Math 357
Long quiz 02

2024–01–29 (M)

Your name: _____

Consider $\mathbf{R}[x, y]$, the polynomial ring in two indeterminates x, y whose ring of coefficients is the field $\mathbf{R}$ of real numbers. Let $f, g \in \mathbf{R}[x, y]$ be the polynomials

$$f(x, y) = x^2y - xy - xy^3 \qquad\qquad g(x, y) = x^2 - xy - 2y^2$$

(a) For each polynomial, state its (total) degree and its number of homogeneous components.

(b) Consider the following statement: "If a polynomial is homogeneous, then the zeros of the induced function are well defined on lines through the origin." Use the polynomials f and g to explain this statement. *Hint:* What is $\{\lambda(x_0, y_0) \mid \lambda \in \mathbf{R}\}$?

(c) Make a conjecture.