

Math 357
Short quiz 07

2024-02-05 (M)

Your name: _____

Let R be a euclidean domain, and let $N : R \rightarrow \mathbf{Z}_{\geq 0}$ be a norm that has a division algorithm. State the division algorithm. Within the division algorithm, what bonus do we get in the case $R = F[t]$, where F is a field and t is an indeterminate?

Solution: The division algorithm states the following: Let $a, b \in R$ such that $b \neq 0$. Then there exist $q, r \in R$ such that

$$a = qb + r$$

and $r = 0$ or $N(r) < N(b)$. In the case $R = F[t]$, we get the bonus that the quotient q (better, the product qb) and the remainder r are unique, if we take N to be \deg with the amendment that $N(0) = 0$.