Math 211
Quiz 19

R 01 Aug 2019

Your name : ________________________________

Exercise

(5 pt) Match each of the homogeneous 1st-order 2×2 linear systems with its corresponding phase plane in Figures 1 and 2. (N.B. In each ODE, $\mathbf{x} = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is a 2×1 matrix of scalar-valued functions.) *Hint:* (Some of the) distinguishing features of the phase plane are associated with

- eigenvalues and their corresponding eigenvectors of the coefficient matrix;
- nullclines, i.e. lines in the phase plane along which $x_1' = 0$ or $x_2' = 0$; and
- evaluating the original ODE at points (x_1, x_2).

Hint: Recall that in the decomposition $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$, if $\mathbf{D}$ is a diagonal matrix, then column j of the matrix $\mathbf{P}$ is an eigenvector corresponding to (the eigenvalue on) the jth diagonal entry of $\mathbf{D}$.

_____(a) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \mathbf{x}$

_____(b) $\mathbf{x}' = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}$

_____(c) $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{x}$

_____(d) $\mathbf{x}' = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \mathbf{x}$

_____(e) $\mathbf{x}' = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \mathbf{x}$

_____(f) $\mathbf{x}' = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \mathbf{x}$

_____(g) $\mathbf{x}' = \frac{1}{4} \begin{bmatrix} 17 & -5 \\ 5 & -9 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}^{-1} \mathbf{x}$

_____(h) $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+2i & 0 \\ 0 & 1-2i \end{bmatrix} \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix}^{-1}$

_____(i) $\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+2i & 0 \\ 0 & 1-2i \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1}$

_____(j) $\mathbf{x}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1}$

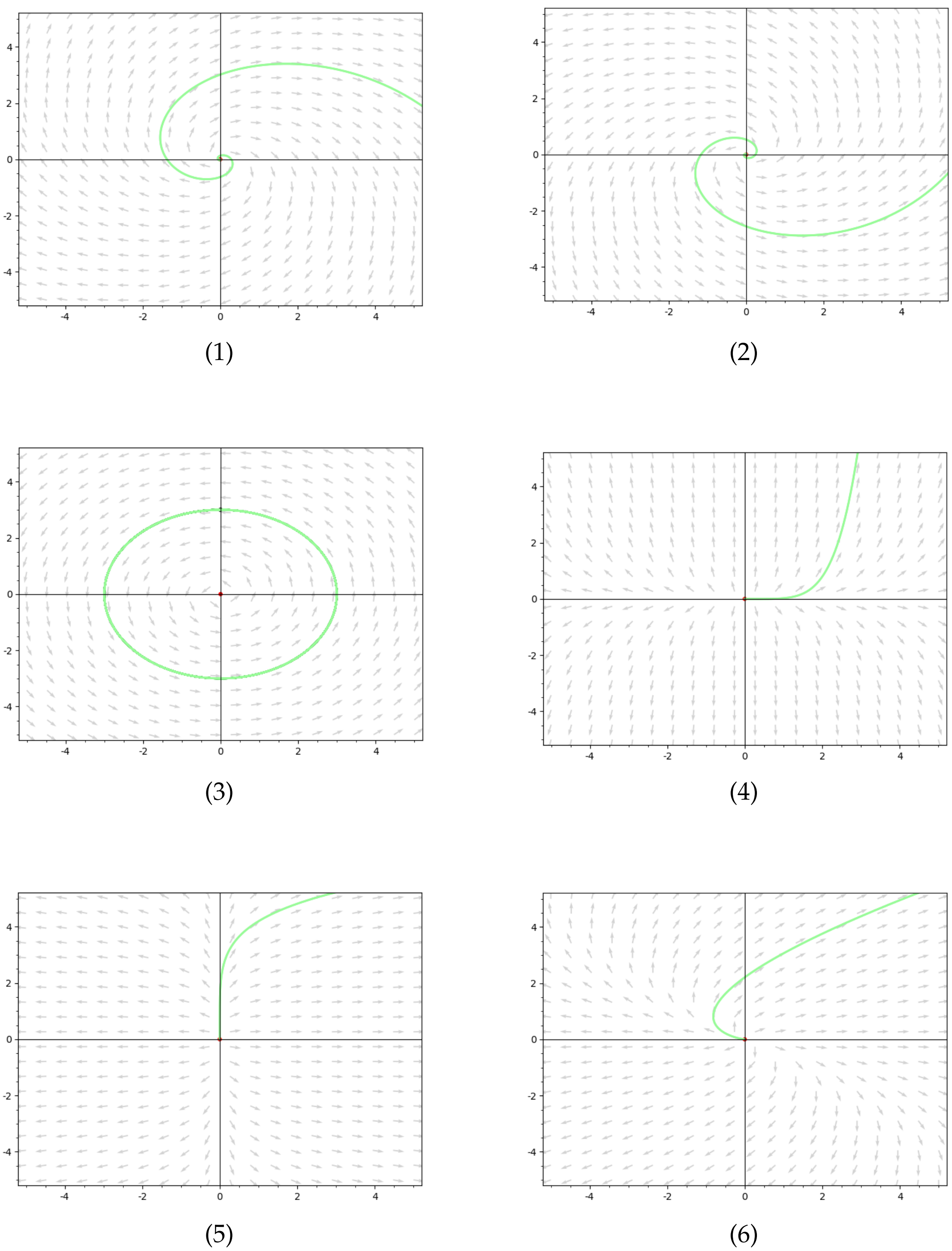
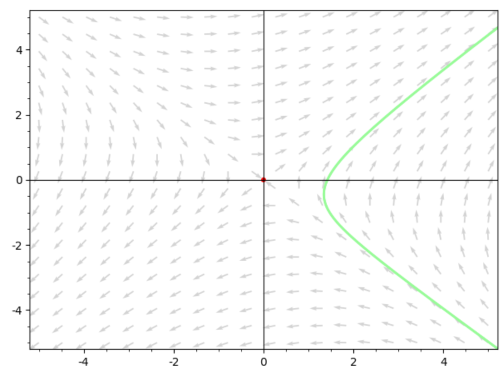


Figure 1: Phase planes for Quiz 19, in the (x_1, x_2) plane.



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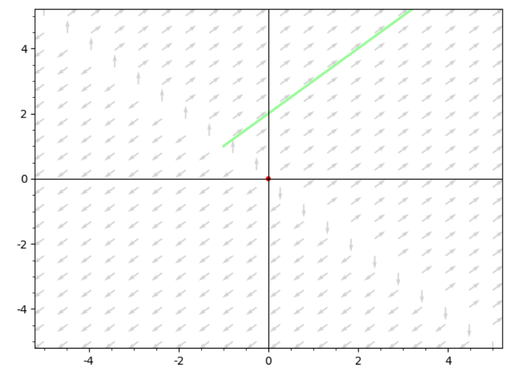

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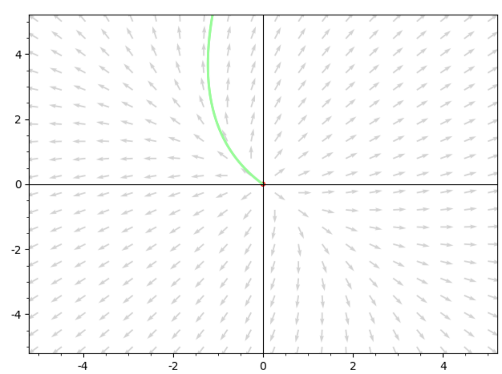

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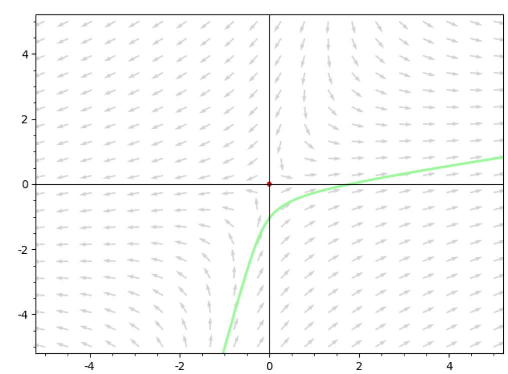

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Figure 2: Phase planes for Quiz 19, in the (x_1, x_2) plane.