# Math 211
# Quiz 19

R 01 Aug 2019

Your name : _____

## Exercise

(5 pt) Match each of the homogeneous 1st-order $2 \times 2$ linear systems with its corresponding phase plane in Figures 1 and 2. (N.B. In each ODE, $\mathbf{x} = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is a $2 \times 1$ matrix of scalar-valued functions.) *Hint:* (Some of the) distinguishing features of the phase plane are associated with

- eigenvalues and their corresponding eigenvectors of the coefficient matrix;
- nullclines, i.e. lines in the phase plane along which $x_1' = 0$ or $x_2' = 0$; and
- evaluating the original ODE at points $(x_1, x_2)$.

*Hint:* Recall that in the decomposition $\mathbf{A} = \mathbf{PDP}^{-1}$, if $\mathbf{D}$ is a diagonal matrix, then column j of the matrix $\mathbf{P}$ is an eigenvector corresponding to (the eigenvalue on) the jth diagonal entry of $\mathbf{D}$.

(a) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{x}$

(d) $\mathbf{x}' = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \mathbf{x}$

(e) $\mathbf{x}' = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \mathbf{x}$

(f) $\mathbf{x}' = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \mathbf{x}$

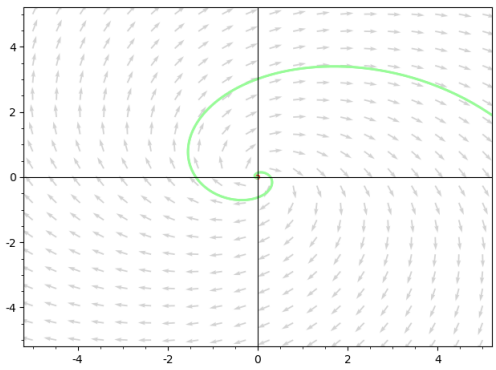
(g) $\mathbf{x}' = \frac{1}{4} \begin{bmatrix} 17 & -5 \\ 5 & -9 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}^{-1} \mathbf{x}$

(h) $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+2i & 0 \\ 0 & 1-2i \end{bmatrix} \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix}^{-1}$

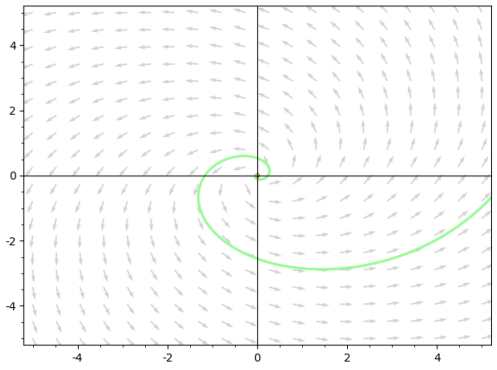
(i) $\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+2i & 0 \\ 0 & 1-2i \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1}$

(j) $\mathbf{x}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1}$

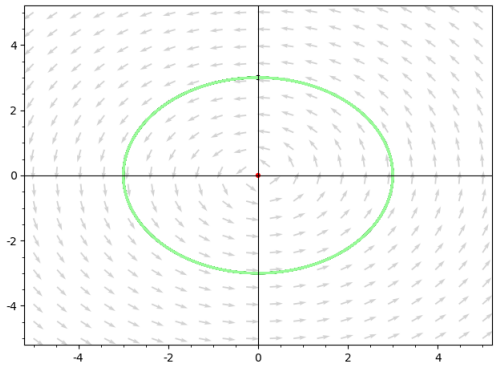
**Solution:** Focusing on the features in the hint, we find
(a) = (4) ; (b) = (5) ; (c) = (6) ; (d) = (9) ; (e) = (8) ;
(f) = (7) ; (g) = (10) ; (h) = (1) ; (i) = (2) ; (j) = (3)




(1)




(2)

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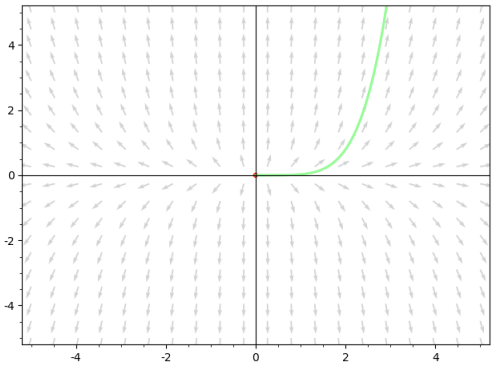
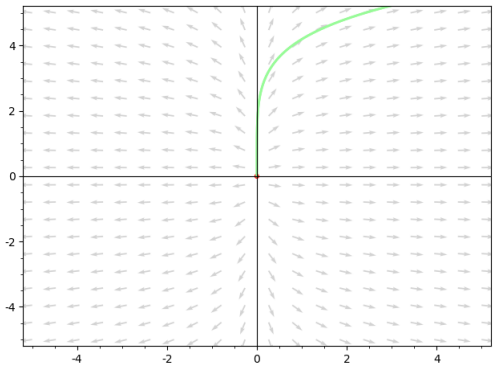
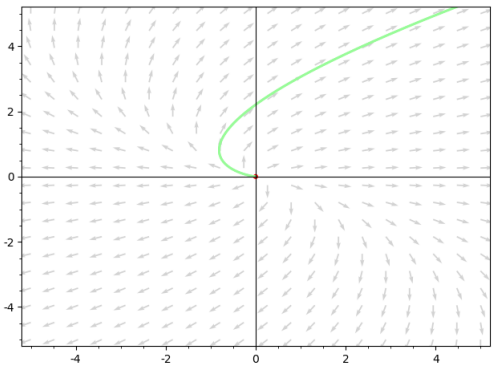



(4)

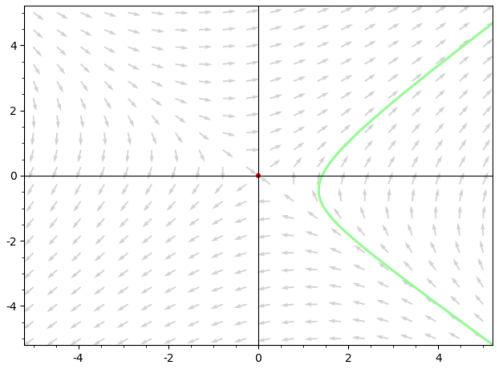



(5)

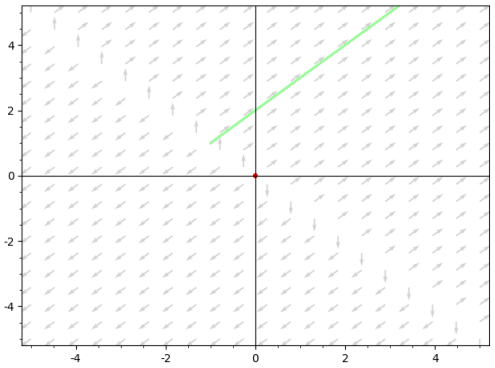



(6)

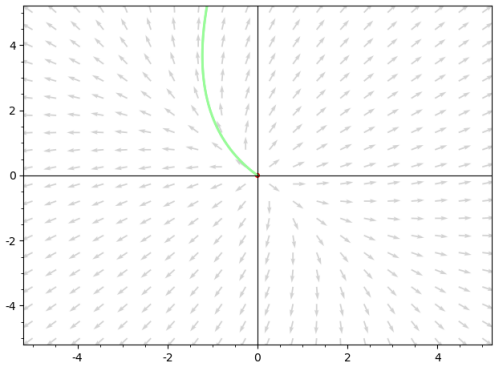
Figure 1: Phase planes for Quiz 19, in the $(x_1, x_2)$ plane.



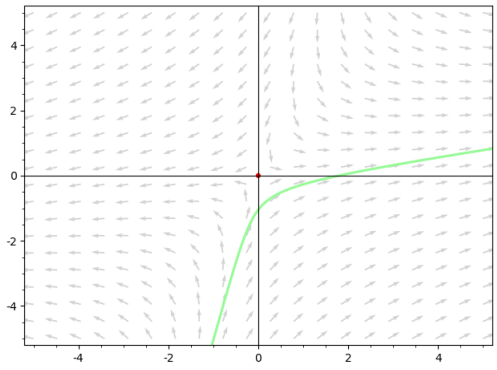

(7)




(8)




(9)




(10)

Figure 2: Phase planes for Quiz 19, in the $(x_1, x_2)$ plane.