

# Math 211

## Quiz 22

T 06 Aug 2019

Your name : \_\_\_\_\_

## Exercise

(5 pt) Consider the following homogeneous 1st-order nonlinear system of ODEs:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} -2x_1 + 2x_1^2 \\ -3x_1 + x_2 + 3x_1^2 \end{bmatrix}, \quad (1)$$

where  $x_1(t), x_2(t)$  are functions of  $t$  (as usual).

(a) (1 pt) Find equations for the  $x_1$ - and  $x_2$ -nullclines. Show that (1) has two equilibrium points.

(b) (2 pt) Write the jacobian matrix  $\mathbf{J}(x_1, x_2)$  for (i.e. the linear approximation to) the system (1), at the general point  $(x_1, x_2)$ . Evaluate this matrix at the equilibrium points found in part (a).

(c) (2 pt) For each equilibrium point  $(x_1, x_2)$ , analyze the stability of the linearized system  $\mathbf{x}' = \mathbf{J}(x_1, x_2)\mathbf{x}$ . *Hint:* One equilibrium should be a saddle, the other, an (unstable) source.