

Math 357
Long quiz 04A

2024-02-19 (M)

Your name: _____

Let \mathbf{Q} denote the field of rational numbers; given a prime $p \in \mathbf{Z}_{>0}$, let $\mathbf{F}_p \cong \mathbf{Z}/(p)$ denote the finite field with p elements; and let t be an indeterminate. For each of the quotient rings below, characterize its algebraic structure as “field”, “integral domain but not field”, or “ring but not integral domain”. Justify your characterization.

$$R_1 = \mathbf{F}_2[t]/(t^4 + t^2 + 1) \quad R_2 = \mathbf{Q}[t]/(t^3 + t^2 - t + 1) \quad R_3 = \mathbf{Q}[t]/(3t^3 + 4t^2 + 2t - 4)$$

Hint: If you feel inclined to do a lot of computation, then I invite you to first check with me.