# Math 357
# Short quiz 04

2024–01–24 (W)

Your name: ______________________________

As you join your friends for another meal, one of them hands you a paper napkin on which the following has been scribbled:

$$f : \mathbf{Z}/(3) \to \mathbf{Z}/(3)$$
$$f(t) = t^3 - t$$

(a) "The function is zero," one of your friends whispers in awe. Briefly provide the most convincing justification you can for this assertion.

**Solution:** "It seems reasonable," I begin, "to define a function to be zero if (i) its codomain has a zero element—language we typically reserve for an additive identity—and (ii) the function maps all elements of its domain to this element. The domain of our function $f$ has three elements, let us call them $-1$, 0, and 1." (A well timed arch of my eyebrow and inclination of my head stays, for the moment, objections to this choice of coset representatives.) "One quickly confirms that $f$ indeed maps all three elements to 0. Therefore, I share in my friend's awful assertion, if not awe." While my friends are momentarily befuddled by the archaic pun, I slip myself a bite to eat.

(b) "But the polynomial is not zero in $(\mathbf{Z}/(3))[t]$!" your particularly opinionated friend declaims. Briefly provide the most convincing justification you can for this assertion.

**Solution:** "It also seems reasonable," I intervene, in less impassioned tones, "to define a polynomial to be zero if all of its coefficients are zero. This is not the case for our polynomial $f$, which has coefficients $\pm 1$. By definition, $1 \neq 0$ in an integral domain; and as a field, $\mathbf{Z}/(3)$ is an integral domain. Therefore, I also share in my friend's louder, yet no less logical, assertion." In a move that would make Elizabeth Bennet smile, I append a musing about truth in many forms and many voices before gracefully turning our mealtime conversation to other topics.