Math 112
LQuiz 13

2022–03–24 (R)

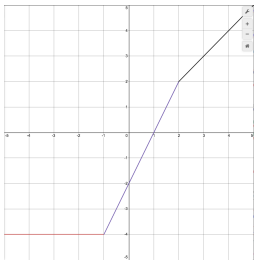
Your name: _____

Exercise

(4 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be the piecewise function whose rule of assignment is

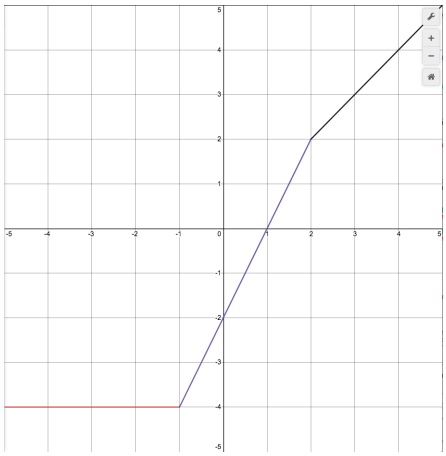
$$f(x) = \begin{cases} -4 & \text{if } x \leqslant -1 \\ 2x - 2 & \text{if } -1 \leqslant x \leqslant 2 \\ x & \text{if } x \geqslant 2 \end{cases}$$

The function f is graphed below. This exercise explores the signed area under the graph of f from $x = -3$ to $x = 4$.

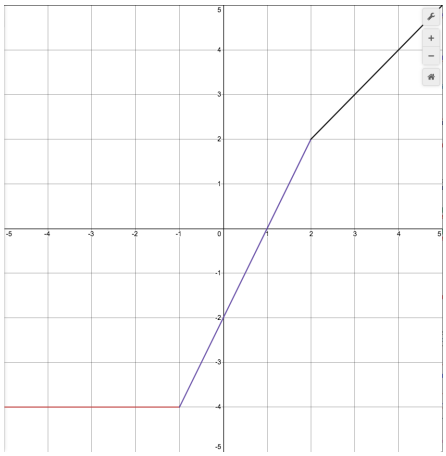


(a) (1 pt) Use geometry to show that $\int_{-3}^{4} f(x)\,dx = -5$.

(b) (2 pt) On separate graphs below, draw a lower sum and an upper sum, each with seven intervals of width 1, to estimate $\int_{-3}^{4} f(x)\,dx$. Show that $L \leqslant \int_{-3}^{4} f(x)\,dx \leqslant U$.



Lower sum (L)



Upper sum (U)

For reference, $f : \mathbf{R} \to \mathbf{R}$ is the piecewise function whose rule of assignment is

$$f(x) = \begin{cases} -4 & \text{if } x \leqslant -1 \\ 2x - 2 & \text{if } -1 \leqslant x \leqslant 2 \\ x & \text{if } x \geqslant 2 \end{cases}$$

(c) (1 pt) Find an antiderivative $F_i(x)$ for each "piece" of $f(x)$. Use these antiderivatives and the fundamental theorem of calculus to compute the value of each integral on the right side of the following equation:

$$\int_{-3}^{4} f(x)\, dx = \int_{-3}^{-1} f(x)\, dx + \int_{-1}^{2} f(x)\, dx + \int_{2}^{4} f(x)\, dx \tag{1}$$

Add your values for the three integrals on the right, and compare the result to part (a).