

# Ray Optics: Observation Range Of Optical Systems & The Dimensionless Approach for DOF

8 May 2023, 14:53 Draft Revision 0.45

Shivam Maurya\*, ORCID 0009-0005-6705-5967

Indian Institute of Technology, Kharagpur

## 1. Introduction

This article lays out the fundamental principles of ray optics and serves as an introduction for readers wanting to characterize refractive changes in observation range of optical systems (including eyes) from external defocus or imaging-screen changes (also termed axial changes). A novel derivation of depth-of-field relations has also been provided for the enthusiastic reader.

Many eye-specialists/eye-care professionals lack intuitive understanding from optics perspective about what actually happens during axial elongation causing an eye to become Myopic. This frequently results in improper characterization of refractive condition and consequently how prescriptions ~~correct~~ compensate for refractive eye conditions. The material in this article is of enormous importance in that regard.

Math worksheets for  $\mu$ Math+ Android application is supplied along with this text containing a workable version of the formulas in this article.

## 2. Ray Optics: Deriving the ideal thin-lens relation

This section focuses on deriving the relationship between object and image distances for the case of (real) image formation only by an ideal converging (also termed convex) thin-lens giving the ideal thin-lens relation. It is trivial to derive the same and the relation remains identical for a concave lens as well but will not be attempted here. The 2D illustrations and ray diagrams of this article extend and apply equally well to the 3-Dimensional real world.

An ideal lens itself can be completely defined with only **two** physical parameters/attributes along with its principal axis – **focal length** and **aperture**. The term focal length or its inverse as lens power always implies the same physical property. These two parameters are sufficient to explain the majority of image formation by actual lenses and their observed behaviour in experiments.

In this sense, an ideal lens is essentially a theoretical object and we needn't immediately concern ourselves with the implementation details of how it achieves 'image formation' and 'focusing' of rays in reality. It also means that these rays in question can be anything – light, radio waves, sound waves etc. reinforcing the purely theoretical nature of the relation. Ideal lenses are interesting objects with properties of their own from a purely theoretical perspective.

32 A lens with curvature only along one axis is known as a Cylindrical (SPH) lens while a lens with identical  
curvature along both independent axes is denoted as Spherical (SPH) lens<sup>1</sup>. It must be noted that words  
34 *cylindrical* and *spherical* were meant with regard to appearance of the lens and have little to do with the  
actual refracting profile (cross-section) of the lens. The axis of a cylinder lies along the direction of constant  
36 curvature with the idea being rotation about its axis should be indistinguishable (due to symmetry). This is  
important from the point of unique determination of cylindrical axis as shown in Figure 1 below.

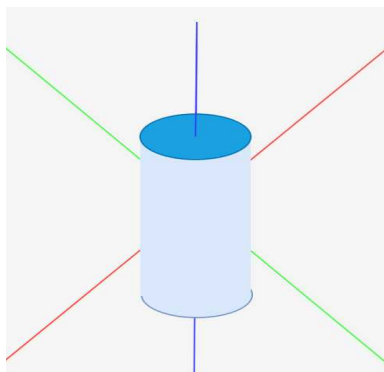


Figure 1: The axis (blue line) of a cylinder is normal to its flat face

## 38 2.1 Definition of an Object and its Image in Ray Optics

An object is considered to be a *collection of object points*. From every object point, rays can be assumed to  
40 **originate/diverge** in all directions. An image is similarly considered to be a *collection of image points*. At  
each image point, rays originating from an object point **terminate/converge**.

42 For an image point, demonstrating convergent intersection of any two rays emerging from an object point is  
sufficient. A point on an object and its image will always have a one-to-one correspondence. It is very  
44 important to note that beyond the image point, away from its object – the rays that converged to form the  
image point start diverging again. An object can be differentiated from its image in the sense of independent  
46 existence. An object can exist independently but an image can not exist without an object. It is immensely  
useful to express Aperture size, object, and image distances in terms of focal length. This dimensionless  
48 representation comes at no loss of generality as can be seen from the later sections.

---

1 <https://www.clzoptics.com/news/cylindrical-vs-spherical-lenses.html>

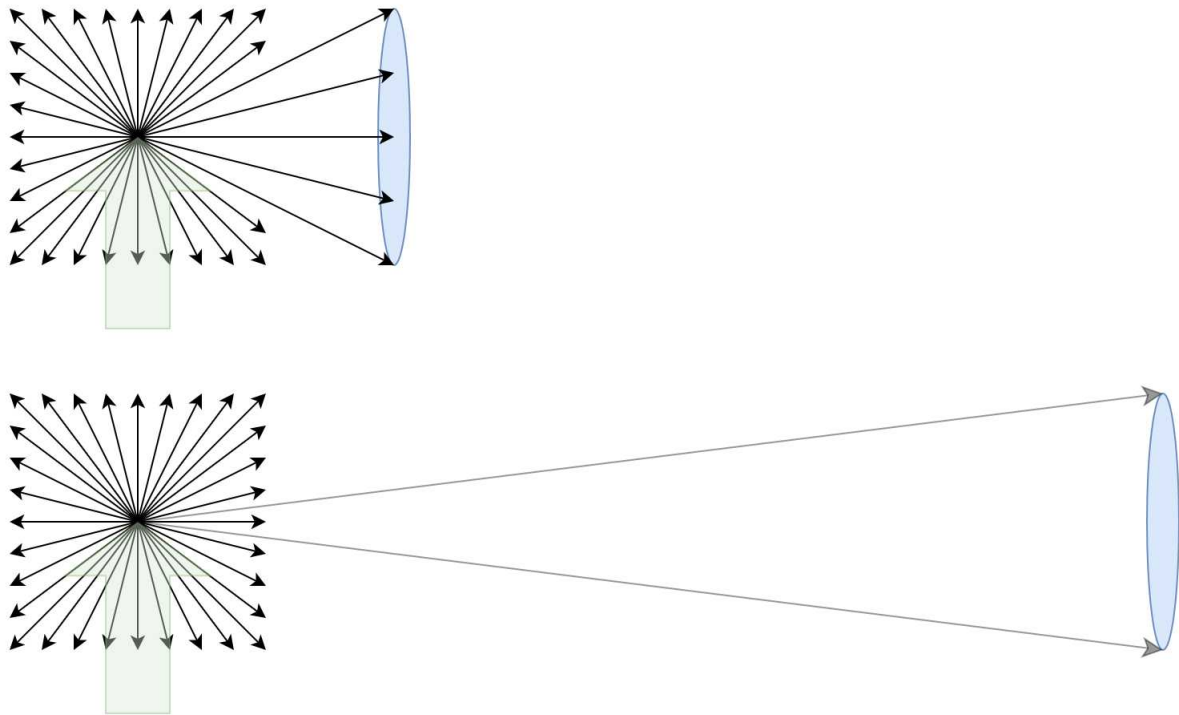


Figure 2: Ray diagram of rays from an object point hitting the lens.

An object point (on an object) emits rays in all directions (Figure 2). At distances become larger, the angle between rays emitted by a point object starts decreasing with the rays themselves appearing almost parallel at very large distances from the lens. However, even at infinity, the rays emitted by an object can only appear diverging or parallel and never converge without help from a lens.

An application of our stricter definition is to consider the commonly mentioned ‘virtual images’<sup>2</sup> in many textbooks as having properties more in line with objects rather than images because rays seem to originate (diverge) from them towards the lens. That a ‘virtual’ image lies on the same side of the lens as a ‘real’ object favours consistency of our argument. For analysis of systems consisting of multiple optical elements, the ‘virtual image’ formed by an object and lens combined can be replaced solely by the virtual object without affecting the rest of the system. Subsequent imaging of such a virtual object by an optical system (whether our eyes or a camera) then results in an actual real image. Henceforth, the word image in this article should always be taken as referring to a real image.

## 2.2 Calculating deviation for a ray incident on an ideal lens

**Assumption A:** The focal length determines the convergence/divergence point (focus) of every ray parallel to the principle axis for a converging/diverging lens respectively shown in Part 1 of Figure 3. This is analogous to a parabolic concave mirror reflecting all incoming rays parallel to its principal axis on its focus. The Aperture size only determines the admission extent for such parallel rays on the lens.

**Assumption B:** Only rays passing parallel to the principle axis obey the previous assumption. All rays incident normal to the principal axis pass unchanged through the lens profile. The word ‘thin’ can be used to

<sup>2</sup> [https://en.wikipedia.org/wiki/Virtual\\_image](https://en.wikipedia.org/wiki/Virtual_image)

68 denote this aforementioned property of not affecting rays normal to the principle axis of the lens. There is the  
 implicit assumption that a ray is to be first decomposed into its normal and parallel constituents. Ray  
 70 diagrams given below demonstrate application of the above two assumptions to show deviations for various  
 incident rays by decomposing them into normal and parallel components with respect to the principal axis.

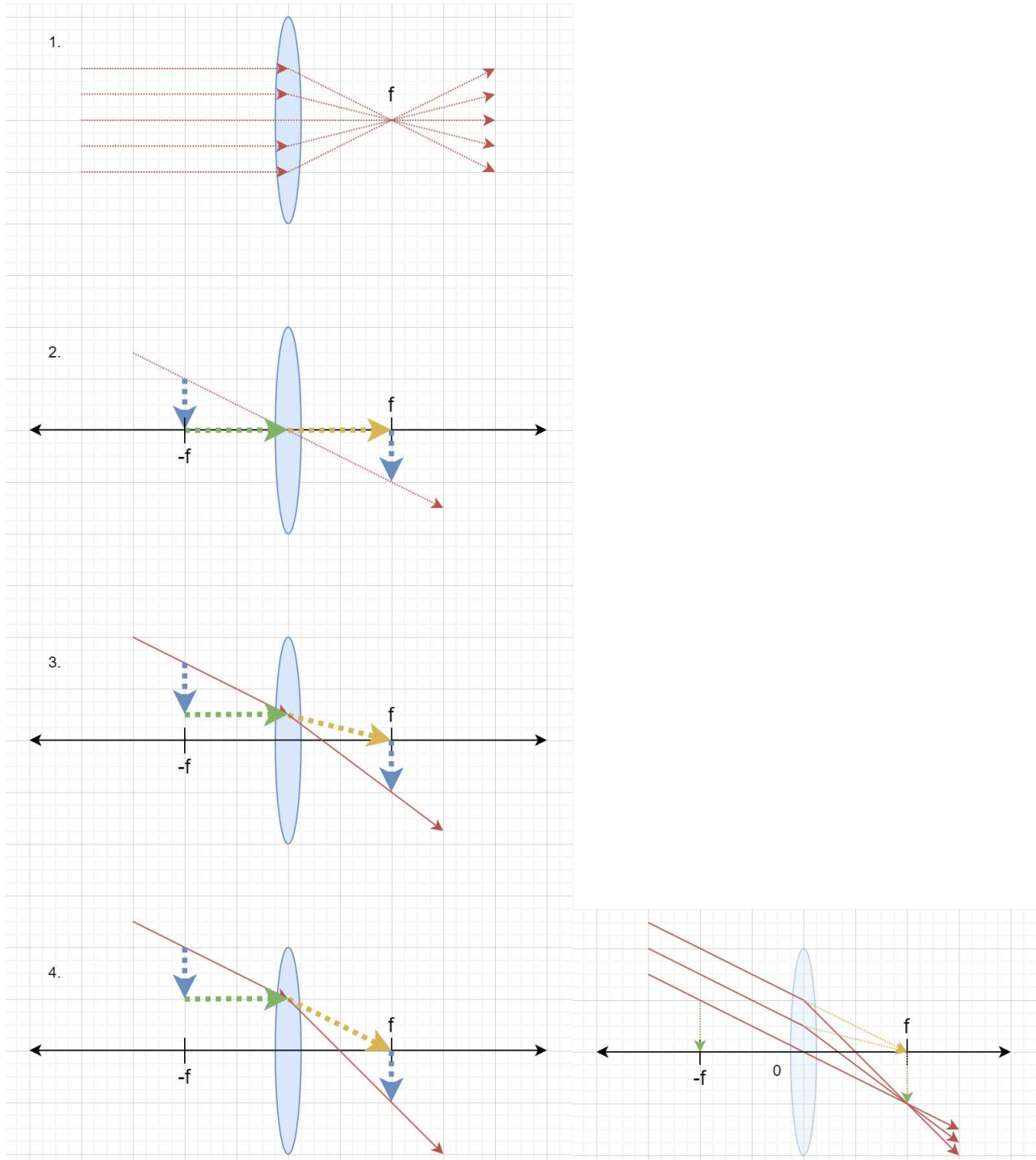


Figure 3: Application of Assumption A (Part 1) and B Figure 4: Combined version of 2, 3, and 4  
 (Part 2 to 4) in Figure 3

72 From Part 2 to 4 shown in Figure 3 and their combined version in Figure 4, we can observe that the length of  
 the vertical component is the same when calculated at the focus. The amount of deviation a ray undergoes  
 74 depends on both where it is hitting the lens Aperture and the angle it makes.

Consideration of these two assumptions results in simplified rules for all rays incident on an ideal converging (+) lens and is sufficient to explain image formation. A verification for an edge case corresponding to image formation at  $2f$  is provided below in Figure 5.

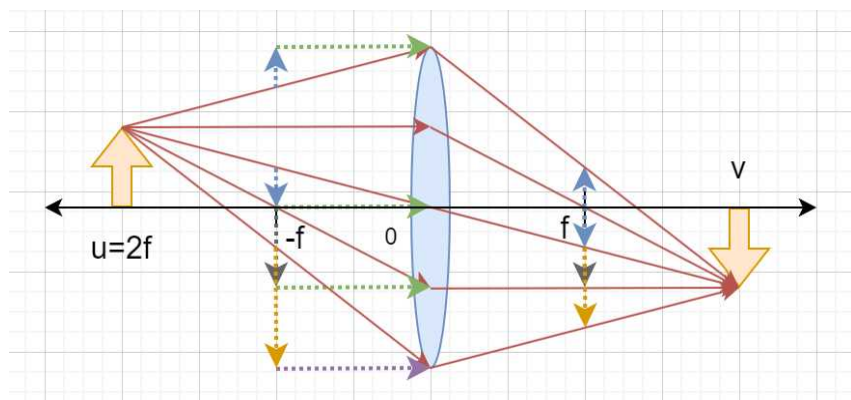


Figure 5: Edge case shown for image formation when  $u=2f$

In this particular edge case, we've shown convergence of five such rays at a single point when demonstrating that only two meet at the image point would have been enough.

The rule regarding rays passing unchanged through the optical centre of the lens (as given in many intermediate textbooks<sup>3</sup>) can be explained as a consequence of fundamental assumptions A and B stated above. It also determines the magnification ratio because the image point must lie on the unaffected ray passing through the optical centre of the lens.

## 2.3 The relation between object and image distance

For a converging lens with a given focal length, two distinct cases of rays intersection are possible according to the assumptions outlined in the previous section 2.2. Both the cases (Part 1 and 2) are shown in Figure 6 below with the same object placed at varying distances with the limiting case (Part 3) of an object placed exactly at the focus.

1. **Image** formation when the object is at a distance farther than the focal length from the lens. Rays from the object point appear to be converging towards the image point. It is important to note beforehand that the image formed in this case is **inverted**.
2. **Virtual object** when the object is at a distance closer than the focal length from the lens. Rays appear to be coming from the virtual object.

<sup>3</sup> Concepts of Physics - Part 1, For Intermediate Level, Publisher: Bharati Bhawan; Revised Reprint 2015 edition, ISBN-13: 978-8177091878

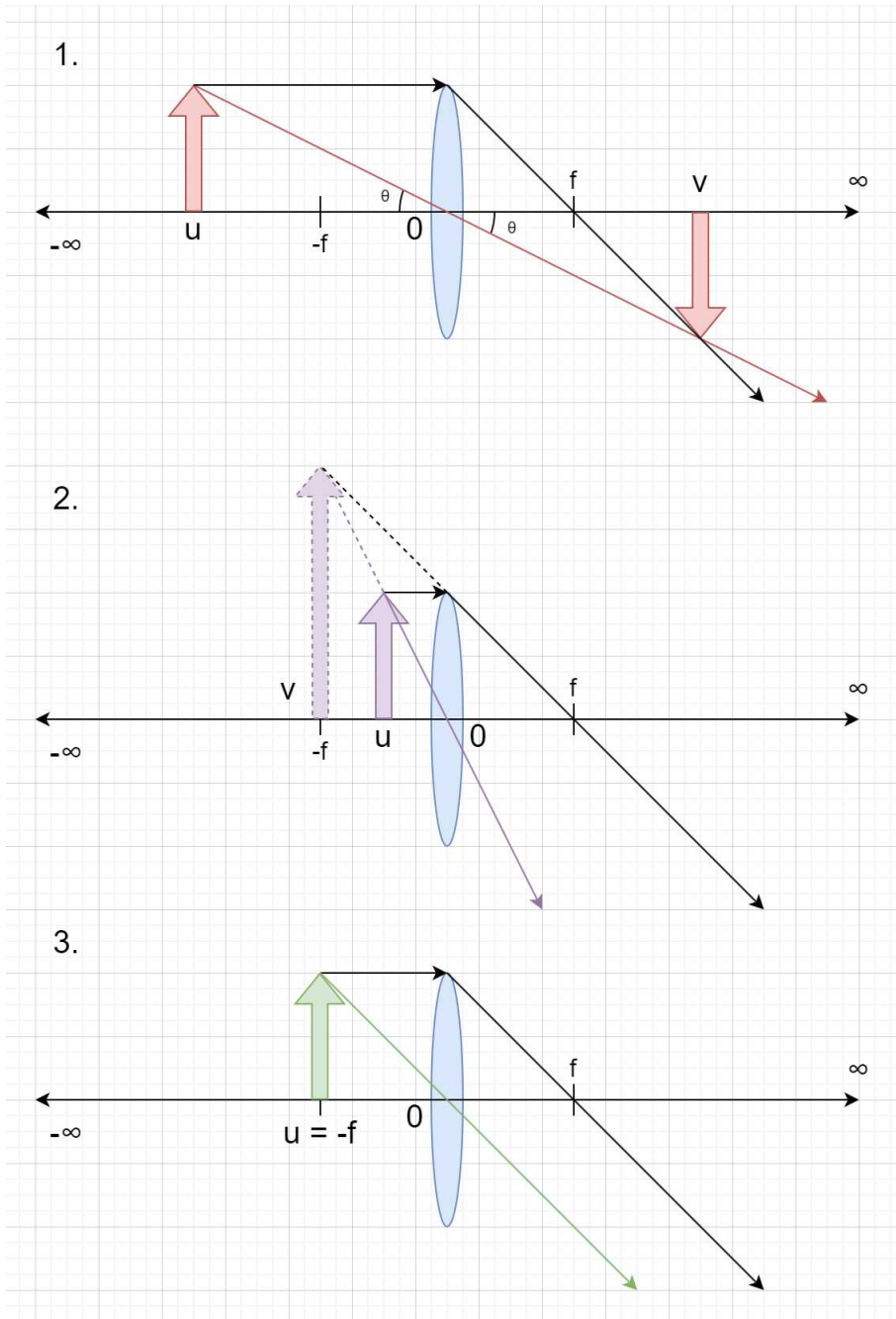


Figure 6: Formation of Image and Virtual Object by a converging lens

- 94 The ray passing through focus has the same slope  $-h/f$  (where  $h$  is the constant height of object point) while  
 96 slope of the ray passing through the optical centre depends on the object distance in all three cases. Whether  
 98 the slope of the latter is greater than or lesser than the former passing through focus determines where these  
 two rays meet and whether an image is formed or not. Precisely, ray from an object point placed farther from  
 the focus and passing through the optical centre will always have a less negative slope.

The derivation of ideal lens relation for a converging lens has been given below (Figure 7) employing  
 100 dimensionless representation.

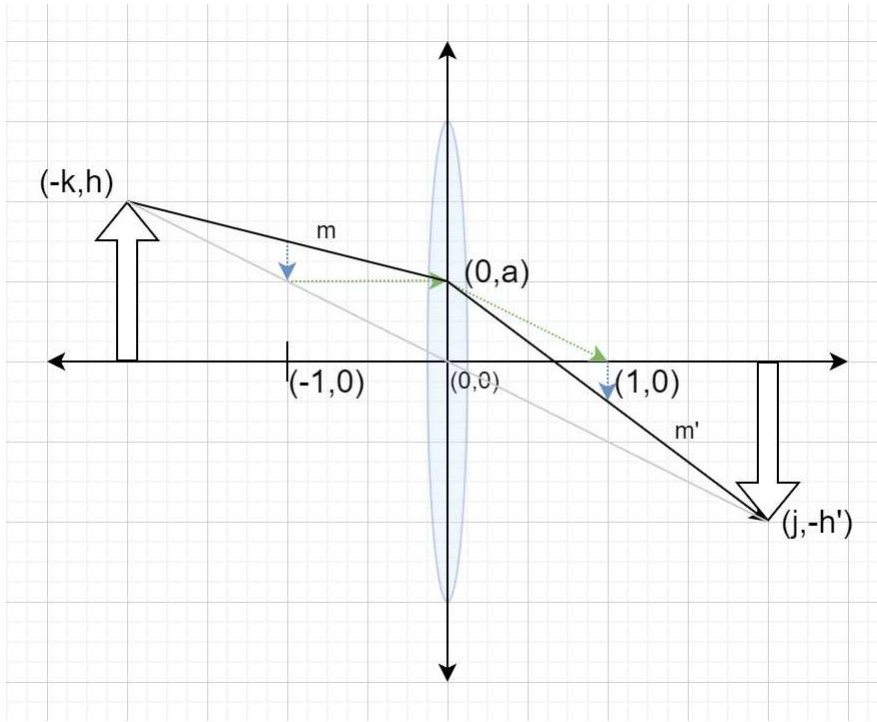


Figure 7: Ray Diagram for dimensionless derivation of relation between image and object distance

For image distance calculation, we assume the origin of the right-handed co-ordinate system centred on the lens. Care must be taken to ensure consistency of units and signs according to conventions while utilizing formulas throughout this article to ensure correctness in calculations. An ideal lens has two foci due to its symmetry. For a converging (+) lens, the focus will always lie on the opposite side of the lens from object. It depends on the signed power (converging or diverging) of lens that determines the sign of focal length and consequently its position relative to the object. For this article, we have only shown images formed by a converging lens.

magnification ( $m$ ) is defined as  $\frac{\text{image height}}{\text{object height}} = \frac{-h'}{h} = \frac{\text{image distance}(v)}{\text{object distance}(u)} = -\frac{j}{k}$

We have normalized object and image distances as follows :

$$u = -kf \text{ and } v = jf$$

for image formation,  $k > 1$  which means  $-u > f$  or  $u < -f$

slope  $m$  when the object point is  $(-k, h)$  is given by

$$m = \frac{a - h}{k}$$

for point  $(0, a)$  on the lens profile ( $y$  - axis)

image formation requires all rays to pass the point  $(j, -h')$  after refraction

The slope after refraction ( $m'$ ) is given by,

$$m' = \frac{a - (-h')}{-j} = \frac{a + h'}{-j}$$

The slope after refraction according to Assumption 2 must be

$$m' = m - a \implies \frac{a + h'}{-j} = \frac{a - h}{k} - a$$

Now, it's already known from the magnification criteria that

$$\begin{aligned} \frac{k}{j} &= \frac{h}{h'} \implies h' = \frac{hj}{k} \\ \implies \frac{a + \frac{hj}{k}}{-j} &= \frac{a - h}{k} - a \end{aligned}$$

$$\begin{aligned} \implies \frac{ka + hj}{k} &= \frac{hj - aj + kaj}{k} \\ \implies ka &= kaj - aj \\ \text{giving, } k + j &= kj \end{aligned}$$

which completes the derivation of  $k$  and  $j$  relation.

To derive the ideal thin lens relation,

we can substitute  $k = -\frac{u}{f}$  &  $j = \frac{v}{f}$  giving

$$-\frac{u}{f} + \frac{v}{f} = -\frac{vu}{f \times f}$$

$$\implies -u + v = -\frac{vu}{f}$$

$$\implies \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

## 108 2.4 Dimensionless object (k) & image (j) distance relation

The relation between dimensionless object and image distance comes out to be  $j = \frac{k}{k-1}$

110 For image formation by a converging lens,  $k > 1$ . This simply encodes the result that the object must always be located beyond the focus for image formation. A plot of this equation results in a hyperbola symmetrical



112 around the line  $x = y$  (Figure 8). The vertical and horizontal asymptotes given by lines  $x = 1$  &  $y = 1$  correspond to the object at focus and  $\infty$  while also corresponding to its image at  $\infty$  and focus respectively.

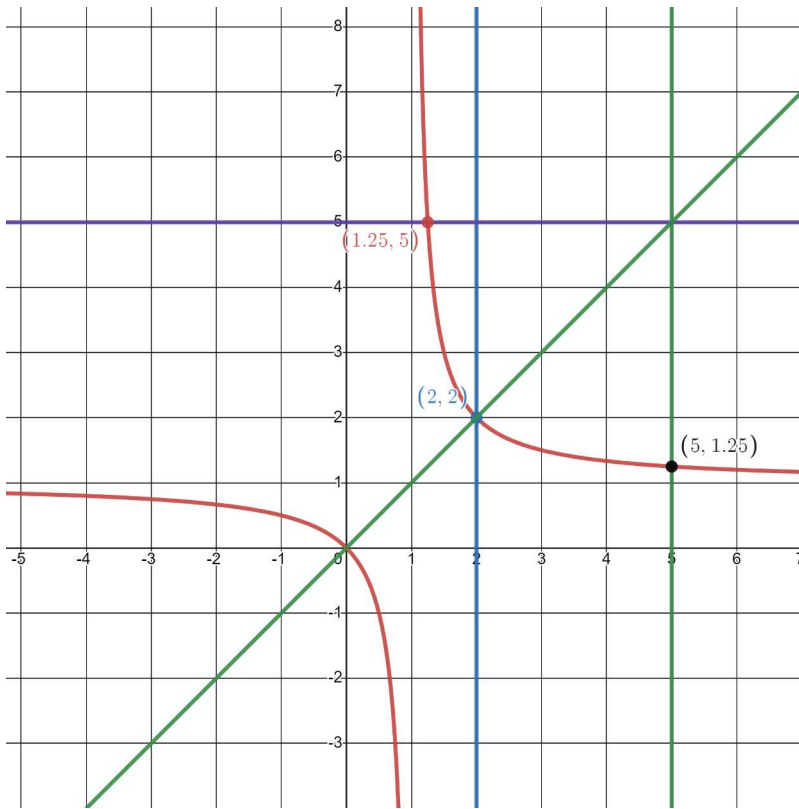


Figure 8: Red Hyperbolic plot representing the dimensionless lens relation

114 It is evident from the symmetry of the hyperbola in Figure 8 that both an object & its image's dimensionless distances form what can be termed as a mirror pair around the  $x = y$  line.

116 It becomes even more obvious if we solve for  $k$  giving  $k = \frac{j}{j-1}$ , which can be understood simply as interchanging  $k$  with  $j$  &  $j$  with  $k$  in the original relation.

118 Adding  $k$  and  $j$  together gives,  $k + j = k + \frac{k}{k-1} = \frac{k^2}{k-1} = k \times j$  which has a minimum of 4 at  $k = j = 2$  for positive values as can be seen from the graph below (Figure 9).

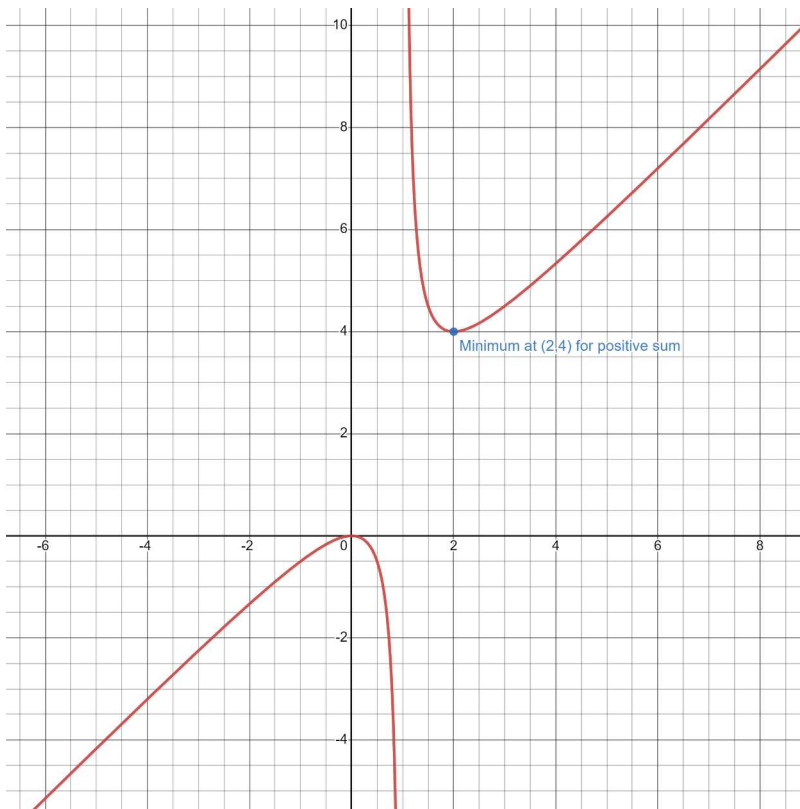


Figure 9: Plot showing variation in sum of dimensionless image and object distance with distance

120 This shows that for a given distance between an object and its required image, the focal length of the system  
 122 able to image it can not be greater than the quarter of the given distance. The  $(k + j) \geq 4$  rule along with the  $k > 1$  rule serve as fundamental constraints for real world implementation of optical systems.

## 2.5 The power addition rule

124 The power (inverse of focal length) addition rule can easily be derived from the ideal lens relation by  
 126 considering two lenses with different focal lengths placed together such that the distance between them can be neglected.

128 Here,  $v$  and  $u$  denote the image and object distances respectively with subscripts denoting which lens they are referring to. The power of a lens is the inverse of its focal length. The Diopetre ( $\text{m}^{-1}$ ) is one such derived SI unit of power. Assuming focal length  $f_1$  for Lens  $L_1$  and  $f_2$  for Lens  $L_2$ . Then the image distance for Lens

130  $L_1$  is given by, 
$$v_1 = \frac{u}{1 + \frac{u}{f_1}}$$

132 Because the distance between the two lenses is negligible, this resultant image now gets further refracted as an object by lens  $L_2$ . The corresponding final image formation distance is given by (where  $u_2 = v_1$ ).

$$v_2 = \frac{v_1}{1 + \frac{v_1}{f_2}} \Rightarrow v_2 = \frac{\frac{u}{1 + \frac{u}{f_1}}}{1 + \frac{\frac{u}{1 + \frac{u}{f_1}}}{f_2}} = \frac{u}{1 + \frac{u}{f_1} + \frac{u}{f_2}} = \frac{u}{1 + u(\frac{1}{f_1} + \frac{1}{f_2})}$$

This implies that the combined lenses together act as a lens of focal length  $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} = P_1 + P_2$  which is

134 basically the law of addition of lens powers. For a combination of more than two lenses closely put together, we can proceed by combining two lenses together at a time until only one remains.

### 136 3. The Ideal Lens System (ILS) Setup

Be it Human eye or an optical system like a camera, the purpose is mostly the same - ‘imaging’ objects  
 138 (which can be located at varying distances) onto the retina/image sensor. For a film camera, the film acts as the image sensor. For a digital camera, an image sensor chip replaces the film. The ray optics involved  
 140 behind image formation remain unchanged. The term ‘image’ should be taken to mean real image and all mentions of the word ‘lens’ refer to an ideal thin-lens.

142 The ideal lens system (ILS from now on) in our case is an optical bench setup consisting of an ideal converging lens and a planar image screen (sensor) together with an object as shown in Figure 10. The  
 144 screen is aligned to the principle axis of the lens. For the ILS, tracing the path of rays from an object to its eventual image is very simple. Rays emanating from an object point encounter the lens aperture and hit the  
 146 image sensor after refraction. The medium in this case can be said to be vacuum/air.

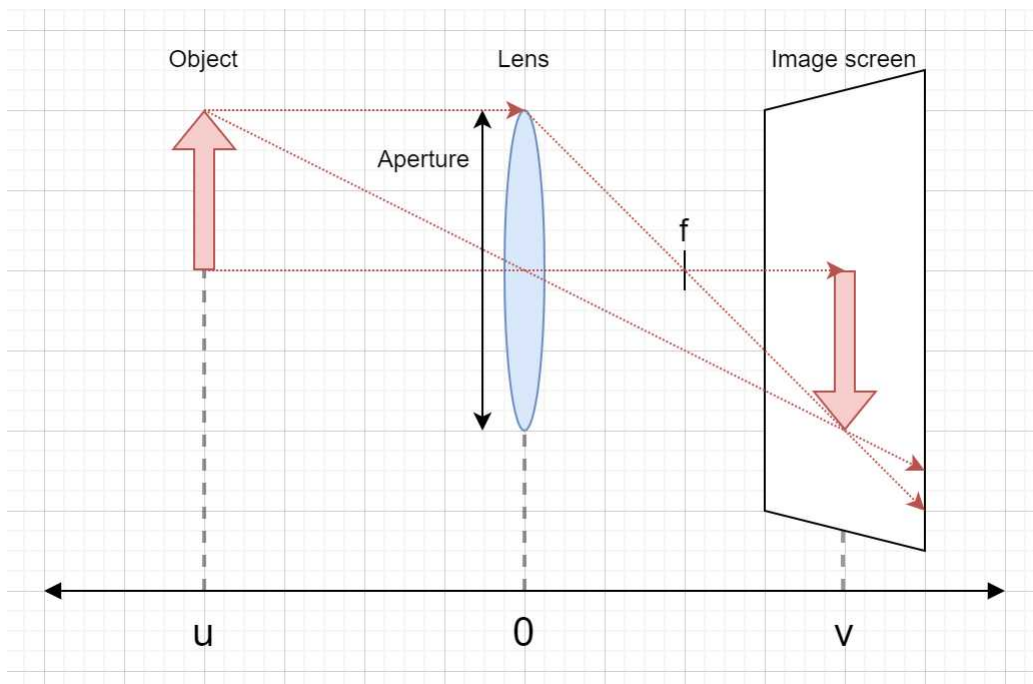


Figure 10: Optical bench diagram of the ILS

Four parameters are necessary to completely describe the setup of an object to be imaged and the ILS, if one also includes Aperture size (having no impact on image distance). The remaining three parameters are object distance (u) with the focal length (f) {together determining the image distance (v) – a dependent variable} and the screen distance (s). Distances are measured using Optical Centre as the origin. The **Field of View (FoV)** is defined as the angular-expanse of objects whose images can be formed on the image sensor/screen. It is easily quantified from the ratio of screen height to the screen distance.

### 3.1 The difference between image distance and screen distance

For image formation, image distance (v) must coincide with the screen distance (s). Image formation in this context refers to the formation of a well focused sharp image and will be the cases for this section and sub-sections.

Unless specified otherwise, it will be assumed that depth of field effects are insignificant. Calculation of distances within which screen distance (s) can vary for a given image distance (v) while still forming appreciably sharp images within a CoC (Circle of Confusion) is given in Section 6.

Thus, we can state that the dependence of screen distance (s) on focal length (f) and object distance (u) is already encoded by the ideal lens relation. Knowing any two, the third is easily calculated.

$$s = \frac{fu}{f+u} = \frac{u}{1+\frac{u}{f}} = \frac{1}{\frac{1}{u}+\frac{1}{f}}$$

### 3.2 The concept of constrained observation range

Fixing screen distance and focal length together also fixes the distance at which an object can be imaged. It is desirable for an optical system to be able to form images of objects located within a distance range instead of fixed distance. Imaging objects can be accomplished by varying either the focal length (f) or the screen distance (s) or some combination of both.

It must be kept in mind that all real-world implementations will always have some physical constraints on the ‘variability’ of independent variables (focal length and screen distance in our case) constraining the values they can take. The range of object distances that can be imaged by a system within constraints will be referred to as its **observation range**. The farthest and nearest distance extremes of the observation range are commonly designated as the far-point ( $d_{\text{far}}$ ) and near-point ( $d_{\text{near}}$ ) for a system respectively. The Far-Point is defined as the distance of **farthest** objects a system is able to image while the Near-Point is defined in a similar manner as the distance of **nearest** objects a system is able to image within constraints.

Our approach for studying changes to a system’s observation range results from fixing one of the two independent variables and studying how varying the other affects the range of object distances ( $d_{\text{near}}$  from  $d_{\text{far}}$ ) that can be imaged. The two cases for our system resulting from fixing focal length (f) and screen distance (s) at a time are discussed below.

### 178 3.3 Observation range of system when imaging screen distance (s) is fixed

Showing that for such a system,

180 the increase in power/decrease in focal length (f) for maintaining **fixed screen distance (s)** solely depends on the object distance (u)

182 can be easily done by rearranging the ideal lens law in terms of focal length.

By the ideal lens relation,  $\frac{1}{f} = \frac{1}{s} - \frac{1}{u}$ , s is constant

For an object at  $\infty$ ,  $u \rightarrow \infty \Rightarrow \frac{1}{f_{\infty}} = \frac{1}{s} - \frac{1}{u} = \frac{1}{s}$

For an object closer than  $\infty$ ,  $\frac{1}{f} = \frac{1}{f_{\infty}} - \frac{1}{u}$

$$\Rightarrow \frac{1}{f} - \frac{1}{f_{\infty}} = \Delta \text{Power} = -\frac{1}{u}$$

The signed term  $-\frac{1}{u}$  is positive because object distance is negative by co-ordinate convention.

For such a system, the increase in lens power (change in focal length) needed to image objects **closer** than a  
184 reference distance depends only on the object distance (u). This lens power increment for imaging closer  
objects is commonly referred to as accommodation of the system with the reference 'un-accommodated'  
186 distance taken at infinity. The accommodation required from far to near point is commonly referred to as a  
particular system's accommodation ability. A system is stated to be fully accommodated once lens power  
188 reaches constrained extreme corresponding to observing objects at near-point.

Fixing screen distance requires the focal length to vary in accordance with the observed object distances. For  
190 instance, a system with a fixed screen distance of 25 cm and lens power constrained in the range +4 D to +9  
D will have its Far and Near-points at  $\infty$  and 20 cm respectively.

192 With lens power at +9 D which amounts to +5 D accommodation from the initial +4 D needed for objects at  
 $\infty$ , the system images objects located at 20 cm. Because screen has been kept fixed for this system, the Field-  
194 of-View (FOV) remains unchanged.

### 3.4 Observation range of the system when the focal length (f) is fixed

196 If the focal length is kept fixed and the screen distance is allowed to vary instead, the dependence of  
observation range on screen distance is given in a similar manner from the ideal lens relation:

ideal lens relation,  $\frac{1}{s} - \frac{1}{u} = \frac{1}{f}$  giving  $\frac{1}{s} = \frac{1}{f} + \frac{1}{u}$

For an object at  $\infty$ ,  $u \rightarrow \infty$

giving  $\frac{1}{s_{\infty}} = \frac{1}{f}$

For an object closer than  $\infty$ ,  $\frac{1}{s} = \frac{1}{s_{\infty}} + \frac{1}{u}$

The signed term  $(\frac{1}{u})$  is negative here.

198 Assuming a system like the one described in section 3.3 above but with a fixed focal length ( $f = 25$  cm)  
instead, we can find out the near and far points of this system in a similar manner.

200 The  $k > 1$  requirement arrived in section 2.4 mandates that screen distances lesser than the (fixed) focal  
length of the system can't result in image formation for any possible object distance. Thus, the screen  
202 distance required in this instance must be always greater than 25 cm and can only be increased up to infinity.  
The far and near point of this system comes out to be  $\infty$  and 25 cm respectively.

204 It is evident that screen distances required for imaging closer objects quickly approach very large values and  
even then the system is unable to image objects closer than 25 cm because it is impossible to physically  
206 locate the screen beyond infinity. Contrast this to the variable focal length ( $f$ ) system described earlier where  
no such physical limit was in place for focal length preventing us from observing closer objects.

208 Changes to the screen distance implies changes to the overall size of the system and thus changes to FOV.

## 210 4. The Relative Dioptr Scale (RDS) and shifting of Observation ranges

From the description of constrained observation ranges, the need for a tool to visualize optical systems and  
212 any subsequent modifications to them whether from defocus or screen distancing was felt. The Relative  
Dioptr Scale was devised for this exact purpose serving as an intuitive way to visualize change in  
214 observation ranges due to induced defocus or changes to the screen distance.

The RDS is simple Cartesian co-ordinate system modified for power (inverse instead of linear distances)  
216 onto which observation ranges for optical systems are represented in this article. The x-axis of the RDS  
represents distances in (D or  $m^{-1}$ ) with the origin representing infinity ( $\infty$ ). The y-axis was chosen to  
218 represent the accommodation ability of the system (also in Dioptr). Thus systems with better  
accommodation ability are vertically ranked/placed higher-up on the Relative Dioptr Scale.

220 The idea of representing observation range of optical systems on an inverse length scale stems from the  
inverse nature of the ideal lens relation itself. This has the important implication of making transformations  
222 on the RDS linear with respect to changes in Power.

The ideal lens relation  $\frac{1}{s} - \frac{1}{u} = \frac{1}{f}$  itself can then be written as Power of Object distance (in  $m^{-1}$ ) = Lens  
224 Power (in D) – Power of Screen distance (in  $m^{-1}$ ).

On the RDS, the left end of the system's observation range represents its far-point while the right end its  
226 near-point. The observation ranges ( $d_{\text{near}}$  to  $d_{\text{far}}$ ) of the systems described in sections 3.3 and 3.4 can be  
represented on the RDS (Figure 11).

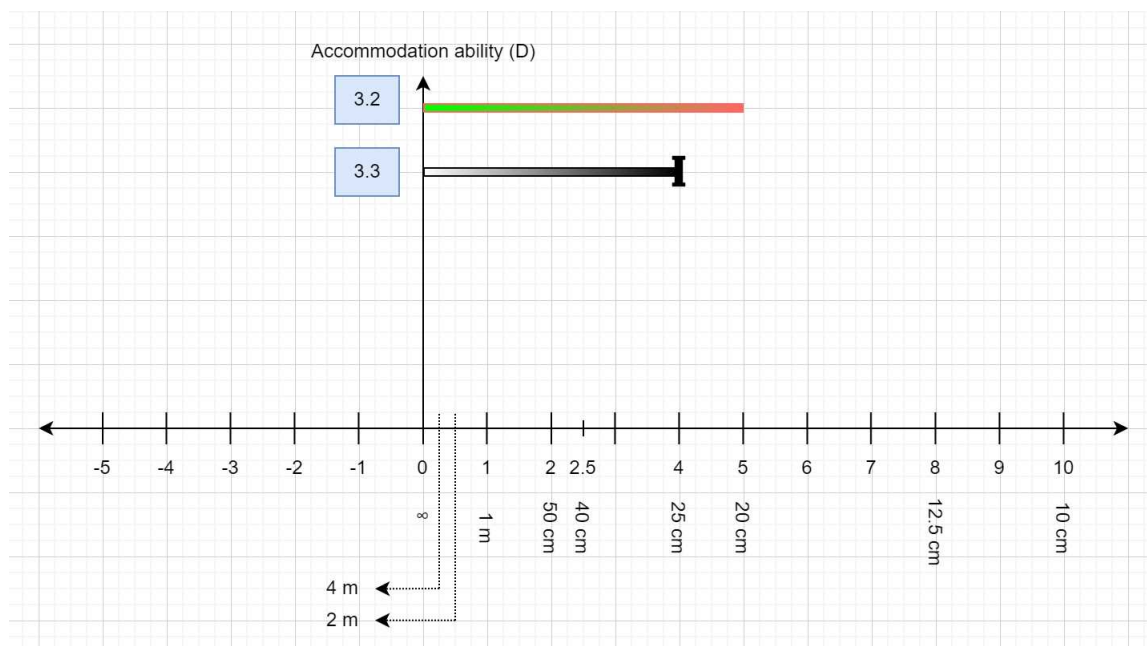


Figure 11 Comparative representation of the observation ranges of the system in section 3.3 and 3.4

228 The vertical rank/height represents accommodation ability of the systems which is also equal to the length of  
 230 of the observation range.

The gradient from red to green and light to dark was used to indicate reaching the end of the system's  
 232 observation range due to constraints. The 'I' symbol was used to indicate the impossibility of any physical  
 234 extension towards the right-hand direction (near-point) for the fixed focal length system described in section  
 3.4. The sign of object distance on the RDS is inverted from the sign for object distance ( $u$ ) used in ideal lens  
 relation due to co-ordinate conventions and the same must be remembered.

#### 236 4.1 Shift in observation range after introducing defocus

What happens to the system in 3.3 when accommodation ability is kept same (+5 D) but the range of values  
 238 themselves are incremented by +1 D on both ends (+5 D to +10 D from +4 D to +9 D)? Such a change can  
 be easily achieved by introducing another +1 D lens such that the power addition law holds (Section 2.5).

240 Just like before, it is sufficient to calculate the shifted far and near-points of the system (now 1 m and 16.66  
 cm respectively). This modified system is represented on the RDS (Figure 12). The original system is shown  
 242 using slightly thicker grey colour for ease of comparison.

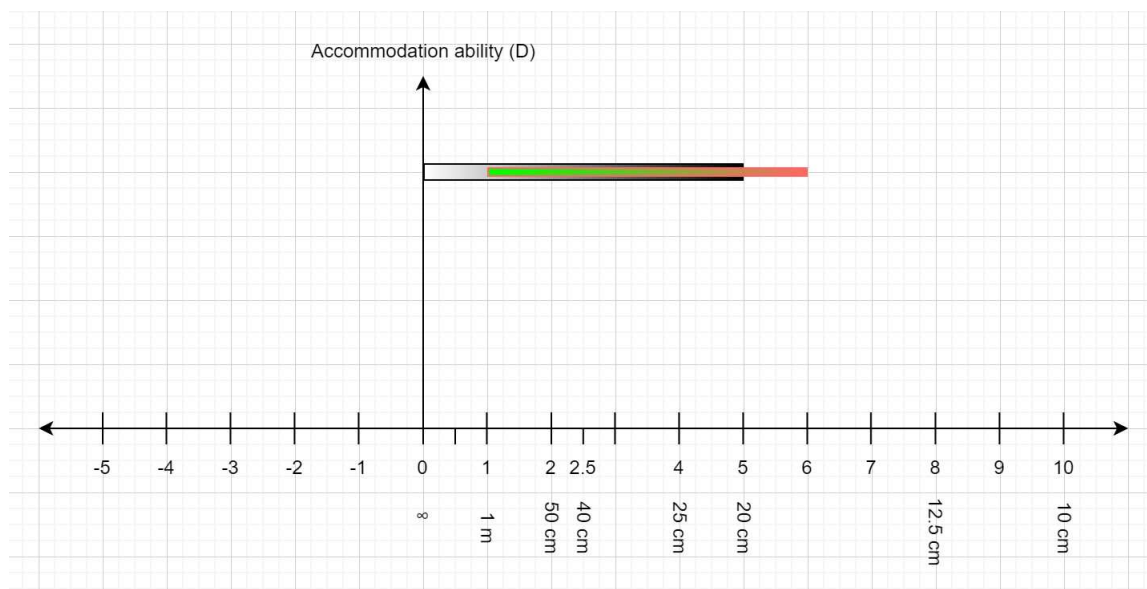


Figure 12: Shift in observation range after introduction of +1 D lens

The modified system's observation range with Lens Power range: +5 D to +10 D gets shifted 1 unit towards the positive/right direction (the sign of introduced lens).

It is evident how an increment in range of lens power values results in right-shifting of the system by the same amount on RDS. Our modifications to the system in 3.3 has rendered the system Myopic (unable to focus on objects farther than 1 m).

The inverse also holds true. It is obvious that introducing a -1 D lens in our newly modified system restores the original accommodation range. This is equivalent to introducing another -1 D lens or removing the previously introduced +1 D lens. Which means that observation range on RDS must also shift towards left by one unit. This is what the word 'Relative' means in the RDS. Relative to our modified system, the system in 3.3 can be said to be hyperopic (unable to focus at objects closer than 20 cm).

It should be now easy to understand why *introducing converging (positive) lenses/removing diverging (negative) lenses* to a system is termed as **Myopic defocus** – because it renders the system myopic (shifts the observation range towards right).

Similarly, *removing converging (positive) lenses/introducing diverging (negative) lenses* to a system is called **Hyperopic defocus** – because it renders the system hyperopic (shifts the observation range toward left).

## 4.2 Shift in observation range after changes to screen distance (Axial changes)

We've previously described shifting of a system's observation range under induced defocus. Calculating shift in Observation range due to screen distance changes is also as easy as recalculating the far and near-points. Changes to the screen distance can also be termed as Axial changes because it results in changes to the overall size of the system.

For instance, the system in 3.3 has the screen fixed at 25 cm from the lens. If we position the screen 5 cm closer maintaining the same lens power range, the near-point recedes farther from 20 cm to 25 cm (1 D



towards left) signifying that the system has become hyperopic. Similarly, if we move the screen 25cm farther from its initial position – the far-point comes closer from  $\infty$  to 50 cm (2 units towards right) rendering the system Myopic. The near-point also shifts closer (by the same unit amount) from 25 cm to 16.66 cm.

## 5. Experimental verification of shift in observation range from external defocus

The shift in observation range due to screen distance increments act opposite to that of increments in focal lengths. Thus, increase in screen distance results in Myopic shift in observation ranges and decrease in screen distance results in Hyperopic shift respectively. It is easy to experimentally observe the shifting in observation range upon introduction of external defocus as described in section 3.3 using a dedicated camera. Experimental verification of the findings of section 3.4 can also be achieved in a similar manner with the help of an optical bench setup where screen distance can be varied. This has probably been observed and documented several times but we still feel the relative unfamiliarity of phenomena deserves a mention.

Regarding shifting of observation range of a system, reducing (increasing) focal length is analogous to increasing (decreasing) screen distance. This is also intuitive from the way we have defined dimensionless image distance  $j = \frac{s}{f}$  where decreasing focal length (f) or increasing screen distance (s) both serve to increase the dimensionless image distance (j) resulting in a decrease in dimensionless object distance (k) signifying Myopic shift and vice-versa. We have also demonstrated how Myopic and hyperopic shift can compensate/cancel each other. It is even possible to compensate observation range shifts from screen distance changes by defocus and vice-versa.

### 5.1 The Camera Setup

The camera (referring to the combination of camera body paired with a lens) used was a Fujifilm X-S10<sup>4</sup> camera body paired with FUJINON XC35mmF2<sup>5</sup> prime lens.

The light rays from an object first encounter the optical elements inside the camera lens and then go on to meet the image sensor just like the ILS. This can be easily verified by simply positioning a converging lens in front of a bare camera sensor and checking for image formation in the viewfinder if **shoot without lens** mode is present and enabled.

The similarities between image-sensor of a Camera body and the imaging screen for ILS is obvious as shown in Figure 13 below.

---

4 <https://fujifilm-x.com/en-in/products/cameras/x-s10/>

5 <https://fujifilm-x.com/en-in/products/lenses/xc35mmf2/specifications/>



Figure 13: The camera body showing the exposed image sensor



Figure 14: The lens showing formation of an inverted image

The camera lens also behaves like an ideal converging lens with a variable aperture as shown in Figure 14.

294 The lens comprises of multiple optical elements – 9 elements in 6 groups (including two aspheric elements) to be exact. But for all our experimental purposes, it still behaves close to ideal. The additional optical  
296 elements are required to adjust focus and minimize aberrations in the image. The larger aperture size of the lens of a dedicated camera permits much shallower Depth-of-Field (DOF) for verification of focus.

298 The Camera (referring to the combination of camera body paired with a lens) can image close objects roughly 31.5 cm from the lens (if we subtract 35 mm focal length from the specified close focusing distance of 35 cm from the image-sensor plane, all the way up to infinity yielding an accommodation ability of ~3.25  
300 D factoring in some inbuilt headroom for proper focusing at optical infinity (Figure 15).

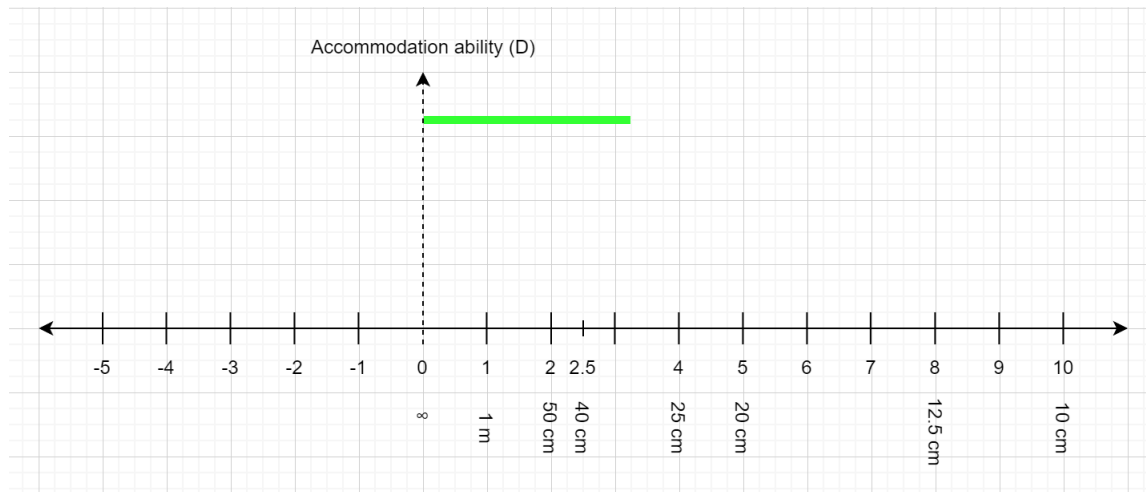


Figure 15: Observation range of XC35mmF2 lens

## 302 5.2 Description of the experimental setup

For the experiment, the camera was positioned outside to image a tree at optical infinity across a field  
304 (Figure 16).



Figure 16: Image showing the positions of camera and the object

The X-S10 camera body used for this experiment has a handy distance scale option which shows the focusing distance estimated from the position of optical elements as reported by the lens. Position of distance scale<sup>6</sup> with the white focus distance and blue DOF indicator<sup>7</sup> is shown in Figure 17 and 18.

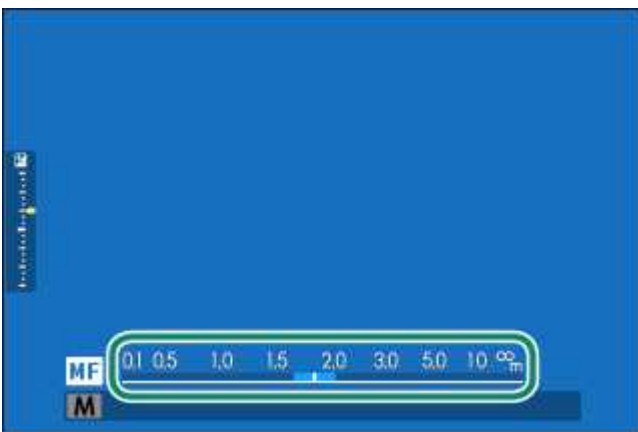


Figure 17: Position of distance scale in the viewfinder



Figure 18: Focus distance and DoF indicators

As can be verified from the white mark near 10m $\infty$  on the Distance scale in the viewfinder (Figure 19), the optical elements inside the camera lens are correctly focused for our subject tree located at optical infinity in

<sup>6</sup> [https://fujifilm-dsc.com/en/manual/x-s10/taking\\_photo/manual-focus/](https://fujifilm-dsc.com/en/manual/x-s10/taking_photo/manual-focus/)

<sup>7</sup> [https://fujifilm-dsc.com/en/manual/x-s10/taking\\_photo/manual-focus/](https://fujifilm-dsc.com/en/manual/x-s10/taking_photo/manual-focus/)

310 the absence of external defocus. Image formation parameters related to image-sensors like exposure or  
311 sensitivity is beyond the scope of this article.



Figure 19: Verification of object distance

### 312 5.3 Shift in observation range after hyperopic defocus

313 What happens if one places a diverging (minus) lens (introduce hyperopic defocus) closely in front of  
314 camera? In section 4.1, we have already outlined how observation range gets shifted after introduction of  
external defocus. For instance, introducing a  $-2.5$  D defocus would shift the observation range 2.5 units  
316 towards left as shown in Figure 20.



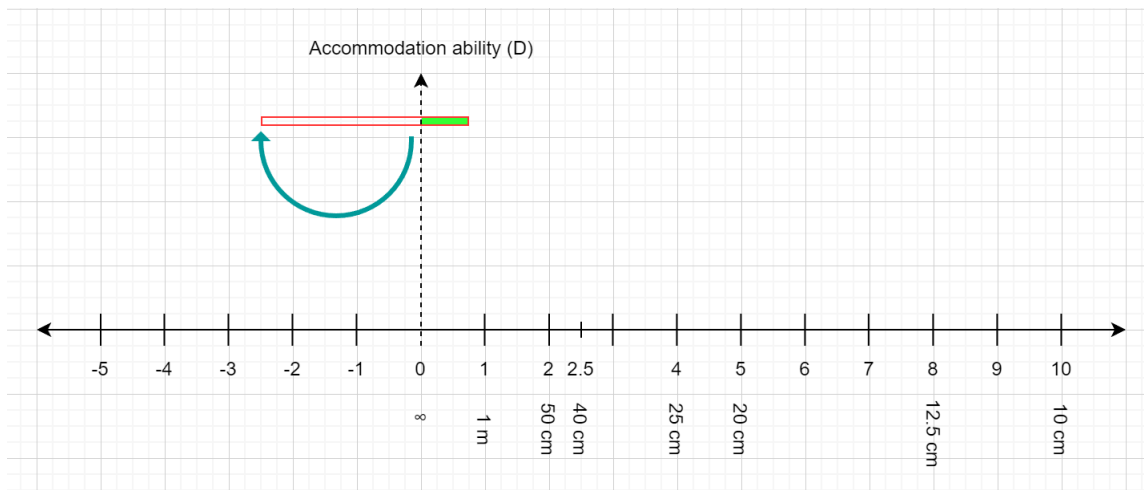


Figure 20: Shift in observation range after introduction of  $-2.5\text{ D}$  defocus

The hollow red part of the observation range to the left of origin indicates unusable part of the observation range because it is physically impossible to place an object beyond  $\infty$ . The remaining useful part of observation range lying closer than infinity is represented with a solid green line. Experimental demonstration and verification for the same using  $-2.5\text{ D}$  defocus is provided in Figure 21.



Figure 21: Verification of reported focus distance with  $-2.5\text{ D}$  defocus

The white focus distance indicator is near the  $0.5\text{ m}$  mark on distance scale. Coincidentally, this is also the focal length ( $0.4\text{ m}$ ) of the introduced defocus lens. Thus, focusing at an object located at infinity with the

camera using hyperopic defocus of -2.5 D results in reported focusing distance of 40 cm at the focus of the  
324 introduced lens. The test subject situated at optical infinity is still in acceptable focus and the same can be  
verified from the blur that results from removing the introduced defocus as shown in Figure 22.



Figure 22: Blur resulting from removing the defocus  
after focusing for  $\infty$

326 Due to the nature of the mapping introduced by -2.5 D defocus, we can state that the Camera is *apparently*  
focused at infinity when the lens optics are *actually* reporting focus around the 40 cm mark. A simple video  
328 clip demonstrating the above is supplied with this article.

In a way, we can say that the introduced -2.5 D defocus ‘eats’ away the usable portion of the effective  
330 observation range of the lens leaving only the remaining part of the observation range available for image  
formation. After introduction of defocus, the camera reports its focus distance as equal to the focal length of  
332 the diverging lens used even when focusing at infinity like before. This can be understood as the introduced  
lens ‘mapping’ object distances to their new apparent distances or alternately as the lens apparently  
334 ‘bringing’ objects from infinity at its focus (virtual object formation). This also translates to the inability of  
the camera to lock focus at objects closer than  $\sim 1.33$  m with the -2.5 D defocus.

336 We have also repeated the experiment with a -3.0 D defocus so that the similar result can be compared as  
shown in Figure 23 below.



*Figure 23: Verification of reported focus distance with -3.0 D defocus*

338 This time, the image distance ( $\sim 33$  cm) is very close to the close-focusing limit of the lens at  $\sim 35$  cm.

340 As one might've already observed, placing a lens in front of camera introduces some optical aberrations and slight crop to the image. But for our limited purposes, this has not affected our ability to verify the shift in observation range due to introduced defocus.

342 External defocus due to an introduced lens can't alter the accommodation ability because the accommodation ability of a camera happens to be an internal physical constraint decided by how the optical elements are  
344 configured to move inside the lens.

#### **5.4 Focusing on objects closer than $\infty$ under hyperopic defocus**

346 The consequence of introducing a diverging lens in front of a camera for formation of image of objects at infinity has been previously demonstrated in previous sections.

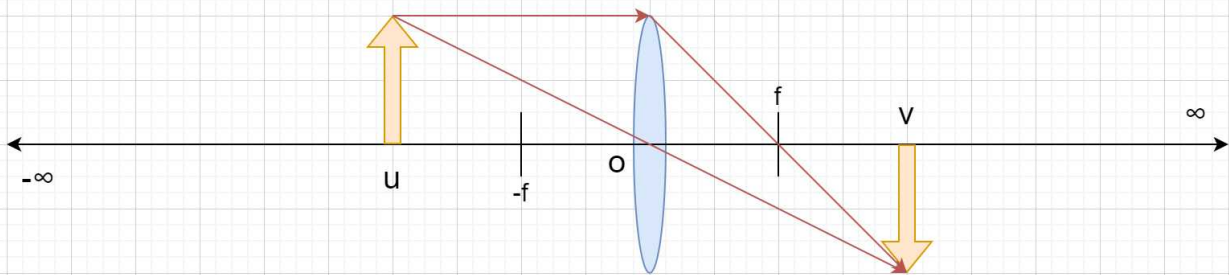
348 In a way, we can say that the introduced -2.5 D defocus decreases usable effective observation range of the lens leaving only a subset of the observation range available for image formation. This translates as the  
350 inability of the camera to achieve focus at objects closer than  $\sim 1.33$  m with the -2.5 D defocus shown earlier by the green part of observation range shown in Figure 20. This close-focusing distance recedes even further  
352 with the -3.0 D defocus rendering the Camera severely hyperopic.

## 6. Depth-of-field (DOF)

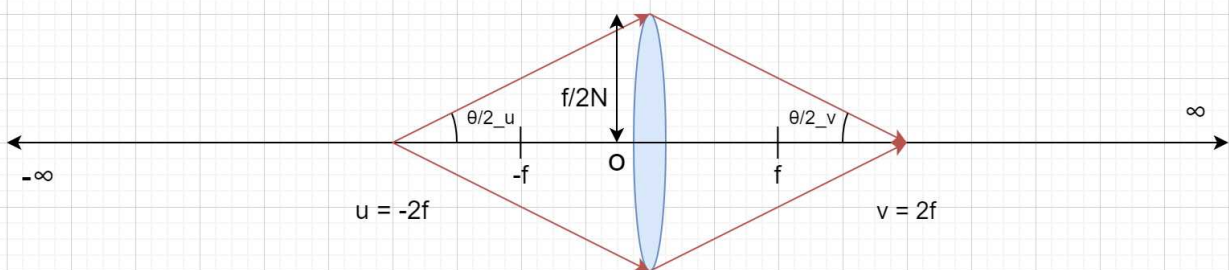
354 DOF diagrams are provided to scale for ease of visualization (Figure 24).



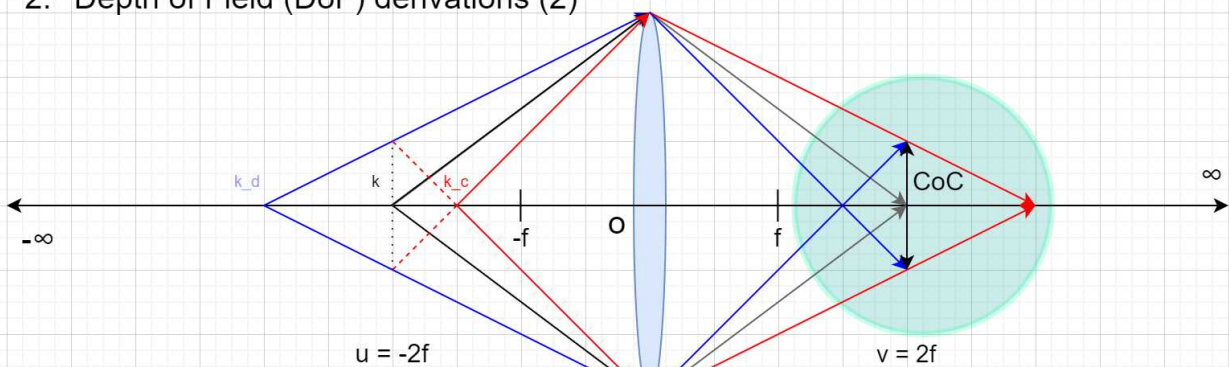
## 0. Image formation by a convex lens



## 1. Depth of Field (DoF) derivations (1)



## 2. Depth of Field (DoF) derivations (2)



## 3. Depth of Field (DoF) derivations (3)

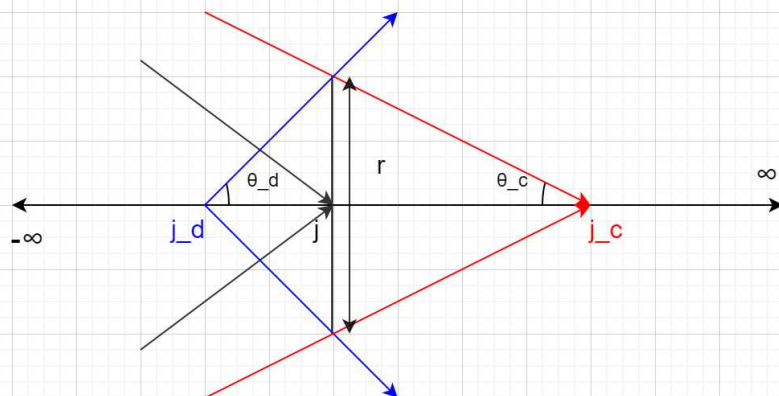


Figure 24: DoF ray diagrams

*An object must be situated at  $(-\infty, -f)$   
for it to form an image at  $(f, \infty)$  on the other side of the convex lens.*

*In common usage, aperture size (A) is usually denoted as F – number or  $\frac{f}{N}$ .*

*$A = \frac{f}{N}$ , where N is some dimensionless number.*

*For instance, F2 or  $\frac{f}{2}$  implies  $N = 2$  in both the cases  
and it denotes that the aperture size / opening is half of the focal length.*

*Following this convention,*

*$\frac{f}{2N}$  denotes the half – width of aperture size / opening  
or radius if the aperture is circular*

*we have accordingly*

$$\theta_u = 2 \tan^{-1} \left( \frac{1}{2Nk} \right)$$

$$\theta_v = 2 \tan^{-1} \left( \frac{1}{2Nj} \right)$$

*To derive D. O. F. we need to find inverse of the problem :*

*for some value of j i. e., imaging distance,  
find the range of k given a particular value of N,  
for which Circle of Confusion (C. o. C. ) is lesser than a threshold value.*

*We can make C. O. C. dimensionless by dividing it by f to get another quantity r*

$$r = \frac{\text{C. O. C.}}{f}$$

*thus obtained both extreme values of u and v obey*

$$u_d > u > u_c :: v_d < v < v_c$$

*where the subscript denotes (c)loser and (d)istant object / image respectively*

*This can be understood as a consequence of the hyperbolic relation between  
object and its image.*

*These same relations must apply to k and j also*

$$k_d > k > k_c :: j_d < j < j_c$$

*From consideration of the quadrilateral formed at image junction,  
we can see that*

$$(j - j_d)\tan \theta_d = (j_c - j)\tan \theta_c$$

$$\implies \frac{j - j_d}{2Nj_d} = \frac{j_c - j}{2Nj_c} = \frac{r}{2}$$

*Three quantities equal to each other means that we will get 3C2 = three equations.*

*equating the first two parts, we get :*

$$\frac{j - j_d}{j_d} = \frac{j_c - j}{j_c}$$

$$\implies j_c \times j - j_c \times j_d = j_c \times j_d - j \times j_d$$

*rearranging we get,*

$$j = \frac{2j_c \times j_d}{j_c + j_d}$$

*This equation can be checked by putting  $j_c$  and  $j_d$  equal to  $j$ .*

Equating the first and second terms separately with  $r$ , we get :

$$\frac{j - j_d}{2Nj_d} = \frac{r}{2}$$

$$\frac{j_c - j}{2Nj_c} = \frac{r}{2}$$

which gives,

$$j_c = \frac{j}{1 - Nr} \text{ \& } j_d = \frac{j}{1 + Nr}$$

we can now invert these according to the relation we got earlier

$$\text{which is } k = \frac{j}{j - 1}, \text{ giving}$$

$$k_c = \frac{j_c}{j_c - 1} = \frac{\frac{j}{1 - Nr}}{\frac{j}{1 - Nr} - 1} = \frac{j}{j - 1 + Nr}$$

$$\text{and } k_d = \frac{j_d}{j_d - 1} \Rightarrow \frac{\frac{j}{1 + Nr}}{\frac{j}{1 + Nr} - 1} = \frac{j}{j - 1 - Nr}$$

$$k_d - k_c = j \times \left( \frac{1}{j - 1 - Nr} - \frac{1}{j - 1 + Nr} \right)$$

$$\text{here, } k_d - k_c \text{ denotes } \frac{D. o. F.}{f}$$

$$\Rightarrow D. o. F. = f \times j \times \left( \frac{j - 1 + Nr - (j - 1 - Nr)}{(j - 1 - Nr) \times (j - 1 + Nr)} \right)$$

$$\Rightarrow D. o. F. = \frac{2f \times j \times Nr}{(j - 1 - Nr) \times (j - 1 + Nr)}$$

$$= \frac{2f \times j \times Nr}{(j - 1)^2 - N^2 r^2}$$

How to use this formula for calculations :  
firstly calculate  $j$  from  $k$ .

then putting the respective values of  $f$ ,  $N$ ,  $j$  &  $r$   
in the above formula will give you the  $D. o. F.$  in the unit of focal length.

Two DOF scenarios of interest are described below:

*Nearest distance in acceptable focus when focus is at  $\infty$  ( $j = 1, k \rightarrow \infty$ ) :*

$$j_c = \frac{1}{1 - Nr}$$

*which gives  $k_c = \frac{1}{Nr}$*

*the nearest distance of focus while still having objects at  $\infty$   
in 'acceptable' focus ( $j_d = 1$ ) :*

$$\text{giving } j = 1 + N \times r$$

$$k = \frac{j}{j-1} = \frac{1 + Nr}{Nr}$$

---FINISHED DOF DERIVATION PART ---

$$\text{we already know that } j = \frac{2j_c \times j_d}{j_c + j_d}$$

$$\text{substituting } j_x = \frac{k_x}{k_x - 1} \text{ we get,}$$

$$j = 2 \times \frac{\frac{k_c}{k_c-1} \times \frac{k_d}{k_d-1}}{\frac{k_c}{k_c-1} + \frac{k_d}{k_d-1}} = 2 \times \frac{\frac{k_c}{k_c-1} \times \frac{k_d}{k_d-1}}{\frac{k_c}{k_c-1} + \frac{k_d}{k_d-1}}$$

$$\text{giving } j = \frac{2k_c \times k_d}{2 \times k_c \times k_d - (k_c + k_d)}$$

*finding the same relationship for k results in*

$$k = \frac{j}{j-1} = \frac{\frac{2k_c \times k_d}{2 \times k_c \times k_d - (k_c + k_d)}}{\frac{2k_c \times k_d}{2 \times k_c \times k_d - (k_c + k_d)} - 1}$$

$$= \frac{2k_c \times k_d}{k_c + k_d}$$

*This result is due to the interchangeable (mirror) nature of the relation.*

## 356 6.1 Experimental verification for depth-of-field calculations

We will demonstrate use of the above DOF formula for calculating DOF in real world scenarios.

358 For the X-S10 and XC35mmF2, the screen distance equivalent can be taken approximately 35mm or 3.5 cm  
equal to the reported focal length of the lens. For a prime lens, the screen distance remains mostly fixed and  
360 it is only the focal length of the lens that changes (otherwise there will be FOV breathing). Using this

362 assumption, one can easily utilize the result from section 3.3 to calculate reduction in focal length for  
distances closer than infinity.

364 The 6240 x 4160 image sensor of the camera measures 23.5 x 15.6 mm (Width x Height). The pixel pitch  
comes out to be approximately  $23.5 \text{ mm} \div 6240 = \sim 0.0037 \text{ mm}$  approximating the CoC. For a manual focus  
366 distance of 2.5 m with F-number of 16, the DOF calculations (Figure 26) of ~60cm front-to-back is in line  
with the value reported in the viewfinder (Figure 25).



Figure 25: Viewfinder indicating DOF of ~60 cm

```

Ac := 0.4  d :=  $\frac{100}{Ac}$   d = 250.0  cm

Power in Dioptre and distance in cm
u := d  u := u  put u for above value
screen := 3.5

f :=  $\frac{100}{\frac{100}{screen} + Ac}$   f = 3.452

Replace d by object distance you want
screen := f ·  $\frac{-u}{f - u}$   screen = 3.5

k :=  $\frac{u}{f}$   k = 72.43

k := k  k > 1, put k for above value

j :=  $\frac{k}{k - 1}$   j = 1.014

sum := k + j  sum = 73.44

magnification :=  $\frac{j}{k}$   magnification = -0.014

DOF derivation part

N := 16  Aperture :=  $\frac{f}{N}$   Aperture = 0.2157  cm

Object and image angles subtended

theta_u :=  $\frac{360}{pi} \cdot atan\left(\frac{1}{2 \cdot N \cdot k}\right)$   theta_u = 0.04944

theta_v :=  $\frac{360}{pi} \cdot atan\left(\frac{1}{2 \cdot N \cdot j}\right)$   theta_v = 3.53

Radius of CoC

CoC :=  $\frac{2.35}{6240}$   cm  r :=  $\frac{CoC}{f}$   r = 1.091E-4

k and j close and distant

j_c :=  $\frac{j}{1 - N \cdot r}$   j_c = 1.016  j_c · f = 3.506

k_c :=  $\frac{j}{j - 1 + N \cdot r}$   k_c = 64.4  k_c · f = 222.3

j_d :=  $\frac{j}{1 + N \cdot r}$   j_d = 1.012  j_d · f = 3.494

k_d :=  $\frac{j}{j - 1 - N \cdot r}$   k_d = 82.75  k_d · f = 285.6

DOF

k_d - k_c = 18.35

DoF := f · (k_d - k_c)  DoF = 63.33  cm

DoF_alt :=  $\frac{2 \cdot f \cdot j \cdot N \cdot r}{(j - 1)^2 - (N \cdot r)^2}$   DoF_alt = 63.33  cm

Nearest distance under DOF keeping focus at ∞ (k_c when j = 1)

 $\frac{f}{N \cdot r}$  = 1977.0  cm

Nearest focus distance keeping ∞ under DOF (k when j_d = 1)

 $\frac{f \cdot (1 + N \cdot r)}{N \cdot r}$  = 1981.0  cm

```

Figure 26: DOF worksheet calculations