Math 211
Quiz 02

T 09 Jul 2019

Your name: ____________________________________

Exercise

(2 pt) Consider the 1st-order linear ODE

$$\frac{dy}{dt} = 2ty + 3te^{t^2}. \tag{1}$$

Confirm that the function[1]

$$\begin{aligned} y : \mathbf{R} &\to \mathbf{R} \\ t &\mapsto \left(\frac{3}{2}t^2 + 1\right) e^{t^2} \end{aligned}$$

is a solution to (1), and that the solution satisfies the initial condition $y(0) = 1$.

Solution: Let's do the easy part first: Check the initial condition. We evaluate

$$y(0) = \left(\frac{3}{2}0^2 + 1\right) e^{0^2} = (0+1)1 = 1,$$

as required by the initial condition. Next, we check that $y(t)$ is a solution to (1), by plugging it into both sides of the ODE. On the left side, we compute the derivative using the product rule and chain rule:

$$\frac{dy}{dt} = (3t)e^{t^2} + \left(\frac{3}{2}t^2 + 1\right)(2t)e^{t^2}.$$

On the right side, we plug in $y(t)$:

$$2t\left(\frac{3}{2}t^2 + 1\right) e^{t^2} + 3te^{t^2}.$$

These expressions are the same,[2] i.e. $y(t)$ satisfies equation (1), so by definition it is a solution.

[1]This notation says that y is a function with domain (input) all real numbers, codomain (output) real numbers, and rule of assignment given by

$$y(t) = \left(\frac{3}{2}t^2 + 1\right) e^{t^2}.$$

[2]Both expressions simplify to

$$te^{t^2}\left(3t^2 + 5\right).$$