Math 211
Quiz 20

F 02 Aug 2019

Your name : ________________________________

Exercise

(2 pt) Match each of the homogeneous 1st-order 2×2 linear systems with its corresponding phase plane in Figure 1. (N.B. In each ODE, $\mathbf{x} = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is a 2×1 matrix of scalar-valued functions.) *Hint:* (Some of the) distinguishing features of the phase plane are associated with

- eigenvalues and their corresponding eigenvectors of the coefficient matrix;
- nullclines, i.e. lines in the phase plane along which $x_1' = 0$ or $x_2' = 0$; and
- evaluating the original ODE at points (x_1, x_2).

Hint: Recall that in the decomposition $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$, if $\mathbf{D}$ is a diagonal matrix, then column j of the matrix $\mathbf{P}$ is an eigenvector corresponding to (the eigenvalue on) the jth diagonal entry of $\mathbf{D}$.

(a) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}^{-1} \mathbf{x}$

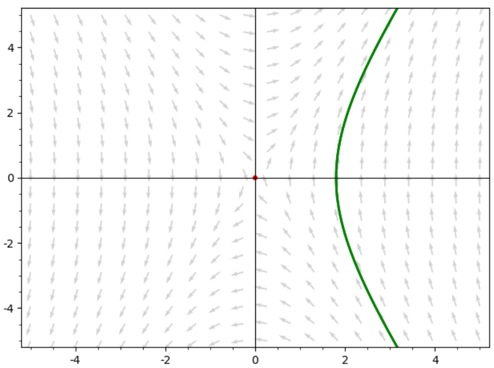
(b) $\mathbf{x}' = \begin{bmatrix} 0 & -1 \\ -4 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}^{-1} \mathbf{x}$

(c) $\mathbf{x}' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+i & 0 \\ 0 & 1-i \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1} \mathbf{x}$

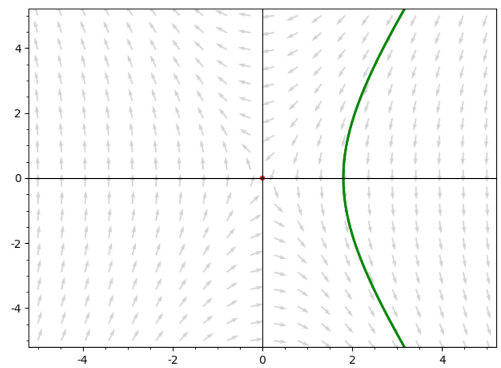
(d) $\mathbf{x}' = \begin{bmatrix} 1 & -10 \\ 10 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+10i & 0 \\ 0 & 1-10i \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1} \mathbf{x}$

(e) $\mathbf{x}' = \begin{bmatrix} -4 & 6 \\ -3 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1-i & 1+i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1+3i & 0 \\ 0 & -1-3i \end{bmatrix} \begin{bmatrix} 1-i & 1+i \\ 1 & 1 \end{bmatrix}^{-1} \mathbf{x}$

(f) $\mathbf{x}' = \begin{bmatrix} 2 & -6 \\ 3 & -4 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1+i & 1-i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1+3i & 0 \\ 0 & -1-3i \end{bmatrix} \begin{bmatrix} 1+i & 1-i \\ 1 & 1 \end{bmatrix}^{-1} \mathbf{x}$



(1)



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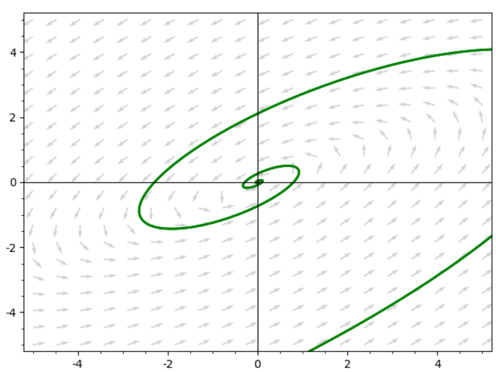

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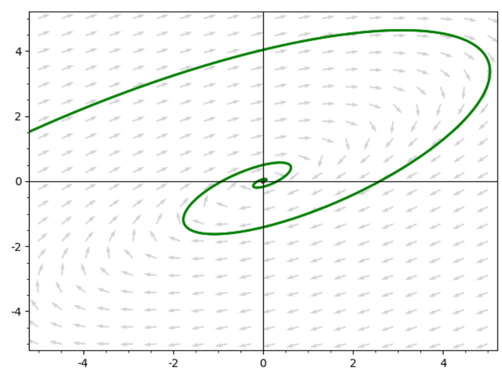

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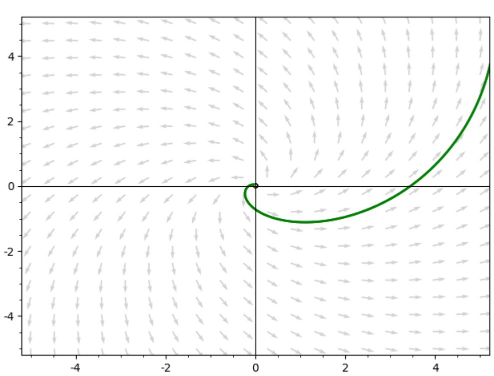

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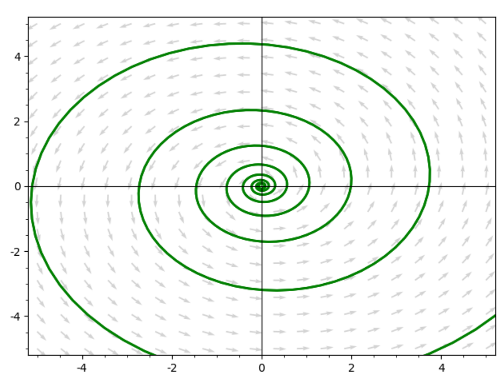

(6)

Figure 1: Phase planes for Quiz 20, in the (x_1, x_2) plane.