Math 357
Long quiz 02A

2024–02–05 (M)

Your name: _____

(a) Consider the polynomial ring $\mathbf{Z}[t_1, t_2, t_3]$ ($\mathbf{Z}$ denotes the integers). For each of the following polynomials, state its (total) degree and its number of (nonzero) homogeneous components.

$$f = t_1^3 + t_1 t_2 t_3 + 2t_2^2 t_3^2 - t_2 t_3^3 + t_3^2$$
$$g = (t_1^2 + t_2^2 + t_3^2)^3 - (t_1^3 + t_2^3 + t_3^3)^2$$

Hint: Think before you compute.

(b) By popular demand, you are explaining ideals to a group of your friends. One of them exclaims, "Ah! So the ideal of all polynomials whose terms all have even (total) degree is an analog, in polynomial rings, to the ideal of even integers in $\mathbf{Z}$." Respond.