

Math 357  
Long quiz 04

2024-02-14 (W)

Your name: \_\_\_\_\_

Let  $\mathbf{Q}$  denote the field of rational numbers; given a prime  $p \in \mathbf{Z}_{>0}$ , let  $\mathbf{F}_p \cong \mathbf{Z}/(p)$  denote the finite field with  $p$  elements; and let  $t$  be an indeterminate. For each of the quotient rings below, characterize its algebraic structure as “field”, “integral domain but not field”, or “ring but not integral domain”. Justify your characterization.

$$R_1 = \mathbf{F}_5[t]/(t^3 - t^2 + 2t - 1) \quad R_2 = \mathbf{Q}[t]/(t^3 - t^2 + 2t - 1) \quad R_3 = \mathbf{Q}[t]/(t^6 - 300)$$