Math 357 Exam 00

2024-01-08 (M)

Instructions

- 1. In the space above, please legibly write your name and the Rice Honor Pledge, then sign.
- 2. Next, please read each exercise and, in the left margin beside each, write an integer 0–4 indicating how confident you feel with that exercise, with 0 indicating "not yet confident" and 4 indicating "very confident".
- 3. Please use the remaining time to work on exercises of your choice.

Exercise	Total	(a)	(b)	(c)
1	/4			
2	/6	/2	/2	/2
3	/6	/2	/4	
4	/4			
5	/6	/2	/4	
6	/6	/2	/4	
7	/6	/2	/4	
8	/4			
9	/4			
10	/4			

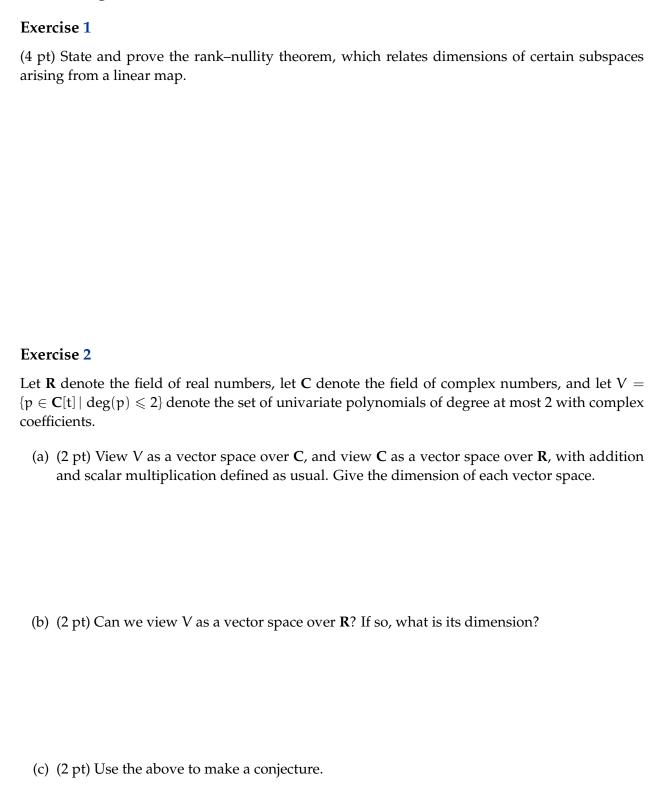
 \neg

 $\overline{}$

 $\overline{}$

 $\overline{}$

Linear algebra



Group theory

•	•	_
Exe	rcica	1
LAC	LCIST	

(a) (2 pt) Define a normal subgroup. Explain its importance	(a)	(2 pt) De	fine a normal	subgroup.	Explain	its importance.
---	-----	-----------	---------------	-----------	---------	-----------------

(b) (4 pt) From the same "parent" group of your choice, give two examples of proper, nontrivial subgroups, one of which is normal, the other of which is not. Justify your assertions.

Exercise 4

(4 pt) From the symmetric group S_4 , give an example of a cyclic subgroup of order 4 and, separately, a noncyclic subgroup of order 4. Justify your assertions.

Ring theory

Exercise 5

(a) (2 pt) Define "integral domain" and, separately, "field".

(b) (4 pt) Prove that every finite integral domain is a field.

Exercise 6

(a) (2 pt) Define "euclidean domain" and, separately, "principal ideal domain".

(b) (4 pt) Prove that a euclidean domain is a principal ideal domain.
Exercise 7
Let A be an integral domain.
(a) (2 pt) Define what it means for an element of A to be prime and, separately, irreducible.
(a) (4 pt) Relate, as fully as possible, the notions of prime and irreducible elements. Provide proof or counterexample for your assertions.

Math 357



(4 pt) Let $p \in \mathbf{Z}_{>0}$ be prime. Prove that the polynomial $f(t) = \sum_{j=0}^{p-1} t^j$ is irreducible in $\mathbf{Q}[t]$.

Exercise 9

(4 pt) Construct a character table for the symmetric group S₃.

Exercise 10

(4 pt) Let $p(t) = t^3 - 2 \in \mathbf{Q}[t]$, and let K denote the splitting field of p over \mathbf{Q} . Draw a diagram of intermediate fields of $K|\mathbf{Q}$ and a diagram of subgroups of the galois group $Gal(K|\mathbf{Q})$. In your diagrams, indicate all normal subgroups and all galois extensions of \mathbf{Q} .