Math 357
Short quiz 05

2024–01–26 (F)

Your name: _____

Let R be a commutative ring with multiplicative identity $1 \neq 0$, and let $I \trianglelefteq R$. Characterize the algebraic structure of the quotient ring R/I in the cases when (i) I is prime and (ii) I is maximal. Briefly describe how the relationship between prime and maximal ideals relates to the relationship between these algebraic structures.

Solution: The ideal I is prime if and only if the quotient ring R/I is an integral domain. The ideal I is maximal if and only if the quotient ring R/I is a field. If an ideal is maximal, then it is prime; the converse is false in general. If a ring is a field, then it is an integral domain; the converse is false in general.