Math 212
Requiz 22B

F 04 Nov / N 06 Nov

Your name: _____

Exercise

(5 pt) You have joined a cadre of mathematicians designing a lottery. The lottery works as follows: Two real numbers X, Y are randomly chosen from the interval $[0, 2]$ according to the probability density function $f : \mathbf{R}^2 \to \mathbf{R}$ given by

$$f(x, y) = \begin{cases} \frac{1}{8}x^3 y & \text{if } 0 \leqslant x \leqslant 2, 0 \leqslant y \leqslant 2; \\ 0 & \text{otherwise.} \end{cases}$$

If the product of X and Y is greater than 1, then the person wins \$1 million. Otherwise, the person wins nothing.

(a) (1 pt) Sketch the relevant region of integration. Label relevant points of intersection.

(b) (2 pt) Set up an iterated (!) integral that gives the total probability that the person wins.

(c) (1.5 pt) Show that the probability that the person wins is $\frac{225}{256} \approx .8789$. *Hint:* One order of integration may be easier than the other.

(d) (0.5 pt) You want to expect to make money. What should you charge people to play this lottery?