Math 211
Quiz 04

R 11 Jul 2019

Your name: _____

Exercise

(5 pt) Evaluate the following integral.[1]

$$\int (\cos t) \ln(\sin t)\, dt$$

Solution: The $\ln(\sin t)$ factor is complicated; differentiating will simplify it. So we try

$$u = \ln(\sin t), \qquad dv = \cos t\, dt.$$

Then

$$du = \frac{\cos t}{\sin t}\, dt, \qquad v = \sin t,$$

and

$$\begin{aligned}\int (\cos t) \ln(\sin t)\, dt &= (\sin t) \ln(\sin t) - \int \sin t \frac{\cos t}{\sin t}\, dt \\ &= (\sin t) \ln(\sin t) - \int \cos t\, dt \\ &= (\sin t) \ln(\sin t) - \sin t + C,\end{aligned}$$

where $C \in \mathbf{R}$. Note: The derivative of our solution is (using the product rule and chain rule)

$$\begin{aligned}\frac{d}{dt}\big((\sin t) \ln(\sin t) - \sin t + C\big) &= (\cos t) \ln(\sin t) + (\sin t)\frac{\cos t}{\sin t} - \cos t + 0 \\ &= (\cos t) \ln(\sin t),\end{aligned}$$

the original integrand, as required.

[1] *Hint:* Let u and v be functions of t. Note that integrating the product rule

$$\frac{d}{dt}(uv) = u'v + v'u$$

gives

$$uv = \int vu'\, dt + \int uv'\, dt,$$

which we can rearrange to yield the "formula" for integration by parts (which now we know is just the product rule, integrated!). Remember that you can check your work: Differentiating your answer should give the original integrand.