Applications of Opinion Dynamics

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May 27, 2024

Introduction

Overview

- **Opinion dynamics:** It is a study of how individual beliefs, attitudes, and perceptions evolve over time through interactions within a social network, giving rise to collective patterns, consensus, or divergence within the group.
- **Models of Social Dynamics:** a model refers to a mathematical or computational representation that simulates how opinions evolve over time within a social network.
- **Types of models:**
  Macroscopic(Statistical) Model
  Microscopic(Agent-Based) Model

Basic Definitions

Definitions

- **Agents:** Individuals in the social network, each holding a specific opinion or set of opinions.
- **Interactions:** The mechanisms through which agents interact with each other, exchanging and possibly adjusting their opinions based on these interactions.
- **Network Structure:** The underlying structure that defines the connections or relationships between agents. This can be represented as a graph, where nodes are agents, and edges represent interactions.

Prerequisite - Basic Notions from Graph Theory

Directed Graph

We define a graph by $G(V, E)$, where V is the set of nodes of graph G and E is the set of edges of graph G. A graph is considered directed if and only if $(v_1, v_2) \neq (v_2, v_1)$ where $v_1, v_2 \in V$ and $(v_1, v_2), (v_2, v_1) \in E$.

Undirected Graph

A graph $G(V, E)$ is termed undirected if and only if $(v_1, v_2) = (v_2, v_1)$, where $v_1, v_2 \in V$ and $(v_1, v_2), (v_2, v_1) \in E$.

Walk of Length L

For any graph given by $G = (V, E)$, a walk of length L is defined as the set of nodes $v_1, v_2, v_3, \ldots, v_L$ such that there exists an edge between v_i and v_{i+1} for $1 \leq i < L$.

Prerequisite - Basic Notions from Graph Theory (contd.)

Definitions

- **Root Node:** A node connected by walks to all other nodes in a graph is referred to as a **root node**.
- **strongly connected graph**: A graph is called **strongly connected** or **strong** if a walk between any two nodes exists.
- **quasi-strongly connected**: A graph is **quasi-strongly connected** or **rooted** if at least one root exists.
- **strong component**: A strong sub-graph G' of the graph G is called a **strong component** if it is not contained by any larger strong sub-graph.

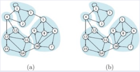

Figure: Strong components of a rooted graph

Prerequisite - Basic Notions from Graph Theory (contd.)

Degree Matrix

For an undirected graph, the degree matrix D is defined as $D = [D_{ij}]$, where $D_{ij} = \deg(v_i)$ if $i = j$ and 0 otherwise.

Adjacency Matrix

For an undirected graph, the adjacency matrix $A = [A_{ij}]$, where $A_{ij} = 1$ if v_i and v_j are connected, and 0 otherwise.

Laplacian Matrix

The Laplacian matrix for an undirected graph $G(V, E)$ is given by $L = D - A$.

Opinion Dynamics Models - French-DeGroot Model

French-DeGroot Model of Opinion Formation

- Model describes a discrete time process of opinion formation among n agents.
- Discrete-time process in a group of n agents with scalar opinions $x_i \in \mathbb{R}$.

$$x(k+1) = Wx(k), \quad k = 0, 1, \ldots \tag{1}$$

$$x_i(k+1) = \sum_{j=1}^{n} w_{ij} x_j(k), \quad \forall i, \; k = 0, 1, \ldots \tag{2}$$

- Stochastic matrix $W = (w_{ij})$ represents influence weights.
- Self-influence weight w_{ii} indicates openness to assimilation.
- Opinions may be scalar ($x_i \in \mathbb{R}$) or vectors.

Opinion Dynamics Models - French-DeGroot Model (contd.)

History of the French-DeGroot model

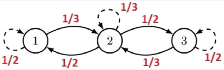

Figure: An example of the French model with n = 3 agents

- . the weighted graph in Fig. corresponds to the dynamics

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

- French formulated without proofs several conditions for reaching a consensus, i.e., the convergence $x_i(k) \xrightarrow{k \to \infty} x^*$ of all opinions to a common "unanimous opinion."

Opinion Dynamics Models - French-DeGroot Model (contd.)

French-DeGroot Model - Algebraic Convergence Criteria

- The model (2) is convergent if, for any initial condition $x(0)$, the limit exists
$$x(\infty) = \lim_{k\to\infty} x(k) = \lim_{k\to\infty} W^k x(0).$$
- A convergent model reaches a consensus if $x_1(\infty) = \ldots = x_n(\infty)$ for any initial opinion vector $x(0)$.
- The model is convergent (i.e. W is regular) if and only if $\lambda = 1$ is the only eigenvalue of W on the unit circle $\{\lambda \in \mathbb{C} : |\lambda| = 1\}$.
- The model reaches consensus (i.e. W is fully regular) if and only if this eigenvalue is simple, i.e. the corresponding eigenspace is spanned by the vector $\mathbb{1}$.

Opinion Dynamics Models - French-DeGroot Model (contd.)

French-DeGroot Model - Graph-theoretic Conditions for Convergence

- The model is convergent if and only if all closed strong components in $G[W]$ are aperiodic.
- The model reaches a consensus if and only if $G[W]$ is quasi-strongly connected and the only closed strong component is aperiodic.

Opinion Dynamics Models - French-DeGroot Model (contd.)

Stubborn Agents in the French-DeGroot Model

- Stubborn agent corresponds to a source node in a graph $G[W]$, i.e., a node having no incoming arcs but for the self-loop.
- The agent with $w_{ii} = 1$ (and $w_{ij} = 0 \; \forall j \neq i$) is a stubborn or zealot agent.
- If more than one stubborn agent exists (i.e., $G[W]$ has several sources), then consensus among them is impossible.

Conclusions

Literature Survey Recap

- Understanding the gap between Social Network Analysis and Control.
- Significance of mathematical models in advancing complex networks theory.

Future Exploration

- Acknowledging the scope for further exploration.
- Need for more research on diverse opinion dynamics models.

Reference

- Proskurnikov, A.V., Tempo, R., 2017. A tutorial on modeling and analysis of dynamic social networks. part i. Annual Reviews in Control 43, 65–79.[**?**]

Thank You