  obtained from Algorithm (1)
987
988 7: Step 3: Compute  $t_j^*$  for each path using Equation (3)
989
990 8: Step 4: Compute Payoff
991
992 9: for each path  $j$ , and each corresponding time  $k$  from  $t$  to  $T$  do
993
994 10:   Determine the payoff according to Table 2 return Corresponding payoff value
995
996 11: end for
997
998 12: Step 5: Compute Present Value of Convertible Bond for each path
999
1000  $V_{t,j} = \sum_{i=1}^{t_j^*} C_i e^{-r_i i} + \text{Payoff}(t_j^*, S_{t_j^*,j}, \text{Conv}_{t_j^*,j}, V'_{t_j^*,j}, \text{Action}_{t_j^*,j}) e^{-r_{t_j^*} t_j^*}$ 
1001
1002
1003 13: Step 6: Estimate Convertible Bond Value at time  $t$ 
1004
1005 14: Compute the expected value of  $V_t$  by averaging the present values  $V_{t,j}$  obtained
1006 from all  $M$  simulated paths:
1007
1008
1009 
$$V_t = \frac{1}{M} \sum_{j=1}^M V_{t,j}$$

1010
1011
1012 15: Output:  $V_t$ 

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### 3 Data

This section describes the dataset used in our empirical analysis. All data in this study are obtained from the Wind database [20].

#### 3.1 Convertible Bonds and Stock Data

The objective of this paper is to develop a convertible bond pricing model that can accurately estimate the theoretical value of convertible bonds. By leveraging the model’s calculated values, the study aims to explore whether mispricing in the Chinese convertible bond market creates opportunities for profit. To achieve this, it is first essential to demonstrate that the model’s calculated values closely approximate the bonds’ true theoretical values.

Since the true theoretical value of an asset is unknown, market trading prices are expected to fluctuate around this value. This paper argues that if a model can consistently and accurately price assets over the long term, the trading prices should align closely with the model’s calculated values and fluctuate around the theoretical value. Conversely, in an efficiently priced market, trading prices should not exhibit significant or persistent deviations from the theoretical values. Therefore, if a trading strategy can generate consistent profits by exploiting discrepancies between the model’s calculated values and actual trading prices, it would suggest potential inefficiencies in market pricing.

To achieve the stated objective, we exclude convertible bonds with particularly short maturities and focus on the gap between the model price and the trading price for each bond 30 trading days after its listing. Specifically, we selected all convertible bonds with a maturity of more than two years and a listing date between January 1, 2015, and December 31, 2019, resulting in a total of 154 bonds. Additionally, we collected the corresponding price series for the underlying stocks associated with these convertible bonds. The decision to focus on bonds issued before 2020 stems from the

**Table 3** Summary of Conditions and Payoffs for Convertible Bonds

| bond<br>id | Call  |         |        | Put   |         |        | Reset   |        |
|------------|-------|---------|--------|-------|---------|--------|---------|--------|
|            | Price | Trigger | Period | Price | Trigger | Period | Trigger | Period |
| 110033     | 100   | 130     | 30     | 100.0 | 70.0    | 30.0   | 90      | 30     |
| 110034     | 100   | 130     | 30     | 103.0 | 70.0    | 30.0   | 85      | 20     |
| 110038     | 100   | 125     | 30     | 100.0 | 50.0    | 30.0   | 80      | 30     |
| 110041     | 100   | 130     | 30     | 100.0 | 70.0    | 30.0   | 90      | 30     |
| 110043     | 100   | 130     | 30     | na    | na      | na     | 80      | 30     |
| ...        | ...   | ...     | ...    | ...   | ...     | ...    | ...     | ...    |
| 128091     | 100   | 130     | 30     | 100.0 | 70.0    | 30.0   | 90      | 20     |

Table 3 presents the key contractual terms embedded in example convertible bonds, specifically focusing on the call, put, and reset clauses. For example, in the case of bond 110033, if during the 30-day Call Period, the stock price exceeds the Call Trigger (130) for a number of consecutive trading days as specified in the bond contract, the issuer has the right to redeem the bond at the Call Price (100). Similarly, if during the 30-day Put Period, the stock price below the Put Trigger (70) for a number of consecutive trading days as specified in the bond contract, the investor has the right to resell the bond at the Put Price (100) to issuer. In the case of the reset clause, if during the 30-day Reset Period, the stock price below the Reset Trigger (90) for a number of consecutive trading days as specified in the bond contract, the issuer has the right to reset the convert price. Some bonds (e.g. 110043) do not include put or reset clauses, in which case the corresponding entries are marked as **na**.

typical six-year listing duration of Chinese convertible bonds. By selecting bonds listed before 2020, we ensure that the vast majority of our sample has completed its full listing cycle by the time of this study, providing a comprehensive dataset for analysis.

After obtaining the convertible bond sample, we analyze the additional clauses embedded in these convertible bonds. Table 3 presents the embedded additional clauses for 15 convertible bonds as examples.  $c_p$  represents the call price, which is usually the face value of the bond plus the accrued interest.  $c_m$  indicates the evaluation period for determining whether the call clause is triggered.  $c_t$  represents the condition under which the call clause is triggered. Similarly,  $p_p$  represents the put price,  $p_t$  represents the condition under which the put clause is triggered, and  $p_m$  is the evaluation period for determining whether the put clause should be triggered. In this sample, all convertible bonds have call clauses, as call clauses are favorable to the bond issuer, i.e., the company. However, not all convertible bonds have put clauses, such as the convertible bond 110043. Therefore, when modeling and estimating the price of convertible bonds, we need to take this factor into consideration.



Compared to call and put clauses, which are predetermined when the bond is issued, the reset clause carries more uncertainty.  $r_t$  represents the condition under which the reset clause is triggered. For example, if  $r_t = 90$ , it means that if the stock price (or average price) falls below  $r_t$  of the convert price during the evaluation period, the reset clause is triggered.  $r_m$  represents the evaluation period for determining whether the reset clause is triggered. When the reset clause is triggered, the company can choose to adjust the convert price or not, so we cannot be certain whether the company will definitely adjust the convert price, which means the model requires parameter adjustments. Even if the company chooses to adjust the convert price, it has the discretion to decide the magnitude of the price adjustment within a certain range. To simplify the analysis, based on recent years' assumptions, we assume that once the reset clause is triggered, the probability that the company will choose to adjust the convert price is 0.4. If the company chooses to reset convert price, the convert price will be reduced to the proportion of  $r_t$ .

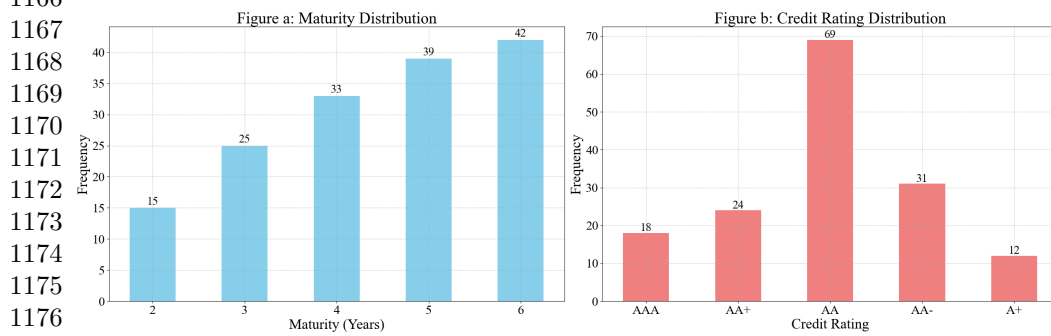
### 3.2 Interest Rate and Credit Rates of Underlying Stock of Convertible Bonds

The interest rates involved in this paper include two aspects: one is the risk-free rate, for which we use the yield of Chinese government bonds. The other is the yield to maturity (YTM) corresponding to the risk of each bond. We identify the market prices of pure bonds issued by each convertible bond's issuing company and then deduce the YTM of each bond as the yield to maturity incorporating default risk for the convertible bonds. Moreover, we need to calculate the data for both the risk-free rate and the bond's YTM for each trading day.

The risk-free rate is used to construct the stock volatility model in this paper, while the bond's yield to maturity is used as the discount rate for discounting future cash

flows in the pricing of convertible bonds. Figure 1 shows the distribution of maturity and credit rating among our sample.

It can be observed that the credit ratings of the companies associated with the convertible bonds in the sample are all A+ or above. This is due to the fact that in the Chinese market, companies issuing convertible bonds are required to meet certain profitability criteria, making convertible bonds a low-risk asset. At the same time, they offer the potential for capital gains if the stock price rises. Therefore, for investors who are optimistic about the company's stock price but do not want to bear the risk of unexpected stock price declines, convertible bonds are a good investment option.



**Fig. 1** Distributions of Maturity and Credit Ratings of Chinese Convertible Bonds. (a) Distribution of issuance durations for the convertible bonds analyzed. Durations range from two to six years; bonds with shorter durations were excluded due to insufficient data, and none exceeded six years. (b) Distribution of credit ratings. All bonds are rated between A+ and AAA, reflecting the stringent requirements for issuing convertible bonds in China. The vertical axis (Frequency) represents the number of bonds (out of 154 total) that fall into each category.

## 4 Empirical Analysis

To test whether Chinese convertible bonds are priced efficiently, we conduct empirical analysis on historical market data introduced in the last section, and compare our model's theoretical valuations to observed trading prices. Drawing on the Efficient

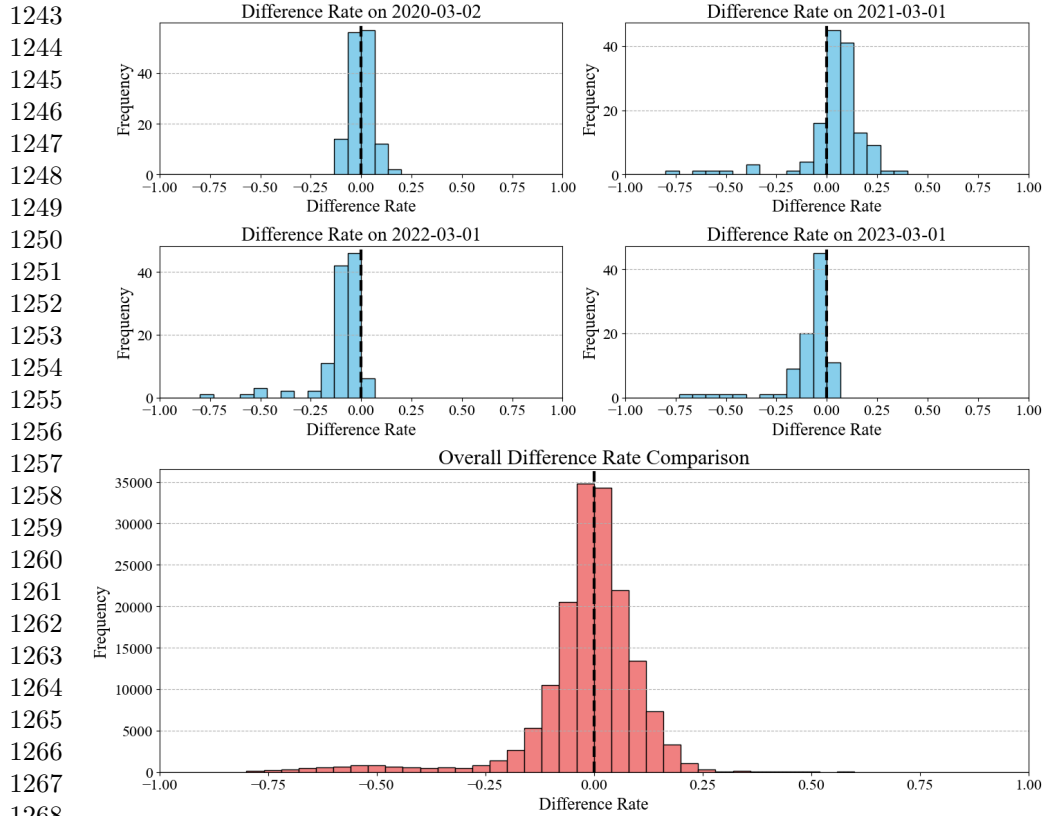
Market Hypothesis (EMH), we assume that any systematic, profit-generating deviations from fair value would indicate partial market inefficiency [21]. If a given pricing method can systematically generate positive excess returns, it suggests some degree of market inefficiency [22]. Conversely, if no persistent excess returns can be obtained, the market is considered efficient [23]. Our empirical investigation proceeds in two stages: analyzing 154 individual bonds and then constructing portfolios to detect broader patterns of mispricing.

#### 4.1 Individual Convertible Bond Example

In this experiment, individual refers to the fact that our pricing and strategy simulations are conducted at the level of an individual convertible bond (instead of a portfolio of multiple bonds). In this subsection, all results presented and performance evaluations are based on the aggregated results from these 154 bonds.

To assess the pricing efficiency of the Chinese convertible bond market, we first examine whether the trading prices of convertible bonds align with the theoretical prices generated by our model at specific time points. This comparison allows us to quantify the extent of deviation between market prices and model-implied values. Figure 2 illustrates the distribution of price deviations, computed as:

$$\text{Difference Rate} = (\text{Simulated Price} - \text{Trading Price}) / \text{Trading Price} \quad (12)$$

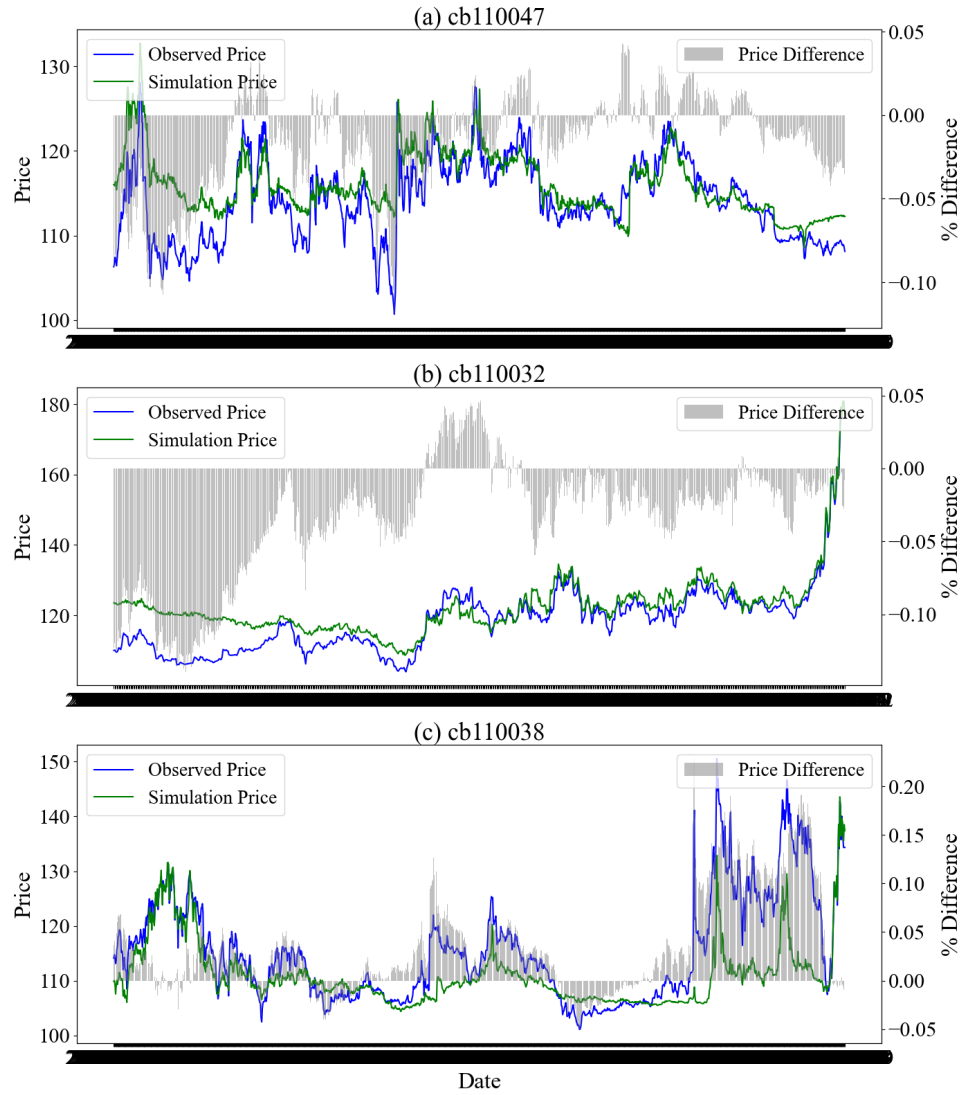


**Fig. 2** Distribution of differences between simulated prices and market trading prices on four selected trading days, as well as across all time points. While deviations are generally within 10%, the overall distribution is slightly left-skewed, suggesting that market prices tend to be marginally higher than simulated prices on average. However, on individual trading days, deviations can be substantially larger in either direction. The vertical axis (Frequency) indicates the number of convertible bonds that fall into each deviation bin. For the first four subplots, the distribution reflects all convertible bonds on each respective trading day. The final subplot aggregates all available Difference Rate values for all sample convertible bonds across the entire sample period, and hence has much higher frequencies.

The first four panels in Figure 2 show the distribution of price deviations for all convertible bonds on early March trading days in each year from 2020 to 2023 (specifically, '2020-03-02', '2021-03-01', '2022-03-01', and '2023-03-01'). These dates represent typical patterns observed throughout the sample period, with similar distributions

found on many other trading days. The fifth panel presents the aggregated distribution across all trading days in the dataset. The results reveal that significant deviations exist between theoretical and market prices, with trading prices deviating by up to 10% in most cases. On average, market prices are 1.58% higher than model-implied values, with a variance of 1.88%.

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**Fig. 3** Price Difference. Comparison between simulated and observed prices for four convertible bonds. (a) shows early-stage volatility post-issuance with eventual convergence to model values. (b) captures a price surge driven by stock rally. (c) illustrate model robustness during high volatility or downward trends.

Figure 3 presents the differences between the simulated prices and the observed market prices for three representative convertible bonds. In Figure 3 (a), the bond

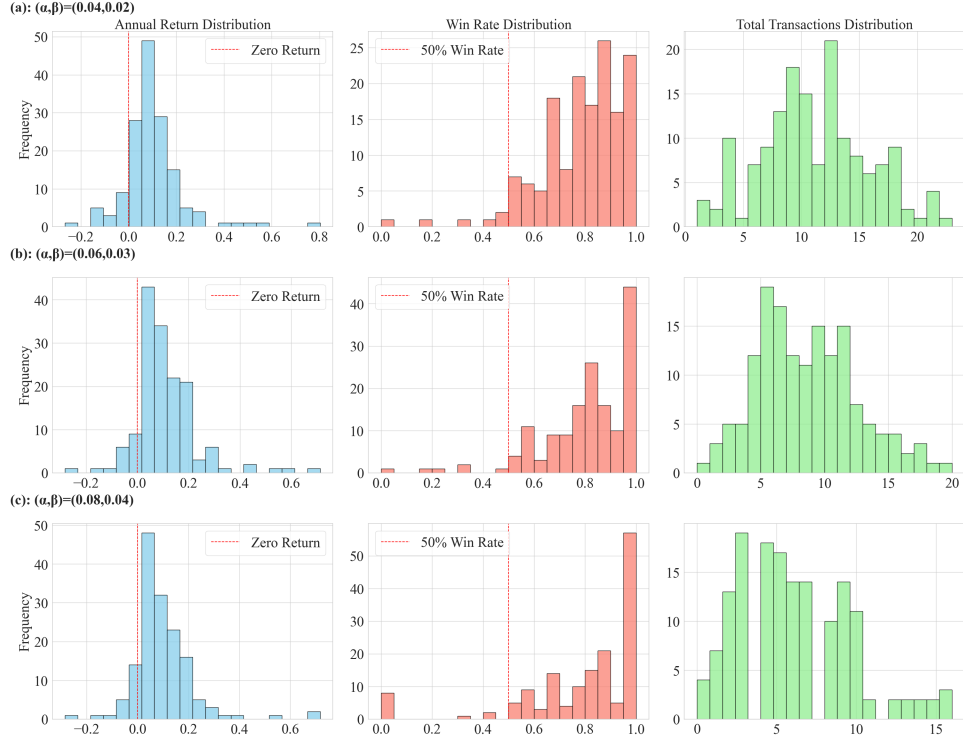
exhibits high price volatility shortly after issuance, while the model remains stable—highlighting its ability to capture intrinsic value without being influenced by short-term market noise. Over time, the market price gradually converges toward the model price. Figure 3 (b) illustrates a case where a sharp rally in the underlying stock causes a surge in the convertible bond price, which is accurately reflected by the model. Figure 3 (c) shows scenarios characterized by pronounced stock price fluctuations or downward trends, where the model continues to provide consistent pricing.

A comparison of the model-predicted price curves and the observed bond price curves in Figure 3 suggests that, while the overall trends are similar—demonstrating the robustness and accuracy of our pricing framework—there are periods with notable deviations. Such deviations are likely driven by short-term market inefficiencies, such as fluctuations in supply and demand. These pricing discrepancies prompt an important question: can they be systematically exploited to generate excess returns through arbitrage strategies?

To investigate whether observed deviations can be systematically exploited, we propose a straightforward "buy-low-sell-high" trading strategy. Specifically, we introduce two thresholds,  $\alpha$  and  $\beta$ , where we buy when the trading price falls below  $1 - \alpha$  times the simulated price, and sell when the trading price exceeds  $1 + \beta$  times the simulated price. This strategy is applied individually to each convertible bond in our data. In this study, we use 3 different settings of  $(\alpha, \beta)$  at  $(4\%, 2\%)$ ,  $(6\%, 3\%)$  and  $(8\%, 4\%)$ .

Figure 4 summarizes the performance of the trading strategy based on the proposed pricing model, applied independently to 154 individual convertible bonds. The figure illustrates the distributions of annualized returns (left column), win rates (middle column), and number of trades (right column) under three parameter configurations, each defined by a distinct combination of entry and exit thresholds  $\alpha$  and  $\beta$ . Across all parameter settings, the strategy demonstrates strong and consistent performance: the mean and median annualized returns range from 8.5% to 10.7%, while win rates

consistently exceed 78%, indicating a high proportion of profitable trades. The number of trades per bond remains moderate, typically between 6 and 11, reflecting a relatively low trading frequency. These results suggest that even a simple rule-based strategy can systematically capture excess returns, highlighting the pricing model's effectiveness in identifying and exploiting mispricing opportunities in the convertible bond market.



**Fig. 4** Performance of the trading strategy based on the proposed pricing model applied to 154 individual convertible bonds. This figure displays the distribution of annualized returns (left column), win rates (middle column), and number of trades (right column) for a trading strategy applied to 154 individual convertible bonds, evaluated under three different combinations of trading decision parameters (entry threshold  $\alpha$  and exit threshold  $\beta$ ), shown across rows. The strategy was applied independently to each bond, and the distributions summarize outcomes across all bonds. The results demonstrate robust performance across parameter settings, with most annual returns being positive, win rates exceeding 0.5 in most cases, and a relatively low number of trades.



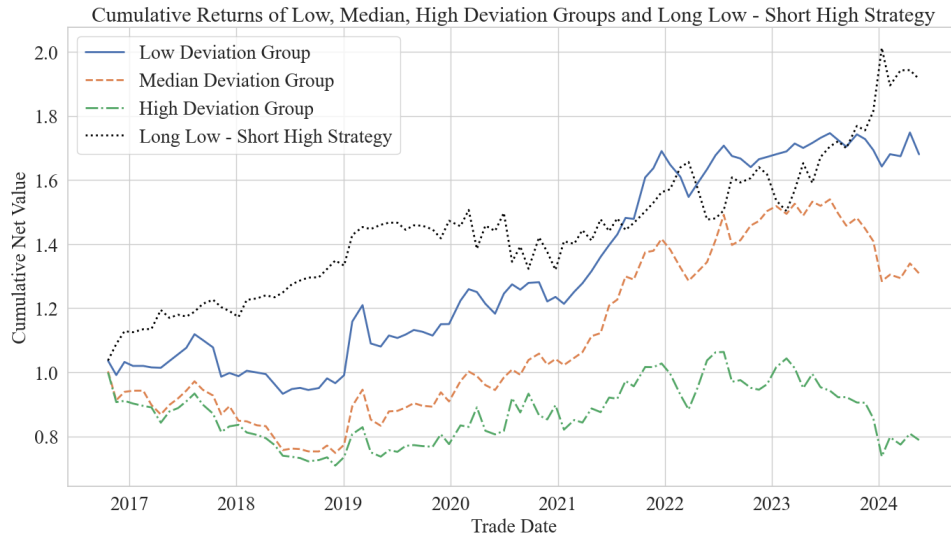
## 4.2 Portfolio-Based Evidence of Arbitrage Opportunities

Beyond individual trade-based strategies, we also evaluate the model’s effectiveness using a portfolio-based approach. Over the entire sample period, we implement a 20-working-day cycle. In each investment cycle, we construct portfolios based on the difference rate between the model-implied price and the market trading price of each convertible bond. Specifically, at the beginning of each cycle, we calculate the difference rate for all eligible convertible bonds in our sample using the formula defined earlier. We then rank the bonds based on their difference rates and divide them into three equal-sized groups: the Low Difference Group (lowest third), the Median Difference Group (middle third), and the High Difference Group (highest third).

The Low Difference Group contains bonds that are relatively undervalued by the market compared to the model, while the High Difference Group includes those that appear relatively overvalued. For each group, we construct an equally weighted portfolio and hold it throughout the investment cycle. Additionally, we form a long-short portfolio by taking a long position in the Low Difference Group and a short position in the High Difference Group, each with equal dollar amounts to ensure market neutrality. This long-short strategy is designed to capture the excess returns arising from mispricing, while minimizing market-wide risk exposure. At the end of the cycle, we rebalance the portfolios by repeating the above steps based on the updated difference rates.

Figure 5 presents the performance metrics of these portfolios. The results reveal a clear and significant monotonic relationship between the difference rate and portfolio returns. The Low Difference Group achieves an annualized return of 7.28%, with volatility and maximum drawdown of 14.82% and -16.62%, respectively. The Median Difference Group records an annualized return of 3.71%, with volatility of 17.37% and a maximum drawdown of -25.37%. Meanwhile, the High Difference Group incurs a negative annualized return of -3.17%, with higher volatility (21.04%) and a deeper

1519 maximum drawdown (-30.69%). These findings indicate that convertible bonds with  
1520 lower difference rates tend to be relatively undervalued by the market, while those  
1521 with higher difference rates are relatively overvalued. As a result, a trading strategy  
1522 that goes long on undervalued (low difference rate) bonds and short on overvalued  
1523 (high difference rate) bonds can generate significant excess returns. Notably, the long-  
1524 short portfolio outperforms all individual groups, delivering an annualized return of  
1525 9.2%, with volatility of 15.36% and a maximum drawdown of -12.41%.



**Fig. 5** Cumulative Returns of Low, Median, High Deviation Groups and Long Low - Short High Strategy. This figure shows the cumulative returns of three deviation groups (Low, Median, High) and the long-short strategy. Convertible bonds are re-ranked every 20 days based on price deviation, and portfolios are updated accordingly. The long-short strategy, going long on Low Deviation Group and short on High Deviation Group, demonstrates the model's effectiveness in portfolio construction.

1555 These results provide compelling evidence that price deviations in the Chinese con-  
1556 vertible bond market can be systematically exploited for arbitrage. The model not  
1557 only offers stable and accurate pricing but also enables the construction of profitable  
1558 portfolios that leverage valuation discrepancies. This highlights its potential in gen-  
1559 erating significant excess returns while offering deeper insights into the efficiency of  
1560 market pricing.

## 5 Conclusion

This paper presents a comprehensive analysis of Chinese convertible bond pricing, based on all 154 bonds issued between 2015 and 2019. We find that market prices broadly align with theoretical values, yet notable and systematic mispricings persist—revealing exploitable inefficiencies in the market. To support this analysis, we develop a novel pricing model based on the Least Squares Monte Carlo (LSM) method, incorporating key contractual features specific to the Chinese market, including reset clauses, call provisions, and put options. While the model successfully replicates overall pricing patterns, the remaining price gaps suggest that investors may overlook or misvalue key contractual elements. For instance, reset triggers—unique to the Chinese convertible bond market—are rarely addressed in the existing literature, leaving limited guidance on how to properly incorporate them into pricing models. Moreover, the relatively small scale of the Chinese convertible bond market makes it more susceptible to speculative capital. This study also finds that convertible bond prices in China frequently exhibit extreme volatility, including sudden surges and sharp declines.

Our empirical analysis confirms that a simple buy-sell strategy keyed to these pricing deviations consistently outperforms baseline returns, revealing partial market inefficiency. Additionally, a long-short approach across multiple bonds shows that relative mispricing can be systematically harnessed for enhanced gains. Consequently, this study provides a roadmap for understanding and quantifying embedded-option complexity in an emerging financial market context. Looking ahead, more advanced calibrations of credit risk, volatility behavior, and issuer decision-making could reduce pricing uncertainty and possibly dampen these arbitrage opportunities. Ultimately, the findings encourage market participants to adopt more comprehensive valuation practices and promote continuing efforts by regulators to foster transparency and liquidity in the Chinese convertible bond sphere.

## 1611 **Declaration**

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## 1613 **Availability of data and material**

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1615 All data in this study are obtained from the Wind database available at:  
1616 <https://www.wind.com.cn/portal/zh/WDS/index.html>. Accessed: March 2025.

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## 1619 **Competing interests**

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1621 The author declares that they have no competing interests.

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## 1631 **Authors' contributions**

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1633 S.L. and X.Z. jointly conceived the study; X.Z. supervised the research, while S.L.  
1634 performed all computer experiments and prepared the initial manuscript draft. All  
1635 author(s) read and approved the final manuscript.

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