Math 212
Quiz 08

F 09 Sep 2016

Your name: _____

Exercise

(5 pt) Let $\mathbf{r} : \mathbf{R} \to \mathbf{R}^3$ be the position function of a particle, given by

$$\mathbf{r}(t) = \left(\cos(2t), \frac{2}{3}\left(t^2+2\right)^{\frac{3}{2}}, \sin(2t)\right).$$

(a) (2 pt) Write the velocity of the particle at time t. *Hint:* Recall that velocity is the rate of change of position with respect to time.

Solution: The related velocity function is

$$\mathbf{v}(t) = \mathbf{r}'(t) = \left(-2\sin(2t), 2t\sqrt{t^2+2}, 2\cos(2t)\right).$$

(b) (3 pt) Find the minimum speed of the particle. *Hint:* Recall that speed is the magnitude of the velocity vector.

Solution: The speed $v(t)$ at time t is

$$\begin{aligned} v(t) &= \|\mathbf{v}(t)\| \\ &= \sqrt{4\sin^2(2t) + 4t^2(t^2+2) + 4\cos^2(2t)} \\ &= \sqrt{4t^4 + 8t^2 + 4} \\ &= \sqrt{4(t^2+1)^2} \\ &= 2(t^2+1). \end{aligned}$$

The minimum value of this function $v(t)$ is 2, occurring at $t = 0$.