

Math 112
LQuiz 12

2022-03-03 (R)

Your name: _____

Exercise

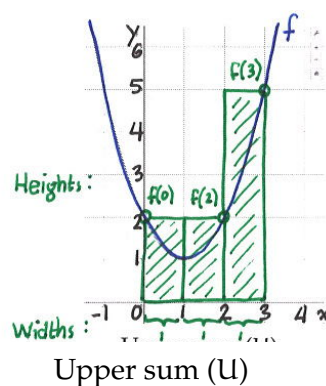
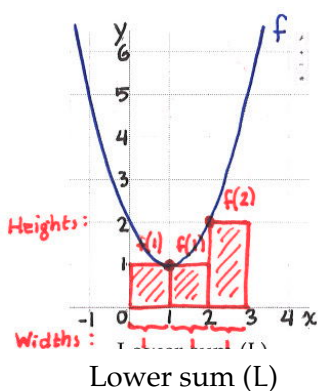
(4 pt) Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be the function whose rule of assignment is

$$f(x) = x^2 - 2x + 2$$

This exercise explores the area under the graph of f from $x = 0$ to $x = 3$, that is,

$$\int_0^3 f(x) \, dx \quad (1)$$

- (a) (2 pt) The function f is graphed below, twice. Draw a lower sum and an upper sum, on separate graphs, each with three subintervals of length 1, for the definite integral in (1). Compute the values L and U , respectively, of these two sums.



Solution: The lower sum L has value (area) 4. More precisely,

$$L = f(1) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 = 1 \cdot 1 + 1 \cdot 1 + 2 \cdot 1 = 4$$

The upper sum U has value (area) 9. More precisely,

$$U = f(0) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 = 2 \cdot 1 + 2 \cdot 1 + 5 \cdot 1 = 9$$

- (b) (2 pt) Find an antiderivative $F(x)$ of $f(x)$. Compute the difference $F(3) - F(0)$. Compare the result to the lower and upper sum you computed in part (a).

Solution: The function $F : \mathbf{R} \rightarrow \mathbf{R}$ given by

$$F(x) = \frac{1}{3}x^3 - x^2 + 2x$$

is an antiderivative of $f(x)$, as we can confirm by differentiating (that is, $F'(x) = f(x)$). We compute

$$F(3) = 6$$

$$F(0) = 0$$

$$F(3) - F(0) = 6 - 0 = 6$$

This value lies between the lower sum L and upper sum U we computed in part (a):

$$L = 4 \leq F(3) - F(0) \leq 9 = U$$

N.B. This difference $F(3) - F(0)$ is the exact area under the graph of f from $x = 0$ to $x = 3$.