Math 357
Long quiz 04B

2024–02–26 (M)

Your name: ____________________________________

Let $\mathbf{Q}$ denote the field of rational numbers; given a prime $p \in \mathbf{Z}_{>0}$, let $\mathbf{F}_p \cong \mathbf{Z}/(p)$ denote the finite field with p elements; and let t be an indeterminate. For each of the quotient rings below, characterize its algebraic structure as "field", "integral domain but not field", or "ring but not integral domain". Justify your characterization.

$$\begin{aligned}
R_1 &= \mathbf{F}_3[t]/(t^4 + t^3 + t^2 + 1) \\
R_2 &= \mathbf{Q}[t]/(3t^3 - 6t^2 + 7t + 8) \\
R_3 &= \mathbf{Q}[t]/(t^4 - 4t^3 + 6t^2 - t + 28)
\end{aligned}$$

Hint: If you feel inclined to do a lot of computation, then I invite you to first check with me.