Math 212
Quiz 34

W 30 Nov 2016

Your name: _____

Exercise

(5 pt) For each of the following integrals, name a theorem that allows you to write an equivalent expression (e.g., integral), write it, and evaluate it. *Guiding light:* Integrating a derivative over a region is related to evaluating the original function on the boundary of that region.

(a) (2.5 pt) Let $\mathbf{F} : \mathbf{R}^3 \to \mathbf{R}^3$ be given by

$$\mathbf{F}(x, y, z) = (xy, 2z, 3y),$$

and let $C \subseteq \mathbf{R}^3$ be the curve of intersection of the plane $x + z = 5$ and the cylinder $x^2 + y^2 = 9$, oriented counterclockwise when viewed from above. Show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 9\pi$.

(b) (2.5 pt) Let $\mathbf{F} : \mathbf{R}^3 \to \mathbf{R}^3$ be given by

$$\mathbf{F}(x, y, z) = \left(x^4, -x^3z^2, 4xy^2z\right),$$

and let $S \subseteq \mathbf{R}^3$ be the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 2$ and $z = 0$, where S is equipped with its outward-pointing unit normal vectors. Show that $\iint_S \mathbf{F} \cdot d\mathbf{S} = \frac{2\pi}{3}$.