Math 212
Quiz 15

M 03 Oct 2016

Your name: ____________________________________

Exercise

(2 pt) Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by

$$f(x, y) = x^4 - x^2y^2 + y^4.$$

Compute the directional derivative of f at the point $(x, y) = (1, 1)$ in the direction of the vector $\mathbf{v} = (2, 2)$. *Hint:* Note that $\mathbf{v}$ is not a unit vector. Recall the relationship between the directional derivative and the gradient.

Solution: The directional derivative of f at the point (x, y) in the direction of the unit vector $\mathbf{u} = (u_1, u_2)$ is

$$(D_{\mathbf{u}}f)(x, y) = ((\nabla f)(x, y)) \cdot \mathbf{u} = \left(\frac{\partial f}{\partial x}(x, y)\right) u_1 + \left(\frac{\partial f}{\partial y}(x, y)\right) u_2. \tag{1}$$

The gradient of the given function f is[1]

$$\nabla f = \left(4x^3 - 2xy^2, 4y^3 - 2yx^2\right),$$

so

$$(\nabla f)(1, 1) = (2, 2).$$

The unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$ is

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{(2)^2 + (2)^2}}(2, 2) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$

Substituting these results into (1), we obtain

$$(D_{\mathbf{u}}f)(1, 1) = (2)\frac{1}{\sqrt{2}} + (2)\frac{1}{\sqrt{2}} = 2\sqrt{2}.$$

[1] Note that the function f is symmetric in x and y (i.e. $f(x, y) = f(y, x)$), so after computing one of the two partial derivatives, we can obtain the other simply by exchanging x and y.