Math 357
Long quiz 01C

2024–04–15 (M)

Your name: _____

Let R be a commutative ring with a multiplicative identity $1_R \neq 0_R$. An element $a \in R$ is **nilpotent** if there exists an $n \in \mathbf{Z}_{>0}$ such that $a^n = 0_R$. The **nilradical** of R is the set

$$\mathrm{Nil}(R) = \{a \in R \mid a \text{ is nilpotent}\}$$

(a) Prove that $\mathrm{Nil}(R)$ is an ideal of R.

(b) Prove that $\mathrm{Nil}(R/\mathrm{Nil}(R)) = \{0\}$.