Math 112
ReQuiz 05A06A

2022–02–01 (T)

Your name: _____

Exercise

(4 pt) Consider the function

$$f : \mathbf{R} \to \mathbf{R} \qquad \text{given by} \qquad f(x) = (x^2 - 1)^5 - \sin(\pi x)$$

(a) (3 pt) Compute the linearization of (aka linear approximation to) f at $x = 1$.

Solution: By definition, the linearization of f at $x = 1$ is the function $L : \mathbf{R} \to \mathbf{R}$ given by

$$L(x) = f(1) + f'(1)(x - 1) \tag{1}$$

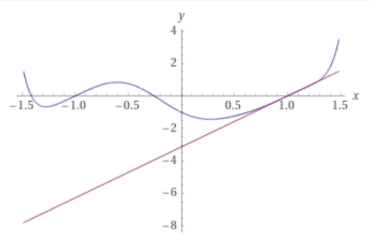
We compute[1]

$$\begin{aligned} f(1) &= (1-1)^5 - \sin(\pi) = 0 - 0 = 0 \\ f'(x) &= 5(x^2-1)^4(2x) - \pi\cos(\pi x) = 10x(x^2-1)^4 - \pi\cos(\pi x) \\ f'(1) &= \pi \end{aligned}$$

Substituting these results into (1), we conclude that the rule of assignment for L is

$$L(x) = 0 + \pi(x - 1) = \pi x - \pi \tag{2}$$

A graph of $f(x)$ (in blue) and $L(x)$ (in red), produced using WolframAlpha, is shown below.



(b) (1 pt) Use your linearization from part (a) to approximate the value $f(0)$. Find the error in this approximation.

Solution: Using the given rule of assignment for $f(x)$ and Equation (2) for $L(x)$, we compute

$$L(0) = -\pi \qquad\qquad f(0) = (-1)^5 - \sin(0) = -1$$

The error ε in the approximation $L(0)$ to $f(0)$ is

$$\varepsilon = L(0) - f(0) = -\pi - (-1) = 1 - \pi$$

This error is negative, indicating that our estimate $L(0)$ is less than the actual value $f(0)$. That is, our estimate is too small. (Our graph of f and L above confirms this.)

[1] In computing $f'(x)$, we use the chain rule twice, once on each term of $f(x)$.