Math 112
MockExam 01

2022–02–01 (T)

Your name: _____

Instructions

Number of exercises : 6
Permitted time : 75 minutes
Permitted resources : None

Remarks:

- Manage your time deliberately.
- If the statement of an exercise is unclear to you, briefly (one sentence) write your understanding of the exercise, then proceed.
- Work hard, do your best, and have fun!

Exercise	Total	(a)	(b)	(c)	(d)	(e)
1	/10	/2	/2	/2	/2	/2
2	/8	/4	/4			
3	/12	/4	/4	/4		
4	/16	/4	/4	/4	/4	
5	/12	/4	/4	/4		
6	/12					
Total	/70					

Exercise 1

(10 pt) True/False. For each of the following statements, circle whether it is true or false. No justification is necessary.

(a) (2 pt) The natural logarithm $\ln a$ of a real number a can be negative.

true false

(b) (2 pt) The exponential e^a of a real number a can be negative.

true false

(c) (2 pt) Let f be a function. The domain (aka set of inputs) of its first-derivative function f' includes all points in the domain of f.

true false

For parts (d) and (e), let f be a function defined on an open set containing a point a.

(d) (2 pt) If f is continuous at $x = a$, then $\lim_{x \to a} f(x)$ exists.

true false

(e) (2 pt) If $\lim_{x \to a} f(x)$ exists, then f is continuous at $x = a$.

true false

Exercise 2

(8 pt) Compute the following. (The answers are integers.)

(a) (4 pt) Let

$$e^a = 4\pi \qquad e^b = 6\pi \qquad e^c = 9\pi$$

Compute

$$e^{-3a-b-c} \cdot \frac{e^{5a-b+4c}}{(e^2)^b e^c}$$

(b) (4 pt) Let

$$\ln a = \frac{1}{3} \qquad \ln b = 3 \qquad \ln c = \frac{1}{2}$$

Compute

$$\ln\left(\frac{a^{15}b^{21}c^6}{a^{12}b^{22}c^2}\right) - \ln\left(a^2 - b^2\right) + \ln\left(\frac{a+b}{c}\right) + \ln(ac - bc)$$

Exercise 3

(12 pt) Consider the piecewise function $f : \mathbf{R} \to \mathbf{R}$ whose rule of assignment is

$$f(x) = \begin{cases} x^3 - 6x^2 + 12x - 11 & \text{if } x < 1 \\ x^2 - 5 & \text{if } x \geqslant 1 \end{cases}$$

(a) (4 pt) Find $\lim_{x \to 1} f(x)$. If the limit does not exist, explain. In either case, show your work.

(b) (4 pt) Is f continuous at $x = 1$? Justify.

(c) (4 pt) Is the first-derivative function f' continuous at $x = 1$? Justify.

Exercise 4

(16 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by

$$f(x) = -3x^4 + 4x^3 + 12x^2 - 10$$

(a) (4 pt) Find the interval(s) on which f is increasing and decreasing.

(b) (4 pt) Find the (x, y)-coördinates of each local minimum and maximum of f. State whether each is a local minimum or maximum of f.

(c) (4 pt) Find the global minimum and maximum of f.

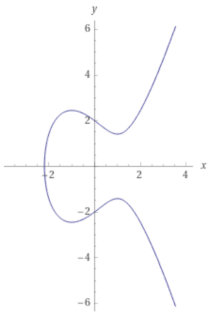
(d) (4 pt) Find the x-coördinate of each inflection point of f.

Exercise 5

(12 pt) The graph of the equation

$$y^2 = x^3 - 3x + 4$$

shown below, is an example of an **elliptic curve**.[1]



(a) (4 pt) Compute the rule of assignment for y'.

(b) (4 pt) The graph suggests that the point $(0, 2)$ is on the elliptic curve, and that the slope of the tangent line there is negative. Show, algebraically, that these statements are true.

(c) (4 pt) Find the linearization to the elliptic curve at the point $(0, 2)$.

[1] Elliptic curves have important applications in digital security (cryptography).

Exercise 6

(12 pt) The base of a triangle is shrinking at a rate of 1 cm/s, and the height of the triangle is increasing at the rate of 5 cm/s. Find the rate at which the area of the triangle changes when the base is 10 cm and the height is 22 cm.