# Math 112
# LQuiz 07

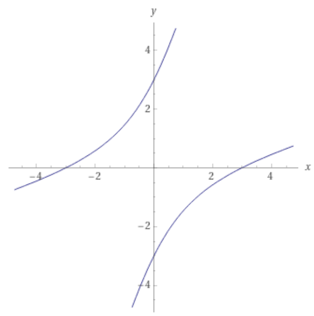
2022–02–01 (T)

Your name: ______________________________

## Exercise

(4 pt) Consider the hyperbola graphed below, which is given by the equation

$$x^2 + y^2 = 4xy + 9 \tag{1}$$



(a) (1 pt) From the graph, the point $(x, y) = (3, 0)$ appears to be on the hyperbola. Prove this, algebraically.

**Solution:** A point $(x, y)$ is on the graph of Equation (1) (a geometric condition) if and only if the point $(x, y)$ satisfies Equation (1) (an algebraic condition). Evaluating Equation (1) at $x = 3$ and $y = 0$, we find

$$3^2 + 0^2 = 4(3)(0) + 9 \qquad \Leftrightarrow \qquad 9 = 9$$

Algebraically, the point $(3, 0)$ satisfies Equation (1). Thus geometrically, the point $(3, 0)$ is on the graph of Equation (1).

(b) (1 pt) From the graph, what can we predict about the slope of the tangent line to the graph at the point $(3, 0)$?

**Solution:** The slope of the tangent line to the graph at the point $(3, 0)$ looks positive, and less than 1. If we sketch the tangent line and estimate two points on it — for example, the point we're given, $(3, 0)$, and the point $(-1, -2)$ — then we can give a numerical approximation for the slope:

$$\text{slope} \approx \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{-1 - 3} = \frac{-2}{-4} = \frac{1}{2}$$

(c) (2 pt) Compute the rule of assignment for $y'$. (Your answer will involve both $x$ and $y$.) Show that $y'$ evaluated at the point $(x, y) = (3, 0)$ equals $\frac{1}{2}$.

**Solution:** We use implicit differentiation. Differentiating both sides of Equation (1) with respect to $x$ (remember that $y$ is an implicit function of $x$) and solving for $y'$, we get

$$2x + 2yy' = 4y + 4xy' + 0 \qquad \Leftrightarrow \qquad y' = \frac{4y - 2x}{2y - 4x} = -\frac{x - 2y}{y - 2x}$$

Evaluating $y'$ at the point $(x, y) = (3, 0)$, we find

$$y' = -\frac{3 - 2(0)}{0 - 2(3)} = -\frac{3}{-6} = \frac{1}{2}$$

We note that our algebraic result here is consistent with our geometric hypothesis in Part (b).