Math 357
Long quiz 05

2024–03–25 (M)

Your name: _____

Let $p = t^3 - 3t - 1 \in \mathbf{Q}[t]$.

(a) Prove that p is irreducible.

(b) Prove that p has three distinct zeros in $\mathbf{R}$. (You need not compute them.)

(c) Let $\alpha \in \mathbf{R}$ be a zero of p. Prove that $\sqrt{2} \notin \mathbf{Q}(\alpha)$.