Math 112
LQuiz 14

2022–03–29 (T)

Your name: _____

Exercise

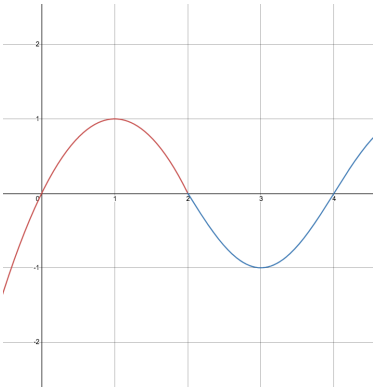
(4 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be the function whose rule of assignment is

$$f(x) = \begin{cases} 2x - x^2 & \text{if } x \leqslant 2 \\ \sin\left(\frac{\pi}{2}x\right) & \text{if } x \geqslant 2 \end{cases}$$

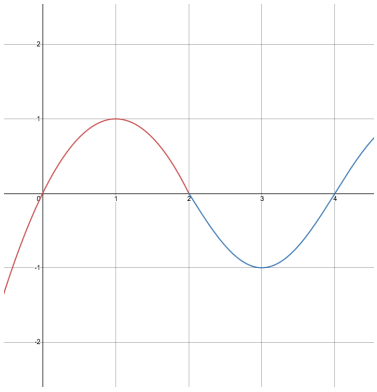
The function f is graphed below. This exercise explores the signed area under the graph of f from $x = 0$ to $x = 4$.

(a) (1 pt) Briefly (!) explain why we can't use finite geometry to find the exact value of $\int_0^4 f(x)\,dx$.

(b) (1 pt) On separate graphs below, draw a lower sum and an upper sum, each with four intervals of width 1. Use these to compute a lower and upper estimate for $\int_0^4 f(x)\,dx$.



Lower sum (L)



Upper sum (U)

(c) (2 pt) Find an antiderivative $F_i(x)$ for each "piece" of $f(x)$. Use these antiderivatives and the fundamental theorem of calculus to compute the integrals on the right side of

$$\int_0^4 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx \tag{1}$$

Add your results to determine the integral on the left side. Show that $L \leqslant \int_0^4 f(x)\,dx \leqslant U$..