

Final Report: Election Prediction

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Introduction

The United States is getting closer to the 2020 presidential and Congressional elections on November 3, 2020. All 435 seats in the United States House of Representatives, 35 of the 100 seats in the United States Senate, and the office of President of the United States are up for election (Wikipedia, 2020). With the current polarizing political landscape, the election outcomes are significant to determine the next stage of this country. Polarization also makes election prediction both less and more difficult. It is less difficult because election results are less subject to election or candidate-specific factors, and it is more difficult as elections become increasingly close and fierce competitions, especially for presidential elections (Gelman, 2020). Therefore, we believe 2020 is a particularly challenging yet interesting year for statisticians to predict election outcomes and compare results with other predictions and the actual outcome to reflect on methodologies and unaccounted predictors. These election predictions not only inform the public about campaign trends and political sentiments but also help political strategists make decisions on allocating campaign resources for different candidates (Linzer et al.). American political pundits have been spending countless hours obtaining and analyzing relevant data to predict the election outcomes, and historical models and predictions and pre-election polls are two of the most important sources of information (Linzer et al.). In this report, similarly, we will also be using both literature review and polls for 2020 to build our prediction models.

We plan to build prediction models for the 2020 U.S. presidential election and the Senate election nationwide. Also, among all states, we decide to further predict the U.S. House election outcome for North Carolina, as it has been a swing state in presidential and Congressional elections for decades. Since 1996, the Republican statewide vote share in Congressional elections has varied “from a low of 45% in 2008 to a high of 55% in 2014” (Perrin et al.). To summarize, this report aims to use statistical models to predict (1) the outcome of the presidential election, (2) whether the US Senate remains in Republican control, (3) the electoral college vote, (4) the outcomes of all NC Congressional elections (the 13 federal Representatives to Congress), and (5) the outcome of the NC Senate election, including characterization of uncertainty in predictions.

Data Description

In literature, election prediction relies on polling data as well as the fundamentals, which are economic indicators (The Economist, 2020) and voter turnout by demographic groups (Hansford, et al.). To forecast the outcomes of presidential election, senate election and house election (for North Carolina only), we obtained 2020 presidential polling data from *The Economist*, 2020 senate and house polling data as well as partisan lean data from *FiveThirtyEight*.

The fundamentals data were retrieved from various online sources, including Andrew Gelman’s presidential election prediction model Github repository (for correlation across states and historical incumbent party’s June approval ratings), Federal Reserve Economic Data (for second quarter real income growth), and NC Board of Elections website (for 2020 NC registered voter demographics). Please refer to Appendix B for a detailed description of all data sets used.

Data Processing and Missing Data

For both the President and Senate models, we had to choose which states we considered to be “battlegrounds” to include in the models. We chose states for the models separately, since some states have a competitive race for President but not for Senate or vice versa.

For the presidential election data, we choose to use polling data from 30 days before the election (as opposed to over 30 days), as this aligns with Andrew Gelman’s observation that polling data becomes more predictive as the day of poll becomes closer to election day (The Economist). Note that state-wide polls 30 days or less are unavailable for Oregon, Idaho, Wyoming, Nebraska, North Dakota, Illinois, Tennessee, Arkansas, Mississippi, and many Northeastern states. But these states are not swing states in recent elections, so we did not include those in our modeling process. For modeling, we will filter out those states with obvious party preferences and focus on those states showing percentages swinging right above or below 50%. For the presidential model, the states chosen are Arizona, Florida, Georgia, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, and Wisconsin. These thirteen states are swing states in the plot, and are rated as either “Toss Up”, “Lean Republican”, or “Lean Democrat” races (the three most competitive categories) by the Cook Political Report (CITE), an nonpartisan elections newsletter, as of October 28. These states are also projected to be competitive in the Presidential race by both the FiveThirtyEight and Economist models. We select poll responses from likely voters only, since it is known that using all responses may overestimate support for the Democratic party, according to FiveThirtyEight (Silver, 2014).

For the Senate model, the states chosen are Alabama, Alaska, Arizona, Colorado, Georgia, Iowa, Kansas, Maine, Michigan, Montana, North Carolina, South Carolina, and Texas. Similarly, for states such as California with strong historical party preferences, state-wide polls within 30 days of election are missing, but the missing data does not hurt our modeling process as we are only focusing on the swing states. There is also a special election for the Senate in Georgia in addition to the regularly scheduled election. These fourteen races are rated as either “Toss Up”, “Lean Republican”, or “Lean Democrat” (the three most competitive categories) by the Cook Political Report (CITE) as of October 29. These states are also projected to have competitive Senate races by both the FiveThirtyEight and Economist models.

For the U.S. House election, due to the scarcity of poll responses for the NC House elections, we included all poll responses within 115 days of the election. We will supplement the polls with the results from our interim report to predict the House election for NC.

Appendix B stuff

Presidential Election Model:

- **2020 US presidential election polls - all_polls.csv**: 2020 U.S. Presidential Election Polls compiled by *The Economist*, publicly available at <https://projects.economist.com/us-2020-forecast/president/how-this-works>.
- **partisan_leans_538.csv**: effect of partisan lean on the Democratic candidate’s presidential election vote share in each state. Available at <https://github.com/fivethirtyeight/data/tree/master/partisan-lean>.
- **abramowitz_data.csv**: downloaded from Andrew Gelman’s Github repository on 2020 Presidential Election. This data set contains information on each 1948-2016 election year’s 2nd quarter GDP performance, incumbent party’s June approval rating and incumbent party’s vote share. Available at <https://github.com/TheEconomist/us-potus-model>.
- **states_cov_matrix_full.csv**: covariance matrix of states which take into account similarity between states based on their demographic and political profiles. Retrieved from Andrew Gelman’s Github repository on 2020 Presidential Election.
- **abramowitz_additional.csv**: Supplemented **abramowitz_data.csv** with the corresponding year’s 2nd quarter real income growth, 3rd quarter real income growth (data from *FRED* <https://fred.stlouisfed.org/series/A067RO1Q156NBEA>) and S&P stock performance 3 months prior to the election (data from <https://www.multpl.com/s-p-500-historical-prices/table/by-month>).

Senate Election Model:

- `senate_polls.csv`: *this is from course website* https://projects.fivethirtyeight.com/polls-page/senate_polls.csv
- `partisan_leans_538.csv`: same as the data set in presidential election model
- We also incorporated incumbency advantage in the model, which as *FiveThirtyEight* suggests, is on average 2.6 for senate incumbents (<https://fivethirtyeight.com/features/how-much-was-incumbency-worth-in-2018/>).

House Election Model (for North Carolina):

- `house_polls.csv`: *this is from course website* https://projects.fivethirtyeight.com/polls-page/house_polls.csv
- `ncvoter_1027_small.rds`: NC registered voter demographics information provided by the NC State Board of Elections (<https://dl.ncsbe.gov/list.html>).

Exploratory Data Analysis

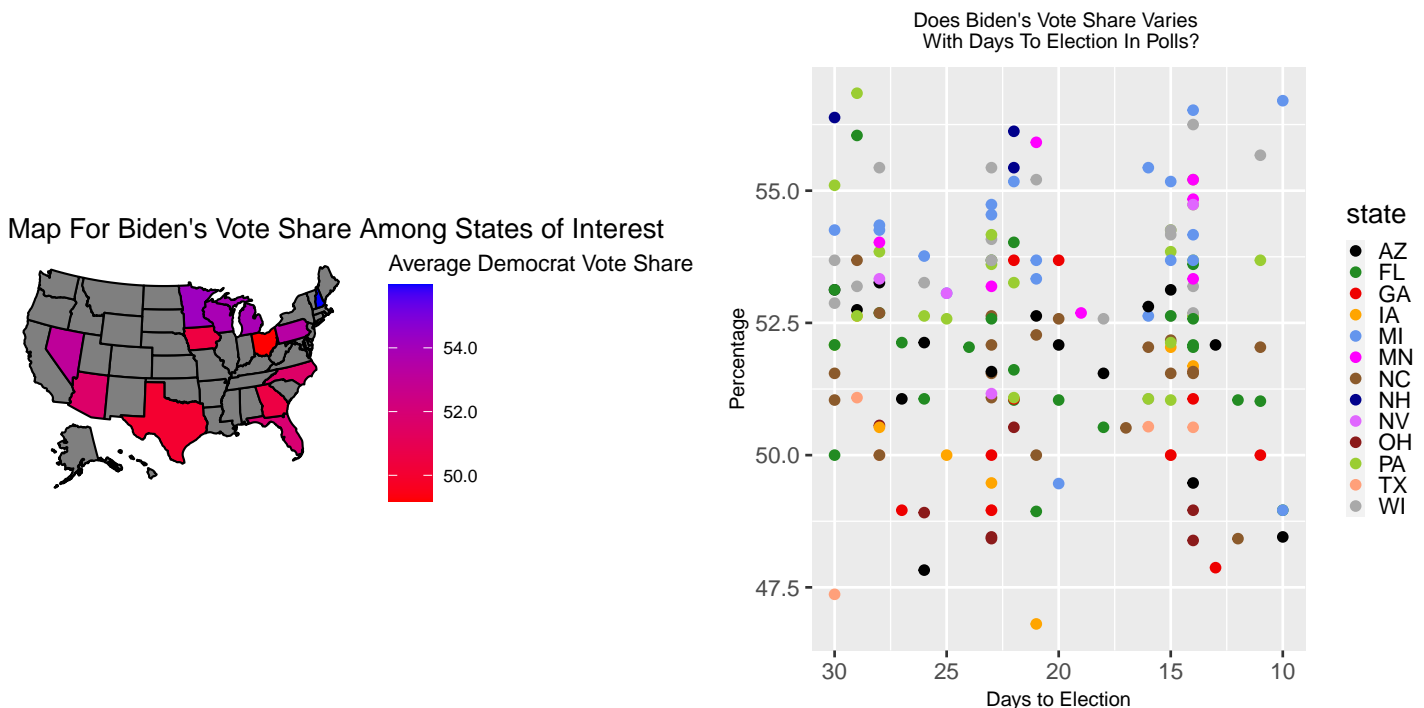


Figure 1: Presidential Election Data Visualization

After initial data cleanup for analysis, Figure 1 aims to explore polling data we have for the 2020 presidential election within 30 days of election, after filtering for the states of interest. The left plot shows that most states, as the polls indicate, have a vote share higher than 50% for Biden, averaging on all the polls available for each state respectively. While unsurprisingly Washington and California turn out to be blue, Louisiana, for example, still shows a deep red color. The right plot shows that within 30 days to election, variations exist for Biden support rate among polls within individual states. Combined with abundant literature that takes time into account for outcome prediction models, we will take days to election as a part of our model prediction as well (Gelman, 2020).

Similarly, Figure 2 visualizes the polling data we have for the 2020 U.S. Senate election, also within 30 days of election and after filtering for the states of interest. The left plot shows that most states, as the polls indicate, have a vote share higher than 50% for the Democratic Party, averaging on all the polls available for each state respectively. Nebraska is the only state with a bright red color in this data set. The right plot shows that within

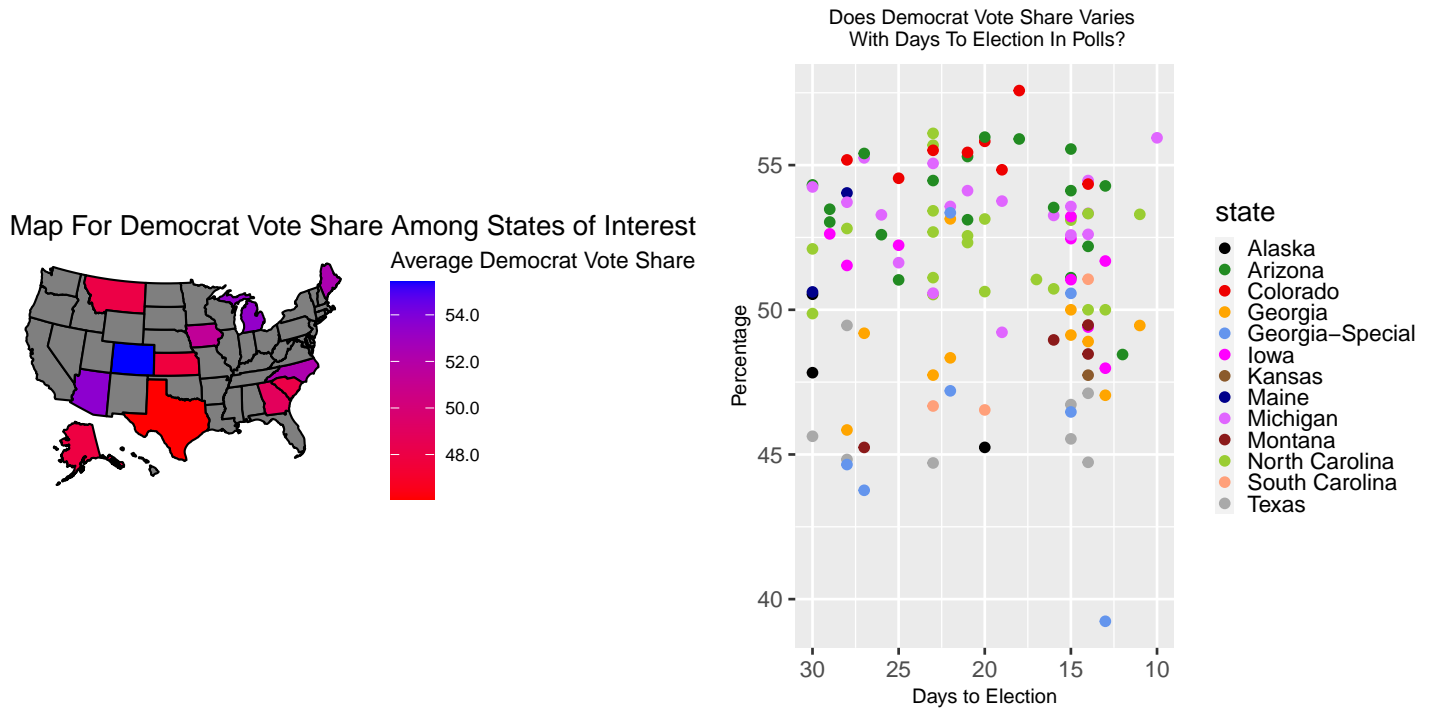


Figure 2: U.S. Senate Election Data Visualization

30 days to election, variations exist for the Democratic party vote share among polls within individual states, such as the trend seen for North Carolina as Democratic Party vote share decreases getting closer to the election. We will take days to election as a part of our model prediction as well.

Figure 3 shows the different Democratic Party vote shares predicted by our interim report for all congressional districts in North Carolina. After predicting for voter turnout in the interim report, we use the party registration for predicted voters to get the vote share for Democratic Party. First, we grouped predicted voters for congressional districts. There are three categories in party registration: Democratic, Republican, and other (indicating a third party or unaffiliated). Within each congressional district, we split the `other` population in half, and add it to both the Democratic Party vote count and the Republican Party vote count, as FiveThirtyEight used the same approach to get vote share for their House predictions, shown in <https://projects.fivethirtyeight.com/2020-election-forecast/house/>. Among 13 districts, 6 of them have a Democratic Party vote share higher than 50%.

Method

Similarly to prior work (Linzer 2013), we use a hierarchical model structure for our election models to overcome the limitation that not every state is polled on every day, allowing the model to borrow data across states (or Congressional districts). This also helps account for the fact that polls from each individual state are correlated data observations.

To specify priors for the mean of two-party vote share for Democrats in each state or district, we used a combination of state partisanship, economic fundamentals, candidate incumbency, and projected voter turnout based on demographics. First, we started with state or district partisan lean calculated by FiveThirtyEight (see “FiveThirtyEight’s Partisan Lean” in references). Partisan lean is the average difference between how a state or district votes and how the country votes overall, based on results from the past two presidential elections as well as statewide elections in 2018. For example, North Carolina has a partisan lean of R+4, meaning that in a 50/50 political environment nationwide, a Republican would be expected to win North Carolina by 4 points. For each partisan lean, we turn the partisan lean into a prior mean of Democratic vote share by adding or subtracting half the partisan lean from 50%. North Carolina is a Republican-leaning state, so we subtract 2 from 50% to get a partisan lean prior of 48% (note that the Republican would then get 52% of the two-party vote, resulting in an

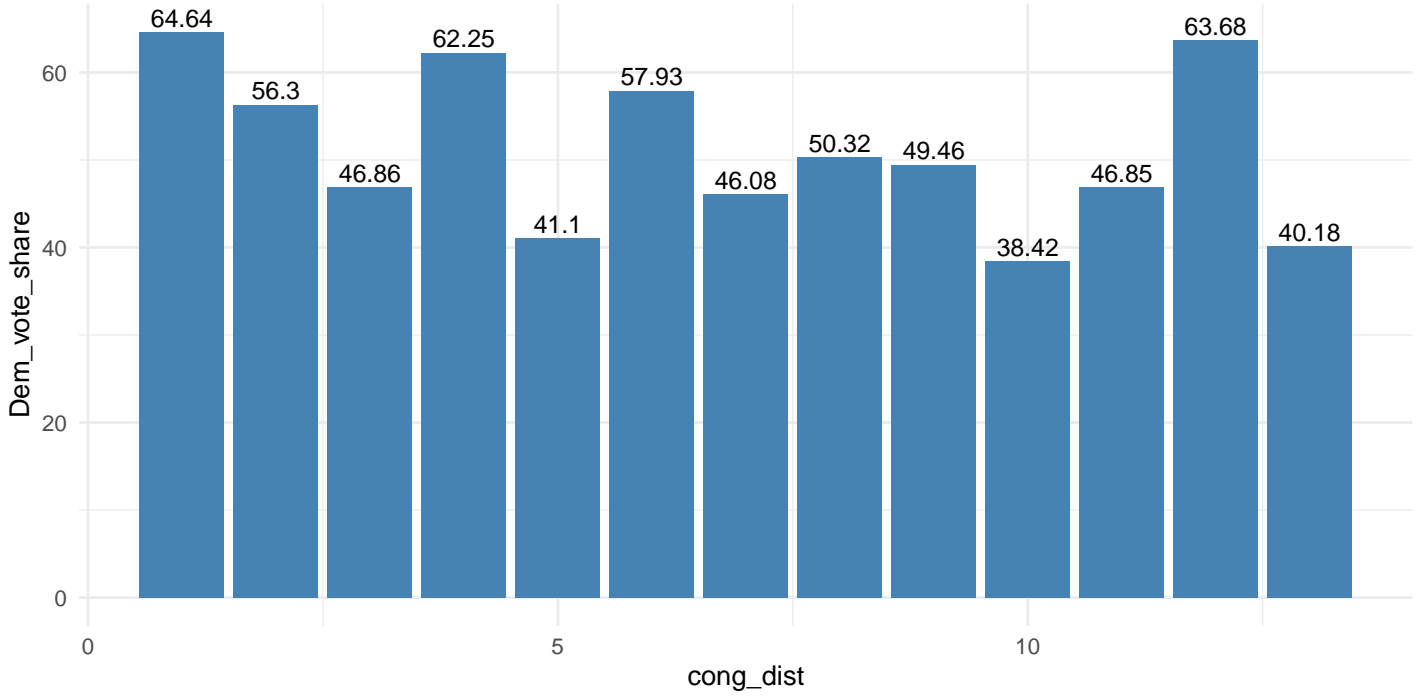


Figure 3: Democratic Vote Share Predicted By The Interim Report

R+4 margin of victory).

Presidential Election Model

For the presidential model, the states chosen are Arizona, Florida, Georgia, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, and Wisconsin. These thirteen states are rated as either “Toss Up”, “Lean Republican”, or “Lean Democrat” races (the three most competitive categories) by the Cook Political Report (as of October 28), an nonpartisan elections newsletter. These states are also projected to be competitive in the Presidential race by both the FiveThirtyEight and Economist models. We select poll responses from likely voters only, since it is known that using all responses may overestimate support for the Democratic party, according to FiveThirtyEight (Silver, 2014). The response we will use is Y_k , Biden’s share of the two-party vote using the following model where the poll k (of $K=3361$ [insert final number] polls) ended t days (of $T = 30$ days) before the election and was conducted on state j (of $J=13$ states of interest). We choose to use polling data from 30 days before the election (as opposed to over 30 days), as this aligns with Andrew Gelman’s observation that polling data becomes more predictive as the day of poll becomes closer to election day (The Economist).

$$Y_k \sim N(\theta_{jt}, \sigma_{yj}^2)$$

$$\theta_{...t} \sim MVN(\theta_{...t-1}, \Sigma)$$

$\Sigma \sim Wishart(S, J + 1)$ where S is the state covariance matrix (obtained from Andrew Gelman’s model) and J is the number of states in the model.

$$\text{Priors for } \sigma_{yj}^2: \sigma_{yj}^2 \sim InvGamma(\nu_u, \nu_y \tau_y)$$

$$\nu_y \sim Uniform(0, 100) \text{ and } \tau_y \sim Uniform(0, 100).$$

$$\text{Priors for } \theta_{j1}: \theta_{j1} \sim N(\mu_j, \sigma^2)$$

$$\sigma^2 \sim InvGamma(0.5, 0.5)$$

$$\mu_j \sim N(h_j, 7.5^2)$$

$$h_j = 0.1 * \text{Presidential Fundamentals}_{2020} + 0.9 * \text{Vote Share from Partisan Lean}_j$$

$$\text{Presidential Fundamentals}_i = \gamma_0 + \gamma_1 \text{June Approval Ratings for Incumbent Party}_i + \gamma_2 \text{Three Month Stock Growth}_i + \gamma_3 \text{2nd Quarter Real Income Growth}_i + \epsilon_i$$

$$\epsilon_i \sim N(0, \phi^2)$$

For our presidential model, we wanted to incorporate state-level correlations that differ between each pair of states. For example, Wisconsin is much more similar to its Great Lakes neighbor Michigan both geographically and demographically than it is to Arizona, which is in the Southwest and has a much larger Hispanic population. So, we would expect that Wisconsin and Michigan have a higher correlation in their election results than Wisconsin and Arizona. In order to incorporate this into our presidential model, we used a multivariate normal distribution to model two-party vote share in each state, which allowed us to specify a covariance matrix for the states. We used a covariance matrix which was downloaded from the Economist forecast model (Gelman, 2020). These similarities between states were calculated by comparing their demographic and political profiles, such as the state’s share of white voters and how urban/rural the state is.

where explain variables later the S&P stock growth three months prior to the election also explain i where i is each year in that Abramowitz data - kinda explained this in Appendix B.

In essence, the presidential model used fundamentals and vote share from partisan lean as a starting point for the expected vote share for the Democratic party, and update the priors with presidential election polling data. We predicted Presidential Fundamentals $_i$ from a linear regression model that used historical election year’s economic data and June approval rating of the incumbent party. When computing the prior, 10% weight is allocated to the fundamentals based on economic data and 90% weight to partisan lean because the effect of economic indicators has shrunk over the years as electorates became increasingly polarized (The Economist, 2020). In addition, economic indicators are particularly volatile in 2020 due to the COVID-19 pandemic. We chose not to do this for the U.S. House and Senate models because these races are generally a bit more localized (Gillespie et al. 2020).

one sentence on hyperparameters

Presidential elections in the United States are decided by the Electoral College, so estimating percentage support in each state alone does not tell us who wins the election. For each set of MCMC samples of two-party vote share by state, we subsequently use the predicted winner in each state to add up the electoral votes of each candidate. For states not in the model, their electoral votes are allocated assuming they vote the same way as in 2016. The probability of President Trump winning re-election is then the probability that he receives 270 or more electoral votes across simulations. Note that although Maine and Nebraska allocate some electoral votes by Congressional District, only two electoral votes are competitive due to this wrinkle, so for simplicity we simply allocate all electoral votes to the statewide winner of each respective state.

Senate Election Model

For the Senate model, the states chosen are Alaska, Arizona, Colorado, Georgia, Iowa, Kansas, Maine, Michigan, Montana, North Carolina, South Carolina, and Texas. There is also a special election for the Senate in Georgia in addition to the regularly scheduled election. These fourteen races are rated as either “Toss Up”, “Lean Republican”, or “Lean Democrat” (the three most competitive categories) by the Cook Political Report (as of October 29). These states are also projected to have competitive Senate races by both the FiveThirtyEight and Economist models. Note that the Georgia special election actually has multiple Republicans and Democrats running on the same ballot. If no candidate wins over 50% of the vote, which is considered likely, the top two finishers will advance to a one-on-one runoff election in January (Ballotpedia, 2020). For simplicity, our model simply sums up the support for Republican candidates and compares that to the sum of the support for Democratic candidates, then treats this election like the others. (This is not the ideal way to simulate this election, but the difficulty of predicting turnout and the political environment for the January runoff election made this decision seem like the best one to make.) Once again, we only included poll responses from likely voters that responded within 30 days of the election.

The response we will use is Y_k , the Democrat candidate’s share of the two-party vote using the following model where the poll k (of K =fill in polls) ended t days (of $T = 60$ days) before the election and was conducted on state j (of J =14 states of interest).

$Y_k \sim N(\beta_{jt}, \sigma_{yj}^2)$
 $\beta_{jt} \sim N(\beta_{jt-1}, \sigma_{\beta j}^2)$
Priors for σ_{yj}^2 : $\sigma_{yj}^2 \sim \text{InvGamma}(\nu_u, \nu_y \tau_y)$
 $\nu_y \sim \text{Uniform}(0, 100)$ and $\tau_y \sim \text{Uniform}(0, 100)$.
Priors for $\sigma_{\beta j}^2$: $\sigma_{\beta j}^2 \sim \text{InvGamma}(\nu_\beta, \nu_\beta \tau_\beta)$
 $\nu_\beta \sim \text{Uniform}(0, 100)$ and $\tau_\beta \sim \text{Uniform}(0, 100)$.
Priors for β_{j1} : $\beta_{j1} \sim N(\mu_j, \sigma^2)$
 $\sigma^2 \sim \text{InvGamma}(0.5, 0.5)$
 $\mu_j \sim N(h_j, 7.5^2)$
 $h_j = \text{Vote Share from Partisan Lean}_j + \text{Incumbency Advantage}_j$

For the U.S. Senate model we also take into account incumbency advantage. Analysis by FiveThirtyEight (Rakich, 2018) found that incumbent senators get a 2.6-point boost. For each incumbent was running a Senate race, depending on whether they were a Democrat or Republican, we either added or subtracted this incumbency adjustment with the prior mean computed from partisan lean.

For the NC-specific prior, we supplemented partisan lean and incumbency adjustment with voter turnout predicted by the model in our Interim Report. When computing the prior for NC only, 10% weight is allocated to predicted Democratic vote share from turnout and demographics, and 90% weight is allocated to partisan lean and incumbency adjustment. The turnout is weighted much less because there is considerable uncertainty surrounding turnout because of the COVID-19 pandemic. See Appendix A.2 for more specifics on how the voter turnout model was used to compute predicted Democratic vote share.

For predicting control of the U.S. Senate, things are complicated slightly by the fact that a 50/50 split of Senate seats is a possible outcome of the elections. In this case, the Vice President breaks the tie, so the party that wins the Presidential election will only need 50 Senate seats for a Senate majority, while the party that loses will need 51. Since our President model gives Democrat Joe Biden a very high chance of winning (discussed more below), we assume in the Senate model that Democrats will control the Senate in the case of a 50/50 split.

House Election Model

The response we will use is Y_k , the Democrat candidate's share of the two-party vote using the following model where the poll k (of $K = \text{fill in}$ polls) ended t days (of $T = 60$ days) before the election and was conducted on district j (where j is in districts 1, \dots , 11, 13 of North Carolina). For district 12, the vote share is coded as 100 because there is only one candidate, and she is a Democrat. Note that, due to the scarcity of poll responses for the NC House elections, we included all poll responses within 115 days of the election.

$Y_k \sim N(\beta_{jt}, \sigma_{yj}^2)$
 $\beta_{jt} \sim N(\beta_{jt-1}, \sigma_{\beta j}^2)$
 $\sigma_{yj}^2 \sim \text{InvGamma}(\nu_u, \nu_y \tau_y)$
 $\nu_y \sim \text{Uniform}(0, 100)$ and $\tau_y \sim \text{Uniform}(0, 100)$.
 $\sigma_{\beta j}^2 \sim \text{InvGamma}(\nu_\beta, \nu_\beta \tau_\beta)$
 $\nu_\beta \sim \text{Uniform}(0, 100)$ and $\tau_\beta \sim \text{Uniform}(0, 100)$.
 $\beta_{j1} \sim N(h_j, 7.5)$
 $h_j = 0.9 * \text{Vote Share from Partisan Lean}_j + 0.1 * \text{Expected Vote Share from Voter Turnout}_j$

According to FiveThirtyEight (Rakich, 2018), incumbent members of the House get a 2.7-point margin boost in their favor. For the U.S. House races, the incumbency adjustment was calculated by the same method as it was for the Senate races.

Expected Vote Share from Voter Turnout _{j} was calculated from the model in the Interim Report *insert citation whatever that ends up being and also say something like our method for splitting undecided voters is in appendix A.3*. Please see Appendix A.3 for more details.

Results

Presidential Election Model

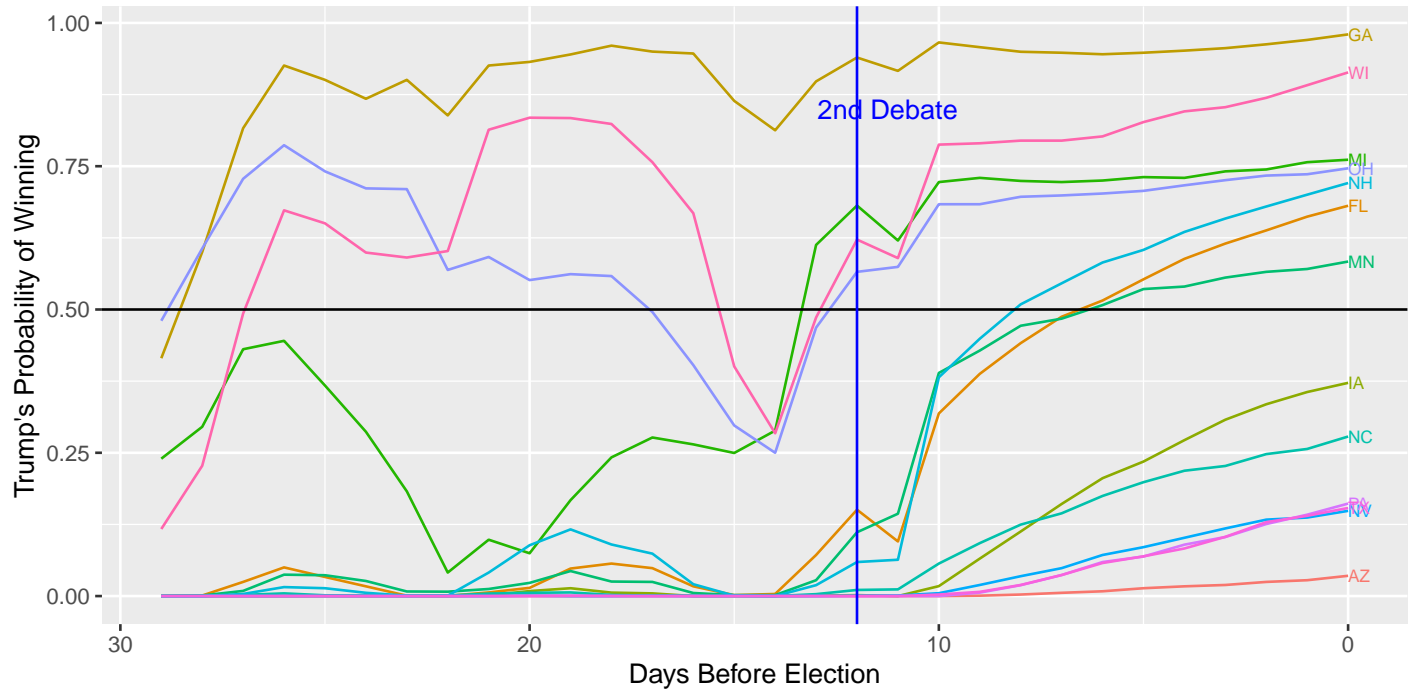


Figure 4: Probability of Re-election for President Trump over the 30 Days Before Election Day

4 shows how Trump’s probability of winning the election changes over the 60 days before the election in each state of interest. It appears that his probability of winning increased after the last presidential debate. *check for final results* From 5 we see that swing states such as Florida and Ohio are predicted to vote Trump while other swing states such as Pennsylvania and North Carolina are predicted to vote Biden. *check for final results*

Table 1: Predicted Probability of Winning and Electoral Vote Count

	Win Probability	Estimated EC Votes	2.5% Quantile EC Votes	97.5% Quantile EC Votes
Trump	0.13	222	152	288
Biden	0.87	316	250	386

The probability that Trump wins re-election is 0.13. The predicted electoral colleges votes for Trump is 222 with a 95% confidence interval of (152, 288).

Senate Election Model

From 6 we see that for the Senate race, states such as Texas are predicted to lean heavily towards the Republican candidate and Democrats have a great than 50% chance of winning states such as Arizona and Michigan. From the histogram in 7 we see the distribution of the predicted number of Democrat seats point to a Democrat majority in the Senate, which is in agreement with the low probability that Republicans retain control of the Senate (0.1).

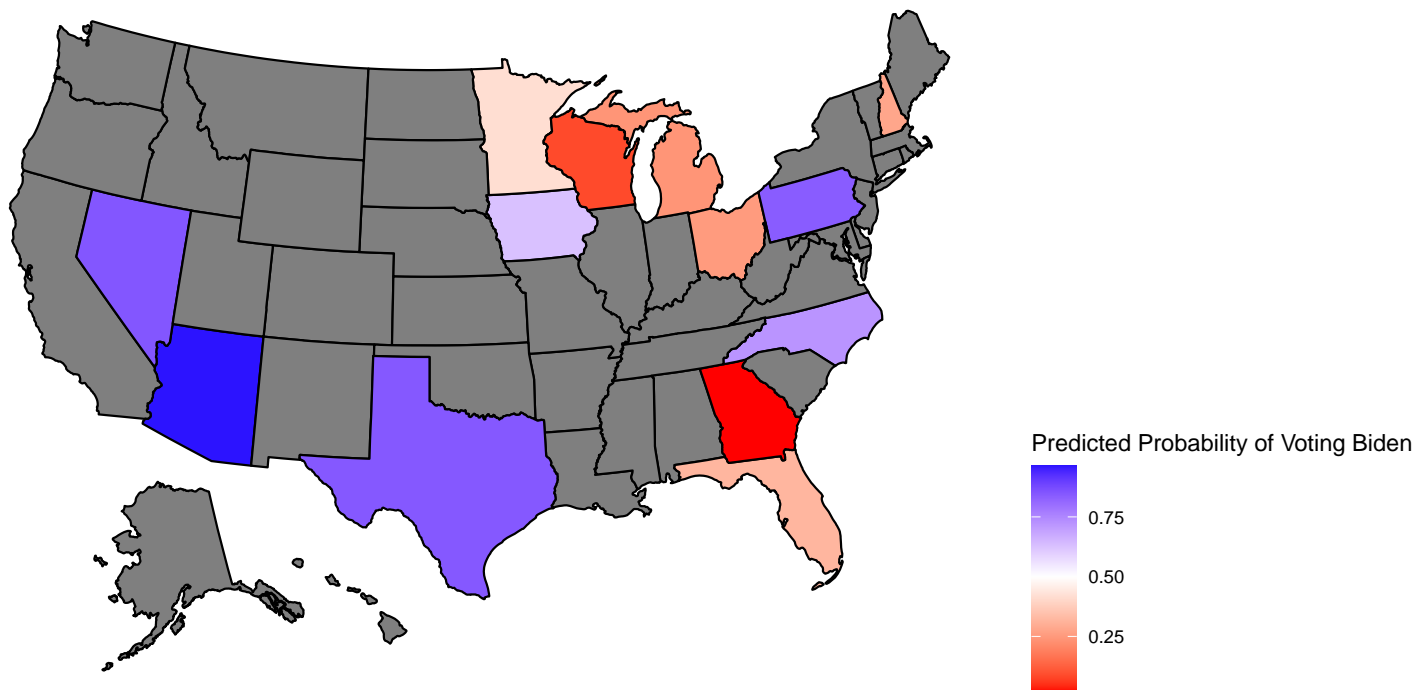


Figure 5: Plot of Predicted Presidential Result for States of Interest on Election Day

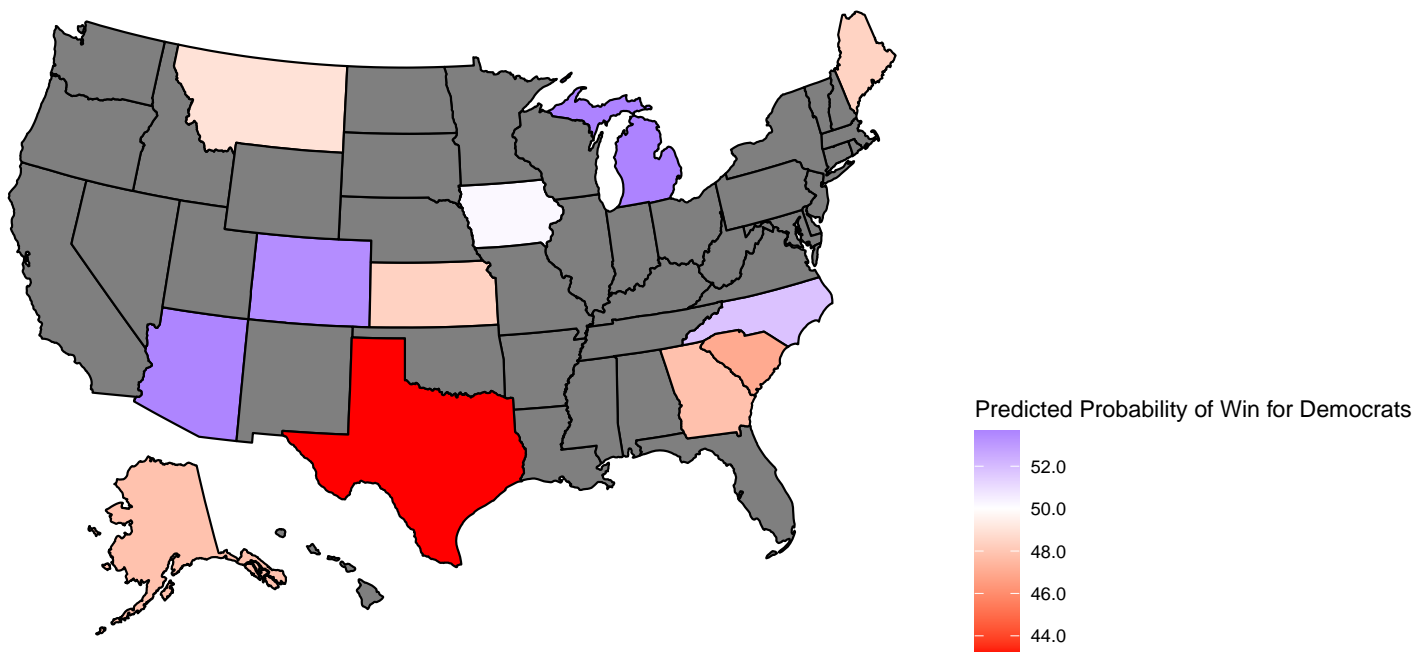


Figure 6: Plot of Predicted Senate Results for States of Interest on Election Day

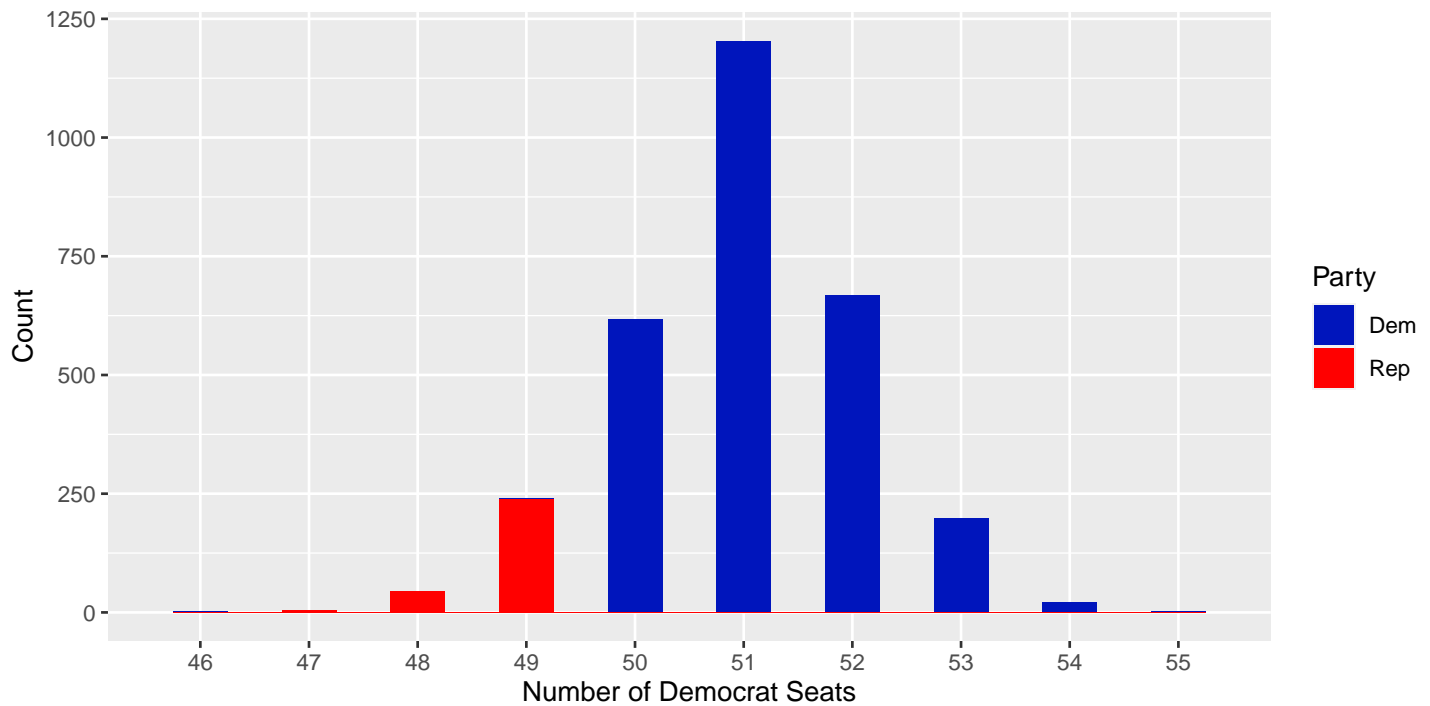


Figure 7: Plot of Predicted Senate Results for States of Interest on Election Day

Table 2: Predicted Vote Share for Tillis-Cunningham Race

	Estimate	2.5% Quantile	97.5% Quantile
Tillis	48.22	45.01	51.80
Cunningham	51.78	48.20	54.99

For the NC Senate race (Tillis vs. Cunningham), the predicted vote share for Cunningham is 51.78% with a 95% confidence interval of (48.2, 54.99).

House Election Model

Table 3: Predicted Republican Vote Share in NC's 13 Districts

	Estimate	2.5% Quantile	97.5% Quantile
District 1	41.69	36.32	47.02
District 2	40.75	35.29	46.17
District 3	63.61	58.32	68.56
District 4	33.29	27.84	38.48
District 5	69.54	64.20	74.93
District 6	41.19	35.87	46.64
District 7	61.81	56.40	67.43
District 8	55.74	50.67	61.01
District 9	56.79	51.40	61.90
District 10	70.68	65.16	75.96
District 11	56.12	51.86	60.59
District 12	0.00	0.00	0.00
District 13	69.60	64.12	75.01

OGR data source with driver: ESRI Shapefile

Source: "/Users/cathylee/Documents/Course 4010/Folders/Fall 2020/STA 440/Case 3/sta440casestudy3/C-Goo

with 12 features

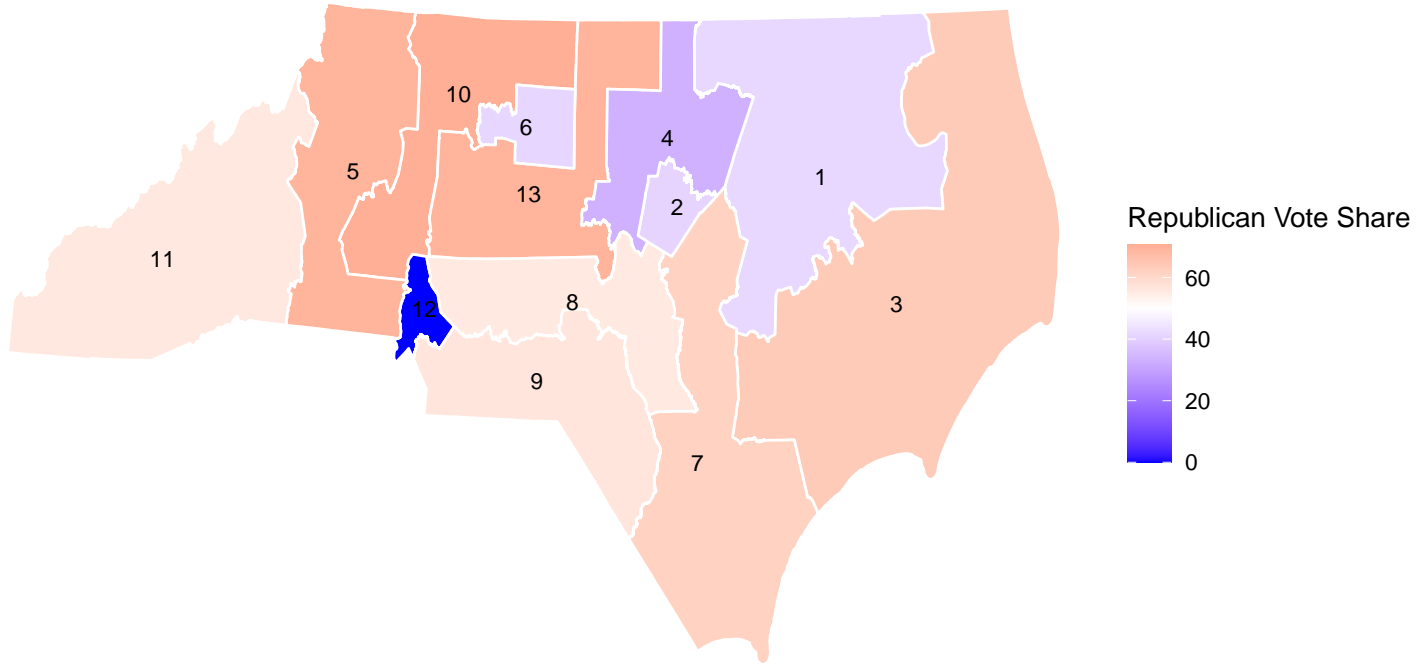


Figure 8: Plot of Predicted Senate Results for States of Interest on Election Day

50% partisan lean multiple times and the predictions for Trump’s re-election probability was generally higher (approximately 0.25) than that from our main model, which makes sense since economic fundamental model contains stock growth and is slightly favored toward Trump. However, the sensitivity analysis models still point to an unlikely re-election.

σ_{yj}

President model: $\text{unif}(0,1)$ – I think this makes it more uninformative for gamma (does this hold for inv gamma?)
 $\Pr(\text{Trump wins}) = 0.128, 0.1176667, 0.2456667$ TLDR: comparable? Senate model: $\mu_0 \sim N(\text{priors}, 1)$ (less variance)
 $\Pr(\text{Rep control}) = 0.1656667$ $\text{unif}(0,1)$ $\Pr(\text{Rep control}) = 0.1436667$ $\sigma^2_{0_inv} \sim \text{gamma}(3,0.5)$ (stronger prior?)
 $\Pr(\text{Rep control}) = 0.07466667$ TLDR: Tillis vote share + confidence intervals have been comparable, probability of republican control more sensitive

To account for shock events,

Diagnostics

Discussion and Limitations

As polls are an important part of our data sources, we acknowledge that with the COVID-19 pandemic, polling has moved from in-person interviews to phone calls, live surveys and internet questions, which could lead to unknown biases with its lower response rates (Gelman, 2020). A recent election prediction study shows that state polls have much higher margins of error than expected (Shirani-Mehr et al., 2018).

Appendix A

A.1 Presidential Election Model

Presidential Model Purpose and Structure

The purpose of this model is to predict the probability that president Trump will win the election, as well as to predict the electoral college vote. The states chosen are Arizona, Florida, Georgia, Iowa, Michigan, Minnesota,

Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, and Wisconsin. These thirteen states are deemed to be competitive in the 2020 presidential race by political analysts.

The response we will use is Y_k , Biden's share of the two-party vote using the following model where the poll k (of $K=3361$ [insert final number] polls) ended t days (of $T = 30$ days) before the election and was conducted on state j (of $J=13$ states of interest). We choose to use polling data from 30 days before the election (as opposed to over 30 days), as this aligns with Andrew Gelman's observation that polling data becomes more predictive as the day of poll becomes closer to election day (The Economist).

$$Y_k \sim N(\theta_{jt}, \sigma_{yj}^2)$$

$$\theta_{...t} \sim MVN(\theta_{...t-1}, \Sigma)$$

$\Sigma \sim Wishart(S, J + 1)$ where S is the state covariance matrix (obtained from Andrew Gelman's model) and J is the number of states in the model.

$$\text{Priors for } \sigma_{yj}^2: \sigma_{yj}^2 \sim InvGamma(\nu_u, \nu_y \tau_y)$$

$$\nu_y \sim Uniform(0, 100) \text{ and } \tau_y \sim Uniform(0, 100).$$

$$\text{Priors for } \theta_{j1}: \theta_{j1} \sim N(\mu_j, \sigma^2)$$

$$\sigma^2 \sim InvGamma(0.5, 0.5)$$

$$\mu_j \sim N(h_j, 7.5^2)$$

$$h_j = 0.1 * \text{Presidential Fundamentals}_{2020} + 0.9 * \text{Vote Share from Partisan Lean}_j$$

$$\text{Presidential Fundamentals}_i = \gamma_0 + \gamma_1 \text{June Approval Ratings for Incumbent Party}_i + \gamma_2 \text{Three Month Stock Growth}_i + \gamma_3 \text{2nd Quarter Real Income Growth}_i + \epsilon_i$$

$$\epsilon_i \sim N(0, \phi^2)$$

A hierarchical model was used, where Biden's share of the two-party vote for poll k on any given day t for state j is modelled as a normal distribution with mean θ_{jt} and variance σ_{yj}^2 . A random walk was used to calculate θ_{jt} for each day t before the election (up until 30 days before the election) from θ_{jt-1} and Σ , a $J \times J$ state covariance matrix. The mean h_j of the normal prior on μ_j (which is the mean of the normal prior on θ_{j1}) was calculated by multiplying the economic fundamentals $\text{Presidential Fundamentals}_i$ for $i = 2020$ by 0.1 and state partisan lean by 0.9, then summing those two values. We predicted $\text{Presidential Fundamentals}_i$ from a linear regression model that had historical election year's June approval rating of the incumbent party, three month stock growth, and 2nd quarter real income growth as predictors. keep?

Raw Model Output

##	mean	sd	2.5%	97.5%
## Sigma[1,1]	0.51987746	0.3933511	0.1156134431	1.6216700
## Sigma[2,1]	0.38953623	0.3309322	-0.0186746854	1.2504349
## Sigma[3,1]	0.23964838	0.3853611	-0.3089195468	1.2260179
## Sigma[4,1]	0.28981049	0.2979531	-0.1443628828	0.9936661
## Sigma[5,1]	0.28184391	0.2537920	-0.0627513603	0.9088115
## Sigma[6,1]	0.23817951	0.2626441	-0.1239967435	0.8596522
## Sigma[7,1]	0.24372195	0.2779328	-0.1173187872	0.9193907
## Sigma[8,1]	0.21989390	0.2805532	-0.2302451059	0.9301554
## Sigma[9,1]	0.41393687	0.3800912	0.0136601243	1.4437415
## Sigma[10,1]	0.27889410	0.2672737	-0.0648931146	0.9194782
## Sigma[11,1]	0.30180210	0.2729730	-0.0588820842	1.0292865
## Sigma[12,1]	0.34233314	0.3712682	-0.0943242228	1.3532501
## Sigma[13,1]	0.26267684	0.2737047	-0.0967705649	0.9595077
## Sigma[1,2]	0.38953623	0.3309322	-0.0186746854	1.2504349
## Sigma[2,2]	0.64618099	0.4754109	0.1411699782	1.9290753
## Sigma[3,2]	0.23999365	0.4266796	-0.4910890857	1.2688728
## Sigma[4,2]	0.41250993	0.3584455	0.0002081135	1.3127487
## Sigma[5,2]	0.42167587	0.3235304	0.0522840840	1.2678316

## Sigma[6,2]	0.29861881	0.2925953	-0.0825987097	1.0034704
## Sigma[7,2]	0.33032929	0.3081441	-0.0666103618	1.1306039
## Sigma[8,2]	0.33853921	0.3764351	-0.1594720933	1.2407230
## Sigma[9,2]	0.41009043	0.3916092	-0.0429140638	1.4282996
## Sigma[10,2]	0.36521129	0.3113452	-0.0029229837	1.1400261
## Sigma[11,2]	0.46144968	0.3514015	0.0524438800	1.3803640
## Sigma[12,2]	0.28440518	0.3602500	-0.1963868704	1.2320766
## Sigma[13,2]	0.34536737	0.2955454	-0.0278631548	1.1069803
## Sigma[1,3]	0.23964838	0.3853611	-0.3089195468	1.2260179
## Sigma[2,3]	0.23999365	0.4266796	-0.4910890857	1.2688728
## Sigma[3,3]	0.67502250	0.6672952	0.1266001482	2.4648287
## Sigma[4,3]	0.18150817	0.3865737	-0.5318948312	1.0561554
## Sigma[5,3]	0.21471471	0.3313413	-0.3213279821	1.0128667
## Sigma[6,3]	0.14763493	0.3043408	-0.3837452873	0.8771322
## Sigma[7,3]	0.37205590	0.3978839	-0.0632602594	1.4694670
## Sigma[8,3]	0.03796519	0.3562941	-0.7852613870	0.7362797
## Sigma[9,3]	0.23722854	0.4488951	-0.3677441661	1.4012362
## Sigma[10,3]	0.26568655	0.3543004	-0.1970817116	1.1333807
## Sigma[11,3]	0.18676033	0.3661480	-0.4756871406	1.0333468
## Sigma[12,3]	0.21189799	0.4640664	-0.4156919373	1.3180710
## Sigma[13,3]	0.16036588	0.3173309	-0.4389436259	0.9149381
## Sigma[1,4]	0.28981049	0.2979531	-0.1443628828	0.9936661
## Sigma[2,4]	0.41250993	0.3584455	0.0002081135	1.3127487
## Sigma[3,4]	0.18150817	0.3865737	-0.5318948312	1.0561554
## Sigma[4,4]	0.58660228	0.4694489	0.1259421829	1.8823080
## Sigma[5,4]	0.44115720	0.3477625	0.0723721784	1.3070032
## Sigma[6,4]	0.40129852	0.3735988	0.0335252124	1.4038863
## Sigma[7,4]	0.33511708	0.3026762	-0.0513876603	1.1305403
## Sigma[8,4]	0.39284650	0.3770341	-0.0359495635	1.4192533
## Sigma[9,4]	0.26187267	0.3262248	-0.1750608468	1.0648012
## Sigma[10,4]	0.39313282	0.3349352	0.0347340134	1.2479423
## Sigma[11,4]	0.45136403	0.3545973	0.0589079187	1.4053985
## Sigma[12,4]	0.20738486	0.3473195	-0.3377567252	0.9937596
## Sigma[13,4]	0.38825834	0.2955196	0.0550926718	1.1547793
## Sigma[1,5]	0.28184391	0.2537920	-0.0627513603	0.9088115
## Sigma[2,5]	0.42167587	0.3235304	0.0522840840	1.2678316
## Sigma[3,5]	0.21471471	0.3313413	-0.3213279821	1.0128667
## Sigma[4,5]	0.44115720	0.3477625	0.0723721784	1.3070032
## Sigma[5,5]	0.47322856	0.3347465	0.1177317504	1.3020556
## Sigma[6,5]	0.35919289	0.3020024	0.0380745492	1.0779491
## Sigma[7,5]	0.32653964	0.2780086	-0.0034189330	1.0491111
## Sigma[8,5]	0.35362894	0.3155137	-0.0217061869	1.1564248
## Sigma[9,5]	0.27207970	0.2982570	-0.1302090554	1.0149302
## Sigma[10,5]	0.37970564	0.2754389	0.0634392771	1.0705666
## Sigma[11,5]	0.43275744	0.3078648	0.0802847343	1.2371083
## Sigma[12,5]	0.20615160	0.2849943	-0.2112426633	0.9390071
## Sigma[13,5]	0.36961542	0.2603924	0.0646016515	1.0158881
## Sigma[1,6]	0.23817951	0.2626441	-0.1239967435	0.8596522
## Sigma[2,6]	0.29861881	0.2925953	-0.0825987097	1.0034704
## Sigma[3,6]	0.14763493	0.3043408	-0.3837452873	0.8771322
## Sigma[4,6]	0.40129852	0.3735988	0.0335252124	1.4038863
## Sigma[5,6]	0.35919289	0.3020024	0.0380745492	1.0779491

## Sigma[6,6]	0.44591736	0.3799608	0.1074059802	1.4573110
## Sigma[7,6]	0.26238398	0.2733913	-0.0886962478	0.9964831
## Sigma[8,6]	0.35125180	0.3115048	0.0139833724	1.1372684
## Sigma[9,6]	0.17159501	0.2702688	-0.2967945064	0.7913024
## Sigma[10,6]	0.33985087	0.2839697	0.0280268989	1.0865598
## Sigma[11,6]	0.37123661	0.3047187	0.0292723536	1.1890576
## Sigma[12,6]	0.14894560	0.2833509	-0.3105226458	0.8523354
## Sigma[13,6]	0.33791768	0.2646669	0.0393755807	1.0221773
## Sigma[1,7]	0.24372195	0.2779328	-0.1173187872	0.9193907
## Sigma[2,7]	0.33032929	0.3081441	-0.0666103618	1.1306039
## Sigma[3,7]	0.37205590	0.3978839	-0.0632602594	1.4694670
## Sigma[4,7]	0.33511708	0.3026762	-0.0513876603	1.1305403
## Sigma[5,7]	0.32653964	0.2780086	-0.0034189330	1.0491111
## Sigma[6,7]	0.26238398	0.2733913	-0.0886962478	0.9964831
## Sigma[7,7]	0.45912539	0.3662298	0.1030619683	1.4917692
## Sigma[8,7]	0.19557188	0.2801388	-0.3211453829	0.8224671
## Sigma[9,7]	0.23477781	0.3005152	-0.1450942728	0.9753830
## Sigma[10,7]	0.32504221	0.2663717	0.0135931916	1.0276670
## Sigma[11,7]	0.30984287	0.2807752	-0.0690891957	1.0199939
## Sigma[12,7]	0.21622495	0.3215119	-0.1910119731	1.0310385
## Sigma[13,7]	0.27118140	0.2427112	-0.0778344914	0.8915151
## Sigma[1,8]	0.21989390	0.2805532	-0.2302451059	0.9301554
## Sigma[2,8]	0.33853921	0.3764351	-0.1594720933	1.2407230
## Sigma[3,8]	0.03796519	0.3562941	-0.7852613870	0.7362797
## Sigma[4,8]	0.39284650	0.3770341	-0.0359495635	1.4192533
## Sigma[5,8]	0.35362894	0.3155137	-0.0217061869	1.1564248
## Sigma[6,8]	0.35125180	0.3115048	0.0139833724	1.1372684
## Sigma[7,8]	0.19557188	0.2801388	-0.3211453829	0.8224671
## Sigma[8,8]	0.52552326	0.4217314	0.1210524603	1.6986723
## Sigma[9,8]	0.18243116	0.3018213	-0.3420622079	0.8816777
## Sigma[10,8]	0.29793801	0.2899143	-0.0863727505	0.9770471
## Sigma[11,8]	0.38786892	0.3449333	-0.0215548102	1.3385041
## Sigma[12,8]	0.16145021	0.3242072	-0.4022376082	0.9515610
## Sigma[13,8]	0.34956655	0.2860953	0.0265283530	1.0861515
## Sigma[1,9]	0.41393687	0.3800912	0.0136601243	1.4437415
## Sigma[2,9]	0.41009043	0.3916092	-0.0429140638	1.4282996
## Sigma[3,9]	0.23722854	0.4488951	-0.3677441661	1.4012362
## Sigma[4,9]	0.26187267	0.3262248	-0.1750608468	1.0648012
## Sigma[5,9]	0.27207970	0.2982570	-0.1302090554	1.0149302
## Sigma[6,9]	0.17159501	0.2702688	-0.2967945064	0.7913024
## Sigma[7,9]	0.23477781	0.3005152	-0.1450942728	0.9753830
## Sigma[8,9]	0.18243116	0.3018213	-0.3420622079	0.8816777
## Sigma[9,9]	0.56704323	0.4842785	0.1169266513	1.8251032
## Sigma[10,9]	0.26023221	0.3013516	-0.0926191781	1.0180868
## Sigma[11,9]	0.29080857	0.3087608	-0.1202711757	1.0517140
## Sigma[12,9]	0.37412652	0.4132303	-0.0673067448	1.5320960
## Sigma[13,9]	0.23512977	0.2904571	-0.1573066264	0.9748217
## Sigma[1,10]	0.27889410	0.2672737	-0.0648931146	0.9194782
## Sigma[2,10]	0.36521129	0.3113452	-0.0029229837	1.1400261
## Sigma[3,10]	0.26568655	0.3543004	-0.1970817116	1.1333807
## Sigma[4,10]	0.39313282	0.3349352	0.0347340134	1.2479423
## Sigma[5,10]	0.37970564	0.2754389	0.0634392771	1.0705666

## Sigma[6,10]	0.33985087	0.2839697	0.0280268989	1.0865598
## Sigma[7,10]	0.32504221	0.2663717	0.0135931916	1.0276670
## Sigma[8,10]	0.29793801	0.2899143	-0.0863727505	0.9770471
## Sigma[9,10]	0.26023221	0.3013516	-0.0926191781	1.0180868
## Sigma[10,10]	0.44565726	0.3296097	0.1076347819	1.3255160
## Sigma[11,10]	0.38489672	0.2934968	0.0471639656	1.1145557
## Sigma[12,10]	0.18937217	0.3196248	-0.2475441006	0.9257476
## Sigma[13,10]	0.33731691	0.2578744	0.0272233074	0.9776188
## Sigma[1,11]	0.30180210	0.2729730	-0.0588820842	1.0292865
## Sigma[2,11]	0.46144968	0.3514015	0.0524438800	1.3803640
## Sigma[3,11]	0.18676033	0.3661480	-0.4756871406	1.0333468
## Sigma[4,11]	0.45136403	0.3545973	0.0589079187	1.4053985
## Sigma[5,11]	0.43275744	0.3078648	0.0802847343	1.2371083
## Sigma[6,11]	0.37123661	0.3047187	0.0292723536	1.1890576
## Sigma[7,11]	0.30984287	0.2807752	-0.0690891957	1.0199939
## Sigma[8,11]	0.38786892	0.3449333	-0.0215548102	1.3385041
## Sigma[9,11]	0.29080857	0.3087608	-0.1202711757	1.0517140
## Sigma[10,11]	0.38489672	0.2934968	0.0471639656	1.1145557
## Sigma[11,11]	0.52624693	0.3669703	0.1267512861	1.5458765
## Sigma[12,11]	0.20525923	0.3080462	-0.2616867650	0.9769332
## Sigma[13,11]	0.38716482	0.2806620	0.0715782952	1.0931656
## Sigma[1,12]	0.34233314	0.3712682	-0.0943242228	1.3532501
## Sigma[2,12]	0.28440518	0.3602500	-0.1963868704	1.2320766
## Sigma[3,12]	0.21189799	0.4640664	-0.4156919373	1.3180710
## Sigma[4,12]	0.20738486	0.3473195	-0.3377567252	0.9937596
## Sigma[5,12]	0.20615160	0.2849943	-0.2112426633	0.9390071
## Sigma[6,12]	0.14894560	0.2833509	-0.3105226458	0.8523354
## Sigma[7,12]	0.21622495	0.3215119	-0.1910119731	1.0310385
## Sigma[8,12]	0.16145021	0.3242072	-0.4022376082	0.9515610
## Sigma[9,12]	0.37412652	0.4132303	-0.0673067448	1.5320960
## Sigma[10,12]	0.18937217	0.3196248	-0.2475441006	0.9257476
## Sigma[11,12]	0.20525923	0.3080462	-0.2616867650	0.9769332
## Sigma[12,12]	0.57262219	0.5471140	0.1128074434	1.9874786
## Sigma[13,12]	0.20078949	0.2875892	-0.1974098981	0.9805224
## Sigma[1,13]	0.26267684	0.2737047	-0.0967705649	0.9595077
## Sigma[2,13]	0.34536737	0.2955454	-0.0278631548	1.1069803
## Sigma[3,13]	0.16036588	0.3173309	-0.4389436259	0.9149381
## Sigma[4,13]	0.38825834	0.2955196	0.0550926718	1.1547793
## Sigma[5,13]	0.36961542	0.2603924	0.0646016515	1.0158881
## Sigma[6,13]	0.33791768	0.2646669	0.0393755807	1.0221773
## Sigma[7,13]	0.27118140	0.2427112	-0.0778344914	0.8915151
## Sigma[8,13]	0.34956655	0.2860953	0.0265283530	1.0861515
## Sigma[9,13]	0.23512977	0.2904571	-0.1573066264	0.9748217
## Sigma[10,13]	0.33731691	0.2578744	0.0272233074	0.9776188
## Sigma[11,13]	0.38716482	0.2806620	0.0715782952	1.0931656
## Sigma[12,13]	0.20078949	0.2875892	-0.1974098981	0.9805224
## Sigma[13,13]	0.42486456	0.2911567	0.1114859171	1.1743461
## sigma2_y[1]	2.12598895	0.4348540	1.4396826156	3.1353584
## sigma2_y[2]	1.99508481	0.3790867	1.3366241902	2.8508695
## sigma2_y[3]	2.14954683	0.4836874	1.4066718591	3.2639373
## sigma2_y[4]	2.12291338	0.4791012	1.4094908691	3.2382886
## sigma2_y[5]	2.27318361	0.4485815	1.5916009707	3.3223183

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## sigma2_y[6]    2.02767123 0.4474855  1.3288265258  2.9592578
## sigma2_y[7]    1.90846193 0.3705648  1.2462995842  2.6894502
## sigma2_y[8]    2.02212075 0.4500000  1.2870426705  3.0265158
## sigma2_y[9]    2.04028775 0.4358620  1.3206861443  3.0054007
## sigma2_y[10]   1.97388568 0.4147912  1.2423428817  2.8739492
## sigma2_y[11]   2.02192418 0.3734588  1.4178493002  2.8695117
## sigma2_y[12]   2.02197890 0.4431448  1.2818505761  2.9924149
## sigma2_y[13]   2.01081577 0.3880073  1.3630827393  2.8686848
## theta[1,1]     49.31630184 2.3278948 43.7485351871 53.0360420
## theta[2,1]     48.61514971 2.1032226 44.5560162536 52.5532443
## theta[3,1]     48.22994230 2.5928733 42.7213744559 53.3772480
## theta[4,1]     47.40204242 2.2122616 42.3460194482 50.7763148
## theta[5,1]     51.56693090 1.5850242 48.5201432099 54.7552930
## theta[6,1]     52.20270823 2.2899325 46.6584622379 55.8549684
## theta[7,1]     49.01046533 2.0525306 43.8636009807 52.6149248
## theta[8,1]     54.18727679 2.4043001 49.8712951848 58.6093133
## theta[9,1]     51.02586199 2.2729597 45.7964128118 55.1415460
## theta[10,1]    46.95904934 1.7401335 43.2990370384 49.8995618
## theta[11,1]    50.48179504 1.9446678 46.7455874785 53.7740001
## theta[12,1]    48.39323646 2.4472631 42.8840092090 53.0718523
## theta[13,1]    51.64710421 1.6335555 48.0695129604 54.5905309
## theta[1,2]     49.40654315 2.2461800 43.8165064922 53.0773708
## theta[2,2]     48.77089537 2.0774177 44.6482719083 52.3898351
## theta[3,2]     48.32150875 2.5438073 43.0520486966 53.3209266
## theta[4,2]     47.55750202 2.2033818 42.5612485241 51.0798024
## theta[5,2]     51.71619627 1.6124299 48.4881634948 54.8828893
## theta[6,2]     52.32501430 2.2662257 46.9426999485 55.9879705
## theta[7,2]     49.12167997 2.0205082 44.3548665501 52.6845680
## theta[8,2]     54.32869808 2.4123592 49.9225036169 58.7167831
## theta[9,2]     51.12525242 2.2355240 45.8019634784 55.1462320
## theta[10,2]    47.08851498 1.7599063 43.3169450248 50.0936811
## theta[11,2]    50.63309745 1.9599056 46.7027216359 53.9472034
## theta[12,2]    48.47123122 2.3939830 43.1837855457 53.0166569
## theta[13,2]    51.80026465 1.6589098 48.1637810354 54.7240439
## theta[1,3]     49.47809777 2.1546212 44.4244546392 53.0691824
## theta[2,3]     48.90385185 2.0256577 44.7744003822 52.5088040
## theta[3,3]     48.41045410 2.4610314 43.1894811637 53.2025771
## theta[4,3]     47.71245928 2.1802426 42.7706872327 51.2417200
## theta[5,3]     51.83751186 1.6208987 48.4986658414 54.9617215
## theta[6,3]     52.44020288 2.2268614 47.3105094252 56.0335989
## theta[7,3]     49.23304307 1.9858403 44.6950489803 52.6361246
## theta[8,3]     54.44532305 2.4056147 50.0062955404 58.9238368
## theta[9,3]     51.19535732 2.1631507 46.0259001460 55.1531345
## theta[10,3]    47.19498927 1.7586411 43.4688361230 50.1763960
## theta[11,3]    50.77977458 1.9361975 46.8476534999 54.1012694
## theta[12,3]    48.52945687 2.3217030 43.3932350723 52.8065348
## theta[13,3]    51.93119018 1.6512899 48.4347553893 54.8014674
## theta[1,4]     49.57329897 2.0566433 44.7975828818 53.1264144
## theta[2,4]     49.04586505 1.9600079 45.0739242397 52.6129464
## theta[3,4]     48.51961208 2.3420746 43.6176639307 53.0375040
## theta[4,4]     47.85472159 2.1243882 43.0534160878 51.2659266
## theta[5,4]     51.97111526 1.5909850 48.6242085835 54.9459959

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## theta[6,4]	52.57216197	2.1749912	47.6747319954	56.0677503
## theta[7,4]	49.36931236	1.9130571	44.8920027713	52.6317569
## theta[8,4]	54.58371708	2.3471309	50.2554632683	58.9677197
## theta[9,4]	51.28041247	2.0958802	46.5139157730	55.1172501
## theta[10,4]	47.32199879	1.7047193	43.6556567758	50.3020444
## theta[11,4]	50.94034591	1.8764190	47.0901027927	54.1743740
## theta[12,4]	48.60472086	2.2427979	43.5270577975	52.6788447
## theta[13,4]	52.08109825	1.6277137	48.6495626388	55.0306631
## theta[1,5]	49.67638221	1.9664470	45.1390796163	53.0781358
## theta[2,5]	49.20252591	1.8792251	45.3187965804	52.5216288
## theta[3,5]	48.63374302	2.2478901	43.8932559813	52.8531787
## theta[4,5]	48.01016872	2.0756996	43.3409915920	51.3217393
## theta[5,5]	52.12176088	1.5617742	48.7763050693	54.9815925
## theta[6,5]	52.70022730	2.1035339	47.8500158719	56.1217543
## theta[7,5]	49.50106527	1.8710757	45.1941122398	52.6939441
## theta[8,5]	54.71708191	2.2687625	50.3800819211	58.9196538
## theta[9,5]	51.38419913	2.0178197	46.6409810697	55.0722316
## theta[10,5]	47.45246423	1.6817376	43.8858441273	50.4094895
## theta[11,5]	51.09467185	1.8252462	47.2943114233	54.2169951
## theta[12,5]	48.73228573	2.1573074	43.8798931370	52.6198554
## theta[13,5]	52.23325927	1.5955226	48.8038702494	55.0443700
## theta[1,6]	49.77002629	1.8794267	45.4294391571	53.1303173
## theta[2,6]	49.35434206	1.8164284	45.4929900651	52.5774962
## theta[3,6]	48.72661849	2.1345666	44.2951303831	52.6567835
## theta[4,6]	48.16553604	2.0276310	43.4900348357	51.4579014
## theta[5,6]	52.26091900	1.5224872	48.9646384227	54.9952045
## theta[6,6]	52.81727554	2.0359542	48.2073089667	56.1598019
## theta[7,6]	49.63225943	1.7973151	45.3391699543	52.6071114
## theta[8,6]	54.85126086	2.2273333	50.5745711589	59.0657551
## theta[9,6]	51.48527411	1.9513136	46.8294522490	55.0097877
## theta[10,6]	47.56518035	1.6370208	44.0770104853	50.4026901
## theta[11,6]	51.25010163	1.7882089	47.4670980697	54.3749701
## theta[12,6]	48.81159893	2.1248769	44.2119385344	52.5702758
## theta[13,6]	52.37740076	1.5702154	48.8645424464	55.1944711
## theta[1,7]	49.85462645	1.7481656	45.7708585163	53.0665105
## theta[2,7]	49.51828831	1.7125596	45.8265143645	52.4591500
## theta[3,7]	48.82968806	2.0279954	44.5783088403	52.6141216
## theta[4,7]	48.33565864	1.9499763	43.8207854269	51.5191232
## theta[5,7]	52.40420252	1.4548051	49.2133357583	55.0421795
## theta[6,7]	52.95785647	1.9504757	48.7493575374	56.2364969
## theta[7,7]	49.79003342	1.6821506	45.9153569334	52.5603374
## theta[8,7]	54.98873658	2.1369539	50.7891840174	58.9646815
## theta[9,7]	51.58001556	1.8582935	47.2845976848	54.8910709
## theta[10,7]	47.69802373	1.5701294	44.3302792129	50.4923750
## theta[11,7]	51.40810558	1.7077977	47.7211081976	54.4027319
## theta[12,7]	48.88571826	2.0063617	44.5545888969	52.4249930
## theta[13,7]	52.53163477	1.5045950	49.2745887819	55.2102001
## theta[1,8]	49.95818910	1.5999568	46.3415892047	52.9377807
## theta[2,8]	49.70490331	1.5635353	46.3446474818	52.4319258
## theta[3,8]	48.92543855	1.8728864	44.8511537821	52.3958939
## theta[4,8]	48.51420447	1.8330275	44.3811309872	51.5288065
## theta[5,8]	52.56592890	1.3471811	49.6867655057	55.1016104

## theta[6,8]	53.09311261	1.8226716	48.9203972282	56.1868169
## theta[7,8]	49.92242663	1.5810810	46.3387547871	52.5635886
## theta[8,8]	55.13870756	2.0137528	51.1458241333	58.8051164
## theta[9,8]	51.70452341	1.7424956	47.7396609791	54.8221902
## theta[10,8]	47.82862822	1.4684973	44.6596473298	50.4483837
## theta[11,8]	51.58945931	1.5745259	48.3116539334	54.3551117
## theta[12,8]	48.98742829	1.8871502	44.8618201908	52.3576303
## theta[13,8]	52.69941359	1.4006820	49.6302320048	55.2471289
## theta[1,9]	50.04289602	1.4450450	46.7666742195	52.7086095
## theta[2,9]	49.86033313	1.3868604	46.8599156194	52.2384131
## theta[3,9]	49.02585352	1.7321859	45.2128049305	52.1740275
## theta[4,9]	48.68157198	1.6526901	45.1458705078	51.5063192
## theta[5,9]	52.71368813	1.1992741	50.1891943601	54.9293748
## theta[6,9]	53.23298665	1.6786064	49.7225285130	56.0995223
## theta[7,9]	50.07284143	1.4325809	46.5677488902	52.5178528
## theta[8,9]	55.26180833	1.9104930	51.5284068922	58.8447094
## theta[9,9]	51.78912880	1.6331432	48.1061152631	54.6776065
## theta[10,9]	47.95086401	1.3488982	45.0632897476	50.4043975
## theta[11,9]	51.75385988	1.4163530	48.8168588015	54.2427752
## theta[12,9]	49.06091896	1.7581566	45.1539708514	52.3045262
## theta[13,9]	52.84911617	1.2873991	50.1761145086	55.2728646
## theta[1,10]	50.13808479	1.2569822	47.2964874595	52.4299996
## theta[2,10]	50.04697529	1.1696917	47.5177743882	52.1211956
## theta[3,10]	49.12292634	1.5406974	45.8085341077	51.9393977
## theta[4,10]	48.85899428	1.4401531	45.7454346437	51.3556660
## theta[5,10]	52.88176978	1.0055696	50.7375468872	54.7805187
## theta[6,10]	53.36833321	1.4917645	50.1953382933	56.0600203
## theta[7,10]	50.21373403	1.2352348	47.3523022995	52.3872406
## theta[8,10]	55.41172488	1.7540172	51.9432105011	58.8360809
## theta[9,10]	51.89965589	1.4699252	48.6596959998	54.5839551
## theta[10,10]	48.08534823	1.1966506	45.6081087519	50.3808419
## theta[11,10]	51.93305587	1.2295147	49.3713103422	54.1492823
## theta[12,10]	49.16366502	1.6127272	45.7137366510	52.1084218
## theta[13,10]	53.00574664	1.1386994	50.6111573843	55.1385497
## theta[1,11]	50.24795297	1.0117758	48.0936512695	52.1418131
## theta[2,11]	50.22247240	0.8599420	48.4483582925	51.7607621
## theta[3,11]	49.26548903	1.2692511	46.6414056391	51.7012032
## theta[4,11]	49.04599635	1.2084090	46.6282561744	51.2006766
## theta[5,11]	53.04597647	0.7358915	51.5940392241	54.5027303
## theta[6,11]	53.50110983	1.2779429	51.0399156964	55.8479870
## theta[7,11]	50.36939780	0.9811612	48.1716593835	52.2038229
## theta[8,11]	55.52657727	1.5735285	52.6077371607	58.7019962
## theta[9,11]	52.01945190	1.2682572	49.4218866245	54.4543557
## theta[10,11]	48.22600301	0.9903190	46.2202626613	50.1291839
## theta[11,11]	52.10937004	0.9507712	50.1620057896	53.9275372
## theta[12,11]	49.26823116	1.4149875	46.3959674015	51.8428757
## theta[13,11]	53.17127919	0.9121631	51.3062041107	54.9424449
## theta[1,12]	50.95775496	0.9257952	49.1182546508	52.7205315
## theta[2,12]	51.08505204	0.7085639	49.6246518126	52.4598532
## theta[3,12]	49.70538724	0.9116882	47.8602231829	51.4788625
## theta[4,12]	49.74888180	1.0641151	47.5610617681	51.6995861
## theta[5,12]	53.71255163	0.7254559	52.2772240933	55.1692649

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## theta[6,12] 54.04730902 1.0766255 51.9262451688 56.1147855
## theta[7,12] 50.93021373 0.7272399 49.4519226683 52.3173074
## theta[8,12] 56.06166384 1.3914286 53.3734039338 58.7336110
## theta[9,12] 52.68050449 1.1086347 50.5106732183 54.7841622
## theta[10,12] 48.83608444 0.8733491 47.1506388039 50.5385284
## theta[11,12] 52.84373199 0.7430620 51.3522490123 54.3088018
## theta[12,12] 49.77163698 1.1697673 47.4574837498 51.9765688
## theta[13,12] 53.76516269 0.7413942 52.3644320391 55.2504908
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## theta[2,13] 51.14318201 0.7361783 49.6430072518 52.5283591
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## theta[6,13] 53.94110097 1.0744827 51.8788301017 56.0131097
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## theta[9,13] 52.75885253 1.1164312 50.5058962669 54.9049904
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## theta[5,14] 53.93666999 0.7212340 52.4729805382 55.3591441
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## theta[2,16] 52.03067167 0.5782913 50.8760642131 53.1837072
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## theta[4,16] 50.18886464 0.7618236 48.6623583308 51.6492456
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## theta[13,16] 53.87279774 0.4837610 52.9110693988 54.8463971
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## theta[2,17] 51.54137554 0.7251419 50.0537934169 52.9841221
## theta[3,17] 50.53185237 0.9060230 48.7541579741 52.3215237
## theta[4,17] 49.60956438 0.8648520 47.9224171405 51.2330986
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## theta[9,17] 53.26009429 0.9462357 51.4957832707 55.1887004
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## theta[3,20] 50.97147296 1.0551437 49.0624115468 53.2189017
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## theta[5,20] 53.14526199 0.7603788 51.4963078952 54.5640135

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## theta[13,20] 53.11262879 0.7804801 51.4416516175 54.5701670
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## theta[2,21] 51.10189857 0.7768818 49.3754597334 52.5318371
## theta[3,21] 51.27333666 0.9512450 49.6202235927 53.3953562
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## theta[2,24] 51.85258026 0.5482085 50.7853917014 52.9063464
## theta[3,24] 50.58089248 0.6531785 49.2952623179 51.8337899
## theta[4,24] 49.85658128 0.7092564 48.4612727084 51.2591000
## theta[5,24] 53.98229983 0.4741996 53.0541577625 54.9376769

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## theta[6,24] 54.20123411 0.7324045 52.8187921935 55.6763131
## theta[7,24] 51.62929198 0.4722989 50.7093913114 52.5659113
## theta[8,24] 56.06202130 0.8472423 54.3473666156 57.6893726
## theta[9,24] 52.79027169 0.7659395 51.3618073409 54.3852592
## theta[10,24] 49.25342199 0.5898458 48.0738798250 50.3995249
## theta[11,24] 53.15232915 0.4638716 52.2420315651 54.0644509
## theta[12,24] 49.56119810 0.7971595 47.9311429294 51.0739261
## theta[13,24] 53.89179815 0.4766578 52.9579819388 54.8412670
## theta[1,25] 51.41330533 0.7577639 49.9789364622 52.8936963
## theta[2,25] 51.80964898 0.6891976 50.4162244797 53.1934351
## theta[3,25] 50.46056384 0.9116127 48.5840273151 52.2105464
## theta[4,25] 49.79668635 0.8384430 48.1416267650 51.4041106
## theta[5,25] 53.89243022 0.6463582 52.6234393778 55.1932915
## theta[6,25] 54.11991391 0.8857963 52.3253253006 55.8304374
## theta[7,25] 51.53687817 0.6806740 50.1926376904 52.7937137
## theta[8,25] 55.99892558 0.9911312 53.9709555967 57.9101619
## theta[9,25] 52.77356430 0.8534005 51.0787789704 54.4727880
## theta[10,25] 49.19080183 0.7450432 47.7499803414 50.6893508
## theta[11,25] 53.08039999 0.6442016 51.7826699120 54.3478200
## theta[12,25] 49.48788336 0.9379009 47.5515458833 51.2521099
## theta[13,25] 53.79430884 0.6627807 52.4886981601 55.1226913
## theta[1,26] 51.28354282 0.7071452 49.8931825432 52.6735620
## theta[2,26] 51.69198463 0.7310242 50.2490835252 53.0795560
## theta[3,26] 50.26417912 0.9682377 48.2619167934 52.1645167
## theta[4,26] 49.70682463 0.7938901 48.0957041417 51.2446513
## theta[5,26] 53.75948550 0.6085659 52.5888306596 54.9643919
## theta[6,26] 54.00673885 0.9101539 52.2472647502 55.7499504
## theta[7,26] 51.39306620 0.7305657 49.8905845935 52.7883192
## theta[8,26] 55.92779084 1.0068644 53.8824624214 57.8620855
## theta[9,26] 52.70245493 0.8061803 51.1131167533 54.2767263
## theta[10,26] 49.08323370 0.7144163 47.6888460275 50.4596450
## theta[11,26] 52.95220598 0.6214680 51.7014863496 54.1632835
## theta[12,26] 49.38637371 0.9619820 47.4027612582 51.1700311
## theta[13,26] 53.66127679 0.6516777 52.3571384228 54.9231280
## theta[1,27] 51.10751197 0.6297612 49.8382170279 52.3240974
## theta[2,27] 51.59557956 0.6889199 50.1742415781 52.9001904
## theta[3,27] 50.06829435 1.0053589 47.9554983212 51.8824707
## theta[4,27] 49.65015382 0.8349751 47.9658041617 51.3178277
## theta[5,27] 53.68106127 0.5961050 52.4921488729 54.8615402
## theta[6,27] 53.93777094 0.9274105 52.1400333011 55.8078190
## theta[7,27] 51.26773885 0.7425197 49.7111310718 52.5936738
## theta[8,27] 55.88485248 1.0170439 53.8716284760 57.8579918
## theta[9,27] 52.54618642 0.8848469 50.7088496373 54.2529611
## theta[10,27] 49.01076638 0.6942045 47.5875057576 50.3089769
## theta[11,27] 52.88243352 0.6016879 51.6726332415 54.0152223
## theta[12,27] 49.20612287 0.9886721 47.1541060735 51.0529016
## theta[13,27] 53.57004950 0.6350478 52.3228395639 54.8008538
## theta[1,28] 51.52521276 0.6542520 50.2366031719 52.8079937
## theta[2,28] 52.05507630 0.6887345 50.6425076391 53.4041565
## theta[3,28] 50.12499043 0.9340830 48.1397356294 51.9130561
## theta[4,28] 50.01536838 0.8509644 48.2781783873 51.7032976
## theta[5,28] 54.00163501 0.6514727 52.7275700498 55.2844871

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## theta[6,28] 54.22122218 0.9355926 52.4263385698 56.0597690
## theta[7,28] 51.45993802 0.6861236 50.0112970650 52.7072014
## theta[8,28] 56.20466660 1.0443394 54.0920129568 58.3059915
## theta[9,28] 52.89991244 0.8949028 51.1540566358 54.6059930
## theta[10,28] 49.32678451 0.7424127 47.7966539299 50.7437920
## theta[11,28] 53.27253695 0.6640182 51.9390898793 54.5361494
## theta[12,28] 49.43351495 0.9078139 47.5618243643 51.1154915
## theta[13,28] 53.85897852 0.6612574 52.5431710612 55.2075711
## theta[1,29] 52.12530292 0.6595801 50.8423487448 53.4707311
## theta[2,29] 52.67502790 0.6244427 51.4718163497 53.9259424
## theta[3,29] 50.52154996 1.0344398 48.3864263495 52.5612354
## theta[4,29] 50.53334731 0.7451807 49.1033066118 52.0260710
## theta[5,29] 54.47683516 0.5637243 53.3454106133 55.5777261
## theta[6,29] 54.61688182 0.8427305 52.9158257358 56.3232763
## theta[7,29] 51.87222964 0.5660203 50.7527529971 53.0156321
## theta[8,29] 56.57034917 0.9457518 54.7122106961 58.4691233
## theta[9,29] 53.42600962 0.8097599 51.8681144577 55.0537921
## theta[10,29] 49.81591533 0.6857500 48.4529458584 51.1721647
## theta[11,29] 53.79586611 0.5433522 52.7079155975 54.8479905
## theta[12,29] 49.79799270 0.7657786 48.2905365850 51.2936409
## theta[13,29] 54.26567422 0.5853673 53.1201524543 55.4225389
## theta[1,30] 52.52593565 0.7540794 51.0772191385 54.0251223
## theta[2,30] 53.21647300 0.7193896 51.8793510504 54.7308610
## theta[3,30] 50.82178011 1.2450955 48.2127004987 53.1899638
## theta[4,30] 50.98839829 0.8450612 49.3407414444 52.6981022
## theta[5,30] 54.88852015 0.6926992 53.5626506778 56.2624302
## theta[6,30] 54.94428988 0.9292286 53.0727065343 56.8014527
## theta[7,30] 52.26674090 0.6506759 50.9816383237 53.6009989
## theta[8,30] 56.87259489 0.9958884 54.8991410248 58.8425715
## theta[9,30] 53.79041736 0.9685055 51.9408237000 55.7191593
## theta[10,30] 50.18104473 0.7972637 48.6460126146 51.7227332
## theta[11,30] 54.22862521 0.5649929 53.1513535402 55.3472148
## theta[12,30] 50.02838402 0.8157884 48.4200384998 51.6320572
## theta[13,30] 54.54344645 0.6367023 53.3176444391 55.8161953
## theta[1,31] 51.94547519 0.7483833 50.5371089322 53.4441194
## theta[2,31] 52.36215642 0.6029952 51.1780485914 53.5150612
## theta[3,31] 50.38912175 1.1796043 47.9054310014 52.6632150
## theta[4,31] 50.28603788 0.8865586 48.5221014368 52.0830561
## theta[5,31] 54.21206266 0.6322007 52.9678891441 55.4232146
## theta[6,31] 54.41930293 0.9607636 52.6045374596 56.4407635
## theta[7,31] 51.65786596 0.6109789 50.4986755972 52.8529058
## theta[8,31] 56.31773966 0.9039508 54.5395522096 58.0594851
## theta[9,31] 53.16384374 1.0044223 51.2322146394 55.1274606
## theta[10,31] 49.57179888 0.7906614 47.9802949704 51.0916952
## theta[11,31] 53.53000965 0.5711645 52.4272722331 54.6956747
## theta[12,31] 49.36719597 0.8625392 47.6442421486 51.0683891
## theta[13,31] 53.94310034 0.5903759 52.8033703048 55.0799736

## mean Rhat: 1.032897

## mean effective sample size: 494.3152

```

Since the outcome of the election is decided by Electoral College vote, for each set of MCMC samples of the

two-party vote share by state, we use the predicted winner in each state j on the day of the election (β_{j1}) and sum the electoral votes from those states for each candidate. For states not in the model, votes were given to the candidates based on how the states voted in 2016. The probability of President Trump winning re-election is the probability that he receives 270 or more electoral votes across simulations. (Note: Maine and Nebraska do not have a “winner-take-all” framework for allocating votes, but for simplicity, we assume that they have the same vote-allocating procedures as the other states).

A.2 Senate Election Model

The purpose of the Senate model is to predict whether the U.S. Senate remains in Republican control (and to predict the outcome for the NC Senate race). The states included in the model are Alaska, Arizona, Colorado, Georgia, Iowa, Kansas, Maine, Michigan, Montana, North Carolina, South Carolina, and Texas. These states were chosen because they are deemed by political analysts as being competitive states for the Senate race in 2020. There is also a special election for the Senate in Georgia in addition to the regularly scheduled election. For the Georgia special election, our model sums up the support for Republican candidates and compares that to the sum of the support for Democratic candidates, and treats this election in the same way as the other states.

The response we will use is Y_k , the Democrat candidate’s share of the two-party vote using the following model where the poll k (of $K = \text{fill in}$ polls) ended t days (of $T = 60$ days) before the election and was conducted on state j (of $J = 14$ states of interest).

$$\begin{aligned}
Y_k &\sim N(\beta_{jt}, \sigma_{yj}^2) \\
\beta_{jt} &\sim N(\beta_{jt-1}, \sigma_{\beta j}^2) \\
\text{Priors for } \sigma_{yj}^2: \sigma_{yj}^2 &\sim \text{InvGamma}(\nu_u, \nu_y \tau_y) \\
\nu_y &\sim \text{Uniform}(0, 100) \text{ and } \tau_y \sim \text{Uniform}(0, 100). \\
\text{Priors for } \sigma_{\beta j}^2: \sigma_{\beta j}^2 &\sim \text{InvGamma}(\nu_\beta, \nu_\beta \tau_\beta) \\
\nu_\beta &\sim \text{Uniform}(0, 100) \text{ and } \tau_\beta \sim \text{Uniform}(0, 100). \\
\text{Priors for } \beta_{j1}: \beta_{j1} &\sim N(\mu_j, \sigma^2) \\
\sigma^2 &\sim \text{InvGamma}(0.5, 0.5) \\
\mu_j &\sim N(h_j, 7.5^2) \\
h_j &= \text{Vote Share from Partisan Lean}_j + \text{Incumbency Advantage}_j
\end{aligned}$$

A hierarchical model was used, where the Democratic candidate’s share of the two-party vote for poll k on any given day t for state j is modeled as a normal distribution with mean β_{jt} and variance σ_{yj}^2 . A random walk was used to calculate β_{jt} for each day t before the election (up until 30 days before the election) from β_{jt-1} and $\sigma_{\beta j}$. The mean h_j of the normal prior on μ_j (which is the mean of the normal prior on β_{j1}), was calculated by the sum of the Vote Share from Partisan Lean $_j$ and Incumbency Advantage $_j$. *keep?*

Raw Model Output

##	mean	sd	2.5%	97.5%
## beta[1,1]	46.4507584	2.2433708	41.59965446	50.6435166
## beta[2,1]	52.3060134	2.1867985	46.72905116	55.6747117
## beta[3,1]	54.3407636	1.7697115	50.52703121	57.6127336
## beta[4,1]	48.6580340	1.6538193	45.60341428	51.9550692
## beta[5,1]	45.5772068	1.9532845	41.67556604	49.6975127
## beta[6,1]	50.7226677	1.6592584	47.18653081	53.7717883
## beta[7,1]	46.2249577	2.6063537	40.90148615	51.1973216
## beta[8,1]	52.0308945	2.2519892	47.08007333	55.9982111
## beta[9,1]	53.9554866	1.7664757	50.67107557	57.9060076
## beta[10,1]	47.5874120	2.0102568	43.45416459	51.5177091
## beta[11,1]	51.7831808	1.6530492	48.20280605	54.9907635
## beta[12,1]	48.8159096	1.9127126	45.15421200	52.8553178

## beta[13,1]	45.7833702	1.8480167	42.38430159	49.9298690
## beta[1,2]	46.4896459	2.2087097	41.73140473	50.7067420
## beta[2,2]	52.3289635	2.1332011	46.76305635	55.6280756
## beta[3,2]	54.3711092	1.7153990	50.70312546	57.5107585
## beta[4,2]	48.6820835	1.5900333	45.55906919	51.7720312
## beta[5,2]	45.5860786	1.8948276	41.78424188	49.4964545
## beta[6,2]	50.7492053	1.6070817	47.32136332	53.6024884
## beta[7,2]	46.2482261	2.5790858	40.95415039	51.1620338
## beta[8,2]	52.0342957	2.2127825	47.17982380	55.9353012
## beta[9,2]	53.9700156	1.7005589	50.93251803	57.7808375
## beta[10,2]	47.6105677	1.9664164	43.60635351	51.4052042
## beta[11,2]	51.8087928	1.5742675	48.42741143	54.8897353
## beta[12,2]	48.8315938	1.8713619	45.27158091	52.8845275
## beta[13,2]	45.8203025	1.8035136	42.50382068	49.8369427
## beta[1,3]	46.4972722	2.1748183	41.79078922	50.6082250
## beta[2,3]	52.3504019	2.0757404	46.89180959	55.5676551
## beta[3,3]	54.4025269	1.6743724	50.83741377	57.4734257
## beta[4,3]	48.7026871	1.5316694	45.66027082	51.6917491
## beta[5,3]	45.5939497	1.8587148	41.96836855	49.4744802
## beta[6,3]	50.7842206	1.5345155	47.56099990	53.5312636
## beta[7,3]	46.2874322	2.5704155	41.03692954	51.1878884
## beta[8,3]	52.0451801	2.1927681	47.31900495	56.0179210
## beta[9,3]	53.9616679	1.6422515	51.02280354	57.7440600
## beta[10,3]	47.6457081	1.9178816	43.67544160	51.3512848
## beta[11,3]	51.8224261	1.5060512	48.57959501	54.7184358
## beta[12,3]	48.8449458	1.8210090	45.44757736	52.9417926
## beta[13,3]	45.8366463	1.7529638	42.65491212	49.7247423
## beta[1,4]	46.5096325	2.1446964	41.93718701	50.5971583
## beta[2,4]	52.3811437	2.0084432	47.18445415	55.4559586
## beta[3,4]	54.4305865	1.6369488	51.02141715	57.4256656
## beta[4,4]	48.7264314	1.4754701	45.70538022	51.5413339
## beta[5,4]	45.5914780	1.8051970	42.05007872	49.3659552
## beta[6,4]	50.8000562	1.4696533	47.72206160	53.3727090
## beta[7,4]	46.3211874	2.5536161	41.19514273	51.1742377
## beta[8,4]	52.0428598	2.1662293	47.43433726	55.9819806
## beta[9,4]	53.9696348	1.5858942	51.09637291	57.6014267
## beta[10,4]	47.6724513	1.8668032	43.89282346	51.2309429
## beta[11,4]	51.8493091	1.4369846	48.83034407	54.6103632
## beta[12,4]	48.8576964	1.7708459	45.71742528	52.7111169
## beta[13,4]	45.8545564	1.6860656	42.79886556	49.5855540
## beta[1,5]	46.5270050	2.1184366	42.04091230	50.5605798
## beta[2,5]	52.3960022	1.9357695	47.40427586	55.3663511
## beta[3,5]	54.4590620	1.6011863	51.10509080	57.3259556
## beta[4,5]	48.7412904	1.4171168	45.81660090	51.4376954
## beta[5,5]	45.5938862	1.7675097	42.11294752	49.1187604
## beta[6,5]	50.8211279	1.4119876	47.90429088	53.3159518
## beta[7,5]	46.3494214	2.5448663	41.13449596	51.2228690
## beta[8,5]	52.0688027	2.1458932	47.40572776	55.9524187
## beta[9,5]	53.9749684	1.5443841	51.16636715	57.5088164
## beta[10,5]	47.7098359	1.8038902	43.97822741	51.1170287
## beta[11,5]	51.8806487	1.3733100	48.98335827	54.5870886
## beta[12,5]	48.8678520	1.7168686	45.75473610	52.6724863

## beta[13,5]	45.8739376	1.6433555	42.91525381	49.5902015
## beta[1,6]	46.5369181	2.0918170	42.30038580	50.5286182
## beta[2,6]	52.4209609	1.8622084	47.65261644	55.3620168
## beta[3,6]	54.4934255	1.5479439	51.22021999	57.3540242
## beta[4,6]	48.7565441	1.3599191	45.99376876	51.3513729
## beta[5,6]	45.5914292	1.7333413	42.14084013	49.1108980
## beta[6,6]	50.8723847	1.3477733	48.15517471	53.2286846
## beta[7,6]	46.3865869	2.5345902	41.11783543	51.2642474
## beta[8,6]	52.0944968	2.1202371	47.53904839	56.0144854
## beta[9,6]	53.9684720	1.4624460	51.34241802	57.3044058
## beta[10,6]	47.7515750	1.7459987	44.24095880	51.1506414
## beta[11,6]	51.9025948	1.3072086	49.13922418	54.5755418
## beta[12,6]	48.8775857	1.6746169	45.86563107	52.5292894
## beta[13,6]	45.9026161	1.5877677	42.99714600	49.4096485
## beta[1,7]	46.5517737	2.0788082	42.20078284	50.5648052
## beta[2,7]	52.4423884	1.8214850	47.83797315	55.2662000
## beta[3,7]	54.5344213	1.5061993	51.42066337	57.3478975
## beta[4,7]	48.7721559	1.3043131	46.11785442	51.2292675
## beta[5,7]	45.5929511	1.6707891	42.26243636	48.9654654
## beta[6,7]	50.9084257	1.2986088	48.29994155	53.1661204
## beta[7,7]	46.4051745	2.5277135	41.18396603	51.2405435
## beta[8,7]	52.1100098	2.0981936	47.66774052	55.9174251
## beta[9,7]	53.9754413	1.3875629	51.51574680	57.0159137
## beta[10,7]	47.7693629	1.6955459	44.26638293	51.0479234
## beta[11,7]	51.9338738	1.2186668	49.50430523	54.4152876
## beta[12,7]	48.8928983	1.6291872	45.94802949	52.3799161
## beta[13,7]	45.9204989	1.5234208	43.04064776	49.3011859
## beta[1,8]	46.5557695	2.0482870	42.29479623	50.5355847
## beta[2,8]	52.4693559	1.7171679	48.18092780	55.1291038
## beta[3,8]	54.5658777	1.4590191	51.64203041	57.3086507
## beta[4,8]	48.7917064	1.2390551	46.29392651	51.1663349
## beta[5,8]	45.6090295	1.6115669	42.32538218	48.7935958
## beta[6,8]	50.9393386	1.2460120	48.40704975	53.1458583
## beta[7,8]	46.4466889	2.5029317	41.22708808	51.2999787
## beta[8,8]	52.1339057	2.0695220	47.70566073	55.8567010
## beta[9,8]	53.9720083	1.3142507	51.60079827	56.9336924
## beta[10,8]	47.8072502	1.6310934	44.51403974	50.9382348
## beta[11,8]	51.9786793	1.1525195	49.61881294	54.4439017
## beta[12,8]	48.8842396	1.5855617	46.06751590	52.3273228
## beta[13,8]	45.9357006	1.4643750	43.17433684	49.1612021
## beta[1,9]	46.5670768	2.0241419	42.28102749	50.5473141
## beta[2,9]	52.5028970	1.6293033	48.32601925	55.0614260
## beta[3,9]	54.6185609	1.3956296	51.75957982	57.2449812
## beta[4,9]	48.8034662	1.1744828	46.48385806	51.0059174
## beta[5,9]	45.6034800	1.5457346	42.46873154	48.6337861
## beta[6,9]	50.9476722	1.1788262	48.51608288	53.0401498
## beta[7,9]	46.4908841	2.4581752	41.26567023	51.1615794
## beta[8,9]	52.1616163	2.0472942	47.62660697	55.8343743
## beta[9,9]	53.9766551	1.2299404	51.74557072	56.6013325
## beta[10,9]	47.8441955	1.5706324	44.69935624	50.8690019
## beta[11,9]	52.0023916	1.0794169	49.80329178	54.2249423
## beta[12,9]	48.8734921	1.5387937	46.15679907	52.3062505

## beta[13,9]	45.9468701	1.3896232	43.27160058	49.0166319
## beta[1,10]	46.5678919	2.0058839	42.32787193	50.4871242
## beta[2,10]	52.5390031	1.5105729	48.69645194	54.8742624
## beta[3,10]	54.6743274	1.3283331	51.95506185	57.1823165
## beta[4,10]	48.8089650	1.1067851	46.59030224	50.9459729
## beta[5,10]	45.6182920	1.4673513	42.62461690	48.4394718
## beta[6,10]	50.9589061	1.1176928	48.65209331	52.8557514
## beta[7,10]	46.5379063	2.4407841	41.33779109	51.2025385
## beta[8,10]	52.1807310	2.0268314	47.64778182	55.7261525
## beta[9,10]	53.9739212	1.1381429	51.91497703	56.3897657
## beta[10,10]	47.8871395	1.5012262	44.87418303	50.8326995
## beta[11,10]	52.0246213	1.0056766	49.98163736	54.0750713
## beta[12,10]	48.8645445	1.5075891	46.23353781	52.2559794
## beta[13,10]	45.9557357	1.3205726	43.33332685	48.9188215
## beta[1,11]	46.5657722	1.9884405	42.47851727	50.4111217
## beta[2,11]	52.5871373	1.3717250	49.06990092	54.7918520
## beta[3,11]	54.7400065	1.2813795	52.04151691	57.2546969
## beta[4,11]	48.8263871	1.0265121	46.80608246	50.7639370
## beta[5,11]	45.6229736	1.3946257	42.81182773	48.3699546
## beta[6,11]	50.9801055	1.0438403	48.76111023	52.8246632
## beta[7,11]	46.5696643	2.4023754	41.27155079	51.1483077
## beta[8,11]	52.2079035	2.0094587	47.72323788	55.5756930
## beta[9,11]	53.9590365	1.0167393	52.09094606	56.1472696
## beta[10,11]	47.9399924	1.4382705	45.08574550	50.8092628
## beta[11,11]	52.0390239	0.9176663	50.21353504	53.8657094
## beta[12,11]	48.8479921	1.4812537	46.24737533	52.1934658
## beta[13,11]	45.9599444	1.2276616	43.47051693	48.6080411
## beta[1,12]	46.5714547	1.9538748	42.56145599	50.2601810
## beta[2,12]	52.6422271	1.2120834	49.65909631	54.6130190
## beta[3,12]	54.7796973	1.2322518	52.24493741	57.1065122
## beta[4,12]	48.8241898	0.9347758	46.96862712	50.5515212
## beta[5,12]	45.6430506	1.3163577	42.89452579	48.1590470
## beta[6,12]	50.9883574	0.9800345	48.91056324	52.7813585
## beta[7,12]	46.6107457	2.3502945	41.38512564	51.1682618
## beta[8,12]	52.2146382	2.0078566	47.80023470	55.5988958
## beta[9,12]	53.8425879	0.9400988	52.10605867	55.7705086
## beta[10,12]	47.9706309	1.3642976	45.24576448	50.5990511
## beta[11,12]	52.0693416	0.8175119	50.41380058	53.6838658
## beta[12,12]	48.8440897	1.4287400	46.28222135	52.0444698
## beta[13,12]	45.9811842	1.1335226	43.68615188	48.3956155
## beta[1,13]	46.5876383	1.9218749	42.58312108	50.3284772
## beta[2,13]	52.6891150	1.0461506	50.27943163	54.4381734
## beta[3,13]	54.8395928	1.1751190	52.48355262	57.0792011
## beta[4,13]	48.7926745	0.8883001	47.04169779	50.4494619
## beta[5,13]	45.6557143	1.2381279	43.13541646	48.0919228
## beta[6,13]	51.0000905	0.8927554	49.09466934	52.5801656
## beta[7,13]	46.6552101	2.2921448	41.57173330	51.1101449
## beta[8,13]	52.2276404	1.9684165	47.85446957	55.4976038
## beta[9,13]	53.7173751	0.8618031	52.08186888	55.5516843
## beta[10,13]	48.0013049	1.2600938	45.42117298	50.4480650
## beta[11,13]	52.0184638	0.7707851	50.49038887	53.5629130
## beta[12,13]	48.8407254	1.3757495	46.33568886	51.8699864

## beta[13,13]	45.9923947	1.0500376	43.91426454	48.1153740
## beta[1,14]	46.6229871	1.8726832	42.85334043	50.2933601
## beta[2,14]	52.9666038	0.8457280	51.15682373	54.4630778
## beta[3,14]	54.9092964	1.1112883	52.66372207	57.0326530
## beta[4,14]	48.7760120	0.8189630	47.17509433	50.3058560
## beta[5,14]	45.6609027	1.1838267	43.23871605	47.9725141
## beta[6,14]	51.0115363	0.7988418	49.40733822	52.4348770
## beta[7,14]	46.7010099	2.2500168	41.64344074	51.0391557
## beta[8,14]	52.2455939	1.9593516	47.90972633	55.5146364
## beta[9,14]	53.6008012	0.7617864	52.06529968	55.0934147
## beta[10,14]	48.0410622	1.1925141	45.53014963	50.2580502
## beta[11,14]	51.9433993	0.7044818	50.51203083	53.3278228
## beta[12,14]	48.8306216	1.3210952	46.45141792	51.6902215
## beta[13,14]	45.9985113	0.9352965	44.09483641	47.8686881
## beta[1,15]	46.6458309	1.8442028	42.95508554	50.2851475
## beta[2,15]	53.1461151	0.7395091	51.61317260	54.4908439
## beta[3,15]	54.9750359	1.0182329	52.99031086	56.9488146
## beta[4,15]	48.8341676	0.7962061	47.29228520	50.3736713
## beta[5,15]	45.9265245	1.1217179	43.56152539	48.1300919
## beta[6,15]	51.1413157	0.7525674	49.60599917	52.5815186
## beta[7,15]	46.7357382	2.1813822	41.77010350	50.9043960
## beta[8,15]	52.2572799	1.9496084	47.88088430	55.3668462
## beta[9,15]	53.4845287	0.6461339	52.20040491	54.7143214
## beta[10,15]	48.0739475	1.1115507	45.72275331	50.1018411
## beta[11,15]	51.9966479	0.6396810	50.72183860	53.2612563
## beta[12,15]	48.8230295	1.2764186	46.45960717	51.5080448
## beta[13,15]	46.0059997	0.8319034	44.25857352	47.6864369
## beta[1,16]	46.6857907	1.7850144	43.15820470	50.1988322
## beta[2,16]	53.3670823	0.6463870	52.03214116	54.5841181
## beta[3,16]	55.0755165	0.9922047	53.09893958	56.9708163
## beta[4,16]	48.9124194	0.7855444	47.36550357	50.5040986
## beta[5,16]	46.1857349	1.0956328	43.84692791	48.3209172
## beta[6,16]	51.3535917	0.7244868	49.89473148	52.7222249
## beta[7,16]	46.7427876	2.2430579	41.80418504	51.0793865
## beta[8,16]	52.2782003	1.9247401	48.06497504	55.3736604
## beta[9,16]	53.3664815	0.6365839	52.05778199	54.5860187
## beta[10,16]	48.0084506	1.1192421	45.65320355	50.1161985
## beta[11,16]	52.0820862	0.6131808	50.91801927	53.3244301
## beta[12,16]	48.7129144	1.2720920	46.37203848	51.3063113
## beta[13,16]	46.0164093	0.8116112	44.36180327	47.6053965
## beta[1,17]	46.7028880	1.7228092	43.35995183	50.0810660
## beta[2,17]	53.5305265	0.6793261	52.16274992	54.8757080
## beta[3,17]	55.1803550	0.9332272	53.28641361	56.9787842
## beta[4,17]	48.9319298	0.8485414	47.31213588	50.6031499
## beta[5,17]	46.2653397	1.1300489	43.89912988	48.4355767
## beta[6,17]	51.3887070	0.8243632	49.70430353	52.9750254
## beta[7,17]	46.7497226	2.2991594	41.63006534	51.0984169
## beta[8,17]	52.2843067	1.9094081	48.08334810	55.3344178
## beta[9,17]	53.2805018	0.6635770	51.91987207	54.5118972
## beta[10,17]	47.9371653	1.1271367	45.66019767	50.0652831
## beta[11,17]	52.0230307	0.6489327	50.75655496	53.3063521
## beta[12,17]	48.5740598	1.2640516	46.18405026	51.0738517

## beta[13,17]	46.0250522	0.8851848	44.24224428	47.7693131
## beta[1,18]	46.7191474	1.6849682	43.37287806	49.9894809
## beta[2,18]	53.6993091	0.7032252	52.30481462	55.0809696
## beta[3,18]	55.2918572	0.8898372	53.51782732	57.0021850
## beta[4,18]	48.9570142	0.8990033	47.26337927	50.7539149
## beta[5,18]	46.3654826	1.1661034	44.02401079	48.6048889
## beta[6,18]	51.4340005	0.8929114	49.57896593	53.1963433
## beta[7,18]	46.7545369	2.3458972	41.57946265	51.2566181
## beta[8,18]	52.2918695	1.8741920	48.21060194	55.3887927
## beta[9,18]	53.1913995	0.7125515	51.70608770	54.5131026
## beta[10,18]	47.8208705	1.1798427	45.45918056	50.1042142
## beta[11,18]	52.0348514	0.6632475	50.65707936	53.3067999
## beta[12,18]	48.4413338	1.2539374	46.05390072	50.8629306
## beta[13,18]	46.0264615	0.9369834	44.12348258	47.9383915
## beta[1,19]	46.7421722	1.6016921	43.54344622	49.7863545
## beta[2,19]	53.8693082	0.7158984	52.52157535	55.3957116
## beta[3,19]	55.4003856	0.8175367	53.78862581	56.9665940
## beta[4,19]	48.9647817	0.9209987	47.21677596	50.7942971
## beta[5,19]	46.4612821	1.2003758	44.06536548	48.7720284
## beta[6,19]	51.4905103	0.9519827	49.52667423	53.3854068
## beta[7,19]	46.7601683	2.3832760	41.63232402	51.3653629
## beta[8,19]	52.2944480	1.8615495	48.19078230	55.3370190
## beta[9,19]	53.1191294	0.7229911	51.64951317	54.4534752
## beta[10,19]	47.7283845	1.2228423	45.28487685	50.1013232
## beta[11,19]	52.1019241	0.6812656	50.67895815	53.4229057
## beta[12,19]	48.3219121	1.2352106	45.94940141	50.7075071
## beta[13,19]	46.0205789	0.9521834	44.04763945	47.9163047
## beta[1,20]	46.7582842	1.5352911	43.73173446	49.7050819
## beta[2,20]	53.9160116	0.7264295	52.57913230	55.4433531
## beta[3,20]	55.3806216	0.7737028	53.82986717	56.8416627
## beta[4,20]	48.9781951	0.9234619	47.22012021	50.8176672
## beta[5,20]	46.5439817	1.2129056	44.19475206	48.8982318
## beta[6,20]	51.5285021	0.9917533	49.48773575	53.5262365
## beta[7,20]	46.7639945	2.4207134	41.58969934	51.3864437
## beta[8,20]	52.3163486	1.8168325	48.26509074	55.2879608
## beta[9,20]	53.0319687	0.7065565	51.50450038	54.3218997
## beta[10,20]	47.6212562	1.2740673	45.08221853	50.1020328
## beta[11,20]	52.1880136	0.6670004	50.86017541	53.4863884
## beta[12,20]	48.1991681	1.2166046	45.88051505	50.4596289
## beta[13,20]	46.0143552	0.9618815	44.05770633	48.0022406
## beta[1,21]	46.7867748	1.4608324	43.78679190	49.5976637
## beta[2,21]	53.9627333	0.7133019	52.64062644	55.4582007
## beta[3,21]	55.3917216	0.7602843	53.86113700	56.8728672
## beta[4,21]	48.9973325	0.9028998	47.28894242	50.8021263
## beta[5,21]	46.6284567	1.2130280	44.27558319	49.1251114
## beta[6,21]	51.5949363	1.0058461	49.51228545	53.6714812
## beta[7,21]	46.7686597	2.4794534	41.48974505	51.5905315
## beta[8,21]	52.3184579	1.7838715	48.31533347	55.2417579
## beta[9,21]	53.1274968	0.7071195	51.64861973	54.4375554
## beta[10,21]	47.5207537	1.3034189	44.89221432	50.0244359
## beta[11,21]	52.2554363	0.6082433	51.07245111	53.4918906
## beta[12,21]	48.0540662	1.1969640	45.75188289	50.2858863

## beta[13,21]	46.0102836	0.9687428	44.05574358	47.8820252
## beta[1,22]	46.9138681	1.4702115	43.87444960	49.7132180
## beta[2,22]	53.9280857	0.6806370	52.66031623	55.3697297
## beta[3,22]	55.3909755	0.7627844	53.89025506	56.8790100
## beta[4,22]	49.0171836	0.8686521	47.29466320	50.7478746
## beta[5,22]	46.7094752	1.2132755	44.28517901	49.0972275
## beta[6,22]	51.6452200	1.0085701	49.56952211	53.6278500
## beta[7,22]	46.7599454	2.5191544	41.44002023	51.6686499
## beta[8,22]	52.3274020	1.7383628	48.40335725	55.2711821
## beta[9,22]	53.2003764	0.6739861	51.82058867	54.4739230
## beta[10,22]	47.4205357	1.3180979	44.78243620	49.8706666
## beta[11,22]	52.3761841	0.5800552	51.24988800	53.5497691
## beta[12,22]	48.0102021	1.2333392	45.61759356	50.3119820
## beta[13,22]	46.0085939	0.9463558	44.11870181	47.8884170
## beta[1,23]	47.0232361	1.4814496	43.91782910	49.9315878
## beta[2,23]	53.8669594	0.7174250	52.50044430	55.3438474
## beta[3,23]	55.3670157	0.8033922	53.77174617	56.9774226
## beta[4,23]	49.0392598	0.8268835	47.40021465	50.6494120
## beta[5,23]	46.7962967	1.2039744	44.43139050	49.1931114
## beta[6,23]	51.6930843	1.0107959	49.71234878	53.8058706
## beta[7,23]	46.7614674	2.5658209	41.35513589	51.8528121
## beta[8,23]	52.3389353	1.6916018	48.58238159	55.2809663
## beta[9,23]	53.2287894	0.6573569	51.86899606	54.5250160
## beta[10,23]	47.3236751	1.3460824	44.58790379	49.8583770
## beta[11,23]	52.4861980	0.5989203	51.37275263	53.7528760
## beta[12,23]	47.9452698	1.2447301	45.45634922	50.2473777
## beta[13,23]	46.0020642	0.9207214	44.15558873	47.8619245
## beta[1,24]	47.1305436	1.4869033	43.88884664	50.0421037
## beta[2,24]	53.8081838	0.7034063	52.45534988	55.2121716
## beta[3,24]	55.3511499	0.8095567	53.71748443	56.9488549
## beta[4,24]	48.8683044	0.8151464	47.23374258	50.4731764
## beta[5,24]	46.6405702	1.1818212	44.32265636	48.9996827
## beta[6,24]	51.7246580	0.9944392	49.79219166	53.8391161
## beta[7,24]	46.7495695	2.6084561	41.44290247	51.8995926
## beta[8,24]	52.3529625	1.6348484	48.77106141	55.1723407
## beta[9,24]	53.2245996	0.6638911	51.87331308	54.5047005
## beta[10,24]	47.2340111	1.3460257	44.42973164	49.6907910
## beta[11,24]	52.6098662	0.5596440	51.58168714	53.7578403
## beta[12,24]	47.8901211	1.2638985	45.43499646	50.2952325
## beta[13,24]	45.9815802	0.8946495	44.19968961	47.8335330
## beta[1,25]	47.2423711	1.4813390	43.96972453	50.0950786
## beta[2,25]	53.6804149	0.7280380	52.21821671	55.1005840
## beta[3,25]	55.3100243	0.8440411	53.62275803	56.9828277
## beta[4,25]	48.7773472	0.8666345	47.08144277	50.4819196
## beta[5,25]	46.4909435	1.1667758	44.24379518	48.7740719
## beta[6,25]	51.7657467	0.9760860	49.79779801	53.7531843
## beta[7,25]	46.7458671	2.6409454	41.43742045	51.9413585
## beta[8,25]	52.3512352	1.5890911	48.92201779	55.1477914
## beta[9,25]	53.2668154	0.7064152	51.82130832	54.6135692
## beta[10,25]	47.1365071	1.3479176	44.32592250	49.6017676
## beta[11,25]	52.5266708	0.6568606	51.33416299	53.9571469
## beta[12,25]	47.9037261	1.3397291	45.30471282	50.3954086

## beta[13,25]	46.0381220	0.9314409	44.21415386	47.9691454
## beta[1,26]	47.3565627	1.4751398	43.93836281	50.3243612
## beta[2,26]	53.5714810	0.7184205	52.12762987	54.9467133
## beta[3,26]	55.2790811	0.8710228	53.49875774	56.9581881
## beta[4,26]	48.6911341	0.9040403	46.88819116	50.4056816
## beta[5,26]	46.3461420	1.1552840	44.08583818	48.5561571
## beta[6,26]	51.8022736	0.9293562	49.97182332	53.6897940
## beta[7,26]	46.7498456	2.6828938	41.39557134	51.9919580
## beta[8,26]	52.3537983	1.5436570	48.99796781	55.1439165
## beta[9,26]	53.2971362	0.6964640	51.88290091	54.6503765
## beta[10,26]	47.0431523	1.3533366	44.26502066	49.4627238
## beta[11,26]	52.4467005	0.7354556	51.04320166	53.9862058
## beta[12,26]	47.9046602	1.4283001	44.93686676	50.5560030
## beta[13,26]	46.0984842	0.9448671	44.28554119	48.0987626
## beta[1,27]	47.4618450	1.4700258	44.09461319	50.3972924
## beta[2,27]	53.6111881	0.7078093	52.24430199	54.9594438
## beta[3,27]	55.2679870	0.9362267	53.37511242	57.1152185
## beta[4,27]	48.6024534	0.9173805	46.74524389	50.3933464
## beta[5,27]	46.2016904	1.1621773	43.91817702	48.3829458
## beta[6,27]	51.8327335	0.9539375	49.97904837	53.7463876
## beta[7,27]	46.7452836	2.7133898	41.29619052	52.0879452
## beta[8,27]	52.3640944	1.4948538	49.20271106	55.1464839
## beta[9,27]	53.4495606	0.7081610	52.07726318	54.8664754
## beta[10,27]	46.9520061	1.3574939	44.08995442	49.4453710
## beta[11,27]	52.3771826	0.7578924	50.91932501	53.8852459
## beta[12,27]	47.9196206	1.5196270	44.77370048	50.7517371
## beta[13,27]	46.1524774	0.9366134	44.33378084	48.0825628
## beta[1,28]	47.5722609	1.4605450	44.23145966	50.4744615
## beta[2,28]	53.6942170	0.7224197	52.29598129	55.1091818
## beta[3,28]	55.2672804	0.9914321	53.25603453	57.2040805
## beta[4,28]	48.5022231	0.9164981	46.66678874	50.2971843
## beta[5,28]	46.0848688	1.1801998	43.82973407	48.3405981
## beta[6,28]	51.8413230	0.9615027	49.95065859	53.7982913
## beta[7,28]	46.7603785	2.7476268	41.24162876	52.2203267
## beta[8,28]	52.3590314	1.4207960	49.40816857	55.0015970
## beta[9,28]	53.6043712	0.7257379	52.16331887	55.0172231
## beta[10,28]	46.8523748	1.3631335	44.02617140	49.3986889
## beta[11,28]	52.3068993	0.7784794	50.74224627	53.8319352
## beta[12,28]	47.9131191	1.6167875	44.54409007	51.0062493
## beta[13,28]	46.1935809	0.9022764	44.56786932	48.1391365
## beta[1,29]	47.6848497	1.4344787	44.45177709	50.5091427
## beta[2,29]	53.6673304	0.7679560	52.13265892	55.1472795
## beta[3,29]	55.2585334	1.0287523	53.13190571	57.2435084
## beta[4,29]	48.3580718	0.9985588	46.30876496	50.2707486
## beta[5,29]	46.0312334	1.2366088	43.54426637	48.3755283
## beta[6,29]	51.8677925	0.9387419	50.03582876	53.7358149
## beta[7,29]	46.7554409	2.7807474	41.15111317	52.2648013
## beta[8,29]	52.3619139	1.3557810	49.48227192	54.8444986
## beta[9,29]	53.6382803	0.7852292	52.09806866	55.1898938
## beta[10,29]	46.8636525	1.4377624	43.83159561	49.6131254
## beta[11,29]	52.2322878	0.7727406	50.59331291	53.7419728
## beta[12,29]	47.9130771	1.7037495	44.21988677	50.9993214

## beta[13,29]	46.2489814	0.8655470	44.69269510	48.0894850
## beta[1,30]	47.7991288	1.4156141	44.69305877	50.5769502
## beta[2,30]	53.6506213	0.7774959	52.13880114	55.1265817
## beta[3,30]	55.2627770	1.1256573	53.01983584	57.3903348
## beta[4,30]	48.3550312	1.1031878	46.08211241	50.4331675
## beta[5,30]	46.0259933	1.3216892	43.37429385	48.5746496
## beta[6,30]	51.9194306	0.9743408	50.02526565	53.8536274
## beta[7,30]	46.7472582	2.8285276	41.14622590	52.2613860
## beta[8,30]	52.2579323	1.3622229	49.49011940	54.8201552
## beta[9,30]	53.6713997	0.8551522	52.03762314	55.4141234
## beta[10,30]	46.8650164	1.5038064	43.80688647	49.6510935
## beta[11,30]	52.1206822	0.8143506	50.38648947	53.6985330
## beta[12,30]	47.9236093	1.7686198	44.21213856	51.1759230
## beta[13,30]	46.2149077	0.9222641	44.50317426	48.1660882
## beta[1,31]	47.8996625	1.3788202	44.76095190	50.4355935
## beta[2,31]	53.6912338	0.8528018	52.00973344	55.3179302
## beta[3,31]	55.2785179	1.2092872	52.90852160	57.6307819
## beta[4,31]	48.3585808	1.2016861	45.86013835	50.6686478
## beta[5,31]	46.0219710	1.3913000	43.19006592	48.7444100
## beta[6,31]	51.9119956	1.0926269	49.75673560	54.1812027
## beta[7,31]	46.7432851	2.8569825	41.01971904	52.3066243
## beta[8,31]	52.1675439	1.3656409	49.40700955	54.7009520
## beta[9,31]	53.6930802	0.9112265	51.84999218	55.5205501
## beta[10,31]	46.8551461	1.5830819	43.61816394	49.7614292
## beta[11,31]	52.0090255	0.8341724	50.25811229	53.5760578
## beta[12,31]	47.9192727	1.8396770	43.94391158	51.3031586
## beta[13,31]	46.1789356	0.9681142	44.33602796	48.1880520
## sigma2_beta[1]	0.2174465	0.1709002	0.03591539	0.6455778
## sigma2_beta[2]	0.2195665	0.1769982	0.03763168	0.6657771
## sigma2_beta[3]	0.2117813	0.1594372	0.03599774	0.6146854
## sigma2_beta[4]	0.2150078	0.1639863	0.03653461	0.6168701
## sigma2_beta[5]	0.2248891	0.1892429	0.03824851	0.6904631
## sigma2_beta[6]	0.2127312	0.1623794	0.03671041	0.6352535
## sigma2_beta[7]	0.2177799	0.1719668	0.03634232	0.6345328
## sigma2_beta[8]	0.2146970	0.1668502	0.03705036	0.6189533
## sigma2_beta[9]	0.2103161	0.1587663	0.03750246	0.6029883
## sigma2_beta[10]	0.2202669	0.1761017	0.03688560	0.6480835
## sigma2_beta[11]	0.2111939	0.1591431	0.03804373	0.5951090
## sigma2_beta[12]	0.2172667	0.1701173	0.03499755	0.6603020
## sigma2_beta[13]	0.2134587	0.1636727	0.03839452	0.6263231
## sigma2_y[1]	4.3393222	2.2137569	2.06291739	8.9778112
## sigma2_y[2]	3.8692044	1.0191342	2.27841397	6.2710692
## sigma2_y[3]	3.4111862	1.1556115	1.48423175	6.0022095
## sigma2_y[4]	3.9842974	1.2215696	2.22593396	6.8663237
## sigma2_y[5]	7.4445583	4.5573201	3.38615960	20.3325823
## sigma2_y[6]	3.7518461	1.1728339	1.95333042	6.6424013
## sigma2_y[7]	4.3641806	3.0370416	1.89056380	10.3323020
## sigma2_y[8]	4.2269202	2.2465433	1.94931827	8.4792981
## sigma2_y[9]	3.5124807	0.9368648	1.89915246	5.5939477
## sigma2_y[10]	3.9496067	1.6799443	1.69059348	7.7787986
## sigma2_y[11]	3.6043268	0.9004233	2.11599421	5.7642380
## sigma2_y[12]	4.2285806	1.8916422	1.99432151	8.2372830


```
## sigma2_y[13]      3.7806794 1.1815197  1.90567452  6.6652333
## mean Rhat: 1.095761
## mean effective sample size: 215.8395
```

For predicting control of the U.S. Senate, if a 50/50 split is predicted, then the VP breaks the tie. In this case, we predict Democrat majority since our presidential model gives Biden a high chance of winning (and a Democrat VP will break the tie).

A.3 House Election Model

House Model Purpose and Structure

The purpose of the House model is to predict which candidate in each of NC's 13 congressional districts will win in their respective elections.

The response we will use is Y_k , the Democrat candidate's share of the two-party vote using the following model where the poll k (of $K = \text{fill in}$ polls) ended t days (of $T = 60$ days) before the election and was conducted on district j (where j is in districts 1, \dots , 11, 13 of North Carolina). For district 12, the vote share is coded as 100 because there is only one candidate, and she is a Democrat. Note that, due to the scarcity of poll responses for the NC House elections, we included all poll responses within 115 days of the election.

$$\begin{aligned}
Y_k &\sim N(\beta_{jt}, \sigma_{yj}^2) \\
\beta_{jt} &\sim N(\beta_{jt-1}, \sigma_{\beta j}^2) \\
\sigma_{yj}^2 &\sim \text{InvGamma}(\nu_u, \nu_y \tau_y) \\
\nu_y &\sim \text{Uniform}(0, 100) \text{ and } \tau_y \sim \text{Uniform}(0, 100). \\
\sigma_{\beta j}^2 &\sim \text{InvGamma}(\nu_\beta, \nu_\beta \tau_\beta) \\
\nu_\beta &\sim \text{Uniform}(0, 100) \text{ and } \tau_\beta \sim \text{Uniform}(0, 100). \\
\beta_{j1} &\sim N(h_j, 7.5) \\
h_j &= 0.9 * \text{Vote Share from Partisan Lean}_j + 0.1 * \text{Expected Vote Share from Voter Turnout}_j
\end{aligned}$$

A hierarchical model was used, where the Democratic candidate's share of the two-party vote for poll k on any given day t for district j is modelled as a normal distribution with mean β_{jt} and variance σ_{yj}^2 . A random walk was used to calculate β_{jt} for each day t before the election (up until 30 days before the election) from β_{jt-1} and $\sigma_{\beta j}$. The mean h_j of the normal prior on μ_{uj} is computed by multiplying Vote Share from Partisan Lean $_j$ by 0.9 and Expected Vote Share from Voter Turnout $_j$ by 0.1, then summing those two values. *keep?*

Interim Report Model

$$\begin{aligned}
\text{Vote}_i &\sim \text{Binomial}(n_i, \pi_i) \\
\text{logit}(\pi_i) &= \alpha_0 + \Sigma_{j=1,2,3} \beta_j * I(\text{Income}_i = \text{Income Category}_s) + \Sigma_{j=5,6} \beta_j * I(\text{Gender}_i = \text{Gender Category}_g) + \\
&\quad \Sigma_{j=7,8} \beta_j * I(\text{Race}_i = \text{Race Category}_r) + \Sigma_{j=9,10,11} \beta_j * I(\text{Age}_i = \text{Age Category}_a) + \\
&\quad \Sigma_{j=12,13} \beta_j * I(\text{Party}_i = \text{Party Category}_p) + \\
&\quad \Sigma_{j=14,\dots,17} \beta_j * I(\text{Gender}_i = \text{Gender Category}_g) I(\text{Party}_i = \text{Party Category}_p) + \\
&\quad \Sigma_{j=18,\dots,21} \beta_j * I(\text{Race}_i = \text{Race Category}_r) I(\text{Party}_i = \text{Party Category}_p) + \\
&\quad \Sigma_{j=22,\dots,27} \beta_j * I(\text{Gender}_i = \text{Gender Category}_g) I(\text{Age}_i = \text{Age Category}_a) + \\
&\quad \Sigma_{j=28,\dots,33} \beta_j * I(\text{Party}_i = \text{Party Category}_p) I(\text{Age}_i = \text{Age Category}_a)
\end{aligned}$$

where:

$Income\ Category_s \in \{Median\ Income\ \$46864 - \$52798, Median\ Income\ \$52798 - \$64509, Median\ Income > \$64509\}$
 $Gender\ Category_g \in \{Male, Unspecified\}$
 $Race\ Category_r \in \{Black, Other\}$
 $Age\ Category_a \in \{Age\ 30 - 44, Age\ 45 - 59, Age\ 60 - 69\}$
 $Party\ Category_p \in \{Republican, Other\}$

Where π_i is the probability that subgroup i votes. $Vote_i$ is the number of people in the i th subgroup that voted in the 2016 election. The baseline is **Median Income** < \$46864 for income, **Female** for gender, **White** for race, 18-29 for age, and **Democrat** for party affiliation.

We fit the voter turnout model from the Interim Report on the 2020 registered voter dataset. The predicted values were the number of people that will vote for each demographic subgroup in each congressional district. We made a reasonable assumption that if someone is registered as a Democrat, that they will indeed vote for the Democratic candidate (and made the same assumption for registered Republicans). If the demographic subgroup had third/unaffiliated party, then we split their predicted number of voters evenly between the Democrat and Republican parties. From this, we calculated Democratic party vote share for each district by taking the ratio of the predicted number of Democratic voters to total number of registered voters in that district.

Raw Output for Appendix

##	mean	sd	2.5%	97.5%
## beta[1,1]	58.306127	2.716740	52.977697539	63.675746
## beta[2,1]	59.253823	2.777571	53.830873329	64.707466
## beta[3,1]	36.385167	2.670935	31.436792575	41.684174
## beta[4,1]	66.711295	2.729256	61.523445759	72.155486
## beta[5,1]	30.463614	2.735714	25.070167216	35.796273
## beta[6,1]	58.811408	2.742312	53.362761435	64.131692
## beta[7,1]	38.194965	2.812420	32.566439603	43.595922
## beta[8,1]	44.256514	2.564823	38.991609441	49.328885
## beta[9,1]	43.211496	2.580971	38.101328337	48.597018
## beta[10,1]	29.324002	2.730484	24.043756131	34.836887
## beta[11,1]	43.880322	2.187492	39.408639848	48.139221
## beta[12,1]	68.969534	2.744467	63.677429845	74.298244
## beta[13,1]	30.396369	2.731562	24.989800885	35.883334
## beta[1,2]	58.352260	3.040225	52.320255196	64.302066
## beta[2,2]	59.267556	3.127547	53.209654701	65.536052
## beta[3,2]	36.362939	3.021067	30.639348890	42.443572
## beta[4,2]	66.670367	3.077007	60.774331456	72.879396
## beta[5,2]	30.484026	3.009829	24.789901778	36.339399
## beta[6,2]	58.837266	3.089096	52.785350981	64.904593
## beta[7,2]	38.250039	3.142514	32.211870346	44.520885
## beta[8,2]	44.391225	2.886468	38.351914439	50.094210
## beta[9,2]	43.362126	2.858532	37.344276524	49.189518
## beta[10,2]	29.309636	3.026344	23.355513128	35.415154
## beta[11,2]	44.172316	2.432975	39.122144150	48.877901
## beta[12,2]	68.944352	3.063979	62.817791974	75.036508
## beta[13,2]	30.438862	3.056301	24.270146352	36.417821
## beta[1,3]	58.368669	3.332791	51.870150051	65.069312
## beta[2,3]	59.276091	3.526879	52.585839537	66.302592
## beta[3,3]	36.311190	3.348195	29.895473390	43.097538

## beta[4,3]	66.643560	3.379636	59.891919751	73.405557
## beta[5,3]	30.509747	3.382913	23.975655355	37.144857
## beta[6,3]	58.835481	3.416625	52.333112910	65.587899
## beta[7,3]	38.270992	3.489868	31.412035147	45.103616
## beta[8,3]	44.527720	3.128332	37.845925520	50.696095
## beta[9,3]	43.556225	3.070066	37.057456871	49.830085
## beta[10,3]	29.268348	3.338790	22.691576703	36.030820
## beta[11,3]	44.449054	2.628877	39.078423450	49.665543
## beta[12,3]	68.965937	3.371608	62.124234052	75.664384
## beta[13,3]	30.413120	3.372929	23.691910985	37.108805
## beta[1,4]	58.365505	3.632878	51.204359548	65.601744
## beta[2,4]	59.247331	3.801727	51.932215509	66.917361
## beta[3,4]	36.324428	3.628670	29.518788188	43.596500
## beta[4,4]	66.643179	3.638378	59.100184032	73.824224
## beta[5,4]	30.524889	3.683148	23.559421377	37.711960
## beta[6,4]	58.827705	3.658566	51.710010972	65.968050
## beta[7,4]	38.288599	3.758513	31.153627517	45.668316
## beta[8,4]	44.679465	3.376841	37.482810933	51.309609
## beta[9,4]	43.751912	3.263212	36.798965705	50.308274
## beta[10,4]	29.255932	3.636745	22.086106219	36.447439
## beta[11,4]	44.764004	2.895262	39.063448015	50.837896
## beta[12,4]	68.929958	3.615904	61.456805864	76.261149
## beta[13,4]	30.387894	3.640859	23.327469316	37.687980
## beta[1,5]	58.338204	3.859056	50.811769371	65.923478
## beta[2,5]	59.304872	4.080726	51.184248393	67.420540
## beta[3,5]	36.328763	3.875191	28.770960718	43.989578
## beta[4,5]	66.607566	3.872030	58.584637237	74.066816
## beta[5,5]	30.536652	3.969847	23.111815665	38.045611
## beta[6,5]	58.801986	3.931892	51.305185383	66.367658
## beta[7,5]	38.303831	4.005214	30.444724652	46.468960
## beta[8,5]	44.773397	3.540718	37.028889643	51.641192
## beta[9,5]	43.976716	3.452329	36.669634874	51.004485
## beta[10,5]	29.244137	3.967901	21.102460081	37.069445
## beta[11,5]	45.018522	3.070197	38.894429749	51.740404
## beta[12,5]	68.949205	3.894044	60.863494047	76.536290
## beta[13,5]	30.420340	3.879809	22.998457762	38.371319
## beta[1,6]	58.304867	4.125022	50.187979967	66.708488
## beta[2,6]	59.298593	4.224963	51.048209626	67.985691
## beta[3,6]	36.296599	4.130048	28.526117351	44.655085
## beta[4,6]	66.611332	4.056136	58.438574939	74.625763
## beta[5,6]	30.540835	4.284264	22.771174840	38.650854
## beta[6,6]	58.798035	4.185780	50.655577589	67.076381
## beta[7,6]	38.278691	4.287222	29.909428996	46.882767
## beta[8,6]	44.859569	3.693646	36.777941715	52.028967
## beta[9,6]	44.177958	3.607561	36.320956602	51.437296
## beta[10,6]	29.234336	4.202400	20.191063565	37.803459
## beta[11,6]	45.344910	3.243098	39.247710483	52.329855
## beta[12,6]	68.955890	4.177474	60.000992617	77.507457
## beta[13,6]	30.437395	4.104944	22.633896225	38.840688
## beta[1,7]	58.260632	4.306509	49.709411964	66.726407
## beta[2,7]	59.296329	4.427612	50.649496907	68.289360
## beta[3,7]	36.290133	4.390128	27.648299375	44.901723

## beta[4,7]	66.587814	4.284334	58.055145300	74.968408
## beta[5,7]	30.550488	4.468446	21.970548559	39.483955
## beta[6,7]	58.785063	4.453932	50.054345089	67.709637
## beta[7,7]	38.232816	4.527592	29.557921255	47.813748
## beta[8,7]	45.012496	3.785056	36.749567232	52.526230
## beta[9,7]	44.386974	3.706472	35.800904827	51.635625
## beta[10,7]	29.210670	4.395346	19.810118586	38.031075
## beta[11,7]	45.643531	3.361235	39.595918578	52.994591
## beta[12,7]	68.881387	4.407658	59.847795152	77.936175
## beta[13,7]	30.427835	4.341382	21.856797765	39.307411
## beta[1,8]	58.268414	4.475594	49.406168860	67.277460
## beta[2,8]	59.289803	4.702609	49.959924066	69.108799
## beta[3,8]	36.292499	4.672751	27.042237016	45.811325
## beta[4,8]	66.587719	4.522559	57.369607330	75.431523
## beta[5,8]	30.536804	4.699414	20.920941442	39.485781
## beta[6,8]	58.820231	4.711010	49.813537225	68.160952
## beta[7,8]	38.252243	4.708654	29.162048851	48.192117
## beta[8,8]	45.170439	3.882223	36.650085483	53.032581
## beta[9,8]	44.573776	3.851951	36.114132518	52.188508
## beta[10,8]	29.236626	4.584843	19.645196332	38.439721
## beta[11,8]	45.918617	3.534732	39.925660440	54.117024
## beta[12,8]	68.925147	4.622607	59.228405231	78.340733
## beta[13,8]	30.458401	4.556214	21.540221068	40.207513
## beta[1,9]	58.227481	4.705658	48.827534270	67.469793
## beta[2,9]	59.300895	4.902999	49.486445285	68.985516
## beta[3,9]	36.300910	4.859348	26.559610569	46.483395
## beta[4,9]	66.595457	4.741732	56.713108738	76.074615
## beta[5,9]	30.514089	4.885376	20.698788360	40.394455
## beta[6,9]	58.815551	4.893609	49.066195290	68.597732
## beta[7,9]	38.219263	4.860930	29.018163419	48.782226
## beta[8,9]	45.337447	4.021454	36.231640253	53.609536
## beta[9,9]	44.811520	3.851604	36.438462750	52.170171
## beta[10,9]	29.238296	4.809784	19.195282646	38.955400
## beta[11,9]	46.232525	3.600083	40.118419854	54.684602
## beta[12,9]	68.932522	4.855149	58.768413473	78.859970
## beta[13,9]	30.448421	4.741623	21.260496585	40.199192
## beta[1,10]	58.247449	4.972730	48.729943726	68.096116
## beta[2,10]	59.271930	5.101482	49.054202063	69.489015
## beta[3,10]	36.288553	5.062371	26.103773020	46.527697
## beta[4,10]	66.613195	5.029024	55.863463490	77.038381
## beta[5,10]	30.529794	5.336355	20.587857122	40.700657
## beta[6,10]	58.820469	5.103717	48.872829775	69.218663
## beta[7,10]	38.203178	5.053553	28.439100941	48.950892
## beta[8,10]	45.505679	4.107839	36.658987968	54.134671
## beta[9,10]	45.013589	3.916728	36.868484116	52.509908
## beta[10,10]	29.230443	4.927455	18.593361748	38.984593
## beta[11,10]	46.492548	3.656128	40.393340003	55.071663
## beta[12,10]	68.973765	5.087724	58.200282340	79.389829
## beta[13,10]	30.435523	4.949181	20.576660287	40.537013
## beta[1,11]	58.266623	5.150298	47.914787416	68.444945
## beta[2,11]	59.261359	5.348439	48.660363549	70.413664
## beta[3,11]	36.290370	5.306424	25.568221934	46.688181

## beta[4,11]	66.608036	5.238743	55.543123369	76.979618
## beta[5,11]	30.567121	5.348584	20.332275024	40.832868
## beta[6,11]	58.783717	5.348421	48.383420142	69.777938
## beta[7,11]	38.163950	5.306011	27.768373582	49.661997
## beta[8,11]	45.631587	4.169952	36.246264533	54.293494
## beta[9,11]	45.259294	3.992133	37.023478633	53.159288
## beta[10,11]	29.197206	5.155472	18.512118346	39.628136
## beta[11,11]	46.792953	3.766233	40.804479041	55.857930
## beta[12,11]	68.962130	5.267353	57.936207098	80.052020
## beta[13,11]	30.415973	5.158791	20.418780079	40.832706
## beta[1,12]	58.302510	5.318072	47.413902113	68.966592
## beta[2,12]	59.254457	5.585898	48.007494430	70.723173
## beta[3,12]	36.339969	5.544313	25.012205351	47.060490
## beta[4,12]	66.627207	5.403722	55.412321982	77.221734
## beta[5,12]	30.557454	5.570089	19.567388996	41.269077
## beta[6,12]	58.769380	5.549584	47.746141566	70.927868
## beta[7,12]	38.166790	5.430003	27.333278172	49.773512
## beta[8,12]	45.783401	4.210739	36.437352864	54.277444
## beta[9,12]	45.508003	4.035619	37.326282803	53.433867
## beta[10,12]	29.166221	5.344817	18.173942715	39.838540
## beta[11,12]	47.065534	3.793021	41.070979469	56.422165
## beta[12,12]	68.972807	5.379676	57.898251724	80.041023
## beta[13,12]	30.449992	5.372069	19.841436368	41.483756
## beta[1,13]	58.262313	5.470788	46.995975079	69.220810
## beta[2,13]	59.234750	5.773217	47.389997901	71.195490
## beta[3,13]	36.337262	5.687907	24.650370831	47.574810
## beta[4,13]	66.681401	5.595423	54.717485504	77.793053
## beta[5,13]	30.571042	5.675612	18.524365707	41.905032
## beta[6,13]	58.766203	5.708592	47.507514407	70.808711
## beta[7,13]	38.154545	5.642853	26.812866801	50.231639
## beta[8,13]	45.946299	4.255413	36.752409829	54.615902
## beta[9,13]	45.740808	4.017914	37.658940787	53.461500
## beta[10,13]	29.180726	5.505926	17.687237104	40.514079
## beta[11,13]	47.335563	3.862901	41.262953767	56.209493
## beta[12,13]	68.959719	5.617269	57.312723831	80.479439
## beta[13,13]	30.421329	5.540451	19.387162344	41.702633
## beta[1,14]	58.301873	5.622821	46.772466916	69.775547
## beta[2,14]	59.256256	5.875500	47.116404303	71.167237
## beta[3,14]	36.321832	5.837515	24.440119288	47.524982
## beta[4,14]	66.664215	5.733737	54.466354809	78.439103
## beta[5,14]	30.584811	5.892816	18.677108888	42.366872
## beta[6,14]	58.786263	5.882636	46.692209525	71.088970
## beta[7,14]	38.155730	5.745581	26.623484389	50.353594
## beta[8,14]	46.065682	4.284096	36.994539345	55.070331
## beta[9,14]	45.967892	4.063710	38.607971485	53.715757
## beta[10,14]	29.178485	5.697925	17.281554521	41.122449
## beta[11,14]	47.624024	3.934260	41.512498814	56.677932
## beta[12,14]	68.946208	5.777823	56.821399985	80.522566
## beta[13,14]	30.397295	5.665580	19.325979259	41.994068
## beta[1,15]	58.338386	5.770202	46.809363654	70.520053
## beta[2,15]	59.267685	6.008962	46.820573422	71.862300
## beta[3,15]	36.324546	6.065624	24.029639291	48.310091

## beta[4,15]	66.652767	5.893977	53.977288833	78.749912
## beta[5,15]	30.562379	5.967742	17.874243540	42.629890
## beta[6,15]	58.771929	5.988793	46.153458767	71.306672
## beta[7,15]	38.148955	5.947232	26.109563892	50.253487
## beta[8,15]	46.171479	4.323291	36.802827741	55.095160
## beta[9,15]	46.157744	4.125107	38.508443584	54.359149
## beta[10,15]	29.197460	5.843681	17.000430591	41.134019
## beta[11,15]	47.927912	3.905768	41.786060481	56.732997
## beta[12,15]	68.943342	5.970391	56.365425554	80.908491
## beta[13,15]	30.339386	5.851190	18.255294150	42.375917
## beta[1,16]	58.336171	6.020326	45.998982492	71.063162
## beta[2,16]	59.285772	6.080984	47.306352062	72.365226
## beta[3,16]	36.318764	6.268369	23.478254023	48.889813
## beta[4,16]	66.611462	6.068660	53.806954659	79.289792
## beta[5,16]	30.598517	6.381035	17.291937897	43.244396
## beta[6,16]	58.755407	6.164153	46.238480893	71.749485
## beta[7,16]	38.117870	6.125044	26.140244127	50.603916
## beta[8,16]	46.307109	4.334710	37.058711739	55.486353
## beta[9,16]	46.349035	4.112395	38.477622404	54.326619
## beta[10,16]	29.183833	6.025569	15.892613625	41.596688
## beta[11,16]	48.233554	3.989519	41.852107165	56.849217
## beta[12,16]	68.966861	6.125410	56.060643175	81.237433
## beta[13,16]	30.342432	6.034239	18.207775029	42.661679
## beta[1,17]	58.321885	6.179098	45.735560302	71.086734
## beta[2,17]	59.277326	6.286190	46.436573437	72.419428
## beta[3,17]	36.322355	6.468998	22.724240017	49.132843
## beta[4,17]	66.647102	6.272970	53.352299422	78.970055
## beta[5,17]	30.614980	6.461924	16.467270534	43.295403
## beta[6,17]	58.753352	6.341515	45.959687652	72.076108
## beta[7,17]	38.128562	6.295447	25.498031197	50.544361
## beta[8,17]	46.443068	4.308399	37.910732732	55.805522
## beta[9,17]	46.528473	4.091345	38.567649964	54.672241
## beta[10,17]	29.150410	6.280176	15.540721166	42.205867
## beta[11,17]	48.514818	3.997312	42.317656921	56.910574
## beta[12,17]	68.975130	6.289467	55.886608126	81.612148
## beta[13,17]	30.305449	6.211051	17.864343045	43.255705
## beta[1,18]	58.306574	6.348285	45.470567881	70.878594
## beta[2,18]	59.268614	6.502635	45.688482924	72.741568
## beta[3,18]	36.322982	6.643092	22.464560254	49.742693
## beta[4,18]	66.630158	6.401904	52.954368029	79.301547
## beta[5,18]	30.634613	6.599235	16.839208491	43.901412
## beta[6,18]	58.836614	6.502314	45.957680319	72.923843
## beta[7,18]	38.132090	6.409905	25.235019125	50.910532
## beta[8,18]	46.544646	4.300672	38.174085608	55.803514
## beta[9,18]	46.747231	4.170000	39.175698347	55.130454
## beta[10,18]	29.165170	6.495744	15.358244357	42.315736
## beta[11,18]	48.669777	4.250977	42.192984959	58.346601
## beta[12,18]	68.954676	6.433772	55.553014150	81.849917
## beta[13,18]	30.287081	6.379433	17.509148367	43.407955
## beta[1,19]	58.317593	6.482085	45.184238176	71.461042
## beta[2,19]	59.261916	6.664147	45.869593571	73.014204
## beta[3,19]	36.309328	6.756385	22.138946429	49.995614

## beta[4,19]	66.618513	6.610978	52.565389495	79.545391
## beta[5,19]	30.637351	6.769377	16.736999562	43.598655
## beta[6,19]	58.899817	6.710655	45.392925886	73.139528
## beta[7,19]	38.147194	6.531765	25.102000748	50.886627
## beta[8,19]	46.643279	4.303104	38.414612893	56.288110
## beta[9,19]	46.945444	4.146384	39.448491536	55.268373
## beta[10,19]	29.171655	6.624807	14.916578822	42.486671
## beta[11,19]	48.807238	4.395896	42.300357935	59.218277
## beta[12,19]	68.914446	6.623577	54.334482328	81.901753
## beta[13,19]	30.258996	6.527071	17.231882583	43.309585
## beta[1,20]	58.384975	6.629628	45.184399152	72.558088
## beta[2,20]	59.262215	6.814608	45.546585146	73.152117
## beta[3,20]	36.329758	6.939561	21.656203568	50.302140
## beta[4,20]	66.599803	6.796110	52.160772009	80.203364
## beta[5,20]	30.621707	6.867378	16.535627750	44.618199
## beta[6,20]	58.843324	6.814834	45.077492033	73.329814
## beta[7,20]	38.150904	6.613662	24.365897975	51.139155
## beta[8,20]	46.801834	4.223229	38.828148397	55.990622
## beta[9,20]	47.139525	4.155936	39.719027087	55.747196
## beta[10,20]	29.165116	6.708796	14.798461824	42.528742
## beta[11,20]	48.953740	4.567724	42.088507479	59.896537
## beta[12,20]	68.932992	6.755592	54.376634353	82.577678
## beta[13,20]	30.239741	6.746749	16.863660637	43.761707
## beta[1,21]	58.333028	6.823553	44.745900753	73.257284
## beta[2,21]	59.277259	6.900809	44.862863959	73.239429
## beta[3,21]	36.338224	7.106053	21.382581325	50.741968
## beta[4,21]	66.618064	6.913746	52.232482411	80.653652
## beta[5,21]	30.622130	6.982153	16.631665016	44.548853
## beta[6,21]	58.787358	7.062388	44.782995340	73.652071
## beta[7,21]	38.123990	6.832153	24.532088954	51.516649
## beta[8,21]	46.948894	4.219319	38.977576236	55.886760
## beta[9,21]	47.362618	4.201026	39.939018308	56.273426
## beta[10,21]	29.154332	6.881683	14.575735504	43.223389
## beta[11,21]	49.027885	4.711607	41.857931032	60.606829
## beta[12,21]	68.911713	6.893027	54.036193279	82.982557
## beta[13,21]	30.173366	7.019513	16.517150409	44.207430
## beta[1,22]	58.379061	6.911633	44.840159036	73.412273
## beta[2,22]	59.301829	7.086533	44.849758762	74.169925
## beta[3,22]	36.334380	7.252386	20.908706664	50.910205
## beta[4,22]	66.658360	7.109628	52.255798392	81.190738
## beta[5,22]	30.669940	6.985330	16.059923887	44.707825
## beta[6,22]	58.778950	7.239992	44.404180745	74.098098
## beta[7,22]	38.142335	7.000180	24.102572084	51.911347
## beta[8,22]	47.111675	4.149624	39.196836491	55.741875
## beta[9,22]	47.603159	4.265260	40.614446872	56.875426
## beta[10,22]	29.132797	7.100869	14.316880108	43.495748
## beta[11,22]	49.128431	4.882646	41.976107304	61.152214
## beta[12,22]	68.932881	7.068263	54.207193517	82.832962
## beta[13,22]	30.166614	7.216423	15.671189128	44.467775
## beta[1,23]	58.351089	7.051408	44.868240496	74.288377
## beta[2,23]	59.288893	7.202677	44.676707287	74.032352
## beta[3,23]	36.310744	7.370730	21.193100408	50.997657

## beta[4,23]	66.711161	7.246430	51.889429279	81.337818
## beta[5,23]	30.641085	7.261301	15.413448042	44.719305
## beta[6,23]	58.799939	7.281320	44.031787435	74.778473
## beta[7,23]	38.098832	7.213834	23.954320030	52.044849
## beta[8,23]	47.307544	4.132758	39.502083029	56.313161
## beta[9,23]	47.790716	4.298330	40.322991956	56.888825
## beta[10,23]	29.119000	7.159204	13.544456447	43.218340
## beta[11,23]	49.212014	5.017169	41.892056284	61.738726
## beta[12,23]	68.935403	7.174223	54.031189201	83.249761
## beta[13,23]	30.115630	7.334874	14.515217728	44.475752
## beta[1,24]	58.336031	7.198778	43.923917841	73.272731
## beta[2,24]	59.270694	7.295185	44.419761293	74.227630
## beta[3,24]	36.314991	7.440503	20.997475824	51.450585
## beta[4,24]	66.708827	7.373735	51.476692114	81.649857
## beta[5,24]	30.685977	7.431120	15.430687298	45.368144
## beta[6,24]	58.812885	7.442241	43.492548274	75.069349
## beta[7,24]	38.055070	7.427031	23.139100328	52.620093
## beta[8,24]	47.527704	4.090125	39.811023449	56.178769
## beta[9,24]	47.954563	4.295914	40.438208619	57.221168
## beta[10,24]	29.142509	7.290425	13.803735115	43.611149
## beta[11,24]	49.245636	5.132736	41.545158973	62.085743
## beta[12,24]	68.990411	7.294981	54.268737105	83.550418
## beta[13,24]	30.140435	7.430950	14.293596367	44.936380
## beta[1,25]	58.359116	7.329318	43.605945472	74.203684
## beta[2,25]	59.271064	7.500391	44.006202915	74.728816
## beta[3,25]	36.334136	7.633326	20.440892035	51.878908
## beta[4,25]	66.700430	7.444504	50.970011594	81.687284
## beta[5,25]	30.689227	7.655568	15.219242366	46.221770
## beta[6,25]	58.813084	7.574588	43.722227739	74.929812
## beta[7,25]	38.080828	7.470243	22.681492650	52.842327
## beta[8,25]	47.713083	3.919387	40.433527135	56.775155
## beta[9,25]	48.079428	4.352540	40.457255275	57.418738
## beta[10,25]	29.130557	7.429949	13.506115896	44.280967
## beta[11,25]	49.290853	5.243420	41.667173561	62.025878
## beta[12,25]	68.966794	7.383241	53.811914763	84.126075
## beta[13,25]	30.160118	7.595330	14.092971878	45.275305
## beta[1,26]	58.360611	7.479748	43.205607838	74.028565
## beta[2,26]	59.232147	7.595289	43.765551891	74.685200
## beta[3,26]	36.314586	7.797782	20.101718783	52.162711
## beta[4,26]	66.685476	7.643953	50.722188456	82.136138
## beta[5,26]	30.680942	7.977261	14.898505814	46.489211
## beta[6,26]	58.787844	7.684557	43.414280882	75.071718
## beta[7,26]	38.114291	7.589911	22.239582752	53.103183
## beta[8,26]	47.895263	3.844534	40.882362570	56.815278
## beta[9,26]	48.260340	4.357672	40.732152402	57.445194
## beta[10,26]	29.124755	7.512582	12.881415916	44.244382
## beta[11,26]	49.408161	5.408229	41.771690631	62.720774
## beta[12,26]	68.948972	7.509845	53.463921133	84.132349
## beta[13,26]	30.133584	7.734837	13.776098867	45.385171
## beta[1,27]	58.352661	7.577134	43.384365724	74.126017
## beta[2,27]	59.213533	7.764217	43.320253250	75.383021
## beta[3,27]	36.327205	7.919219	19.929613001	52.171283

## beta[4,27]	66.702326	7.768731	50.515620001	82.360221
## beta[5,27]	30.664580	8.011198	13.989054365	46.819722
## beta[6,27]	58.781013	7.864430	42.957864910	75.284220
## beta[7,27]	38.154901	7.757022	22.025412803	53.359401
## beta[8,27]	48.059529	3.747971	41.404372109	56.688012
## beta[9,27]	48.381731	4.306679	41.169647913	57.342771
## beta[10,27]	29.120428	7.686989	12.719665265	44.763436
## beta[11,27]	49.461466	5.477838	41.822762660	63.932493
## beta[12,27]	68.968277	7.675911	53.083295671	84.497324
## beta[13,27]	30.109131	7.836403	13.845008502	45.501562
## beta[1,28]	58.388813	7.749344	43.126231736	74.114999
## beta[2,28]	59.221344	7.886189	43.094458205	75.254985
## beta[3,28]	36.325693	8.130292	19.330446330	52.314718
## beta[4,28]	66.704719	7.907875	49.217919743	82.686228
## beta[5,28]	30.647430	7.878266	13.348651640	46.757838
## beta[6,28]	58.770549	7.953038	42.896862188	75.768467
## beta[7,28]	38.176783	7.859607	22.124564158	53.623693
## beta[8,28]	48.180134	3.536938	42.151152631	56.040795
## beta[9,28]	48.543158	4.354064	41.426635951	57.451410
## beta[10,28]	29.076100	7.784649	12.518390421	45.048406
## beta[11,28]	49.538702	5.573821	41.729774462	64.569799
## beta[12,28]	69.016248	7.725038	53.192741102	85.110034
## beta[13,28]	30.112135	7.955354	13.758173225	45.728022
## beta[1,29]	58.388375	7.895937	42.912426552	74.772536
## beta[2,29]	59.209923	8.037368	42.330713311	75.337637
## beta[3,29]	36.329592	8.136441	19.091861127	52.356614
## beta[4,29]	66.693147	8.056034	49.686300795	82.997463
## beta[5,29]	30.661621	8.051739	13.586113546	47.113767
## beta[6,29]	58.786726	8.042698	42.789835667	75.772358
## beta[7,29]	38.121583	8.011942	22.261782318	53.609690
## beta[8,29]	48.374186	3.343004	42.666469128	55.689562
## beta[9,29]	48.675968	4.291979	41.636738451	57.494187
## beta[10,29]	29.076926	7.940232	12.168736672	45.255600
## beta[11,29]	49.619604	5.729946	41.795605221	65.059051
## beta[12,29]	69.006914	7.903579	53.233256734	85.516148
## beta[13,29]	30.113794	8.038976	13.931821996	46.138378
## beta[1,30]	58.405280	7.985358	42.409859745	75.375195
## beta[2,30]	59.190132	8.121986	42.414946881	75.714370
## beta[3,30]	36.324500	8.257989	18.505170231	52.519473
## beta[4,30]	66.717932	8.279229	48.587751944	83.738281
## beta[5,30]	30.665387	8.125884	13.108117849	47.456315
## beta[6,30]	58.809266	8.241909	42.766113711	76.314390
## beta[7,30]	38.136258	8.113950	21.605238611	53.838367
## beta[8,30]	48.374920	3.469840	42.472431181	56.262291
## beta[9,30]	48.858458	4.321865	41.742379798	57.650443
## beta[10,30]	29.068039	8.061060	11.651731872	45.545969
## beta[11,30]	49.676875	5.836066	41.725145417	65.500220
## beta[12,30]	68.984087	8.023210	52.746320250	85.452682
## beta[13,30]	30.099370	8.099412	13.371774534	45.976690
## beta[1,31]	58.405322	8.141417	42.219517033	75.767319
## beta[2,31]	59.202163	8.240867	41.428756071	76.259253
## beta[3,31]	36.330624	8.318322	18.474305923	52.465169

## beta[4,31]	66.767165	8.409721	48.561604539	84.166468
## beta[5,31]	30.715365	8.161749	12.896407598	48.113251
## beta[6,31]	58.770578	8.336039	42.180950777	75.876092
## beta[7,31]	38.141041	8.281447	20.753533164	54.228917
## beta[8,31]	48.352146	3.591761	42.522402831	56.547823
## beta[9,31]	48.978578	4.344048	41.827528315	57.965734
## beta[10,31]	29.087480	8.203606	11.815270435	45.626210
## beta[11,31]	49.766943	5.867911	41.699531137	65.662993
## beta[12,31]	69.001572	8.110464	52.811669334	85.757175
## beta[13,31]	30.043771	8.164211	12.790561647	45.798382
## beta[1,32]	58.397283	8.327742	41.754774571	75.982748
## beta[2,32]	59.246670	8.354888	41.310874283	76.355037
## beta[3,32]	36.379510	8.494378	18.216258228	53.359564
## beta[4,32]	66.717809	8.491812	48.188657130	84.293506
## beta[5,32]	30.691713	8.319938	12.536224390	47.799236
## beta[6,32]	58.749662	8.442745	42.234912385	76.185943
## beta[7,32]	38.171561	8.363757	21.210772986	54.146962
## beta[8,32]	48.337925	3.652381	42.018881707	57.012004
## beta[9,32]	49.197333	4.253861	42.081565198	57.846987
## beta[10,32]	29.101255	8.323924	11.581156230	46.233340
## beta[11,32]	49.760127	5.910531	41.478098841	66.011081
## beta[12,32]	69.002466	8.346700	52.206337790	86.132964
## beta[13,32]	30.057511	8.293582	12.521668364	46.299268
## beta[1,33]	58.363050	8.515607	41.803872189	76.780161
## beta[2,33]	59.259829	8.510742	40.988121226	77.221818
## beta[3,33]	36.357808	8.639713	17.787882918	53.958855
## beta[4,33]	66.702033	8.650870	48.669019007	84.441895
## beta[5,33]	30.690800	8.508055	12.309171787	48.083626
## beta[6,33]	58.738126	8.479283	42.122644404	75.922231
## beta[7,33]	38.188739	8.564912	20.622662113	54.501034
## beta[8,33]	48.336329	3.620403	42.115415592	56.800534
## beta[9,33]	49.385765	4.251209	42.021157236	57.793415
## beta[10,33]	29.129031	8.438221	10.715903408	46.283507
## beta[11,33]	49.782489	5.897437	41.199014102	65.772357
## beta[12,33]	69.015222	8.476621	51.883178649	86.552070
## beta[13,33]	30.056081	8.435342	11.935030707	46.595457
## beta[1,34]	58.337080	8.585368	41.250944281	76.227461
## beta[2,34]	59.240301	8.626346	41.114094045	77.206329
## beta[3,34]	36.337712	8.749139	17.777673824	54.126746
## beta[4,34]	66.733223	8.718471	49.012065320	84.902938
## beta[5,34]	30.696055	8.617647	11.972031662	48.263148
## beta[6,34]	58.690203	8.601654	41.788576846	77.036900
## beta[7,34]	38.152323	8.823461	20.538360018	54.864974
## beta[8,34]	48.327681	3.661307	41.724390849	56.810018
## beta[9,34]	49.386044	4.491684	41.855367443	58.049463
## beta[10,34]	29.116853	8.587442	10.510940846	46.748104
## beta[11,34]	49.753827	5.831538	41.100226162	65.096384
## beta[12,34]	69.021027	8.630769	51.688666137	87.282493
## beta[13,34]	30.038949	8.533841	11.542266387	46.922635
## beta[1,35]	58.301167	8.676634	41.391949657	76.495223
## beta[2,35]	59.239932	8.936085	40.114102584	77.601488
## beta[3,35]	36.375970	8.968403	17.820692170	54.768484

## beta[4,35]	66.707187	8.772537	49.030541861	85.027266
## beta[5,35]	30.738280	8.669593	12.272459321	48.876473
## beta[6,35]	58.691741	8.685950	41.421417352	77.411867
## beta[7,35]	38.179575	8.954459	20.402896140	55.122419
## beta[8,35]	48.326164	3.592566	42.098410192	56.683027
## beta[9,35]	49.385504	4.656056	41.464022839	58.689330
## beta[10,35]	29.089098	8.695084	10.685081980	46.845660
## beta[11,35]	49.734665	5.763414	41.148496195	64.073444
## beta[12,35]	68.956366	8.779555	50.967050708	87.263363
## beta[13,35]	29.994411	8.704314	11.022609108	47.259752
## beta[1,36]	58.322351	8.786967	40.933249212	76.883964
## beta[2,36]	59.270942	9.032085	39.860439848	78.529761
## beta[3,36]	36.362008	9.088661	17.601532312	55.174064
## beta[4,36]	66.752500	8.920629	48.845998864	85.994887
## beta[5,36]	30.764507	8.803632	12.629435694	49.131105
## beta[6,36]	58.702858	8.836718	40.971385361	77.716841
## beta[7,36]	38.163203	9.002164	20.205639274	55.511474
## beta[8,36]	48.321837	3.524404	42.052537616	56.383302
## beta[9,36]	49.399843	4.873875	41.347955702	59.500296
## beta[10,36]	29.047275	8.894310	10.064662584	46.861783
## beta[11,36]	49.756600	5.642103	41.228426815	64.068104
## beta[12,36]	68.951891	8.874267	51.020552750	87.590803
## beta[13,36]	30.002838	8.850375	11.200756574	47.942744
## beta[1,37]	58.341108	8.840590	41.264843599	77.051938
## beta[2,37]	59.287404	9.157310	40.322821289	78.633457
## beta[3,37]	36.366248	9.252908	17.627972176	55.296685
## beta[4,37]	66.757056	9.043156	47.926249689	85.193607
## beta[5,37]	30.773170	8.844649	11.760803474	49.231633
## beta[6,37]	58.704776	8.950223	41.133307694	78.306443
## beta[7,37]	38.149012	9.195791	20.160715882	56.041741
## beta[8,37]	48.335032	3.320553	42.530093976	56.179212
## beta[9,37]	49.415194	5.086141	40.750152182	60.281467
## beta[10,37]	29.055831	9.028082	10.065773373	47.103764
## beta[11,37]	49.736746	5.492626	41.056461263	64.073448
## beta[12,37]	68.953397	8.939985	51.127304349	87.830202
## beta[13,37]	30.016123	8.999085	10.699783852	48.176534
## beta[1,38]	58.340468	8.915533	40.180364778	77.327484
## beta[2,38]	59.286290	9.297610	40.067979567	78.734585
## beta[3,38]	36.368730	9.367791	17.491857229	55.832696
## beta[4,38]	66.788116	9.225656	47.588104682	85.396471
## beta[5,38]	30.767453	8.905424	11.422559599	49.843618
## beta[6,38]	58.720409	9.082314	41.162307046	78.581337
## beta[7,38]	38.172983	9.242998	19.182914386	56.549156
## beta[8,38]	48.387789	3.498412	42.168388358	56.413607
## beta[9,38]	49.444886	5.306420	40.800231582	60.749636
## beta[10,38]	29.035435	9.142563	9.418918387	47.089320
## beta[11,38]	49.715431	5.352785	41.035532568	63.293734
## beta[12,38]	68.920898	9.070841	50.354393036	88.211045
## beta[13,38]	30.038596	9.063264	10.567711050	48.120986
## beta[1,39]	58.359120	9.055597	40.087699119	77.355351
## beta[2,39]	59.284449	9.469253	39.502874714	78.399574
## beta[3,39]	36.408078	9.519165	16.818198437	56.536652

## beta[4,39]	66.776303	9.306232	47.168488626	85.992522
## beta[5,39]	30.758422	9.050036	11.085774912	49.653635
## beta[6,39]	58.710203	9.205787	40.576298194	78.587820
## beta[7,39]	38.140829	9.300548	18.894201298	56.998035
## beta[8,39]	48.429538	3.622395	41.858979011	56.709754
## beta[9,39]	49.473816	5.450238	40.357940479	60.951173
## beta[10,39]	28.995772	9.302294	9.130487427	47.417964
## beta[11,39]	49.709755	5.169734	41.182487243	62.277429
## beta[12,39]	68.942451	9.235935	49.783307056	88.434486
## beta[13,39]	30.003333	9.221598	10.393625232	47.771017
## beta[1,40]	58.353360	9.231575	39.431915295	77.871004
## beta[2,40]	59.290483	9.635168	38.411617382	78.866058
## beta[3,40]	36.399430	9.659436	16.597760585	56.977430
## beta[4,40]	66.768518	9.424373	47.229364921	85.992447
## beta[5,40]	30.727088	9.094256	11.412599951	49.889352
## beta[6,40]	58.709054	9.289561	39.995344031	78.472721
## beta[7,40]	38.146629	9.373491	19.164503954	57.627618
## beta[8,40]	48.451595	3.813805	41.368817089	57.200151
## beta[9,40]	49.448852	5.644211	39.983535608	61.891590
## beta[10,40]	28.999952	9.442012	8.618916745	47.525784
## beta[11,40]	49.685392	4.965266	41.061955231	61.435476
## beta[12,40]	68.931455	9.355081	49.594773334	88.582473
## beta[13,40]	30.051289	9.299604	9.718960143	48.051397
## beta[1,41]	58.336727	9.363785	39.071837410	78.805922
## beta[2,41]	59.286191	9.722480	38.720183462	79.558980
## beta[3,41]	36.418498	9.713833	16.493530288	57.493731
## beta[4,41]	66.831560	9.482985	47.615645885	86.931792
## beta[5,41]	30.701429	9.188257	11.048517588	49.874717
## beta[6,41]	58.741832	9.421281	39.671102795	78.852236
## beta[7,41]	38.183375	9.411449	19.164451608	57.790796
## beta[8,41]	48.495216	3.912999	41.193255320	57.663554
## beta[9,41]	49.441621	5.840838	39.995993475	62.198043
## beta[10,41]	29.026590	9.435413	8.788286374	47.850572
## beta[11,41]	49.671192	4.730102	41.188750018	60.665107
## beta[12,41]	68.911594	9.396281	49.524821191	88.890187
## beta[13,41]	30.080213	9.401652	10.507102014	48.092876
## beta[1,42]	58.327299	9.464999	38.494441114	79.135529
## beta[2,42]	59.315197	9.904535	38.758678715	79.756946
## beta[3,42]	36.431269	9.855190	16.702552703	57.422336
## beta[4,42]	66.794619	9.618841	47.270909739	87.279727
## beta[5,42]	30.724200	9.270947	11.697667477	49.581526
## beta[6,42]	58.709875	9.539052	39.072112399	78.369050
## beta[7,42]	38.200600	9.499330	19.127685432	58.811867
## beta[8,42]	48.485498	4.018009	40.977861641	58.217997
## beta[9,42]	49.489778	5.953679	39.464632502	62.525827
## beta[10,42]	28.976214	9.587119	8.456011989	48.057925
## beta[11,42]	49.606954	4.854200	40.985117783	61.233931
## beta[12,42]	68.922454	9.483470	49.213112779	88.231759
## beta[13,42]	30.113020	9.519371	10.527053415	48.956384
## beta[1,43]	58.394157	9.561621	38.319952365	79.931830
## beta[2,43]	59.262977	9.878818	38.906056136	78.935442
## beta[3,43]	36.390810	9.981834	16.160352123	57.869348

## beta[4,43]	66.806487	9.716313	47.240231384	88.402134
## beta[5,43]	30.755100	9.442915	11.307810825	50.556291
## beta[6,43]	58.698953	9.636199	38.785208086	78.747404
## beta[7,43]	38.189758	9.633878	19.518471948	58.635497
## beta[8,43]	48.496378	4.160272	40.617589007	58.535339
## beta[9,43]	49.483359	6.064949	39.055451967	63.309265
## beta[10,43]	28.983426	9.576681	8.159164170	47.874831
## beta[11,43]	49.552887	5.009932	40.541488469	61.582569
## beta[12,43]	68.939115	9.540294	49.246824195	88.995207
## beta[13,43]	30.138320	9.697521	9.535885444	49.358725
## beta[1,44]	58.405969	9.633922	38.387239157	79.697591
## beta[2,44]	59.302714	9.957654	38.498677602	79.710075
## beta[3,44]	36.412800	10.035501	16.343707023	57.240067
## beta[4,44]	66.854301	9.828002	46.902249065	88.266295
## beta[5,44]	30.755484	9.481572	10.800641827	50.965044
## beta[6,44]	58.702254	9.784251	39.029395870	78.735606
## beta[7,44]	38.203245	9.651559	18.652743731	58.949260
## beta[8,44]	48.529961	4.245846	40.282785902	59.027521
## beta[9,44]	49.453420	6.148721	38.741434479	62.775051
## beta[10,44]	29.014147	9.656701	8.626992705	48.055538
## beta[11,44]	49.427021	5.099508	39.999163899	61.011901
## beta[12,44]	68.923515	9.702870	48.915360632	89.206240
## beta[13,44]	30.141496	9.814635	10.204458277	50.130765
## beta[1,45]	58.430856	9.717739	38.972051096	79.758418
## beta[2,45]	59.312852	10.088833	38.256471921	80.287300
## beta[3,45]	36.378715	10.062634	15.659445261	57.202105
## beta[4,45]	66.802716	9.866866	47.024185883	88.505745
## beta[5,45]	30.766169	9.586867	9.965656773	50.708335
## beta[6,45]	58.714490	9.897630	38.570894024	79.643478
## beta[7,45]	38.219739	9.718123	19.029486536	59.264245
## beta[8,45]	48.549159	4.309225	39.720507688	58.072259
## beta[9,45]	49.478769	6.373041	38.560404723	63.430349
## beta[10,45]	29.032992	9.689625	8.090693482	48.261811
## beta[11,45]	49.344680	5.196111	39.642010590	61.275254
## beta[12,45]	68.878893	9.754842	49.325852234	89.337864
## beta[13,45]	30.166113	9.855824	10.166690817	50.701074
## beta[1,46]	58.499507	9.761177	38.409384574	80.097271
## beta[2,46]	59.330301	10.269850	38.493708436	80.978253
## beta[3,46]	36.410325	10.179832	15.619467395	57.070546
## beta[4,46]	66.721869	10.040144	45.963368365	88.335803
## beta[5,46]	30.790977	9.689562	9.849343843	50.858197
## beta[6,46]	58.752577	9.997818	38.282581329	79.618938
## beta[7,46]	38.224216	9.806980	18.222606321	58.963267
## beta[8,46]	48.568829	4.392153	39.555711824	58.544346
## beta[9,46]	49.478814	6.492325	38.766877622	63.394037
## beta[10,46]	29.081176	9.858958	7.923523351	48.403863
## beta[11,46]	49.269274	5.350192	39.254654944	61.349672
## beta[12,46]	68.918785	9.965804	48.085270324	89.964039
## beta[13,46]	30.181177	9.998305	9.735086322	50.057624
## beta[1,47]	58.423416	9.879746	38.576325587	80.338348
## beta[2,47]	59.283561	10.292897	38.303267450	81.235734
## beta[3,47]	36.379006	10.324383	15.081852478	57.482141

## beta[4,47]	66.729498	10.071167	45.610250117	88.876759
## beta[5,47]	30.815304	9.835193	9.599901756	51.673917
## beta[6,47]	58.758131	10.071746	37.866078057	79.835772
## beta[7,47]	38.212291	9.921624	17.729374945	59.259893
## beta[8,47]	48.608688	4.478398	39.193238629	57.986176
## beta[9,47]	49.442414	6.546517	38.139298413	64.214771
## beta[10,47]	29.121687	9.988454	7.296830671	49.486167
## beta[11,47]	49.197262	5.449391	38.616841354	60.913929
## beta[12,47]	68.936028	10.148096	47.907419430	89.631177
## beta[13,47]	30.190164	10.029180	10.412857376	50.130141
## beta[1,48]	58.376473	9.966157	38.088012056	79.725484
## beta[2,48]	59.316804	10.499138	38.339091728	81.578694
## beta[3,48]	36.383144	10.409268	14.380657387	57.154426
## beta[4,48]	66.735541	10.190436	45.536808312	88.417871
## beta[5,48]	30.821549	9.952443	10.212783464	51.782073
## beta[6,48]	58.763379	10.163826	37.563366017	79.826758
## beta[7,48]	38.192056	10.039275	18.057363093	59.112538
## beta[8,48]	48.643146	4.443255	39.692474090	57.852687
## beta[9,48]	49.456994	6.703917	37.905764264	64.284976
## beta[10,48]	29.111836	10.032908	6.897197424	49.585019
## beta[11,48]	49.073080	5.475640	37.974862071	61.387713
## beta[12,48]	68.953508	10.210129	48.101115995	90.397385
## beta[13,48]	30.215184	10.143215	10.135639255	50.678913
## beta[1,49]	58.371748	10.076427	37.610775850	80.675610
## beta[2,49]	59.262730	10.537179	38.194307498	81.923461
## beta[3,49]	36.401477	10.576266	14.374298813	57.507530
## beta[4,49]	66.791020	10.392486	44.781314836	89.294955
## beta[5,49]	30.861386	10.016867	9.679411445	52.220877
## beta[6,49]	58.727007	10.214012	37.561745616	79.754401
## beta[7,49]	38.173544	10.068986	17.952996392	58.910903
## beta[8,49]	48.665191	4.589358	39.611942349	58.550163
## beta[9,49]	49.393515	6.896712	36.959567670	64.542820
## beta[10,49]	29.149170	10.113054	6.766574563	49.558695
## beta[11,49]	48.979910	5.479934	37.953116067	61.139598
## beta[12,49]	68.975411	10.222189	47.854051187	90.485636
## beta[13,49]	30.183653	10.215848	9.427334993	50.598959
## beta[1,50]	58.359633	10.173904	37.726434046	80.278073
## beta[2,50]	59.254114	10.660825	37.828848998	81.586989
## beta[3,50]	36.373283	10.709186	14.387938771	57.587661
## beta[4,50]	66.780769	10.471904	44.737988893	89.022409
## beta[5,50]	30.846789	10.105321	9.152477380	52.042250
## beta[6,50]	58.759141	10.279610	37.756613081	80.158068
## beta[7,50]	38.121702	10.177687	17.780112791	59.767881
## beta[8,50]	48.679945	4.567504	39.497142616	57.927683
## beta[9,50]	49.385363	6.986503	36.891909846	64.707378
## beta[10,50]	29.142695	10.250042	6.658297429	49.806549
## beta[11,50]	48.894773	5.482486	37.388418318	60.792996
## beta[12,50]	68.939555	10.367675	47.644376610	91.177732
## beta[13,50]	30.187666	10.255948	8.777585890	50.407425
## beta[1,51]	58.360508	10.253196	37.391792101	80.466785
## beta[2,51]	59.214193	10.762827	37.563021517	81.792932
## beta[3,51]	36.372563	10.788348	14.107608358	57.888447

## beta[4,51]	66.832387	10.530477	44.684969452	89.726578
## beta[5,51]	30.842377	10.285400	9.259517983	52.490213
## beta[6,51]	58.771224	10.342302	37.582263220	80.131550
## beta[7,51]	38.124566	10.325040	17.343901195	59.462827
## beta[8,51]	48.666946	4.547974	39.576742231	58.176938
## beta[9,51]	49.403413	7.142299	36.147592232	65.078941
## beta[10,51]	29.173840	10.417500	6.709338557	50.130286
## beta[11,51]	48.796643	5.539586	37.460855979	60.910156
## beta[12,51]	68.933561	10.493311	47.371023231	91.104252
## beta[13,51]	30.177794	10.336718	8.327671177	50.996954
## beta[1,52]	58.358538	10.238379	37.396494319	80.939294
## beta[2,52]	59.234402	10.865255	37.556279355	82.498279
## beta[3,52]	36.379327	10.957520	13.548887188	57.988779
## beta[4,52]	66.896477	10.540694	45.137626078	89.427730
## beta[5,52]	30.812262	10.383023	8.924519973	53.395845
## beta[6,52]	58.737873	10.494347	36.513691614	80.817242
## beta[7,52]	38.129867	10.375952	17.131317373	60.727658
## beta[8,52]	48.648500	4.516445	39.721257965	57.914559
## beta[9,52]	49.408848	7.220367	36.147935818	64.860569
## beta[10,52]	29.167035	10.390763	6.073941805	50.503183
## beta[11,52]	48.734515	5.636700	37.446339036	60.993169
## beta[12,52]	68.880616	10.535383	47.012084178	90.937786
## beta[13,52]	30.193530	10.389880	8.003942688	51.143873
## beta[1,53]	58.341440	10.260756	36.870105757	80.802492
## beta[2,53]	59.218473	10.946704	38.010436406	81.834745
## beta[3,53]	36.386089	11.063236	13.740314043	59.118357
## beta[4,53]	66.874743	10.623580	45.084557438	88.915980
## beta[5,53]	30.807665	10.542474	9.332499709	53.797092
## beta[6,53]	58.789201	10.545334	36.597044566	80.953247
## beta[7,53]	38.128572	10.490000	16.936744619	60.634267
## beta[8,53]	48.564014	4.514155	39.632684987	58.633418
## beta[9,53]	49.402590	7.391540	35.937221252	65.269496
## beta[10,53]	29.157230	10.403835	6.254603198	50.162928
## beta[11,53]	48.644189	5.629157	36.752845703	61.395883
## beta[12,53]	68.867769	10.657177	46.960059281	91.292683
## beta[13,53]	30.235656	10.580251	7.845516186	51.964238
## beta[1,54]	58.351241	10.423225	37.282258057	81.733496
## beta[2,54]	59.222953	11.046093	37.419624873	82.533546
## beta[3,54]	36.414459	11.100828	13.226809567	59.347321
## beta[4,54]	66.900985	10.686572	45.026811314	89.628045
## beta[5,54]	30.773649	10.644238	8.799265359	54.224195
## beta[6,54]	58.755876	10.612440	36.207909358	81.116149
## beta[7,54]	38.121836	10.591404	16.351813840	60.820745
## beta[8,54]	48.564066	4.464247	39.684957586	58.439100
## beta[9,54]	49.432602	7.581063	35.819965337	66.004459
## beta[10,54]	29.159083	10.544954	6.541942937	51.352615
## beta[11,54]	48.582843	5.714138	36.756121162	61.381583
## beta[12,54]	68.880772	10.740436	46.541235569	91.511470
## beta[13,54]	30.232632	10.707013	7.507528645	51.545120
## beta[1,55]	58.343371	10.547482	36.562238189	81.479082
## beta[2,55]	59.229158	11.097615	36.900865752	82.705022
## beta[3,55]	36.365920	11.176318	12.864779686	59.466620

## beta[4,55]	66.917908	10.803984	45.010219945	89.941779
## beta[5,55]	30.780107	10.864908	7.786811227	53.919234
## beta[6,55]	58.805079	10.744315	36.586321595	81.370895
## beta[7,55]	38.134680	10.726170	16.027044153	61.503235
## beta[8,55]	48.510220	4.407804	39.514798471	57.916517
## beta[9,55]	49.460040	7.780201	35.172964231	66.315828
## beta[10,55]	29.164294	10.530930	6.561905336	50.806524
## beta[11,55]	48.556084	5.763693	36.819071593	61.521158
## beta[12,55]	68.863385	10.851512	46.136422023	91.602013
## beta[13,55]	30.232908	10.832946	7.900152233	51.800465
## beta[1,56]	58.371922	10.665882	36.116386784	81.913443
## beta[2,56]	59.267522	11.182612	37.492784085	83.161382
## beta[3,56]	36.382676	11.345059	12.963487104	60.321347
## beta[4,56]	66.910817	10.900907	44.729870439	90.445138
## beta[5,56]	30.793502	10.982388	7.895552844	53.916957
## beta[6,56]	58.835567	10.851720	36.431103803	81.554842
## beta[7,56]	38.163235	10.752045	16.026432348	60.646408
## beta[8,56]	48.513139	4.419162	39.451658984	57.990484
## beta[9,56]	49.452286	7.901976	34.663283088	66.420171
## beta[10,56]	29.169069	10.648283	6.478439505	51.585557
## beta[11,56]	48.484691	5.854181	36.702473733	61.666074
## beta[12,56]	68.844701	10.991723	46.272332667	91.873197
## beta[13,56]	30.267814	10.897896	7.626706282	52.009110
## beta[1,57]	58.356785	10.820326	36.309412844	82.617113
## beta[2,57]	59.218785	11.207454	37.275134821	82.897199
## beta[3,57]	36.366838	11.413242	13.057425364	60.329897
## beta[4,57]	66.934335	10.901233	44.588549850	90.700158
## beta[5,57]	30.808424	11.120891	7.637684719	54.254930
## beta[6,57]	58.846877	10.993248	36.490434651	81.787732
## beta[7,57]	38.143969	10.809271	16.376183478	61.043716
## beta[8,57]	48.465420	4.422176	39.367811830	57.935443
## beta[9,57]	49.453209	8.008460	33.884218092	66.651680
## beta[10,57]	29.194361	10.776201	6.490554527	51.299729
## beta[11,57]	48.381313	5.945537	36.673099218	61.690526
## beta[12,57]	68.849023	11.130463	45.255149913	92.088191
## beta[13,57]	30.285477	10.956350	8.390324596	52.622708
## beta[1,58]	58.329161	11.000672	35.470286955	82.345793
## beta[2,58]	59.206358	11.190222	37.808780208	83.013399
## beta[3,58]	36.380671	11.531063	12.910055425	60.636513
## beta[4,58]	66.931938	10.959597	43.796551416	90.622406
## beta[5,58]	30.786600	11.258816	7.020274303	54.419597
## beta[6,58]	58.828922	11.026541	35.863443852	81.851471
## beta[7,58]	38.137641	10.875510	15.960975008	61.244778
## beta[8,58]	48.393890	4.464992	39.102932054	58.065218
## beta[9,58]	49.421673	8.139247	34.164584881	66.743577
## beta[10,58]	29.142205	10.914206	6.707854288	51.584996
## beta[11,58]	48.268719	6.069698	36.436398721	62.264508
## beta[12,58]	68.841191	11.217644	45.055445507	92.216677
## beta[13,58]	30.263699	11.063687	8.004322381	52.672884
## beta[1,59]	58.367483	11.096307	35.187186501	83.697803
## beta[2,59]	59.208403	11.255258	37.442798539	82.954968
## beta[3,59]	36.398560	11.657224	13.115135881	60.861212

## beta[4,59]	66.896607	11.024604	44.344113119	90.822548
## beta[5,59]	30.822151	11.357791	7.259622816	54.458574
## beta[6,59]	58.854387	11.030556	36.276026604	82.682631
## beta[7,59]	38.137141	11.039464	15.874004305	61.027153
## beta[8,59]	48.337983	4.427708	38.844276032	57.868409
## beta[9,59]	49.423118	8.348622	33.655405610	66.933881
## beta[10,59]	29.146749	10.966565	6.162773092	50.860331
## beta[11,59]	48.200853	6.185793	36.399784986	62.582954
## beta[12,59]	68.800269	11.283705	45.041936902	92.512809
## beta[13,59]	30.233704	11.034258	7.467569945	52.322719
## beta[1,60]	58.399385	11.216877	35.299687117	83.912895
## beta[2,60]	59.221091	11.384827	36.293106530	82.595534
## beta[3,60]	36.407102	11.697158	13.333197127	61.079396
## beta[4,60]	66.928589	11.098653	43.383818775	90.714565
## beta[5,60]	30.787482	11.349042	6.824251845	54.764170
## beta[6,60]	58.858079	11.013371	36.868076389	82.592953
## beta[7,60]	38.165060	11.156202	15.786224385	61.782034
## beta[8,60]	48.267115	4.438814	38.898205129	57.504716
## beta[9,60]	49.413152	8.526230	33.105198061	68.323963
## beta[10,60]	29.199060	11.088639	5.441454211	51.715760
## beta[11,60]	48.077798	6.195604	36.350945112	62.273026
## beta[12,60]	68.837330	11.351994	45.059385147	92.416020
## beta[13,60]	30.190974	11.126890	7.239842949	53.175586
## beta[1,61]	58.371852	11.356562	34.701425186	83.659054
## beta[2,61]	59.219874	11.487686	36.450351763	83.746174
## beta[3,61]	36.437960	11.735315	12.703765676	61.793046
## beta[4,61]	66.916942	11.121217	43.104250789	91.228247
## beta[5,61]	30.781223	11.545389	6.576052289	54.329584
## beta[6,61]	58.853556	11.117743	36.304430192	82.952119
## beta[7,61]	38.158854	11.218346	15.518861044	61.261440
## beta[8,61]	48.256132	4.430269	39.239965241	57.361407
## beta[9,61]	49.420281	8.603216	32.650724922	68.153958
## beta[10,61]	29.148674	11.172535	5.309437026	52.322882
## beta[11,61]	47.976439	6.233249	35.826713427	61.475557
## beta[12,61]	68.837590	11.398324	45.624958727	92.615233
## beta[13,61]	30.162911	11.213700	6.334993787	53.166707
## beta[1,62]	58.391343	11.447857	34.461657051	82.773453
## beta[2,62]	59.238652	11.534219	36.204043700	83.918613
## beta[3,62]	36.429648	11.802938	12.458547333	61.602292
## beta[4,62]	66.953771	11.290503	42.995189708	91.811529
## beta[5,62]	30.777061	11.719088	6.643962964	54.700657
## beta[6,62]	58.822960	11.239776	36.133674386	82.527150
## beta[7,62]	38.192201	11.337867	14.988273257	61.409143
## beta[8,62]	48.153490	4.306677	39.115646432	56.658797
## beta[9,62]	49.431759	8.723153	32.894364583	68.274806
## beta[10,62]	29.151681	11.306333	5.169229961	52.452953
## beta[11,62]	47.866026	6.251211	34.834361162	61.797696
## beta[12,62]	68.848084	11.504274	45.165849442	92.573702
## beta[13,62]	30.166750	11.290535	6.122693283	53.399221
## beta[1,63]	58.373729	11.515945	34.079818559	84.043405
## beta[2,63]	59.202479	11.628876	35.727945320	84.208520
## beta[3,63]	36.417399	11.930145	11.885525227	61.881696

## beta[4,63]	66.954686	11.349544	42.698047641	91.513979
## beta[5,63]	30.810003	11.822721	6.933093799	54.825385
## beta[6,63]	58.880541	11.368932	35.940211580	83.389061
## beta[7,63]	38.203453	11.496070	13.864259274	62.037468
## beta[8,63]	48.116468	4.324737	38.586997265	57.050284
## beta[9,63]	49.420086	8.879292	32.314616284	68.501876
## beta[10,63]	29.112195	11.457877	4.775617753	52.606172
## beta[11,63]	47.772666	6.241172	34.831031978	61.623744
## beta[12,63]	68.849640	11.618509	45.129750446	93.230882
## beta[13,63]	30.149187	11.327947	5.711058387	53.599843
## beta[1,64]	58.375628	11.520699	33.872698478	83.599130
## beta[2,64]	59.177138	11.763269	35.605706196	84.651303
## beta[3,64]	36.427187	11.994397	12.360076599	61.910626
## beta[4,64]	66.942923	11.447698	42.963776574	91.565736
## beta[5,64]	30.829439	11.696516	6.622654464	55.359942
## beta[6,64]	58.895988	11.467617	35.351837032	84.106282
## beta[7,64]	38.213059	11.631348	13.813603747	62.804926
## beta[8,64]	47.992343	4.335173	38.520324624	56.755129
## beta[9,64]	49.397296	9.073611	32.534713759	68.376852
## beta[10,64]	29.107985	11.607488	4.081686529	52.564719
## beta[11,64]	47.694091	6.249770	34.814495849	61.420876
## beta[12,64]	68.870790	11.735939	44.863761849	93.904641
## beta[13,64]	30.178077	11.407891	5.821665232	53.480482
## beta[1,65]	58.360293	11.615244	33.923621724	83.323372
## beta[2,65]	59.152876	11.851874	35.142731077	85.146911
## beta[3,65]	36.462653	12.092097	11.707490776	61.670257
## beta[4,65]	66.967801	11.554573	42.683096204	91.463034
## beta[5,65]	30.896622	11.819904	6.511232906	55.885813
## beta[6,65]	58.843573	11.538920	34.853962209	84.591943
## beta[7,65]	38.223277	11.735928	14.018747249	62.881761
## beta[8,65]	47.938952	4.317256	38.274920125	57.092983
## beta[9,65]	49.416578	9.197400	32.083009851	68.838766
## beta[10,65]	29.087871	11.675726	4.266908684	52.742621
## beta[11,65]	47.639925	6.274316	34.509333380	60.919568
## beta[12,65]	68.871959	11.789986	45.340007609	94.105321
## beta[13,65]	30.194340	11.510016	5.337938975	54.040600
## beta[1,66]	58.319068	11.740632	33.433443007	83.614582
## beta[2,66]	59.153497	11.874212	34.720834645	84.958936
## beta[3,66]	36.467368	12.177388	11.887423043	62.034647
## beta[4,66]	66.964838	11.623368	43.095558132	91.223000
## beta[5,66]	30.882659	11.820394	6.591294988	55.942858
## beta[6,66]	58.876684	11.578933	35.000832680	84.249143
## beta[7,66]	38.213197	11.823833	14.037734423	63.043339
## beta[8,66]	47.828085	4.290763	38.277693346	56.159101
## beta[9,66]	49.433882	9.350091	31.521365898	69.135348
## beta[10,66]	29.071038	11.765064	3.798024744	52.709295
## beta[11,66]	47.582478	6.293944	34.431942064	61.430073
## beta[12,66]	68.843516	11.845838	44.454908881	94.459279
## beta[13,66]	30.166756	11.654989	5.591366970	54.338490
## beta[1,67]	58.335350	11.863195	33.593979298	83.070067
## beta[2,67]	59.145296	11.935702	35.110508035	85.251156
## beta[3,67]	36.490503	12.214464	11.822771719	62.087071

## beta[4,67]	66.955054	11.710533	43.636065803	91.682543
## beta[5,67]	30.911322	12.170245	5.635220508	56.720341
## beta[6,67]	58.880914	11.657765	34.889448986	84.544273
## beta[7,67]	38.229864	11.899796	14.117395868	63.434659
## beta[8,67]	47.704448	4.283660	38.268340695	55.915032
## beta[9,67]	49.457194	9.423271	31.340991634	69.631722
## beta[10,67]	29.083442	11.892838	3.614520539	52.397398
## beta[11,67]	47.550898	6.239501	33.837221721	60.831068
## beta[12,67]	68.824949	11.981327	43.763197087	95.013018
## beta[13,67]	30.204274	11.704860	4.155919560	54.274206
## beta[1,68]	58.338325	11.984597	32.761158696	83.665478
## beta[2,68]	59.167650	11.970022	34.454614518	86.098268
## beta[3,68]	36.483560	12.314699	11.850445126	62.589779
## beta[4,68]	67.004598	11.854641	42.587606149	92.807817
## beta[5,68]	30.951491	12.311378	5.820253640	57.063566
## beta[6,68]	58.831470	11.750276	34.038952511	84.533575
## beta[7,68]	38.266988	11.959153	13.903926425	63.739515
## beta[8,68]	47.647819	4.303650	38.250003639	56.235636
## beta[9,68]	49.423036	9.534090	31.534536076	69.782768
## beta[10,68]	29.139129	12.069566	2.914690032	53.526170
## beta[11,68]	47.513857	6.232526	33.774650920	60.842811
## beta[12,68]	68.820723	12.079601	43.832960337	94.977532
## beta[13,68]	30.232503	11.772844	4.815861901	54.330568
## beta[1,69]	58.340948	12.141020	32.308162336	83.671443
## beta[2,69]	59.180642	11.972796	34.794681196	84.628579
## beta[3,69]	36.517004	12.364581	11.180227182	63.024401
## beta[4,69]	67.074346	11.904119	42.591502206	93.206374
## beta[5,69]	30.980940	12.468565	5.372143052	57.498884
## beta[6,69]	58.827283	11.835322	34.368102837	84.737354
## beta[7,69]	38.276345	11.954292	13.672977752	64.366874
## beta[8,69]	47.582802	4.374362	37.674961692	56.003573
## beta[9,69]	49.440078	9.673483	31.160361510	70.014294
## beta[10,69]	29.137937	12.065679	3.517033752	53.795796
## beta[11,69]	47.497258	6.176592	34.060121599	60.997473
## beta[12,69]	68.835773	12.113050	43.921416916	94.873925
## beta[13,69]	30.271566	11.860393	5.118499679	54.615339
## beta[1,70]	58.376557	12.113884	32.307987744	84.039839
## beta[2,70]	59.180403	11.994052	34.317278123	84.715138
## beta[3,70]	36.532871	12.493138	10.847719694	63.308818
## beta[4,70]	67.045646	12.039169	42.391243970	93.138444
## beta[5,70]	30.971603	12.793942	4.982994407	57.275895
## beta[6,70]	58.825254	11.849343	34.594148388	84.647890
## beta[7,70]	38.257487	12.069729	13.910830265	64.611359
## beta[8,70]	47.499951	4.363396	37.361483064	56.000985
## beta[9,70]	49.462794	9.778698	30.726174630	70.453000
## beta[10,70]	29.141884	12.062563	3.109836522	53.955494
## beta[11,70]	47.512724	6.143290	34.206399401	61.177899
## beta[12,70]	68.878035	12.193529	44.242526446	95.134336
## beta[13,70]	30.295872	11.995297	4.955058760	55.732865
## beta[1,71]	58.389135	12.171906	32.094280559	84.138663
## beta[2,71]	59.139176	12.034133	34.170543889	84.868332
## beta[3,71]	36.512162	12.617404	10.104695942	63.608130

## beta[4,71]	67.073922	12.111629	42.616533721	93.493815
## beta[5,71]	30.991149	13.092239	4.279579041	57.150367
## beta[6,71]	58.821858	11.867665	34.551054006	84.285714
## beta[7,71]	38.253813	12.207121	12.485185945	64.915939
## beta[8,71]	47.469760	4.376404	37.278858315	55.803421
## beta[9,71]	49.517369	9.920173	30.063638875	71.616799
## beta[10,71]	29.124487	12.126597	3.328032843	54.763195
## beta[11,71]	47.477054	6.149547	34.299553257	61.363167
## beta[12,71]	68.899953	12.265686	43.669368042	94.927925
## beta[13,71]	30.368542	12.080691	4.998923155	56.128048
## beta[1,72]	58.414393	12.298819	31.666610346	84.186927
## beta[2,72]	59.200237	12.180809	34.115466880	85.234820
## beta[3,72]	36.507679	12.752761	9.715142074	64.297358
## beta[4,72]	67.044334	12.133782	41.785651851	93.558911
## beta[5,72]	30.949765	13.209077	4.594940175	56.814419
## beta[6,72]	58.802256	12.001162	34.052073739	84.539552
## beta[7,72]	38.278682	12.312897	13.036511822	64.869772
## beta[8,72]	47.410875	4.399645	37.113256583	55.943761
## beta[9,72]	49.516255	9.991996	30.322847470	72.141849
## beta[10,72]	29.120122	12.193277	3.477082916	54.102608
## beta[11,72]	47.462080	6.109807	34.517458640	61.028475
## beta[12,72]	68.919687	12.334761	43.163687501	95.580325
## beta[13,72]	30.358773	12.203530	4.401692206	55.667976
## beta[1,73]	58.449060	12.372700	30.798546104	85.222698
## beta[2,73]	59.238416	12.271963	34.275660387	85.614054
## beta[3,73]	36.521323	12.887359	9.981926698	63.729871
## beta[4,73]	67.040703	12.202768	41.548395113	93.486045
## beta[5,73]	30.963490	13.312499	4.609267070	56.619057
## beta[6,73]	58.758012	12.023991	32.920769845	84.581795
## beta[7,73]	38.261928	12.384401	12.577460540	64.197204
## beta[8,73]	47.356301	4.342371	37.152438616	55.215584
## beta[9,73]	49.543790	9.998745	30.851432615	71.710668
## beta[10,73]	29.151020	12.195170	3.476246834	54.522922
## beta[11,73]	47.465257	6.057054	34.323694181	60.893485
## beta[12,73]	68.916134	12.465056	42.488397083	95.467160
## beta[13,73]	30.330845	12.258066	4.500789090	55.023851
## beta[1,74]	58.463040	12.501422	30.294223787	84.434830
## beta[2,74]	59.259105	12.357756	34.266725650	85.591715
## beta[3,74]	36.520008	12.942036	9.465940951	64.439258
## beta[4,74]	67.011043	12.250510	42.102533514	93.556858
## beta[5,74]	30.950975	13.247195	3.862107925	56.923023
## beta[6,74]	58.792671	12.099198	32.649687665	84.857500
## beta[7,74]	38.231942	12.470705	12.165510732	64.076279
## beta[8,74]	47.348046	4.417501	37.251180494	55.568351
## beta[9,74]	49.459961	10.060066	30.657190794	71.021424
## beta[10,74]	29.184352	12.281924	3.420815648	54.323875
## beta[11,74]	47.463473	6.039339	34.628183337	61.251493
## beta[12,74]	68.889007	12.601500	42.615350553	95.895221
## beta[13,74]	30.331375	12.363635	4.960443263	55.846072
## beta[1,75]	58.434137	12.638000	30.394348276	84.352801
## beta[2,75]	59.263301	12.399018	33.439190924	85.487078
## beta[3,75]	36.506027	13.024933	9.125295795	64.452571

## beta[4,75]	67.027579	12.294680	41.817997232	94.363861
## beta[5,75]	30.957306	13.526832	3.553272651	57.181566
## beta[6,75]	58.806512	12.187032	32.904725204	85.320917
## beta[7,75]	38.244119	12.578479	11.746589641	63.913766
## beta[8,75]	47.361629	4.387525	37.441233029	55.237735
## beta[9,75]	49.487540	10.020795	29.914458405	71.205841
## beta[10,75]	29.199321	12.284474	2.863219821	55.071708
## beta[11,75]	47.469991	5.968461	34.302432028	61.262171
## beta[12,75]	68.850578	12.730594	41.985190023	96.106345
## beta[13,75]	30.339860	12.389446	4.247127217	56.113875
## beta[1,76]	58.474796	12.772836	30.159055192	85.414651
## beta[2,76]	59.253370	12.504782	32.921076769	86.101379
## beta[3,76]	36.471748	13.134295	8.315161521	64.852105
## beta[4,76]	67.071777	12.361497	41.465248810	93.968322
## beta[5,76]	30.989195	13.577187	2.961328379	58.097902
## beta[6,76]	58.792099	12.284230	32.671086447	86.096819
## beta[7,76]	38.228862	12.593232	11.142634379	65.086725
## beta[8,76]	47.380522	4.366588	37.476536224	55.116488
## beta[9,76]	49.512021	10.181306	29.345030677	71.536225
## beta[10,76]	29.246713	12.329187	3.676782078	54.261305
## beta[11,76]	47.449339	5.942041	34.406261691	61.103637
## beta[12,76]	68.818546	12.787133	42.340259523	96.016407
## beta[13,76]	30.348127	12.484313	4.029957544	56.098686
## beta[1,77]	58.440115	12.839389	30.194062579	84.842276
## beta[2,77]	59.285687	12.612232	33.074322410	85.726805
## beta[3,77]	36.458178	13.145206	9.857862274	65.602520
## beta[4,77]	67.075767	12.439711	41.643106008	93.944982
## beta[5,77]	30.995338	13.668198	2.777438536	58.093984
## beta[6,77]	58.803491	12.300718	32.929037354	85.831217
## beta[7,77]	38.217071	12.671162	11.537155924	65.494304
## beta[8,77]	47.354370	4.361285	37.327727416	55.048040
## beta[9,77]	49.489741	10.318856	28.625434689	71.959150
## beta[10,77]	29.231871	12.392294	3.612038822	54.266606
## beta[11,77]	47.451691	5.989854	34.211588926	61.428655
## beta[12,77]	68.821310	12.801531	42.445271876	97.086950
## beta[13,77]	30.361160	12.497925	3.894510184	56.419447
## beta[1,78]	58.442399	12.945947	30.286695099	84.665727
## beta[2,78]	59.297688	12.637853	33.438602249	86.420344
## beta[3,78]	36.501079	13.260490	8.946310993	65.639211
## beta[4,78]	67.079286	12.470051	41.254533868	94.069775
## beta[5,78]	31.030724	14.130040	2.330889531	58.781413
## beta[6,78]	58.710647	12.355916	32.579096109	85.523627
## beta[7,78]	38.227864	12.742695	10.985115421	66.055869
## beta[8,78]	47.336597	4.387756	37.359393957	55.136432
## beta[9,78]	49.499875	10.496797	29.162125590	72.437681
## beta[10,78]	29.231761	12.487892	2.873191906	54.678556
## beta[11,78]	47.480486	5.902921	34.457122278	61.027930
## beta[12,78]	68.774596	12.927679	41.726635381	97.127861
## beta[13,78]	30.326017	12.526485	3.673811482	56.419497
## beta[1,79]	58.418095	13.078510	30.262801135	84.313174
## beta[2,79]	59.281973	12.710055	32.299275965	86.800214
## beta[3,79]	36.462680	13.365958	8.404600955	66.662058

## beta[4,79]	67.117670	12.594917	41.711344061	94.735824
## beta[5,79]	30.982659	13.717177	1.829955961	58.189314
## beta[6,79]	58.750004	12.537748	32.701258175	85.517729
## beta[7,79]	38.264444	12.820554	10.454754609	65.887888
## beta[8,79]	47.315194	4.401318	37.180134967	55.269292
## beta[9,79]	49.532062	10.542824	29.136100369	72.491871
## beta[10,79]	29.222979	12.502015	3.044710347	54.166543
## beta[11,79]	47.547716	5.872066	34.735857209	60.955238
## beta[12,79]	68.773562	12.962920	41.316164105	96.779547
## beta[13,79]	30.303660	12.574035	3.202266224	56.557982
## beta[1,80]	58.450192	13.202012	31.153967472	85.372306
## beta[2,80]	59.270896	12.739603	32.643423179	86.528572
## beta[3,80]	36.447746	13.374385	8.607800548	65.990425
## beta[4,80]	67.133856	12.682451	41.207539943	95.340824
## beta[5,80]	30.996376	13.982576	1.414040497	59.429336
## beta[6,80]	58.764803	12.611418	31.640037146	86.333946
## beta[7,80]	38.243988	12.896864	10.788548998	66.071397
## beta[8,80]	47.271060	4.452798	37.109150453	55.201199
## beta[9,80]	49.545967	10.597010	28.982333137	72.511281
## beta[10,80]	29.295029	12.514982	2.754774552	54.495338
## beta[11,80]	47.514720	5.725496	34.965498472	60.871616
## beta[12,80]	68.792624	13.032590	41.493209905	96.587079
## beta[13,80]	30.306317	12.608364	3.062780582	56.320468
## beta[1,81]	58.442139	13.340047	30.628266799	85.095463
## beta[2,81]	59.307031	12.922564	32.496577079	86.561729
## beta[3,81]	36.471298	13.445158	8.511642093	66.355875
## beta[4,81]	67.132254	12.744687	40.886507530	94.692614
## beta[5,81]	31.010128	14.031865	1.068535156	58.413731
## beta[6,81]	58.720344	12.717022	31.496459500	86.777700
## beta[7,81]	38.263884	13.018900	10.594698851	66.461683
## beta[8,81]	47.287926	4.482460	37.291124907	55.367806
## beta[9,81]	49.567884	10.651740	28.480436150	72.808832
## beta[10,81]	29.294580	12.680438	2.603868098	55.451300
## beta[11,81]	47.536844	5.599482	35.140365008	59.818959
## beta[12,81]	68.831353	13.101126	41.547322992	97.338746
## beta[13,81]	30.298639	12.665924	2.226043615	57.201053
## beta[1,82]	58.446079	13.313209	30.104044759	85.376755
## beta[2,82]	59.344437	12.957502	32.188058657	86.701779
## beta[3,82]	36.474577	13.599161	8.633338176	66.686279
## beta[4,82]	67.100393	12.843710	41.158613904	94.848676
## beta[5,82]	31.014529	14.226937	2.045232788	59.299384
## beta[6,82]	58.734116	12.831532	31.087698640	87.498141
## beta[7,82]	38.244843	13.046531	10.190872996	66.530521
## beta[8,82]	47.325448	4.442578	37.191517119	55.069577
## beta[9,82]	49.627856	10.755914	28.994929267	74.046594
## beta[10,82]	29.353550	12.726875	1.712609381	55.530820
## beta[11,82]	47.526263	5.448350	35.421864097	58.877770
## beta[12,82]	68.800796	13.138503	41.169607961	97.028130
## beta[13,82]	30.295392	12.743703	1.841897214	56.323586
## beta[1,83]	58.451397	13.309849	29.804767288	85.255314
## beta[2,83]	59.318403	13.002163	31.920984149	87.272427
## beta[3,83]	36.495897	13.632266	8.216115795	67.438267

## beta[4,83]	67.078619	12.860300	40.502382408	95.625360
## beta[5,83]	31.064226	14.614560	1.838264011	59.385550
## beta[6,83]	58.773653	12.910748	31.733657930	86.939983
## beta[7,83]	38.196548	13.170629	9.736291597	67.105815
## beta[8,83]	47.320611	4.421482	37.124546160	54.806194
## beta[9,83]	49.632754	10.920383	28.457661251	74.219645
## beta[10,83]	29.357860	12.809497	1.491561971	56.300331
## beta[11,83]	47.551638	5.370239	35.332850154	59.287343
## beta[12,83]	68.828486	13.217327	41.510878793	96.420626
## beta[13,83]	30.320098	12.809140	1.519127453	56.574302
## beta[1,84]	58.479495	13.369851	30.577309577	85.751641
## beta[2,84]	59.328970	13.099365	31.718523608	86.892667
## beta[3,84]	36.501320	13.733241	8.427217125	67.274308
## beta[4,84]	67.117476	12.955959	40.185280012	96.244169
## beta[5,84]	31.074040	14.692053	2.177909512	59.044887
## beta[6,84]	58.779312	12.969305	31.435276279	87.202697
## beta[7,84]	38.199893	13.253178	9.564712697	66.728874
## beta[8,84]	47.310601	4.383836	36.908493658	54.861786
## beta[9,84]	49.658444	11.016255	28.170202380	74.361974
## beta[10,84]	29.344164	12.910673	1.653413421	56.376747
## beta[11,84]	47.549821	5.276159	35.367699322	59.122216
## beta[12,84]	68.863841	13.243987	41.304902907	96.575641
## beta[13,84]	30.315187	12.912650	1.373321751	57.488981
## beta[1,85]	58.450981	13.446973	30.170860361	85.594329
## beta[2,85]	59.319050	13.257428	32.360578257	87.450596
## beta[3,85]	36.482719	13.740438	7.957199609	67.556561
## beta[4,85]	67.164678	13.013702	40.719959879	96.006662
## beta[5,85]	31.073472	14.555819	1.888448595	59.792780
## beta[6,85]	58.779733	12.999525	31.890488762	87.790030
## beta[7,85]	38.192673	13.453433	9.154076625	66.328917
## beta[8,85]	47.349584	4.353061	36.756278448	54.969211
## beta[9,85]	49.617940	11.169704	27.719251817	74.494411
## beta[10,85]	29.346789	13.061989	1.594366474	56.869497
## beta[11,85]	47.561990	5.121236	35.386786642	58.332326
## beta[12,85]	68.866892	13.303604	41.998672753	97.146323
## beta[13,85]	30.254567	13.016166	1.939608534	58.090779
## beta[1,86]	58.472047	13.515010	29.719558080	86.016975
## beta[2,86]	59.315879	13.331707	31.537552222	87.756812
## beta[3,86]	36.463548	13.888965	7.182969407	68.353288
## beta[4,86]	67.176013	13.109186	40.870965281	95.888008
## beta[5,86]	31.086835	14.560016	1.553743888	60.146822
## beta[6,86]	58.781629	13.069621	31.365959452	87.812980
## beta[7,86]	38.191801	13.444150	8.838913672	66.563783
## beta[8,86]	47.351458	4.361169	36.713539351	54.929296
## beta[9,86]	49.640554	11.280721	27.944028588	74.673949
## beta[10,86]	29.338459	13.094629	1.181589780	56.322007
## beta[11,86]	47.517310	4.959121	35.321525812	57.853291
## beta[12,86]	68.888690	13.292049	41.688129982	97.103812
## beta[13,86]	30.297120	13.069863	1.922848588	58.858301
## beta[1,87]	58.450824	13.591812	30.343039121	86.450893
## beta[2,87]	59.299618	13.449319	30.804740647	88.040584
## beta[3,87]	36.417415	13.934157	6.979624160	68.039295

## beta[4,87]	67.225310	13.221147	40.792196519	96.938613
## beta[5,87]	31.075939	14.709691	1.771144810	59.991173
## beta[6,87]	58.782616	13.157511	31.286870200	87.842434
## beta[7,87]	38.186399	13.533528	8.294664597	66.734514
## beta[8,87]	47.363533	4.321938	36.652024423	54.483616
## beta[9,87]	49.613345	11.365495	27.603298939	74.483422
## beta[10,87]	29.327194	13.180760	1.279026979	56.392973
## beta[11,87]	47.512452	4.821468	35.279910431	56.765063
## beta[12,87]	68.900888	13.388940	41.240044693	96.975178
## beta[13,87]	30.321100	13.177257	1.948253278	58.525529
## beta[1,88]	58.455364	13.657766	29.963649679	87.392755
## beta[2,88]	59.298919	13.499998	31.243585377	88.184607
## beta[3,88]	36.415218	13.969182	7.108364831	67.769917
## beta[4,88]	67.232183	13.254010	40.419797044	97.475984
## beta[5,88]	31.066556	14.521748	1.099408845	60.309538
## beta[6,88]	58.773534	13.312460	30.061747979	88.519206
## beta[7,88]	38.163138	13.630254	8.136026309	67.202803
## beta[8,88]	47.379579	4.337303	36.483659657	54.707998
## beta[9,88]	49.583559	11.499147	26.873136594	74.419207
## beta[10,88]	29.370933	13.289700	0.982845701	56.486055
## beta[11,88]	47.463292	4.672813	35.360682726	56.014810
## beta[12,88]	68.921394	13.439166	41.491560507	97.139183
## beta[13,88]	30.284860	13.267151	2.251015829	58.779831
## beta[1,89]	58.490435	13.734478	29.073572307	86.508526
## beta[2,89]	59.275649	13.684315	30.701411466	88.392121
## beta[3,89]	36.409046	14.010696	6.486718581	67.311473
## beta[4,89]	67.228914	13.344590	40.438214172	97.156912
## beta[5,89]	31.015448	14.512065	1.188575826	60.600105
## beta[6,89]	58.797814	13.427819	30.570428238	88.359945
## beta[7,89]	38.160310	13.717254	8.042544992	66.394773
## beta[8,89]	47.419292	4.331050	36.572815906	54.474855
## beta[9,89]	49.600551	11.655872	26.556160027	74.833353
## beta[10,89]	29.379659	13.334638	0.777739093	56.611651
## beta[11,89]	47.442683	4.527536	35.550487136	55.237011
## beta[12,89]	68.911740	13.533632	41.390790421	97.602479
## beta[13,89]	30.269347	13.395703	2.107129687	58.666754
## beta[1,90]	58.503702	13.828103	28.524807485	87.025442
## beta[2,90]	59.256433	13.797696	30.277394685	88.306656
## beta[3,90]	36.381748	14.195180	5.450641477	67.550584
## beta[4,90]	67.239086	13.452026	39.787298646	97.315084
## beta[5,90]	31.026342	14.588261	0.538517549	60.441714
## beta[6,90]	58.786394	13.496273	30.352589842	88.382582
## beta[7,90]	38.125724	13.842857	7.501782909	66.891963
## beta[8,90]	47.437551	4.368079	36.694080236	54.809547
## beta[9,90]	49.619771	11.739493	25.578959090	74.673209
## beta[10,90]	29.459543	13.452899	1.096111340	57.344588
## beta[11,90]	47.384902	4.289008	35.605570979	54.545550
## beta[12,90]	68.910305	13.601626	41.475952669	97.699939
## beta[13,90]	30.263290	13.504422	2.207613559	58.962556
## beta[1,91]	58.481698	13.814705	28.720929554	87.079811
## beta[2,91]	59.284588	13.872279	29.956252266	88.149971
## beta[3,91]	36.419896	14.316089	5.942044071	68.488022

## beta[4,91]	67.223129	13.492837	40.068851229	97.497651
## beta[5,91]	31.089511	14.773280	0.147549870	61.019326
## beta[6,91]	58.767858	13.612250	29.353352911	88.540321
## beta[7,91]	38.137107	13.894436	7.219636029	67.046431
## beta[8,91]	47.501993	4.273436	37.015406041	54.803213
## beta[9,91]	49.602096	11.804760	25.656752845	75.005149
## beta[10,91]	29.457050	13.526091	-0.431972067	57.249462
## beta[11,91]	47.374163	4.465825	35.372902753	54.704423
## beta[12,91]	68.898816	13.654034	40.756598685	98.653165
## beta[13,91]	30.269491	13.551687	1.570205521	59.404653
## beta[1,92]	58.487072	13.886613	28.144193822	86.565171
## beta[2,92]	59.300380	13.917326	28.815908071	88.213779
## beta[3,92]	36.465951	14.350817	5.866131724	68.794601
## beta[4,92]	67.218516	13.605910	39.361666354	97.947839
## beta[5,92]	31.096583	14.926044	0.790871303	60.589752
## beta[6,92]	58.779828	13.692409	29.352800520	89.323443
## beta[7,92]	38.157595	13.924427	7.072891348	67.230771
## beta[8,92]	47.556838	4.259204	37.261004492	54.799448
## beta[9,92]	49.591745	11.858030	26.315650160	75.688881
## beta[10,92]	29.421657	13.540965	-0.401557400	58.776704
## beta[11,92]	47.371563	4.592290	35.313944451	55.166417
## beta[12,92]	68.904757	13.756313	40.350982444	98.438777
## beta[13,92]	30.255230	13.593713	0.731245872	59.175457
## beta[1,93]	58.505022	13.991163	28.882161558	87.859738
## beta[2,93]	59.331251	13.928071	29.200141303	89.019961
## beta[3,93]	36.487717	14.405551	6.165259593	69.177613
## beta[4,93]	67.254018	13.705624	39.422730850	98.075588
## beta[5,93]	31.083305	14.940060	0.784822563	60.977175
## beta[6,93]	58.813315	13.676760	30.078270812	88.748184
## beta[7,93]	38.159992	13.901255	7.103293309	67.443064
## beta[8,93]	47.629225	4.158204	37.366183868	54.953121
## beta[9,93]	49.551770	12.046538	24.786687215	75.260830
## beta[10,93]	29.404384	13.511006	0.053257895	58.729657
## beta[11,93]	47.329129	4.711456	35.110188947	55.259214
## beta[12,93]	68.863087	13.899593	39.755665925	98.824786
## beta[13,93]	30.238805	13.663890	1.424466846	59.465303
## beta[1,94]	58.467914	14.059569	28.485090541	87.514468
## beta[2,94]	59.345293	14.008480	29.280251184	88.909074
## beta[3,94]	36.461428	14.473528	6.013463359	68.277224
## beta[4,94]	67.299892	13.732530	39.418030383	97.799708
## beta[5,94]	31.132974	14.972969	0.116464872	61.279563
## beta[6,94]	58.779761	13.734265	30.422117162	88.864566
## beta[7,94]	38.186749	14.012908	7.577535614	67.682480
## beta[8,94]	47.732827	3.990566	38.463843577	55.041498
## beta[9,94]	49.519404	12.144891	24.549838750	75.022243
## beta[10,94]	29.371810	13.569355	-0.147888897	58.056595
## beta[11,94]	47.361416	4.808430	35.244023668	55.564992
## beta[12,94]	68.861086	14.014480	39.897491680	99.272416
## beta[13,94]	30.222478	13.698995	0.755021410	59.035740
## beta[1,95]	58.482808	14.175845	28.485783910	87.631161
## beta[2,95]	59.351240	14.132131	28.652784587	89.370345
## beta[3,95]	36.462526	14.512482	5.751421249	67.557621

## beta[4,95]	67.291735	13.835573	38.635527609	98.114346
## beta[5,95]	31.157622	14.798585	0.423109146	61.808388
## beta[6,95]	58.797763	13.760962	30.428505922	88.359082
## beta[7,95]	38.219543	14.111109	7.659775227	68.122932
## beta[8,95]	47.887089	3.881354	39.044230588	54.968705
## beta[9,95]	49.479677	12.289840	24.946801583	75.346613
## beta[10,95]	29.365906	13.664192	-0.213591842	58.101443
## beta[11,95]	47.347834	4.890665	35.075989258	55.801208
## beta[12,95]	68.907631	14.009427	39.882085597	98.965252
## beta[13,95]	30.250757	13.794187	0.966625646	59.564176
## beta[1,96]	58.470147	14.341229	28.007354065	88.063465
## beta[2,96]	59.383379	14.237811	29.223881702	89.960092
## beta[3,96]	36.475569	14.603313	5.961839464	67.878356
## beta[4,96]	67.350706	13.927575	37.466005696	98.781810
## beta[5,96]	31.162449	14.892975	0.126986471	62.037761
## beta[6,96]	58.780786	13.841211	30.246819785	88.557202
## beta[7,96]	38.221740	14.146828	7.631020547	68.156196
## beta[8,96]	47.991230	3.794238	39.499304572	55.092858
## beta[9,96]	49.464369	12.330496	24.615125709	75.509600
## beta[10,96]	29.368463	13.689555	-0.216200567	58.211732
## beta[11,96]	47.339879	4.907446	35.113948413	55.753835
## beta[12,96]	68.898168	14.006979	40.346360446	99.261886
## beta[13,96]	30.230053	13.894176	0.599638460	59.102963
## beta[1,97]	58.422051	14.347631	27.440604529	87.708792
## beta[2,97]	59.398398	14.426555	29.321168830	89.977041
## beta[3,97]	36.505868	14.654644	6.446507279	68.574365
## beta[4,97]	67.429649	13.999151	37.503238721	98.338374
## beta[5,97]	31.144112	14.839137	0.589855152	62.221399
## beta[6,97]	58.761657	13.933559	29.533231026	87.842365
## beta[7,97]	38.225421	14.214132	7.572076666	68.382226
## beta[8,97]	48.076125	3.620083	39.799179406	54.741156
## beta[9,97]	49.474629	12.407843	24.576602727	75.102741
## beta[10,97]	29.372509	13.779314	-0.445664576	58.307552
## beta[11,97]	47.347251	4.941268	34.937497435	55.833838
## beta[12,97]	68.906167	14.087631	39.478769479	99.224588
## beta[13,97]	30.274044	13.866641	-0.427158997	59.006927
## beta[1,98]	58.463782	14.395777	28.194444764	88.065125
## beta[2,98]	59.413152	14.516360	28.938822420	90.159595
## beta[3,98]	36.505273	14.664654	5.487817382	68.163572
## beta[4,98]	67.406780	14.070760	37.725935495	98.268650
## beta[5,98]	31.138782	14.779542	0.261473277	61.977819
## beta[6,98]	58.772726	13.974799	28.728758701	88.659861
## beta[7,98]	38.242399	14.276853	7.442526449	67.969882
## beta[8,98]	48.043589	3.860131	39.149747167	55.251722
## beta[9,98]	49.450973	12.425590	23.712587454	75.378349
## beta[10,98]	29.401973	13.869587	-0.009670088	58.381403
## beta[11,98]	47.395392	4.943761	35.110375281	56.209231
## beta[12,98]	68.932335	14.140465	39.908564843	99.154203
## beta[13,98]	30.268511	13.976989	-0.305729195	59.415597
## beta[1,99]	58.499174	14.510359	28.097884337	89.147595
## beta[2,99]	59.418196	14.563652	29.156445006	90.238250
## beta[3,99]	36.490699	14.699476	5.878880701	69.284636

## beta[4,99]	67.419464	14.146375	37.704625936	98.283254
## beta[5,99]	31.165018	14.708415	0.301018704	61.902421
## beta[6,99]	58.843354	14.035628	29.205962075	88.766827
## beta[7,99]	38.220541	14.355419	7.338553207	69.052431
## beta[8,99]	48.042217	4.127688	38.219562586	55.815799
## beta[9,99]	49.478334	12.547817	23.555054498	75.477932
## beta[10,99]	29.414188	13.925284	-0.218125998	58.993668
## beta[11,99]	47.465187	4.987996	34.776077602	56.146041
## beta[12,99]	68.923579	14.223529	39.798434781	99.758196
## beta[13,99]	30.246880	14.110841	-0.572031303	60.071933
## beta[1,100]	58.534356	14.670160	27.492948858	89.026205
## beta[2,100]	59.460839	14.643741	29.342075834	91.052187
## beta[3,100]	36.435958	14.797745	5.795247637	69.327556
## beta[4,100]	67.454667	14.189188	37.535697856	98.773128
## beta[5,100]	31.148354	14.851306	0.798445748	62.431481
## beta[6,100]	58.827908	14.055433	29.144694590	89.412444
## beta[7,100]	38.208685	14.532615	7.872218580	68.988919
## beta[8,100]	48.073761	4.366229	38.166698215	56.229272
## beta[9,100]	49.466808	12.539796	23.947357982	75.294324
## beta[10,100]	29.371451	14.032972	-0.209541674	59.131644
## beta[11,100]	47.501948	4.987343	34.834993438	56.538364
## beta[12,100]	68.922751	14.295474	39.827583202	99.836571
## beta[13,100]	30.348948	14.185084	-0.816649133	60.599818
## beta[1,101]	58.529822	14.724542	27.827608597	89.306480
## beta[2,101]	59.453514	14.723742	28.725260014	91.317838
## beta[3,101]	36.423404	14.836332	5.729896311	68.904759
## beta[4,101]	67.410830	14.314282	37.623476356	98.391202
## beta[5,101]	31.174707	15.053658	0.462350540	62.434975
## beta[6,101]	58.832297	14.126485	28.907162929	89.093957
## beta[7,101]	38.222217	14.623938	7.644012334	69.202068
## beta[8,101]	48.107596	4.542158	37.428722570	57.044382
## beta[9,101]	49.413702	12.665056	23.668460312	75.561908
## beta[10,101]	29.401776	14.115092	-1.331262971	58.578592
## beta[11,101]	47.561980	5.012752	34.776263139	56.554404
## beta[12,101]	68.901387	14.392575	39.213791642	100.067515
## beta[13,101]	30.360479	14.249116	-0.712555535	60.370076
## beta[1,102]	58.521702	14.913876	27.187243468	89.117030
## beta[2,102]	59.457708	14.812161	28.746759008	91.098138
## beta[3,102]	36.410562	14.844185	5.917651220	69.210490
## beta[4,102]	67.415961	14.337409	37.846790360	98.265397
## beta[5,102]	31.162766	15.211181	0.413221855	62.510389
## beta[6,102]	58.859362	14.100761	28.848892298	90.078939
## beta[7,102]	38.238055	14.680565	6.986384293	69.468690
## beta[8,102]	48.106959	4.724519	37.344831468	57.142700
## beta[9,102]	49.375945	12.804696	23.648705129	75.467155
## beta[10,102]	29.440138	14.205465	-0.855583642	59.038375
## beta[11,102]	47.617317	4.978334	34.971509545	56.656813
## beta[12,102]	68.889719	14.448748	39.662058718	99.865979
## beta[13,102]	30.369069	14.258307	-0.266364472	61.134376
## beta[1,103]	58.517287	15.010239	28.930458941	89.328765
## beta[2,103]	59.486934	14.763821	29.014057882	91.565093
## beta[3,103]	36.422907	14.861680	5.989911688	69.054668

## beta[4,103]	67.423203	14.424688	37.246279328	98.364621
## beta[5,103]	31.144332	15.166507	0.158879528	63.396573
## beta[6,103]	58.851272	14.164084	29.200453354	89.785004
## beta[7,103]	38.313594	14.807774	7.456951065	69.719509
## beta[8,103]	48.158270	4.896836	37.094423327	57.823683
## beta[9,103]	49.388000	12.867716	23.677683052	76.523458
## beta[10,103]	29.425878	14.260382	-1.136299170	59.025516
## beta[11,103]	47.672236	4.986336	34.770470366	56.700244
## beta[12,103]	68.871270	14.537572	38.418041527	100.122615
## beta[13,103]	30.411603	14.306189	-1.139135510	60.704431
## beta[1,104]	58.544594	15.100534	26.909723825	90.038070
## beta[2,104]	59.515983	14.852131	28.966174975	91.372763
## beta[3,104]	36.434451	14.952258	5.650089117	68.880497
## beta[4,104]	67.437531	14.462891	37.345534276	98.581485
## beta[5,104]	31.174100	15.365335	0.163437593	63.238677
## beta[6,104]	58.854444	14.231139	28.978202559	90.022006
## beta[7,104]	38.313777	14.902038	7.353152015	71.417893
## beta[8,104]	48.148655	5.052438	36.621283000	58.037955
## beta[9,104]	49.389890	12.927949	24.090460257	76.376971
## beta[10,104]	29.421340	14.274005	-1.510781794	59.131230
## beta[11,104]	47.744162	4.990501	34.818052656	56.747213
## beta[12,104]	68.878453	14.583227	37.113884658	99.586788
## beta[13,104]	30.373950	14.371021	-0.757236832	61.354466
## beta[1,105]	58.552910	15.130429	27.162968167	91.401876
## beta[2,105]	59.496220	14.948010	28.535133470	92.355080
## beta[3,105]	36.455978	15.047226	5.363157804	68.925238
## beta[4,105]	67.405996	14.530614	37.242263226	98.466239
## beta[5,105]	31.145710	15.594986	-0.460060907	63.726685
## beta[6,105]	58.855607	14.311385	28.212065465	89.021862
## beta[7,105]	38.300764	14.910170	6.918394815	70.386569
## beta[8,105]	48.127553	5.329854	35.713633388	58.377534
## beta[9,105]	49.417462	13.011171	23.405940699	76.120228
## beta[10,105]	29.423359	14.326569	-1.418406720	59.317289
## beta[11,105]	47.799051	5.029693	34.817944439	57.072574
## beta[12,105]	68.872178	14.590462	37.946993600	99.943065
## beta[13,105]	30.388877	14.463342	0.262617665	61.282308
## beta[1,106]	58.570802	15.163902	27.067249277	90.523098
## beta[2,106]	59.518113	14.994128	28.695201146	92.130398
## beta[3,106]	36.460448	15.132319	5.156811011	69.605806
## beta[4,106]	67.435537	14.628972	36.769868924	99.538277
## beta[5,106]	31.171708	15.566392	-0.025628651	63.378785
## beta[6,106]	58.876191	14.396628	28.221159745	90.362644
## beta[7,106]	38.303126	14.996612	6.677285931	70.075741
## beta[8,106]	48.156693	5.603683	35.936097054	58.794730
## beta[9,106]	49.415531	13.073247	23.124763105	76.460347
## beta[10,106]	29.448561	14.408729	-1.463237263	59.269382
## beta[11,106]	47.846381	5.015558	34.818447099	57.062207
## beta[12,106]	68.882775	14.554444	38.077909283	99.770044
## beta[13,106]	30.389892	14.566982	-0.223573344	61.912249
## beta[1,107]	58.552472	15.250208	27.241626457	90.104349
## beta[2,107]	59.526814	15.083464	27.775031205	91.781373
## beta[3,107]	36.460840	15.142161	4.844790553	68.865694

## beta[4,107]	67.422996	14.631275	36.997562514	98.903184
## beta[5,107]	31.192744	15.677314	-0.619976216	63.784903
## beta[6,107]	58.871325	14.442019	28.173915577	91.128825
## beta[7,107]	38.288381	15.009083	6.950172900	71.139246
## beta[8,107]	48.186743	5.741478	35.762459031	59.568449
## beta[9,107]	49.423711	13.141094	23.184197734	76.383584
## beta[10,107]	29.443143	14.496671	-0.869191926	60.155940
## beta[11,107]	47.885350	5.024630	34.886000230	56.963134
## beta[12,107]	68.861550	14.673255	37.733814740	99.641203
## beta[13,107]	30.348519	14.604726	-1.198661825	61.660106
## beta[1,108]	58.544221	15.232621	26.911381778	90.849436
## beta[2,108]	59.538989	15.111997	27.644695957	92.331898
## beta[3,108]	36.441877	15.262197	5.113834547	69.764514
## beta[4,108]	67.432595	14.679484	36.469311440	98.880791
## beta[5,108]	31.195703	15.943643	-1.750979455	63.992782
## beta[6,108]	58.874570	14.441085	28.256350076	90.777014
## beta[7,108]	38.292938	15.101103	6.421582645	71.870207
## beta[8,108]	48.176146	5.895478	35.677776457	59.373725
## beta[9,108]	49.454760	13.165819	22.750586273	76.848945
## beta[10,108]	29.440327	14.565796	-1.485287948	60.396572
## beta[11,108]	47.918438	5.009081	35.156977527	57.158629
## beta[12,108]	68.891212	14.786904	37.807289181	100.019358
## beta[13,108]	30.318894	14.673154	-0.990335556	61.679571
## beta[1,109]	58.578381	15.260406	25.788257857	90.848784
## beta[2,109]	59.485097	15.189719	27.339304653	91.909942
## beta[3,109]	36.447677	15.292005	4.098015234	69.574413
## beta[4,109]	67.426084	14.801718	36.850682105	99.044158
## beta[5,109]	31.204160	15.885864	-1.725583908	64.639333
## beta[6,109]	58.889569	14.458076	28.393632931	90.777817
## beta[7,109]	38.247543	15.130284	6.155371904	71.547239
## beta[8,109]	48.131687	6.046735	35.095250614	59.560235
## beta[9,109]	49.485247	13.176027	22.734438962	77.459559
## beta[10,109]	29.402153	14.615564	-1.914017531	59.461746
## beta[11,109]	47.912860	5.001895	35.426172746	57.025462
## beta[12,109]	68.898512	14.886954	37.652883068	99.654569
## beta[13,109]	30.314435	14.769204	-1.886341589	62.172336
## beta[1,110]	58.574114	15.352939	25.963782478	90.609463
## beta[2,110]	59.534774	15.234252	26.731660150	92.569710
## beta[3,110]	36.457748	15.386269	4.908989639	69.652404
## beta[4,110]	67.440380	14.887063	36.171478342	98.904479
## beta[5,110]	31.222180	15.820166	-1.641929183	63.127376
## beta[6,110]	58.884785	14.579627	28.454970948	91.234244
## beta[7,110]	38.264974	15.241585	6.114061278	71.745790
## beta[8,110]	48.110088	6.204888	34.834746404	59.883539
## beta[9,110]	49.454403	13.256462	22.160783119	76.941123
## beta[10,110]	29.360868	14.675493	-2.299724194	59.530251
## beta[11,110]	47.938202	4.933461	35.368095977	56.810279
## beta[12,110]	68.909035	14.916958	37.107266150	101.050474
## beta[13,110]	30.342436	14.851165	-1.492861174	61.936053
## beta[1,111]	58.564797	15.462214	25.634529326	91.233327
## beta[2,111]	59.515223	15.240979	27.475753312	92.300253
## beta[3,111]	36.383021	15.454878	4.128870601	69.400622

## beta[4,111]	67.456707	14.999565	36.481841054	99.719814
## beta[5,111]	31.205580	15.867948	-1.406752424	64.086389
## beta[6,111]	58.855824	14.655752	27.785173340	90.562325
## beta[7,111]	38.241728	15.349047	6.496787798	71.807272
## beta[8,111]	48.146661	6.314744	34.753354661	59.857590
## beta[9,111]	49.473490	13.343273	22.106760491	77.137463
## beta[10,111]	29.344498	14.784262	-1.870301499	60.299285
## beta[11,111]	48.031646	4.863332	35.506032665	56.791706
## beta[12,111]	68.891588	14.983982	36.596948151	100.596558
## beta[13,111]	30.348096	14.939825	-1.533104954	61.755255
## beta[1,112]	58.532854	15.463239	25.150847947	90.577494
## beta[2,112]	59.553275	15.282585	27.963490818	92.695369
## beta[3,112]	36.371279	15.455754	3.941339189	70.007357
## beta[4,112]	67.424977	15.144009	35.933950358	100.222911
## beta[5,112]	31.181657	15.790971	-1.704779282	63.032831
## beta[6,112]	58.893787	14.724141	28.350776947	91.308534
## beta[7,112]	38.224304	15.448736	6.068094473	72.499084
## beta[8,112]	48.153561	6.450529	35.044014384	60.361278
## beta[9,112]	49.445104	13.443656	22.066419673	77.500670
## beta[10,112]	29.340805	14.821354	-1.805016737	59.783853
## beta[11,112]	48.046158	4.810704	35.410971090	56.877441
## beta[12,112]	68.940182	15.061226	36.627503261	100.815146
## beta[13,112]	30.340149	15.001039	-2.501783385	62.138620
## beta[1,113]	58.512854	15.582554	24.798178375	91.472426
## beta[2,113]	59.573891	15.307105	26.817808832	92.201276
## beta[3,113]	36.355777	15.500318	3.907971815	70.483131
## beta[4,113]	67.447400	15.173743	36.180667980	100.069040
## beta[5,113]	31.145040	15.862183	-1.813990146	63.311154
## beta[6,113]	58.881602	14.801884	27.984599577	91.282050
## beta[7,113]	38.267546	15.570668	6.043142050	72.800092
## beta[8,113]	48.169177	6.586409	34.417454327	60.752258
## beta[9,113]	49.416787	13.472163	21.451649705	77.396961
## beta[10,113]	29.337697	14.842759	-2.201555402	60.306090
## beta[11,113]	48.047547	4.731630	35.519314497	56.775413
## beta[12,113]	68.942640	15.096326	36.587673985	100.506048
## beta[13,113]	30.323328	15.056708	-2.513245288	61.655171
## beta[1,114]	58.480654	15.633596	25.004711826	90.792900
## beta[2,114]	59.563926	15.379890	27.208771989	92.592575
## beta[3,114]	36.394915	15.556948	4.013376121	71.004690
## beta[4,114]	67.452730	15.255744	36.665513183	100.311589
## beta[5,114]	31.171045	15.915308	-2.260333397	64.094180
## beta[6,114]	58.854556	14.861429	28.193465128	91.301716
## beta[7,114]	38.265627	15.716984	5.714380488	72.526767
## beta[8,114]	48.214652	6.815766	33.682449793	62.125410
## beta[9,114]	49.377833	13.589180	21.054750172	77.534027
## beta[10,114]	29.355287	14.929723	-1.528255157	59.873379
## beta[11,114]	48.084391	4.697465	35.690524648	56.631079
## beta[12,114]	68.949386	15.132711	36.527058511	100.720395
## beta[13,114]	30.338341	15.208953	-2.996330970	61.734911
## beta[1,115]	58.461322	15.672772	24.470528695	90.919361
## beta[2,115]	59.569209	15.531058	26.443103930	93.410377
## beta[3,115]	36.398617	15.687246	3.893716597	70.869696

```

## beta[4,115]      67.428692   15.337208  36.012638208 100.386296
## beta[5,115]      31.192249   16.068145 -2.367557346  64.529868
## beta[6,115]      58.890005   14.895428  28.953342686  90.781675
## beta[7,115]      38.261816   15.709511   5.967598825  72.553018
## beta[8,115]      48.258049    6.950195  33.652875577  62.324705
## beta[9,115]      49.404457   13.716344  20.621321785  77.487928
## beta[10,115]     29.338346   14.959042 -1.555941243  60.506202
## beta[11,115]     48.075969    4.590628  35.691189070  56.318053
## beta[12,115]     68.966326   15.217738  37.266791466 101.520492
## beta[13,115]     30.353083   15.265654 -2.345823449  61.604019
## sigma2_beta[1]    1.983584    2.619386   0.067432770   9.811024
## sigma2_beta[2]    2.123004    6.608926   0.068747630   9.154619
## sigma2_beta[3]    1.998874    2.700569   0.067607334   9.785534
## sigma2_beta[4]    1.990037    2.628090   0.069899162   9.827270
## sigma2_beta[5]    2.177180   10.277233   0.068706091   9.399325
## sigma2_beta[6]    2.009647    2.882833   0.070212655   9.610445
## sigma2_beta[7]    2.016162    2.950729   0.070508205   9.488029
## sigma2_beta[8]    1.922471    2.481571   0.074708213   9.502068
## sigma2_beta[9]    1.990683    2.643962   0.062246212   9.629270
## sigma2_beta[10]   2.011775    3.124264   0.069299203   9.668897
## sigma2_beta[11]   1.903645    2.461996   0.079021045   9.060829
## sigma2_beta[12]   2.002740    2.699599   0.069533108   9.788253
## sigma2_beta[13]   1.980313    2.623479   0.070235126   9.407093
## sigma2_y[1]       110.421134  4140.772763  3.630945679   98.700218
## sigma2_y[2]       37.433096   219.283024  3.657232440  100.172850
## sigma2_y[3]       32.606741    32.253440  3.605711682   97.711550
## sigma2_y[4]       32.953798    63.934587  3.779732662   97.786865
## sigma2_y[5]       37.302642   226.543715  3.702411676   98.689607
## sigma2_y[6]       32.556122    31.099612  3.834331138   99.768413
## sigma2_y[7]       43.575421   555.350283  3.686516698  100.398992
## sigma2_y[8]       31.245772    26.743780  3.666795868   94.702051
## sigma2_y[9]       32.297343    28.016058  3.940521011   97.735543
## sigma2_y[10]      32.578443    31.998891  3.694215054   99.966396
## sigma2_y[11]      31.093843    27.015430  3.591506985   95.332058
## sigma2_y[12]      32.599110    38.049204  3.647017778   98.692937
## sigma2_y[13]      35.672709    90.873938  3.742074546  101.658292

## mean Rhat: 1.12697

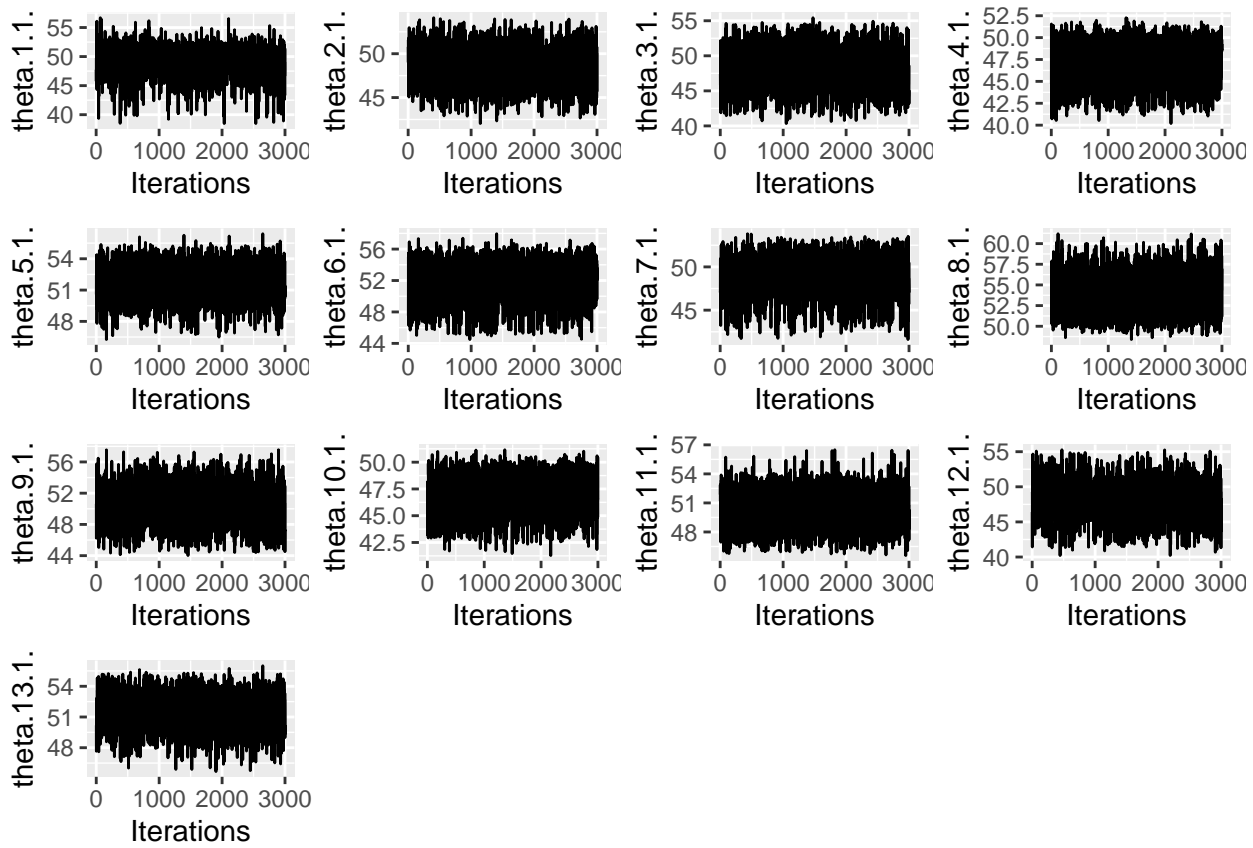
## mean effective sample size: 1698.656

```

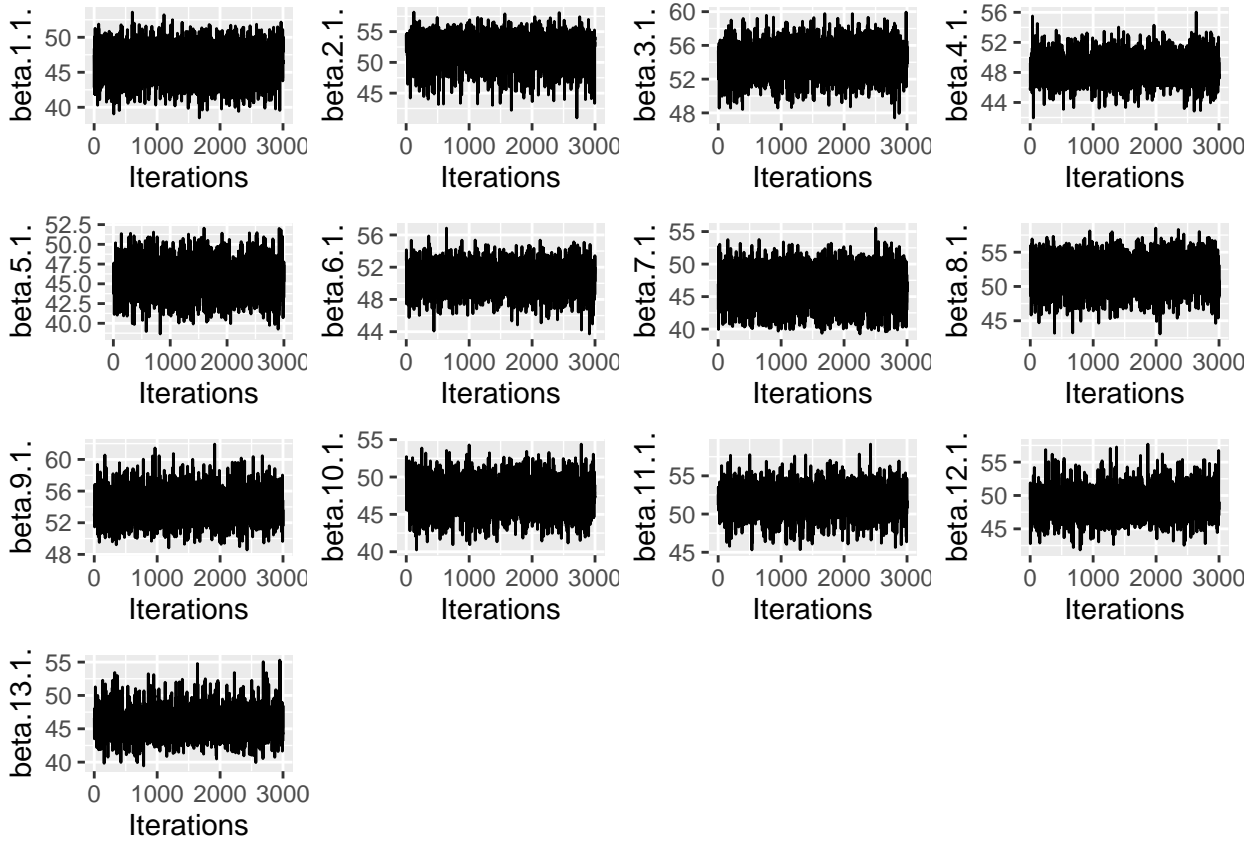
Appendix B

Appendix C

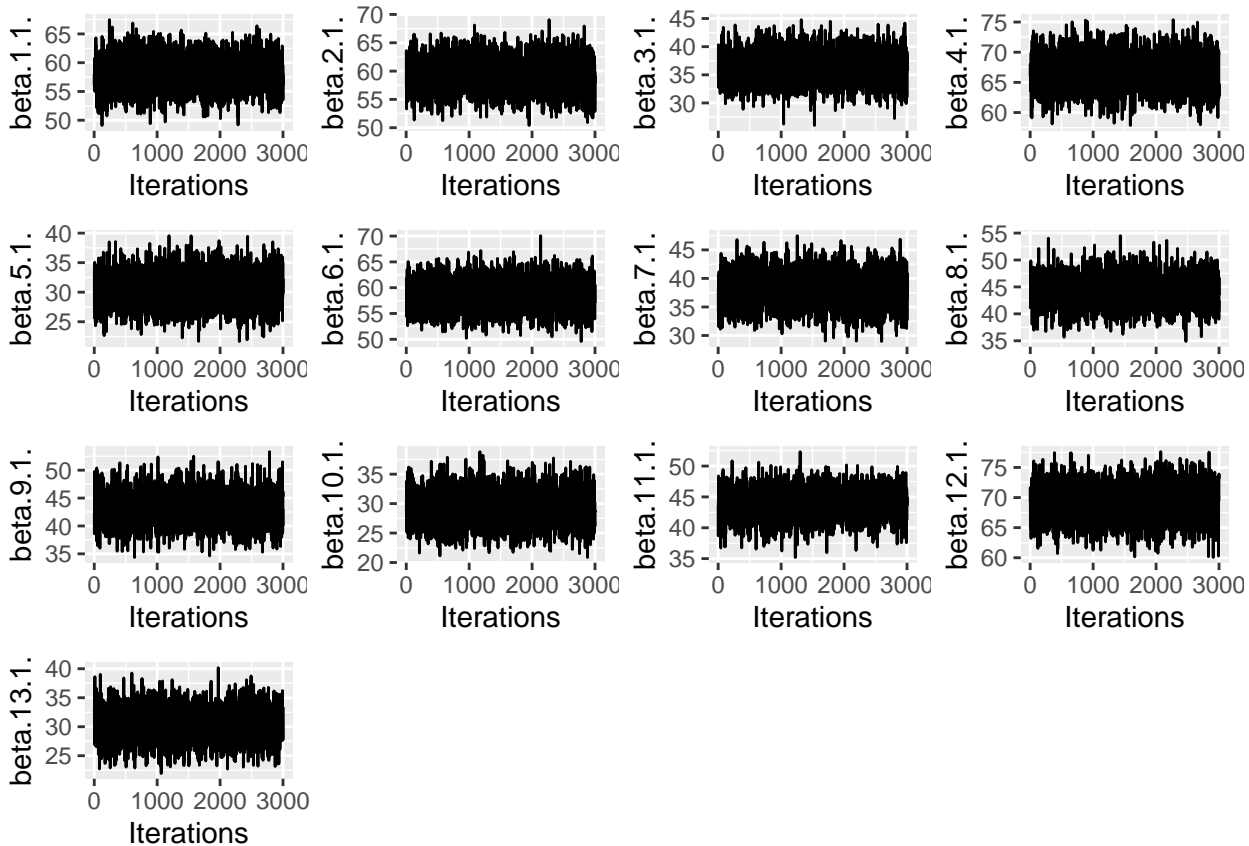
Traceplots for Presidential Model



Traceplots for Senate Model



Traceplots for House Model



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Additional EDA

Map For The Number of Filtered Polls Among States

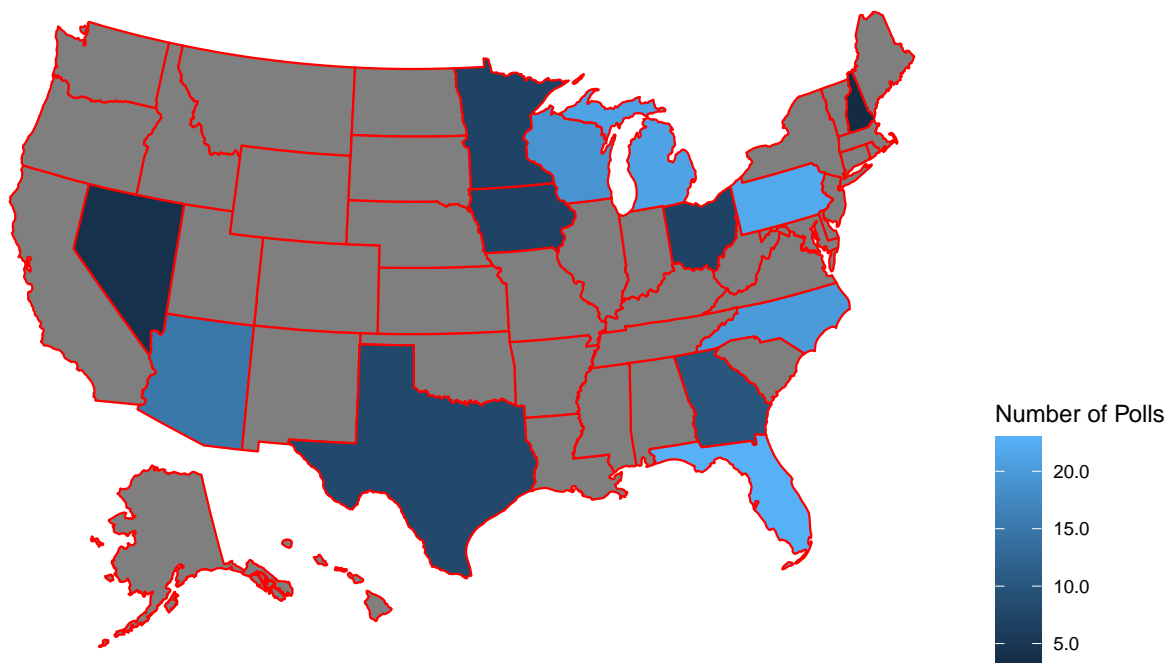


Figure 9: Additional Presidential Election Data Visualization

Map For The Number of Filtered Polls Among States

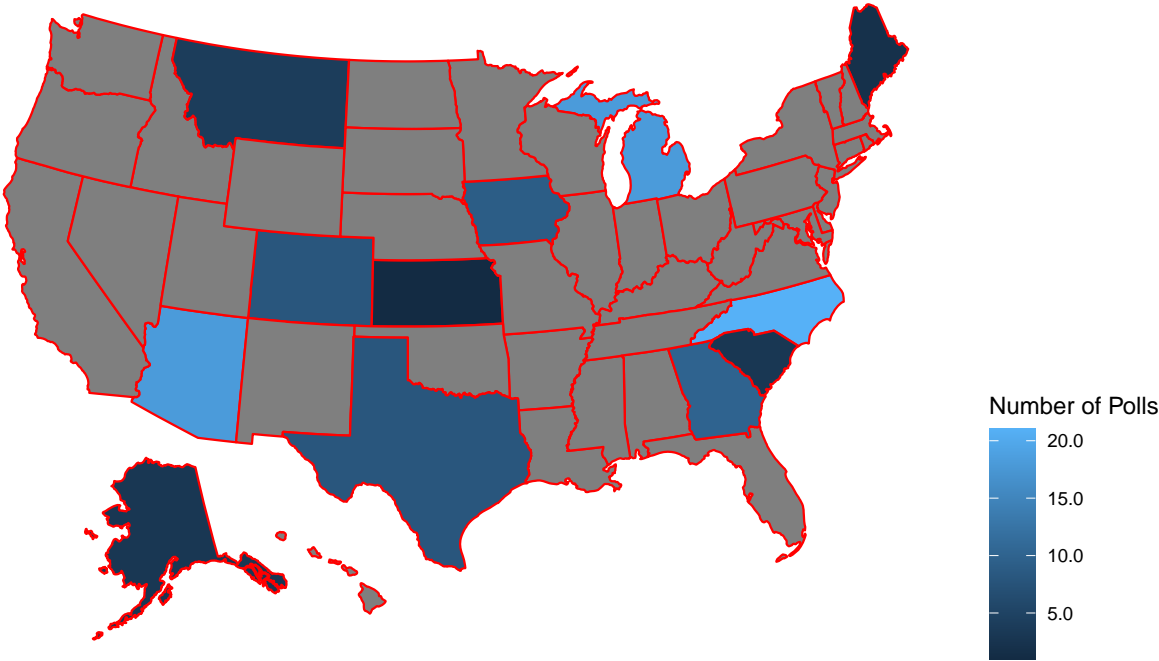


Figure 10: Additional Senate Election Data Visualization