Math 211
Quiz 14

R 25 Jul 2019

Your name : ______________________________

Exercise

(2 pt) Consider the 3rd-order linear ODE

$$y^{(3)} + 6y^{(2)} + 11y^{(1)} + 6y = 0, \tag{1}$$

where $y^{(n)}$ denotes the nth (ordinary) derivative $\frac{d^n y}{dt^n}$. Using the change of variables

$$x_n = y^{(n)},$$

for $n = 0, 1, 2$, translate the 3rd-order ODE (1) into a 1st-order linear system

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}' = A \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix},$$

where A is a 3×3 matrix of constants. *Hint:* The entries in the third row of A should be related to the coefficients of the original ODE (1); the entries in the first two rows of A should all be 0 or 1.

(Not required : Compute the characteristic polynomial of the coefficient matrix A, i.e. the polynomial $\det(A - \lambda I)$. (This is the polynomial we've met before, whose roots are the eigenvalues of A.) How does this compare to the equation we get by replacing each $y^{(n)}$ in the original ODE (1) with λ^n? How do the roots of these two polynomials compare?)