Math 357
Long quiz 05B

2024–04–10 (W)

Your name: _____

Let $f = t^5 + 8t^3 + 2t - 2 \in \mathbf{Q}[t]$.

(a) Prove that f is irreducible in $\mathbf{Q}[t]$.

(b) Prove that f has exactly one real zero. (You need not compute its value.)

(c) Let $\alpha \in \mathbf{C}$ be a zero of f, and let $\beta \in \mathbf{C}$ such that the minimal polynomial of β over $\mathbf{Q}$ has degree 3. (Note that α or β may be real.) Can $\beta \in \mathbf{Q}(\alpha)$? in a splitting field of f? Justify your assertions.