Math 112
LQuiz 06

2022–01–27 (R)

Your name: _____

Exercise

(4 pt) Consider the function

$$f : \mathbf{R} \to \mathbf{R} \qquad \text{given by} \qquad f(x) = x^3 - x + \sin x$$

(a) (3 pt) Compute the linearization of (aka linear approximation to) f at $x = 0$.

Solution: By definition, the linearization of f at $x = 0$ is the function $L : \mathbf{R} \to \mathbf{R}$ given by

$$L(x) = f(0) + f'(0)(x - 0) \tag{1}$$

We compute

$$f(0) = 0 \qquad f'(x) = 3x^2 - 1 + \cos x \qquad f'(0) = 0$$

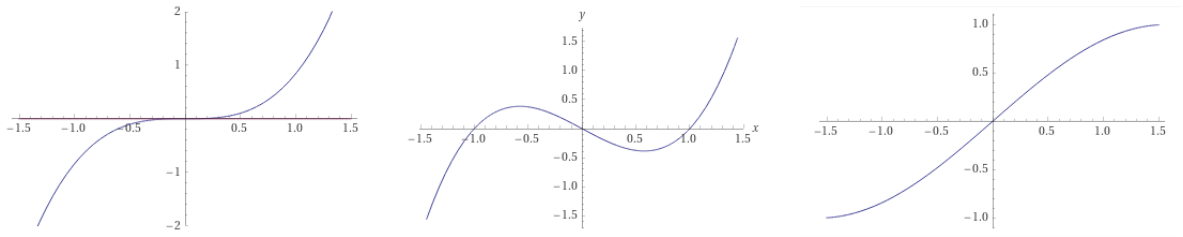
Substituting this results into (1), we conclude that the rule of assignment for L is

$$L(x) = 0$$

(b) (1 pt) Sketch a graph of your linearization of f at $x = 0$. Clearly label the point $(0, f(0))$ and the slope. (While you will not be graded on the following, if you have spare time, try to sketch the graph of f near $x = 0$. Can you weave a coherent story from these parts?)

Solution: The graph of our linearization L is a horizontal line through the point $(0, 0)$, shown in Figure 1(i).

Figure 1 also shows graphs of the function f, as well as two of its "parts", $f_1(x) = x^3 - x$ and $f_2(x) = \sin x$. It is interesting to note that the slope of the tangent line to the graph of f_1 at $x = 0$ seems to cancel the slope of the tangent line to the graph of f_2 at $x = 0$. Does this make sense? Can you make this more precise?



(i) (ii) (iii)

Figure 1: Graphs. (i) The function $f(x) = x^3 - x + \sin x$ (in blue) and the linearization L of f at $x = 0$, given by $L(x) = 0$ (in red). (ii) The part $f_1(x) = x^3 - x$. (iii) The part $f_2(x) = \sin x$.