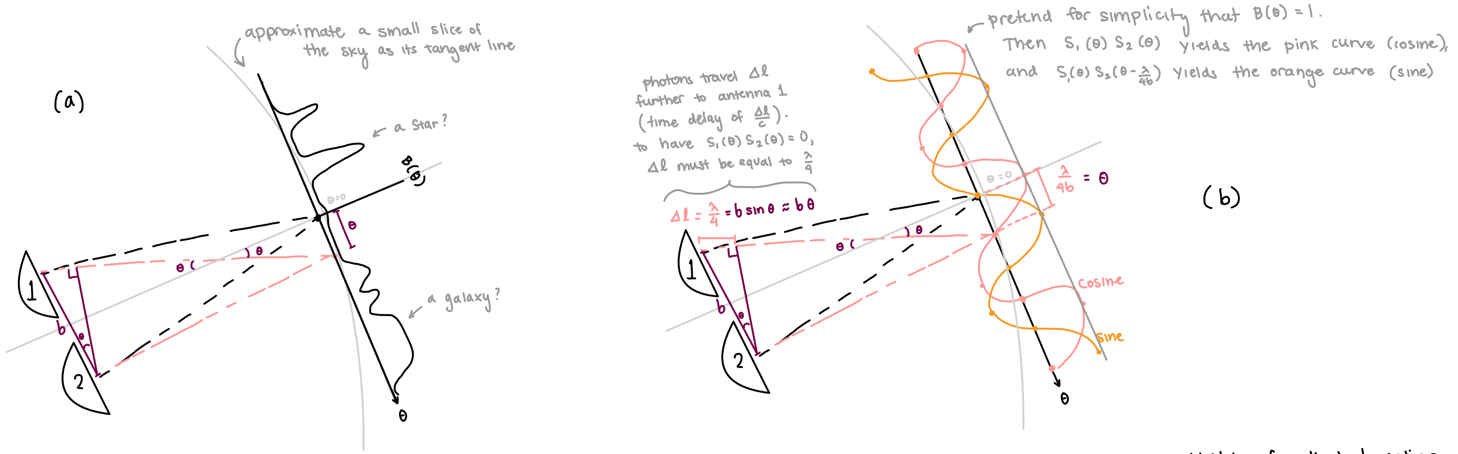
Inteferometers (pt. 1)

Let 1 and 2 be two antenna separated by baseline and measuring the strength of received EM signal over time as a voltage $S_x(t)$.
Let $B(\theta)$ be the brightness of the sky as a function of θ (a). The path length to the two antennas differs as a function of θ and baseline (b).
"brightness" = power per unit collecting area, per unit bandwidth (Hz), per unit solid angle



Taking two time-averaged cross correlations yields the real (cosine) and imaginary (sine) components of the visibility for that baseline

(a) for some pointsource at θ, the antenna the antennas would measure S_1, S_2, where $E_0 = E_0(\theta)$

(b) let $u = \frac{b}{\lambda}$ (dimensionless). the correlator stores only the real part of both correlations, since voltage is a real quantity, so we combine them to get a complex number.

(c) Integrate over all θ in the antennas beams to get the complex visibility (the measurement we make by cross-correlating). Notice that $V(u)$ is a sample of $\tilde{B}(\kappa)$ at $\kappa = u$ * and in fact, that $V = \tilde{B}$, so our cross-correlation measures the Fourier transform of brightness $B(\theta)$!

1

$$S_1(t,\theta) = E_0 e^{i 2\pi \nu t}$$
$$S_2(t,\theta) = E_0 e^{i 2\pi\nu(t + \frac{\Delta\ell}{c})} = E_0 e^{i[2\pi\nu t + 2\pi \frac{b}{\lambda}\theta]}$$
$$\langle S_1(t,\theta)\, S_2^*(t,\theta)\rangle_t = |E_0|^2 e^{-i(2\pi \frac{b}{\lambda}\theta)} \longrightarrow C = \mathrm{Re}(S_1(t,\theta)\, S_2^*(t,\theta)) = |E_0(\theta)|^2 \cos(2\pi u\theta)$$

$$V(u,\theta) = C + i(-S) \longrightarrow V(u) = \int |E_0(\theta)|^2 e^{-i 2\pi u\theta}\, d\theta$$

this is a simple, 1D version of the full van Cittert-Zernike integral. note that u is "angular frequency

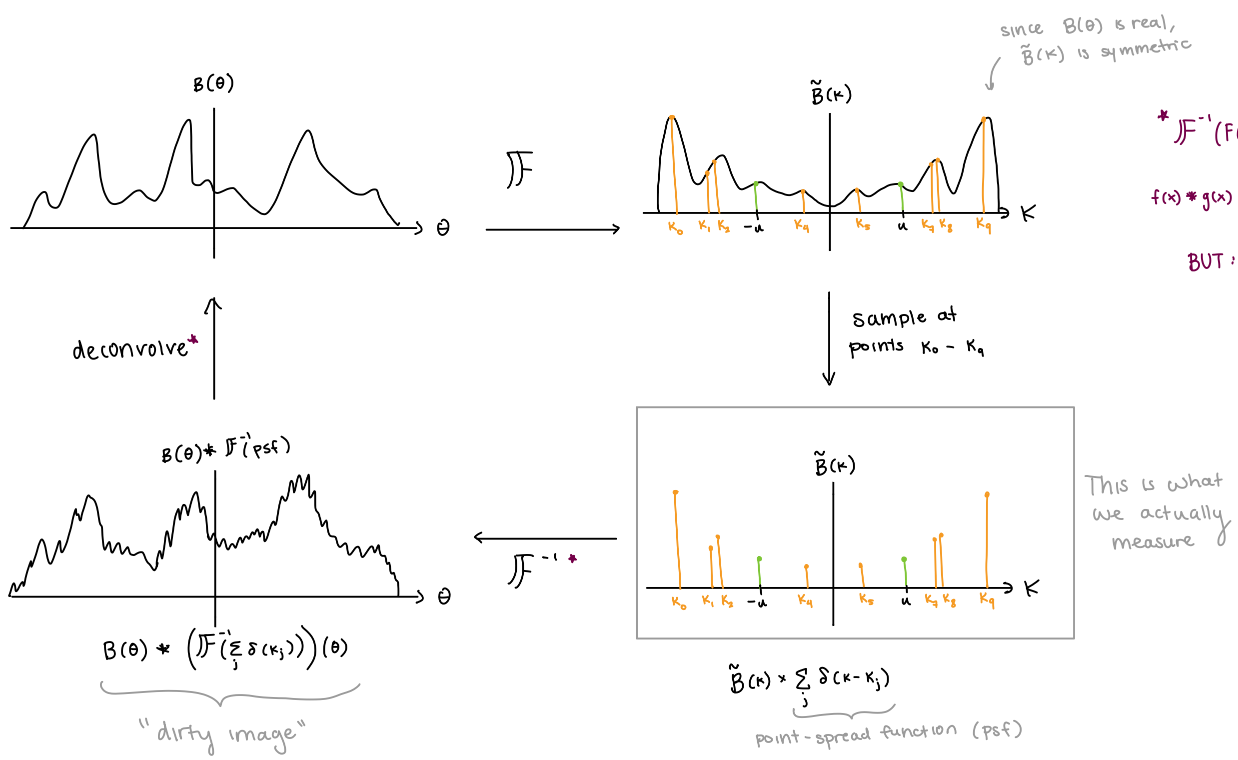
2

$$S_1(t,\theta) = E_0 e^{i 2\pi\nu t}$$
$$S_2(t - \tfrac{T}{4}, \theta) = E_0 e^{i[2\pi\nu(t - \frac{T}{4}) + 2\pi\frac{b}{\lambda}\theta]}$$
$$\langle S_1(t,\theta)\, S_2^*(t - \tfrac{T}{4},\theta)\rangle_t = |E_0|^2 e^{-i(\frac{\pi}{2} + 2\pi\frac{b}{\lambda}\theta)} = -i\,|E_0|^2 e^{-i(2\pi\frac{b}{\lambda}\theta)} \longrightarrow S = \mathrm{Re}(S_1(t,\theta)\, S_2^*(t - \tfrac{T}{4},\theta)) = -|E_0(\theta)|^2 \sin(2\pi u\theta)$$

Averaging over many T strips the high-freq $e^{i 2\pi\nu t}$ oscillations, leaving $e^{-i 2\pi \frac{b}{\lambda}\theta}$

* u is angular frequency. thus, large b probe fine structure (large u) and small b probe large structure (small u).

A single baseline (i.e. one pair of antennas) samples $\tilde{B}(\kappa)$ at two points corresponding to (u) and $(-u)$, $u = \frac{b}{\lambda}$. Increasing the number of antennas (more baselines) or the range of frequencies we measure over (more λ) allows us to sample more of $\tilde{B}(\kappa)$. We can then inverse Fourier transform and deconvolve to recover $B(\theta)$.



$$^*\ \mathbb{F}^{-1}(F(\kappa)\,G(\kappa)) = f(x) * g(x)$$

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(\xi)\, g(x - \xi)\, d\xi$$

BUT: IFT and deconvolving are <u>hard</u>