

Math 112
ReQuiz 05B06B

2022-02-13 (N)

Your name: _____

Exercise

(4 pt) Consider the function

$$f : \mathbf{R} \rightarrow \mathbf{R} \quad \text{given by} \quad f(x) = e^{-2x} + x^3 + 4x + 3$$

(a) (3 pt) Compute the linearization of (aka linear approximation to) f at $x = 0$.

Solution: By definition, the linearization of f at $x = 0$ is the function $L : \mathbf{R} \rightarrow \mathbf{R}$ given by

$$L(x) = f(0) + f'(0)(x - 0) \tag{1}$$

We compute

$$f(0) = e^{-2(0)} + 0 + 0 + 3 = 4$$

$$f'(x) = -2e^{-2x} + 3x^2 + 4$$

$$f'(0) = -2e^{-2(0)} + 0 + 4 = 2$$

Substituting these results into Equation (1), we find that the rule of assignment for L is

$$L(x) = 4 + 2x$$

(b) (1 pt) Use your linearization from part (a) to approximate the value $f(1)$. Find the error in this approximation. (Where relevant, you may leave computations in terms of e .)

Solution: We compute

$$L(1) = 4 + 2(1) = 6 \quad \text{and} \quad f(1) = e^{-2(1)} + (1)^3 + 4(1) + 3 = 8 + e^{-2}$$

The error ε in the approximation $L(1)$ to $f(1)$ is

$$\varepsilon = L(1) - f(1) = 6 - (8 + e^{-2}) = -2 - e^{-2}$$

This error is negative, indicating that our estimate $L(1)$ is less than the actual value $f(1)$. That is, our estimate is too small.