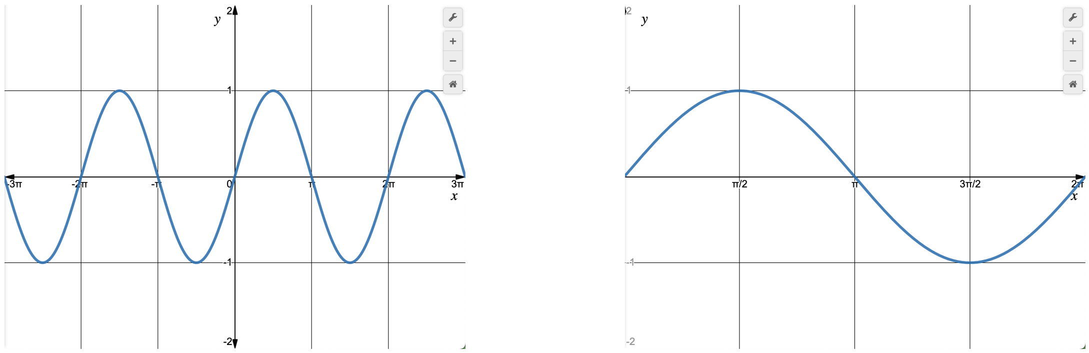
Math 112
Exam 03

2022–04–14 (R)

Your name: ______________________________

Exercise	Total	(a)	(b)	(c)	(d)	(e)
1	/10	/2	/2	/2	/2	/2
2	/12	/4	/4	/4		
3	/12	/4	/4	/4		
4	/8	/4	/4			
5	/10	/4	/4	/2		
6	/8	/4	/4			
Total	/60					



Graph of $g(x)$ for parts (a)–(b). Graph of $F(x)$ for parts (c)–(e).

Figure 1: Graphs for Exercise 1.

Exercise 1

(10 pt) True/False. For each of the following statements, circle whether it is true or false. No justification is necessary.

For parts (a)–(b), let $g : \mathbf{R} \to \mathbf{R}$ be the function given by $g(x) = \sin x$, graphed in Figure 1.

(a) (2 pt) $\displaystyle\int_{-\pi}^{\pi} g(x)\,dx = 0$

true false

(b) (2 pt) For every positive real number a, $\displaystyle\int_{-a}^{a} g(x)\,dx = 0$.

true false

For parts (c)–(e), let $f : [0, 2\pi] \to \mathbf{R}$ be a continuous function, and let $F : [0, 2\pi] \to \mathbf{R}$ be the cumulative signed area function graphed in Figure 1, given by

$$F(x) = \int_0^x f(t)\,dt$$

(c) (2 pt) For all x in the range $0 \leqslant x \leqslant \pi$, $f(x) > 0$.

true false

(d) (2 pt) On the interval $[0, 2\pi]$, $f(x) = 0$ at exactly two points.

true false

(e) (2 pt) The average value of f on the interval $[0, 2\pi]$ equals 0.

true false

Exercise 2

(12 pt) Evaluate each indefinite integral. Clearly communicate your approach.

(a) (4 pt) $\int e^{2x} + 3x^2 - 4x \, dx$

(b) (4 pt) $\int t^2 (\sin(t^3))^2 \cos(t^3) \, dt$

(c) (4 pt) $\int x^2 \cos x \, dx$

Exercise 3

(12 pt) Evaluate each definite integral. Clearly communicate your approach.

(a) (4 pt) $\int_0^1 (2x-1)^3 \, dx$

(b) (4 pt) $\int_0^4 \sqrt{4x - x^2} \, dx$

Hint: Set the integrand equal to y. Massage this equation into the form $(x-a)^2+(y-b)^2 = r^2$, an equation of a circle with center (a, b) and radius r. Use geometry to deduce the value of the integral.

(c) (4 pt) $\int_0^2 (x^2-1)(x^3-3x)^3 \, dx$

Exercise 4

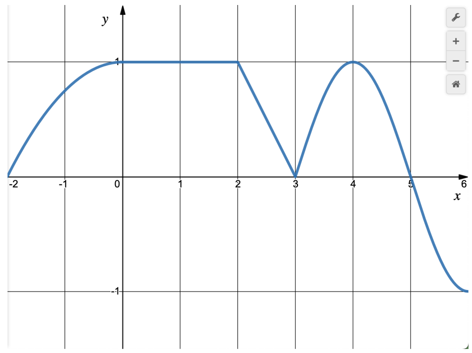
(8 pt) Use the fundamental theorem of calculus to compute each derivative. Assume $x \geqslant 0$.

(a) (4 pt) $\dfrac{d}{dx}\displaystyle\int_0^x e^{-t^2}\,dt$

(b) (4 pt) $\dfrac{d}{dx}\displaystyle\int_{x^2}^{4x^2} \sin(\sqrt{t})\,dt$

Exercise 5

(10 pt) Let $f : [-2, 6] \to \mathbf{R}$ be a piecewise function. A graph of $F(x) = \int_{-2}^{x} f(t)\,dt$ is shown below.



(a) (4 pt) On which intervals is f positive? negative? equal to zero?

(b) (4 pt) On which intervals is f increasing? decreasing? constant?

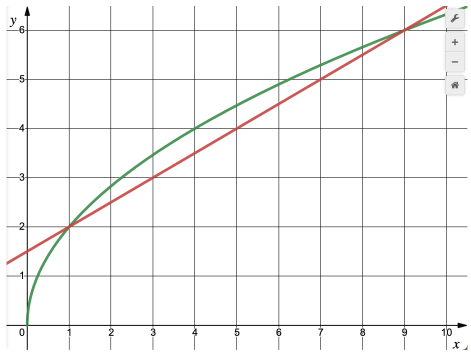
(c) (2 pt) What is the average value of f on the interval $[-2, 6]$?

Exercise 6

(8 pt) Consider the functions $f : [0, +\infty) \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ given by

$$f(x) = 2\sqrt{x} \qquad\qquad g(x) = 2 + \frac{1}{2}(x - 1)$$

respectively. Graphs of f and g appear below.



(a) (4 pt) Using the graph, write the two points (x, y) of intersection of f and g. Using the equations for f and g, show that, for each point (x, y) of intersection, $f(x) = y$ and $g(x) = y$. That is, the intersection points (x, y) are on the graphs of both f and g.

(b) (4 pt) Recall that linearity of the integral implies that

$$\int_a^b f(x) - g(x)\, dx = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

Use this to help explain, geometrically, why the area between the graphs of $f(x)$ and $g(x)$ equals $\int_1^9 f(x) - g(x)\, dx$.