Math 212 Requiz 28A

F 11 Nov 2016 / N 13 Nov 2016

Exercise

(5 pt) Let $D \subseteq \mathbf{R}^2$ be the region bounded by the curves

$$y - x = 0,$$
 $y - x = 1,$ $xy = 1,$ $xy = 2.$

Show that

$$\iint_D \left(x^2-y^2\right) dA = -\frac{1}{2}.$$

Hint: Apply a change of variables. Let the equations of the boundary of the region R guide your definition of new variables u, v as functions of the given variables x, y. To compute the Jacobian determinant, either (i) solve for x, y as functions of u, v; or (ii) use the fact that

$$\det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \left(\det\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} \right)^{-1}.$$