Math 357
Long quiz 03B

2024–02–19 (M)

Your name: _____

Let $\mathbf{Z}$ denote the ring of integers, let $\mathbf{Q}$ denote the field of rational numbers, and let t be an indeterminate.

(a) Let $n \in \mathbf{Z}_{>0}$; let $f \in \mathbf{Z}[t]$, say

$$f = a_n t^n + \ldots + a_0$$

with $a_n \neq 0$; and let $\frac{r}{s} \in \mathbf{Q}$ such that $\gcd(r, s) = 1$. Show that if $f(\frac{r}{s}) = 0$, then $r \mid a_0$ and $s \mid a_n$.

(b) Prove that $f = 2t^3 + 7t + 8 \in \mathbf{Z}[t]$ is irreducible. *Hint:* Think before you compute.