Math 212
Quiz 17

F 07 Oct 2016

Your name: ____

Exercise

(5 pt) Find the global minimum and maximum values of the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by

$$f(x, y) = x^2 y + xy^2 - xy$$

on the closed set $D \subseteq \mathbf{R}^2$ given by

$$D = \left\{ (x, y) \in \mathbf{R}^2 \,|\, x \geqslant 0, y \geqslant 0, x + y \leqslant 2 \right\},$$

a 45-45-90 right triangle with side length 2 in the first quadrant of the xy-plane. We'll do this in steps.

(a) (1 pt) Justify why a global minimum and maximum exist in this case. *Hint:* Name that theorem, and validate its hypotheses.

(b) (2 pt) Find all critical points on the interior of D. *Hint:* As in single-variable optimization, do this by setting the appropriate notion of "derivative of f" equal to (the appropriate notion of) zero. Note that in the interior of D, $x \neq 0$ and $y \neq 0$. The derivative equal to zero gives a system of two equations, which will yield a unique solution — our critical point.

(c) (1 pt) Find all critical points on the boundary of D. *Hint:* Note that $f(x, y) = 0$ along the boundary components of D where $x = 0$ or $y = 0$. Thus we need only consider the boundary component $x + y = 2$. Solve for y as a function of x (or vice versa), substitute into f to obtain a function of a single variable, and optimize this using single-variable calculus. Again you should find a unique critical point.

(d) (1 pt) State the global minimum and maximum values of f on D. *Hint:* Compare values of f at points from (b) and (c).