Math 211
Quiz 08

W 17 Jul 2019

Your name : _____

Exercise

(2 pt) Consider the 1st-order autonomous ODE

$$\frac{dy}{dt} = -(y+1)(y-1)^2(y-3). \tag{1}$$

Identify the equilibrium solutions, and classify the stability of each. (Mind the leading negative sign in this ODE!)

Solution: By definition, equilibrium solutions are constant functions $y(t) \equiv c$ that solve the given ODE. For the ODE (1), the equilibrium solutions are $y \equiv -1$, $y \equiv 1$, and $y \equiv 3$. By using the ODE (1) to compute the sign of the slope (e.g., for each factor, then multiplying all the results to get the slope $\frac{dy}{dt}$; or computing the value of (1) all at once using y-values in the open intervals cut out by the equilibrium solutions), and being careful with the coefficient -1 in (1), we get the phase line *(insert graphic of phase line)*

$$-\infty \qquad (-) \qquad -1 \qquad (+) \qquad 1 \qquad (+) \qquad 3 \qquad (-) \qquad +\infty.$$

We conclude that the equilibrium solutions and their stability are

- $y \equiv -1$: unstable
- $y \equiv 1$: semistable
- $y \equiv 3$: stable