Math 211
Quiz 01

M 08 Jul 2019

Your name: ______________________________

Exam instructions

Number of exercises	: 12
Permitted time	: 30 minutes
Permitted resources	: None

Instructor's note:

- BEFORE SOLVING any exercises, go through the entire quiz and write a "confidence number" 1–5 to the LEFT of each exercise, denoting how confident you are that you can solve the exercise (1 = "Not at all confident", 5 = "Very confident").

1.1	/2	2.1	/2	3.1	/2
1.2	/2	2.2	/2	3.2	/2
1.3	/2	2.3	/2	3.3	/2
1.4	/2	2.4	/2	3.4	/2
Total	/8		/8		/8

Single-Variable Calculus

1.1 Exercise 1.1

(2 pt) Let $a, s \in \mathbf{R}$ such that $s > a$. Evaluate the definite integral $\int_0^{+\infty} e^{-st} e^{at} \, dt$.

1.2 Exercise 1.2

(2 pt) Let $p(t)$ be a continuous function of t, let $v(t)$ be a differentiable function of t, and let

$$y(t) = v(t) e^{-\int p(t) \, dt}.$$

Compute $y'(t)$.

1.3 Exercise 1.3

(2 pt) Evaluate the integral $\int \frac{x^2 + 3x + 2}{(x-3)(x^2+1)}\,dx$.

1.4 Exercise 1.4

(2 pt) Evaluate the integral $3\int x^2 \ln x \, dx$.

Algebra

2.1 Exercise 2.1

(2 pt) Let i satisfy $i^2 = -1$. Write the following in the form $A + iB$, for $A, B \in \mathbf{R}$:

$$(a + ib)e^{\alpha + i\beta}.$$

2.2 Exercise 2.2

(2 pt) Let $f(x) = mx$, where $m \neq 0$. Let x_h be a solution to $f(x) = 0$, and let x_p be a solution to $f(x) = 1$. Let $a \in \mathbf{R}$. Show that $ax_h + x_p$ is also a solution to $f(x) = 1$.

2.3 Exercise 2.3

(2 pt) Write the following system of equations as a matrix equation:

$$x_1 + x_2 - 2x_3 = b_1, \qquad -x_1 + 2x_2 + x_3 = b_2, \qquad x_2 - x_3 = b_3.$$

2.4 Exercise 2.4

(2 pt) Show that for all b_1, b_2, b_3, there exists a unique solution to the system of equations in Exercise 2.3.

Differential Equations

3.1 Exercise 3.1

(2 pt) For the following homogeneous 1st-order autonomous ODE, determine the equilibrium values and classify the stability of each.

$$y' = (y-1)(y-3)(y-5)^2.$$

3.2 Exercise 3.2

(2 pt) Find the general solution to the homogeneous 1st-order linear system of ODEs

$$\mathbf{x}' = \begin{bmatrix} -3 & 2 \\ -1 & -5 \end{bmatrix} \mathbf{x}.$$

3.3 Exercise 3.3

(2 pt) Find the general solution $y \in \mathbf{R}[t]$ to the homogeneous 4th-order ODE

$$y^{(4)} - 3y^{(2)} - 4y = 0.$$

3.4 Exercise 3.4

(2 pt) Using the laplace transform, solve the following homogeneous 3rd-order ODE IVP:

$$y^{(3)} + 4y^{(2)} - 5y^{(1)} = 0, \qquad y(0) = 4, \qquad y'(0) = -7, \qquad y''(0) = 23.$$