Math 211
Quiz 21

M 05 Aug 2019

Your name : _____

Exercise

(5 pt) Consider the homogeneous 1st-order 2×2 linear system of ODEs

$$\mathbf{x}' = \begin{bmatrix} -1 & -4 \\ 4 & -1 \end{bmatrix} \mathbf{x}. \tag{1}$$

(a) (2 pt) Diagonalize the coefficient matrix (i.e. compute the eigenvalues and corresponding eigenvectors of the given matrix $\mathbf{A}$, and write it in the form $\mathbf{A} = \mathbf{PDP}^{-1}$).

(b) (2 pt) Use this information to write the general real solution $\mathbf{x}(t)$ to (1). *Hint:* Recall that when the eigenvalues of a homogeneous 2×2 linear system are complex, we can compute one solution $e^{\lambda t}v$, just as we did with real eigenvalues; decompose it into real and imaginary parts; then use these parts as our basis for the solution space. The formulas

$$e^{a+ib} = e^a e^{ib}, \qquad\qquad e^{ib} = \cos b + i\sin b,$$

will be useful.

(c) (1 pt) Quickly sketch the phase plane. Your sketch need not be exact; focus on whether trajectories move toward or away from the origin, and the direction of rotation.