Math 212
Quiz 18

W 12 Oct 2016

Your name: ______________________________

Exercise

(2 pt) Let $f : \mathbf{R}^2 \to \mathbf{R}$ be given by

$$f(x, y) = 3x^2 - 6xy^2 + 2y,$$

and let $R \subseteq \mathbf{R}^2$ be the rectangle

$$R = \left\{(x, y) \in \mathbf{R}^2 \,|\, 0 \leqslant x \leqslant 2, 0 \leqslant y \leqslant 1\right\}.$$

We wish to evaluate the double integral $\iint_R f(x, y)\, dA$.

(a) (0.5 pt) Justify why you can write the double integral as an iterated integral. *Hint:* Two words, rhymes with "Houdini's serum". Heart points for noting a sufficient condition on f.

Solution: Fubini's theorem. More precisely, the integrand $f(x, y)$ is a polynomial, hence continuous everywhere on its domain, and in particular on the region R of integration.

(b) (1.5 pt) Evaluate the integral $\iint_R f(x, y)\, dA$.

Solution: By Fubini's theorem, we may evaluate the double integral as an iterated integral in either order. Using the order $dx\, dy$, we compute

$$\begin{aligned}
\iint_R f(x, y)\, dA &= \int_{y=0}^{y=1} \int_{x=0}^{x=2} \left(3x^2 - 6xy^2 + 2y\right) dx\, dy \\
&= \int_{y=0}^{y=1} \left[x^3 - 3x^2y^2 + 2xy\right]_{x=0}^{x=2} dy \\
&= \int_{y=0}^{y=1} \left(8 - 12y^2 + 4y\right)\, dy \\
&= \left[8y - 4y^3 + 2y^2\right]_{y=0}^{y=1} \\
&= 6.
\end{aligned}$$

If instead we integrate using the order $dy\, dx$, then we obtain

$$\begin{aligned}
\iint_R f(x, y)\, dA &= \int_{x=0}^{x=2} \int_{y=0}^{y=1} \left(3x^2 - 6xy^2 + 2y\right)\, dy\, dx \\
&= \int_{x=0}^{x=2} \left[3x^2y - 2xy^3 + y^2\right]_{y=0}^{y=1} dx \\
&= \int_{x=0}^{x=2} \left(3x^2 - 2x + 1\right)\, dx \\
&= \left[x^3 - x^2 + x\right]_{x=0}^{x=2} \\
&= 6.
\end{aligned}$$

Note that both orders of integration yield the same result, as required by Fubini's theorem.