Math 212
Quiz 12

M 19 Sep 2016

Your name: ______________________________

Exercise

(2 pt) Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by

$$f(x, y) = x^4y^2 - 2xy^5.$$

(a) (1 pt) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Solution: Partial derivatives are computed by holding all variables but one constant:

$$\frac{\partial f}{\partial x} = 4x^3y^2 - 2y^5, \qquad \frac{\partial f}{\partial y} = 2x^4y - 10xy^4.$$

(b) (1 pt) Compute $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$. Verify that the Clairaut–Schwarz theorem (equality of mixed partials) holds.

Solution: We compute

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 8x^3y - 10y^4 = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}.$$

These two second-order mixed partial derivatives are equal, as asserted by the Clairaut–Schwarz theorem. (Note that these second-order mixed partial derivatives are polynomials and therefore continuous everywhere, so the hypothesis of the theorem is satisfied.)