Math 112
Exam 01

2022–02–08 (T)

Your name: ______________________________

Instructions

Number of exercises	: 6
Permitted time	: 75 minutes
Permitted resources	: None

Remarks:

- Manage your time deliberately.
- If the statement of an exercise is unclear to you, briefly (one sentence) write your understanding of the exercise, then proceed.
- Work hard, do your best, and have fun!

Exercise	Total	(a)	(b)	(c)	(d)	(e)	(f)
1	/12	/2	/2	/2	/2	/2	/2
2	/8	/4	/4				
3	/18	/2	/4	/4	/4	/4	
4	/16	/4	/4	/4	/4		
5	/14	/2	/4	/4	/4		
6	/12	/4	/4	/4			
Total	/80						

Exercise 1

(12 pt) True/False. For each of the following statements, circle whether it is true or false. No justification is necessary.

(a) (2 pt) Let $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be given by

$$f(x) = 2x - 2 \qquad\qquad g(x) = \frac{1}{2}x + 1$$

The functions f and g are inverse functions.

true false

(b) (2 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be a function. If $f'(a) = 0$ for some input $a \in \mathbf{R}$, then $x = a$ is either a local minimum or a local maximum of f.

true false

(c) (2 pt) For the graph of the unit circle $x^2 + y^2 = 1$, we cannot give an equation of the tangent line at every point on the circle.

true false

(d) (2 pt) Let f be a function. Suppose that the domain (aka the set of inputs) of the second-derivative function f'' equals the domain of f. Then the domain of the first-derivative function f' also equals the domain of f.

true false

For parts (e) and (f), let f be a function defined on an open set containing a point a.

(e) (2 pt) If $\lim_{x \uparrow a} f(x)$ (that is, the limit from the left) and $\lim_{x \downarrow a} f(x)$ (that is, the limit from the right) exist, then $\lim_{x \to a} f(x)$ exists.

true false

(f) (2 pt) If $\lim_{x \to a} f(x)$ exists, then $\lim_{x \uparrow a} f(x)$ (that is, the limit from the left) and $\lim_{x \downarrow a} f(x)$ (that is, the limit from the right) exist.

true false

Exercise 2

(8 pt) Compute the following. (The answers are integers.)

(a) (4 pt) Let

$$e^a = 2 \qquad e^b = 3 \qquad e^c = 16$$

Compute

$$\sqrt{\frac{e^{4a+c}}{e^{2b}}} \cdot \frac{e^{6a-b+c}}{e^{4a+2c}} \cdot e^{2\ln 3}$$

(b) (4 pt) Let

$$\ln a = \frac{1}{3} \qquad \ln b = \frac{1}{5} \qquad \ln c = -\frac{1}{2}$$

Compute

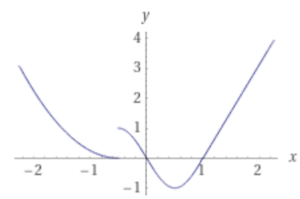
$$\ln\left((ac+bc)^2\right) - \ln\left(a^2 + 2ab + b^2\right) + \ln\left(\frac{a^3b^5}{c^{-2}}\right)$$

Exercise 3

(18 pt) Consider the piecewise function $f : \mathbf{R} \to \mathbf{R}$ whose rule of assignment is

$$f(x) = \begin{cases} \dfrac{1}{4}(2x+1)^2 & \text{if } x < -\frac{1}{2} \\ -\sin(\pi x) & \text{if } -\frac{1}{2} \leqslant x \leqslant 1 \\ \pi x - \pi & \text{if } x > 1 \end{cases}$$

A graph of f is shown below.



(a) (2 pt) Using the graph, identify the values of x at which $f(x)$ is not continuous.

(b) (4 pt) Justify, algebraically, that $f(x)$ is not continuous at the value(s) of x you identified in part (a).

(c) (4 pt) Is the first-derivative function $f'(x)$ continuous at $x = 1$? Justify.

(d) (4 pt) Is the second-derivative function $f''(x)$ continuous at $x = 1$? Justify.

(e) (4 pt) Is the first-derivative function $f'(x)$ continuous at $x = -\frac{1}{2}$? Justify.

Exercise 4

(16 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by

$$f(x) = 2x^3 - x^2 - 4x + 2$$

(a) (4 pt) Find the interval(s) on which f is increasing and decreasing.

(b) (4 pt) Find the x-coördinate of each local minimum and maximum of f. State whether each is a local minimum or maximum of f. Justify.

(c) (4 pt) Find the global minimum and maximum of f.

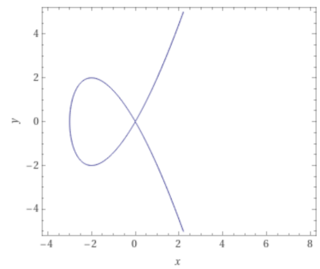
(d) (4 pt) Find the x-coördinate of each inflection point of f.

Exercise 5

(14 pt) The graph of the equation

$$y^2 = x^3 + 3x^2 \tag{1}$$

shown below, is an elliptic curve. It is said to be “singular” (due to its behavior at the origin).



(a) (2 pt) Using the graph, estimate the (x, y)-coördinates of the two points on the graph at which the tangent line to the graph is horizontal.

(b) (4 pt) Compute the rule of assignment for y'.

(c) (4 pt) Using the rule of assignment you computed in part (b), determine, algebraically, the points (x, y) on the graph at which the tangent line is horizontal. Compare these results to your estimates in part (a).

(d) (4 pt) Find an equation for the tangent line to the graph at the point $(6, -18)$.

Exercise 6

(12 pt) Two cyclists, training for Beer Bike, start at the intersection by the Rec Center, rear wheels touching.[1] They start pedaling simultaneously, one cyclist hurtling southwest (toward Rice Stadium) at 9 meters per second (about 21 miles per hour), the other cyclist flying southeast (toward Tudor Fieldhouse and the track, ish) at 12 meters per second (about 28 miles per hour).[2]

(a) (4 pt) Sketch a diagram. Label relevant information.

(b) (4 pt) Write an equation that relates relevant variables and does not (!) involve rates. Justify briefly why this equation is true. Using implicit differentiation, differentiate the equation.

(c) (4 pt) How fast (in meters per second) are the cyclists moving apart 10 seconds after they started? Measure "moving apart" along the straight line connecting the two cyclists at that point in time.

[1] Yes — somehow, their colleges convinced RUPD to close off campus roads to give us (and them) this exercise.

[2] Yes — somehow, these cyclists are able to accelerate instantaneously to racing speed, and hold it perfectly.