# Math 212
# Quiz 24

W 26 Oct 2016

Your name: ______________________________

## Exercise

(2 pt) Let $E \subseteq \mathbf{R}^3$ be the region bounded by (i.e. inside) the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 2$.

(a) (0.5 pt) Sketch the region $E$.

**Solution:** The region $E$ is a solid (classical) cylinder, centered along the $z$-axis, with radius 2, height 2, and base in the $xy$-plane. *(add graphic)*

(b) (1.5 pt) Evaluate the triple integral

$$\iiint_E z e^{x^2+y^2}\, dV.$$

*Hint:* Use cylindrical coordinates. Mind the integration factor.

**Solution:** The integrand $f(x, y, z) = z e^{x^2+y^2}$ is continuous everywhere; in particular, $f$ is continuous on the region $E$ of integration. Hence by Fubini's theorem, we may evaluate the triple integral as an iterated integral using any order of integration. In cylindrical coordinates,

- region of integration: $E = \{(r, \theta, z) \mid 0 \leqslant r \leqslant 2, 0 \leqslant \theta \leqslant 2\pi, 0 \leqslant z \leqslant 2\}$,
- integrand: $z e^{x^2+y^2} = z e^{r^2}$,
- differential: $dV = r\, dr\, d\theta\, dz$ (or any other order of the variables),

so the triple integral writes as

$$\begin{aligned}
\iiint_E z e^{x^2+y^2}\, dV &= \int_{z=0}^{z=2} \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} z e^{r^2}\, r\, dr\, d\theta\, dz \\
&= \int_{z=0}^{z=2} z\, dz \int_{\theta=0}^{\theta=2\pi} d\theta \frac{1}{2} \int_{r=0}^{r=2} e^{r^2} 2r\, dr \\
&= \left[\frac{1}{2} z^2\right]_{z=0}^{z=2} [\theta]_{\theta=0}^{\theta=2\pi} \frac{1}{2} \left[e^{r^2}\right]_{r=0}^{r=2} \\
&= (2)(2\pi)\frac{1}{2}(e^4 - 1) \\
&= 2\pi(e^4 - 1).
\end{aligned}$$