Math 357
Long quiz 05C

2024–04–19 (F)

Your name: _____

Let $f = 6000t^3 + 45t^2 - 60t - 1 \in \mathbf{Q}[t]$.

(a) Prove that f is irreducible in $\mathbf{Q}[t]$.

(b) Show that $f(-\frac{1}{20}) = \frac{109}{80}$. Use this to deduce the number of zeros of f in $\mathbf{Q}$, in $\mathbf{R} - \mathbf{Q}$, and in $\mathbf{C} - \mathbf{R}$. (You need not compute the values of these zeros.) Explain your logic.

(c) Let $\alpha \in \mathbf{C}$ be a zero of f, and let $\beta \in \mathbf{C}$ such that the minimal polynomial of β over $\mathbf{Q}$ has degree 2. (Note that α or β may be real.) Can $\beta \in \mathbf{Q}(\alpha)$? in a splitting field of f? Justify your assertions.