

Math 357
Long quiz 04C

2024-04-10 (W)

Your name: _____

Let \mathbf{Q} denote the field of rational numbers; given a prime $p \in \mathbf{Z}_{>0}$, let $\mathbf{F}_p \cong \mathbf{Z}/(p)$, a finite field with p elements; and let t be an indeterminate. For each of the quotient rings below, characterize its algebraic structure as “field”, “integral domain but not field”, or “ring but not integral domain”. Justify your characterization.

$$R_1 = \mathbf{F}_5[t]/(t^3 + 2)$$

$$R_2 = \mathbf{Q}[t]/(t^4 - 8t^2 + 20)$$

$$R_3 = \mathbf{Q}[t]/(t^3 - 11t^2 + 57t + 12)$$