

Math 212

Quiz 22

F 21 Oct 2016

Your name: _____

Exercise

(5 pt) A thin washer (i.e. O-shaped piece of material) is described by the region $D \subseteq \mathbf{R}^2$ lying between the circles

$$C_1 : x^2 + y^2 = 1,$$

$$C_2 : x^2 + y^2 = 4.$$

The charge density of the washer is given by the function $\sigma : D \rightarrow \mathbf{R}$ defined by

$$\sigma(x, y) = \frac{2xy}{x^2 + y^2}.$$

We want to find the total (net) charge of the washer.

- (a) (1 pt) Recall that we recover a quantity (e.g., mass, charge, etc.) by integrating a density. Sketch the relevant region of integration.

- (b) (3 pt) Set up an iterated (!) integral that gives the total (net) charge Q of the washer. *Hint:* Use polar coordinates. Mind the integration factor.

- (c) (1 pt) Evaluate the integral in part (b) to show that the total (net) charge $Q = 0$. *Hint:* Recall that $\sin(2\theta) = 2 \sin \theta \cos \theta$.