Shivam Singh
220911596
Roll no - 55

MATHS ASSIGNMENT

1) $X \sim U(0,1)$

$\therefore f(x) = \begin{cases} 1 & 0 < x \leq 1 \\ 0 & \text{lwd} \end{cases}$

Now : $Y = e^{-x}$, $g(y)$

$G(y) = P(Y \leq y)$
$= P(e^{-x} \leq y)$
$= P(-x \leq \ln y)$
$= P(x \geq -\ln y) = P(y \geq \ln 1/y)$

$= \int_{\ln y}^{1} f(x)\, dx$

$= \int_{\ln 1/y} 1 . dx = 1 - \ln \frac{1}{y} = 1 + \ln y \quad \ldots (2)$

$\therefore g(y) = \frac{d}{dy} G(y) = \frac{d}{dy} (1 + \ln y) = \frac{1}{y}$

R_Y : $x \in (0, 1)$
$\therefore y = e^{-x} \rightarrow e^{-0} = 1$
$y = e^{-1} = \frac{1}{e}$

So, $g(y) = \begin{cases} 1/y & 1/e \leq y \leq 1 \\ 0 & \text{else} \end{cases}$

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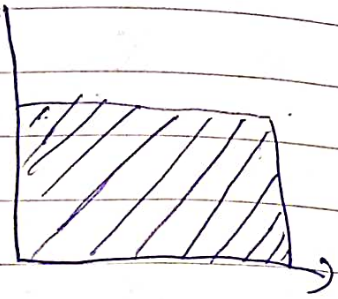
2) $f(x,y) = \begin{cases} x+y & 0 \le x,\ y \le 1 \\ 0 & \text{else} \end{cases}$



$z = x - y$

let $w = y$

then $x = z + w$

$y = w$

Calculating Jacobian

$$|J| = \begin{vmatrix} \partial x/\partial z & \partial x/\partial w \\ \partial y/\partial z & \partial y/\partial w \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$= \underline{\underline{1}}$

Joint pdf of z, w

$k(z,w) = f(x,y)\,|J|$

$= x + y$

$= z + 2w$

$0 \le x \le 1$

$0 \le z + w \le 1$ $\qquad$ $0 \le y \le 1$

$-w \le z \le 1 - w$ $\qquad$ $0 \le w \le 1$

$0 \le z \le 1 - w$

Since $w > 0$, 0 is infimum for z

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for $0 \leq z \leq 1$

$$w(z) = \int_{0}^{1-z} k(z,w)\, dw$$

$$= \int_{0}^{1-z} (z+2w)\, dw$$

$$= \left[zw + w^2\right]_{0}^{1-z}$$

$$= z(1-z) + (1-z)^2$$

$$= z - z^2 + z^2 + 1 - 2z$$

$$= 1 - z$$

for $-1 \leq z \leq 0$

$$h(z) = \int_{w=-z}^{1} k(z,w)\, dw$$

$$= \int_{-z}^{1} (z+2w)\, dw$$

$$= \left[zw + w^2\right]_{-z}^{1}$$

$$= (z+1) - (-z^2 + z^2)$$

$$= z+1$$

$$w(z) = \begin{cases} 1-z & 0 \leq z \leq 1 \\ 1+z & -1 \leq z \leq 0 \\ 0 & \text{else} \end{cases}$$

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3) Given $X \sim N(\mu, 10)$

we know that

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{\omega}\right)$$

i.e $N\left(\mu, \frac{10}{\omega}\right)$

let $a \in R$ be $\ni$

$$P(-a < Z < a) = 0.95$$

where $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$$P(Z<a) - P(Z<-a) = 0.95$$

$$2\Phi(a) - 1 = 0.95$$

$$a = 1.96$$

now, $P(-a < Z < a) = 0.95$

$$P\left(-a < \frac{\sqrt{n}}{\sigma}(x-\mu) < a\right)$$

$$P\left(\bar{x} - \frac{a\sigma}{\sqrt{n}} < \mu < \bar{x} + \frac{a\sigma}{\sqrt{n}}\right) = 0.95$$

required $n \ni$

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$$\frac{a\sigma}{\sqrt{n}} = \frac{1}{2}$$

$$\frac{1.96 \times \sqrt{10}}{\sqrt{n}} = \frac{1}{2}$$

$$(2 \times 1.96 \times \sqrt{10})^2 = n$$

$$n = 154$$

4) Given $n = 196$

$\sigma = 4000$

we know that $z = \frac{\bar{x} - \mu \sqrt{n}}{\sigma} \sim N(0, 1)$

$\therefore$ required probability :

$$P(-700 \leq \bar{x} - \mu \leq 700)$$

$$= P\left(\frac{-700 \times 14}{4000} \leq \frac{\bar{x} - \mu}{4000} \times 14 \leq \frac{700}{4000} \times 14\right)$$

$$= P(-2.45 \leq z < 2.45)$$

$$= 2 \times \Phi(2.45) - 1$$

$$= 2 \times 0.9929 - 1$$

$$= 0.9858$$

$\Rightarrow$

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5) $k = 7$ $n = 2200$

$P_1 = 0.17$ $P_2 = 2.2\%$ $P_3 = 13.6\%$

$P_4 = 34.1\%$ $P_5 = 34.1\%$ $P_6 = 13.6\%$

$P_7 = 2.3\%$

$$Q_{k-1} = \frac{(X_1 - nP_1)^2}{nP_1} + \frac{(X_2 - nP_2)^2}{nP_2}$$

$$+ \ldots + \frac{(X_7 - nP_7)^2}{nP_7}$$

$$= \frac{(5-2)^2}{2} + \frac{(45-44)^2}{44} + \frac{(290-272)^2}{277}$$

$$+ \frac{(640-682)^2}{682} + \frac{(720-682)^2}{682} +$$

$$\frac{(282-272)^2}{272} + \frac{(18-46)^2}{46} = 27.8073$$

$C = X_6 @ 5\% = 12.592$

$Q_{k-1} \geq C$

So the grade distribution doesn't fit the ideal curve