

Math 357
Short quiz 02

2024-01-12 (F)

Your name: _____

You join your friends for a meal. You mention that you're studying ring theory.

- (a) One (particularly opinionated) friend advises, "You don't need to study rings. Just work with abelian groups and define multiplication to always be zero." What do you respond? In particular, is what your friend proposes a ring?

Solution: It is straightforward to check that the proposed structure indeed satisfies the ring axioms. Such rings are sometimes called **trivial rings**, as the multiplication operation does not add any new results beyond those of abelian groups. I thank my friend for the unsolicited advice, which led to this perhaps surprising conclusion, and proceed peacefully with my meal.

- (b) Another friend muses aloud, "Could the structure just proposed—be it a ring or something else—have a multiplicative identity?" What do you respond?

Solution: Yes, it could, but only if $1 = 0$, in which case we necessarily have the zero ring. (What contradiction arises if we assume a multiplicative identity $1 \neq 0$?) I invite both friends to join me in my study of ring theory before gracefully turning our mealtime conversation to other topics.