

Math 211
Quiz 04A

R 25 Jul 2019

Your name : _____

Exercise

(5 pt) Evaluate the following integral.

$$\int \sin^3 t \, dt.$$

Hint: View the integral as $\int (\sin^2 t)(\sin t) dt$. After an integration, use the identity $\sin^2 t + \cos^2 t = 1$. You should get an equation involving the given integral on both the left and the right. Now treat the integral like an unknown variable (e.g., x) and solve for it.

Solution: Using integration by parts with

$$u = \sin^2 t, \quad dv = \sin t \, dt,$$

so that

$$du = 2 \sin t \cos t \, dt, \quad v = -\cos t,$$

we get

$$\int \sin^3 t \, dt = -\sin^2 t \cos t + 2 \int \sin t \cos^2 t \, dt.$$

Using the identity $\sin^2 t + \cos^2 t = 1$, we can write this last expression as

$$\begin{aligned} \int \sin^3 t \, dt &= -\sin^2 t \cos t + 2 \int \sin t (1 - \sin^2 t) \, dt \\ &= -\sin^2 t \cos t + 2 \int \sin t \, dt - 2 \int \sin^3 t \, dt. \end{aligned}$$

Solving for $\int \sin^3 t \, dt$, we find

$$\int \sin^3 t \, dt = -\frac{1}{3} \sin^2 t \cos t - \frac{2}{3} \cos t + c,$$

where $c \in \mathbf{R}$.