Math 211
Quiz 17A

M 05 Aug 2019

Your name : _____

Exercise

(5 pt) For each of the following linear maps $T : \mathbf{R}^n \to \mathbf{R}^m$, where $\mathbf{R}^n, \mathbf{R}^m$ are viewed as vector spaces over $\mathbf{R}$,

(i) write a basis for the image $\mathrm{im}(T)$ and a basis for the kernel $\ker(T)$, and

(ii) confirm the rank–nullity theorem.

(a) (2.5 pt) The linear map T_1 given by

$$T_1 : \mathbf{R}^3 \to \mathbf{R}^5$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_1 & + & & & x_3 \\ x_1 & + & x_2 & - & x_3 \\ & & x_2 & - & 2x_3 \\ & & x_2 & - & 2x_3 \\ & & -2x_2 & + & 4x_3 \end{bmatrix}.$$

(b) (2.5 pt) The linear map T_2 given by

$$T_2 : \mathbf{R}^4 \to \mathbf{R}^4$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} x_1 & + & & & x_3 & & \\ x_1 & + & x_2 & + & 2x_3 & + & 3x_4 \\ 2x_1 & + & & & 2x_3 & + & 2x_4 \\ & & & & & & 4x_4 \end{bmatrix}.$$