The optics behind refractive conditions based on the single ideal thin-lens system

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1. Background

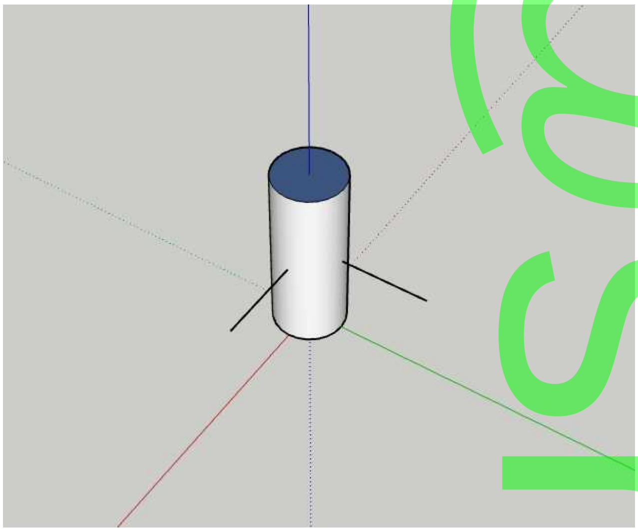
Multiple eye-specialist lack the precise awareness about what physically happens during axial elongation causing an eye to become Myopic. This may result in wrong understanding of what refractive condition of an eye really is and what it isn't.

This article tackles the basic concepts of geometric optics from the ground-up and serves as a stand-alone reference text for anyone wanting to characterize refractive changes in optical systems due to defocus or imaging screen shift along with depth-of-field changes.

2. Geometrical Optics and the ideal thin-lens relation

This section focuses on deriving the relationship between object and image distances for the case of (real) image formation by converging (convex) lens only yielding the ideal thin-lens relation. Although, all the illustrations and considerations in this article are in 2D, they extend and apply equally well to the 3-Dimensional real world also.

A lens with curvature only along one axis is known as a Cylindrical (SPH) lens while a lens with uniform curvature along both independent axes is denoted as Spherical (SPH) lens. It is important to note that the words *cylindrical* and *spherical* were meant only with regard to appearance of the lens and have little to do with the actual refracting profile (cross-section) of the lens. The axis of a cylinder lies along the direction of constant curvature with the idea being rotation about its axis should be indistinguishable (due to symmetry). This is important from the point of unique determination of cylindrical axis shown in Figure 7.1 below.



The blue line perpendicular to the flat end is the axis for this CYL lens.

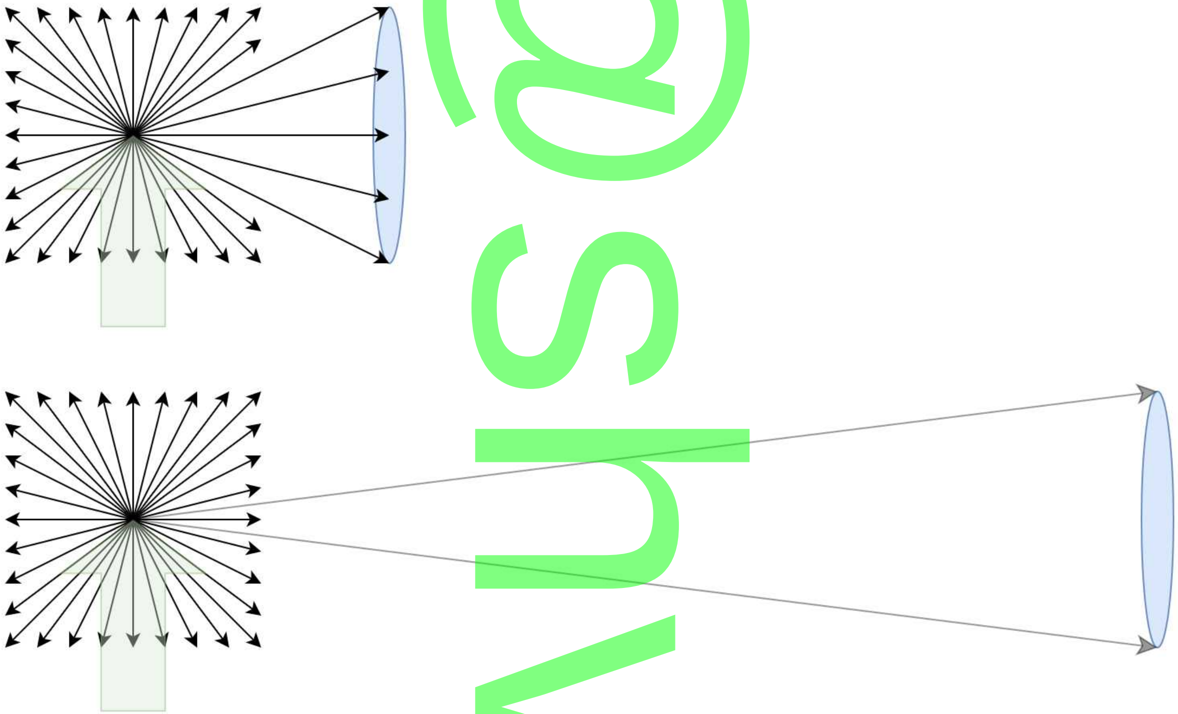
An ideal lens itself can be completely described with only **two** physical parameters/attributes along with the principal axis – its **focal length** and **aperture**. The term focal length or its inverse in lens power always implies the same physical property. These two parameters explain the majority of image formation by real lenses and their observed behaviour in experiments.

2.1 Definition of an Object and its Image in Geometric Optics

An object can be considered to be a *collection of object points*. From every object point, rays can be assumed to **originate/diverge** in all directions.

An image is similarly considered to be a *collection of image points*. At each image point, rays originating from an object point can be assumed to **terminate/converge.** An object can be differentiated from its image in the sense of independent existence. An object can exist independently but an image can not exist without an object.

It is immensely useful to express Aperture size, object, and image distances in terms of focal length. This dimensionless representation comes at no loss of generality.



Schematic of light rays hitting the same lens at different distance from an object point.

An object point (on an object) emits rays of light in all directions. At distances become larger, the angle between rays emitted by a point object starts decreasing with the rays themselves appearing almost parallel at very large distances from the lens. However, even at infinity, the rays emitted by an object can only appear diverging or parallel and never converge without a lens.

An application of our stricter definition is to consider the commonly mentioned *'virtual images'* in many textbooks as having properties more like objects rather than images because light rays seem to (diverge) originate from them towards the lens. That a 'virtual' image lies on the same side of the lens as a 'real' object only strengthens the consistency of our argument. For analysis of systems consisting of multiple optical elements, the '*virtual image*' formed by an object and lens combination can be replaced by the virtual object alone without affecting the rest of the system. Subsequent imaging of such a virtual object by an optical system (whether our eyes or a camera) then results in the formation of an actual real image.

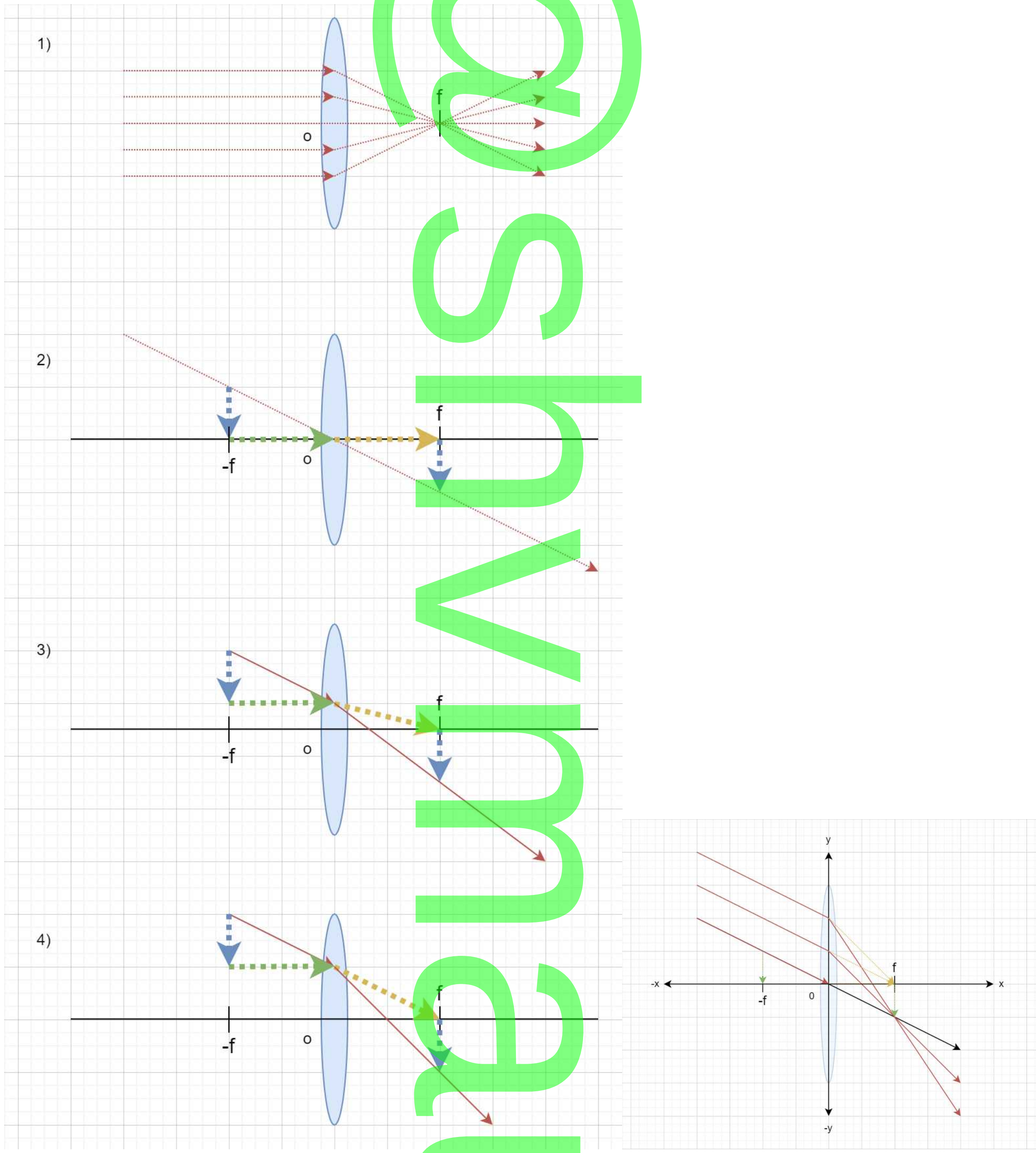
The use of the word image in this article should always be taken to mean 'real' image.
For formation of an image point, demonstrating the intersection of any two rays emerging from an object point is sufficient. A point on an object and its image will always have a one-to-one correspondence.

2.2 Calculating deviation for a ray incident on an ideal lens

Assumption A: The focal length defines the convergence/divergence point (focus) for all rays parallel to the principle axis of the converging/diverging lens. This is analogues to a parabolic concave mirror reflecting all incoming rays parallel to its principal axis on its focus. The Aperture size only determines the admission extent for such parallel rays on the lens profile.

In this sense, an ideal lens is essentially a theoretical object and we needn't immediately concern ourselves with the implementation details of how it achieve such 'focusing' of rays in reality. It also means that these rays in question can be anything – light, radio waves, sound waves etc. reinforcing the purely theoretical nature of the relation. Ideal lenses are interesting objects with properties of their own from a purely theoretical perspective.

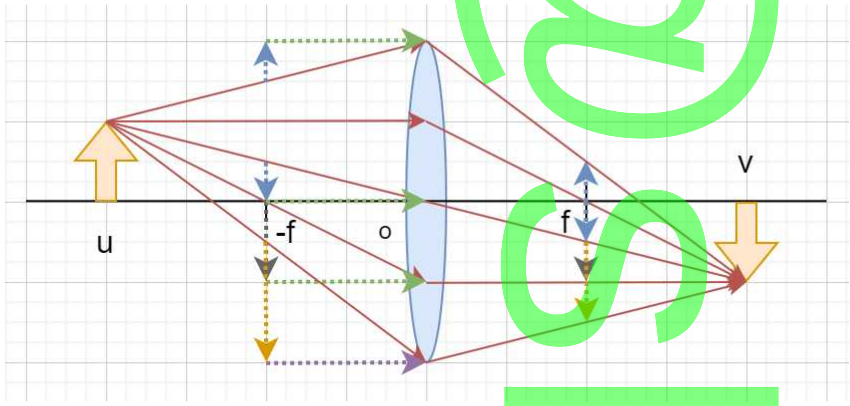
Assumption B: Only and only rays of light passing parallel to the principle axis obey the previous assumption. Any ray incident 'vertically' on the lens profile (normal to the principal axis) passes through the lens profile unchanged. The word 'thin' is used to denote this aforementioned property of not affecting vertical rays parallel with the lens. This assumption also involves the implicit assumption that a ray can be first decomposed into its normal and vertical constituents The ray diagrams given below demonstrate application of the two assumptions to calculate deviations for various incident rays by breaking them into their parallel and normal components with respect to the principal axis.



Application of Assumption A (1) and B (2 to 4). Demonstration of focusing of oblique rays (combined version of 2, 3, and 4 in Figure 7.3)

In all three cases 2 to 4 shown in Figure 7.3 and their combined version in Figure 7.4, we can observe that the length of the vertical component is the same when determined at at the focus. The amount of deviation a ray undergoes depends on both where it is hitting the Aperture and the angle it makes with the lens profile.

Consideration of these two assumptions results in simplified rules for all rays incident on an ideal converging (+) lens and is sufficient to explain image formation by ideal lenses. A verification of this simple rule for an edge case corresponding to image formation at 2f is provided below in Figure 9.5.



Edge case shown for image formation at 2f.

In this particular edge case, we've shown the convergence of five such rays at a single point when demonstrating that only two meet at the image point would have sufficed.

The rule describing ray of light passing unchanged through the optical centre of the lens (as given in many elementary school textbooks) can be explained as a consequence of assumptions A and B stated above. It also determines the magnification ratio because the image point lies on the ray passing unaffected through the optical centre of the lens.

2.3 Deriving the relation between object and image distances:

For a converging lens with a given focal length, two distinct cases of rays intersection are possible according to rules outlined in the previous section 9.1.b:

1. **Virtual *object*** formation when the object is at a distance closer than the focal length from the lens. Light rays appear to be coming from a virtual object point on the virtual object.

2. **Image** formation when the object is at a distance farther than the focal length from the lens. Light rays from the object point appear to be converging towards the image point.

Both of these cases can be seen in Figure 9.6 below with an object of same height placed at varying distances with additional illustration of the limiting case of an object situated exactly at the focus resulting in no intersection of light rays:

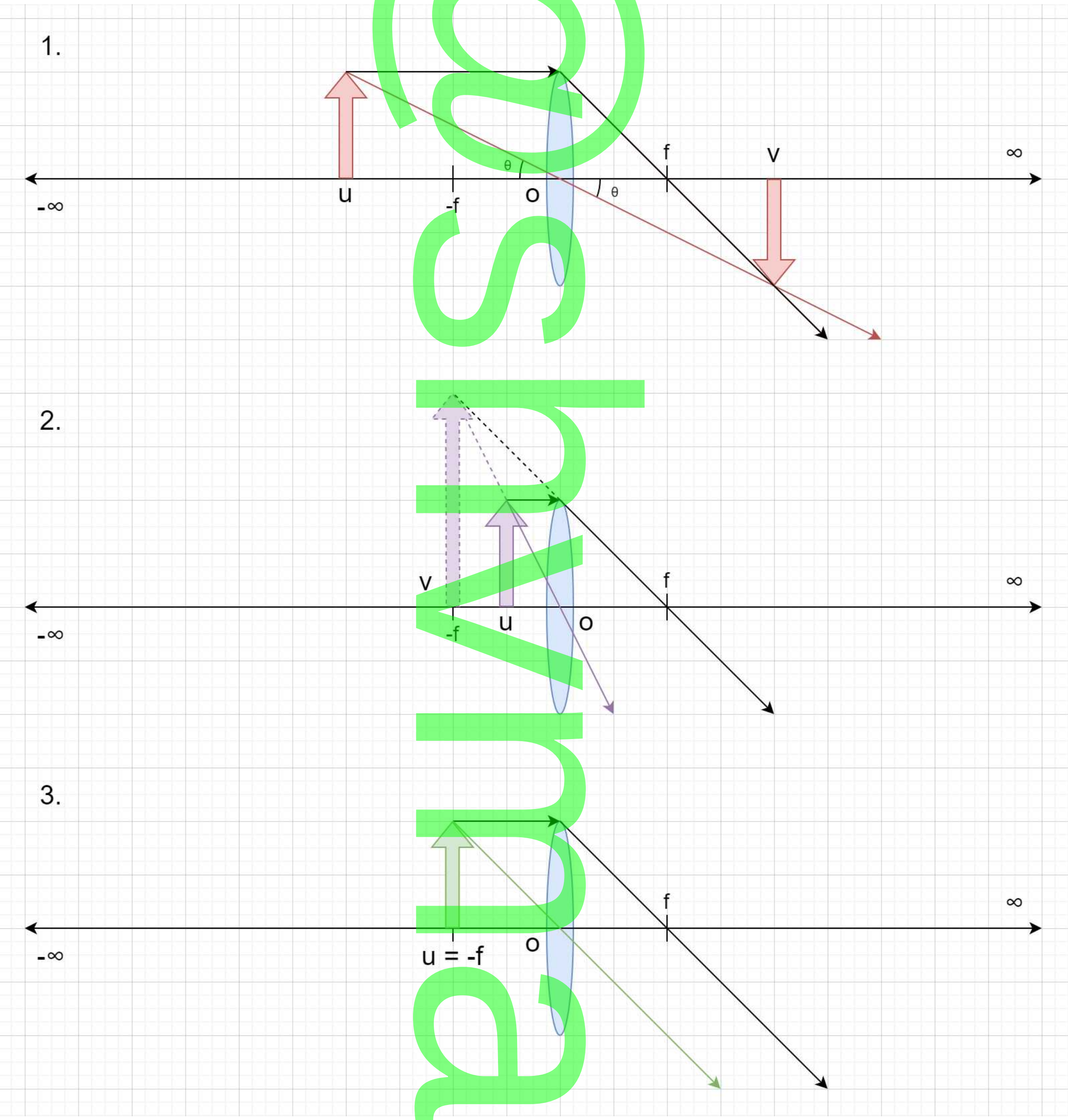
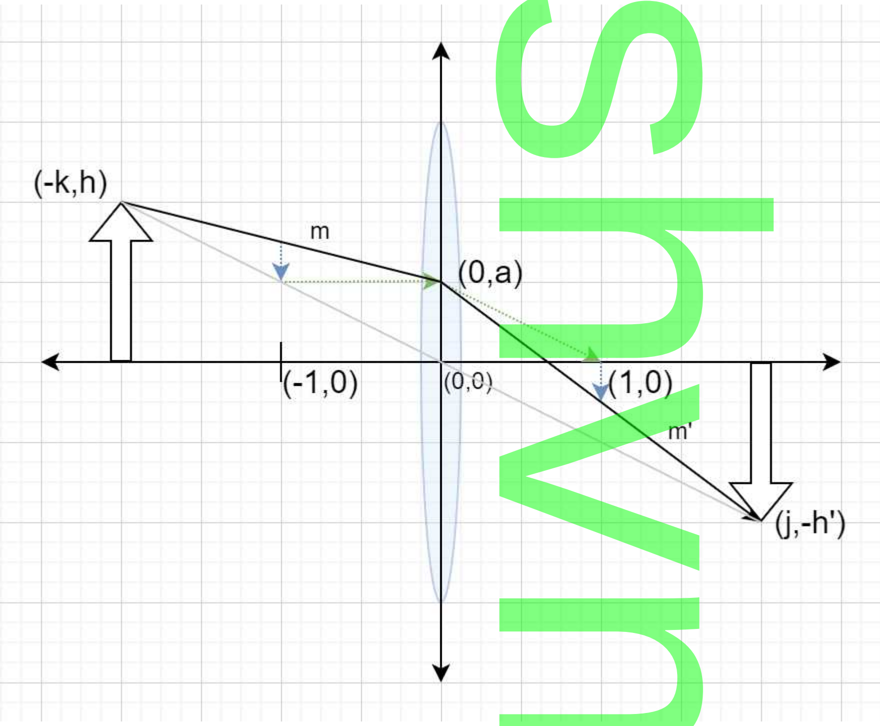


Image formation and virtual image formation by converging lens

In all the three situations, the ray that passes through focus has the same slope *-h/f* where *h* is the height of object point. But, the slope of line passing through the optical centre depends on the object distance in all three cases.

Whether this slope is greater than or lesser than the line passing through focus determines where these two lines meet and whether an image is formed or not. Precisely, a light ray emerging from a point on an object of height *h* located farther than the focus through the optical centre will always have a less negative slope.

The derivation of ideal lens relation for a converging lens has been given below. It is important to note beforehand that the image formed by all such systems is **inverted**.



Ray Diagram for derivation of image and object distance relation.

For calculation of image distance, we assume the origin of the right-handed co-ordinate system centred on the lens. An ideal lens will always has two foci due to its symmetry. For a converging (+) lens, the focus is on the opposite side of the lens from object. It depends on the signed power (converging or diverging) of lens that determines the sign of focal length and consequently its actual position relative to the object. For this article, we are concerned with 'real' images formed by the converging lens but the relation is equally applicable for both.

$$magnification(m)\,is\,defined\,as\,\frac{image\,height}{object\,height}=\frac{-h'}{h}=\frac{image\,distance(v)}{object\,distance(u)}=-\frac{j}{k}$$

$$\textit{We have normalized object and image distances as follows:}$$
$$u = -kf \text{ and } v = jf$$

$$\textit{for image formation, } k > 1 \textit{ which means } -u > f \textit{ or } u < -f$$
$$\textit{slope } m \textit{ when the object point is } (-k, h) \textit{ is given by}$$
$$m = \frac{a-h}{k}$$
$$\textit{for point } (0, a) \textit{ on the lens profile } (y - axis)$$
$$\textit{image formation requires all rays to pass the point } (j, -h') \textit{ after refraction}$$
$$\textit{The slope after refraction } (m') \textit{ is given by,}$$
$$m' = \frac{a-(-h')}{-j} = \frac{a+h'}{-j}$$
$$\textit{The slope after refraction according to Assumption 2 must be}$$
$$m' = m - a \implies \frac{a+h'}{-j} = \frac{a-h}{k} - a$$
$$\textit{Now, it's already known from the magnification criteria that}$$
$$\frac{k}{j} = \frac{h}{h'} \implies h' = \frac{hj}{k}$$
$$\implies \frac{a+\frac{hj}{k}}{-j} = \frac{a-h}{k} - a$$

$$\implies \frac{ka+hj}{k} = \frac{hj-aj+kaj}{k}$$
$$\implies ka = kaj - aj$$
$$\textit{giving, } k + j = kj$$
$$\textit{which completes the derivation of k and j relation.}$$

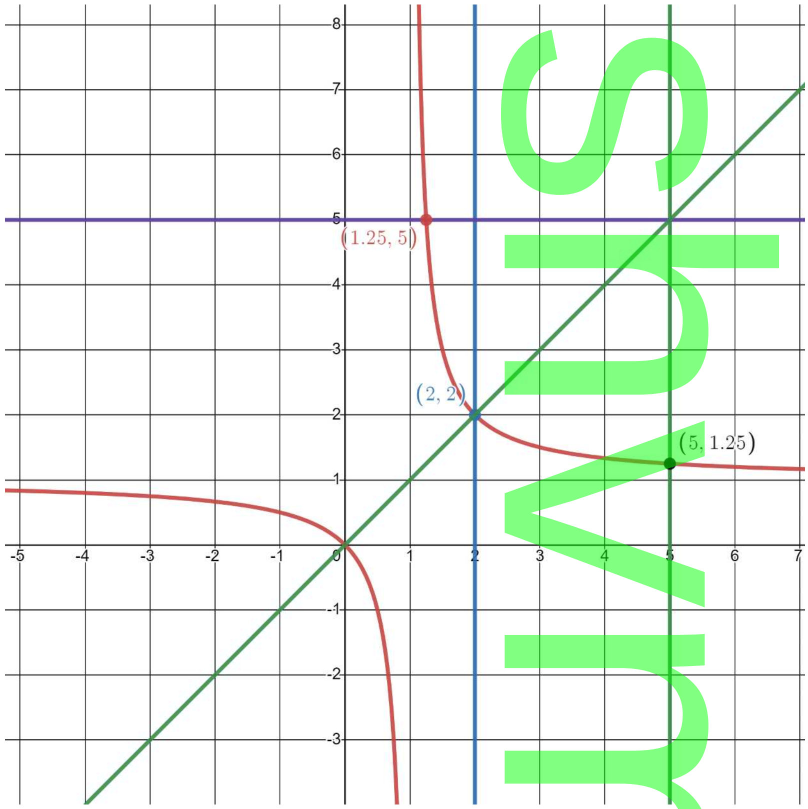
$$\textit{To derive the ideal thin lens relation,}$$
$$\textit{we can substitute } k = -\frac{u}{f} \ \& \ j = \frac{v}{f} \textit{ giving}$$
$$-\frac{u}{f} + \frac{v}{f} = -\frac{vu}{f \times f}$$

$$\implies -u + v = -\frac{vu}{f}$$
$$\implies \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

2.4 Physical results from dimensionless object (k) & image distance (j)

The relation between dimensionless object and image distance is $j = \frac{k}{k-1}$

For image formation by a converging lens, k > 1. This simply means that the object must always be located beyond the focus of the converging lens. A plot of this equation results in a hyperbola symmetrical around the line x = y as shown in Figure 2.1. The vertical and horizontal asymptotes given by lines x = 1 & y = 1 correspond to the object at focus and ∞ while also corresponding to its image at ∞ and focus respectively.

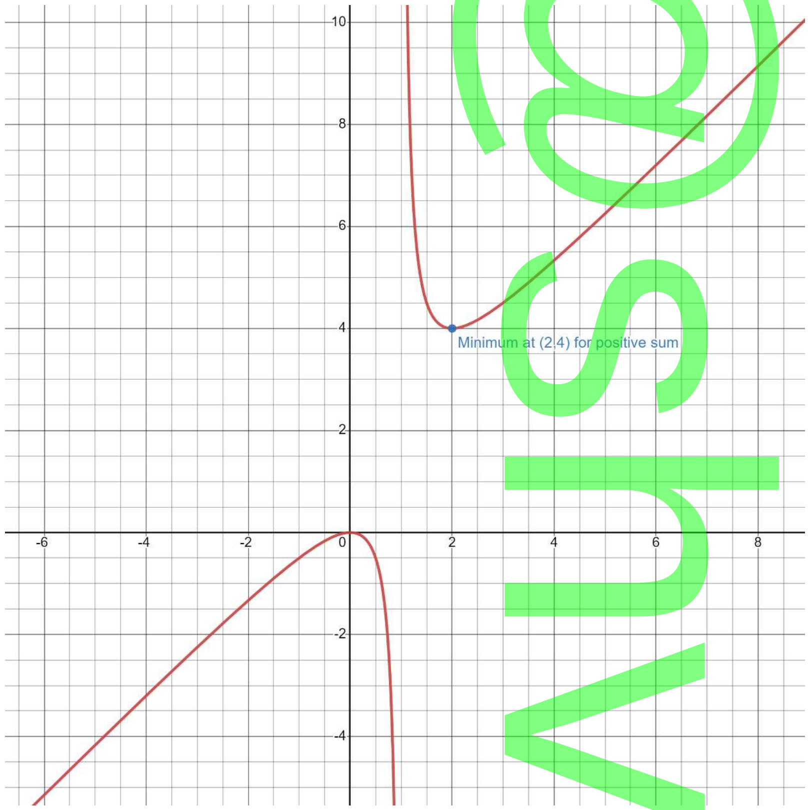


The hyperbolic plot for the dimensionless lens relation is shown in red

It is evident from the symmetry of the hyperbola in Figure 2.1 that both an object & its image's dimensionless distances form what can be termed as a mirror pair around the x = y line.

This becomes even more evident if we solve for k giving $k=\frac{j}{j-1}$, which can be understood simply as interchanging k with j & j with k in the original relation.

Adding k and j together gives, $k+j=k+\frac{k}{k-1}=\frac{k^2}{k-1}=\frac{j^2}{j-1}$ which has a minimum of 4 at k = j = 2 for positive values as can be seen from the graph below in Figure 2.2.



Plot showing how the sum of dimensionless image and object distance varies with image/object distance

This relation shows that for a given distance between an object and its required image, the focal length of the system able to image it must be lesser than one-fourth of said distance.

Thus, the $(k + j) \geq 4$ rule along with the $k > 1$ rule for image formation serve as baseline conditions for implementations of real world optical imaging systems involving lenses.

2.5 The power addition rule

The power (inverse of focal length) addition rule can easily be derived from the ideal lens relation by considering two lenses with different focal lengths placed together such that the distance between them can be neglected.

Here, v and u denote the image and object distances respectively with subscripts denoting which lens they are referring to.

The power of a lens is the inverse of its focal length. The Dioptre (m^{-1}) is one such derived SI unit of power.

Assuming focal length f_1 for Lens L_1 and f_2 for Lens L_2. Then the image distance for Lens L_1 is given by,

$$v_1 = \frac{u}{1 + \frac{u}{f_1}}$$

Because the distance between the two lenses is negligible, this resultant image now gets further refracted as an object by lens L_2. The corresponding final image formation distance is given by (where $u_2 = v_1$).

$$v_2=\frac{v_1}{1+\frac{v_1}{f_2}} \Rightarrow v_2=\frac{\frac{u}{1+\frac{u}{f_1}}}{1+\frac{\frac{u}{f_2}}{1+\frac{u}{f_1}}}=\frac{u}{1+\frac{u}{f_1}+\frac{u}{f_2}}=\frac{u}{1+u(\frac{1}{f_1}+\frac{1}{f_2})}$$

This implies that the combined lenses together act as a lens of focal length $\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}=P_1+P_2$ which is basically the law of addition of lens powers.

For a combination of more than two lenses closely put together, we can proceed by combining two lenses together at a time until only one remains.

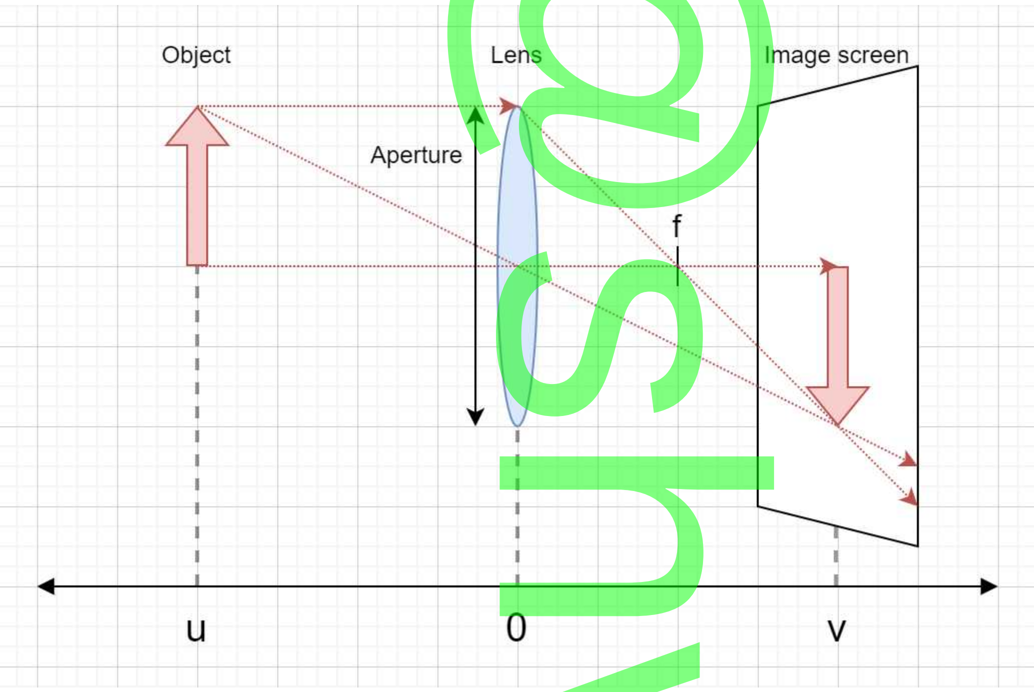
3. The Simple Lens system

Care must be taken to ensure consistency of units and signs according to conventions while utilizing formulas throughout this article to ensure correctness in calculations. The term 'image' should be taken to mean real image and all mentions of the word 'lens' should be assumed as referring to an ideal thin-lens.

Be it Human eye or an optical system like a camera, the purpose is mostly the same - 'imaging' objects (which can be located at varying distances) onto the retina/image sensor. For a film camera, the film acts as the image sensor. For a digital camera, an image sensor chip replaces the film. The physical principles involved behind image formation remain unchanged.

We will start with an optical bench setup consisting of a converging lens and an image sensor (planar screen) as shown in Figure 2.3. The screen is aligned perpendicular to the principle axis of the lens.

For the simple lens system, tracing the path of light rays from an object to its eventual image is very simple. Light rays emanating from an object point encounter the lens aperture and hit the image sensor after refraction. The medium in this case can be said to be vacuum/air.



Four parameters completely describe the combined setup of an object to be imaged and our simple lens system, if one also includes Aperture size (having no impact on image distance). The remaining three parameters are object distance (u) with the focal length (f) {both determining the image distance (v) – a dependent variable} and the screen distance (s). Distances are measured with Lens Optical Centre as origin.

For the formation of sharpest possible image, image distance (v) must coincide with the screen distance (s). This will be applicable for all the cases discussed in this section and sub-sections. Image formation in this context refers to the formation of a clearly focused image with the screen placed accordingly. The procedure for calculating distances within which screen distance (s) can vary for a given image distance (v) while still forming appreciably sharp images within a CoC (Circle of Confusion) is provided in the DoF derivation Section 8.4 of the Appendix accompanying this article.

Assuming the above, we can say that dependence of screen distance (s) on focal length (f) and object distance (u) is already encoded by the ideal lens relation. Knowing any two, the third is easily determined.

$$s=\frac{fu}{f+u}=\frac{u}{1+\frac{u}{f}}=\frac{1}{\frac{1}{u}+\frac{1}{f}}$$

Fixed screen distance and fixed focal length also fixes the distance at which an object can be imaged. It is desirable for a system to be able to form images of objects located within a distance range. Imaging objects can be achieved by varying either the focal length (f) or the screen distance (s) or some combination of both. It is important to remember that all real-world implementations always have some physical constraints on the ‘variability’ of these independent variables restricting their values.

The range of object distances that can be imaged by a system will be referred to as ***observation range*** throughout this article. The farthest and nearest distance extremes of the observation range are commonly designated as its far-point (d_{far}) and near-point (d_{near}) respectively. The Far-Point is defined as the distance of ***far***thest objects a system is able to image while the Near-Point is similarly defined as the distance of ***near***est objects a system is able to image within its constraints.

The **Field of View (FoV)** is defined as the angular-expanse of objects whose images can be formed on the image sensor/screen. It is easily quantified from the ratio of screen height to the screen distance.

Our approach for studying changes to a system's observation range results from fixing one variable and studying how variation of the other affects the object distances (d_{near} from d_{far}) that can be imaged. The two resulting cases for our system corresponding to fixing independent variables focal length (f) and screen distance (s) one at a time are discussed below.

3.1 Constrained Observation range of system when imaging screen distance (s) is fixed

Showing that for such a system,

*the increase in power/decrease in focal length (f) for maintaining **fixed screen distance (s)** solely depends on the object distance (u)*

can be easily done by rearranging the ideal lens law in terms of focal length.

$$\text{By the ideal lens relation}, \frac{1}{f}=\frac{1}{s}-\frac{1}{u}, s \text{ is constant}$$

$$\text{For an object at } \infty, u \rightarrow \infty \Rightarrow \frac{1}{f_\infty}=\frac{1}{s}-\frac{1}{u}=\frac{1}{s}$$

$$\text{For an object closer than } \infty, \frac{1}{f}=\frac{1}{f_\infty}-\frac{1}{u}$$

$$\Rightarrow \frac{1}{f}-\frac{1}{f_\infty}=\Delta Power=-\frac{1}{u}$$

$$\text{The signed term} -\frac{1}{u} \text{ is positive because object distance is negative by co-ordinate convention.}$$

For such a system, the increase in lens power needed to image objects **closer** than a reference distance depends only on the object distance (u). This increase in lens power for imaging closer distances is commonly referred to as *accommodation* with reference 'un-accommodated' distance taken as optical infinity. Similarly, the power difference between far and near point of a particular system will be referred to as that system's accommodation ability. A system is said to be fully accommodated if the extreme of lens power corresponding to observing objects at its near-point is reached.

Fixing screen distance requires the focal length to vary in accordance with the observed object distances. For instance, a system with a fixed screen distance (s) of 25 cm and lens power ranging between +4 D and +9 D will have its Far and Near-points at ∞ and 20 cm respectively.

With lens power at +9 D which amounts to +5 D increase from the initial +4 D needed for objects at ∞, the system images objects located at 20 cm. Thus, it can be stated that this particular system accommodates +5 D in order to observe objects at its near-point of 20 cm.

Because screen distance (s) is fixed for this system, the Field-of-View (FOV) remains unchanged.

3.2 Constrained Observation range of the system when the focal length (f) is fixed

If the focal length is kept fixed and the screen distance is allowed to vary instead, the dependence of observation range on screen distance is given in a similar manner by the ideal lens relation:

$$\text{By ideal lens relation}, \frac{1}{s} - \frac{1}{u} = \frac{1}{f} \text{ giving } \frac{1}{s} = \frac{1}{f} + \frac{1}{u}$$

$$\text{For an object at } \infty, u \rightarrow \infty$$

$$\text{giving } \frac{1}{s_\infty} = \frac{1}{f}$$

$$\text{For an object closer than } \infty, \frac{1}{s} = \frac{1}{s_\infty} + \frac{1}{u}$$

$$\text{The signed term } \left(\frac{1}{u}\right) \text{ is negative here.}$$

Assuming a system like the one described in section 2.2.a above but with a fixed focal length (f = 25 cm) instead, we can find out the near and far points of this system in a similar manner.

The k > 1 requirement arrived in section 2.1 ensures that screen distances lesser than the (fixed) focal length of the system can't result in image formation for any possible physical value of object distance. Thus, the screen distance required is always greater than 25 cm and can only be increased up to infinity resulting in Far and Near-point at ∞ and 25 cm respectively.

It is evident that increments in screen distances required for imaging closer objects quickly approach very large values and even then the system is unable to image objects closer than 25 cm because there's no such physical thing as 'screen distance beyond infinity'. Contrast this to the variable focal length (f) system where no such physical limit on focal length was in place preventing us from observing closer objects.

Changes to the screen distance implies changes to the overall size of the system and impact the FOV also.

4. Representing observation ranges on the Relative Dioptre Scale (RDS)

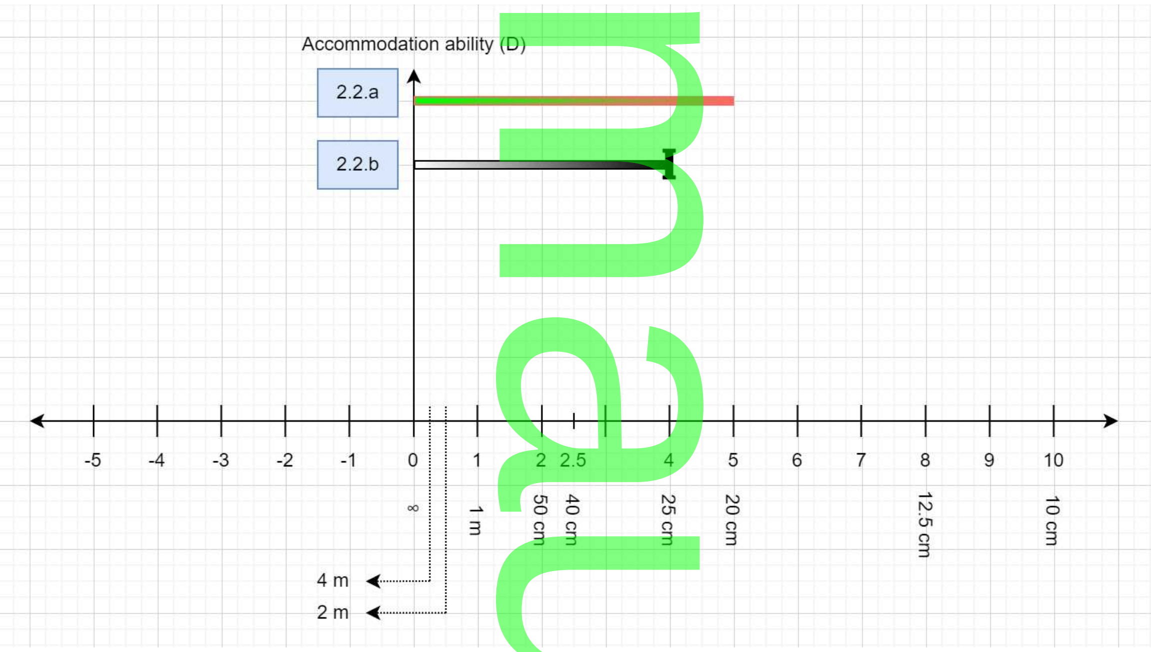
From the description of constrained observation ranges, the need for a tool to visualize various systems and any subsequent modifications to them was felt. We have devised Relative Dioptre Scale for this purpose which also serves as an intuitive way to visualize change in observation ranges due to induced defocus or changes to the screen distance.

The RDS is just Cartesian co-ordinate system modified for power (inverse instead of linear distances) onto which we will represent observation ranges of optical systems in this article. The x-axis of the RDS represents distances in (D or m^{-1}) with the origin representing infinity (∞). The y-axis was chosen to represent the accommodation ability of the system (also in Dioptre). Thus systems with better accommodation ability are vertically ranked/placed higher-up on the Relative Dioptre Scale.

This idea of representing observation range of optical systems on an inverse length scale stems from the inverse nature of the ideal lens relation itself. This has the important simplification of making transformations on the RDS linear with respect to changes in Power.

The ideal lens relation for the simple lens system $\frac{1}{s}-\frac{1}{u}=\frac{1}{f}$ itself can then be written as Power of Object distance (in m^{-1}) = Lens Power (in D) – Power of Screen distance (in m^{-1}).

On the RDS, the left end of the system's observation range represents its far-point (the farthest a system can focus) while the right end represents its near-point. The observation ranges (d_{near} to d_{far}) of the systems described in section 2.2.a and 2.2.b can be represented on the RDS as shown in Figure 2.4.



The vertical rank/height represents accommodation ability of the systems which is also equal to the length of line segments. Accommodation ability of a system can be stated as the power difference between the two extremes (near and far-point) of the observation range.

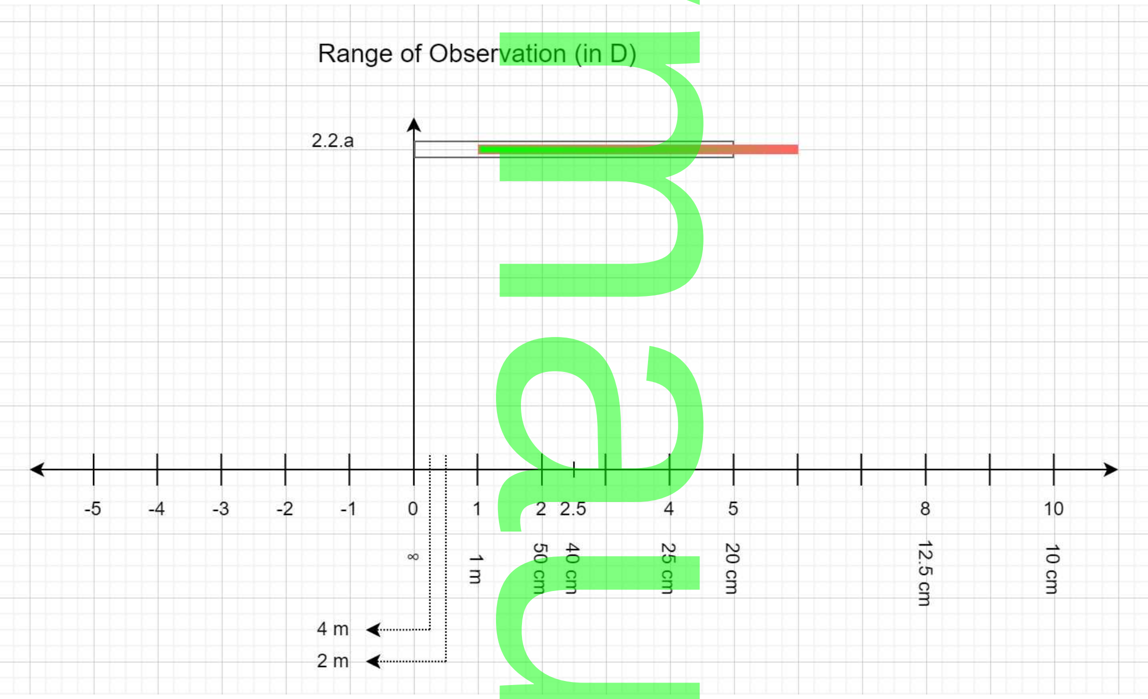
A line segment of some finite length on the RDS denotes a system with non-zero accommodation ability. A system with both – fixed focal length and fixed screen distance would be reduced to a point on the x-axis because of its fixed observation distance with no accommodation ability.

The gradient from red to green and light to dark in this instance was used to indicate limiting of the system's observation range due to physical constraints. The presence of an 'I' indicates the impossibility of any physical extension towards the right-hand direction (near-point) for the fixed focal length system described in section 2.2.b. It is very important to note that sign of object distance (d) on the RDS is inverted from the sign for object distance (u) used in ideal thin lens relation due to co-ordinate conventions.

4.1 Shift in observation range after external defocus

What happens to the system in 3.1.a when difference in lens power is kept same (+5 D) but increased by +1 D on both extremes (+5 D to +10 D)? Such a change can be termed as 'accommodation-shift' and the same can be easily achieved by introducing another +1 D lens ensuring that the power addition law holds (Section 8.2).

Just like before, it is sufficient to calculate the shifted far and near-points of the system (which are now 1 m and 16.66 cm). This modified system is represented on the RDS as shown in Figure 2.5. The former system in section 2.2.a is represented using slightly thicker grey colour for comparison.



The modified system's observation range with Lens Power = +5 D to +10 D gets shifted 1 unit towards the positive/right direction (the sign of introduced accommodation-shift).

It is obvious that an increment in both extremes of the lens power range results in right-shifting of the system by the same amount on RDS. Our modifications to system in 2.2.a has resulted in the system becoming Myopic (unable to form images of objects beyond 1 m).

The inverse also holds. It is trivial to point out that introducing a -1 D accommodation shift in our now modified system restores the original accommodation range. This is equivalent to introducing a -1 D lens or removing the previously introduced +1 D lens. Which means that the system on RDS must also move one unit towards the left direction. This is what the word ‘Relative’ in the RDS stands for. Relative to our modified system, the system in 2.2.a can be said to be hyperopic (unable to image objects closer than 20 cm).

It should be now trivial to understand why *introducing converging (positive) lenses/removing diverging (negative) lenses* to a system is termed as ***Myopic defocus*** – because it makes the system myopic (causes shift towards right in the observation range).
Similarly, *removing converging (positive) lenses/introducing diverging (negative) lenses* to a system is called ***Hyperopic defocus*** – because it results in the system becoming hyperopic (causes shift towards left in the observation range).

4.2 Shift in observation range due to change in screen distance

We’ve already described shifting of a system’s observation range under induced defocus. Calculating shift in Observation range due to screen distance changes is also as easy as recalculating the far and near-points.

For instance, the system in 2.2.a has the screen positioned at 25 cm from the lens – If we now position the screen 5 cm closer, the near-point recedes farther from 20 cm to 25 cm (1 D towards left) signifying that the system has turned hyperopic.

Similarly, if we move the screen 25cm farther from its initial position – the far-point comes closer from infinity to 50 cm (2 D towards right) rendering the system Myopic. The near-point also shifts closer (by the same power) from 25 cm to 16.66 cm.

The shift in observation range due to screen distance increments act opposite to that of increments in focal lengths. Thus, increase in screen distance results in Myopic shift in observation ranges and decrease in screen distance results in Hyperopic shift respectively.

5. Experimental verification: shift in observation range due to hyperopic defocus

It is easy to experimentally observe the physical shifting of observation range upon introduction of external defocus as described in section 2.3.b using a camera and lens setup.

Experimental verification of the findings of section 2.3.c can also be achieved in a similar manner with the help of an optical bench setup. This phenomena has probably been observed and mentioned multiple times already but we still feel its inclusion is important because of the relative unfamiliarity of the concept.

Regarding shifting of observation ranges of a system, **reducing** (increasing) focal length is analogous to **increasing** (decreasing) screen distance. This is intuitive from the way we have defined dimensionless image distance $j = \frac{s}{f}$ where decreasing focal length (f) or increasing screen distance (s) both serve to increase the dimensionless image distance (j) resulting in a decrease in dimensionless object distance (k) signifying Myopic shift and vice-versa.

5.1 The Camera Setup

The camera (referring to the combination of camera body paired with a lens) used was a Fujifilm X-S10[1] camera body paired with FUJINON XC35mmF2[2] prime lens.

The light rays from an object first encounter the optical elements inside the camera lens and then go on to meet the image sensor just like the simple lens system described in section 2.2. This can be easily verified by simply positioning a converging lens in front of a bare camera sensor and checking for image formation in the viewfinder ensuring that ‘shoot without lens’ mode is enabled.

In the Figure 3.1 provided below, similarities between a Camera body as the image-sensor equivalent in the simple lens model is obvious.

The camera lens can also be assumed to behave like an ideal lens with a variable aperture as shown in Figure 3.2. The lens comprises multiple optical elements – 9 elements in 6 groups (incl. two aspheric elements) to be exact. But for our experimental purposes, it still behaves close to an ideal converging lens evident from the inverted images it forms. The extra optical *elements* are needed to adjust focus and minimize aberrations in the image.

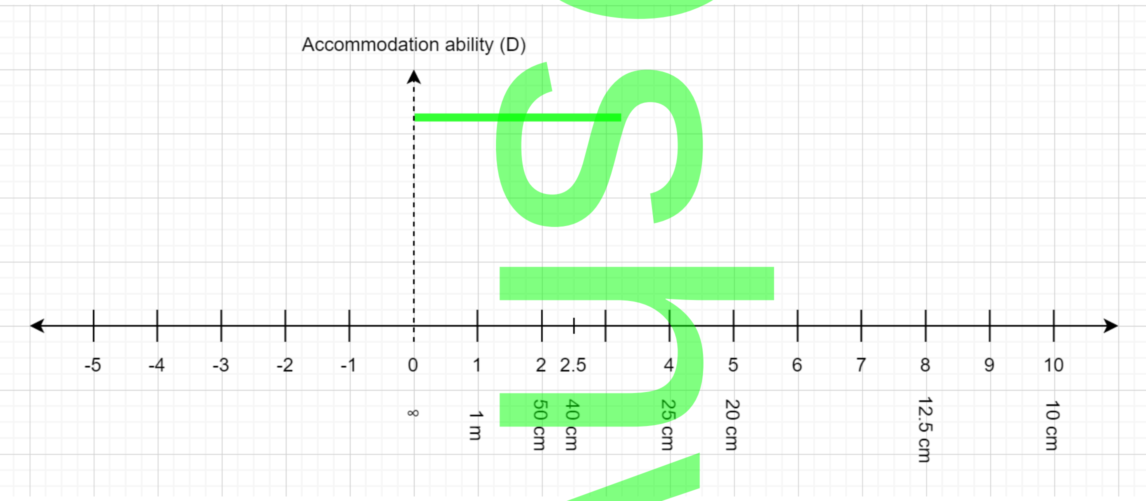
A dedicated Mirror-less camera offers extra features such as manual mode and advance image information like distance indication for focusing which are hard to find in a modern camera phone. The larger aperture size of the lens of a dedicated camera also results in much shallower Depth-of-Field (DOF) useful towards checking focus.

The Camera (referring to the combination of camera body paired with a lens) can image close objects roughly 31.5 cm from the lens (if we subtract 35 mm focal length from the specified close focusing distance

1 https://fujifilm-x.com/en-in/products/cameras/x-s10/
2 https://fujifilm-x.com/en-in/products/lenses/xc35mmf2/specifications/

of 35 cm from the image-sensor plane, all the way up to ∞ yielding an accommodation ability of roughly 3.25 D factoring in some inbuilt headroom for focusing at infinity.

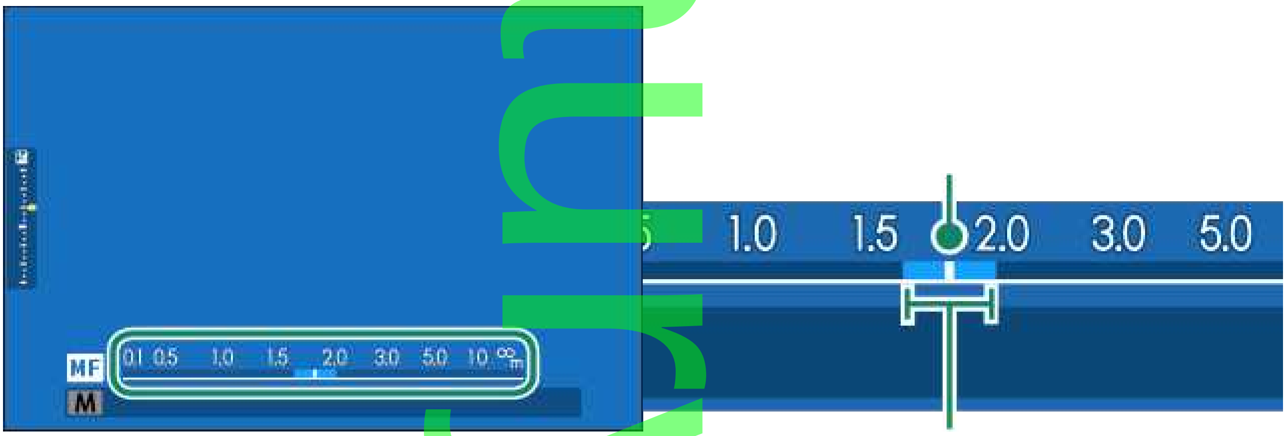
This is represented in the Figure 3.3 below.



5.2 Description of the experimental setup

For the experiment, the camera was positioned outside to image a tree at optical infinity across a field.

The X-S10 camera body used for this experiment has a handy distance scale option which shows the focusing distance reported by the lens estimated from the position of optical elements. The position of distance scale[3] in the Viewfinder with the white focus distance and blue DOF indicator is shown in Figure 3.4 and 3.5 [[4]]. Image formation parameters related to image-sensors like exposure or sensitivity is beyond the scope of this article.



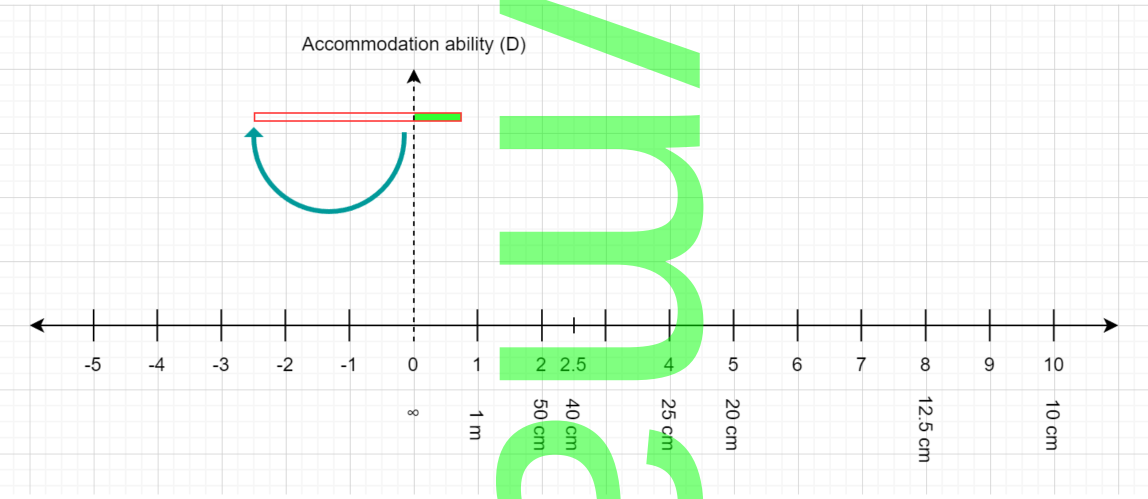
3 https://fujifilm-dsc.com/en/manual/x-s10/taking_photo/manual-focus/
4 https://fujifilm-dsc.com/en/manual/x-s10/taking_photo/manual-focus/

As can be verified clearly from the white mark near 10m∞ on the Distance scale in the viewfinder, the optical elements inside the camera lens are focused for our subject tree located at optical infinity in the absence of any defocus.

5.3 Shift in observation range after inducing hyperopic defocus

What happens if we place a diverging (minus) lens (induce hyperopic defocus) very close to the front element of the camera lens?

In section 2.3.b, we have already shown how the observation range would get shifted towards the left depending on the defocus. For instance, introducing a -2.5 D defocus would shift the observation range of the camera 2.5 units towards the left as shown in figure 3.8.



This can be understood as the introduced lens ‘mapping’ object distances to their new apparent distances or alternately as the lens apparently ‘bringing’ objects from infinity at its focus. The hollow red part of the observation range to the left of origin indicates unusable part of the observation range because it is physically impossible to have an object beyond infinity. The remaining useful part of observation range lying closer than infinity is represented with a solid green line. Experimental demonstration and verification for the same using the same defocus is provided in Figure 3.9.

The white focus distance indicator is close to 0.5 m mark on the distance scale clearly indicating the focus distance. Coincidentally, this is also the focal length (0.4 m) of the introduced defocus lens. Thus, focusing at an object located at infinity with the camera using hyperopic defocus of -2.5 D results in reported focusing distance of 40 cm. The test subject situated at optical infinity is still in acceptable focus like before and can be verified from the blur that results from removing the introduced lens as shown in Figure 3.10.

Due to the nature of the introduced -2.5 D defocus, we can say that the Camera is *apparently* focused at infinity even when the lens optics are *actually* reporting focus around the 40 cm mark.

A simple video verification using a camera (Fujifilm X-S10 with FUJINON XC35mmF2) and -2.5 D defocus is supplied with this article.

In a way, we can say that the introduced -2.5 D defocus ‘eats’ away the usable portion of the effective observation range of the lens leaving only the remaining part of the observation range available for image formation. In the absence of defocus, the camera focusing at distant tree reports its focus distance at optical infinity. After introduction of defocus, the camera reports its focus distance as equal to the focal length of the diverging lens used even when showing focus at infinity like before. This also translates to the inability of the camera to lock focus at objects closer than ~1.33 m with the -2.5 D defocus.

We have also repeated the experiment with a -3.0 D defocus so that the similar result can be compared as shown in Figure 3.11 below.

The camera still has our subject situated at optical infinity in focus like before. This time, the focus distance (~33 cm) approaches the close-focusing limit of the Camera lens at ~35 cm.

As one might've already observed, placing a lens in front of camera introduces some optical aberrations and some slight crop to the image. But for our limited purposes, this has not affected our ability to verify the shift in observation range under introduced defocus.

External defocus due to an introduced lens can't affect the accommodation ability because the accommodation ability of a camera is a physical constraint imposed by how its optical elements are configured to move inside the lens.

5.4 Focusing on objects closer than ∞ under hyperopic defocus

The consequence of introducing a diverging lens in front of a camera for formation of image of objects at infinity has been previously demonstrated in section 3.3.

In a way, we can say that the introduced -2.5 D defocus 'eats' away the usable portion of the effective observation range of the lens leaving only the remaining part of the observation range available for image formation. This translates as the inability of the camera to achieve *apparent* focus at objects closer than ~1.33 m with the -2.5 D defocus shown earlier by the green part of observation range shown in Figure 3.8. This close-focusing distance increases even further with the -3.0 D defocus rendering the Camera severely hyperopic in both of the cases.

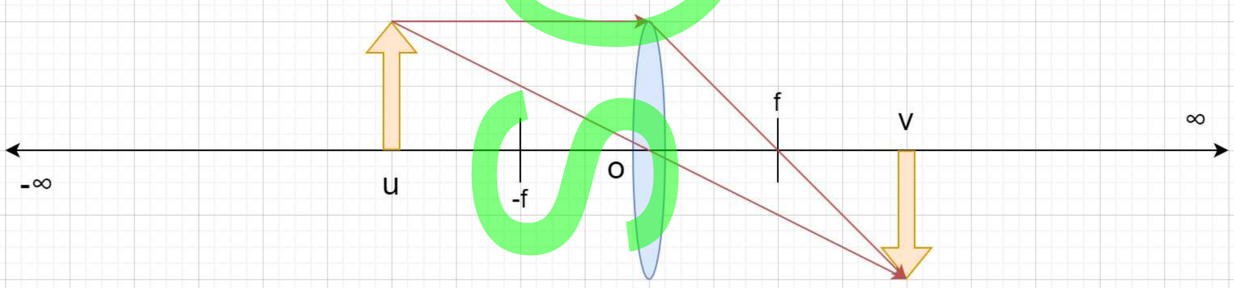
Because we know the accommodation ability of a camera lens is a physical constraint imposed by how its optical elements are allowed to move inside, external defocus due to an introduced lens can't affect the accommodation ability.

6. Appendix

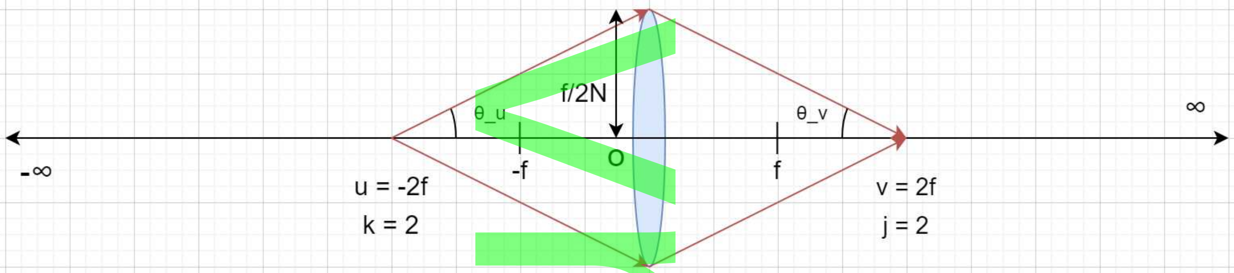
6.1 Depth-of-field (DOF) Derivations using dimensionless relation

DOF diagrams are provided to scale for ease of visualization.

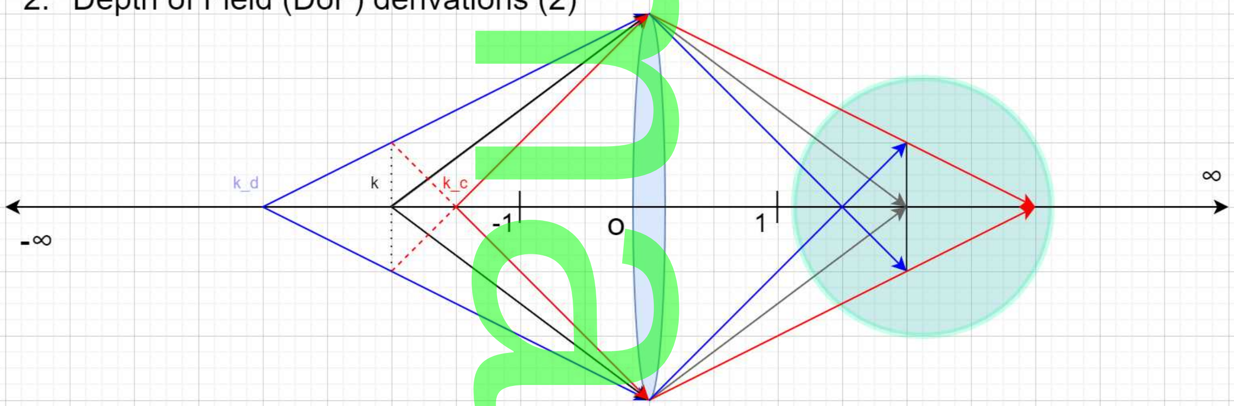
0. Image formation by a convex lens



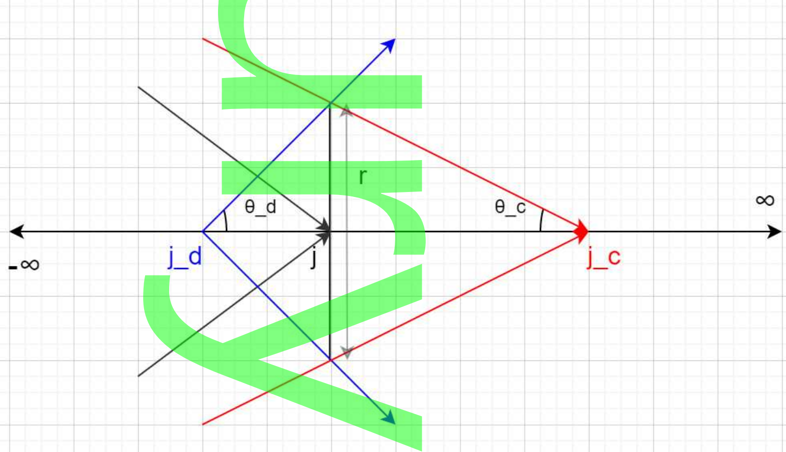
1. Depth of Field (DoF) derivations (1)



2. Depth of Field (DoF) derivations (2)



3. Depth of Field (DoF) derivations (3)



$$An\ object\ must\ be\ situated\ at\ (-\infty, -f)$$
$$for\ it\ to\ form\ an\ image\ at\ (f,\ \infty)\ on\ the\ other\ side\ of\ the\ convex\ lens.$$

$$In\ common\ usage,\ aperture\ size\ (A)\ is\ usually\ denoted\ as\ F-number\ or\ \frac{f}{N}.$$

$$A = \frac{f}{N},\ where\ N\ is\ some\ dimensionless\ number.$$

$$For\ instance,\ F2\ or\ \frac{f}{2}\ implies\ N = 2\ in\ both\ the\ cases$$
$$and\ it\ denotes\ that\ the\ aperture\ size\,/\,opening\ is\ half\ of\ the\ focal\ length.$$

$$Following\ this\ convention,$$
$$\frac{f}{2N}\ denotes\ the\ half-width\ of\ aperture\ size\,/\,opening$$
$$or\ radius\ if\ the\ aperture\ is\ circular$$

$$we\ have\ accordingly$$

$$\theta_u = \tan^{-1}\left(\frac{1}{2Nk}\right)$$
$$\theta_v = \tan^{-1}\left(\frac{1}{2Nj}\right)$$

$$To\ derive\ D.O.F.\ we\ need\ to\ solve\ the\ inverse\ of\ this\ problem:$$

$$for\ a\ fixed\ value\ of\ j\ i.e.,\ imaging\ distance,$$
$$find\ the\ range\ of\ k\ for\ a\ particular\ value\ of\ N,$$
$$for\ which\ Circle\ of\ Confusion\ (C.o.C.)\ is\ less\ than\ a\ threshold\ value.$$

$$We\ can\ make\ C.O.C.\ dimensionless\ by\ dividing\ it\ by\ f\ to\ get\ another\ quantity\ r$$

$$r = \frac{C.O.C.}{f}$$

$$thus\ obtained\ both\ extreme\ values\ of\ u\ and\ v\ obey$$

$$u_d > u > u_c :: v_d < v < v_c$$

$$where\ the\ subscript\ denotes\ (c)loser\ and\ (d)istant\ object\,/\,image\ respectively$$

$$This\ can\ be\ understood\ as\ a\ consequence\ of\ the\ hyperbolic\ relation\ between$$
$$object\ and\ its\ image.$$

$$These\ same\ relations\ must\ apply\ to\ k\ and\ j\ also$$

$$k_d > k > k_c :: j_d < j < j_c$$

From consideration of the quadrilateral formed at image junction, we can see that

$$(j - j_d)\tan\theta_d = (j_c - j)\tan\theta c$$
$$\Longrightarrow \frac{j - j_d}{2Nj_d} = \frac{j_c - j}{2Nj_c} = \frac{r}{2}$$

Three quantities equal to each other means that we will get 3C2 = *three equations.*

equating the first two parts, we get :

$$\frac{j - j_d}{j_d} = \frac{j_c - j}{j_c}$$
$$\Longrightarrow j_c \times j - j_c \times j_d = j_c \times jd - j \times j_d$$

rearrranging we get,

$$j = \frac{2j_c \times j_d}{j_c + j_d}$$

This equation can be checked by putting j_c *and* j_d *equal to* j.

Equating the first and second terms separately with r, we get :

$$\frac{j - j_d}{2Nj_d} = \frac{r}{2}$$
$$\frac{j_c - j}{2Nj_c} = \frac{r}{2}$$

which gives,

$$j_c = \frac{j}{1 - Nr} \;\&\; j_d = \frac{j}{1 + Nr}$$

we can now invert these according to the relation we got earlier

$$\text{which is } k = \frac{j}{j-1}, \text{ giving}$$

$$k_c = \frac{j_c}{j_c - 1} = \frac{\frac{j}{1-Nr}}{\frac{j}{1-Nr} - 1} = \frac{j}{j - 1 + Nr}$$

$$\text{and } k_d = \frac{j_d}{j_d - 1} \implies \frac{\frac{j}{1+Nr}}{\frac{j}{1+Nr} - 1} = \frac{j}{j - 1 - Nr}$$

$$k_d - k_c = j \times \left(\frac{1}{j - 1 - Nr} - \frac{1}{j - 1 + Nr}\right)$$

$$\text{here, } k_d - k_f \text{ denotes } \frac{D.o.F.}{f}$$

$$\implies D.o.F. = f \times j \times \left(\frac{j - 1 + Nr - (j - 1 - Nr)}{(j - 1 - Nr) \times (j - 1 + Nr)}\right)$$

$$\implies D.o.F. = \frac{2f \times j \times Nr}{(j - 1 - Nr) \times (j - 1 + Nr)}$$

$$= \frac{2f \times j \times Nr}{(j-1)^2 - N^2r^2}$$

How to use this formula for calculations :
firstly calculate j from k.

then putting the respective values of f, N, j & r
in the above formula will give you the D.o.F. in the unit of focal length.

Two DOF scenarios of interest are described below:

$$\textit{Nearest distance in acceptable focus when focus is at } \infty \ (j = 1,\ k \to \infty):$$

$$j_c = \frac{1}{1 - Nr}$$

$$\textit{which gives } k_c = \frac{1}{Nr}$$

$$\textit{the nearest distance of focus while still having objects at } \infty$$
$$\textit{in 'acceptable' focus } (j_d = 1):$$

$$\textit{giving } j = 1 + N \times r$$

$$k = \frac{j}{j-1} = \frac{1 + Nr}{Nr}$$

---FINISHED DOF DERIVATION PART---

$$\textit{we already know that } j = \frac{2j_c \times j_d}{j_c + j_d}$$

$$\textit{substituting } j_x = \frac{k_x}{k_x - 1} \textit{ we get,}$$

$$j = 2 \times \frac{\frac{k_c}{k_c-1} \times \frac{k_d}{k_d-1}}{\frac{k_c}{k_c-1} + \frac{k_d}{k_d-1}} = 2 \times \frac{\frac{k_c}{k_c-1} \times \frac{k_d}{k_d-1}}{\frac{k_c}{k_c-1} + \frac{k_d}{k_d-1}}$$

$$\textit{giving } j = \frac{2k_c \times k_d}{2 \times k_c \times k_d - (k_c + k_d)}$$

$$\textit{finding the same relationship for } k \textit{ results in}$$

$$k = \frac{j}{j-1} = \frac{\frac{2k_c \times k_d}{2 \times k_c \times k_d - (k_c + k_d)}}{\frac{2k_c \times k_d}{2 \times k_c \times k_d - (k_c + k_d)} - 1}$$
$$= \frac{2k_c \times k_d}{k_c + k_d}$$

This result is due to the interchangable (mirror) nature of the relation.

6.2 Experimental verification for depth-of-field calculations