# Final Report: Election Prediction

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## Introduction

The United States is getting closer to the 2020 presidential and Congressional elections on November 3, 2020. All 435 seats in the United States House of Representatives, 35 of the 100 seats in the United States Senate, and the office of President of the United States are up for election (Wikipedia, 2020). With the current polarizing political landscape, the election outcomes are significant to determine the next stage of this country. Polarization also makes election prediction both less and more difficult. It is less difficult because election results are less subject to election or candidate-specific factors, and it is more difficult as elections become increasingly close and fierce competitions, especially for presidential elections (Gelman, 2020). Therefore, we believe 2020 is a particularly challenging yet interesting year for statisticians to predict election outcomes and compare results with other predictions and the actual outcome to reflect on methodologies and unaccounted predictors. These election predictions not only inform the public about campaign trends and political sentiments but also help political strategists make decisions on allocating campaign resources for different candidates (Linzer et al.). American political pundits have been spending countless hours obtaining and analyzing relevant data to predict the election outcomes, and historical models and predictions and pre-election polls are two of the most important sources of information (Linzer et al.). In this report, similarly, we will also be using both literature review and polls for 2020 to build our prediction models.

We plan to build prediction models for the 2020 U.S. presidential election and the Senate election nationwide. Also, among all states, we decide to further predict the U.S. House election outcome for North Carolina, as it has been a swing state in presidential and Congressional elections for decades. Since 1996, the Republican statewide vote share in Congressional elections has varied "from a low of 45% in 2008 to a high of 55% in 2014" (Perrin et al.). To summarize, this report aims to use statistical models to predict (1) the outcome of the presidential election, (2) whether the US Senate remains in Republican control, (3) the electoral college vote, (4) the outcomes of all NC Congressional elections (the 13 federal Representatives to Congress), and (5) the outcome of the NC Senate election, including characterization of uncertainty in predictions.

## Data Description

In literature, election prediction relies on polling data as well as the fundamentals, which are economic indicators (The Economist, 2020) and voter turnout by demographic groups (Hansford, et al.). To forecast the outcomes of presidential election, senate election and house election (for North Carolina only), we obtained 2020 presidential polling data from *The Economist*, 2020 senate and house polling data as well as partisan lean data from *FiveThirtyEight*.

The fundamentals data were retrieved from various online sources, including Andrew Gelman's presidential election prediction model Github repository (for correlation across states and historical incumbent party's June approval ratings), Federal Reserve Economic Data (for second quarter real income growth), and NC Board of Elections website (for 2020 NC registered voter demographics). Please refer to Appendix B for a detailed description of all data sets used.

## Data Processing and Missing Data

For both the President and Senate models, we had to choose which states we considered to be "battlegrounds" to include in the models. We chose states for the models separately, since some states have a competitive race for President but not for Senate or vice versa.

For the presidential election data, we choose to use polling data from 30 days before the election (as opposed to over 30 days), as this aligns with Andrew Gelman's observation that polling data becomes more predictive as the day of poll becomes closer to election day (The Economist). Note that state-wide polls 30 days or less are unavailable for Oregon, Idaho, Wyoming, Nebraska, North Dakota, Illinois, Tennessee, Arkansas, Mississippi, and many Northeastern states. But these states are not swing states in recent elections, so we did not include those in our modeling process. For modeling, we will filter out those states with obvious party preferences and focus on those states showing percentages swinging right above or below 50%. For the presidential model, the states chosen are Arizona, Florida, Georgia, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, and Wisconsin. These thirteen states are swing states in the plot, and are rated as either "Toss Up", "Lean Republican", or "Lean Democrat" races (the three most competitive categories) by the Cook Political Report (CITE), an nonpartisan elections newsletter, as of October 28. These states are also projected to be competitive in the Presidential race by both the FiveThirtyEight and Economist models. We select poll responses from likely voters only, since it is known that using all responses may overestimate support for the Democratic party, according to FiveThirtyEight (Silver, 2014).

For the Senate model, the states chosen are Alabama, Alaska, Arizona, Colorado, Georgia, Iowa, Kansas, Maine, Michigan, Montana, North Carolina, South Carolina, and Texas. Similarly, for states such as California with strong historical party preferences, state-wide polls within 30 days of election are missing, but the missing data does not hurt our modeling process as we are only focusing on the swing states. There is also a special election for the Senate in Georgia in addition to the regularly scheduled election. These fourteen races are rated as either "Toss Up", "Lean Republican", or "Lean Democrat" (the three most competitive categories) by the Cook Political Report (CITE) as of October 29. These states are also projected to have competitive Senate races by both the FiveThirtyEight and Economist models.

For the U.S. House election, due to the scarcity of poll responses for the NC House elections, we included all poll responses within 115 days of the election. We will supplement the polls with the voter turnout results from our interim report to predict the House election for NC.

## Exploratory Data Analysis

After initial data cleanup for analysis, Figure 1 aims to explore polling data we have for the 2020 presidential election within 30 days of election, after filtering for the states of interest. The left plot shows that most states, as the polls indicate, have a vote share higher than 50% for Biden, averaging on all the polls available for each state respectively. While unsurprisingly Washington and California turn out to be blue, Louisiana, for example, still shows a deep red color. The right plot shows that within 30 days to election, variations exist for Biden support rate among polls within individual states. Combined with abundant literature that takes time into account for outcome prediction models, we will take days to election as a part of our model prediction as well (Gelman, 2020).

Similarly, Figure 2 visualizes the polling data we have for the 2020 U.S. Senate election, also within 30 days of election and after filtering for the states of interest. The left plot shows that most states, as the polls indicate, have a vote share higher than 50% for the Democratic Party, averaging on all the polls available for each state respectively. Nebraska is the only state with a bright red color in this data set. The right plot shows that within 30 days to election, variations exist for the Democratic party vote share among polls within individual states, such as the trend seen for North Carolina as Democratic Party vote share decreases getting closer to the election. We will take days to election as a part of our model prediction as well.

Figure 3 shows the different Democratic Party vote shares predicted by our interim report for all congressional districts in North Carolina. After predicting for voter turnout in the interim report, we use the party registration for predicted voters to get the vote share for Democratic Party. First, we grouped predicted voters for congressional districts. There are three categories in party registration: Democratic, Republican, and other (indicating a

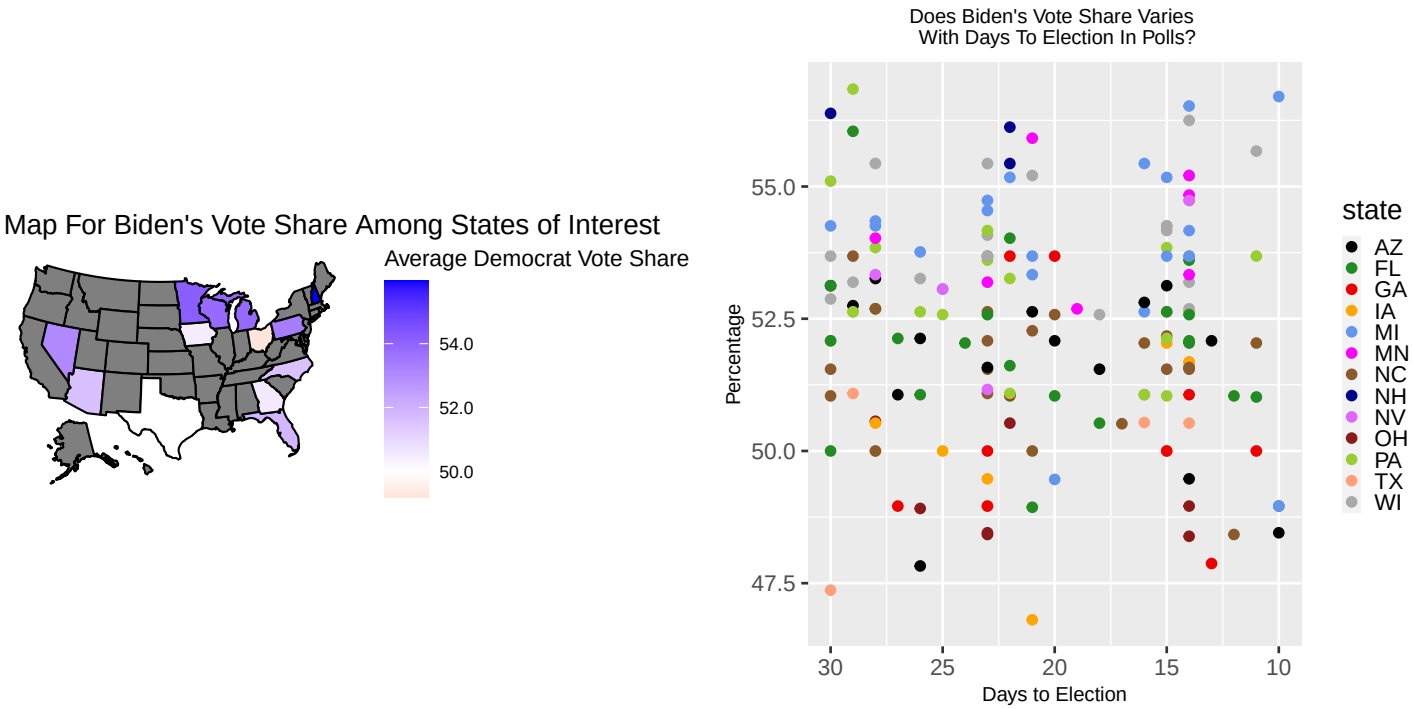



Figure 1: Presidential Election Data Visualization

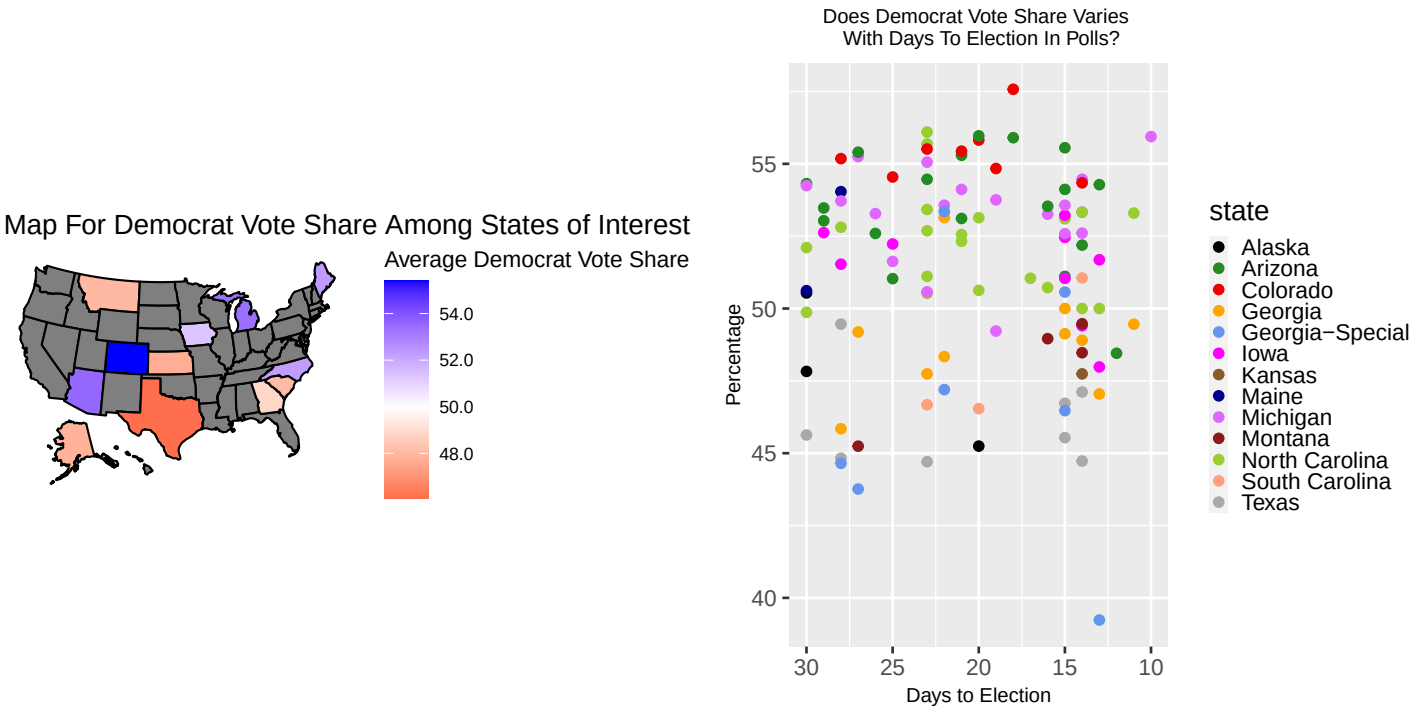



Figure 2: U.S. Senate Election Data Visualization

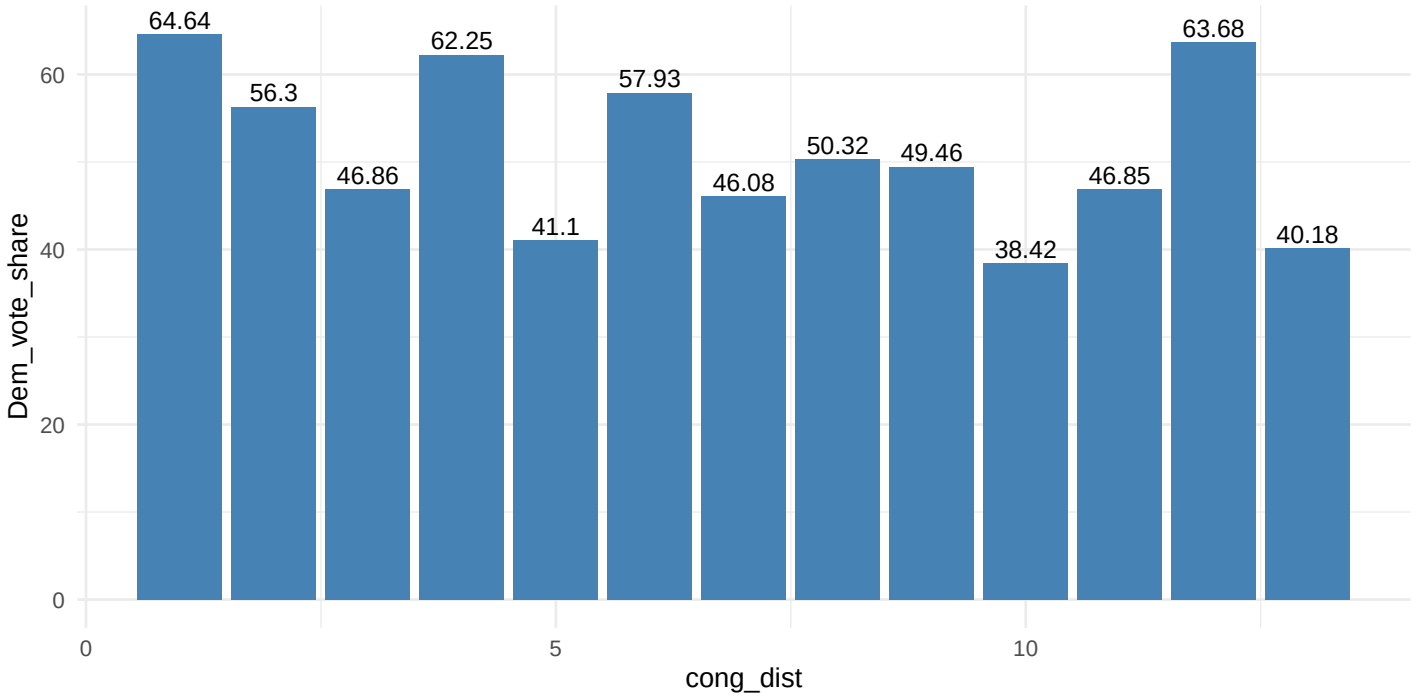



Figure 3: Democratic Vote Share Predicted By The Interim Report

third party or unaffiliated). Within each congressional district, we split the `other` population in half, and add it to both the Democratic Party vote count and the Republican Party vote count, as FiveThirtyEight used the same approach to get vote share for their House predictions, shown in[https://projects.fivethirtyeight.com/2020-election-forecast/house/]. Among 13 districts, 6 of them have a Democratic Party vote share higher than 50%.

## Method

Similarly to prior work (Linzer 2013), we use a hierarchical model structure for our election models to overcome the limitation that not every state is polled on every day, allowing the model to borrow data across states (or Congressional districts). This also helps account for the fact that polls from each individual state are correlated data observations. We believe that vote share should be modeled as a normal distribution because, historically, vote shares among candidates have mode toward the center of the distribution (landslides are unlikely, especially in the case of a presidential election). In addition, we chose for our random walk to be lag-1 because it's more sensical to have updated predicted dependent on the previous prediction as opposed to being from a longer time period ago (see model specification below).

To specify priors for the mean of two-party vote share for Democrats in each state or district, we used a combination of state partisanship, economic fundamentals, candidate incumbency, and projected voter turnout based on demographics. First, we started with state or district partisan lean calculated by FiveThirtyEight (see "FiveThirtyEight's Partisan Lean" in references). Partisan lean is the average difference between how a state or district votes and how the country votes overall, based on results from the past two presidential elections as well as statewide elections in 2018. For example, North Carolina has a partisan lean of R+4, meaning that in a 50/50 political environment nationwide, a Republican would be expected to win North Carolina by 4 points. For each partisan lean, we turn the parsian lean into a prior mean of Democratic vote share by adding or subtracting half the partisan lean from 50%. North Carolina is a Republican-leaning state, so we subtract 2 from 50% to get a partisan lean prior of 48% (note that the Republican would then get 52% of the two-party vote, resulting in an R+4 margin of victory).

**Presidential Election Model**

For the presidential model, the states chosen are Arizona, Florida, Georgia, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, and Wisconsin. These thirteen states are rated as either "Toss Up", "Lean Republican", or "Lean Democrat" races (the three most competitive categories) by the Cook Political Report (as of October 28), an nonpartisan elections newsletter. These states are also projected to be competitive in the Presidential race by both the FiveThirtyEight and Economist models. We select poll responses from likely voters only, since it is known that using all responses may overestimate support for the Democratic party, according to FiveThirtyEight (Silver, 2014).

The response we will use is $Y_k$, Biden's share of the two-party vote using the following model where the poll $k$ (of $K$=*3361 [insert final number]* polls) ended $t$ days (of $T = 30$ days) before the election and was conducted on state $j$ (of $J$=13 states of interest). We choose to use polling data from 30 days before the election (as opposed to over 30 days), as this aligns with Andrew Gelman's observation that polling data becomes more predictive as the day of poll becomes closer to election day (The Economist).

$Y_k \sim N(\theta_{jt}, \sigma^2_{yj})$
$\theta_{...t} \sim MVN(\theta_{...t-1}, \Sigma)$
$\Sigma \sim Wishart(S, J+1)$ where $S$ is the state covariance matrix (obtained from Andrew Gelman's model) and $J$ is the number of states in the model.
Priors for $\sigma^2_{yj}$: $\sigma^2_{yj} \sim InvGamma(\nu_u, \nu_y\tau_y)$
$\nu_y \sim Uniform(0, 100)$ and $\tau_y \sim Uniform(0, 100)$.
Priors for $\theta_{j1}$: $\theta_{j1} \sim N(\mu_j, \sigma^2)$
$\sigma^2 \sim InvGamma(0.5, 0.5)$
$\mu_j \sim N(h_j, 7.5^2)$
$h_j = 0.1 * \text{Presidential Fundamentals}_{2020} + 0.9 * \text{Vote Share from Partisan Lean}_j$
$\text{Presidential Fundamentals}_i = 100 - \text{Incumbent Party Vote Share}_i$
$\text{Incumbent Party Vote Share}_i = \gamma_0 + \gamma_1 \text{June Approval Rating for Incumbent Party}_i + \gamma_2 \text{Three Month Stock Growth}_i + \gamma_3 \text{2nd Quarter Real Income Growth}_i + \epsilon_i$
$\epsilon_i \sim N(0, \phi^2)$

For our presidential model, we wanted to incorporate state-level correlations that differ between each pair of states. For example, Wisconsin is much more similar to its Great Lakes neighbor Michigan both geographically and demographically than it is to Arizona, which is in the Southwest and has a much larger Hispanic population. So, we would expect that Wisconsin and Michigan have a higher correlation in their election results than Wisconsin and Arizona. In order to incorporate this into our presidential model, we used a multivariate normal distribution to model two-party vote share in each state, which allowed us to specify a covariance matrix for the states. We used a covariance matrix which was downloaded from *The Economist* forecast model (Gelman, 2020). These similarities between states were calculated by comparing their demographic and political profiles, such as the state's share of white voters and how urban/rural the state is.

In essence, the presidential model used fundamentals and vote share from partisan lean as a starting point for the expected vote share for the Democratic party, and update the priors with presidential election polling data. We first predicted incument party's national vote share ($\text{Incumbent Party Vote Share}_i$) based on the corresponding election year $i$'s economic data and June net approval rating of the incumbent party. Since in the 2020 presidential election Biden's Democratic party is not the incumbent party, $\text{Presidental Fundamentals}_{2020}$, which can be though of as the national level prior of Biden's vote share, is 100 minus $\text{Incumbent Party Vote Share}_{2020}$. When computing the state specific prior, 10% weight is allocated to the fundamentals based on economic data and 90% weight to the state's partisan lean because the effect of economic indicators has shrunk over the years as electorates became increasingly polarized (The Economist, 2020). In addition, economic indicators are particularly volatile in 2020 due to the COVID-19 pandemic. We chose not to do this for the U.S. House and Senate models because these races are generally more localized (Gillespie et al. 2020).

*one sentence on hyperparameters*

Presidential elections in the United States are decided by the Electoral College, so estimating percentage support in each state alone does not tell us who wins the election. For each set of MCMC samples of two-party vote share by state, we subsequently use the predicted winner in each state to add up the electoral votes of each candidate. For states not in the model, their electoral votes are allocated assuming they vote the same way as in 2016. The probability of President Trump winning re-election is then the probability that he receives 270 or more electoral votes across simulations. Note that although Maine and Nebraska allocate some electoral votes by Congressional District, only two electoral votes are competitive due to this wrinkle, so for simplicity we simply allocate all electoral votes to the statewide winner of each respective state.

**Senate Election Model**

For the Senate model, the states chosen are Alaska, Arizona, Colorado, Georgia, Iowa, Kansas, Maine, Michigan, Montana, North Carolina, South Carolina, and Texas. There is also a special election for the Senate in Georgia in addition to the regularly scheduled election. These fourteen races are rated as either "Toss Up", "Lean Republican", or "Lean Democrat" (the three most competitive categories) by the Cook Political Report (as of October 29). These states are also projected to have competitive Senate races by both the FiveThirtyEight and Economist models. Note that the Georgia special election actually has multiple Republicans and Democrats running on the same ballot. If no candidate wins over 50% of the vote, which is considered likely, the top two finishers will advance to a one-on-one runoff election in January (Ballotpedia, 2020). For simplicity, our model simply sums up the support for Republican candidates and compares that to the sum of the support for Democratic candidates, then treats this election like the others. By summing the Republican and Democratic support, we essentially assume that partisan support will consolidate around one candidate for each party and will remain about balanced between the two parties. (This is not the ideal way to simulate this election, but the difficulty of predicting turnout and the political environment for the January runoff election made this decision seem like the best one to make.) Once again, we only included poll respones from likely voters that responded within 30 days of the election.

The response we will use is $Y_k$, the Democrat candidate's share of the two-party vote using the following model where the poll $k$ (of $K$=*fill in* polls) ended $t$ days (of $T = 60$ days) before the election and was conducted on state $j$ (of $J$=14 states of interest).

$Y_k \sim N(\beta_{jt}, \sigma^2_{yj})$
$\beta_{jt} \sim N(\beta_{jt-1}, \sigma^2_{\beta j})$
Priors for $\sigma^2_{yj}$: $\sigma^2_{yj} \sim InvGamma(\nu_u, \nu_y \tau_y)$
$\nu_y \sim Uniform(0, 100)$ and $\tau_y \sim Uniform(0, 100)$.
Priors for $\sigma^2_{\beta j}$ $\sigma^2_{\beta j} \sim InvGamma(\nu_\beta, \nu_\beta \tau_\beta))$
$\nu_\beta \sim Uniform(0, 100)$ and $\tau_\beta \sim Uniform(0, 100)$.
Priors for $\beta_{j1}$ $\beta_{j1} \sim N(\mu_j, \sigma^2)$
$\sigma^2 \sim InvGamma(0.5, 0.5)$
$\mu_j \sim N(h_j, 7.5^2)$
$h_j =$ Vote Share from Partisan Lean$_j$ + Incumbency Advantage$_j$

For the U.S. Senate model we also take into account incumbency advantage. Analysis by FiveThirtyEight (Rakich, 2018) found that incumbent senators get a 2.6-point boost. For each incumbent was running a Senate race, depending on whether they were a Democrat or Republican, we either added or subtracted this incumbency adjustment with the prior mean computed from partisan lean.

For the NC-specific prior, we supplemented partisan lean and incumbency adjustment with voter turnout predicted by the model in our Interim Report. When computing the prior for NC only, 10% weight is allocated to predicted Democratic vote share from turnout and demographics, and 90% weight is allocated to partisan lean and incumbency adjustment. The turnout is weighted much less because there is considerable uncertainty surrounding turnout because of the COVID-19 pandemic. See Appendix A.2 for more specifics on how the voter turnout model was used to compute predicted Democratic vote share.

For predicting control of the U.S. Senate, things are complicated slightly by the fact that a 50/50 split of Senate seats is a possible outcome of the elections. In this case, the Vice President breaks the tie, so the party that wins the Presidential election will only need 50 Senate seats for a Senate majority, while the party that loses will need 51. Since our President model gives Democrat Joe Biden a very high chance of winning (discussed more below), we assume in the Senate model that Democrats will control the Senate in the case of a 50/50 split.

### House Election Model

The response we will use is $Y_k$, the Democrat candidate's share of the two-party vote using the following model where the poll $k$ (of $K$=*fill in* polls) ended $t$ days (of $T = 30$ days) before the election and was conducted on district $j$ (where $j$ is in districts $1, \cdots, 11, 13$ of North Carolina). For district 12, the vote share is coded as 100 because there is only one candidate, and she is a Democrat. Note that, due to the scarcity of poll responses for the NC House elections, we included all poll responses within 115 days of the election.

$Y_k \sim N(\beta_{jt}, \sigma^2_{yj})$
$\beta_{jt} \sim N(\beta_{jt-1}, \sigma^2_{\beta j})$
$\sigma^2_{yj} \sim InvGamma(\nu_u, \nu_y \tau_y)$
$\nu_y \sim Uniform(0, 100)$ and $\tau_y \sim Uniform(0, 100)$.
$\sigma^2_{\beta j} \sim InvGamma(\nu_\beta, \nu_\beta \tau_\beta))$
$\nu_\beta \sim Uniform(0, 100)$ and $\tau_\beta \sim Uniform(0, 100)$.
$\beta_{j1} \sim N(h_j, 7.5)$
$h_j = 0.9 * \text{Vote Share from Partisan Lean}_j + 0.1 * \text{Expected Vote Share from Voter Turnout}_j$

According to FiveThirtyEight (Rakich, 2018),incumbent members of the House get a 2.7-point margin boost in their favor. For the U.S. House races, the incumbency adjustment was calculated by the same method as it was for the Senate races. Expected Vote Share from Voter Turnout$_j$ was calculated from the model in the Interim Report. Please see Appendix A.3 for more details.

## Results

### Presidential Election Model

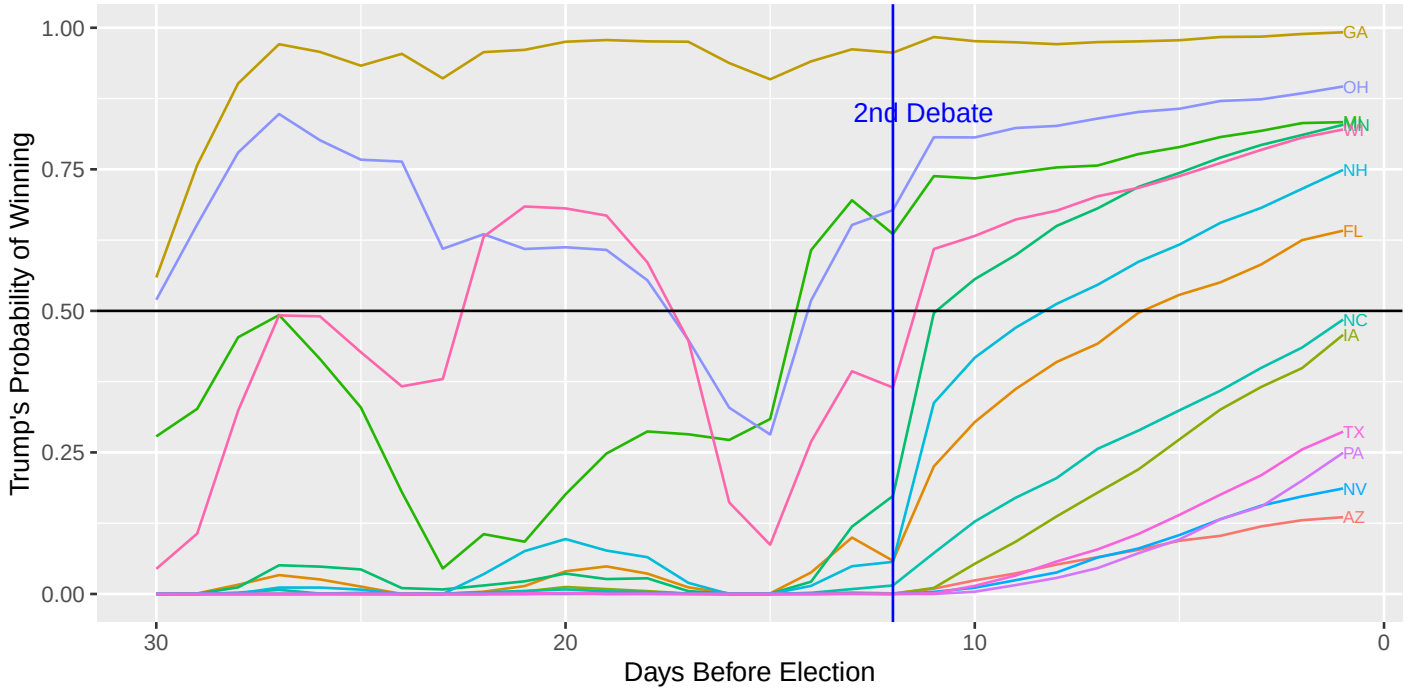



Figure 4: Probability of Re-election for President Trump over the 30 Days Before Election Day

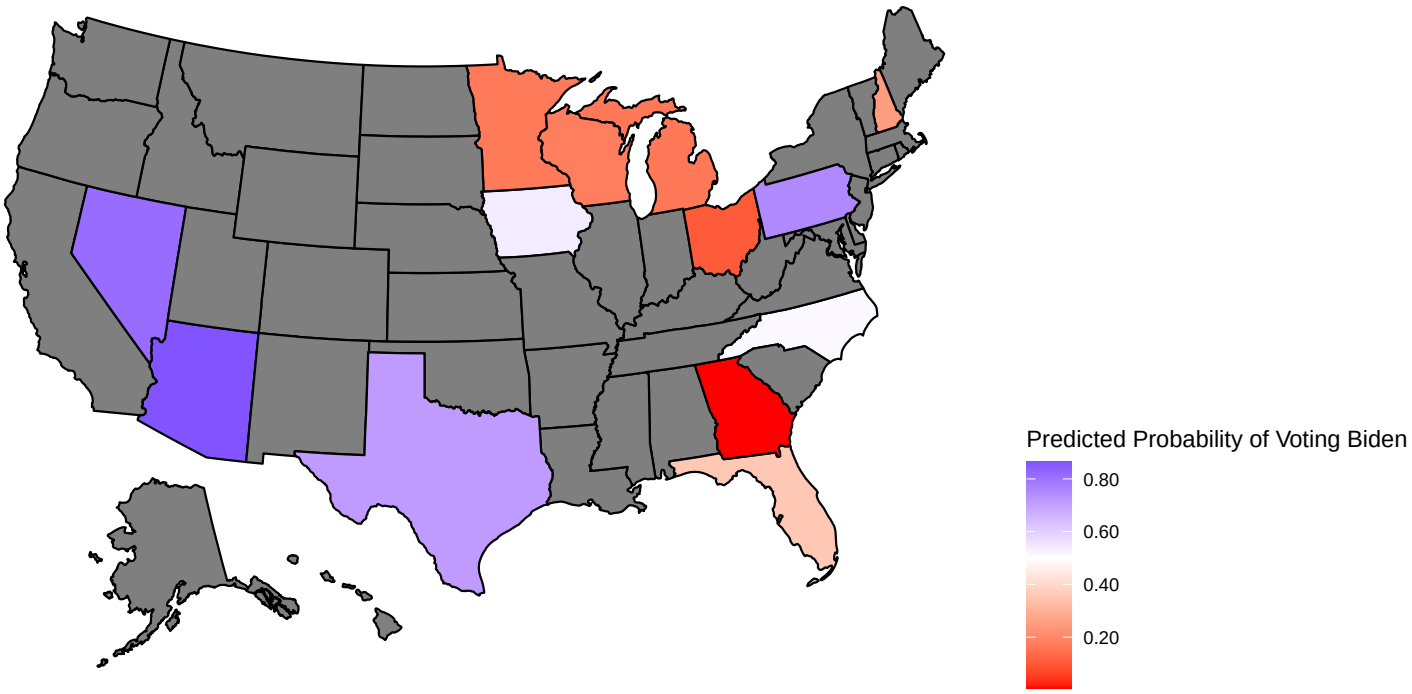



Figure 5: Plot of Predicted Presidential Result for States of Interest on Election Day

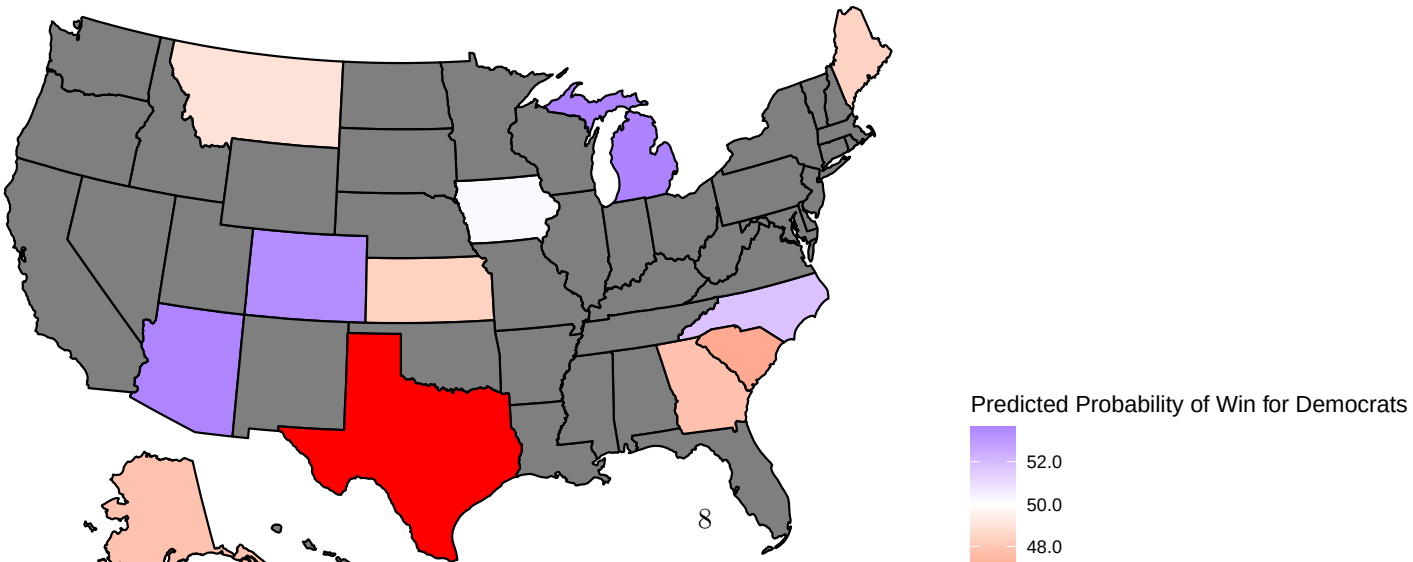
4 shows how Trump's probability of winning the election changes over the 60 days before the election in each state of interest. It appears that his probability of winning increased after the last presidential debate. *check for final results* From 5 we see that swing states such as Florida and Ohio are predicted to vote Trump while other swing states such as Pennsylvania and North Carolina are predicted to vote Biden. *check for final results*

Table 1: Predicted Probability of Winning and Electoral Vote Count

| | Win Probability | Estimated EC Votes | 2.5% Quantile EC Votes | 97.5% Quantile EC Votes |
|---|---|---|---|---|
| Trump | 0.23 | 238 | 168 | 325 |
| Biden | 0.77 | 300 | 213 | 370 |

The probability that Trump wins re-election is 0.23. The predicted electoral colleges votes for Trump is 238 with a 95% confidence interval of (168, 325).

**Senate Election Model**



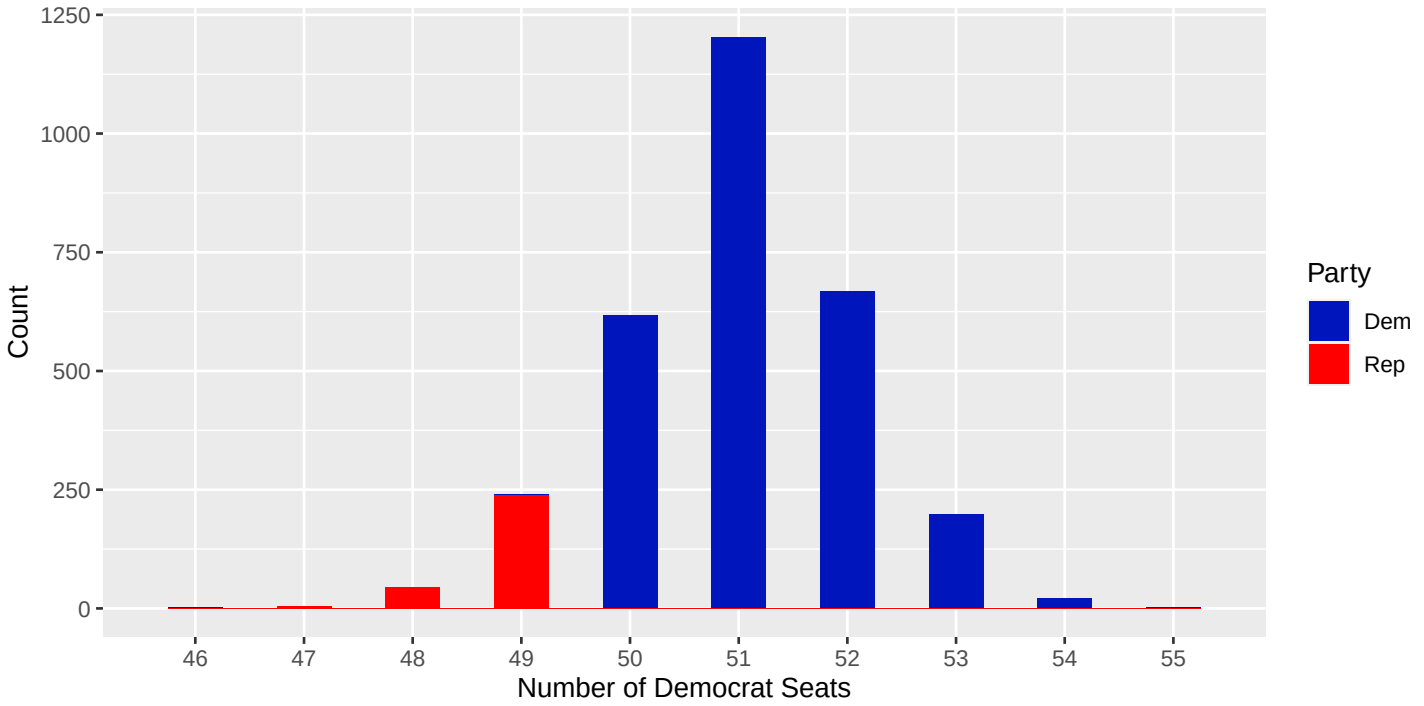



Figure 7: Plot of Predicted Senate Results for States of Interest on Election Day

For the NC Senate race (Tillis vs. Cunningham), the predicted vote share for Cunningham is 51.78% with a 95% confidence interval of (48.2, 54.99).

### House Election Model

Table 3: Predicted Republican Vote Share in NC's 13 Districts

| | Estimate | 2.5% Quantile | 97.5% Quantile |
|---|---|---|---|
| District 1 | 41.69 | 36.32 | 47.02 |
| District 2 | 40.75 | 35.29 | 46.17 |
| District 3 | 63.61 | 58.32 | 68.56 |
| District 4 | 33.29 | 27.84 | 38.48 |
| District 5 | 69.54 | 64.20 | 74.93 |
| District 6 | 41.19 | 35.87 | 46.64 |
| District 7 | 61.81 | 56.40 | 67.43 |
| District 8 | 55.74 | 50.67 | 61.01 |
| District 9 | 56.79 | 51.40 | 61.90 |
| District 10 | 70.68 | 65.16 | 75.96 |
| District 11 | 56.12 | 51.86 | 60.59 |
| District 12 | 0.00 | 0.00 | 0.00 |
| District 13 | 69.60 | 64.12 | 75.01 |

```
## OGR data source with driver: ESRI Shapefile
## Source: "/Users/cathylee/Documents/Course Folders/Fall 2020/STA 440/Case 3/sta440casestudy3/C-Goo
## with 13 features
## It has 2 fields
## Integer64 fields read as strings:  POPULATION
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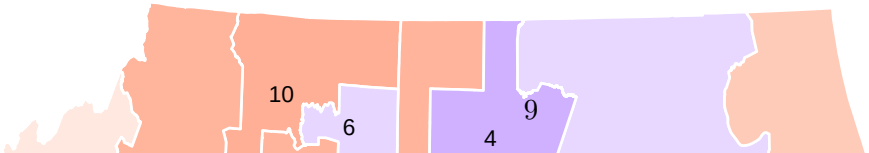

confidence intervals for predicted vote share are also all above 50). From 8 we can see that the vote share for districts 1, 2, 4, and 6 lean toward Democrat (note that the only candidate on the ballot for district 12 is a Democrat).

### Model Validation and Sensitivity Analysis

For model validation, we trained our 2020 presidential election model on 2016 polling data and adjusted fundamentals accordingly. For instance, instead of predicting on 2020 economic indicators, we removed the year 2016 from the fundamentals training data and predicted on 2016 economic indicators. This 2016 model produced a 7% probability that Trump would win the presidency, which is within the range of predictions by respected models available in 2016 (The Economist, 2016). Polls were skewed in 2016, so it is expected that the model would not be entirely accurate.

For sensitivity analysis, we adjusted the relative weights placed on the components of prior for the presidential election model. Instead of 10% fundamentals and 90% partisan lean, we ran a model with 50% fundamentals and 50% partisan lean multiple times and the predictions for Trump's re-election probability was generally higher (approximately 0.25) than that from our main model, which makes sense since economic fundamental model contains stock growth and is slightly favored toward Trump. However, the sensitivity analysis models still point to an unlikely re-election.

We also adjusted the prior parameters for sensitivity analysis. In both the senate and house model, changing the priors from $\nu_y, \tau_y, \nu_\beta, \tau_\beta \sim Uniform(0, 100)$ to $\nu_y, \nu_\beta \sim Uniform(0, 10), \tau_y, \tau_\beta \sim Uniform(0, 1000)$ to increase the variance had little change on the estimates. Changing the relevant uniform priors in the presidential model resulted in a slightly higher probability of Trump's re-election (the probability was about 0.23), but again, this still indicates Using a stronger prior for the variance for the estimate on the day of election ($\sigma^2 \sim InvGamma(10, 0.5)$) for the senate and presidential models resulted in comparable performance to our main senate model while the presidential model had higher win probabilities and slightly narrower credible intervals for Trump compared to our main model (the win probability was about 0.34). Similarly, reducing the variance for the house model ($\beta_{ji} \sim N(h_j, 1)$) also resulted in comparable performance, though the quantiles were more narrow, as expected (Linzer, 2013). To account for shock events, we could modify the priors to increase the variance associated with $Y_k$ to reflect the increased uncertainty due to the shock event in each relevant model.

### Diagnostics

```
## No id variables; using all as measure variables
```

As shown in ref{presresid}, the residuals for the estimates of the probability that Biden wins the election are distributed around 0 for all of the states of interest.

```
## No id variables; using all as measure variables
```

```
## No id variables; using all as measure variables
```

### Discussion and Limitations

One place where we made decisions regarding predictive ability of variables was in presidential election model. We followed Alan Abramowitz's "Time for Change" model where annualized second quarter GDP growth rates and incumbent party's June approval ratings were incorporated as the fundamentals data (Abramowitz 2008). However, given the economic impact of COVID-19, this year's annualized second quarter GDP growth rate is abnormally low (-34%) (), which subsequently resulted in a very low predicted probability of Trump's re-election (approximately 5%). Therefore we felt annualized second quarter GDP growth rate is not an appropriate predictor and substituted it with second quarter real income growth from one year ago and stock performance three month prior to the election date. As seen in the Results section, the updated predicted probability of Trump winning the election is more sensible.

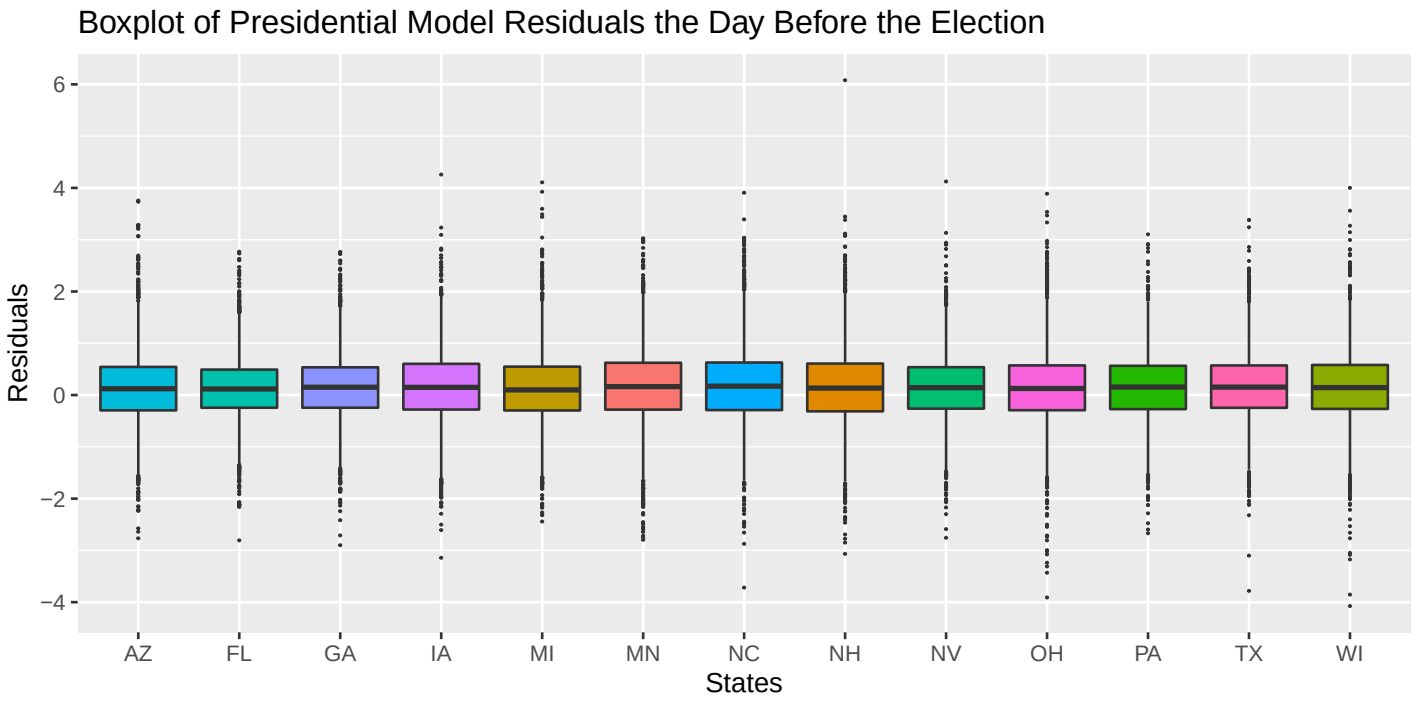


Figure 9: Presidential Model Residual Plot

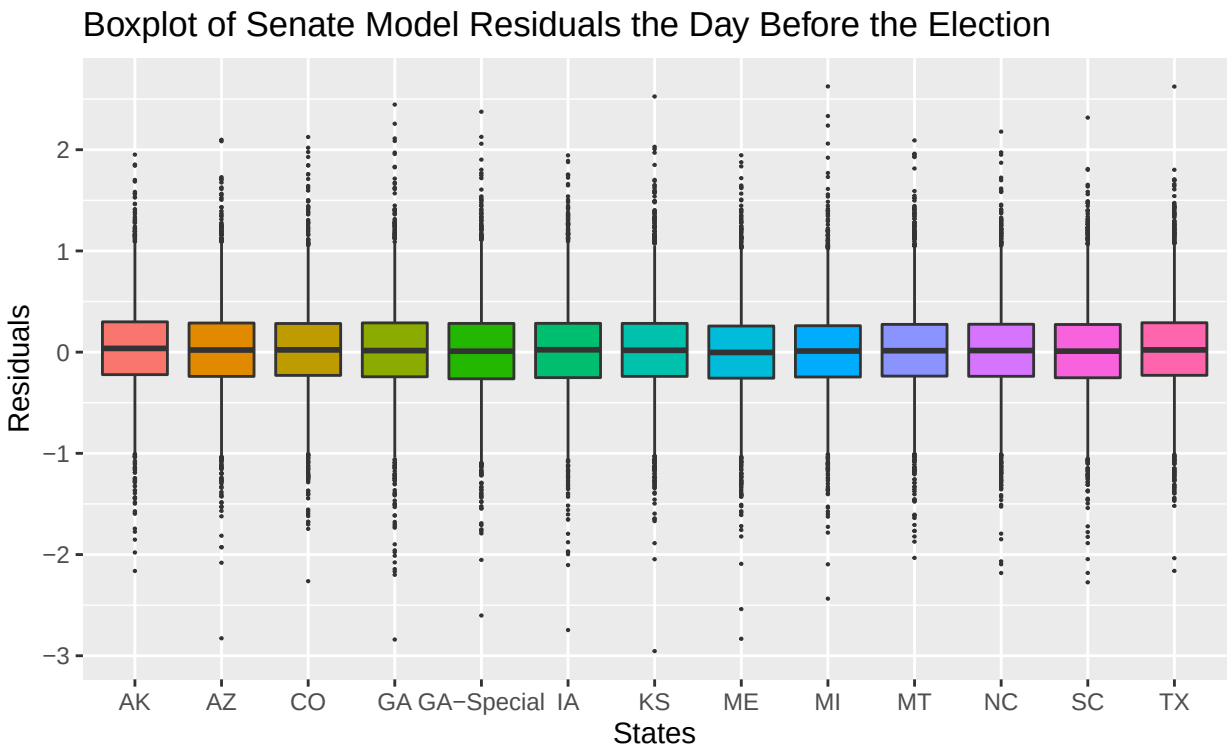


Figure 10: Senate Model Residual Plot

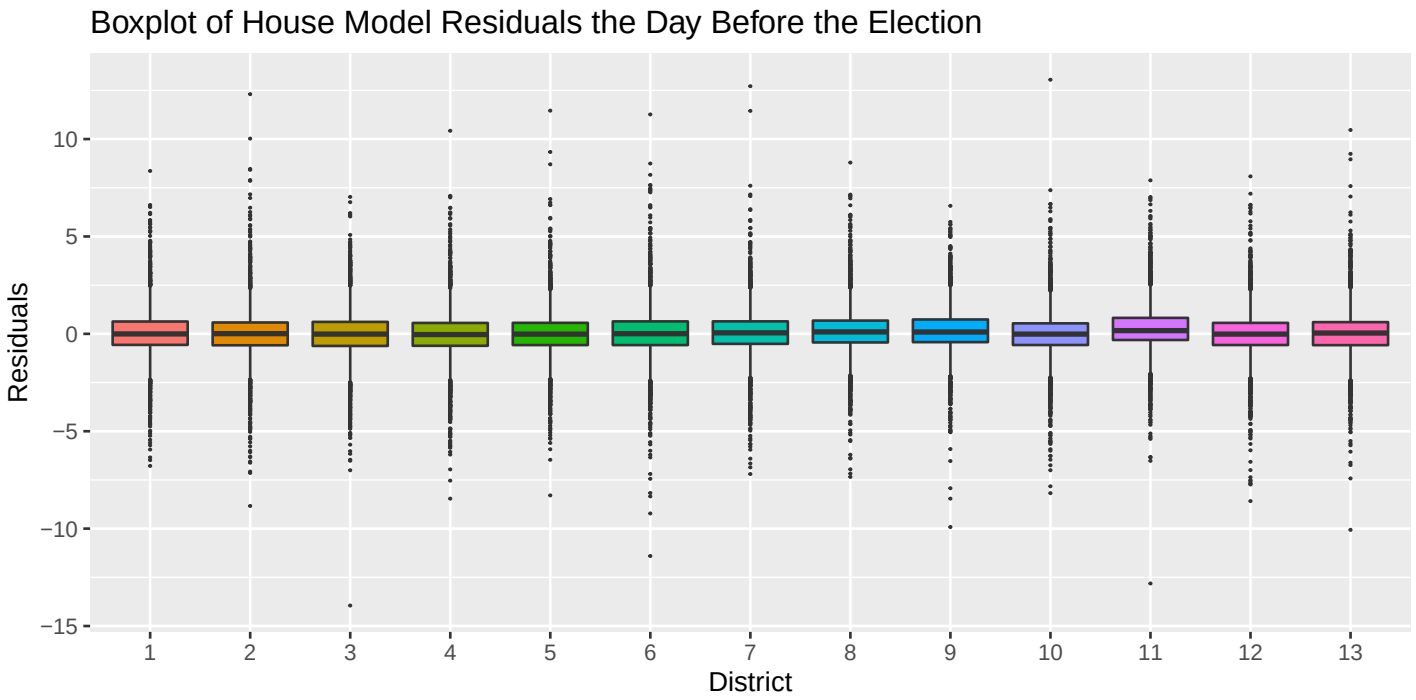


Figure 11: House Model Residual Plot

For the NC House election model and NC Senate election model we incorporated voter turnout prediction based on registered voters' demographics from the Interim Report. We believe that even if we had abundant polling data, there's a difference between how people respond to a poll and whether or not they actually turn out to vote. Therefore, we included turnout information from our interim model.

A limitation of our model is that we did not explicitly weight any polls more strongly than others. Poll-based models often weight polls based on recency of the poll, sample size, and pollster quality (Silver, 2014). Our model actually does take into account poll recency, since our prior specification assumes that polls closer to election day have less variance (noise) around the true level of candidate support. However, we do not weight based on sample size or pollster quality. While it would have been possible to do this, it would have required making many arbitrary decisions. For example, what is a "good" sample size, and how heavily should sample size be weighted? How do we determine pollster quality, especially if we don't just want to blindly use FiveThirtyEight's pollster ratings? We decided to avoid these issues, especially since it has been observed that such adjustments rarely have a large impact on models (Silver, 2014).

There are inherent limitations to modeling elections based on polls. In some cases, as with the U.S. House races in North Carolina, there may be few or no polls for a given district. Even when polling data is plentiful, we are essentially assuming that the polls are reasonably accurate measures of candidate support. It has been well-documented that systematic polling errors during the 2016 election cycle underestimated the support of then-candidate Trump. While many pollsters have adjusted their sample weighting methods (for example, to adjust for education level), it is not impossible that we could witness a 2016-level polling error again (Skelley, 2020). A recent election prediction study shows that state polls have much higher margins of error than expected (Shirani-Mehr et al., 2018). It has also been noted that registered Democrats have been overrepresented in recent polling samples compared to their proportion of the population overall (Cohn, 2020). In addition, we acknowledge that with the COVID-19 pandemic, polling has moved from in-person interviews to phone calls, live surveys and internet questions, which could lead to unknown biases with its lower response rates (Gelman, 2020). In addition, COVID-19 posted significant uncertainty to election outcomes. For example, rejected absentee ballots due to mailing issues were not as significant in previous elections. But in 2020, so much more voters are mailing their votes, and an unknown number of them will be rejected, potentially changing voter turnout (Rakich, 2020). We could have also used early voting data, but we excluded them because abundant literature has proven early voting data to have strong partisan bias (McDonald, 2020). Another factor that could significantly affect

election results is potential shock events - terrorist attacks, natural disasters, power outages. Although literature proves that sometimes elections could be vulnerable to those events, because our predictions are so close to the actual day of election, we did not account for specific shock events in our analysis (Morley, 1905). However, we did sensitivity analysis by increasing variance to account for unpredictable events.

One feature that we did not include is correlation between simulations of the Presidential, House, and Senate elections. For instance, if Joe Biden wins the Presidential election by a wide margin, we would also expect Democrats to do better in the House and Senate elections comapred to if Biden did not win (Desilver, 2020). This is especially true by state; if Biden wins North Carolina by a wide margin, this would likely help Democrat Cal Cunningham win the NC Senate race. Additionally, since the Vice President can determine Senate control in the case of a 50/50 split in seats, having model correlations could help account for this dependence between Senate control and Presidential results. Instead, all of our models simply run independently.

We do account for state-by-state correlations in our President model, but we do not for the Senate model or the House model. Although there is likely still some correlation in these cases, such races are generally a bit more localized (Gillespie et al. 2020) and incumbency advantages can contradict national trends. As a result, we treated these races as being uncorrelated. Possible ideas for addressing this in future studies could be to try to compute our own covariances between districts based on voter demographics, or for the Senate to try predicting covariance between states that is similar to that in the Presidential model but "weaker".

Furthermore, for making our prior of Democrat vote share in the House and Senate model (for NC), we assume that registered Democrats and Republicans will vote for their party's candidate, and then we split unaffiliated voters evenly between the two parties. FiveThirtyEight's models such as the 2020 President and Senate models allocate undecided voters in polls 50/50 as well, so our assumption is in line with standard approaches. However, other reasonable approaches could have been to allocate unaffiliated voters proportionally to partisan support within the state of interest, or even to calculate the split based on Trump's overperformance of polls in 2016 based on winning over undecided voters (Golshan, 2016). Such approaches have been explored in the past by FiveThirtyEight (Silver, 2008) but are not the norm.

## Conclusion

In conclusion, the probability that Trump wins re-election is 0.23. The predicted electoral colleges votes for Trump is 238. We predict that the US Senate is unlikely to remain in Republican control (in fact, we predict there is only a 0.1 chance that Republicans retain control) For the House elections in NC's 13 districts, it appears that the Republican candidate will win in districts 3, 5, 7, 8, 9, 10, 11, and 13, and that the Democratic candidate will win districts 1, 2, 4, and 6. For the NC Senate race (Tillis vs. Cunningham), the predicted vote share for Cunningham is 51.78%, and we predict that he will win by a small margin. *check to see if nothing changes*

Key parts of our model include: the incorporation of a state covariance matrix and use of economic fundamentals in our presidential model, the incumbency adjustment in our Senate model, and the inclusion of voter turnout information in our NC House model. As mentioned before, there are several limitations, but overall, we believe our analysis to be thorough and robust.

# Appendix A

## A.1 Presidential Election Model

### Presidential Model Purpose and Structure

The purpose of this model is to predict the probability that president Trump will win the election, as well as to predict the electoral college vote. The states chosen are Arizona, Florida, Georgia, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, and Wisconsin. These thirteen states are deemed to be competitive in the 2020 presidential race by political analysts.

The response we will use is $Y_k$, Biden's share of the two-party vote using the following model where the poll $k$ (of $K$=*3361 [insert final number]* polls) ended $t$ days (of $T = 30$ days) before the election and was conducted on state $j$ (of $J$=13 states of interest). We choose to use polling data from 30 days before the election (as opposed to over 30 days), as this aligns with Andrew Gelman's observation that polling data becomes more predictive as the day of poll becomes closer to election day (The Economist).

$Y_k \sim N(\theta_{jt}, \sigma^2_{yj})$
$\theta_{\cdots t} \sim MVN(\theta_{\cdots t-1}, \Sigma)$
$\Sigma \sim Wishart(S, J+1)$ where $S$ is the state covariance matrix (obtained from Andrew Gelman's model) and $J$ is the number of states in the model.
Priors for $\sigma^2_{yj}$: $\sigma^2_{yj} \sim InvGamma(\nu_u, \nu_y\tau_y)$
$\nu_y \sim Uniform(0, 100)$ and $\tau_y \sim Uniform(0, 100)$.
Priors for $\theta_{j1}$: $\theta_{j1} \sim N(\mu_j, \sigma^2)$
$\sigma^2 \sim InvGamma(0.5, 0.5)$
$\mu_j \sim N(h_j, 7.5^2)$
$h_j = 0.1 * \text{Presidential } \hat{\text{F}}\text{undamentals}_{2020} + 0.9 * \text{Vote Share from Partisan Lean}_j$
$\text{Presidential Fundamentals}_i = 100 - \text{Incumbent Party Vote Share}_i$
$\text{Incumbent Party Vote Share}_i = \gamma_0 + \gamma_1 \text{June Approval Rating for Incumbent Party}_i + \gamma_2 \text{Three Month Stock Growth}_i + \gamma_3 \text{2nd Quarter Real Income Growth}_i + \epsilon_i$
$\epsilon_i \sim N(0, \phi^2)$

A hierarchical model was used, where Biden's share of the two-party vote for poll $k$ on any given day $t$ for state $j$ is modelled as a normal distribution with mean $\theta_{jt}$ and variance $\sigma^2_{yj}$. A random walk was used to calculate $\theta_{jt}$ for each day $t$ before the election (up until 30 days before the election) from $\theta_{jt-1}$ and $\Sigma$, a $JxJ$ state covariance matrix. The mean $h_j$ of the normal prior on $\mu_j$ (which is the mean of the normal prior on $\theta_{j1}$) was calculated by multiplying the economic fundamentals $\text{Presidental Fundamentals}_i$ for $i = 2020$ by 0.1 and state partisan lean by 0.9, then summing those two values.

We predicted $\text{Incumbent Party Vote Share}_{2020}$ (i.e. Republican Party's national vote share in the 2020 election) from a linear regression model that had historical election year's June approval rating of the incumbent party, three month stock growth, and 2nd quarter real income growth as predictors. We borrowed this idea from the "Time for Change" model which assumes that presidential election is referendum on the performance of incumbent president (Alan Abramowitz, 2008). Then we used 100 minus $\text{Incumbent Party Vote Share}_{2020}$ to obtain the predicted vote share for Democrats ($\text{Presidential Fundamentals}_{2020}$).

### Raw Model Output

```
##                    mean        sd          2.5%      97.5%
## Sigma[1,1]    0.5926277 0.4169338  0.1401836021  1.7274851
## Sigma[2,1]    0.4251874 0.3387688 -0.0002283602  1.2701249
## Sigma[3,1]    0.2440198 0.3449571 -0.2321267767  1.1510161
## Sigma[4,1]    0.3132443 0.2933798 -0.0848882218  1.0270067
## Sigma[5,1]    0.3266411 0.2721255 -0.0305401182  1.0137364
```

```
## Sigma[6,1]    0.3040325 0.2888737 -0.0476364569  1.0713749
## Sigma[7,1]    0.2275937 0.2554548 -0.1548570273  0.8166067
## Sigma[8,1]    0.2820362 0.3265992 -0.1752488754  1.0902519
## Sigma[9,1]    0.4943119 0.4183646  0.0524378585  1.5207589
## Sigma[10,1]   0.3178951 0.2708774 -0.0233583632  1.0321280
## Sigma[11,1]   0.3589893 0.2818443 -0.0119504340  1.1193115
## Sigma[12,1]   0.4009916 0.3857631 -0.0477329538  1.3872376
## Sigma[13,1]   0.3061326 0.2816699 -0.0846494342  1.0119030
## Sigma[1,2]    0.4251874 0.3387688 -0.0002283602  1.2701249
## Sigma[2,2]    0.6284036 0.4727188  0.1453472902  1.8809586
## Sigma[3,2]    0.1956539 0.3666133 -0.3847343035  1.1170539
## Sigma[4,2]    0.3845201 0.3332699  0.0008520579  1.2503345
## Sigma[5,2]    0.4174954 0.3186108  0.0545438564  1.2107249
## Sigma[6,2]    0.3220976 0.2888364 -0.0473801960  1.0693034
## Sigma[7,2]    0.2918273 0.2863269 -0.0675107533  1.0166839
## Sigma[8,2]    0.3521511 0.3605598 -0.1198860302  1.2946100
## Sigma[9,2]    0.4418112 0.3873470 -0.0269089642  1.3853154
## Sigma[10,2]   0.3762308 0.3015611  0.0268614225  1.1961907
## Sigma[11,2]   0.4609680 0.3381195  0.0744841422  1.3865660
## Sigma[12,2]   0.3059777 0.3749321 -0.2145964918  1.2486763
## Sigma[13,2]   0.3581039 0.2993956 -0.0231285944  1.1027838
## Sigma[1,3]    0.2440198 0.3449571 -0.2321267767  1.1510161
## Sigma[2,3]    0.1956539 0.3666133 -0.3847343035  1.1170539
## Sigma[3,3]    0.5615323 0.5228857  0.1159923302  2.0302963
## Sigma[4,3]    0.1460615 0.3168338 -0.4461193730  0.8921857
## Sigma[5,3]    0.2050582 0.3072212 -0.2677881563  0.9652456
## Sigma[6,3]    0.1599571 0.2863692 -0.3170963647  0.9039876
## Sigma[7,3]    0.2895306 0.2892311 -0.0669233363  1.1112296
## Sigma[8,3]    0.1030601 0.3600340 -0.5429747553  0.9347838
## Sigma[9,3]    0.2033842 0.4127043 -0.4100539557  1.1625597
## Sigma[10,3]   0.2314251 0.3251117 -0.2175799164  1.0570924
## Sigma[11,3]   0.1799239 0.3059580 -0.3750970240  0.9211500
## Sigma[12,3]   0.2051472 0.3715473 -0.4130821536  1.1013664
## Sigma[13,3]   0.1598724 0.2796824 -0.3101837479  0.8603737
## Sigma[1,4]    0.3132443 0.2933798 -0.0848882218  1.0270067
## Sigma[2,4]    0.3845201 0.3332699  0.0008520579  1.2503345
## Sigma[3,4]    0.1460615 0.3168338 -0.4461193730  0.8921857
## Sigma[4,4]    0.5531209 0.4611783  0.1189037976  1.7968496
## Sigma[5,4]    0.4247840 0.3346626  0.0701215819  1.2988731
## Sigma[6,4]    0.3732796 0.3059434  0.0470746496  1.1820607
## Sigma[7,4]    0.3160422 0.2931324 -0.0312562443  1.0494564
## Sigma[8,4]    0.3651697 0.3406580 -0.0328973357  1.2633326
## Sigma[9,4]    0.3033084 0.3504380 -0.1487407521  1.1789423
## Sigma[10,4]   0.3869015 0.3047337  0.0464524121  1.1546177
## Sigma[11,4]   0.4262938 0.3172997  0.0647142114  1.2820444
## Sigma[12,4]   0.2657040 0.4000015 -0.2300013413  1.3090124
## Sigma[13,4]   0.4067303 0.3554714  0.0488870334  1.3077756
## Sigma[1,5]    0.3266411 0.2721255 -0.0305401182  1.0137364
## Sigma[2,5]    0.4174954 0.3186108  0.0545438564  1.2107249
## Sigma[3,5]    0.2050582 0.3072212 -0.2677881563  0.9652456
## Sigma[4,5]    0.4247840 0.3346626  0.0701215819  1.2988731
## Sigma[5,5]    0.4813892 0.3280486  0.1284036536  1.3577086
```

```
## Sigma[6,5]    0.3538441 0.2730992  0.0439592847  1.0772349
## Sigma[7,5]    0.3185697 0.2591231  0.0047367015  0.9961968
## Sigma[8,5]    0.3480787 0.3005127 -0.0294529672  1.0997538
## Sigma[9,5]    0.3267678 0.3076279 -0.0640653402  1.0958794
## Sigma[10,5]   0.3919361 0.2828736  0.0721733143  1.1451009
## Sigma[11,5]   0.4325306 0.2909003  0.0921156048  1.1724666
## Sigma[12,5]   0.2527650 0.3124424 -0.1687341159  1.0423785
## Sigma[13,5]   0.3926725 0.2899983  0.0643848731  1.1564069
## Sigma[1,6]    0.3040325 0.2888737 -0.0476364569  1.0713749
## Sigma[2,6]    0.3220976 0.2888364 -0.0473801960  1.0693034
## Sigma[3,6]    0.1599571 0.2863692 -0.3170963647  0.9039876
## Sigma[4,6]    0.3732796 0.3059434  0.0470746496  1.1820607
## Sigma[5,6]    0.3538441 0.2730992  0.0439592847  1.0772349
## Sigma[6,6]    0.4479472 0.3320102  0.1143899911  1.3378654
## Sigma[7,6]    0.2611669 0.2375416 -0.0420831327  0.8503527
## Sigma[8,6]    0.3725363 0.3305535  0.0277772075  1.2645876
## Sigma[9,6]    0.2486553 0.2823299 -0.1569249338  0.9551400
## Sigma[10,6]   0.3560221 0.2731348  0.0603535392  1.1225273
## Sigma[11,6]   0.3847001 0.2780287  0.0616680229  1.1358873
## Sigma[12,6]   0.2426971 0.3381054 -0.1685774626  1.1419630
## Sigma[13,6]   0.3844812 0.2936880  0.0740909417  1.1803948
## Sigma[1,7]    0.2275937 0.2554548 -0.1548570273  0.8166067
## Sigma[2,7]    0.2918273 0.2863269 -0.0675107533  1.0166839
## Sigma[3,7]    0.2895306 0.2892311 -0.0669233363  1.1112296
## Sigma[4,7]    0.3160422 0.2931324 -0.0312562443  1.0494564
## Sigma[5,7]    0.3185697 0.2591231  0.0047367015  0.9961968
## Sigma[6,7]    0.2611669 0.2375416 -0.0420831327  0.8503527
## Sigma[7,7]    0.3996740 0.2825407  0.1005733042  1.1042876
## Sigma[8,7]    0.2331518 0.2688362 -0.1533396025  0.8874913
## Sigma[9,7]    0.2172113 0.2772271 -0.1574032481  0.8784702
## Sigma[10,7]   0.3106344 0.2510559  0.0107480615  0.9606069
## Sigma[11,7]   0.3031140 0.2592399 -0.0332402903  0.9789891
## Sigma[12,7]   0.2193556 0.2997994 -0.1699836537  0.9870879
## Sigma[13,7]   0.2916892 0.2554411 -0.0194041517  0.9498543
## Sigma[1,8]    0.2820362 0.3265992 -0.1752488754  1.0902519
## Sigma[2,8]    0.3521511 0.3605598 -0.1198860302  1.2946100
## Sigma[3,8]    0.1030601 0.3600340 -0.5429747553  0.9347838
## Sigma[4,8]    0.3651697 0.3406580 -0.0328973357  1.2633326
## Sigma[5,8]    0.3480787 0.3005127 -0.0294529672  1.0997538
## Sigma[6,8]    0.3725363 0.3305535  0.0277772075  1.2645876
## Sigma[7,8]    0.2331518 0.2688362 -0.1533396025  0.8874913
## Sigma[8,8]    0.5481863 0.4555987  0.1150046040  1.7984107
## Sigma[9,8]    0.2322145 0.3299794 -0.2663684193  1.0547727
## Sigma[10,8]   0.3236458 0.3054760 -0.0998811413  1.1261311
## Sigma[11,8]   0.3895159 0.3216926 -0.0221792702  1.2321954
## Sigma[12,8]   0.2370998 0.3864578 -0.2651359762  1.1599556
## Sigma[13,8]   0.3875809 0.3254926  0.0276139110  1.2341771
## Sigma[1,9]    0.4943119 0.4183646  0.0524378585  1.5207589
## Sigma[2,9]    0.4418112 0.3873470 -0.0269089642  1.3853154
## Sigma[3,9]    0.2033842 0.4127043 -0.4100539557  1.1625597
## Sigma[4,9]    0.3033084 0.3504380 -0.1487407521  1.1789423
## Sigma[5,9]    0.3267678 0.3076279 -0.0640653402  1.0958794
```

```
## Sigma[6,9]    0.2486553 0.2823299 -0.1569249338  0.9551400
## Sigma[7,9]    0.2172113 0.2772271 -0.1574032481  0.8784702
## Sigma[8,9]    0.2322145 0.3299794 -0.2663684193  1.0547727
## Sigma[9,9]    0.6539936 0.5863633  0.1367808333  2.1184583
## Sigma[10,9]   0.2887912 0.2929506 -0.0820150392  1.0193222
## Sigma[11,9]   0.3451387 0.3059127 -0.0598508639  1.1328531
## Sigma[12,9]   0.4071460 0.4155616 -0.0576674367  1.4614689
## Sigma[13,9]   0.2788308 0.3053013 -0.1420735774  1.0041301
## Sigma[1,10]   0.3178951 0.2708774 -0.0233583632  1.0321280
## Sigma[2,10]   0.3762308 0.3015611  0.0268614225  1.1961907
## Sigma[3,10]   0.2314251 0.3251117 -0.2175799164  1.0570924
## Sigma[4,10]   0.3869015 0.3047337  0.0464524121  1.1546177
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## Sigma[6,10]   0.3560221 0.2731348  0.0603535392  1.1225273
## Sigma[7,10]   0.3106344 0.2510559  0.0107480615  0.9606069
## Sigma[8,10]   0.3236458 0.3054760 -0.0998811413  1.1261311
## Sigma[9,10]   0.2887912 0.2929506 -0.0820150392  1.0193222
## Sigma[10,10]  0.4575235 0.3234459  0.1171387406  1.3386603
## Sigma[11,10]  0.4057823 0.2775314  0.0719267095  1.1372539
## Sigma[12,10]  0.2210826 0.3000177 -0.1870721665  0.9442876
## Sigma[13,10]  0.3766181 0.2757837  0.0671847989  1.0786055
## Sigma[1,11]   0.3589893 0.2818443 -0.0119504340  1.1193115
## Sigma[2,11]   0.4609680 0.3381195  0.0744841422  1.3865660
## Sigma[3,11]   0.1799239 0.3059580 -0.3750970240  0.9211500
## Sigma[4,11]   0.4262938 0.3172997  0.0647142114  1.2820444
## Sigma[5,11]   0.4325306 0.2909003  0.0921156048  1.1724666
## Sigma[6,11]   0.3847001 0.2780287  0.0616680229  1.1358873
## Sigma[7,11]   0.3031140 0.2592399 -0.0332402903  0.9789891
## Sigma[8,11]   0.3895159 0.3216926 -0.0221792702  1.2321954
## Sigma[9,11]   0.3451387 0.3059127 -0.0598508639  1.1328531
## Sigma[10,11]  0.4057823 0.2775314  0.0719267095  1.1372539
## Sigma[11,11]  0.5235450 0.3399323  0.1358923790  1.4453185
## Sigma[12,11]  0.2625726 0.3243554 -0.1881208387  1.0742784
## Sigma[13,11]  0.4142582 0.2871448  0.0728608906  1.1408192
## Sigma[1,12]   0.4009916 0.3857631 -0.0477329538  1.3872376
## Sigma[2,12]   0.3059777 0.3749321 -0.2145964918  1.2486763
## Sigma[3,12]   0.2051472 0.3715473 -0.4130821536  1.1013664
## Sigma[4,12]   0.2657040 0.4000015 -0.2300013413  1.3090124
## Sigma[5,12]   0.2527650 0.3124424 -0.1687341159  1.0423785
## Sigma[6,12]   0.2426971 0.3381054 -0.1685774626  1.1419630
## Sigma[7,12]   0.2193556 0.2997994 -0.1699836537  0.9870879
## Sigma[8,12]   0.2370998 0.3864578 -0.2651359762  1.1599556
## Sigma[9,12]   0.4071460 0.4155616 -0.0576674367  1.4614689
## Sigma[10,12]  0.2210826 0.3000177 -0.1870721665  0.9442876
## Sigma[11,12]  0.2625726 0.3243554 -0.1881208387  1.0742784
## Sigma[12,12]  0.6280694 0.5767020  0.1243429154  2.2033130
## Sigma[13,12]  0.2544873 0.3577528 -0.1868714233  1.1438745
## Sigma[1,13]   0.3061326 0.2816699 -0.0846494342  1.0119030
## Sigma[2,13]   0.3581039 0.2993956 -0.0231285944  1.1027838
## Sigma[3,13]   0.1598724 0.2796824 -0.3101837479  0.8603737
## Sigma[4,13]   0.4067303 0.3554714  0.0488870334  1.3077756
## Sigma[5,13]   0.3926725 0.2899983  0.0643848731  1.1564069
```

```
## Sigma[6,13]   0.3844812 0.2936880  0.0740909417  1.1803948
## Sigma[7,13]   0.2916892 0.2554411 -0.0194041517  0.9498543
## Sigma[8,13]   0.3875809 0.3254926  0.0276139110  1.2341771
## Sigma[9,13]   0.2788308 0.3053013 -0.1420735774  1.0041301
## Sigma[10,13]  0.3766181 0.2757837  0.0671847989  1.0786055
## Sigma[11,13]  0.4142582 0.2871448  0.0728608906  1.1408192
## Sigma[12,13]  0.2544873 0.3577528 -0.1868714233  1.1438745
## Sigma[13,13]  0.4732744 0.3452920  0.1123685507  1.3645763
## sigma2_y[1]   2.0631807 0.4200509  1.3870137066  2.9433971
## sigma2_y[2]   1.9571972 0.3755882  1.3275332882  2.8096934
## sigma2_y[3]   2.1106819 0.4542211  1.4196045776  3.2142118
## sigma2_y[4]   2.0630892 0.4452591  1.3583745536  3.1091737
## sigma2_y[5]   2.2417661 0.4562729  1.5720275705  3.2751637
## sigma2_y[6]   1.9866924 0.4300040  1.2666244420  2.9180791
## sigma2_y[7]   1.8909327 0.3703109  1.2359783462  2.6689607
## sigma2_y[8]   1.9989117 0.4777182  1.2531722490  2.9656064
## sigma2_y[9]   2.0168268 0.4603962  1.2751520466  3.0331654
## sigma2_y[10]  1.9445118 0.4149592  1.2135413636  2.8749040
## sigma2_y[11]  1.9940210 0.3782283  1.3578814364  2.8688157
## sigma2_y[12]  1.9642988 0.4210706  1.2395456762  2.8454045
## sigma2_y[13]  1.9782000 0.3692536  1.3651748113  2.8174252
## theta[1,1]   48.0194135 2.0836772 43.6142452243 51.9545187
## theta[2,1]   48.7149570 2.1791798 44.1220274640 53.1787715
## theta[3,1]   48.0160544 2.1220506 43.5233782490 51.8645864
## theta[4,1]   47.8690570 2.2645322 42.7809010828 51.4733646
## theta[5,1]   51.4117743 1.7444836 48.3715016168 54.8335647
## theta[6,1]   51.4797346 1.7345245 47.7037921433 54.2858014
## theta[7,1]   49.3298396 1.7453197 46.0742712849 52.7612071
## theta[8,1]   53.1836658 2.8757671 46.5072760843 58.1831573
## theta[9,1]   50.1423759 2.6569660 44.6985116032 55.1245100
## theta[10,1]  46.0793044 1.7337032 42.8994507747 49.2815498
## theta[11,1]  50.1685276 1.6677493 46.8360317555 53.3025424
## theta[12,1]  47.1694021 2.5359719 41.2455675411 51.5600100
## theta[13,1]  51.0595498 1.8395423 47.2871151522 54.2248898
## theta[1,2]   48.2005858 2.0420682 43.8034771616 51.9797879
## theta[2,2]   48.8684490 2.1569324 44.3032530345 53.0808257
## theta[3,2]   48.1479844 2.0889905 43.9121391212 51.9027261
## theta[4,2]   48.0275491 2.2322704 42.9369767614 51.6170428
## theta[5,2]   51.5683780 1.7453916 48.3746868359 54.9701871
## theta[6,2]   51.6227860 1.7148365 47.7898120819 54.3690313
## theta[7,2]   49.4577184 1.7248713 46.2599464251 52.7922985
## theta[8,2]   53.3276536 2.8151959 46.7906349607 58.2701408
## theta[9,2]   50.3317166 2.5936220 44.9718371297 54.9993546
## theta[10,2]  46.2298405 1.7300599 42.9683348256 49.5578765
## theta[11,2]  50.3255967 1.6732408 46.9658550466 53.3246325
## theta[12,2]  47.3206680 2.4867933 41.3438755931 51.6125176
## theta[13,2]  51.2237174 1.8131468 47.3180633686 54.2517714
## theta[1,3]   48.3867365 1.9873948 44.1203522786 52.0785041
## theta[2,3]   49.0337120 2.1062678 44.5593745095 53.0271219
## theta[3,3]   48.2622922 2.0435096 43.8308527437 51.9508104
## theta[4,3]   48.2152660 2.2012105 43.3628798760 51.8157023
## theta[5,3]   51.7534159 1.7055923 48.5233937737 54.9733359
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## theta[6,3]   51.7905340 1.6992881 48.1352722040 54.6024736
## theta[7,3]   49.6023035 1.6784783 46.4397515349 52.9722285
## theta[8,3]   53.4769845 2.7739353 47.1295992703 58.4456949
## theta[9,3]   50.5230305 2.5429008 45.0546645921 55.0281471
## theta[10,3]  46.4011767 1.7129774 43.1411531336 49.7113688
## theta[11,3]  50.5183460 1.6449181 47.0833111543 53.4588188
## theta[12,3]  47.4767831 2.4206002 41.5497196225 51.6475773
## theta[13,3]  51.4014135 1.7674557 47.5695651031 54.4440405
## theta[1,4]   48.5707330 1.9353794 44.4168449993 52.0322179
## theta[2,4]   49.1961993 2.0074119 44.8131742379 52.9146955
## theta[3,4]   48.3983131 2.0027347 44.2165300133 52.0064287
## theta[4,4]   48.3861388 2.1555568 43.5736171324 51.9106645
## theta[5,4]   51.9200685 1.6694016 48.6002339856 55.1127822
## theta[6,4]   51.9508465 1.6713573 48.3284662104 54.7783641
## theta[7,4]   49.7365093 1.6491807 46.4720644545 52.9396431
## theta[8,4]   53.6427043 2.7431981 47.4308555796 58.6304251
## theta[9,4]   50.7035760 2.4612564 45.3328080620 55.1008541
## theta[10,4]  46.5663475 1.6749146 43.3351439244 49.8136818
## theta[11,4]  50.6891647 1.6064510 47.2988606305 53.5170652
## theta[12,4]  47.6330833 2.3158721 42.1023612914 51.6338072
## theta[13,4]  51.5811698 1.7089813 47.9988245538 54.5081859
## theta[1,5]   48.7647074 1.8546090 44.6546625466 52.1391024
## theta[2,5]   49.3747381 1.9185482 45.2611049983 52.8848838
## theta[3,5]   48.5366756 1.9559349 44.3183639250 52.0966332
## theta[4,5]   48.5712361 2.0871839 44.0876541414 52.1052023
## theta[5,5]   52.0951286 1.6147949 48.9554744031 55.1769550
## theta[6,5]   52.1273479 1.6280528 48.5685423901 54.9758469
## theta[7,5]   49.8756669 1.6023593 46.6957057742 52.9290806
## theta[8,5]   53.8070479 2.6850806 47.7086681293 58.5789683
## theta[9,5]   50.9007698 2.3514359 45.8796805805 55.1071589
## theta[10,5]  46.7294354 1.6437330 43.5016720028 49.9597897
## theta[11,5]  50.8804829 1.5573108 47.4925741384 53.6961799
## theta[12,5]  47.8121703 2.2182067 42.5900871060 51.5624664
## theta[13,5]  51.7603999 1.6556004 48.2127187839 54.6172484
## theta[1,6]   48.9726927 1.7766647 44.9986255502 52.1191735
## theta[2,6]   49.5631812 1.8239610 45.6019822505 52.7613317
## theta[3,6]   48.6740369 1.8415674 44.7198556631 51.9331442
## theta[4,6]   48.7540986 2.0006101 44.4250094773 52.1654704
## theta[5,6]   52.2623727 1.5285166 49.2250276275 55.1395534
## theta[6,6]   52.3059776 1.5728479 48.8130941969 55.1088310
## theta[7,6]   50.0207367 1.5244430 46.9727550983 52.9726310
## theta[8,6]   53.9819940 2.5988676 48.0248996011 58.6935114
## theta[9,6]   51.0899932 2.2454450 46.1817758868 55.0506378
## theta[10,6]  46.9045736 1.5761369 43.8755068091 49.9669511
## theta[11,6]  51.0808320 1.4893209 47.8839140304 53.7451700
## theta[12,6]  47.9923703 2.1235128 42.9712522088 51.6088889
## theta[13,6]  51.9522960 1.5719358 48.5451984487 54.8210730
## theta[1,7]   49.1585068 1.6566612 45.5432487287 52.1905979
## theta[2,7]   49.7041678 1.6996595 46.0558367172 52.8118195
## theta[3,7]   48.8109139 1.7423934 45.0349420454 51.8865980
## theta[4,7]   48.9122033 1.8972867 44.8714682037 52.2334420
## theta[5,7]   52.4112395 1.4422377 49.6681793564 55.3206037
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## theta[6,7]   52.4685807 1.5077468 49.2332393246 55.1339865
## theta[7,7]   50.1343777 1.4571926 47.1959278506 52.9616529
## theta[8,7]   54.1304708 2.4886633 48.5637582786 58.6564767
## theta[9,7]   51.2740041 2.1202984 46.7060924740 55.1718800
## theta[10,7]  47.0637671 1.5154936 44.1088333783 50.0027533
## theta[11,7]  51.2496517 1.4095868 48.2462000194 53.9040233
## theta[12,7]  48.1691269 2.0260135 43.4506779460 51.7618420
## theta[13,7]  52.1297994 1.4923165 48.9402837829 54.7887794
## theta[1,8]   49.3574878 1.5367899 46.0540282041 52.1754454
## theta[2,8]   49.8576965 1.5847662 46.4147912871 52.6681518
## theta[3,8]   48.9297587 1.6296276 45.3967750908 51.7754042
## theta[4,8]   49.0888348 1.7645509 45.3218468678 52.1413260
## theta[5,8]   52.5790224 1.3235070 49.9366673373 55.1006069
## theta[6,8]   52.6616338 1.4341366 49.6662983896 55.2005105
## theta[7,8]   50.2605664 1.3530552 47.4786929640 52.8114127
## theta[8,8]   54.3035110 2.3846770 48.9164215314 58.6020836
## theta[9,8]   51.4713096 1.9699409 47.1989364039 55.0950369
## theta[10,8]  47.2299275 1.4509282 44.4138790209 50.0503719
## theta[11,8]  51.4337962 1.3407777 48.6137556427 53.9119833
## theta[12,8]  48.3331351 1.9175354 43.8908989763 51.7227505
## theta[13,8]  52.3160909 1.3987612 49.3736788625 54.8695010
## theta[1,9]   49.5536406 1.4040749 46.4545278834 52.0657258
## theta[2,9]   50.0129344 1.3928513 46.9352400519 52.5109378
## theta[3,9]   49.0531188 1.4921050 45.9026779466 51.8943749
## theta[4,9]   49.2736179 1.5780902 45.9060511384 52.0322617
## theta[5,9]   52.7385434 1.1731980 50.3014535411 54.9957947
## theta[6,9]   52.8448606 1.3399070 50.0108149698 55.1646446
## theta[7,9]   50.3790665 1.2272601 47.7641405628 52.7635325
## theta[8,9]   54.4809611 2.2518099 49.5302889243 58.4684450
## theta[9,9]   51.6395833 1.8218429 47.7967844896 55.0159388
## theta[10,9]  47.4029544 1.3395265 44.7209261322 50.0211180
## theta[11,9]  51.6160344 1.2057453 49.1044171624 53.7757336
## theta[12,9]  48.5014563 1.7616794 44.5381343241 51.5062061
## theta[13,9]  52.5171991 1.2572524 49.8448385076 54.8048237
## theta[1,10]  49.7594271 1.2310446 47.0728821306 51.9668135
## theta[2,10]  50.1535281 1.1917890 47.6593874490 52.3595717
## theta[3,10]  49.1832413 1.3280663 46.2393595106 51.5725689
## theta[4,10]  49.4442818 1.4098870 46.3593486120 51.9885365
## theta[5,10]  52.8936362 0.9976061 50.8675166362 54.7795306
## theta[6,10]  53.0177448 1.2077936 50.4605090190 55.2500567
## theta[7,10]  50.4882818 1.0751332 48.1914565591 52.5514301
## theta[8,10]  54.6403878 2.1007692 50.0859093764 58.3446227
## theta[9,10]  51.8189360 1.6493946 48.4482965913 54.9082434
## theta[10,10] 47.5667879 1.2116454 45.2093736670 49.9930620
## theta[11,10] 51.7907474 1.0568310 49.5873790253 53.7475703
## theta[12,10] 48.6817655 1.5925164 45.1271847886 51.5641189
## theta[13,10] 52.6944925 1.1032371 50.2936180521 54.8021702
## theta[1,11]  49.9661692 0.9516389 47.9960097412 51.8140815
## theta[2,11]  50.3224773 0.8554144 48.4965007745 51.9403330
## theta[3,11]  49.2927379 1.1106281 46.9445843580 51.2724890
## theta[4,11]  49.6264932 1.1707811 47.1293585564 51.8375866
## theta[5,11]  53.0691029 0.7312868 51.6573689528 54.4427241
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## theta[6,11]  53.2120903 1.0418536 51.0475684793 55.1102195
## theta[7,11]  50.6079341 0.8711309 48.7445406313 52.2437767
## theta[8,11]  54.8163646 1.9210365 50.8017819536 58.2739088
## theta[9,11]  51.9930305 1.4463397 49.1690714657 54.8439116
## theta[10,11] 47.7497729 1.0288299 45.7476826106 49.8254993
## theta[11,11] 51.9958741 0.8356289 50.3247715116 53.5675725
## theta[12,11] 48.8412235 1.3873969 45.8444838106 51.3800816
## theta[13,11] 52.8997410 0.8910864 51.0721090416 54.6489848
## theta[1,12]  50.8073194 0.8929034 48.9266478953 52.5351180
## theta[2,12]  51.1581960 0.7145099 49.7424451081 52.5391973
## theta[3,12]  49.6929103 0.8478207 47.9931275191 51.3597887
## theta[4,12]  50.3099410 0.9966297 48.3167033370 52.2492006
## theta[5,12]  53.7760523 0.7213931 52.3799706009 55.2211548
## theta[6,12]  53.8461306 0.8993057 52.0771206535 55.5965702
## theta[7,12]  51.0961063 0.6910615 49.7490171584 52.4678429
## theta[8,12]  55.4461173 1.7127478 51.8084536712 58.4912699
## theta[9,12]  52.7611782 1.3158479 50.2906203938 55.3645717
## theta[10,12] 48.4202830 0.9075974 46.6915739160 50.2460817
## theta[11,12] 52.7746706 0.6959655 51.3957556656 54.1688134
## theta[12,12] 49.4597801 1.1538186 47.0274446589 51.6314529
## theta[13,12] 53.6008666 0.7407441 52.2044904378 55.1405270
## theta[1,13]  50.9886633 0.8663505 49.2364143716 52.6319682
## theta[2,13]  51.2202051 0.7274560 49.7382350821 52.6187791
## theta[3,13]  49.5530022 0.8816141 47.6828061972 51.1856682
## theta[4,13]  50.2219953 1.0008149 48.1698201332 52.1474025
## theta[5,13]  53.6885556 0.7634024 52.2036032741 55.2448972
## theta[6,13]  53.7740556 0.8902820 52.0076817238 55.4585143
## theta[7,13]  50.8912552 0.6831546 49.4555847172 52.2137851
## theta[8,13]  55.4040566 1.7018004 51.7664354113 58.4559123
## theta[9,13]  52.9169946 1.2596596 50.5299571975 55.4515686
## theta[10,13] 48.3401017 0.9151566 46.5542840023 50.1526908
## theta[11,13] 52.7161556 0.7650869 51.2035727439 54.1543804
## theta[12,13] 49.5525350 1.1079575 47.1682928815 51.6012106
## theta[13,13] 53.4778166 0.7696114 51.9435932070 55.1012395
## theta[1,14]  51.4648000 0.7550408 50.0386970331 52.9710481
## theta[2,14]  51.6938683 0.7509458 50.1936359590 53.1815212
## theta[3,14]  49.7764494 0.7947843 48.1425408737 51.2645833
## theta[4,14]  50.5324189 0.9214005 48.6613692002 52.3982419
## theta[5,14]  54.0187490 0.7320607 52.6038395665 55.5152281
## theta[6,14]  54.0370062 0.8205407 52.3316131183 55.5872900
## theta[7,14]  51.2007786 0.6502664 49.8356729464 52.4780861
## theta[8,14]  55.6764636 1.6310859 52.1982587413 58.7151787
## theta[9,14]  53.3623411 1.1325959 51.2420723234 55.6790194
## theta[10,14] 48.6482817 0.8661439 46.9345189925 50.3479949
## theta[11,14] 53.0588751 0.7190628 51.6170233598 54.4562143
## theta[12,14] 49.9219786 0.9460095 47.9439401942 51.6501094
## theta[13,14] 53.7380478 0.7231375 52.3367963259 55.2312475
## theta[1,15]  51.9843888 0.6366638 50.7800520750 53.2719258
## theta[2,15]  52.2388752 0.5098974 51.2399715292 53.2856610
## theta[3,15]  50.3602718 0.7330088 48.9313724954 51.8379573
## theta[4,15]  50.9068835 0.6589941 49.6327789732 52.2475710
## theta[5,15]  54.4612427 0.5264018 53.4628913761 55.5564513
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## theta[6,15]  54.3760957 0.6390550 53.1256250487 55.5923037
## theta[7,15]  51.6792130 0.5028657 50.7009380484 52.6752387
## theta[8,15]  56.0071125 1.4150643 52.9827246655 58.6840602
## theta[9,15]  53.8475165 0.9462986 52.0281172072 55.7907119
## theta[10,15] 49.0907926 0.6681314 47.8128269946 50.4299165
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## theta[13,15] 54.0877773 0.4918699 53.1306765469 55.0738723
## theta[1,16]  51.9906093 0.6497328 50.7509923896 53.3313426
## theta[2,16]  52.0354065 0.5708449 50.8987419165 53.1968899
## theta[3,16]  50.4473033 0.7371756 49.0295467021 51.8759288
## theta[4,16]  50.6375945 0.6605114 49.3263812554 51.9429314
## theta[5,16]  54.2159494 0.5201690 53.2234944322 55.2844758
## theta[6,16]  54.1421187 0.6961035 52.8107241475 55.4408400
## theta[7,16]  51.5943173 0.4843420 50.6704570955 52.5678609
## theta[8,16]  55.7518340 1.3886867 52.6568035089 58.4407107
## theta[9,16]  53.7427518 0.9572301 51.8677886518 55.6546891
## theta[10,16] 48.9048154 0.7061873 47.5233714926 50.3030763
## theta[11,16] 53.1678894 0.4487227 52.2733772901 54.0187198
## theta[12,16] 50.2898247 0.6906292 48.8725849282 51.5723212
## theta[13,16] 53.8039946 0.4824179 52.8776348305 54.7682133
## theta[1,17]  51.7332601 0.6892863 50.3941481108 53.2351292
## theta[2,17]  51.5454396 0.7249223 50.1068935213 52.9509078
## theta[3,17]  50.4760674 0.8514468 48.7313478923 52.1529665
## theta[4,17]  50.0857268 0.7816024 48.5171647075 51.5972024
## theta[5,17]  53.7110147 0.5738594 52.5402125035 54.8091502
## theta[6,17]  53.6977011 0.7707939 52.2280847386 55.1887042
## theta[7,17]  51.3325232 0.5581719 50.2242238682 52.3981928
## theta[8,17]  55.3017289 1.4170360 52.2268607997 57.9155884
## theta[9,17]  53.4256523 1.0172235 51.4683013610 55.4987954
## theta[10,17] 48.4841483 0.7871893 46.9305375125 49.9930699
## theta[11,17] 52.6164071 0.6198776 51.3134354804 53.8135786
## theta[12,17] 50.0837323 0.7772785 48.5436797013 51.6372720
## theta[13,17] 53.2897730 0.5994404 52.0498342489 54.4107594
## theta[1,18]  51.5319983 0.8110356 49.9369969280 53.1917433
## theta[2,18]  51.2633804 0.8492491 49.4984979931 52.8669601
## theta[3,18]  50.5053319 0.9561380 48.6267728773 52.3857961
## theta[4,18]  49.7780071 0.9245334 47.8100496732 51.4505537
## theta[5,18]  53.4559564 0.7326230 52.0097925796 54.8332963
## theta[6,18]  53.5116973 0.8727760 51.7471028362 55.1078994
## theta[7,18]  51.1977971 0.6465334 49.8683302941 52.4321006
## theta[8,18]  55.1082869 1.4681030 51.9156356114 57.8167960
## theta[9,18]  53.1578704 1.1340415 50.9944348408 55.4586737
## theta[10,18] 48.3052204 0.8877976 46.5253484196 49.9956075
## theta[11,18] 52.3797765 0.7944337 50.6622725117 53.8572919
## theta[12,18] 49.8488775 1.0174648 47.7442113491 51.8933214
## theta[13,18] 53.0907395 0.7397186 51.5219066925 54.4113526
## theta[1,19]  51.4112435 0.7530038 49.9884928835 52.9127197
## theta[2,19]  51.0848405 0.7666204 49.5083543085 52.5563658
## theta[3,19]  50.6285924 0.9820212 48.7867534135 52.5092913
## theta[4,19]  49.5832993 0.9162309 47.6311433671 51.2645995
## theta[5,19]  53.2885484 0.7385071 51.8076657078 54.6835269
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## theta[6,19]  53.4132538 0.8292212 51.7831150124 55.0027772
## theta[7,19]  51.1843101 0.7034635 49.7830698889 52.5434106
## theta[8,19]  55.0110523 1.4103815 51.9822522776 57.5496586
## theta[9,19]  52.9687501 1.0943290 50.8838709147 55.1894117
## theta[10,19] 48.2347860 0.8990597 46.4383018092 49.9248686
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## theta[13,19] 52.9990299 0.7149180 51.5141784076 54.3312910
## theta[1,20]  51.4363897 0.8279347 49.8065030272 53.0859428
## theta[2,20]  51.0618494 0.8362894 49.3397932314 52.6404751
## theta[3,20]  50.8529294 0.9785894 49.0702200433 52.9047971
## theta[4,20]  49.5086687 0.9721984 47.4853062769 51.2174899
## theta[5,20]  53.2524566 0.7706357 51.6494097501 54.7078328
## theta[6,20]  53.4361201 0.7874091 51.8724728248 54.9333443
## theta[7,20]  51.2781130 0.7241406 49.7965520756 52.6980280
## theta[8,20]  55.0190734 1.3961085 52.0102943507 57.5483981
## theta[9,20]  52.9129274 1.1382420 50.7232243223 55.2216628
## theta[10,20] 48.3003476 0.8976707 46.4941810566 49.9986712
## theta[11,20] 52.2436512 0.8419333 50.4688790212 53.7203226
## theta[12,20] 49.6812418 1.1768295 47.2348016535 51.9269707
## theta[13,20] 53.0319683 0.7719897 51.3714415040 54.4494659
## theta[1,21]  51.5259999 0.7625470 50.0579335068 53.0955008
## theta[2,21]  51.1191979 0.7525809 49.5614824211 52.5245915
## theta[3,21]  51.1537702 0.9091806 49.5389195781 53.0557340
## theta[4,21]  49.5105923 0.9331490 47.4851707956 51.1322069
## theta[5,21]  53.2929945 0.7071102 51.8394212027 54.5553159
## theta[6,21]  53.5676389 0.8042537 51.9638274299 55.0388052
## theta[7,21]  51.4431859 0.6467815 50.1682124724 52.7343776
## theta[8,21]  55.1251170 1.3002144 52.3769004712 57.5098160
## theta[9,21]  52.8921251 1.1051643 50.7591167040 55.0540054
## theta[10,21] 48.4402802 0.8653932 46.6960820411 50.1434248
## theta[11,21] 52.3489724 0.7915330 50.6783385685 53.7660853
## theta[12,21] 49.6881129 1.2127657 47.1927635822 52.0635334
## theta[13,21] 53.1548705 0.7639108 51.5507618815 54.5703110
## theta[1,22]  51.5868785 0.7363148 50.1658695419 53.0646195
## theta[2,22]  51.2815895 0.6647398 49.8657452398 52.5273098
## theta[3,22]  50.9962069 0.8433777 49.4694744556 52.8811155
## theta[4,22]  49.7014650 0.7793702 48.0690831793 51.1163274
## theta[5,22]  53.5560011 0.5567881 52.4130276892 54.5756240
## theta[6,22]  53.8330083 0.7107329 52.4183129876 55.1846331
## theta[7,22]  51.4936138 0.5549349 50.3951047730 52.5522372
## theta[8,22]  55.4172739 1.1621214 52.9613096517 57.6505545
## theta[9,22]  52.9064576 1.0369670 50.9067323216 54.9860789
## theta[10,22] 48.6740872 0.7519562 47.2236615105 50.1708506
## theta[11,22] 52.6281003 0.6692615 51.2546974527 53.8628249
## theta[12,22] 49.6173002 1.1270405 47.3080772973 51.7439460
## theta[13,22] 53.4535056 0.6360031 52.2326039502 54.7065241
## theta[1,23]  51.8192804 0.7281157 50.4201378636 53.3115141
## theta[2,23]  51.8667693 0.5400564 50.7910665279 52.9243708
## theta[3,23]  51.1055523 0.6784736 49.8365544765 52.5528090
## theta[4,23]  50.1962654 0.7338292 48.6514181722 51.5958571
## theta[5,23]  54.0613261 0.5541310 53.0145111627 55.1708954
```

```
## theta[6,23]  54.1365776 0.7375408 52.7477679001 55.6267147
## theta[7,23]  51.8378407 0.5612503 50.7657576432 52.9742289
## theta[8,23]  55.7923405 0.9664923 53.8141598331 57.7272965
## theta[9,23]  53.1457819 0.9544325 51.3418132698 55.0801706
## theta[10,23] 49.1141268 0.6510162 47.8667176803 50.4146761
## theta[11,23] 53.1201264 0.5491930 52.0384976958 54.1978069
## theta[12,23] 49.7177657 0.9931884 47.7368664054 51.7102701
## theta[13,23] 53.8397406 0.5910736 52.7299709628 55.0532780
## theta[1,24]  51.5492772 0.6385654 50.2839227406 52.7920190
## theta[2,24]  51.8726119 0.5521938 50.8228146817 52.9722832
## theta[3,24]  50.5359033 0.6114479 49.3323793053 51.7378299
## theta[4,24]  50.2268804 0.6423640 48.9873864016 51.4607594
## theta[5,24]  54.0315503 0.4996609 53.0788040499 55.0487275
## theta[6,24]  54.0597810 0.6650996 52.7373957998 55.3449930
## theta[7,24]  51.6569591 0.4647910 50.7545299855 52.5526924
## theta[8,24]  55.8437700 1.0986010 53.5317627324 58.0474071
## theta[9,24]  52.9040264 0.8078593 51.3332866259 54.5291211
## theta[10,24] 49.0075115 0.5688452 47.9357690482 50.1296406
## theta[11,24] 53.1664214 0.4589237 52.2174262912 54.0199720
## theta[12,24] 49.4000406 0.8270114 47.7274189637 50.9977105
## theta[13,24] 53.8319036 0.4586615 52.9688083517 54.7950311
## theta[1,25]  51.4161459 0.7823299 49.7587410785 52.9414598
## theta[2,25]  51.7856005 0.6789754 50.4342421831 53.0969589
## theta[3,25]  50.3562644 0.8346449 48.6640000828 51.9673082
## theta[4,25]  50.1316661 0.7847099 48.5558918205 51.6414296
## theta[5,25]  53.9046061 0.6592192 52.6147404239 55.2354657
## theta[6,25]  53.9306520 0.8013663 52.3564723659 55.4802355
## theta[7,25]  51.5105954 0.6482916 50.1949239762 52.7988983
## theta[8,25]  55.7583231 1.2369338 53.2272881405 58.1928356
## theta[9,25]  52.8445492 0.9395917 50.9496308352 54.6655769
## theta[10,25] 48.8985322 0.7407623 47.4159154625 50.3795218
## theta[11,25] 53.0554860 0.6609424 51.7374484929 54.3334404
## theta[12,25] 49.2880811 0.9992422 47.1913403396 51.1763443
## theta[13,25] 53.7010649 0.6621943 52.3866203332 54.9959922
## theta[1,26]  51.2578217 0.7374468 49.7333438807 52.6269270
## theta[2,26]  51.6526151 0.7070511 50.2434446602 53.0002236
## theta[3,26]  50.1680329 0.8944496 48.3102448939 51.8940947
## theta[4,26]  50.0117037 0.7321633 48.5012431018 51.3888958
## theta[5,26]  53.7606359 0.6203088 52.5622114919 54.9912395
## theta[6,26]  53.7752044 0.8048841 52.1366297419 55.3160056
## theta[7,26]  51.3622759 0.6748189 49.9894453727 52.6479511
## theta[8,26]  55.6420302 1.2683458 53.0355163181 58.1434638
## theta[9,26]  52.7411454 0.8677459 51.0328931087 54.4102899
## theta[10,26] 48.7613891 0.7134260 47.3539976406 50.1746084
## theta[11,26] 52.9113312 0.6218117 51.6748892055 54.1146326
## theta[12,26] 49.1591198 1.0064712 47.0745755042 51.0076855
## theta[13,26] 53.5379659 0.6446720 52.2701952571 54.7533763
## theta[1,27]  51.0761519 0.6329387 49.7590152061 52.2787242
## theta[2,27]  51.5691879 0.6580916 50.2274212486 52.8573972
## theta[3,27]  49.9819352 0.9096610 48.1144229547 51.6812423
## theta[4,27]  49.9825919 0.7794595 48.3841411581 51.4590167
## theta[5,27]  53.7059698 0.6145023 52.5140261039 54.9484744
```

```
## theta[6,27]  53.7194988 0.8187078 52.0974938824 55.2855512
## theta[7,27]  51.2660625 0.6774158 49.9297796190 52.6476564
## theta[8,27]  55.6240885 1.2804306 52.9538623472 58.1581180
## theta[9,27]  52.5859364 0.9400319 50.7139141982 54.3607232
## theta[10,27] 48.7102985 0.6710318 47.3704757327 50.0587400
## theta[11,27] 52.8594153 0.6042102 51.6525985224 54.0211021
## theta[12,27] 48.9866382 1.0320735 46.8254517351 50.8796392
## theta[13,27] 53.4660960 0.6136055 52.2597307155 54.6650362
## theta[1,28]  51.5504928 0.6515730 50.2195802384 52.8044283
## theta[2,28]  52.0446445 0.6604395 50.6701862343 53.2908029
## theta[3,28]  50.0643153 0.8797872 48.3189210860 51.7583340
## theta[4,28]  50.3563603 0.8254974 48.6735307980 51.9770465
## theta[5,28]  54.0646953 0.6836498 52.7483713645 55.4150987
## theta[6,28]  54.0345069 0.8389081 52.4013246846 55.6330151
## theta[7,28]  51.4741955 0.6637727 50.1215745301 52.7935611
## theta[8,28]  55.9489555 1.2686818 53.3848696152 58.4723131
## theta[9,28]  53.0096538 0.9613699 51.0716045561 54.9065521
## theta[10,28] 49.0543641 0.7274197 47.6200186987 50.4871733
## theta[11,28] 53.2739532 0.6684137 51.9187258058 54.6267775
## theta[12,28] 49.2808782 0.9616694 47.2321161189 51.0507042
## theta[13,28] 53.7884832 0.6711769 52.5424433688 55.1245163
## theta[1,29]  52.2300313 0.6280084 51.0291279148 53.4824088
## theta[2,29]  52.6727052 0.5915961 51.5375073941 53.8390905
## theta[3,29]  50.4177109 0.9930148 48.3634715690 52.2804135
## theta[4,29]  50.8473771 0.6815458 49.5323893414 52.2214199
## theta[5,29]  54.5539922 0.5726671 53.4672541922 55.7197350
## theta[6,29]  54.4534418 0.7636820 52.9279717082 55.9315270
## theta[7,29]  51.8331739 0.5597589 50.7341733765 52.9580707
## theta[8,29]  56.3622541 1.1610181 54.0598460306 58.7554720
## theta[9,29]  53.5952545 0.8716067 51.9392624476 55.3151240
## theta[10,29] 49.5378014 0.6606843 48.2250410570 50.8636896
## theta[11,29] 53.8120100 0.5370700 52.7929250441 54.8907263
## theta[12,29] 49.7010820 0.7606935 48.2024838933 51.2224253
## theta[13,29] 54.2166747 0.5531435 53.1477910962 55.3230307
## theta[1,30]  52.6513217 0.7359755 51.2679539259 54.1950704
## theta[2,30]  53.1784391 0.6839792 51.9122151520 54.6086305
## theta[3,30]  50.6733003 1.2030382 48.2386582814 52.9964342
## theta[4,30]  51.2543950 0.7549703 49.8177018446 52.7661563
## theta[5,30]  54.9492055 0.7143217 53.6343631251 56.3946687
## theta[6,30]  54.7639979 0.8824018 53.0084222770 56.4971519
## theta[7,30]  52.1725989 0.6471917 50.9183814392 53.4633062
## theta[8,30]  56.6686858 1.1766394 54.3527931306 59.1077607
## theta[9,30]  54.0103216 1.0231429 51.9654248586 56.0954251
## theta[10,30] 49.8964483 0.7596871 48.4486490462 51.4337556
## theta[11,30] 54.2407679 0.5747335 53.1613084893 55.4045407
## theta[12,30] 49.9714304 0.8088389 48.3697194163 51.5963996
## theta[13,30] 54.4975915 0.6343205 53.3278977151 55.8138512
## theta[1,31]  52.0256256 0.7330838 50.6325453506 53.4796399
## theta[2,31]  52.3509488 0.5761369 51.2228410016 53.4832495
## theta[3,31]  50.3009877 1.1619101 47.9521356913 52.5247082
## theta[4,31]  50.5819357 0.8174118 48.9501642601 52.1616251
## theta[5,31]  54.2746414 0.6455829 53.0194406469 55.5164824
```

```
## theta[6,31]  54.2055323 0.8627026 52.5183836623 55.9216718
## theta[7,31]  51.6138961 0.6158961 50.3639977573 52.8505602
## theta[8,31]  56.1075072 1.0957691 53.9451150085 58.2713166
## theta[9,31]  53.3416942 1.0658056 51.1963946869 55.4712585
## theta[10,31] 49.2899299 0.7633165 47.8325871726 50.8524960
## theta[11,31] 53.5364504 0.5709050 52.4025325693 54.6639845
## theta[12,31] 49.2737366 0.8458534 47.5907394420 50.9015152
## theta[13,31] 53.8634156 0.5834134 52.7181439098 55.0251801

## mean Rhat: 1.017614

## mean effective sample size: 860.6576
```

Since the outcome of the election is decided by Electoral College vote, for each set of MCMC samples of the two-party vote share by state, we use the predicted winner in each state $j$ on the day of the election ($\beta_{j1}$) and sum the electoral votes from those states for each candidate. For states not in the model, votes were given to the candidates based on how the states voted in 2016. The probability of President Trump winning re-election is the probability that he receives 270 or more electoral votes across simulations. (Note: Maine and Nebraska do not have a "winner-take-all" framework for allocating votes, but for simplicity, we assume that they have the same vote-allocating procedures as the other states).

## A.2 Senate Election Model

The purpose of the Senate model is to predict whether the U.S. Senate remains in Republican control (and to predict the outcome for the NC Senate race). The states included in the model are Alaska, Arizona, Colorado, Georgia, Iowa, Kansas, Maine, Michigan, Montana, North Carolina, South Carolina, and Texas. These states were chosen because they are deemed by political analysts as being competitive states for the Senate race in 2020. There is also a special election for the Senate in Georgia in addition to the regularly scheduled election. For the Georgia special election, our model sums up the support for Republican candidates and compares that to the sum of the support for Democratic candidates, and treats this election in the same way as the other states.

The response we will use is $Y_k$, the Democrat candidate's share of the two-party vote using the following model where the poll $k$ (of $K$=*fill in* polls) ended $t$ days (of $T = 30$ days) before the election and was conducted on state $j$ (of $J$=14 states of interest).

$Y_k \sim N(\beta_{jt}, \sigma^2_{yj})$
$\beta_{jt} \sim N(\beta_{jt-1}, \sigma^2_{\beta j})$
Priors for $\sigma^2_{yj}$: $\sigma^2_{yj} \sim InvGamma(\nu_u, \nu_y\tau_y)$
$\nu_y \sim Uniform(0, 100)$ and $\tau_y \sim Uniform(0, 100)$.
Priors for $\sigma^2_{\beta j}$ $\sigma^2_{\beta j} \sim InvGamma(\nu_\beta, \nu_\beta\tau_\beta))$
$\nu_\beta \sim Uniform(0, 100)$ and $\tau_\beta \sim Uniform(0, 100)$.
Priors for $\beta_{j1}$ $\beta_{j1} \sim N(\mu_j, \sigma^2)$
$\sigma^2 \sim InvGamma(0.5, 0.5)$
$\mu_j \sim N(h_j, 7.5^2)$
$h_j =$ Vote Share from Partisan Lean$_j$ + Incumbency Advantage$_j$

A hierarchical model was used, where the Democratic candidate's share of the two-party vote for poll $k$ on any given day $t$ for state $j$ is modeled as a normal distribution with mean $\beta_{jt}$ and variance $\sigma^2_{yj}$. A random walk was used to calculate $\beta_{jt}$ for each day $t$ before the election (up until 30 days before the election) from $\beta_{jt-1}$ and $\sigma_{\beta j}$. The mean $h_j$ of the normal prior on $mu_j$ (which is the mean of the normal prior on $\beta_{j1}$), was calculated by the sum of the Vote Share from Partisan Lean$_j$ and Incumbency Advantage$_j$.

### Raw Model Output

```
##                       mean        sd        2.5%      97.5%
```

```
## beta[1,1]       46.4507584 2.2433708 41.59965446 50.6435166
## beta[2,1]       52.3060134 2.1867985 46.72905116 55.6747117
## beta[3,1]       54.3407636 1.7697115 50.52703121 57.6127336
## beta[4,1]       48.6580340 1.6538193 45.60341428 51.9550692
## beta[5,1]       45.5772068 1.9532845 41.67556604 49.6975127
## beta[6,1]       50.7226677 1.6592584 47.18653081 53.7717883
## beta[7,1]       46.2249577 2.6063537 40.90148615 51.1973216
## beta[8,1]       52.0308945 2.2519892 47.08007333 55.9982111
## beta[9,1]       53.9554866 1.7664757 50.67107557 57.9060076
## beta[10,1]      47.5874120 2.0102568 43.45416459 51.5177091
## beta[11,1]      51.7831808 1.6530492 48.20280605 54.9907635
## beta[12,1]      48.8159096 1.9127126 45.15421200 52.8553178
## beta[13,1]      45.7833702 1.8480167 42.38430159 49.9298690
## beta[1,2]       46.4896459 2.2087097 41.73140473 50.7067420
## beta[2,2]       52.3289635 2.1332011 46.76305635 55.6280756
## beta[3,2]       54.3711092 1.7153990 50.70312546 57.5107585
## beta[4,2]       48.6820835 1.5900333 45.55906919 51.7720312
## beta[5,2]       45.5860786 1.8948276 41.78424188 49.4964545
## beta[6,2]       50.7492053 1.6070817 47.32136332 53.6024884
## beta[7,2]       46.2482261 2.5790858 40.95415039 51.1620338
## beta[8,2]       52.0342957 2.2127825 47.17982380 55.9353012
## beta[9,2]       53.9700156 1.7005589 50.93251803 57.7808375
## beta[10,2]      47.6105677 1.9664164 43.60635351 51.4052042
## beta[11,2]      51.8087928 1.5742675 48.42741143 54.8897353
## beta[12,2]      48.8315938 1.8713619 45.27158091 52.8845275
## beta[13,2]      45.8203025 1.8035136 42.50382068 49.8369427
## beta[1,3]       46.4972722 2.1748183 41.79078922 50.6082250
## beta[2,3]       52.3504019 2.0757404 46.89180959 55.5676551
## beta[3,3]       54.4025269 1.6743724 50.83741377 57.4734257
## beta[4,3]       48.7026871 1.5316694 45.66027082 51.6917491
## beta[5,3]       45.5939497 1.8587148 41.96836855 49.4744802
## beta[6,3]       50.7842206 1.5345155 47.56099990 53.5312636
## beta[7,3]       46.2874322 2.5704155 41.03692954 51.1878884
## beta[8,3]       52.0451801 2.1927681 47.31900495 56.0179210
## beta[9,3]       53.9616679 1.6422515 51.02280354 57.7440600
## beta[10,3]      47.6457081 1.9178816 43.67544160 51.3512848
## beta[11,3]      51.8224261 1.5060512 48.57959501 54.7184358
## beta[12,3]      48.8449458 1.8210090 45.44757736 52.9417926
## beta[13,3]      45.8366463 1.7529638 42.65491212 49.7247423
## beta[1,4]       46.5096325 2.1446964 41.93718701 50.5971583
## beta[2,4]       52.3811437 2.0084432 47.18445415 55.4559586
## beta[3,4]       54.4305865 1.6369488 51.02141715 57.4256656
## beta[4,4]       48.7264314 1.4754701 45.70538022 51.5413339
## beta[5,4]       45.5914780 1.8051970 42.05007872 49.3659552
## beta[6,4]       50.8000562 1.4696533 47.72206160 53.3727090
## beta[7,4]       46.3211874 2.5536161 41.19514273 51.1742377
## beta[8,4]       52.0428598 2.1662293 47.43433726 55.9819806
## beta[9,4]       53.9696348 1.5858942 51.09637291 57.6014267
## beta[10,4]      47.6724513 1.8668032 43.89282346 51.2309429
## beta[11,4]      51.8493091 1.4369846 48.83034407 54.6103632
## beta[12,4]      48.8576964 1.7708459 45.71742528 52.7111169
## beta[13,4]      45.8545564 1.6860656 42.79886556 49.5855540
```

```
## beta[1,5]       46.5270050 2.1184366 42.04091230 50.5605798
## beta[2,5]       52.3960022 1.9357695 47.40427586 55.3663511
## beta[3,5]       54.4590620 1.6011863 51.10509080 57.3259556
## beta[4,5]       48.7412904 1.4171168 45.81660090 51.4376954
## beta[5,5]       45.5938862 1.7675097 42.11294752 49.1187604
## beta[6,5]       50.8211279 1.4119876 47.90429088 53.3159518
## beta[7,5]       46.3494214 2.5448663 41.13449596 51.2228690
## beta[8,5]       52.0688027 2.1458932 47.40572776 55.9524187
## beta[9,5]       53.9749684 1.5443841 51.16636715 57.5088164
## beta[10,5]      47.7098359 1.8038902 43.97822741 51.1170287
## beta[11,5]      51.8806487 1.3733100 48.98335827 54.5870886
## beta[12,5]      48.8678520 1.7168686 45.75473610 52.6724863
## beta[13,5]      45.8739376 1.6433555 42.91525381 49.5902015
## beta[1,6]       46.5369181 2.0918170 42.30038580 50.5286182
## beta[2,6]       52.4209609 1.8622084 47.65261644 55.3620168
## beta[3,6]       54.4934255 1.5479439 51.22021999 57.3540242
## beta[4,6]       48.7565441 1.3599191 45.99376876 51.3513729
## beta[5,6]       45.5914292 1.7333413 42.14084013 49.1108980
## beta[6,6]       50.8723847 1.3477733 48.15517471 53.2286846
## beta[7,6]       46.3865869 2.5345902 41.11783543 51.2642474
## beta[8,6]       52.0944968 2.1202371 47.53904839 56.0144854
## beta[9,6]       53.9684720 1.4624460 51.34241802 57.3044058
## beta[10,6]      47.7515750 1.7459987 44.24095880 51.1506414
## beta[11,6]      51.9025948 1.3072086 49.13922418 54.5755418
## beta[12,6]      48.8775857 1.6746169 45.86563107 52.5292894
## beta[13,6]      45.9026161 1.5877677 42.99714600 49.4096485
## beta[1,7]       46.5517737 2.0788082 42.20078284 50.5648052
## beta[2,7]       52.4423884 1.8214850 47.83797315 55.2662000
## beta[3,7]       54.5344213 1.5061993 51.42066337 57.3478975
## beta[4,7]       48.7721559 1.3043131 46.11785442 51.2292675
## beta[5,7]       45.5929511 1.6707891 42.26243636 48.9654654
## beta[6,7]       50.9084257 1.2986088 48.29994155 53.1661204
## beta[7,7]       46.4051745 2.5277135 41.18396603 51.2405435
## beta[8,7]       52.1100098 2.0981936 47.66774052 55.9174251
## beta[9,7]       53.9754413 1.3875629 51.51574680 57.0159137
## beta[10,7]      47.7693629 1.6955459 44.26638293 51.0479234
## beta[11,7]      51.9338738 1.2186668 49.50430523 54.4152876
## beta[12,7]      48.8928983 1.6291872 45.94802949 52.3799161
## beta[13,7]      45.9204989 1.5234208 43.04064776 49.3011859
## beta[1,8]       46.5557695 2.0482870 42.29479623 50.5355847
## beta[2,8]       52.4693559 1.7171679 48.18092780 55.1291038
## beta[3,8]       54.5658777 1.4590191 51.64203041 57.3086507
## beta[4,8]       48.7917064 1.2390551 46.29392651 51.1663349
## beta[5,8]       45.6090295 1.6115669 42.32538218 48.7935958
## beta[6,8]       50.9393386 1.2460120 48.40704975 53.1458583
## beta[7,8]       46.4466889 2.5029317 41.22708808 51.2999787
## beta[8,8]       52.1339057 2.0695220 47.70566073 55.8567010
## beta[9,8]       53.9720083 1.3142507 51.60079827 56.9336924
## beta[10,8]      47.8072502 1.6310934 44.51403974 50.9382348
## beta[11,8]      51.9786793 1.1525195 49.61881294 54.4439017
## beta[12,8]      48.8842396 1.5855617 46.06751590 52.3273228
## beta[13,8]      45.9357006 1.4643750 43.17433684 49.1612021
```

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## beta[1,9]       46.5670768 2.0241419 42.28102749 50.5473141
## beta[2,9]       52.5028970 1.6293033 48.32601925 55.0614260
## beta[3,9]       54.6185609 1.3956296 51.75957982 57.2449812
## beta[4,9]       48.8034662 1.1744828 46.48385806 51.0059174
## beta[5,9]       45.6034800 1.5457346 42.46873154 48.6337861
## beta[6,9]       50.9476722 1.1788262 48.51608288 53.0401498
## beta[7,9]       46.4908841 2.4581752 41.26567023 51.1615794
## beta[8,9]       52.1616163 2.0472942 47.62660697 55.8343743
## beta[9,9]       53.9766551 1.2299404 51.74557072 56.6013325
## beta[10,9]      47.8441955 1.5706324 44.69935624 50.8690019
## beta[11,9]      52.0023916 1.0794169 49.80329178 54.2249423
## beta[12,9]      48.8734921 1.5387937 46.15679907 52.3062505
## beta[13,9]      45.9468701 1.3896232 43.27160058 49.0166319
## beta[1,10]      46.5678919 2.0058839 42.32787193 50.4871242
## beta[2,10]      52.5390031 1.5105729 48.69645194 54.8742624
## beta[3,10]      54.6743274 1.3283331 51.95506185 57.1823165
## beta[4,10]      48.8089650 1.1067851 46.59030224 50.9459729
## beta[5,10]      45.6182920 1.4673513 42.62461690 48.4394718
## beta[6,10]      50.9589061 1.1176928 48.65209331 52.8557514
## beta[7,10]      46.5379063 2.4407841 41.33779109 51.2025385
## beta[8,10]      52.1807310 2.0268314 47.64778182 55.7261525
## beta[9,10]      53.9739212 1.1381429 51.91497703 56.3897657
## beta[10,10]     47.8871395 1.5012262 44.87418303 50.8326995
## beta[11,10]     52.0246213 1.0056766 49.98163736 54.0750713
## beta[12,10]     48.8645445 1.5075891 46.23353781 52.2559794
## beta[13,10]     45.9557357 1.3205726 43.33332685 48.9188215
## beta[1,11]      46.5657722 1.9884405 42.47851727 50.4111217
## beta[2,11]      52.5871373 1.3717250 49.06990092 54.7918520
## beta[3,11]      54.7400065 1.2813795 52.04151691 57.2546969
## beta[4,11]      48.8263871 1.0265121 46.80608246 50.7639370
## beta[5,11]      45.6229736 1.3946257 42.81182773 48.3699546
## beta[6,11]      50.9801055 1.0438403 48.76111023 52.8246632
## beta[7,11]      46.5696643 2.4023754 41.27155079 51.1483077
## beta[8,11]      52.2079035 2.0094587 47.72323788 55.5756930
## beta[9,11]      53.9590365 1.0167393 52.09094606 56.1472696
## beta[10,11]     47.9399924 1.4382705 45.08574550 50.8092628
## beta[11,11]     52.0390239 0.9176663 50.21353504 53.8657094
## beta[12,11]     48.8479921 1.4812537 46.24737533 52.1934658
## beta[13,11]     45.9599444 1.2276616 43.47051693 48.6080411
## beta[1,12]      46.5714547 1.9538748 42.56145599 50.2601810
## beta[2,12]      52.6422271 1.2120834 49.65909631 54.6130190
## beta[3,12]      54.7796973 1.2322518 52.24493741 57.1065122
## beta[4,12]      48.8241898 0.9347758 46.96862712 50.5515212
## beta[5,12]      45.6430506 1.3163577 42.89452579 48.1590470
## beta[6,12]      50.9883574 0.9800345 48.91056324 52.7813585
## beta[7,12]      46.6107457 2.3502945 41.38512564 51.1682618
## beta[8,12]      52.2146382 2.0078566 47.80023470 55.5988958
## beta[9,12]      53.8425879 0.9400988 52.10605867 55.7705086
## beta[10,12]     47.9706309 1.3642976 45.24576448 50.5990511
## beta[11,12]     52.0693416 0.8175119 50.41380058 53.6838658
## beta[12,12]     48.8440897 1.4287400 46.28222135 52.0444698
## beta[13,12]     45.9811842 1.1335226 43.68615188 48.3956155
```

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## beta[1,13]      46.5876383 1.9218749 42.58312108 50.3284772
## beta[2,13]      52.6891150 1.0461506 50.27943163 54.4381734
## beta[3,13]      54.8395928 1.1751190 52.48355262 57.0792011
## beta[4,13]      48.7926745 0.8883001 47.04169779 50.4494619
## beta[5,13]      45.6557143 1.2381279 43.13541646 48.0919228
## beta[6,13]      51.0000905 0.8927554 49.09466934 52.5801656
## beta[7,13]      46.6552101 2.2921448 41.57173330 51.1101449
## beta[8,13]      52.2276404 1.9684165 47.85446957 55.4976038
## beta[9,13]      53.7173751 0.8618031 52.08186888 55.5516843
## beta[10,13]     48.0013049 1.2600938 45.42117298 50.4480650
## beta[11,13]     52.0184638 0.7707851 50.49038887 53.5629130
## beta[12,13]     48.8407254 1.3757495 46.33568886 51.8699864
## beta[13,13]     45.9923947 1.0500376 43.91426454 48.1153740
## beta[1,14]      46.6229871 1.8726832 42.85334043 50.2933601
## beta[2,14]      52.9666038 0.8457280 51.15682373 54.4630778
## beta[3,14]      54.9092964 1.1112883 52.66372207 57.0326530
## beta[4,14]      48.7760120 0.8189630 47.17509433 50.3058560
## beta[5,14]      45.6609027 1.1838267 43.23871605 47.9725141
## beta[6,14]      51.0115363 0.7988418 49.40733822 52.4348770
## beta[7,14]      46.7010099 2.2500168 41.64344074 51.0391557
## beta[8,14]      52.2455939 1.9593516 47.90972633 55.5146364
## beta[9,14]      53.6008012 0.7617864 52.06529968 55.0934147
## beta[10,14]     48.0410622 1.1925141 45.53014963 50.2580502
## beta[11,14]     51.9433993 0.7044818 50.51203083 53.3278228
## beta[12,14]     48.8306216 1.3210952 46.45141792 51.6902215
## beta[13,14]     45.9985113 0.9352965 44.09483641 47.8686881
## beta[1,15]      46.6458309 1.8442028 42.95508554 50.2851475
## beta[2,15]      53.1461151 0.7395091 51.61317260 54.4908439
## beta[3,15]      54.9750359 1.0182329 52.99031086 56.9488146
## beta[4,15]      48.8341676 0.7962061 47.29228520 50.3736713
## beta[5,15]      45.9265245 1.1217179 43.56152539 48.1300919
## beta[6,15]      51.1413157 0.7525674 49.60599917 52.5815186
## beta[7,15]      46.7357382 2.1813822 41.77010350 50.9043960
## beta[8,15]      52.2572799 1.9496084 47.88088430 55.3668462
## beta[9,15]      53.4845287 0.6461339 52.20040491 54.7143214
## beta[10,15]     48.0739475 1.1115507 45.72275331 50.1018411
## beta[11,15]     51.9966479 0.6396810 50.72183860 53.2612563
## beta[12,15]     48.8230295 1.2764186 46.45960717 51.5080448
## beta[13,15]     46.0059997 0.8319034 44.25857352 47.6864369
## beta[1,16]      46.6857907 1.7850144 43.15820470 50.1988322
## beta[2,16]      53.3670823 0.6463870 52.03214116 54.5841181
## beta[3,16]      55.0755165 0.9922047 53.09893958 56.9708163
## beta[4,16]      48.9124194 0.7855444 47.36550357 50.5040986
## beta[5,16]      46.1857349 1.0956328 43.84692791 48.3209172
## beta[6,16]      51.3535917 0.7244868 49.89473148 52.7222249
## beta[7,16]      46.7427876 2.2430579 41.80418504 51.0793865
## beta[8,16]      52.2782003 1.9247401 48.06497504 55.3736604
## beta[9,16]      53.3664815 0.6365839 52.05778199 54.5860187
## beta[10,16]     48.0084506 1.1192421 45.65320355 50.1161985
## beta[11,16]     52.0820862 0.6131808 50.91801927 53.3244301
## beta[12,16]     48.7129144 1.2720920 46.37203848 51.3063113
## beta[13,16]     46.0164093 0.8116112 44.36180327 47.6053965
```

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## beta[1,17]      46.7028880 1.7228092 43.35995183 50.0810660
## beta[2,17]      53.5305265 0.6793261 52.16274992 54.8757080
## beta[3,17]      55.1803550 0.9332272 53.28641361 56.9787842
## beta[4,17]      48.9319298 0.8485414 47.31213588 50.6031499
## beta[5,17]      46.2653397 1.1300489 43.89912988 48.4355767
## beta[6,17]      51.3887070 0.8243632 49.70430353 52.9750254
## beta[7,17]      46.7497226 2.2991594 41.63006534 51.0984169
## beta[8,17]      52.2843067 1.9094081 48.08334810 55.3344178
## beta[9,17]      53.2805018 0.6635770 51.91987207 54.5118972
## beta[10,17]     47.9371653 1.1271367 45.66019767 50.0652831
## beta[11,17]     52.0230307 0.6489327 50.75655496 53.3063521
## beta[12,17]     48.5740598 1.2640516 46.18405026 51.0738517
## beta[13,17]     46.0250522 0.8851848 44.24224428 47.7693131
## beta[1,18]      46.7191474 1.6849682 43.37287806 49.9894809
## beta[2,18]      53.6993091 0.7032252 52.30481462 55.0809696
## beta[3,18]      55.2918572 0.8898372 53.51782732 57.0021850
## beta[4,18]      48.9570142 0.8990033 47.26337927 50.7539149
## beta[5,18]      46.3654826 1.1661034 44.02401079 48.6048889
## beta[6,18]      51.4340005 0.8929114 49.57896593 53.1963433
## beta[7,18]      46.7545369 2.3458972 41.57946265 51.2566181
## beta[8,18]      52.2918695 1.8741920 48.21060194 55.3887927
## beta[9,18]      53.1913995 0.7125515 51.70608770 54.5131026
## beta[10,18]     47.8208705 1.1798427 45.45918056 50.1042142
## beta[11,18]     52.0348514 0.6632475 50.65707936 53.3067999
## beta[12,18]     48.4413338 1.2539374 46.05390072 50.8629306
## beta[13,18]     46.0264615 0.9369834 44.12348258 47.9383915
## beta[1,19]      46.7421722 1.6016921 43.54344622 49.7863545
## beta[2,19]      53.8693082 0.7158984 52.52157535 55.3957116
## beta[3,19]      55.4003856 0.8175367 53.78862581 56.9665940
## beta[4,19]      48.9647817 0.9209987 47.21677596 50.7942971
## beta[5,19]      46.4612821 1.2003758 44.06536548 48.7720284
## beta[6,19]      51.4905103 0.9519827 49.52667423 53.3854068
## beta[7,19]      46.7601683 2.3832760 41.63232402 51.3653629
## beta[8,19]      52.2944480 1.8615495 48.19078230 55.3370190
## beta[9,19]      53.1191294 0.7229911 51.64951317 54.4534752
## beta[10,19]     47.7283845 1.2228423 45.28487685 50.1013232
## beta[11,19]     52.1019241 0.6812656 50.67895815 53.4229057
## beta[12,19]     48.3219121 1.2352106 45.94940141 50.7075071
## beta[13,19]     46.0205789 0.9521834 44.04763945 47.9163047
## beta[1,20]      46.7582842 1.5352911 43.73173446 49.7050819
## beta[2,20]      53.9160116 0.7264295 52.57913230 55.4433531
## beta[3,20]      55.3806216 0.7737028 53.82986717 56.8416627
## beta[4,20]      48.9781951 0.9234619 47.22012021 50.8176672
## beta[5,20]      46.5439817 1.2129056 44.19475206 48.8982318
## beta[6,20]      51.5285021 0.9917533 49.48773575 53.5262365
## beta[7,20]      46.7639945 2.4207134 41.58969934 51.3864437
## beta[8,20]      52.3163486 1.8168325 48.26509074 55.2879608
## beta[9,20]      53.0319687 0.7065565 51.50450038 54.3218997
## beta[10,20]     47.6212562 1.2740673 45.08221853 50.1020328
## beta[11,20]     52.1880136 0.6670004 50.86017541 53.4863884
## beta[12,20]     48.1991681 1.2166046 45.88051505 50.4596289
## beta[13,20]     46.0143552 0.9618815 44.05770633 48.0022406
```

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## beta[1,21]      46.7867748 1.4608324 43.78679190 49.5976637
## beta[2,21]      53.9627333 0.7133019 52.64062644 55.4582007
## beta[3,21]      55.3917216 0.7602843 53.86113700 56.8728672
## beta[4,21]      48.9973325 0.9028998 47.28894242 50.8021263
## beta[5,21]      46.6284567 1.2130280 44.27558319 49.1251114
## beta[6,21]      51.5949363 1.0058461 49.51228545 53.6714812
## beta[7,21]      46.7686597 2.4794534 41.48974505 51.5905315
## beta[8,21]      52.3184579 1.7838715 48.31533347 55.2417579
## beta[9,21]      53.1274968 0.7071195 51.64861973 54.4375554
## beta[10,21]     47.5207537 1.3034189 44.89221432 50.0244359
## beta[11,21]     52.2554363 0.6082433 51.07245111 53.4918906
## beta[12,21]     48.0540662 1.1969640 45.75188289 50.2858863
## beta[13,21]     46.0102836 0.9687428 44.05574358 47.8820252
## beta[1,22]      46.9138681 1.4702115 43.87444960 49.7132180
## beta[2,22]      53.9280857 0.6806370 52.66031623 55.3697297
## beta[3,22]      55.3909755 0.7627844 53.89025506 56.8790100
## beta[4,22]      49.0171836 0.8686521 47.29466320 50.7478746
## beta[5,22]      46.7094752 1.2132755 44.28517901 49.0972275
## beta[6,22]      51.6452200 1.0085701 49.56952211 53.6278500
## beta[7,22]      46.7599454 2.5191544 41.44002023 51.6686499
## beta[8,22]      52.3274020 1.7383628 48.40335725 55.2711821
## beta[9,22]      53.2003764 0.6739861 51.82058867 54.4739230
## beta[10,22]     47.4205357 1.3180979 44.78243620 49.8706666
## beta[11,22]     52.3761841 0.5800552 51.24988800 53.5497691
## beta[12,22]     48.0102021 1.2333392 45.61759356 50.3119820
## beta[13,22]     46.0085939 0.9463558 44.11870181 47.8884170
## beta[1,23]      47.0232361 1.4814496 43.91782910 49.9315878
## beta[2,23]      53.8669594 0.7174250 52.50044430 55.3438474
## beta[3,23]      55.3670157 0.8033922 53.77174617 56.9774226
## beta[4,23]      49.0392598 0.8268835 47.40021465 50.6494120
## beta[5,23]      46.7962967 1.2039744 44.43139050 49.1931114
## beta[6,23]      51.6930843 1.0107959 49.71234878 53.8058706
## beta[7,23]      46.7614674 2.5658209 41.35513589 51.8528121
## beta[8,23]      52.3389353 1.6916018 48.58238159 55.2809663
## beta[9,23]      53.2287894 0.6573569 51.86899606 54.5250160
## beta[10,23]     47.3236751 1.3460824 44.58790379 49.8583770
## beta[11,23]     52.4861980 0.5989203 51.37275263 53.7528760
## beta[12,23]     47.9452698 1.2447301 45.45634922 50.2473777
## beta[13,23]     46.0020642 0.9207214 44.15558873 47.8619245
## beta[1,24]      47.1305436 1.4869033 43.88884664 50.0421037
## beta[2,24]      53.8081838 0.7034063 52.45534988 55.2121716
## beta[3,24]      55.3511499 0.8095567 53.71748443 56.9488549
## beta[4,24]      48.8683044 0.8151464 47.23374258 50.4731764
## beta[5,24]      46.6405702 1.1818212 44.32265636 48.9996827
## beta[6,24]      51.7246580 0.9944392 49.79219166 53.8391161
## beta[7,24]      46.7495695 2.6084561 41.44290247 51.8995926
## beta[8,24]      52.3529625 1.6348484 48.77106141 55.1723407
## beta[9,24]      53.2245996 0.6638911 51.87331308 54.5047005
## beta[10,24]     47.2340111 1.3460257 44.42973164 49.6907910
## beta[11,24]     52.6098662 0.5596440 51.58168714 53.7578403
## beta[12,24]     47.8901211 1.2638985 45.43499646 50.2952325
## beta[13,24]     45.9815802 0.8946495 44.19968961 47.8335330
```

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## beta[1,25]      47.2423711 1.4813390 43.96972453 50.0950786
## beta[2,25]      53.6804149 0.7280380 52.21821671 55.1005840
## beta[3,25]      55.3100243 0.8440411 53.62275803 56.9828277
## beta[4,25]      48.7773472 0.8666345 47.08144277 50.4819196
## beta[5,25]      46.4909435 1.1667758 44.24379518 48.7740719
## beta[6,25]      51.7657467 0.9760860 49.79779801 53.7531843
## beta[7,25]      46.7458671 2.6409454 41.43742045 51.9413585
## beta[8,25]      52.3512352 1.5890911 48.92201779 55.1477914
## beta[9,25]      53.2668154 0.7064152 51.82130832 54.6135692
## beta[10,25]     47.1365071 1.3479176 44.32592250 49.6017676
## beta[11,25]     52.5266708 0.6568606 51.33416299 53.9571469
## beta[12,25]     47.9037261 1.3397291 45.30471282 50.3954086
## beta[13,25]     46.0381220 0.9314409 44.21415386 47.9691454
## beta[1,26]      47.3565627 1.4751398 43.93836281 50.3243612
## beta[2,26]      53.5714810 0.7184205 52.12762987 54.9467133
## beta[3,26]      55.2790811 0.8710228 53.49875774 56.9581881
## beta[4,26]      48.6911341 0.9040403 46.88819116 50.4056816
## beta[5,26]      46.3461420 1.1552840 44.08583818 48.5561571
## beta[6,26]      51.8022736 0.9293562 49.97182332 53.6897940
## beta[7,26]      46.7498456 2.6828938 41.39557134 51.9919580
## beta[8,26]      52.3537983 1.5436570 48.99796781 55.1439165
## beta[9,26]      53.2971362 0.6964640 51.88290091 54.6503765
## beta[10,26]     47.0431523 1.3533366 44.26502066 49.4627238
## beta[11,26]     52.4467005 0.7354556 51.04320166 53.9862058
## beta[12,26]     47.9046602 1.4283001 44.93686676 50.5560030
## beta[13,26]     46.0984842 0.9448671 44.28554119 48.0987626
## beta[1,27]      47.4618450 1.4700258 44.09461319 50.3972924
## beta[2,27]      53.6111881 0.7078093 52.24430199 54.9594438
## beta[3,27]      55.2679870 0.9362267 53.37511242 57.1152185
## beta[4,27]      48.6024534 0.9173805 46.74524389 50.3933464
## beta[5,27]      46.2016904 1.1621773 43.91817702 48.3829458
## beta[6,27]      51.8327335 0.9539375 49.97904837 53.7463876
## beta[7,27]      46.7452836 2.7133898 41.29619052 52.0879452
## beta[8,27]      52.3640944 1.4948538 49.20271106 55.1464839
## beta[9,27]      53.4495606 0.7081610 52.07726318 54.8664754
## beta[10,27]     46.9520061 1.3574939 44.08995442 49.4453710
## beta[11,27]     52.3771826 0.7578924 50.91932501 53.8852459
## beta[12,27]     47.9196206 1.5196270 44.77370048 50.7517371
## beta[13,27]     46.1524774 0.9366134 44.33378084 48.0825628
## beta[1,28]      47.5722609 1.4605450 44.23145966 50.4744615
## beta[2,28]      53.6942170 0.7224197 52.29598129 55.1091818
## beta[3,28]      55.2672804 0.9914321 53.25603453 57.2040805
## beta[4,28]      48.5022231 0.9164981 46.66678874 50.2971843
## beta[5,28]      46.0848688 1.1801998 43.82973407 48.3405981
## beta[6,28]      51.8413230 0.9615027 49.95065859 53.7982913
## beta[7,28]      46.7603785 2.7476268 41.24162876 52.2203267
## beta[8,28]      52.3590314 1.4207960 49.40816857 55.0015970
## beta[9,28]      53.6043712 0.7257379 52.16331887 55.0172231
## beta[10,28]     46.8523748 1.3631335 44.02617140 49.3986889
## beta[11,28]     52.3068993 0.7784794 50.74224627 53.8319352
## beta[12,28]     47.9131191 1.6167875 44.54409007 51.0062493
## beta[13,28]     46.1935809 0.9022764 44.56786932 48.1391365
```

```
## beta[1,29]      47.6848497 1.4344787 44.45177709 50.5091427
## beta[2,29]      53.6673304 0.7679560 52.13265892 55.1472795
## beta[3,29]      55.2585334 1.0287523 53.13190571 57.2435084
## beta[4,29]      48.3580718 0.9985588 46.30876496 50.2707486
## beta[5,29]      46.0312334 1.2366088 43.54426637 48.3755283
## beta[6,29]      51.8677925 0.9387419 50.03582876 53.7358149
## beta[7,29]      46.7554409 2.7807474 41.15111317 52.2648013
## beta[8,29]      52.3619139 1.3557810 49.48227192 54.8444986
## beta[9,29]      53.6382803 0.7852292 52.09806866 55.1898938
## beta[10,29]     46.8636525 1.4377624 43.83159561 49.6131254
## beta[11,29]     52.2322878 0.7727406 50.59331291 53.7419728
## beta[12,29]     47.9130771 1.7037495 44.21988677 50.9993214
## beta[13,29]     46.2489814 0.8655470 44.69269510 48.0894850
## beta[1,30]      47.7991288 1.4156141 44.69305877 50.5769502
## beta[2,30]      53.6506213 0.7774959 52.13880114 55.1265817
## beta[3,30]      55.2627770 1.1256573 53.01983584 57.3903348
## beta[4,30]      48.3550312 1.1031878 46.08211241 50.4331675
## beta[5,30]      46.0259933 1.3216892 43.37429385 48.5746496
## beta[6,30]      51.9194306 0.9743408 50.02526565 53.8536274
## beta[7,30]      46.7472582 2.8285276 41.14622590 52.2613860
## beta[8,30]      52.2579323 1.3622229 49.49011940 54.8201552
## beta[9,30]      53.6713997 0.8551522 52.03762314 55.4141234
## beta[10,30]     46.8650164 1.5038064 43.80688647 49.6510935
## beta[11,30]     52.1206822 0.8143506 50.38648947 53.6985330
## beta[12,30]     47.9236093 1.7686198 44.21213856 51.1759230
## beta[13,30]     46.2149077 0.9222641 44.50317426 48.1660882
## beta[1,31]      47.8996625 1.3788202 44.76095190 50.4355935
## beta[2,31]      53.6912338 0.8528018 52.00973344 55.3179302
## beta[3,31]      55.2785179 1.2092872 52.90852160 57.6307819
## beta[4,31]      48.3585808 1.2016861 45.86013835 50.6686478
## beta[5,31]      46.0219710 1.3913000 43.19006592 48.7444100
## beta[6,31]      51.9119956 1.0926269 49.75673560 54.1812027
## beta[7,31]      46.7432851 2.8569825 41.01971904 52.3066243
## beta[8,31]      52.1675439 1.3656409 49.40700955 54.7009520
## beta[9,31]      53.6930802 0.9112265 51.84999218 55.5205501
## beta[10,31]     46.8551461 1.5830819 43.61816394 49.7614292
## beta[11,31]     52.0090255 0.8341724 50.25811229 53.5760578
## beta[12,31]     47.9192727 1.8396770 43.94391158 51.3031586
## beta[13,31]     46.1789356 0.9681142 44.33602796 48.1880520
## sigma2_beta[1]   0.2174465 0.1709002  0.03591539  0.6455778
## sigma2_beta[2]   0.2195665 0.1769982  0.03763168  0.6657771
## sigma2_beta[3]   0.2117813 0.1594372  0.03599774  0.6146854
## sigma2_beta[4]   0.2150078 0.1639863  0.03653461  0.6168701
## sigma2_beta[5]   0.2248891 0.1892429  0.03824851  0.6904631
## sigma2_beta[6]   0.2127312 0.1623794  0.03671041  0.6352535
## sigma2_beta[7]   0.2177799 0.1719668  0.03634232  0.6345328
## sigma2_beta[8]   0.2146970 0.1668502  0.03705036  0.6189533
## sigma2_beta[9]   0.2103161 0.1587663  0.03750246  0.6029883
## sigma2_beta[10]  0.2202669 0.1761017  0.03688560  0.6480835
## sigma2_beta[11]  0.2111939 0.1591431  0.03804373  0.5951090
## sigma2_beta[12]  0.2172667 0.1701173  0.03499755  0.6603020
## sigma2_beta[13]  0.2134587 0.1636727  0.03839452  0.6263231
```

```
## sigma2_y[1]      4.3393222 2.2137569  2.06291739  8.9778112
## sigma2_y[2]      3.8692044 1.0191342  2.27841397  6.2710692
## sigma2_y[3]      3.4111862 1.1556115  1.48423175  6.0022095
## sigma2_y[4]      3.9842974 1.2215696  2.22593396  6.8663237
## sigma2_y[5]      7.4445583 4.5573201  3.38615960 20.3325823
## sigma2_y[6]      3.7518461 1.1728339  1.95333042  6.6424013
## sigma2_y[7]      4.3641806 3.0370416  1.89056380 10.3323020
## sigma2_y[8]      4.2269202 2.2465433  1.94931827  8.4792981
## sigma2_y[9]      3.5124807 0.9368648  1.89915246  5.5939477
## sigma2_y[10]     3.9496067 1.6799443  1.69059348  7.7787986
## sigma2_y[11]     3.6043268 0.9004233  2.11599421  5.7642380
## sigma2_y[12]     4.2285806 1.8916422  1.99432151  8.2372830
## sigma2_y[13]     3.7806794 1.1815197  1.90567452  6.6652333

## mean Rhat: 1.077681

## mean effective sample size: 710.0193
```

For predicting control of the U.S. Senate, if a 50/50 split is predicted, then the VP breaks the tie. In this case, we predict Democrat majority since our presidential model gives Biden a high chance of winning (and a Democrat VP will break the tie).

## A.3 House Election Model

### House Model Purpose and Structure

The purpose of the House model is to predict which candidate in each of NC's 13 congressional districts will win in their respective elections.

The response we will use is $Y_k$, the Democrat candidate's share of the two-party vote using the following model where the poll $k$ (of $K$=*fill in* polls) ended $t$ days (of $T = 60$ days) before the election and was conducted on district $j$ (where $j$ is in districts $1, \cdots, 11, 13$ of North Carolina). For district 12, the vote share is coded as 100 because there is only one candidate, and she is a Democrat. Note that, due to the scarcity of poll responses for the NC House elections, we included all poll responses within 115 days of the election.

$Y_k \sim N(\beta_{jt}, \sigma^2_{yj})$
$\beta_{jt} \sim N(\beta_{jt-1}, \sigma^2_{\beta j})$
$\sigma^2_{yj} \sim InvGamma(\nu_u, \nu_y \tau_y)$
$\nu_y \sim Uniform(0, 100)$ and $\tau_y \sim Uniform(0, 100)$.
$\sigma^2_{\beta j} \sim InvGamma(\nu_\beta, \nu_\beta \tau_\beta))$
$\nu_\beta \sim Uniform(0, 100)$ and $\tau_\beta \sim Uniform(0, 100)$.
$\beta_{j1} \sim N(h_j, 7.5)$
$h_j = 0.9 * \text{Vote Share from Partisan Lean}_j + 0.1 * \text{Expected Vote Share from Voter Turnout}_j$

A hierarchical model was used, where the Democratic candidate's share of the two-party vote for poll $k$ on any given day $t$ for district $j$ is modelled as a normal distribution with mean $\beta_{jt}$ and variance $\sigma^2_{yj}$. A random walk was used to calculate $\beta_{jt}$ for each day $t$ before the election (up until 30 days before the election) from $\beta_{jt-1}$ and $\sigma_{\beta j}$. The mean $h_j$ of the normal prior on $mu_j$ is computed by multiplying Vote Share from Partisan Lean$_j$ by 0.9 and Expected Vote Share from Voter Turnout$_j$ by 0.1, then summing those two values.

**Interim Report Model**

$$Vote_i \sim Binomial(n_i, \pi_i)$$

$$\begin{aligned} logit(\pi_i) = \alpha_0 &+ \Sigma_{j=1,2,3}\beta_j * I(Income_i = Income\ Category_s) + \Sigma_{j=5,6}\beta_j * I(Gender_i = Gender\ Category_g) + \\ &\Sigma_{j=7,8}\beta_j * I(Race_i = Race\ Category_r) + \Sigma_{j=9,10,11}\beta_j * I(Age_i = Age\ Category_a) + \\ &\Sigma_{j=12,13}\beta_j * I(Party_i = Party\ Category_p) + \\ &\Sigma_{j=14,...,17}\beta_j * I(Gender_i = Gender\ Category_g)I(Party_i = Party\ Category_p) + \\ &\Sigma_{j=18,...,21}\beta_j * I(Race_i = Race\ Category_r)I(Party_i = Party\ Category_p) + \\ &\Sigma_{j=22,...,27}\beta_j * I(Gender_i = Gender\ Category_g)I(Age_i = Age\ Category_a) + \\ &\Sigma_{j=28,...,33}\beta_j * I(Party_i = Party\ Category_p)I(Age_i = Age\ Category_a) \end{aligned}$$

where:

$$\begin{aligned} Income\ Category_s &\in \{Median\ Income\ \$46864 - \$52798,\ Median\ Income\ \$52798 - \$64509, \\ &\quad Median\ Income > \$64509\} \\ Gender\ Category_g &\in \{Male, Unspecified\} \\ Race\ Category_r &\in \{Black, Other\} \\ Age\ Category_a &\in \{Age\ 30-44,\ Age\ 45-59,\ Age\ 60+\} \\ Party\ Category_p &\in \{Republican,\ Other\} \end{aligned}$$

Where $\pi_i$ is the probability that subgroup $i$ votes. $Vote_i$ is the number of people in the $i$th subgroup that voted in the 2016 election. The baseline is `Median Income < $46864` for income, `Female` for gender, `White` for race, `18-29` for age, and `Democrat` for party affiliation.

We fit the voter turnout model from the Interim Report on the 2020 registered voter dataset. The predicted values were the number of people that will vote for each demographic subgroup in each congressional district. We made a reasonable assumption that if someone is registered as a Democrat, that they will indeed vote for the Democratic candidate (and made the same assumption for registered Republicans). If the demographic subgroup had third/unaffiliated party, then we split their predicted number of voters evenly between the Democrat and Republican parties. From this, we calculated Democratic party vote share for each district by taking the ratio of the predicted number of Democratic voters to total number of registered voters in that district.

**Raw Output for Appendix**

```
##                    mean         sd        2.5%      97.5%
## beta[1,1]       58.306127   2.716740 52.977697539  63.675746
## beta[2,1]       59.253823   2.777571 53.830873329  64.707466
## beta[3,1]       36.385167   2.670935 31.436792575  41.684174
## beta[4,1]       66.711295   2.729256 61.523445759  72.155486
## beta[5,1]       30.463614   2.735714 25.070167216  35.796273
## beta[6,1]       58.811408   2.742312 53.362761435  64.131692
## beta[7,1]       38.194965   2.812420 32.566439603  43.595922
## beta[8,1]       44.256514   2.564823 38.991609441  49.328885
## beta[9,1]       43.211496   2.580971 38.101328337  48.597018
## beta[10,1]      29.324002   2.730484 24.043756131  34.836887
## beta[11,1]      43.880322   2.187492 39.408639848  48.139221
## beta[12,1]      68.969534   2.744467 63.677429845  74.298244
## beta[13,1]      30.396369   2.731562 24.989800885  35.883334
## beta[1,2]       58.352260   3.040225 52.320255196  64.302066
## beta[2,2]       59.267556   3.127547 53.209654701  65.536052
## beta[3,2]       36.362939   3.021067 30.639348890  42.443572
```

```
## beta[4,2]       66.670367    3.077007 60.774331456  72.879396
## beta[5,2]       30.484026    3.009829 24.789901778  36.339399
## beta[6,2]       58.837266    3.089096 52.785350981  64.904593
## beta[7,2]       38.250039    3.142514 32.211870346  44.520885
## beta[8,2]       44.391225    2.886468 38.351914439  50.094210
## beta[9,2]       43.362126    2.858532 37.344276524  49.189518
## beta[10,2]      29.309636    3.026344 23.355513128  35.415154
## beta[11,2]      44.172316    2.432975 39.122144150  48.877901
## beta[12,2]      68.944352    3.063979 62.817791974  75.036508
## beta[13,2]      30.438862    3.056301 24.270146352  36.417821
## beta[1,3]       58.368669    3.332791 51.870150051  65.069312
## beta[2,3]       59.276091    3.526879 52.585839537  66.302592
## beta[3,3]       36.311190    3.348195 29.895473390  43.097538
## beta[4,3]       66.643560    3.379636 59.891919751  73.405557
## beta[5,3]       30.509747    3.382913 23.975655355  37.144857
## beta[6,3]       58.835481    3.416625 52.333112910  65.587899
## beta[7,3]       38.270992    3.489868 31.412035147  45.103616
## beta[8,3]       44.527720    3.128332 37.845925520  50.696095
## beta[9,3]       43.556225    3.070066 37.057456871  49.830085
## beta[10,3]      29.268348    3.338790 22.691576703  36.030820
## beta[11,3]      44.449054    2.628877 39.078423450  49.665543
## beta[12,3]      68.965937    3.371608 62.124234052  75.664384
## beta[13,3]      30.413120    3.372929 23.691910985  37.108805
## beta[1,4]       58.365505    3.632878 51.204359548  65.601744
## beta[2,4]       59.247331    3.801727 51.932215509  66.917361
## beta[3,4]       36.324428    3.628670 29.518788188  43.596500
## beta[4,4]       66.643179    3.638378 59.100184032  73.824224
## beta[5,4]       30.524889    3.683148 23.559421377  37.711960
## beta[6,4]       58.827705    3.658566 51.710010972  65.968050
## beta[7,4]       38.288599    3.758513 31.153627517  45.668316
## beta[8,4]       44.679465    3.376841 37.482810933  51.309609
## beta[9,4]       43.751912    3.263212 36.798965705  50.308274
## beta[10,4]      29.255932    3.636745 22.086106219  36.447439
## beta[11,4]      44.764004    2.895262 39.063448015  50.837896
## beta[12,4]      68.929958    3.615904 61.456805864  76.261149
## beta[13,4]      30.387894    3.640859 23.327469316  37.687980
## beta[1,5]       58.338204    3.859056 50.811769371  65.923478
## beta[2,5]       59.304872    4.080726 51.184248393  67.420540
## beta[3,5]       36.328763    3.875191 28.770960718  43.989578
## beta[4,5]       66.607566    3.872030 58.584637237  74.066816
## beta[5,5]       30.536652    3.969847 23.111815665  38.045611
## beta[6,5]       58.801986    3.931892 51.305185383  66.367658
## beta[7,5]       38.303831    4.005214 30.444724652  46.468960
## beta[8,5]       44.773397    3.540718 37.028889643  51.641192
## beta[9,5]       43.976716    3.452329 36.669634874  51.004485
## beta[10,5]      29.244137    3.967901 21.102460081  37.069445
## beta[11,5]      45.018522    3.070197 38.894429749  51.740404
## beta[12,5]      68.949205    3.894044 60.863494047  76.536290
## beta[13,5]      30.420340    3.879809 22.998457762  38.371319
## beta[1,6]       58.304867    4.125022 50.187979967  66.708488
## beta[2,6]       59.298593    4.224963 51.048209626  67.985691
## beta[3,6]       36.296599    4.130048 28.526117351  44.655085
```

```
## beta[4,6]       66.611332    4.056136 58.438574939  74.625763
## beta[5,6]       30.540835    4.284264 22.771174840  38.650854
## beta[6,6]       58.798035    4.185780 50.655577589  67.076381
## beta[7,6]       38.278691    4.287222 29.909428996  46.882767
## beta[8,6]       44.859569    3.693646 36.777941715  52.028967
## beta[9,6]       44.177958    3.607561 36.320956602  51.437296
## beta[10,6]      29.234336    4.202400 20.191063565  37.803459
## beta[11,6]      45.344910    3.243098 39.247710483  52.329855
## beta[12,6]      68.955890    4.177474 60.000992617  77.507457
## beta[13,6]      30.437395    4.104944 22.633896225  38.840688
## beta[1,7]       58.260632    4.306509 49.709411964  66.726407
## beta[2,7]       59.296329    4.427612 50.649496907  68.289360
## beta[3,7]       36.290133    4.390128 27.648299375  44.901723
## beta[4,7]       66.587814    4.284334 58.055145300  74.968408
## beta[5,7]       30.550488    4.468446 21.970548559  39.483955
## beta[6,7]       58.785063    4.453932 50.054345089  67.709637
## beta[7,7]       38.232816    4.527592 29.557921255  47.813748
## beta[8,7]       45.012496    3.785056 36.749567232  52.526230
## beta[9,7]       44.386974    3.706472 35.800904827  51.635625
## beta[10,7]      29.210670    4.395346 19.810118586  38.031075
## beta[11,7]      45.643531    3.361235 39.595918578  52.994591
## beta[12,7]      68.881387    4.407658 59.847795152  77.936175
## beta[13,7]      30.427835    4.341382 21.856797765  39.307411
## beta[1,8]       58.268414    4.475594 49.406168860  67.277460
## beta[2,8]       59.289803    4.702609 49.959924066  69.108799
## beta[3,8]       36.292499    4.672751 27.042237016  45.811325
## beta[4,8]       66.587719    4.522559 57.369607330  75.431523
## beta[5,8]       30.536804    4.699414 20.920941442  39.485781
## beta[6,8]       58.820231    4.711010 49.813537225  68.160952
## beta[7,8]       38.252243    4.708654 29.162048851  48.192117
## beta[8,8]       45.170439    3.882223 36.650085483  53.032581
## beta[9,8]       44.573776    3.851951 36.114132518  52.188508
## beta[10,8]      29.236626    4.584843 19.645196332  38.439721
## beta[11,8]      45.918617    3.534732 39.925660440  54.117024
## beta[12,8]      68.925147    4.622607 59.228405231  78.340733
## beta[13,8]      30.458401    4.556214 21.540221068  40.207513
## beta[1,9]       58.227481    4.705658 48.827534270  67.469793
## beta[2,9]       59.300895    4.902999 49.486445285  68.985516
## beta[3,9]       36.300910    4.859348 26.559610569  46.483395
## beta[4,9]       66.595457    4.741732 56.713108738  76.074615
## beta[5,9]       30.514089    4.885376 20.698788360  40.394455
## beta[6,9]       58.815551    4.893609 49.066195290  68.597732
## beta[7,9]       38.219263    4.860930 29.018163419  48.782226
## beta[8,9]       45.337447    4.021454 36.231640253  53.609536
## beta[9,9]       44.811520    3.851604 36.438462750  52.170171
## beta[10,9]      29.238296    4.809784 19.195282646  38.955400
## beta[11,9]      46.232525    3.600083 40.118419854  54.684602
## beta[12,9]      68.932522    4.855149 58.768413473  78.859970
## beta[13,9]      30.448421    4.741623 21.260496585  40.199192
## beta[1,10]      58.247449    4.972730 48.729943726  68.096116
## beta[2,10]      59.271930    5.101482 49.054202063  69.489015
## beta[3,10]      36.288553    5.062371 26.103773020  46.527697
```

```
## beta[4,10]       66.613195    5.029024 55.863463490  77.038381
## beta[5,10]       30.529794    5.336355 20.587857122  40.700657
## beta[6,10]       58.820469    5.103717 48.872829775  69.218663
## beta[7,10]       38.203178    5.053553 28.439100941  48.950892
## beta[8,10]       45.505679    4.107839 36.658987968  54.134671
## beta[9,10]       45.013589    3.916728 36.868484116  52.509908
## beta[10,10]      29.230443    4.927455 18.593361748  38.984593
## beta[11,10]      46.492548    3.656128 40.393340003  55.071663
## beta[12,10]      68.973765    5.087724 58.200282340  79.389829
## beta[13,10]      30.435523    4.949181 20.576660287  40.537013
## beta[1,11]       58.266623    5.150298 47.914787416  68.444945
## beta[2,11]       59.261359    5.348439 48.660363549  70.413664
## beta[3,11]       36.290370    5.306424 25.568221934  46.688181
## beta[4,11]       66.608036    5.238743 55.543123369  76.979618
## beta[5,11]       30.567121    5.348584 20.332275024  40.832868
## beta[6,11]       58.783717    5.348421 48.383420142  69.777938
## beta[7,11]       38.163950    5.306011 27.768373582  49.661997
## beta[8,11]       45.631587    4.169952 36.246264533  54.293494
## beta[9,11]       45.259294    3.992133 37.023478633  53.159288
## beta[10,11]      29.197206    5.155472 18.512118346  39.628136
## beta[11,11]      46.792953    3.766233 40.804479041  55.857930
## beta[12,11]      68.962130    5.267353 57.936207098  80.052020
## beta[13,11]      30.415973    5.158791 20.418780079  40.832706
## beta[1,12]       58.302510    5.318072 47.413902113  68.966592
## beta[2,12]       59.254457    5.585898 48.007494430  70.723173
## beta[3,12]       36.339969    5.544313 25.012205351  47.060490
## beta[4,12]       66.627207    5.403722 55.412321982  77.221734
## beta[5,12]       30.557454    5.570089 19.567388996  41.269077
## beta[6,12]       58.769380    5.549584 47.746141566  70.927868
## beta[7,12]       38.166790    5.430003 27.333278172  49.773512
## beta[8,12]       45.783401    4.210739 36.437352864  54.277444
## beta[9,12]       45.508003    4.035619 37.326282803  53.433867
## beta[10,12]      29.166221    5.344817 18.173942715  39.838540
## beta[11,12]      47.065534    3.793021 41.070979469  56.422165
## beta[12,12]      68.972807    5.379676 57.898251724  80.041023
## beta[13,12]      30.449992    5.372069 19.841436368  41.483756
## beta[1,13]       58.262313    5.470788 46.995975079  69.220810
## beta[2,13]       59.234750    5.773217 47.389997901  71.195490
## beta[3,13]       36.337262    5.687907 24.650370831  47.574810
## beta[4,13]       66.681401    5.595423 54.717485504  77.793053
## beta[5,13]       30.571042    5.675612 18.524365707  41.905032
## beta[6,13]       58.766203    5.708592 47.507514407  70.808711
## beta[7,13]       38.154545    5.642853 26.812866801  50.231639
## beta[8,13]       45.946299    4.255413 36.752409829  54.615902
## beta[9,13]       45.740808    4.017914 37.658940787  53.461500
## beta[10,13]      29.180726    5.505926 17.687237104  40.514079
## beta[11,13]      47.335563    3.862901 41.262953767  56.209493
## beta[12,13]      68.959719    5.617269 57.312723831  80.479439
## beta[13,13]      30.421329    5.540451 19.387162344  41.702633
## beta[1,14]       58.301873    5.622821 46.772466916  69.775547
## beta[2,14]       59.256256    5.875500 47.116404303  71.167237
## beta[3,14]       36.321832    5.837515 24.440119288  47.524982
```

```
## beta[4,14]       66.664215    5.733737 54.466354809  78.439103
## beta[5,14]       30.584811    5.892816 18.677108888  42.366872
## beta[6,14]       58.786263    5.882636 46.692209525  71.088970
## beta[7,14]       38.155730    5.745581 26.623484389  50.353594
## beta[8,14]       46.065682    4.284096 36.994539345  55.070331
## beta[9,14]       45.967892    4.063710 38.607971485  53.715757
## beta[10,14]      29.178485    5.697925 17.281554521  41.122449
## beta[11,14]      47.624024    3.934260 41.512498814  56.677932
## beta[12,14]      68.946208    5.777823 56.821399985  80.522566
## beta[13,14]      30.397295    5.665580 19.325979259  41.994068
## beta[1,15]       58.338386    5.770202 46.809363654  70.520053
## beta[2,15]       59.267685    6.008962 46.820573422  71.862300
## beta[3,15]       36.324546    6.065624 24.029639291  48.310091
## beta[4,15]       66.652767    5.893977 53.977288833  78.749912
## beta[5,15]       30.562379    5.967742 17.874243540  42.629890
## beta[6,15]       58.771929    5.988793 46.153458767  71.306672
## beta[7,15]       38.148955    5.947232 26.109563892  50.253487
## beta[8,15]       46.171479    4.323291 36.802827741  55.095160
## beta[9,15]       46.157744    4.125107 38.508443584  54.359149
## beta[10,15]      29.197460    5.843681 17.000430591  41.134019
## beta[11,15]      47.927912    3.905768 41.786060481  56.732997
## beta[12,15]      68.943342    5.970391 56.365425554  80.908491
## beta[13,15]      30.339386    5.851190 18.255294150  42.375917
## beta[1,16]       58.336171    6.020326 45.998982492  71.063162
## beta[2,16]       59.285772    6.080984 47.306352062  72.365226
## beta[3,16]       36.318764    6.268369 23.478254023  48.889813
## beta[4,16]       66.611462    6.068660 53.806954659  79.289792
## beta[5,16]       30.598517    6.381035 17.291937897  43.244396
## beta[6,16]       58.755407    6.164153 46.238480893  71.749485
## beta[7,16]       38.117870    6.125044 26.140244127  50.603916
## beta[8,16]       46.307109    4.334710 37.058711739  55.486353
## beta[9,16]       46.349035    4.112395 38.477622404  54.326619
## beta[10,16]      29.183833    6.025569 15.892613625  41.596688
## beta[11,16]      48.233554    3.989519 41.852107165  56.849217
## beta[12,16]      68.966861    6.125410 56.060643175  81.237433
## beta[13,16]      30.342432    6.034239 18.207775029  42.661679
## beta[1,17]       58.321885    6.179098 45.735560302  71.086734
## beta[2,17]       59.277326    6.286190 46.436573437  72.419428
## beta[3,17]       36.322355    6.468998 22.724240017  49.132843
## beta[4,17]       66.647102    6.272970 53.352299422  78.970055
## beta[5,17]       30.614980    6.461924 16.467270534  43.295403
## beta[6,17]       58.753352    6.341515 45.959687652  72.076108
## beta[7,17]       38.128562    6.295447 25.498031197  50.544361
## beta[8,17]       46.443068    4.308399 37.910732732  55.805522
## beta[9,17]       46.528473    4.091345 38.567649964  54.672241
## beta[10,17]      29.150410    6.280176 15.540721166  42.205867
## beta[11,17]      48.514818    3.997312 42.317656921  56.910574
## beta[12,17]      68.975130    6.289467 55.886608126  81.612148
## beta[13,17]      30.305449    6.211051 17.864343045  43.255705
## beta[1,18]       58.306574    6.348285 45.470567881  70.878594
## beta[2,18]       59.268614    6.502635 45.688482924  72.741568
## beta[3,18]       36.322982    6.643092 22.464560254  49.742693
```

```
## beta[4,18]       66.630158    6.401904 52.954368029  79.301547
## beta[5,18]       30.634613    6.599235 16.839208491  43.901412
## beta[6,18]       58.836614    6.502314 45.957680319  72.923843
## beta[7,18]       38.132090    6.409905 25.235019125  50.910532
## beta[8,18]       46.544646    4.300672 38.174085608  55.803514
## beta[9,18]       46.747231    4.170000 39.175698347  55.130454
## beta[10,18]      29.165170    6.495744 15.358244357  42.315736
## beta[11,18]      48.669777    4.250977 42.192984959  58.346601
## beta[12,18]      68.954676    6.433772 55.553014150  81.849917
## beta[13,18]      30.287081    6.379433 17.509148367  43.407955
## beta[1,19]       58.317593    6.482085 45.184238176  71.461042
## beta[2,19]       59.261916    6.664147 45.869593571  73.014204
## beta[3,19]       36.309328    6.756385 22.138946429  49.995614
## beta[4,19]       66.618513    6.610978 52.565389495  79.545391
## beta[5,19]       30.637351    6.769377 16.736999562  43.598655
## beta[6,19]       58.899817    6.710655 45.392925886  73.139528
## beta[7,19]       38.147194    6.531765 25.102000748  50.886627
## beta[8,19]       46.643279    4.303104 38.414612893  56.288110
## beta[9,19]       46.945444    4.146384 39.448491536  55.268373
## beta[10,19]      29.171655    6.624807 14.916578822  42.486671
## beta[11,19]      48.807238    4.395896 42.300357935  59.218277
## beta[12,19]      68.914446    6.623577 54.334482328  81.901753
## beta[13,19]      30.258996    6.527071 17.231882583  43.309585
## beta[1,20]       58.384975    6.629628 45.184399152  72.558088
## beta[2,20]       59.262215    6.814608 45.546585146  73.152117
## beta[3,20]       36.329758    6.939561 21.656203568  50.302140
## beta[4,20]       66.599803    6.796110 52.160772009  80.203364
## beta[5,20]       30.621707    6.867378 16.535627750  44.618199
## beta[6,20]       58.843324    6.814834 45.077492033  73.329814
## beta[7,20]       38.150904    6.613662 24.365897975  51.139155
## beta[8,20]       46.801834    4.223229 38.828148397  55.990622
## beta[9,20]       47.139525    4.155936 39.719027087  55.747196
## beta[10,20]      29.165116    6.708796 14.798461824  42.528742
## beta[11,20]      48.953740    4.567724 42.088507479  59.896537
## beta[12,20]      68.932992    6.755592 54.376634353  82.577678
## beta[13,20]      30.239741    6.746749 16.863660637  43.761707
## beta[1,21]       58.333028    6.823553 44.745900753  73.257284
## beta[2,21]       59.277259    6.900809 44.862863959  73.239429
## beta[3,21]       36.338224    7.106053 21.382581325  50.741968
## beta[4,21]       66.618064    6.913746 52.232482411  80.653652
## beta[5,21]       30.622130    6.982153 16.631665016  44.548853
## beta[6,21]       58.787358    7.062388 44.782995340  73.652071
## beta[7,21]       38.123990    6.832153 24.532088954  51.516649
## beta[8,21]       46.948894    4.219319 38.977576236  55.886760
## beta[9,21]       47.362618    4.201026 39.939018308  56.273426
## beta[10,21]      29.154332    6.881683 14.575735504  43.223389
## beta[11,21]      49.027885    4.711607 41.857931032  60.606829
## beta[12,21]      68.911713    6.893027 54.036193279  82.982557
## beta[13,21]      30.173366    7.019513 16.517150409  44.207430
## beta[1,22]       58.379061    6.911633 44.840159036  73.412273
## beta[2,22]       59.301829    7.086533 44.849758762  74.169925
## beta[3,22]       36.334380    7.252386 20.908706664  50.910205
```

```
## beta[4,22]       66.658360    7.109628 52.255798392  81.190738
## beta[5,22]       30.669940    6.985330 16.059923887  44.707825
## beta[6,22]       58.778950    7.239992 44.404180745  74.098098
## beta[7,22]       38.142335    7.000180 24.102572084  51.911347
## beta[8,22]       47.111675    4.149624 39.196836491  55.741875
## beta[9,22]       47.603159    4.265260 40.614446872  56.875426
## beta[10,22]      29.132797    7.100869 14.316880108  43.495748
## beta[11,22]      49.128431    4.882646 41.976107304  61.152214
## beta[12,22]      68.932881    7.068263 54.207193517  82.832962
## beta[13,22]      30.166614    7.216423 15.671189128  44.467775
## beta[1,23]       58.351089    7.051408 44.868240496  74.288377
## beta[2,23]       59.288893    7.202677 44.676707287  74.032352
## beta[3,23]       36.310744    7.370730 21.193100408  50.997657
## beta[4,23]       66.711161    7.246430 51.889429279  81.337818
## beta[5,23]       30.641085    7.261301 15.413448042  44.719305
## beta[6,23]       58.799939    7.281320 44.031787435  74.778473
## beta[7,23]       38.098832    7.213834 23.954320030  52.044849
## beta[8,23]       47.307544    4.132758 39.502083029  56.313161
## beta[9,23]       47.790716    4.298330 40.322991956  56.888825
## beta[10,23]      29.119000    7.159204 13.544456447  43.218340
## beta[11,23]      49.212014    5.017169 41.892056284  61.738726
## beta[12,23]      68.935403    7.174223 54.031189201  83.249761
## beta[13,23]      30.115630    7.334874 14.515217728  44.475752
## beta[1,24]       58.336031    7.198778 43.923917841  73.272731
## beta[2,24]       59.270694    7.295185 44.419761293  74.227630
## beta[3,24]       36.314991    7.440503 20.997475824  51.450585
## beta[4,24]       66.708827    7.373735 51.476692114  81.649857
## beta[5,24]       30.685977    7.431120 15.430687298  45.368144
## beta[6,24]       58.812885    7.442241 43.492548274  75.069349
## beta[7,24]       38.055070    7.427031 23.139100328  52.620093
## beta[8,24]       47.527704    4.090125 39.811023449  56.178769
## beta[9,24]       47.954563    4.295914 40.438208619  57.221168
## beta[10,24]      29.142509    7.290425 13.803735115  43.611149
## beta[11,24]      49.245636    5.132736 41.545158973  62.085743
## beta[12,24]      68.990411    7.294981 54.268737105  83.550418
## beta[13,24]      30.140435    7.430950 14.293596367  44.936380
## beta[1,25]       58.359116    7.329318 43.605945472  74.203684
## beta[2,25]       59.271064    7.500391 44.006202915  74.728816
## beta[3,25]       36.334136    7.633326 20.440892035  51.878908
## beta[4,25]       66.700430    7.444504 50.970011594  81.687284
## beta[5,25]       30.689227    7.655568 15.219242366  46.221770
## beta[6,25]       58.813084    7.574588 43.722227739  74.929812
## beta[7,25]       38.080828    7.470243 22.681492650  52.842327
## beta[8,25]       47.713083    3.919387 40.433527135  56.775155
## beta[9,25]       48.079428    4.352540 40.457255275  57.418738
## beta[10,25]      29.130557    7.429949 13.506115896  44.280967
## beta[11,25]      49.290853    5.243420 41.667173561  62.025878
## beta[12,25]      68.966794    7.383241 53.811914763  84.126075
## beta[13,25]      30.160118    7.595330 14.092971878  45.275305
## beta[1,26]       58.360611    7.479748 43.205607838  74.028565
## beta[2,26]       59.232147    7.595289 43.765551891  74.685200
## beta[3,26]       36.314586    7.797782 20.101718783  52.162711
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```
## beta[4,26]       66.685476    7.643953 50.722188456  82.136138
## beta[5,26]       30.680942    7.977261 14.898505814  46.489211
## beta[6,26]       58.787844    7.684557 43.414280882  75.071718
## beta[7,26]       38.114291    7.589911 22.239582752  53.103183
## beta[8,26]       47.895263    3.844534 40.882362570  56.815278
## beta[9,26]       48.260340    4.357672 40.732152402  57.445194
## beta[10,26]      29.124755    7.512582 12.881415916  44.244382
## beta[11,26]      49.408161    5.408229 41.771690631  62.720774
## beta[12,26]      68.948972    7.509845 53.463921133  84.132349
## beta[13,26]      30.133584    7.734837 13.776098867  45.385171
## beta[1,27]       58.352661    7.577134 43.384365724  74.126017
## beta[2,27]       59.213533    7.764217 43.320253250  75.383021
## beta[3,27]       36.327205    7.919219 19.929613001  52.171283
## beta[4,27]       66.702326    7.768731 50.515620001  82.360221
## beta[5,27]       30.664580    8.011198 13.989054365  46.819722
## beta[6,27]       58.781013    7.864430 42.957864910  75.284220
## beta[7,27]       38.154901    7.757022 22.025412803  53.359401
## beta[8,27]       48.059529    3.747971 41.404372109  56.688012
## beta[9,27]       48.381731    4.306679 41.169647913  57.342771
## beta[10,27]      29.120428    7.686989 12.719665265  44.763436
## beta[11,27]      49.461466    5.477838 41.822762660  63.932493
## beta[12,27]      68.968277    7.675911 53.083295671  84.497324
## beta[13,27]      30.109131    7.836403 13.845008502  45.501562
## beta[1,28]       58.388813    7.749344 43.126231736  74.114999
## beta[2,28]       59.221344    7.886189 43.094458205  75.254985
## beta[3,28]       36.325693    8.130292 19.330446330  52.314718
## beta[4,28]       66.704719    7.907875 49.217919743  82.686228
## beta[5,28]       30.647430    7.878266 13.348651640  46.757838
## beta[6,28]       58.770549    7.953038 42.896862188  75.768467
## beta[7,28]       38.176783    7.859607 22.124564158  53.623693
## beta[8,28]       48.180134    3.536938 42.151152631  56.040795
## beta[9,28]       48.543158    4.354064 41.426635951  57.451410
## beta[10,28]      29.076100    7.784649 12.518390421  45.048406
## beta[11,28]      49.538702    5.573821 41.729774462  64.569799
## beta[12,28]      69.016248    7.725038 53.192741102  85.110034
## beta[13,28]      30.112135    7.955354 13.758173225  45.728022
## beta[1,29]       58.388375    7.895937 42.912426552  74.772536
## beta[2,29]       59.209923    8.037368 42.330713311  75.337637
## beta[3,29]       36.329592    8.136441 19.091861127  52.356614
## beta[4,29]       66.693147    8.056034 49.686300795  82.997463
## beta[5,29]       30.661621    8.051739 13.586113546  47.113767
## beta[6,29]       58.786726    8.042698 42.789835667  75.772358
## beta[7,29]       38.121583    8.011942 22.261782318  53.609690
## beta[8,29]       48.374186    3.343004 42.666469128  55.689562
## beta[9,29]       48.675968    4.291979 41.636738451  57.494187
## beta[10,29]      29.076926    7.940232 12.168736672  45.255600
## beta[11,29]      49.619604    5.729946 41.795605221  65.059051
## beta[12,29]      69.006914    7.903579 53.233256734  85.516148
## beta[13,29]      30.113794    8.038976 13.931821996  46.138378
## beta[1,30]       58.405280    7.985358 42.409859745  75.375195
## beta[2,30]       59.190132    8.121986 42.414946881  75.714370
## beta[3,30]       36.324500    8.257989 18.505170231  52.519473
```

```
## beta[4,30]       66.717932    8.279229 48.587751944  83.738281
## beta[5,30]       30.665387    8.125884 13.108117849  47.456315
## beta[6,30]       58.809266    8.241909 42.766113711  76.314390
## beta[7,30]       38.136258    8.113950 21.605238611  53.838367
## beta[8,30]       48.374920    3.469840 42.472431181  56.262291
## beta[9,30]       48.858458    4.321865 41.742379798  57.650443
## beta[10,30]      29.068039    8.061060 11.651731872  45.545969
## beta[11,30]      49.676875    5.836066 41.725145417  65.500220
## beta[12,30]      68.984087    8.023210 52.746320250  85.452682
## beta[13,30]      30.099370    8.099412 13.371774534  45.976690
## beta[1,31]       58.405322    8.141417 42.219517033  75.767319
## beta[2,31]       59.202163    8.240867 41.428756071  76.259253
## beta[3,31]       36.330624    8.318322 18.474305923  52.465169
## beta[4,31]       66.767165    8.409721 48.561604539  84.166468
## beta[5,31]       30.715365    8.161749 12.896407598  48.113251
## beta[6,31]       58.770578    8.336039 42.180950777  75.876092
## beta[7,31]       38.141041    8.281447 20.753533164  54.228917
## beta[8,31]       48.352146    3.591761 42.522402831  56.547823
## beta[9,31]       48.978578    4.344048 41.827528315  57.965734
## beta[10,31]      29.087480    8.203606 11.815270435  45.626210
## beta[11,31]      49.766943    5.867911 41.699531137  65.662993
## beta[12,31]      69.001572    8.110464 52.811669334  85.757175
## beta[13,31]      30.043771    8.164211 12.790561647  45.798382
## beta[1,32]       58.397283    8.327742 41.754774571  75.982748
## beta[2,32]       59.246670    8.354888 41.310874283  76.355037
## beta[3,32]       36.379510    8.494378 18.216258228  53.359564
## beta[4,32]       66.717809    8.491812 48.188657130  84.293506
## beta[5,32]       30.691713    8.319938 12.536224390  47.799236
## beta[6,32]       58.749662    8.442745 42.234912385  76.185943
## beta[7,32]       38.171561    8.363757 21.210772986  54.146962
## beta[8,32]       48.337925    3.652381 42.018881707  57.012004
## beta[9,32]       49.197333    4.253861 42.081565198  57.846987
## beta[10,32]      29.101255    8.323924 11.581156230  46.233340
## beta[11,32]      49.760127    5.910531 41.478098841  66.011081
## beta[12,32]      69.002466    8.346700 52.206337790  86.132964
## beta[13,32]      30.057511    8.293582 12.521668364  46.299268
## beta[1,33]       58.363050    8.515607 41.803872189  76.780161
## beta[2,33]       59.259829    8.510742 40.988121226  77.221818
## beta[3,33]       36.357808    8.639713 17.787882918  53.958855
## beta[4,33]       66.702033    8.650870 48.669019007  84.441895
## beta[5,33]       30.690800    8.508055 12.309171787  48.083626
## beta[6,33]       58.738126    8.479283 42.122644404  75.922231
## beta[7,33]       38.188739    8.564912 20.622662113  54.501034
## beta[8,33]       48.336329    3.620403 42.115415592  56.800534
## beta[9,33]       49.385765    4.251209 42.021157236  57.793415
## beta[10,33]      29.129031    8.438221 10.715903408  46.283507
## beta[11,33]      49.782489    5.897437 41.199014102  65.772357
## beta[12,33]      69.015222    8.476621 51.883178649  86.552070
## beta[13,33]      30.056081    8.435342 11.935030707  46.595457
## beta[1,34]       58.337080    8.585368 41.250944281  76.227461
## beta[2,34]       59.240301    8.626346 41.114094045  77.206329
## beta[3,34]       36.337712    8.749139 17.777673824  54.126746
```

```
## beta[4,34]       66.733223    8.718471 49.012065320  84.902938
## beta[5,34]       30.696055    8.617647 11.972031662  48.263148
## beta[6,34]       58.690203    8.601654 41.788576846  77.036900
## beta[7,34]       38.152323    8.823461 20.538360018  54.864974
## beta[8,34]       48.327681    3.661307 41.724390849  56.810018
## beta[9,34]       49.386044    4.491684 41.855367443  58.049463
## beta[10,34]      29.116853    8.587442 10.510940846  46.748104
## beta[11,34]      49.753827    5.831538 41.100226162  65.096384
## beta[12,34]      69.021027    8.630769 51.688666137  87.282493
## beta[13,34]      30.038949    8.533841 11.542266387  46.922635
## beta[1,35]       58.301167    8.676634 41.391949657  76.495223
## beta[2,35]       59.239932    8.936085 40.114102584  77.601488
## beta[3,35]       36.375970    8.968403 17.820692170  54.768484
## beta[4,35]       66.707187    8.772537 49.030541861  85.027266
## beta[5,35]       30.738280    8.669593 12.272459321  48.876473
## beta[6,35]       58.691741    8.685950 41.421417352  77.411867
## beta[7,35]       38.179575    8.954459 20.402896140  55.122419
## beta[8,35]       48.326164    3.592566 42.098410192  56.683027
## beta[9,35]       49.385504    4.656056 41.464022839  58.689330
## beta[10,35]      29.089098    8.695084 10.685081980  46.845660
## beta[11,35]      49.734665    5.763414 41.148496195  64.073444
## beta[12,35]      68.956366    8.779555 50.967050708  87.263363
## beta[13,35]      29.994411    8.704314 11.022609108  47.259752
## beta[1,36]       58.322351    8.786967 40.933249212  76.883964
## beta[2,36]       59.270942    9.032085 39.860439848  78.529761
## beta[3,36]       36.362008    9.088661 17.601532312  55.174064
## beta[4,36]       66.752500    8.920629 48.845998864  85.994887
## beta[5,36]       30.764507    8.803632 12.629435694  49.131105
## beta[6,36]       58.702858    8.836718 40.971385361  77.716841
## beta[7,36]       38.163203    9.002164 20.205639274  55.511474
## beta[8,36]       48.321837    3.524404 42.052537616  56.383302
## beta[9,36]       49.399843    4.873875 41.347955702  59.500296
## beta[10,36]      29.047275    8.894310 10.064662584  46.861783
## beta[11,36]      49.756600    5.642103 41.228426815  64.068104
## beta[12,36]      68.951891    8.874267 51.020552750  87.590803
## beta[13,36]      30.002838    8.850375 11.200756574  47.942744
## beta[1,37]       58.341108    8.840590 41.264843599  77.051938
## beta[2,37]       59.287404    9.157310 40.322821289  78.633457
## beta[3,37]       36.366248    9.252908 17.627972176  55.296685
## beta[4,37]       66.757056    9.043156 47.926249689  85.193607
## beta[5,37]       30.773170    8.844649 11.760803474  49.231633
## beta[6,37]       58.704776    8.950223 41.133307694  78.306443
## beta[7,37]       38.149012    9.195791 20.160715882  56.041741
## beta[8,37]       48.335032    3.320553 42.530093976  56.179212
## beta[9,37]       49.415194    5.086141 40.750152182  60.281467
## beta[10,37]      29.055831    9.028082 10.065773373  47.103764
## beta[11,37]      49.736746    5.492626 41.056461263  64.073448
## beta[12,37]      68.953397    8.939985 51.127304349  87.830202
## beta[13,37]      30.016123    8.999085 10.699783852  48.176534
## beta[1,38]       58.340468    8.915533 40.180364778  77.327484
## beta[2,38]       59.286290    9.297610 40.067979567  78.734585
## beta[3,38]       36.368730    9.367791 17.491857229  55.832696
```

```
## beta[4,38]       66.788116    9.225656 47.588104682  85.396471
## beta[5,38]       30.767453    8.905424 11.422559599  49.843618
## beta[6,38]       58.720409    9.082314 41.162307046  78.581337
## beta[7,38]       38.172983    9.242998 19.182914386  56.549156
## beta[8,38]       48.387789    3.498412 42.168388358  56.413607
## beta[9,38]       49.444886    5.306420 40.800231582  60.749636
## beta[10,38]      29.035435    9.142563  9.418918387  47.089320
## beta[11,38]      49.715431    5.352785 41.035532568  63.293734
## beta[12,38]      68.920898    9.070841 50.354393036  88.211045
## beta[13,38]      30.038596    9.063264 10.567711050  48.120986
## beta[1,39]       58.359120    9.055597 40.087699119  77.355351
## beta[2,39]       59.284449    9.469253 39.502874714  78.399574
## beta[3,39]       36.408078    9.519165 16.818198437  56.536652
## beta[4,39]       66.776303    9.306232 47.168488626  85.992522
## beta[5,39]       30.758422    9.050036 11.085774912  49.653635
## beta[6,39]       58.710203    9.205787 40.576298194  78.587820
## beta[7,39]       38.140829    9.300548 18.894201298  56.998035
## beta[8,39]       48.429538    3.622395 41.858979011  56.709754
## beta[9,39]       49.473816    5.450238 40.357940479  60.951173
## beta[10,39]      28.995772    9.302294  9.130487427  47.417964
## beta[11,39]      49.709755    5.169734 41.182487243  62.277429
## beta[12,39]      68.942451    9.235935 49.783307056  88.434486
## beta[13,39]      30.003333    9.221598 10.393625232  47.771017
## beta[1,40]       58.353360    9.231575 39.431915295  77.871004
## beta[2,40]       59.290483    9.635168 38.411617382  78.866058
## beta[3,40]       36.399430    9.659436 16.597760585  56.977430
## beta[4,40]       66.768518    9.424373 47.229364921  85.992447
## beta[5,40]       30.727088    9.094256 11.412599951  49.889352
## beta[6,40]       58.709054    9.289561 39.995344031  78.472721
## beta[7,40]       38.146629    9.373491 19.164503954  57.627618
## beta[8,40]       48.451595    3.813805 41.368817089  57.200151
## beta[9,40]       49.448852    5.644211 39.983535608  61.891590
## beta[10,40]      28.999952    9.442012  8.618916745  47.525784
## beta[11,40]      49.685392    4.965266 41.061955231  61.435476
## beta[12,40]      68.931455    9.355081 49.594773334  88.582473
## beta[13,40]      30.051289    9.299604  9.718960143  48.051397
## beta[1,41]       58.336727    9.363785 39.071837410  78.805922
## beta[2,41]       59.286191    9.722480 38.720183462  79.558980
## beta[3,41]       36.418498    9.713833 16.493530288  57.493731
## beta[4,41]       66.831560    9.482985 47.615645885  86.931792
## beta[5,41]       30.701429    9.188257 11.048517588  49.874717
## beta[6,41]       58.741832    9.421281 39.671102795  78.852236
## beta[7,41]       38.183375    9.411449 19.164451608  57.790796
## beta[8,41]       48.495216    3.912999 41.193255320  57.663554
## beta[9,41]       49.441621    5.840838 39.995993475  62.198043
## beta[10,41]      29.026590    9.435413  8.788286374  47.850572
## beta[11,41]      49.671192    4.730102 41.188750018  60.665107
## beta[12,41]      68.911594    9.396281 49.524821191  88.890187
## beta[13,41]      30.080213    9.401652 10.507102014  48.092876
## beta[1,42]       58.327299    9.464999 38.494441114  79.135529
## beta[2,42]       59.315197    9.904535 38.758678715  79.756946
## beta[3,42]       36.431269    9.855190 16.702552703  57.422336
```

```
## beta[4,42]       66.794619    9.618841 47.270909739  87.279727
## beta[5,42]       30.724200    9.270947 11.697667477  49.581526
## beta[6,42]       58.709875    9.539052 39.072112399  78.369050
## beta[7,42]       38.200600    9.499330 19.127685432  58.811867
## beta[8,42]       48.485498    4.018009 40.977861641  58.217997
## beta[9,42]       49.489778    5.953679 39.464632502  62.525827
## beta[10,42]      28.976214    9.587119  8.456011989  48.057925
## beta[11,42]      49.606954    4.854200 40.985117783  61.233931
## beta[12,42]      68.922454    9.483470 49.213112779  88.231759
## beta[13,42]      30.113020    9.519371 10.527053415  48.956384
## beta[1,43]       58.394157    9.561621 38.319952365  79.931830
## beta[2,43]       59.262977    9.878818 38.906056136  78.935442
## beta[3,43]       36.390810    9.981834 16.160352123  57.869348
## beta[4,43]       66.806487    9.716313 47.240231384  88.402134
## beta[5,43]       30.755100    9.442915 11.307810825  50.556291
## beta[6,43]       58.698953    9.636199 38.785208086  78.747404
## beta[7,43]       38.189758    9.633878 19.518471948  58.635497
## beta[8,43]       48.496378    4.160272 40.617589007  58.535339
## beta[9,43]       49.483359    6.064949 39.055451967  63.309265
## beta[10,43]      28.983426    9.576681  8.159164170  47.874831
## beta[11,43]      49.552887    5.009932 40.541488469  61.582569
## beta[12,43]      68.939115    9.540294 49.246824195  88.995207
## beta[13,43]      30.138320    9.697521  9.535885444  49.358725
## beta[1,44]       58.405969    9.633922 38.387239157  79.697591
## beta[2,44]       59.302714    9.957654 38.498677602  79.710075
## beta[3,44]       36.412800   10.035501 16.343707023  57.240067
## beta[4,44]       66.854301    9.828002 46.902249065  88.266295
## beta[5,44]       30.755484    9.481572 10.800641827  50.965044
## beta[6,44]       58.702254    9.784251 39.029395870  78.735606
## beta[7,44]       38.203245    9.651559 18.652743731  58.949260
## beta[8,44]       48.529961    4.245846 40.282785902  59.027521
## beta[9,44]       49.453420    6.148721 38.741434479  62.775051
## beta[10,44]      29.014147    9.656701  8.626992705  48.055538
## beta[11,44]      49.427021    5.099508 39.999163899  61.011901
## beta[12,44]      68.923515    9.702870 48.915360632  89.206240
## beta[13,44]      30.141496    9.814635 10.204458277  50.130765
## beta[1,45]       58.430856    9.717739 38.972051096  79.758418
## beta[2,45]       59.312852   10.088833 38.256471921  80.287300
## beta[3,45]       36.378715   10.062634 15.659445261  57.202105
## beta[4,45]       66.802716    9.866866 47.024185883  88.505745
## beta[5,45]       30.766169    9.586867  9.965656773  50.708335
## beta[6,45]       58.714490    9.897630 38.570894024  79.643478
## beta[7,45]       38.219739    9.718123 19.029486536  59.264245
## beta[8,45]       48.549159    4.309225 39.720507688  58.072259
## beta[9,45]       49.478769    6.373041 38.560404723  63.430349
## beta[10,45]      29.032992    9.689625  8.090693482  48.261811
## beta[11,45]      49.344680    5.196111 39.642010590  61.275254
## beta[12,45]      68.878893    9.754842 49.325852234  89.337864
## beta[13,45]      30.166113    9.855824 10.166690817  50.701074
## beta[1,46]       58.499507    9.761177 38.409384574  80.097271
## beta[2,46]       59.330301   10.269850 38.493708436  80.978253
## beta[3,46]       36.410325   10.179832 15.619467395  57.070546
```

```
## beta[4,46]       66.721869   10.040144 45.963368365  88.335803
## beta[5,46]       30.790977    9.689562  9.849343843  50.858197
## beta[6,46]       58.752577    9.997818 38.282581329  79.618938
## beta[7,46]       38.224216    9.806980 18.222606321  58.963267
## beta[8,46]       48.568829    4.392153 39.555711824  58.544346
## beta[9,46]       49.478814    6.492325 38.766877622  63.394037
## beta[10,46]      29.081176    9.858958  7.923523351  48.403863
## beta[11,46]      49.269274    5.350192 39.254654944  61.349672
## beta[12,46]      68.918785    9.965804 48.085270324  89.964039
## beta[13,46]      30.181177    9.998305  9.735086322  50.057624
## beta[1,47]       58.423416    9.879746 38.576325587  80.338348
## beta[2,47]       59.283561   10.292897 38.303267450  81.235734
## beta[3,47]       36.379006   10.324383 15.081852478  57.482141
## beta[4,47]       66.729498   10.071167 45.610250117  88.876759
## beta[5,47]       30.815304    9.835193  9.599901756  51.673917
## beta[6,47]       58.758131   10.071746 37.866078057  79.835772
## beta[7,47]       38.212291    9.921624 17.729374945  59.259893
## beta[8,47]       48.608688    4.478398 39.193238629  57.986176
## beta[9,47]       49.442414    6.546517 38.139298413  64.214771
## beta[10,47]      29.121687    9.988454  7.296830671  49.486167
## beta[11,47]      49.197262    5.449391 38.616841354  60.913929
## beta[12,47]      68.936028   10.148096 47.907419430  89.631177
## beta[13,47]      30.190164   10.029180 10.412857376  50.130141
## beta[1,48]       58.376473    9.966157 38.088012056  79.725484
## beta[2,48]       59.316804   10.499138 38.339091728  81.578694
## beta[3,48]       36.383144   10.409268 14.380657387  57.154426
## beta[4,48]       66.735541   10.190436 45.536808312  88.417871
## beta[5,48]       30.821549    9.952443 10.212783464  51.782073
## beta[6,48]       58.763379   10.163826 37.563366017  79.826758
## beta[7,48]       38.192056   10.039275 18.057363093  59.112538
## beta[8,48]       48.643146    4.443255 39.692474090  57.852687
## beta[9,48]       49.456994    6.703917 37.905764264  64.284976
## beta[10,48]      29.111836   10.032908  6.897197424  49.585019
## beta[11,48]      49.073080    5.475640 37.974862071  61.387713
## beta[12,48]      68.953508   10.210129 48.101115995  90.397385
## beta[13,48]      30.215184   10.143215 10.135639255  50.678913
## beta[1,49]       58.371748   10.076427 37.610775850  80.675610
## beta[2,49]       59.262730   10.537179 38.194307498  81.923461
## beta[3,49]       36.401477   10.576266 14.374298813  57.507530
## beta[4,49]       66.791020   10.392486 44.781314836  89.294955
## beta[5,49]       30.861386   10.016867  9.679411445  52.220877
## beta[6,49]       58.727007   10.214012 37.561745616  79.754401
## beta[7,49]       38.173544   10.068986 17.952996392  58.910903
## beta[8,49]       48.665191    4.589358 39.611942349  58.550163
## beta[9,49]       49.393515    6.896712 36.959567670  64.542820
## beta[10,49]      29.149170   10.113054  6.766574563  49.558695
## beta[11,49]      48.979910    5.479934 37.953116067  61.139598
## beta[12,49]      68.975411   10.222189 47.854051187  90.485636
## beta[13,49]      30.183653   10.215848  9.427334993  50.598959
## beta[1,50]       58.359633   10.173904 37.726434046  80.278073
## beta[2,50]       59.254114   10.660825 37.828848998  81.586989
## beta[3,50]       36.373283   10.709186 14.387938771  57.587661
```

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## beta[4,50]       66.780769   10.471904 44.737988893  89.022409
## beta[5,50]       30.846789   10.105321  9.152477380  52.042250
## beta[6,50]       58.759141   10.279610 37.756613081  80.158068
## beta[7,50]       38.121702   10.177687 17.780112791  59.767881
## beta[8,50]       48.679945    4.567504 39.497142616  57.927683
## beta[9,50]       49.385363    6.986503 36.891909846  64.707378
## beta[10,50]      29.142695   10.250042  6.658297429  49.806549
## beta[11,50]      48.894773    5.482486 37.388418318  60.792996
## beta[12,50]      68.939555   10.367675 47.644376610  91.177732
## beta[13,50]      30.187666   10.255948  8.777585890  50.407425
## beta[1,51]       58.360508   10.253196 37.391792101  80.466785
## beta[2,51]       59.214193   10.762827 37.563021517  81.792932
## beta[3,51]       36.372563   10.788348 14.107608358  57.888447
## beta[4,51]       66.832387   10.530477 44.684969452  89.726578
## beta[5,51]       30.842377   10.285400  9.259517983  52.490213
## beta[6,51]       58.771224   10.342302 37.582263220  80.131550
## beta[7,51]       38.124566   10.325040 17.343901195  59.462827
## beta[8,51]       48.666946    4.547974 39.576742231  58.176938
## beta[9,51]       49.403413    7.142299 36.147592232  65.078941
## beta[10,51]      29.173840   10.417500  6.709338557  50.130286
## beta[11,51]      48.796643    5.539586 37.460855979  60.910156
## beta[12,51]      68.933561   10.493311 47.371023231  91.104252
## beta[13,51]      30.177794   10.336718  8.327671177  50.996954
## beta[1,52]       58.358538   10.238379 37.396494319  80.939294
## beta[2,52]       59.234402   10.865255 37.556279355  82.498279
## beta[3,52]       36.379327   10.957520 13.548887188  57.988779
## beta[4,52]       66.896477   10.540694 45.137626078  89.427730
## beta[5,52]       30.812262   10.383023  8.924519973  53.395845
## beta[6,52]       58.737873   10.494347 36.513691614  80.817242
## beta[7,52]       38.129867   10.375952 17.131317373  60.727658
## beta[8,52]       48.648500    4.516445 39.721257965  57.914559
## beta[9,52]       49.408848    7.220367 36.147935818  64.860569
## beta[10,52]      29.167035   10.390763  6.073941805  50.503183
## beta[11,52]      48.734515    5.636700 37.446339036  60.993169
## beta[12,52]      68.880616   10.535383 47.012084178  90.937786
## beta[13,52]      30.193530   10.389880  8.003942688  51.143873
## beta[1,53]       58.341440   10.260756 36.870105757  80.802492
## beta[2,53]       59.218473   10.946704 38.010436406  81.834745
## beta[3,53]       36.386089   11.063236 13.740314043  59.118357
## beta[4,53]       66.874743   10.623580 45.084557438  88.915980
## beta[5,53]       30.807665   10.542474  9.332499709  53.797092
## beta[6,53]       58.789201   10.545334 36.597044566  80.953247
## beta[7,53]       38.128572   10.490000 16.936744619  60.634267
## beta[8,53]       48.564014    4.514155 39.632684987  58.633418
## beta[9,53]       49.402590    7.391540 35.937221252  65.269496
## beta[10,53]      29.157230   10.403835  6.254603198  50.162928
## beta[11,53]      48.644189    5.629157 36.752845703  61.395883
## beta[12,53]      68.867769   10.657177 46.960059281  91.292683
## beta[13,53]      30.235656   10.580251  7.845516186  51.964238
## beta[1,54]       58.351241   10.423225 37.282258057  81.733496
## beta[2,54]       59.222953   11.046093 37.419624873  82.533546
## beta[3,54]       36.414459   11.100828 13.226809567  59.347321
```

```
## beta[4,54]       66.900985   10.686572 45.026811314  89.628045
## beta[5,54]       30.773649   10.644238  8.799265359  54.224195
## beta[6,54]       58.755876   10.612440 36.207909358  81.116149
## beta[7,54]       38.121836   10.591404 16.351813840  60.820745
## beta[8,54]       48.564066    4.464247 39.684957586  58.439100
## beta[9,54]       49.432602    7.581063 35.819965337  66.004459
## beta[10,54]      29.159083   10.544954  6.541942937  51.352615
## beta[11,54]      48.582843    5.714138 36.756121162  61.381583
## beta[12,54]      68.880772   10.740436 46.541235569  91.511470
## beta[13,54]      30.232632   10.707013  7.507528645  51.545120
## beta[1,55]       58.343371   10.547482 36.562238189  81.479082
## beta[2,55]       59.229158   11.097615 36.900865752  82.705022
## beta[3,55]       36.365920   11.176318 12.864779686  59.466620
## beta[4,55]       66.917908   10.803984 45.010219945  89.941779
## beta[5,55]       30.780107   10.864908  7.786811227  53.919234
## beta[6,55]       58.805079   10.744315 36.586321595  81.370895
## beta[7,55]       38.134680   10.726170 16.027044153  61.503235
## beta[8,55]       48.510220    4.407804 39.514798471  57.916517
## beta[9,55]       49.460040    7.780201 35.172964231  66.315828
## beta[10,55]      29.164294   10.530930  6.561905336  50.806524
## beta[11,55]      48.556084    5.763693 36.819071593  61.521158
## beta[12,55]      68.863385   10.851512 46.136422023  91.602013
## beta[13,55]      30.232908   10.832946  7.900152233  51.800465
## beta[1,56]       58.371922   10.665882 36.116386784  81.913443
## beta[2,56]       59.267522   11.182612 37.492784085  83.161382
## beta[3,56]       36.382676   11.345059 12.963487104  60.321347
## beta[4,56]       66.910817   10.900907 44.729870439  90.445138
## beta[5,56]       30.793502   10.982388  7.895552844  53.916957
## beta[6,56]       58.835567   10.851720 36.431103803  81.554842
## beta[7,56]       38.163235   10.752045 16.026432348  60.646408
## beta[8,56]       48.513139    4.419162 39.451658984  57.990484
## beta[9,56]       49.452286    7.901976 34.663283088  66.420171
## beta[10,56]      29.169069   10.648283  6.478439505  51.585557
## beta[11,56]      48.484691    5.854181 36.702473733  61.666074
## beta[12,56]      68.844701   10.991723 46.272332667  91.873197
## beta[13,56]      30.267814   10.897896  7.626706282  52.009110
## beta[1,57]       58.356785   10.820326 36.309412844  82.617113
## beta[2,57]       59.218785   11.207454 37.275134821  82.897199
## beta[3,57]       36.366838   11.413242 13.057425364  60.329897
## beta[4,57]       66.934335   10.901233 44.588549850  90.700158
## beta[5,57]       30.808424   11.120891  7.637684719  54.254930
## beta[6,57]       58.846877   10.993248 36.490434651  81.787732
## beta[7,57]       38.143969   10.809271 16.376183478  61.043716
## beta[8,57]       48.465420    4.422176 39.367811830  57.935443
## beta[9,57]       49.453209    8.008460 33.884218092  66.651680
## beta[10,57]      29.194361   10.776201  6.490554527  51.299729
## beta[11,57]      48.381313    5.945537 36.673099218  61.690526
## beta[12,57]      68.849023   11.130463 45.255149913  92.088191
## beta[13,57]      30.285477   10.956350  8.390324596  52.622708
## beta[1,58]       58.329161   11.000672 35.470286955  82.345793
## beta[2,58]       59.206358   11.190222 37.808780208  83.013399
## beta[3,58]       36.380671   11.531063 12.910055425  60.636513
```

```
## beta[4,58]       66.931938   10.959597 43.796551416  90.622406
## beta[5,58]       30.786600   11.258816  7.020274303  54.419597
## beta[6,58]       58.828922   11.026541 35.863443852  81.851471
## beta[7,58]       38.137641   10.875510 15.960975008  61.244778
## beta[8,58]       48.393890    4.464992 39.102932054  58.065218
## beta[9,58]       49.421673    8.139247 34.164584881  66.743577
## beta[10,58]      29.142205   10.914206  6.707854288  51.584996
## beta[11,58]      48.268719    6.069698 36.436398721  62.264508
## beta[12,58]      68.841191   11.217644 45.055445507  92.216677
## beta[13,58]      30.263699   11.063687  8.004322381  52.672884
## beta[1,59]       58.367483   11.096307 35.187186501  83.697803
## beta[2,59]       59.208403   11.255258 37.442798539  82.954968
## beta[3,59]       36.398560   11.657224 13.115135881  60.861212
## beta[4,59]       66.896607   11.024604 44.344113119  90.822548
## beta[5,59]       30.822151   11.357791  7.259622816  54.458574
## beta[6,59]       58.854387   11.030556 36.276026604  82.682631
## beta[7,59]       38.137141   11.039464 15.874004305  61.027153
## beta[8,59]       48.337983    4.427708 38.844276032  57.868409
## beta[9,59]       49.423118    8.348622 33.655405610  66.933881
## beta[10,59]      29.146749   10.966565  6.162773092  50.860331
## beta[11,59]      48.200853    6.185793 36.399784986  62.582954
## beta[12,59]      68.800269   11.283705 45.041936902  92.512809
## beta[13,59]      30.233704   11.034258  7.467569945  52.322719
## beta[1,60]       58.399385   11.216877 35.299687117  83.912895
## beta[2,60]       59.221091   11.384827 36.293106530  82.595534
## beta[3,60]       36.407102   11.697158 13.333197127  61.079396
## beta[4,60]       66.928589   11.098653 43.383818775  90.714565
## beta[5,60]       30.787482   11.349042  6.824251845  54.764170
## beta[6,60]       58.858079   11.013371 36.868076389  82.592953
## beta[7,60]       38.165060   11.156202 15.786224385  61.782034
## beta[8,60]       48.267115    4.438814 38.898205129  57.504716
## beta[9,60]       49.413152    8.526230 33.105198061  68.323963
## beta[10,60]      29.199060   11.088639  5.441454211  51.715760
## beta[11,60]      48.077798    6.195604 36.350945112  62.273026
## beta[12,60]      68.837330   11.351994 45.059385147  92.416020
## beta[13,60]      30.190974   11.126890  7.239842949  53.175586
## beta[1,61]       58.371852   11.356562 34.701425186  83.659054
## beta[2,61]       59.219874   11.487686 36.450351763  83.746174
## beta[3,61]       36.437960   11.735315 12.703765676  61.793046
## beta[4,61]       66.916942   11.121217 43.104250789  91.228247
## beta[5,61]       30.781223   11.545389  6.576052289  54.329584
## beta[6,61]       58.853556   11.117743 36.304430192  82.952119
## beta[7,61]       38.158854   11.218346 15.518861044  61.261440
## beta[8,61]       48.256132    4.430269 39.239965241  57.361407
## beta[9,61]       49.420281    8.603216 32.650724922  68.153958
## beta[10,61]      29.148674   11.172535  5.309437026  52.322882
## beta[11,61]      47.976439    6.233249 35.826713427  61.475557
## beta[12,61]      68.837590   11.398324 45.624958727  92.615233
## beta[13,61]      30.162911   11.213700  6.334993787  53.166707
## beta[1,62]       58.391343   11.447857 34.461657051  82.773453
## beta[2,62]       59.238652   11.534219 36.204043700  83.918613
## beta[3,62]       36.429648   11.802938 12.458547333  61.602292
```

```
## beta[4,62]       66.953771   11.290503 42.995189708  91.811529
## beta[5,62]       30.777061   11.719088  6.643962964  54.700657
## beta[6,62]       58.822960   11.239776 36.133674386  82.527150
## beta[7,62]       38.192201   11.337867 14.988273257  61.409143
## beta[8,62]       48.153490    4.306677 39.115646432  56.658797
## beta[9,62]       49.431759    8.723153 32.894364583  68.274806
## beta[10,62]      29.151681   11.306333  5.169229961  52.452953
## beta[11,62]      47.866026    6.251211 34.834361162  61.797696
## beta[12,62]      68.848084   11.504274 45.165849442  92.573702
## beta[13,62]      30.166750   11.290535  6.122693283  53.399221
## beta[1,63]       58.373729   11.515945 34.079818559  84.043405
## beta[2,63]       59.202479   11.628876 35.727945320  84.208520
## beta[3,63]       36.417399   11.930145 11.885525227  61.881696
## beta[4,63]       66.954686   11.349544 42.698047641  91.513979
## beta[5,63]       30.810003   11.822721  6.933093799  54.825385
## beta[6,63]       58.880541   11.368932 35.940211580  83.389061
## beta[7,63]       38.203453   11.496070 13.864259274  62.037468
## beta[8,63]       48.116468    4.324737 38.586997265  57.050284
## beta[9,63]       49.420086    8.879292 32.314616284  68.501876
## beta[10,63]      29.112195   11.457877  4.775617753  52.606172
## beta[11,63]      47.772666    6.241172 34.831031978  61.623744
## beta[12,63]      68.849640   11.618509 45.129750446  93.230882
## beta[13,63]      30.149187   11.327947  5.711058387  53.599843
## beta[1,64]       58.375628   11.520699 33.872698478  83.599130
## beta[2,64]       59.177138   11.763269 35.605706196  84.651303
## beta[3,64]       36.427187   11.994397 12.360076599  61.910626
## beta[4,64]       66.942923   11.447698 42.963776574  91.565736
## beta[5,64]       30.829439   11.696516  6.622654464  55.359942
## beta[6,64]       58.895988   11.467617 35.351837032  84.106282
## beta[7,64]       38.213059   11.631348 13.813603747  62.804926
## beta[8,64]       47.992343    4.335173 38.520324624  56.755129
## beta[9,64]       49.397296    9.073611 32.534713759  68.376852
## beta[10,64]      29.107985   11.607488  4.081686529  52.564719
## beta[11,64]      47.694091    6.249770 34.814495849  61.420876
## beta[12,64]      68.870790   11.735939 44.863761849  93.904641
## beta[13,64]      30.178077   11.407891  5.821665232  53.480482
## beta[1,65]       58.360293   11.615244 33.923621724  83.323372
## beta[2,65]       59.152876   11.851874 35.142731077  85.146911
## beta[3,65]       36.462653   12.092097 11.707490776  61.670257
## beta[4,65]       66.967801   11.554573 42.683096204  91.463034
## beta[5,65]       30.896622   11.819904  6.511232906  55.885813
## beta[6,65]       58.843573   11.538920 34.853962209  84.591943
## beta[7,65]       38.223277   11.735928 14.018747249  62.881761
## beta[8,65]       47.938952    4.317256 38.274920125  57.092983
## beta[9,65]       49.416578    9.197400 32.083009851  68.838766
## beta[10,65]      29.087871   11.675726  4.266908684  52.742621
## beta[11,65]      47.639925    6.274316 34.509333380  60.919568
## beta[12,65]      68.871959   11.789986 45.340007609  94.105321
## beta[13,65]      30.194340   11.510016  5.337938975  54.040600
## beta[1,66]       58.319068   11.740632 33.433443007  83.614582
## beta[2,66]       59.153497   11.874212 34.720834645  84.958936
## beta[3,66]       36.467368   12.177388 11.887423043  62.034647
```

```
## beta[4,66]       66.964838   11.623368 43.095558132  91.223000
## beta[5,66]       30.882659   11.820394  6.591294988  55.942858
## beta[6,66]       58.876684   11.578933 35.000832680  84.249143
## beta[7,66]       38.213197   11.823833 14.037734423  63.043339
## beta[8,66]       47.828085    4.290763 38.277693346  56.159101
## beta[9,66]       49.433882    9.350091 31.521365898  69.135348
## beta[10,66]      29.071038   11.765064  3.798024744  52.709295
## beta[11,66]      47.582478    6.293944 34.431942064  61.430073
## beta[12,66]      68.843516   11.845838 44.454908881  94.459279
## beta[13,66]      30.166756   11.654989  5.591366970  54.338490
## beta[1,67]       58.335350   11.863195 33.593979298  83.070067
## beta[2,67]       59.145296   11.935702 35.110508035  85.251156
## beta[3,67]       36.490503   12.214464 11.822771719  62.087071
## beta[4,67]       66.955054   11.710533 43.636065803  91.682543
## beta[5,67]       30.911322   12.170245  5.635220508  56.720341
## beta[6,67]       58.880914   11.657765 34.889448986  84.544273
## beta[7,67]       38.229864   11.899796 14.117395868  63.434659
## beta[8,67]       47.704448    4.283660 38.268340695  55.915032
## beta[9,67]       49.457194    9.423271 31.340991634  69.631722
## beta[10,67]      29.083442   11.892838  3.614520539  52.397398
## beta[11,67]      47.550898    6.239501 33.837221721  60.831068
## beta[12,67]      68.824949   11.981327 43.763197087  95.013018
## beta[13,67]      30.204274   11.704860  4.155919560  54.274206
## beta[1,68]       58.338325   11.984597 32.761158696  83.665478
## beta[2,68]       59.167650   11.970022 34.454614518  86.098268
## beta[3,68]       36.483560   12.314699 11.850445126  62.589779
## beta[4,68]       67.004598   11.854641 42.587606149  92.807817
## beta[5,68]       30.951491   12.311378  5.820253640  57.063566
## beta[6,68]       58.831470   11.750276 34.038952511  84.533575
## beta[7,68]       38.266988   11.959153 13.903926425  63.739515
## beta[8,68]       47.647819    4.303650 38.250003639  56.235636
## beta[9,68]       49.423036    9.534090 31.534536076  69.782768
## beta[10,68]      29.139129   12.069566  2.914690032  53.526170
## beta[11,68]      47.513857    6.232526 33.774650920  60.842811
## beta[12,68]      68.820723   12.079601 43.832960337  94.977532
## beta[13,68]      30.232503   11.772844  4.815861901  54.330568
## beta[1,69]       58.340948   12.141020 32.308162336  83.671443
## beta[2,69]       59.180642   11.972796 34.794681196  84.628579
## beta[3,69]       36.517004   12.364581 11.180227182  63.024401
## beta[4,69]       67.074346   11.904119 42.591502206  93.206374
## beta[5,69]       30.980940   12.468565  5.372143052  57.498884
## beta[6,69]       58.827283   11.835322 34.368102837  84.737354
## beta[7,69]       38.276345   11.954292 13.672977752  64.366874
## beta[8,69]       47.582802    4.374362 37.674961692  56.003573
## beta[9,69]       49.440078    9.673483 31.160361510  70.014294
## beta[10,69]      29.137937   12.065679  3.517033752  53.795796
## beta[11,69]      47.497258    6.176592 34.060121599  60.997473
## beta[12,69]      68.835773   12.113050 43.921416916  94.873925
## beta[13,69]      30.271566   11.860393  5.118499679  54.615339
## beta[1,70]       58.376557   12.113884 32.307987744  84.039839
## beta[2,70]       59.180403   11.994052 34.317278123  84.715138
## beta[3,70]       36.532871   12.493138 10.847719694  63.308818
```

```
## beta[4,70]       67.045646   12.039169 42.391243970  93.138444
## beta[5,70]       30.971603   12.793942  4.982994407  57.275895
## beta[6,70]       58.825254   11.849343 34.594148388  84.647890
## beta[7,70]       38.257487   12.069729 13.910830265  64.611359
## beta[8,70]       47.499951    4.363396 37.361483064  56.000985
## beta[9,70]       49.462794    9.778698 30.726174630  70.453000
## beta[10,70]      29.141884   12.062563  3.109836522  53.955494
## beta[11,70]      47.512724    6.143290 34.206399401  61.177899
## beta[12,70]      68.878035   12.193529 44.242526446  95.134336
## beta[13,70]      30.295872   11.995297  4.955058760  55.732865
## beta[1,71]       58.389135   12.171906 32.094280559  84.138663
## beta[2,71]       59.139176   12.034133 34.170543889  84.868332
## beta[3,71]       36.512162   12.617404 10.104695942  63.608130
## beta[4,71]       67.073922   12.111629 42.616533721  93.493815
## beta[5,71]       30.991149   13.092239  4.279579041  57.150367
## beta[6,71]       58.821858   11.867665 34.551054006  84.285714
## beta[7,71]       38.253813   12.207121 12.485185945  64.915939
## beta[8,71]       47.469760    4.376404 37.278858315  55.803421
## beta[9,71]       49.517369    9.920173 30.063638875  71.616799
## beta[10,71]      29.124487   12.126597  3.328032843  54.763195
## beta[11,71]      47.477054    6.149547 34.299553257  61.363167
## beta[12,71]      68.899953   12.265686 43.669368042  94.927925
## beta[13,71]      30.368542   12.080691  4.998923155  56.128048
## beta[1,72]       58.414393   12.298819 31.666610346  84.186927
## beta[2,72]       59.200237   12.180809 34.115466880  85.234820
## beta[3,72]       36.507679   12.752761  9.715142074  64.297358
## beta[4,72]       67.044334   12.133782 41.785651851  93.558911
## beta[5,72]       30.949765   13.209077  4.594940175  56.814419
## beta[6,72]       58.802256   12.001162 34.052073739  84.539552
## beta[7,72]       38.278682   12.312897 13.036511822  64.869772
## beta[8,72]       47.410875    4.399645 37.113256583  55.943761
## beta[9,72]       49.516255    9.991996 30.322847470  72.141849
## beta[10,72]      29.120122   12.193277  3.477082916  54.102608
## beta[11,72]      47.462080    6.109807 34.517458640  61.028475
## beta[12,72]      68.919687   12.334761 43.163687501  95.580325
## beta[13,72]      30.358773   12.203530  4.401692206  55.667976
## beta[1,73]       58.449060   12.372700 30.798546104  85.222698
## beta[2,73]       59.238416   12.271963 34.275660387  85.614054
## beta[3,73]       36.521323   12.887359  9.981926698  63.729871
## beta[4,73]       67.040703   12.202768 41.548395113  93.486045
## beta[5,73]       30.963490   13.312499  4.609267070  56.619057
## beta[6,73]       58.758012   12.023991 32.920769845  84.581795
## beta[7,73]       38.261928   12.384401 12.577460540  64.197204
## beta[8,73]       47.356301    4.342371 37.152438616  55.215584
## beta[9,73]       49.543790    9.998745 30.851432615  71.710668
## beta[10,73]      29.151020   12.195170  3.476246834  54.522922
## beta[11,73]      47.465257    6.057054 34.323694181  60.893485
## beta[12,73]      68.916134   12.465056 42.488397083  95.467160
## beta[13,73]      30.330845   12.258066  4.500789090  55.023851
## beta[1,74]       58.463040   12.501422 30.294223787  84.434830
## beta[2,74]       59.259105   12.357756 34.266725650  85.591715
## beta[3,74]       36.520008   12.942036  9.465940951  64.439258
```

```
## beta[4,74]       67.011043   12.250510 42.102533514  93.556858
## beta[5,74]       30.950975   13.247195  3.862107925  56.923023
## beta[6,74]       58.792671   12.099198 32.649687665  84.857500
## beta[7,74]       38.231942   12.470705 12.165510732  64.076279
## beta[8,74]       47.348046    4.417501 37.251180494  55.568351
## beta[9,74]       49.459961   10.060066 30.657190794  71.021424
## beta[10,74]      29.184352   12.281924  3.420815648  54.323875
## beta[11,74]      47.463473    6.039339 34.628183337  61.251493
## beta[12,74]      68.889007   12.601500 42.615350553  95.895221
## beta[13,74]      30.331375   12.363635  4.960443263  55.846072
## beta[1,75]       58.434137   12.638000 30.394348276  84.352801
## beta[2,75]       59.263301   12.399018 33.439190924  85.487078
## beta[3,75]       36.506027   13.024933  9.125295795  64.452571
## beta[4,75]       67.027579   12.294680 41.817997232  94.363861
## beta[5,75]       30.957306   13.526832  3.553272651  57.181566
## beta[6,75]       58.806512   12.187032 32.904725204  85.320917
## beta[7,75]       38.244119   12.578479 11.746589641  63.913766
## beta[8,75]       47.361629    4.387525 37.441233029  55.237735
## beta[9,75]       49.487540   10.020795 29.914458405  71.205841
## beta[10,75]      29.199321   12.284474  2.863219821  55.071708
## beta[11,75]      47.469991    5.968461 34.302432028  61.262171
## beta[12,75]      68.850578   12.730594 41.985190023  96.106345
## beta[13,75]      30.339860   12.389446  4.247127217  56.113875
## beta[1,76]       58.474796   12.772836 30.159055192  85.414651
## beta[2,76]       59.253370   12.504782 32.921076769  86.101379
## beta[3,76]       36.471748   13.134295  8.315161521  64.852105
## beta[4,76]       67.071777   12.361497 41.465248810  93.968322
## beta[5,76]       30.989195   13.577187  2.961328379  58.097902
## beta[6,76]       58.792099   12.284230 32.671086447  86.096819
## beta[7,76]       38.228862   12.593232 11.142634379  65.086725
## beta[8,76]       47.380522    4.366588 37.476536224  55.116488
## beta[9,76]       49.512021   10.181306 29.345030677  71.536225
## beta[10,76]      29.246713   12.329187  3.676782078  54.261305
## beta[11,76]      47.449339    5.942041 34.406261691  61.103637
## beta[12,76]      68.818546   12.787133 42.340259523  96.016407
## beta[13,76]      30.348127   12.484313  4.029957544  56.098686
## beta[1,77]       58.440115   12.839389 30.194062579  84.842276
## beta[2,77]       59.285687   12.612232 33.074322410  85.726805
## beta[3,77]       36.458178   13.145206  9.857862274  65.602520
## beta[4,77]       67.075767   12.439711 41.643106008  93.944982
## beta[5,77]       30.995338   13.668198  2.777438536  58.093984
## beta[6,77]       58.803491   12.300718 32.929037354  85.831217
## beta[7,77]       38.217071   12.671162 11.537155924  65.494304
## beta[8,77]       47.354370    4.361285 37.327727416  55.048040
## beta[9,77]       49.489741   10.318856 28.625434689  71.959150
## beta[10,77]      29.231871   12.392294  3.612038822  54.266606
## beta[11,77]      47.451691    5.989854 34.211588926  61.428655
## beta[12,77]      68.821310   12.801531 42.445271876  97.086950
## beta[13,77]      30.361160   12.497925  3.894510184  56.419447
## beta[1,78]       58.442399   12.945947 30.286695099  84.665727
## beta[2,78]       59.297688   12.637853 33.438602249  86.420344
## beta[3,78]       36.501079   13.260490  8.946310993  65.639211
```

```
## beta[4,78]       67.079286   12.470051 41.254533868  94.069775
## beta[5,78]       31.030724   14.130040  2.330889531  58.781413
## beta[6,78]       58.710647   12.355916 32.579096109  85.523627
## beta[7,78]       38.227864   12.742695 10.985115421  66.055869
## beta[8,78]       47.336597    4.387756 37.359393957  55.136432
## beta[9,78]       49.499875   10.496797 29.162125590  72.437681
## beta[10,78]      29.231761   12.487892  2.873191906  54.678556
## beta[11,78]      47.480486    5.902921 34.457122278  61.027930
## beta[12,78]      68.774596   12.927679 41.726635381  97.127861
## beta[13,78]      30.326017   12.526485  3.673811482  56.419497
## beta[1,79]       58.418095   13.078510 30.262801135  84.313174
## beta[2,79]       59.281973   12.710055 32.299275965  86.800214
## beta[3,79]       36.462680   13.365958  8.404600955  66.662058
## beta[4,79]       67.117670   12.594917 41.711344061  94.735824
## beta[5,79]       30.982659   13.717177  1.829955961  58.189314
## beta[6,79]       58.750004   12.537748 32.701258175  85.517729
## beta[7,79]       38.264444   12.820554 10.454754609  65.887888
## beta[8,79]       47.315194    4.401318 37.180134967  55.269292
## beta[9,79]       49.532062   10.542824 29.136100369  72.491871
## beta[10,79]      29.222979   12.502015  3.044710347  54.166543
## beta[11,79]      47.547716    5.872066 34.735857209  60.955238
## beta[12,79]      68.773562   12.962920 41.316164105  96.779547
## beta[13,79]      30.303660   12.574035  3.202266224  56.557982
## beta[1,80]       58.450192   13.202012 31.153967472  85.372306
## beta[2,80]       59.270896   12.739603 32.643423179  86.528572
## beta[3,80]       36.447746   13.374385  8.607800548  65.990425
## beta[4,80]       67.133856   12.682451 41.207539943  95.340824
## beta[5,80]       30.996376   13.982576  1.414040497  59.429336
## beta[6,80]       58.764803   12.611418 31.640037146  86.333946
## beta[7,80]       38.243988   12.896864 10.788548998  66.071397
## beta[8,80]       47.271060    4.452798 37.109150453  55.201199
## beta[9,80]       49.545967   10.597010 28.982333137  72.511281
## beta[10,80]      29.295029   12.514982  2.754774552  54.495338
## beta[11,80]      47.514720    5.725496 34.965498472  60.871616
## beta[12,80]      68.792624   13.032590 41.493209905  96.587079
## beta[13,80]      30.306317   12.608364  3.062780582  56.320468
## beta[1,81]       58.442139   13.340047 30.628266799  85.095463
## beta[2,81]       59.307031   12.922564 32.496577079  86.561729
## beta[3,81]       36.471298   13.445158  8.511642093  66.355875
## beta[4,81]       67.132254   12.744687 40.886507530  94.692614
## beta[5,81]       31.010128   14.031865  1.068535156  58.413731
## beta[6,81]       58.720344   12.717022 31.496459500  86.777700
## beta[7,81]       38.263884   13.018900 10.594698851  66.461683
## beta[8,81]       47.287926    4.482460 37.291124907  55.367806
## beta[9,81]       49.567884   10.651740 28.480436150  72.808832
## beta[10,81]      29.294580   12.680438  2.603868098  55.451300
## beta[11,81]      47.536844    5.599482 35.140365008  59.818959
## beta[12,81]      68.831353   13.101126 41.547322992  97.338746
## beta[13,81]      30.298639   12.665924  2.226043615  57.201053
## beta[1,82]       58.446079   13.313209 30.104044759  85.376755
## beta[2,82]       59.344437   12.957502 32.188058657  86.701779
## beta[3,82]       36.474577   13.599161  8.633338176  66.686279
```

```
## beta[4,82]       67.100393   12.843710 41.158613904  94.848676
## beta[5,82]       31.014529   14.226937  2.045232788  59.299384
## beta[6,82]       58.734116   12.831532 31.087698640  87.498141
## beta[7,82]       38.244843   13.046531 10.190872996  66.530521
## beta[8,82]       47.325448    4.442578 37.191517119  55.069577
## beta[9,82]       49.627856   10.755914 28.994929267  74.046594
## beta[10,82]      29.353550   12.726875  1.712609381  55.530820
## beta[11,82]      47.526263    5.448350 35.421864097  58.877770
## beta[12,82]      68.800796   13.138503 41.169607961  97.028130
## beta[13,82]      30.295392   12.743703  1.841897214  56.323586
## beta[1,83]       58.451397   13.309849 29.804767288  85.255314
## beta[2,83]       59.318403   13.002163 31.920984149  87.272427
## beta[3,83]       36.495897   13.632266  8.216115795  67.438267
## beta[4,83]       67.078619   12.860300 40.502382408  95.625360
## beta[5,83]       31.064226   14.614560  1.838264011  59.385550
## beta[6,83]       58.773653   12.910748 31.733657930  86.939983
## beta[7,83]       38.196548   13.170629  9.736291597  67.105815
## beta[8,83]       47.320611    4.421482 37.124546160  54.806194
## beta[9,83]       49.632754   10.920383 28.457661251  74.219645
## beta[10,83]      29.357860   12.809497  1.491561971  56.300331
## beta[11,83]      47.551638    5.370239 35.332850154  59.287343
## beta[12,83]      68.828486   13.217327 41.510878793  96.420626
## beta[13,83]      30.320098   12.809140  1.519127453  56.574302
## beta[1,84]       58.479495   13.369851 30.577309577  85.751641
## beta[2,84]       59.328970   13.099365 31.718523608  86.892667
## beta[3,84]       36.501320   13.733241  8.427217125  67.274308
## beta[4,84]       67.117476   12.955959 40.185280012  96.244169
## beta[5,84]       31.074040   14.692053  2.177909512  59.044887
## beta[6,84]       58.779312   12.969305 31.435276279  87.202697
## beta[7,84]       38.199893   13.253178  9.564712697  66.728874
## beta[8,84]       47.310601    4.383836 36.908493658  54.861786
## beta[9,84]       49.658444   11.016255 28.170202380  74.361974
## beta[10,84]      29.344164   12.910673  1.653413421  56.376747
## beta[11,84]      47.549821    5.276159 35.367699322  59.122216
## beta[12,84]      68.863841   13.243987 41.304902907  96.575641
## beta[13,84]      30.315187   12.912650  1.373321751  57.488981
## beta[1,85]       58.450981   13.446973 30.170860361  85.594329
## beta[2,85]       59.319050   13.257428 32.360578257  87.450596
## beta[3,85]       36.482719   13.740438  7.957199609  67.556561
## beta[4,85]       67.164678   13.013702 40.719959879  96.006662
## beta[5,85]       31.073472   14.555819  1.888448595  59.792780
## beta[6,85]       58.779733   12.999525 31.890488762  87.790030
## beta[7,85]       38.192673   13.453433  9.154076625  66.328917
## beta[8,85]       47.349584    4.353061 36.756278448  54.969211
## beta[9,85]       49.617940   11.169704 27.719251817  74.494411
## beta[10,85]      29.346789   13.061989  1.594366474  56.869497
## beta[11,85]      47.561990    5.121236 35.386786642  58.332326
## beta[12,85]      68.866892   13.303604 41.998672753  97.146323
## beta[13,85]      30.254567   13.016166  1.939608534  58.090779
## beta[1,86]       58.472047   13.515010 29.719558080  86.016975
## beta[2,86]       59.315879   13.331707 31.537552222  87.756812
## beta[3,86]       36.463548   13.888965  7.182969407  68.353288
```

```
## beta[4,86]       67.176013   13.109186 40.870965281  95.888008
## beta[5,86]       31.086835   14.560016  1.553743888  60.146822
## beta[6,86]       58.781629   13.069621 31.365959452  87.812980
## beta[7,86]       38.191801   13.444150  8.838913672  66.563783
## beta[8,86]       47.351458    4.361169 36.713539351  54.929296
## beta[9,86]       49.640554   11.280721 27.944028588  74.673949
## beta[10,86]      29.338459   13.094629  1.181589780  56.322007
## beta[11,86]      47.517310    4.959121 35.321525812  57.853291
## beta[12,86]      68.888690   13.292049 41.688129982  97.103812
## beta[13,86]      30.297120   13.069863  1.922848588  58.858301
## beta[1,87]       58.450824   13.591812 30.343039121  86.450893
## beta[2,87]       59.299618   13.449319 30.804740647  88.040584
## beta[3,87]       36.417415   13.934157  6.979624160  68.039295
## beta[4,87]       67.225310   13.221147 40.792196519  96.938613
## beta[5,87]       31.075939   14.709691  1.771144810  59.991173
## beta[6,87]       58.782616   13.157511 31.286870200  87.842434
## beta[7,87]       38.186399   13.533528  8.294664597  66.734514
## beta[8,87]       47.363533    4.321938 36.652024423  54.483616
## beta[9,87]       49.613345   11.365495 27.603298939  74.483422
## beta[10,87]      29.327194   13.180760  1.279026979  56.392973
## beta[11,87]      47.512452    4.821468 35.279910431  56.765063
## beta[12,87]      68.900888   13.388940 41.240044693  96.975178
## beta[13,87]      30.321100   13.177257  1.948253278  58.525529
## beta[1,88]       58.455364   13.657766 29.963649679  87.392755
## beta[2,88]       59.298919   13.499998 31.243585377  88.184607
## beta[3,88]       36.415218   13.969182  7.108364831  67.769917
## beta[4,88]       67.232183   13.254010 40.419797044  97.475984
## beta[5,88]       31.066556   14.521748  1.099408845  60.309538
## beta[6,88]       58.773534   13.312460 30.061747979  88.519206
## beta[7,88]       38.163138   13.630254  8.136026309  67.202803
## beta[8,88]       47.379579    4.337303 36.483659657  54.707998
## beta[9,88]       49.583559   11.499147 26.873136594  74.419207
## beta[10,88]      29.370933   13.289700  0.982845701  56.486055
## beta[11,88]      47.463292    4.672813 35.360682726  56.014810
## beta[12,88]      68.921394   13.439166 41.491560507  97.139183
## beta[13,88]      30.284860   13.267151  2.251015829  58.779831
## beta[1,89]       58.490435   13.734478 29.073572307  86.508526
## beta[2,89]       59.275649   13.684315 30.701411466  88.392121
## beta[3,89]       36.409046   14.010696  6.486718581  67.311473
## beta[4,89]       67.228914   13.344590 40.438214172  97.156912
## beta[5,89]       31.015448   14.512065  1.188575826  60.600105
## beta[6,89]       58.797814   13.427819 30.570428238  88.359945
## beta[7,89]       38.160310   13.717254  8.042544992  66.394773
## beta[8,89]       47.419292    4.331050 36.572815906  54.474855
## beta[9,89]       49.600551   11.655872 26.556160027  74.833353
## beta[10,89]      29.379659   13.334638  0.777739093  56.611651
## beta[11,89]      47.442683    4.527536 35.550487136  55.237011
## beta[12,89]      68.911740   13.533632 41.390790421  97.602479
## beta[13,89]      30.269347   13.395703  2.107129687  58.666754
## beta[1,90]       58.503702   13.828103 28.524807485  87.025442
## beta[2,90]       59.256433   13.797696 30.277394685  88.306656
## beta[3,90]       36.381748   14.195180  5.450641477  67.550584
```

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## beta[4,90]       67.239086   13.452026 39.787298646  97.315084
## beta[5,90]       31.026342   14.588261  0.538517549  60.441714
## beta[6,90]       58.786394   13.496273 30.352589842  88.382582
## beta[7,90]       38.125724   13.842857  7.501782909  66.891963
## beta[8,90]       47.437551    4.368079 36.694080236  54.809547
## beta[9,90]       49.619771   11.739493 25.578959090  74.673209
## beta[10,90]      29.459543   13.452899  1.096111340  57.344588
## beta[11,90]      47.384902    4.289008 35.605570979  54.545550
## beta[12,90]      68.910305   13.601626 41.475952669  97.699939
## beta[13,90]      30.263290   13.504422  2.207613559  58.962556
## beta[1,91]       58.481698   13.814705 28.720929554  87.079811
## beta[2,91]       59.284588   13.872279 29.956252266  88.149971
## beta[3,91]       36.419896   14.316089  5.942044071  68.488022
## beta[4,91]       67.223129   13.492837 40.068851229  97.497651
## beta[5,91]       31.089511   14.773280  0.147549870  61.019326
## beta[6,91]       58.767858   13.612250 29.353352911  88.540321
## beta[7,91]       38.137107   13.894436  7.219636029  67.046431
## beta[8,91]       47.501993    4.273436 37.015406041  54.803213
## beta[9,91]       49.602096   11.804760 25.656752845  75.005149
## beta[10,91]      29.457050   13.526091 -0.431972067  57.249462
## beta[11,91]      47.374163    4.465825 35.372902753  54.704423
## beta[12,91]      68.898816   13.654034 40.756598685  98.653165
## beta[13,91]      30.269491   13.551687  1.570205521  59.404653
## beta[1,92]       58.487072   13.886613 28.144193822  86.565171
## beta[2,92]       59.300380   13.917326 28.815908071  88.213779
## beta[3,92]       36.465951   14.350817  5.866131724  68.794601
## beta[4,92]       67.218516   13.605910 39.361666354  97.947839
## beta[5,92]       31.096583   14.926044  0.790871303  60.589752
## beta[6,92]       58.779828   13.692409 29.352800520  89.323443
## beta[7,92]       38.157595   13.924427  7.072891348  67.230771
## beta[8,92]       47.556838    4.259204 37.261004492  54.799448
## beta[9,92]       49.591745   11.858030 26.315650160  75.688881
## beta[10,92]      29.421657   13.540965 -0.401557400  58.776704
## beta[11,92]      47.371563    4.592290 35.313944451  55.166417
## beta[12,92]      68.904757   13.756313 40.350982444  98.438777
## beta[13,92]      30.255230   13.593713  0.731245872  59.175457
## beta[1,93]       58.505022   13.991163 28.882161558  87.859738
## beta[2,93]       59.331251   13.928071 29.200141303  89.019961
## beta[3,93]       36.487717   14.405551  6.165259593  69.177613
## beta[4,93]       67.254018   13.705624 39.422730850  98.075588
## beta[5,93]       31.083305   14.940060  0.784822563  60.977175
## beta[6,93]       58.813315   13.676760 30.078270812  88.748184
## beta[7,93]       38.159992   13.901255  7.103293309  67.443064
## beta[8,93]       47.629225    4.158204 37.366183868  54.953121
## beta[9,93]       49.551770   12.046538 24.786687215  75.260830
## beta[10,93]      29.404384   13.511006  0.053257895  58.729657
## beta[11,93]      47.329129    4.711456 35.110188947  55.259214
## beta[12,93]      68.863087   13.899593 39.755665925  98.824786
## beta[13,93]      30.238805   13.663890  1.424466846  59.465303
## beta[1,94]       58.467914   14.059569 28.485090541  87.514468
## beta[2,94]       59.345293   14.008480 29.280251184  88.909074
## beta[3,94]       36.461428   14.473528  6.013463359  68.277224
```

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## beta[4,94]       67.299892   13.732530 39.418030383  97.799708
## beta[5,94]       31.132974   14.972969  0.116464872  61.279563
## beta[6,94]       58.779761   13.734265 30.422117162  88.864566
## beta[7,94]       38.186749   14.012908  7.577535614  67.682480
## beta[8,94]       47.732827    3.990566 38.463843577  55.041498
## beta[9,94]       49.519404   12.144891 24.549838750  75.022243
## beta[10,94]      29.371810   13.569355 -0.147888897  58.056595
## beta[11,94]      47.361416    4.808430 35.244023668  55.564992
## beta[12,94]      68.861086   14.014480 39.897491680  99.272416
## beta[13,94]      30.222478   13.698995  0.755021410  59.035740
## beta[1,95]       58.482808   14.175845 28.485783910  87.631161
## beta[2,95]       59.351240   14.132131 28.652784587  89.370345
## beta[3,95]       36.462526   14.512482  5.751421249  67.557621
## beta[4,95]       67.291735   13.835573 38.635527609  98.114346
## beta[5,95]       31.157622   14.798585  0.423109146  61.808388
## beta[6,95]       58.797763   13.760962 30.428505922  88.359082
## beta[7,95]       38.219543   14.111109  7.659775227  68.122932
## beta[8,95]       47.887089    3.881354 39.044230588  54.968705
## beta[9,95]       49.479677   12.289840 24.946801583  75.346613
## beta[10,95]      29.365906   13.664192 -0.213591842  58.101443
## beta[11,95]      47.347834    4.890665 35.075989258  55.801208
## beta[12,95]      68.907631   14.009427 39.882085597  98.965252
## beta[13,95]      30.250757   13.794187  0.966625646  59.564176
## beta[1,96]       58.470147   14.341229 28.007354065  88.063465
## beta[2,96]       59.383379   14.237811 29.223881702  89.960092
## beta[3,96]       36.475569   14.603313  5.961839464  67.878356
## beta[4,96]       67.350706   13.927575 37.466005696  98.781810
## beta[5,96]       31.162449   14.892975  0.126986471  62.037761
## beta[6,96]       58.780786   13.841211 30.246819785  88.557202
## beta[7,96]       38.221740   14.146828  7.631020547  68.156196
## beta[8,96]       47.991230    3.794238 39.499304572  55.092858
## beta[9,96]       49.464369   12.330496 24.615125709  75.509600
## beta[10,96]      29.368463   13.689555 -0.216200567  58.211732
## beta[11,96]      47.339879    4.907446 35.113948413  55.753835
## beta[12,96]      68.898168   14.006979 40.346360446  99.261886
## beta[13,96]      30.230053   13.894176  0.599638460  59.102963
## beta[1,97]       58.422051   14.347631 27.440604529  87.708792
## beta[2,97]       59.398398   14.426555 29.321168830  89.977041
## beta[3,97]       36.505868   14.654644  6.446507279  68.574365
## beta[4,97]       67.429649   13.999151 37.503238721  98.338374
## beta[5,97]       31.144112   14.839137  0.589855152  62.221399
## beta[6,97]       58.761657   13.933559 29.533231026  87.842365
## beta[7,97]       38.225421   14.214132  7.572076666  68.382226
## beta[8,97]       48.076125    3.620083 39.799179406  54.741156
## beta[9,97]       49.474629   12.407843 24.576602727  75.102741
## beta[10,97]      29.372509   13.779314 -0.445664576  58.307552
## beta[11,97]      47.347251    4.941268 34.937497435  55.833838
## beta[12,97]      68.906167   14.087631 39.478769479  99.224588
## beta[13,97]      30.274044   13.866641 -0.427158997  59.006927
## beta[1,98]       58.463782   14.395777 28.194444764  88.065125
## beta[2,98]       59.413152   14.516360 28.938822420  90.159595
## beta[3,98]       36.505273   14.664654  5.487817382  68.163572
```

```
## beta[4,98]       67.406780   14.070760 37.725935495  98.268650
## beta[5,98]       31.138782   14.779542  0.261473277  61.977819
## beta[6,98]       58.772726   13.974799 28.728758701  88.659861
## beta[7,98]       38.242399   14.276853  7.442526449  67.969882
## beta[8,98]       48.043589    3.860131 39.149747167  55.251722
## beta[9,98]       49.450973   12.425590 23.712587454  75.378349
## beta[10,98]      29.401973   13.869587 -0.009670088  58.381403
## beta[11,98]      47.395392    4.943761 35.110375281  56.209231
## beta[12,98]      68.932335   14.140465 39.908564843  99.154203
## beta[13,98]      30.268511   13.976989 -0.305729195  59.415597
## beta[1,99]       58.499174   14.510359 28.097884337  89.147595
## beta[2,99]       59.418196   14.563652 29.156445006  90.238250
## beta[3,99]       36.490699   14.699476  5.878880701  69.284636
## beta[4,99]       67.419464   14.146375 37.704625936  98.283254
## beta[5,99]       31.165018   14.708415  0.301018704  61.902421
## beta[6,99]       58.843354   14.035628 29.205962075  88.766827
## beta[7,99]       38.220541   14.355419  7.338553207  69.052431
## beta[8,99]       48.042217    4.127688 38.219562586  55.815799
## beta[9,99]       49.478334   12.547817 23.555054498  75.477932
## beta[10,99]      29.414188   13.925284 -0.218125998  58.993668
## beta[11,99]      47.465187    4.987996 34.776077602  56.146041
## beta[12,99]      68.923579   14.223529 39.798434781  99.758196
## beta[13,99]      30.246880   14.110841 -0.572031303  60.071933
## beta[1,100]      58.534356   14.670160 27.492948858  89.026205
## beta[2,100]      59.460839   14.643741 29.342075834  91.052187
## beta[3,100]      36.435958   14.797745  5.795247637  69.327556
## beta[4,100]      67.454667   14.189188 37.535697856  98.773128
## beta[5,100]      31.148354   14.851306  0.798445748  62.431481
## beta[6,100]      58.827908   14.055433 29.144694590  89.412444
## beta[7,100]      38.208685   14.532615  7.872218580  68.988919
## beta[8,100]      48.073761    4.366229 38.166698215  56.229272
## beta[9,100]      49.466808   12.539796 23.947357982  75.294324
## beta[10,100]     29.371451   14.032972 -0.209541674  59.131644
## beta[11,100]     47.501948    4.987343 34.834993438  56.538364
## beta[12,100]     68.922751   14.295474 39.827583202  99.836571
## beta[13,100]     30.348948   14.185084 -0.816649133  60.599818
## beta[1,101]      58.529822   14.724542 27.827608597  89.306480
## beta[2,101]      59.453514   14.723742 28.725260014  91.317838
## beta[3,101]      36.423404   14.836332  5.729896311  68.904759
## beta[4,101]      67.410830   14.314282 37.623476356  98.391202
## beta[5,101]      31.174707   15.053658  0.462350540  62.434975
## beta[6,101]      58.832297   14.126485 28.907162929  89.093957
## beta[7,101]      38.222217   14.623938  7.644012334  69.202068
## beta[8,101]      48.107596    4.542158 37.428722570  57.044382
## beta[9,101]      49.413702   12.665056 23.668460312  75.561908
## beta[10,101]     29.401776   14.115092 -1.331262971  58.578592
## beta[11,101]     47.561980    5.012752 34.776263139  56.554404
## beta[12,101]     68.901387   14.392575 39.213791642 100.067515
## beta[13,101]     30.360479   14.249116 -0.712555535  60.370076
## beta[1,102]      58.521702   14.913876 27.187243468  89.117030
## beta[2,102]      59.457708   14.812161 28.746759008  91.098138
## beta[3,102]      36.410562   14.844185  5.917651220  69.210490
```

```
## beta[4,102]      67.415961   14.337409 37.846790360  98.265397
## beta[5,102]      31.162766   15.211181  0.413221855  62.510389
## beta[6,102]      58.859362   14.100761 28.848892298  90.078939
## beta[7,102]      38.238055   14.680565  6.986384293  69.468690
## beta[8,102]      48.106959    4.724519 37.344831468  57.142700
## beta[9,102]      49.375945   12.804696 23.648705129  75.467155
## beta[10,102]     29.440138   14.205465 -0.855583642  59.038375
## beta[11,102]     47.617317    4.978334 34.971509545  56.656813
## beta[12,102]     68.889719   14.448748 39.662058718  99.865979
## beta[13,102]     30.369069   14.258307 -0.266364472  61.134376
## beta[1,103]      58.517287   15.010239 28.930458941  89.328765
## beta[2,103]      59.486934   14.763821 29.014057882  91.565093
## beta[3,103]      36.422907   14.861680  5.989911688  69.054668
## beta[4,103]      67.423203   14.424688 37.246279328  98.364621
## beta[5,103]      31.144332   15.166507  0.158879528  63.396573
## beta[6,103]      58.851272   14.164084 29.200453354  89.785004
## beta[7,103]      38.313594   14.807774  7.456951065  69.719509
## beta[8,103]      48.158270    4.896836 37.094423327  57.823683
## beta[9,103]      49.388000   12.867716 23.677683052  76.523458
## beta[10,103]     29.425878   14.260382 -1.136299170  59.025516
## beta[11,103]     47.672236    4.986336 34.770470366  56.700244
## beta[12,103]     68.871270   14.537572 38.418041527 100.122615
## beta[13,103]     30.411603   14.306189 -1.139135510  60.704431
## beta[1,104]      58.544594   15.100534 26.909723825  90.038070
## beta[2,104]      59.515983   14.852131 28.966174975  91.372763
## beta[3,104]      36.434451   14.952258  5.650089117  68.880497
## beta[4,104]      67.437531   14.462891 37.345534276  98.581485
## beta[5,104]      31.174100   15.365335  0.163437593  63.238677
## beta[6,104]      58.854444   14.231139 28.978202559  90.022006
## beta[7,104]      38.313777   14.902038  7.353152015  71.417893
## beta[8,104]      48.148655    5.052438 36.621283000  58.037955
## beta[9,104]      49.389890   12.927949 24.090460257  76.376971
## beta[10,104]     29.421340   14.274005 -1.510781794  59.131230
## beta[11,104]     47.744162    4.990501 34.818052656  56.747213
## beta[12,104]     68.878453   14.583227 37.113884658  99.586788
## beta[13,104]     30.373950   14.371021 -0.757236832  61.354466
## beta[1,105]      58.552910   15.130429 27.162968167  91.401876
## beta[2,105]      59.496220   14.948010 28.535133470  92.355080
## beta[3,105]      36.455978   15.047226  5.363157804  68.925238
## beta[4,105]      67.405996   14.530614 37.242263226  98.466239
## beta[5,105]      31.145710   15.594986 -0.460060907  63.726685
## beta[6,105]      58.855607   14.311385 28.212065465  89.021862
## beta[7,105]      38.300764   14.910170  6.918394815  70.386569
## beta[8,105]      48.127553    5.329854 35.713633388  58.377534
## beta[9,105]      49.417462   13.011171 23.405940699  76.120228
## beta[10,105]     29.423359   14.326569 -1.418406720  59.317289
## beta[11,105]     47.799051    5.029693 34.817944439  57.072574
## beta[12,105]     68.872178   14.590462 37.946993600  99.943065
## beta[13,105]     30.388877   14.463342  0.262617665  61.282308
## beta[1,106]      58.570802   15.163902 27.067249277  90.523098
## beta[2,106]      59.518113   14.994128 28.695201146  92.130398
## beta[3,106]      36.460448   15.132319  5.156811011  69.605806
```

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## beta[4,106]      67.435537   14.628972 36.769868924  99.538277
## beta[5,106]      31.171708   15.566392 -0.025628651  63.378785
## beta[6,106]      58.876191   14.396628 28.221159745  90.362644
## beta[7,106]      38.303126   14.996612  6.677285931  70.075741
## beta[8,106]      48.156693    5.603683 35.936097054  58.794730
## beta[9,106]      49.415531   13.073247 23.124763105  76.460347
## beta[10,106]     29.448561   14.408729 -1.463237263  59.269382
## beta[11,106]     47.846381    5.015558 34.818447099  57.062207
## beta[12,106]     68.882775   14.554444 38.077909283  99.770044
## beta[13,106]     30.389892   14.566982 -0.223573344  61.912249
## beta[1,107]      58.552472   15.250208 27.241626457  90.104349
## beta[2,107]      59.526814   15.083464 27.775031205  91.781373
## beta[3,107]      36.460840   15.142161  4.844790553  68.865694
## beta[4,107]      67.422996   14.631275 36.997562514  98.903184
## beta[5,107]      31.192744   15.677314 -0.619976216  63.784903
## beta[6,107]      58.871325   14.442019 28.173915577  91.128825
## beta[7,107]      38.288381   15.009083  6.950172900  71.139246
## beta[8,107]      48.186743    5.741478 35.762459031  59.568449
## beta[9,107]      49.423711   13.141094 23.184197734  76.383584
## beta[10,107]     29.443143   14.496671 -0.869191926  60.155940
## beta[11,107]     47.885350    5.024630 34.886000230  56.963134
## beta[12,107]     68.861550   14.673255 37.733814740  99.641203
## beta[13,107]     30.348519   14.604726 -1.198661825  61.660106
## beta[1,108]      58.544221   15.232621 26.911381778  90.849436
## beta[2,108]      59.538989   15.111997 27.644695957  92.331898
## beta[3,108]      36.441877   15.262197  5.113834547  69.764514
## beta[4,108]      67.432595   14.679484 36.469311440  98.880791
## beta[5,108]      31.195703   15.943643 -1.750979455  63.992782
## beta[6,108]      58.874570   14.441085 28.256350076  90.777014
## beta[7,108]      38.292938   15.101103  6.421582645  71.870207
## beta[8,108]      48.176146    5.895478 35.677776457  59.373725
## beta[9,108]      49.454760   13.165819 22.750586273  76.848945
## beta[10,108]     29.440327   14.565796 -1.485287948  60.396572
## beta[11,108]     47.918438    5.009081 35.156977527  57.158629
## beta[12,108]     68.891212   14.786904 37.807289181 100.019358
## beta[13,108]     30.318894   14.673154 -0.990335556  61.679571
## beta[1,109]      58.578381   15.260406 25.788257857  90.848784
## beta[2,109]      59.485097   15.189719 27.339304653  91.909942
## beta[3,109]      36.447677   15.292005  4.098015234  69.574413
## beta[4,109]      67.426084   14.801718 36.850682105  99.044158
## beta[5,109]      31.204160   15.885864 -1.725583908  64.639333
## beta[6,109]      58.889569   14.458076 28.393632931  90.777817
## beta[7,109]      38.247543   15.130284  6.155371904  71.547239
## beta[8,109]      48.131687    6.046735 35.095250614  59.560235
## beta[9,109]      49.485247   13.176027 22.734438962  77.459559
## beta[10,109]     29.402153   14.615564 -1.914017531  59.461746
## beta[11,109]     47.912860    5.001895 35.426172746  57.025462
## beta[12,109]     68.898512   14.886954 37.652883068  99.654569
## beta[13,109]     30.314435   14.769204 -1.886341589  62.172336
## beta[1,110]      58.574114   15.352939 25.963782478  90.609463
## beta[2,110]      59.534774   15.234252 26.731660150  92.569710
## beta[3,110]      36.457748   15.386269  4.908989639  69.652404
```

```
## beta[4,110]      67.440380   14.887063 36.171478342  98.904479
## beta[5,110]      31.222180   15.820166 -1.641929183  63.127376
## beta[6,110]      58.884785   14.579627 28.454970948  91.234244
## beta[7,110]      38.264974   15.241585  6.114061278  71.745790
## beta[8,110]      48.110088    6.204888 34.834746404  59.883539
## beta[9,110]      49.454403   13.256462 22.160783119  76.941123
## beta[10,110]     29.360868   14.675493 -2.299724194  59.530251
## beta[11,110]     47.938202    4.933461 35.368095977  56.810279
## beta[12,110]     68.909035   14.916958 37.107266150 101.050474
## beta[13,110]     30.342436   14.851165 -1.492861174  61.936053
## beta[1,111]      58.564797   15.462214 25.634529326  91.233327
## beta[2,111]      59.515223   15.240979 27.475753312  92.300253
## beta[3,111]      36.383021   15.454878  4.128870601  69.400622
## beta[4,111]      67.456707   14.999565 36.481841054  99.719814
## beta[5,111]      31.205580   15.867948 -1.406752424  64.086389
## beta[6,111]      58.855824   14.655752 27.785173340  90.562325
## beta[7,111]      38.241728   15.349047  6.496787798  71.807272
## beta[8,111]      48.146661    6.314744 34.753354661  59.857590
## beta[9,111]      49.473490   13.343273 22.106760491  77.137463
## beta[10,111]     29.344498   14.784262 -1.870301499  60.299285
## beta[11,111]     48.031646    4.863332 35.506032665  56.791706
## beta[12,111]     68.891588   14.983982 36.596948151 100.596558
## beta[13,111]     30.348096   14.939825 -1.533104954  61.755255
## beta[1,112]      58.532854   15.463239 25.150847947  90.577494
## beta[2,112]      59.553275   15.282585 27.963490818  92.695369
## beta[3,112]      36.371279   15.455754  3.941339189  70.007357
## beta[4,112]      67.424977   15.144009 35.933950358 100.222911
## beta[5,112]      31.181657   15.790971 -1.704779282  63.032831
## beta[6,112]      58.893787   14.724141 28.350776947  91.308534
## beta[7,112]      38.224304   15.448736  6.068094473  72.499084
## beta[8,112]      48.153561    6.450529 35.044014384  60.361278
## beta[9,112]      49.445104   13.443656 22.066419673  77.500670
## beta[10,112]     29.340805   14.821354 -1.805016737  59.783853
## beta[11,112]     48.046158    4.810704 35.410971090  56.877441
## beta[12,112]     68.940182   15.061226 36.627503261 100.815146
## beta[13,112]     30.340149   15.001039 -2.501783385  62.138620
## beta[1,113]      58.512854   15.582554 24.798178375  91.472426
## beta[2,113]      59.573891   15.307105 26.817808832  92.201276
## beta[3,113]      36.355777   15.500318  3.907971815  70.483131
## beta[4,113]      67.447400   15.173743 36.180667980 100.069040
## beta[5,113]      31.145040   15.862183 -1.813990146  63.311154
## beta[6,113]      58.881602   14.801884 27.984599577  91.282050
## beta[7,113]      38.267546   15.570668  6.043142050  72.800092
## beta[8,113]      48.169177    6.586409 34.417454327  60.752258
## beta[9,113]      49.416787   13.472163 21.451649705  77.396961
## beta[10,113]     29.337697   14.842759 -2.201555402  60.306090
## beta[11,113]     48.047547    4.731630 35.519314497  56.775413
## beta[12,113]     68.942640   15.096326 36.587673985 100.506048
## beta[13,113]     30.323328   15.056708 -2.513245288  61.655171
## beta[1,114]      58.480654   15.633596 25.004711826  90.792900
## beta[2,114]      59.563926   15.379890 27.208771989  92.592575
## beta[3,114]      36.394915   15.556948  4.013376121  71.004690
```

```
## beta[4,114]      67.452730   15.255744 36.665513183 100.311589
## beta[5,114]      31.171045   15.915308 -2.260333397  64.094180
## beta[6,114]      58.854556   14.861429 28.193465128  91.301716
## beta[7,114]      38.265627   15.716984  5.714380488  72.526767
## beta[8,114]      48.214652    6.815766 33.682449793  62.125410
## beta[9,114]      49.377833   13.589180 21.054750172  77.534027
## beta[10,114]     29.355287   14.929723 -1.528255157  59.873379
## beta[11,114]     48.084391    4.697465 35.690524648  56.631079
## beta[12,114]     68.949386   15.132711 36.527058511 100.720395
## beta[13,114]     30.338341   15.208953 -2.996330970  61.734911
## beta[1,115]      58.461322   15.672772 24.470528695  90.919361
## beta[2,115]      59.569209   15.531058 26.443103930  93.410377
## beta[3,115]      36.398617   15.687246  3.893716597  70.869696
## beta[4,115]      67.428692   15.337208 36.012638208 100.386296
## beta[5,115]      31.192249   16.068145 -2.367557346  64.529868
## beta[6,115]      58.890005   14.895428 28.953342686  90.781675
## beta[7,115]      38.261816   15.709511  5.967598825  72.553018
## beta[8,115]      48.258049    6.950195 33.652875577  62.324705
## beta[9,115]      49.404457   13.716344 20.621321785  77.487928
## beta[10,115]     29.338346   14.959042 -1.555941243  60.506202
## beta[11,115]     48.075969    4.590628 35.691189070  56.318053
## beta[12,115]     68.966326   15.217738 37.266791466 101.520492
## beta[13,115]     30.353083   15.265654 -2.345823449  61.604019
## sigma2_beta[1]    1.983584    2.619386  0.067432770   9.811024
## sigma2_beta[2]    2.123004    6.608926  0.068747630   9.154619
## sigma2_beta[3]    1.998874    2.700569  0.067607334   9.785534
## sigma2_beta[4]    1.990037    2.628090  0.069899162   9.827270
## sigma2_beta[5]    2.177180   10.277233  0.068706091   9.399325
## sigma2_beta[6]    2.009647    2.882833  0.070212655   9.610445
## sigma2_beta[7]    2.016162    2.950729  0.070508205   9.488029
## sigma2_beta[8]    1.922471    2.481571  0.074708213   9.502068
## sigma2_beta[9]    1.990683    2.643962  0.062246212   9.629270
## sigma2_beta[10]   2.011775    3.124264  0.069299203   9.668897
## sigma2_beta[11]   1.903645    2.461996  0.079021045   9.060829
## sigma2_beta[12]   2.002740    2.699599  0.069533108   9.788253
## sigma2_beta[13]   1.980313    2.623479  0.070235126   9.407093
## sigma2_y[1]     110.421134 4140.772763  3.630945679  98.700218
## sigma2_y[2]      37.433096  219.283024  3.657232440 100.172850
## sigma2_y[3]      32.606741   32.253440  3.605711682  97.711550
## sigma2_y[4]      32.953798   63.934587  3.779732662  97.786865
## sigma2_y[5]      37.302642  226.543715  3.702411676  98.689607
## sigma2_y[6]      32.556122   31.099612  3.834331138  99.768413
## sigma2_y[7]      43.575421  555.350283  3.686516698 100.398992
## sigma2_y[8]      31.245772   26.743780  3.666795868  94.702051
## sigma2_y[9]      32.297343   28.016058  3.940521011  97.735543
## sigma2_y[10]     32.578443   31.998891  3.694215054  99.966396
## sigma2_y[11]     31.093843   27.015430  3.591506985  95.332058
## sigma2_y[12]     32.599110   38.049204  3.647017778  98.692937
## sigma2_y[13]     35.672709   90.873938  3.742074546 101.658292

## mean Rhat: 1.116908

## mean effective sample size: 2018.325
```

# Appendix B

**Presidential Election Model:**

- `2020 US presidential election polls - all_polls.csv`: 2020 U.S. Presidential Election Polls compiled by *The Economist*. Available at https://projects.economist.com/us-2020-forecast/president/how-this-works. + We only used poll responses from likely voters in the battleground states within 30 days prior to the election date. From the poll responses we calculated Biden's vote share in the two-party race in each poll ($Y_k$).
- `partisan_leans_538.csv`: effect of partisan lean on the Democratic candidate's presidential election vote share in each state. Available at https://github.com/fivethirtyeight/data/tree/master/partisan-lean. + 90% weight is allocated to this variable Vote Share from Partisan Lean$_j$ when computing hyperparameter $h_j$ for each state $j$.
- `abramowitz_data.csv`: downloaded from Andrew Gelman's Github repository on the 2020 Presidential Election. Available at https://github.com/TheEconomist/us-potus-model. + This data set contains information on each of the 1948-2016 election year $i$'s annualized 2nd quarter GDP performance (not used due to its abnormally low value in 2020), incumbent party's June net approval rating (June Approval Ratings for Incumbent Party$_i$) and incumbent party's national vote share (Presidental Fundamentals$_i$).
- `states_cov_matrix_full.csv`: covariance matrix of states which takes into account similarity between states based on their demographic and political profiles. Retrieved from Andrew Gelman's Github repository on the 2020 Presidential Election (https://github.com/TheEconomist/us-potus-model). + The covariance matrix corresponds to $S$, the scale matrix in the prior distribution of $\Sigma$
- `abramowitz_additional.csv`: we supplemented `abramowitz_data.csv` with the corresponding year $i$'s 2nd quarter real income growth compared to one year ago (2nd Quarter Real Income Growth$_i$) and S&P stock performance 3 months prior to the election date (Three Month Stock Growth$_i$). Data sets are from https://fred.stlouisfed.org/series/A067RO1Q156NBEA and https://www.multpl.com/s-p-500-historical-prices/table/by-month. A subset of the combined data set is displayed below. *check where the table is* + We fitted a linear regression model on historical incumbent party's national vote share with June Approval Ratings for Incumbent Party, 2nd Quarter Real Income Growth, and Three Month Stock Growth as predictors. We then predicted this year's incumbent party's national vote share (i.e. Republican's national vote share) and used 100 minus the predicted value to obtain Democrat's national vote share (Presidental Fundamentals$_{2020}$). We can think of Presidental Fundamentals$_{2020}$ as a national level prior. 10% weight is allocated to the "national prior" when computing the state specific prior $h_j$.

| Year | Incumbent Party Vote Share | Democrat Incumbency | Incumbent Party June Net Approval | 2nd Quarter GDP Growth | 3 Month Stock Growth | 2nd Quarter Real Income Growth |
|---|---|---|---|---|---|---|
| 2016 | 51.10 | Yes | 4 | 1.1 | 0.0033832 | 1.8 |
| 2012 | 52.00 | Yes | 1 | 1.3 | -0.0338848 | 2.4 |
| 2008 | 46.30 | No | -40 | 0.6 | -0.2743827 | 0.0 |
| 2004 | 51.24 | No | -1 | 2.9 | 0.0458816 | 2.5 |
| 2000 | 50.30 | Yes | 15 | 8.0 | -0.0613126 | 5.8 |
| 1996 | 54.70 | Yes | 10 | 7.1 | 0.0900753 | 3.3 |

**Senate Election Model:**

- `senate_polls.csv`: Senate election polls compiled by *FiveThirtyEight*. Available at https://projects.fivethirtyeight.com/polls-page/senate_polls.csv. + Similar to the presidential election prediction model,

we only included poll responses from likely voters in the battleground states within 30 days prior to the senate election date. From the poll responses we calculated Democratic candidates' vote share in the two-party race in each poll ($Y_k$).

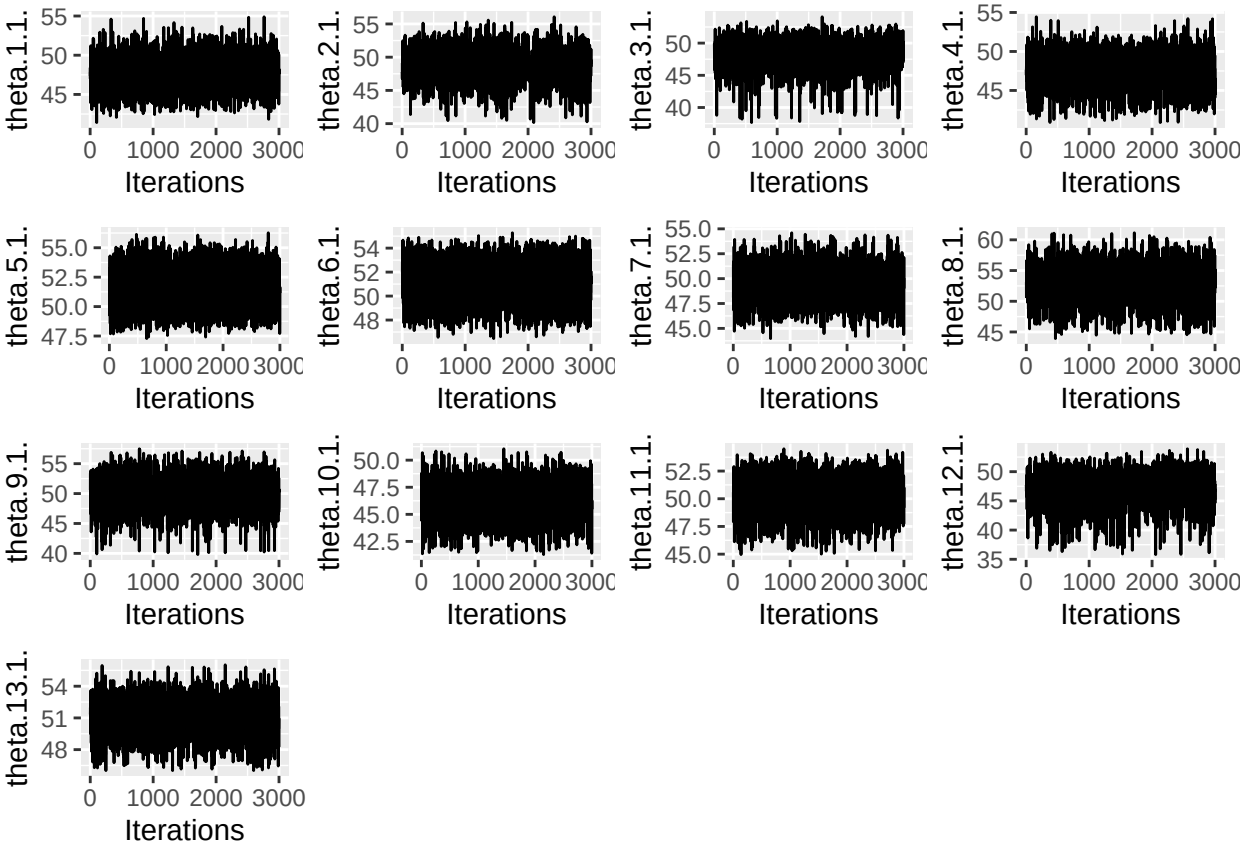
- `partisan_leans_538.csv`: same as the data set in presidential election model and we used this partisan lean in the calculation of prior hyperparameter $h_j$.
- We also incroporated incumbency advantage in calculating $h_j$. As *FiveThirtyEight* suggests, incumbency advantage is on average 2.6 for senate incumbents (https://fivethirtyeight.com/features/how-much-was-incumbency-worth-in-2018/).
- For NC Senate race only, we supplemented the prior with predicted voter turnout from the Interim Report model.

**House Election Model (for North Carolina):**

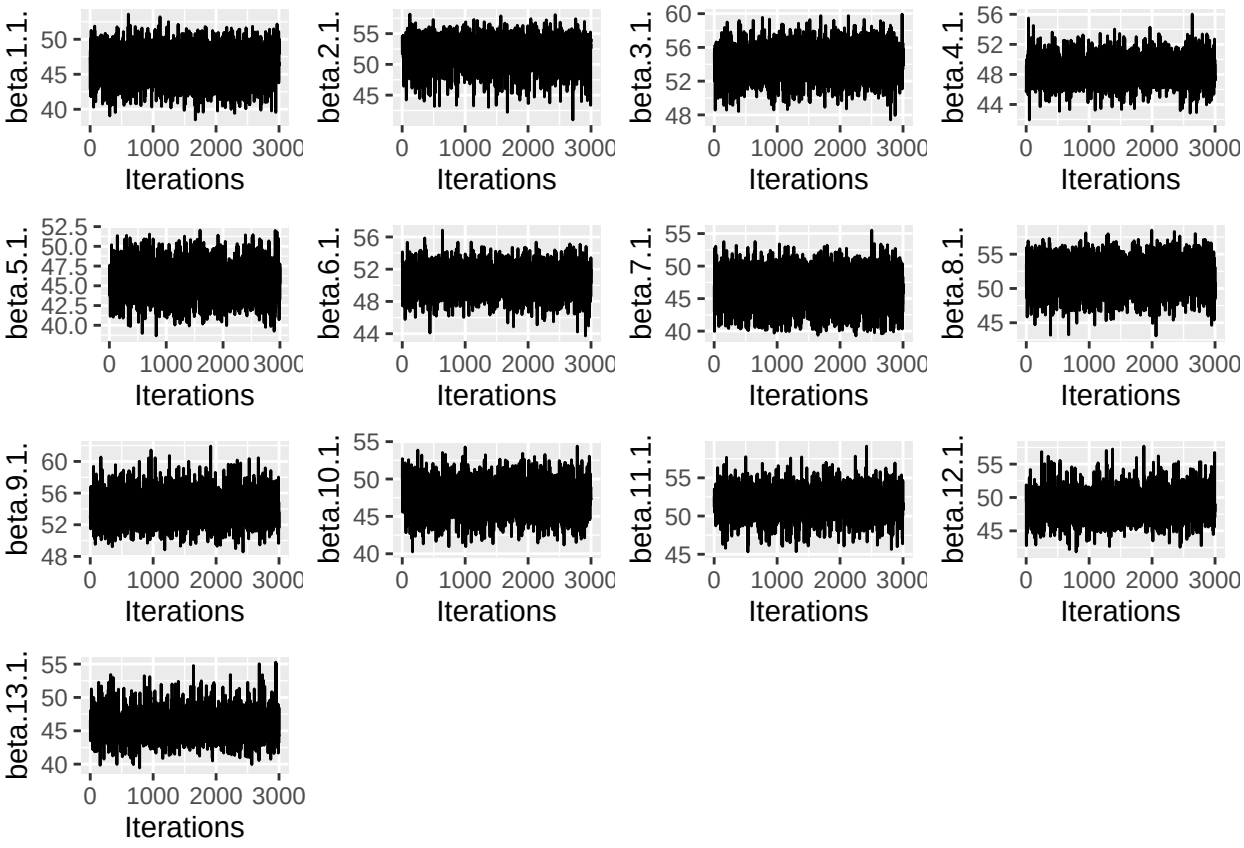
- `house_polls.csv`: House election polls compiled by *FiveThirtyEight*. Available at https://projects.fivethirtyeight.com/polls-page/house_polls.csv. + We only used polls related to the NC House race. Due to the lack of poll responses for the NC House elections, we included all poll responses within 115 days of the election.
- `ncvoter_1027_small.rds`: NC registered voter demographics information provided by the NC State Board of Elections (https://dl.ncsbe.gov/list.html). + Please refer to the Interim Report to see a detailed explanation of how voter turnout is modeled by registered voters' race, age, party affiliation, income and gender.
- Partisan lean and incumbency information was taken from *FiveThirtyEight* at the following links, respectively: https://fivethirtyeight.com/features/north-carolinas-new-house-map-hands-democrats-two-seats-but-it-still-leans-republican/, https://fivethirtyeight.com/features/how-much-was-incumbency-worth-in-2018/

# Appendix C

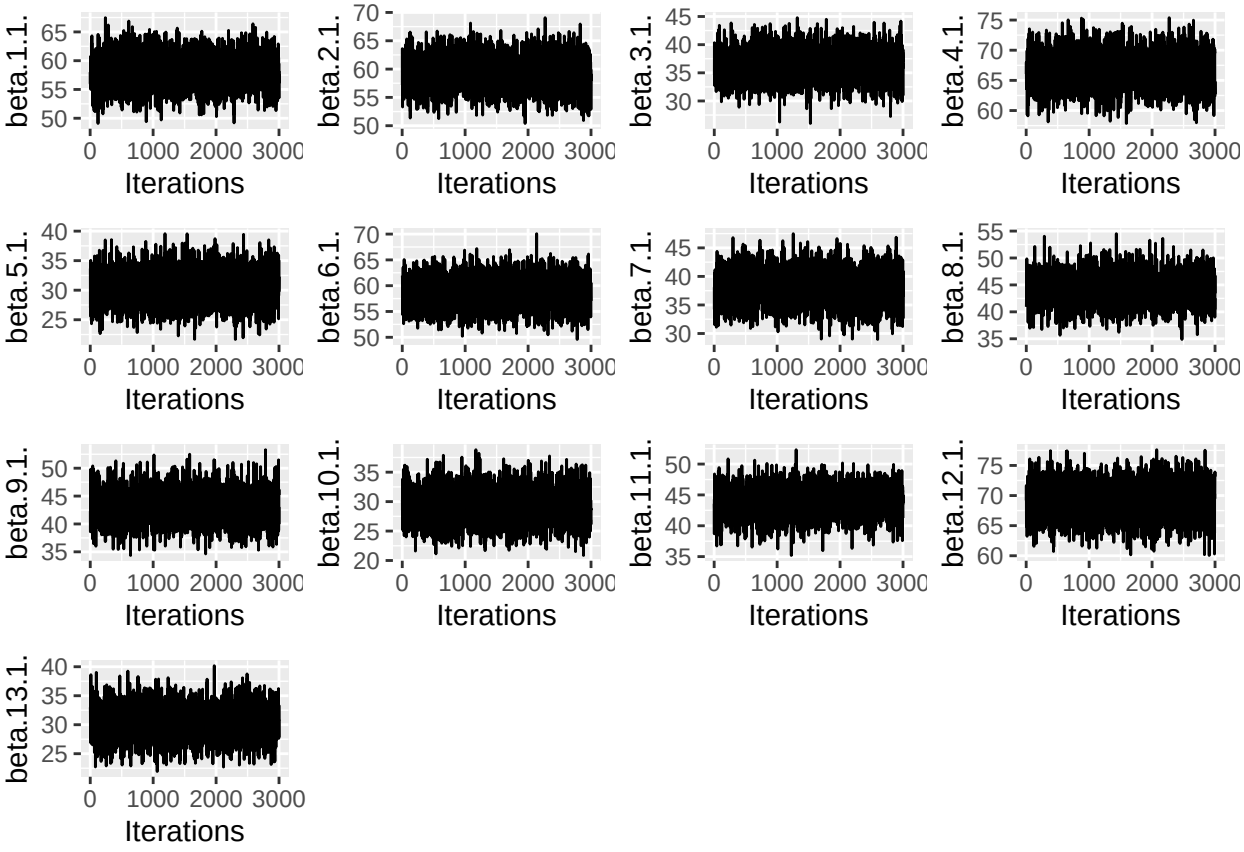
### Traceplots for Presidential Model



### Traceplots for Senate Model




### Traceplots for House Model



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## Additional EDA

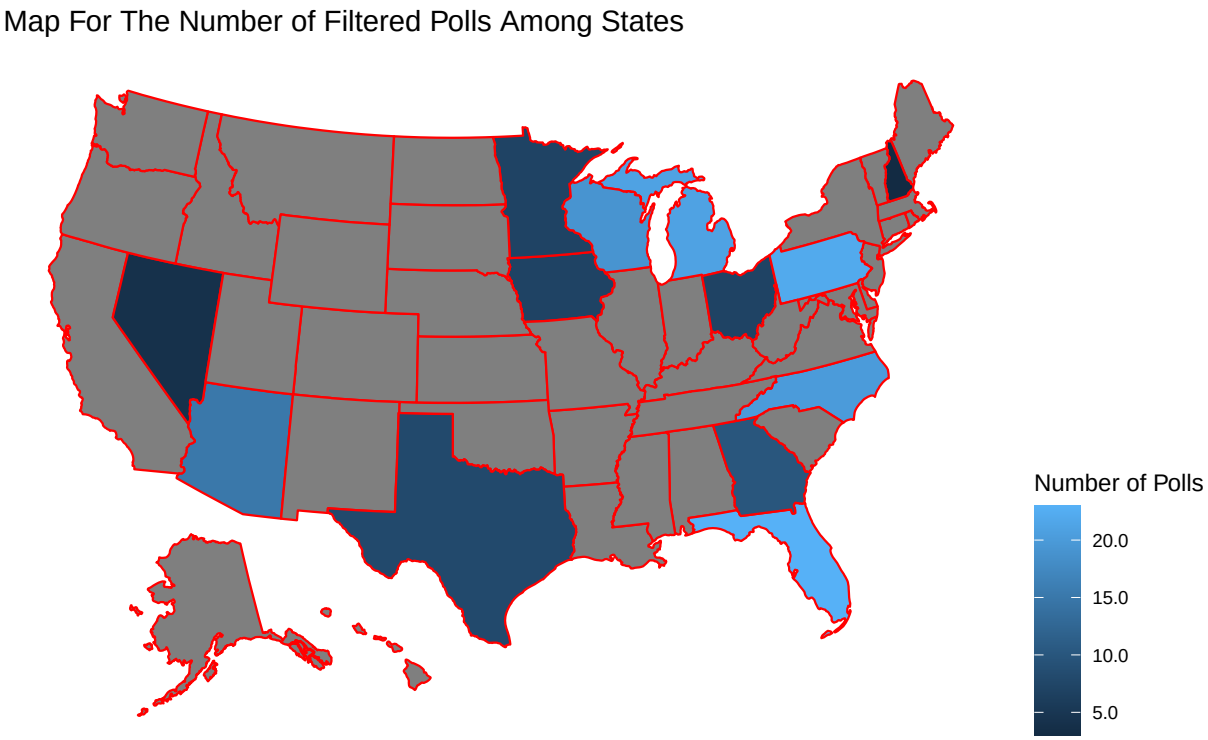


Figure 12: Additional Presidential Election Data Visualization

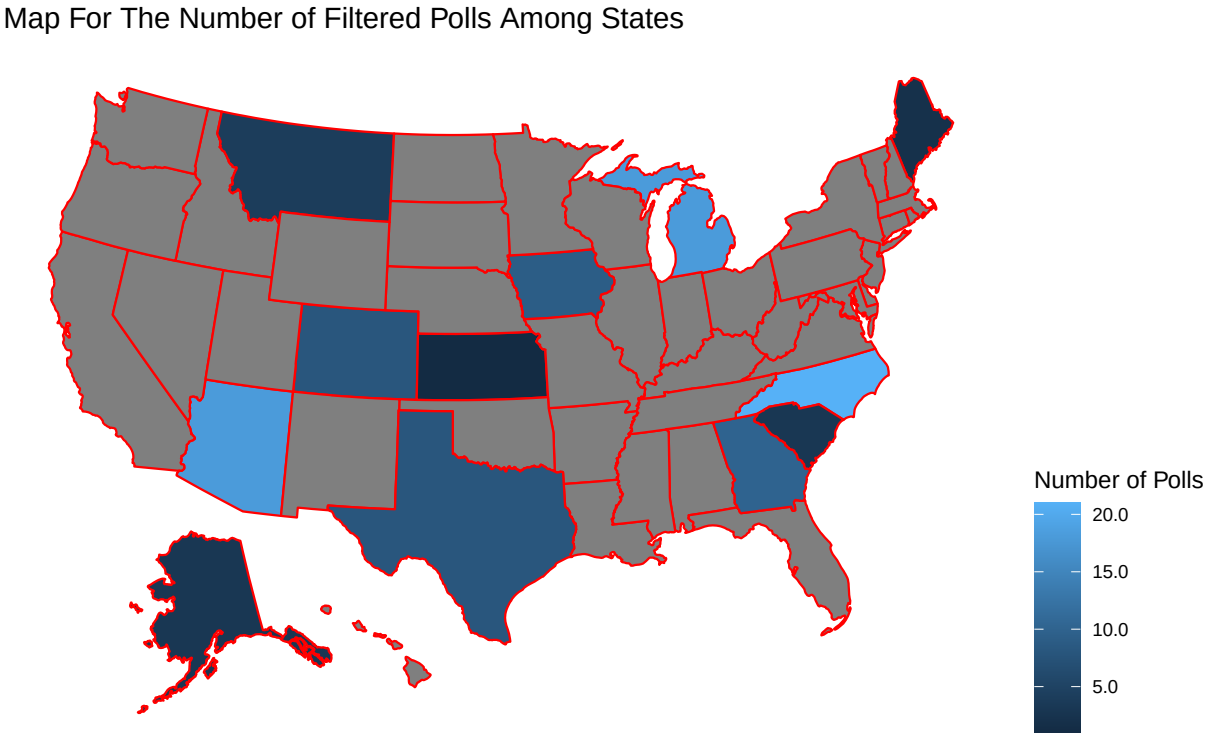


Figure 13: Additional Senate Election Data Visualization