

$$\begin{array}{r} 5 \quad 3 \quad 3 \\ 2 \quad 1 \quad 1 \\ \hline 2 \quad 2 \quad 1 \\ 1 \quad 1 \quad 2 \\ \hline 7 \quad 7 \quad 7 \end{array}$$

$2L + M$

$2K+M$

$$\text{incV} = L + R + 2M$$

based on sum:4 one of these can be the ans  
 $1+18$  +1,  $1+18+2$ ,  $1+18+3$ ,  $1+18+4$

What else?

$$\begin{array}{ccc}
 2 & 1 & 2 \\
 \swarrow & \downarrow & \searrow \\
 a_1 & a_2 & a_3 \\
 \swarrow & \downarrow & \searrow \\
 a_3 + a_2 & a_1 + a_3 & a_1 + a_2 \\
 + 2a_1 & + 2a_2 & + 2a_3 \\
 = 5 & = 6 & = 5
 \end{array}$$

$$a_1 = a_3$$
$$a_1 + a_2 = 2$$
$$4a_1 + a_2 = 5$$
$$a_1 = a_2 = 1$$

N equations and N variables and we need to solve this

Let's look at example for n = 5

$$\underline{a_1} \quad \underline{a_2} \quad \underline{a_3} \quad \underline{a_4} \quad \underline{a_5}$$

$$2a_i + a_{i-1} + a_{i+1} = x_i$$

$$\begin{bmatrix} a_{i-1} & a_i & a_{i+1} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x_i \end{bmatrix}$$

$$\begin{bmatrix} a_5 & a_1 & a_2 \\ a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \\ a_3 & a_4 & a_5 \\ a_4 & a_5 & a_6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

How can we solve this, using a compute program?

$$4 \sum a_i = \sum x_i$$

$$\sum a_i = \frac{\sum x_i}{4}$$

$$a_1 \quad a_2 \quad \frac{1}{4} a_3 \quad \frac{2}{4} a_4 \quad \frac{1}{4} a_5$$

$$d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5$$

$$d_i = a_{i+1} - a_i$$

$$d_1 = a_2 - a_1$$

$$d_2 = a_3 - a_2 + 1$$

$$d_3 = a_4 - a_3 + 1$$

$$d_4 = a_5 - a_4 - 1$$

$$d_5 = a_6 - a_5 - 1$$

$$d_4 =$$

You are given an array you can choose any i and do  $a_i += 1$ ,  $a_{i+1} += 1$ ,  $a_{i+2} -= 1$  and  $a_{i+3} -= 1$

Can you turn the array to all zeroes?

Sum must be zero initially.

It is always zero initially.

How to solve?

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

I know exactly what the final value is..

it is zero

How to solve it if it was non cyclic?

$$\begin{array}{cccccc} & & & 1 & & -1 & -1 \\ & & & 0 & & 0 & \\ a_1 & a_2 & a_3 & a_4 & a_5 & & \\ 0_1 & & & & & & \\ 0_2 & 1 & 1 & -1 & -1 & 0_1 & \\ & & & & & & \end{array}$$

$\rightarrow a_4 = -1, +0_1$   
 $\rightarrow a_5 = -0_2$

$$\begin{array}{cccccc} a_1 & a_2 & a_3 & 0 & 0 \\ 1 & 1 & -1 & -1 & & \end{array}$$

$$\begin{array}{cccccc} 1 & 1 & -1 & -1 & & \\ & 1 & 1 & -1 & -1 & \\ & & & 1 & 1 & -1 & -1 \\ \hline 1 & 1 & -1 & -1 & & \end{array}$$

The more general version of this operation will be choose  $i, j$  such that

$$1 \leq i, j \leq n$$

and do

$$\left. \begin{array}{l} a_i += 1 \\ a_{i+1} += 1 \\ a_j -= 1 \\ a_{j+1} -= 1 \end{array} \right\}$$

ok

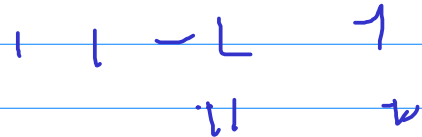
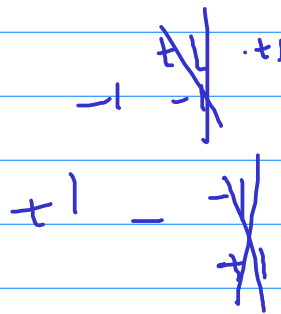
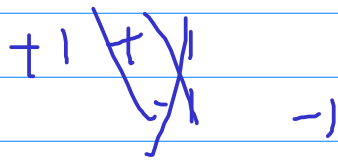
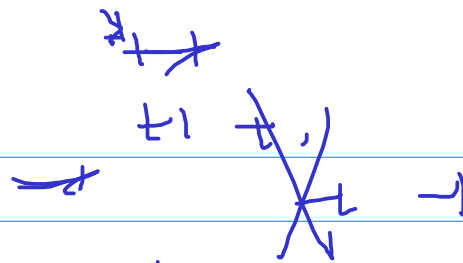
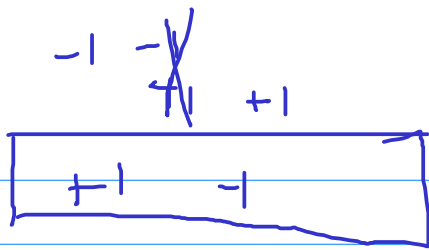
Well if all the final numbers have to be zero and above condition will have to satisfy

So the op reduces to  
such  $i$

$$\begin{array}{l} a_i += 1 \\ a_{i+1} += 1 \end{array}$$

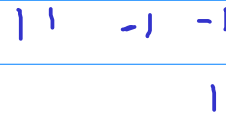
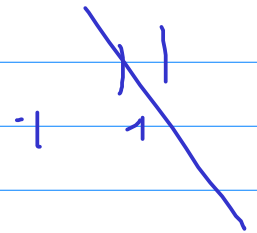
OR

$$\begin{array}{l} a_j += -1 \\ a_{j+1} += -1 \end{array}$$



It further reduces to sum at each parity must be zero

Let's look at some test cases

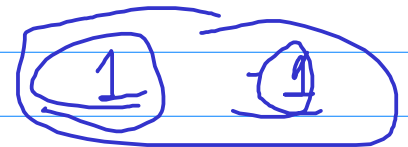


1 2 1 2 1

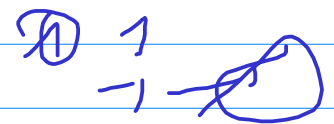
(1) -1 (1) -1 0 0

2 -1 0 -1 0

0 -2 1 -1



1 2 1 2 1 3 1



(1) -1 (1) -1 (2) -2 0



sum of p1 = - sum of p2

The operation is

$a_i += 1$

$a_{i+1} += -1$

Then we will be done