Math 357
Long quiz 05A

2024–03–27 (W)

Your name: _____

Let $p = 3t^4 + 4t^3 - 12t^2 + 18 \in \mathbf{Q}[t]$.

(a) Prove that p is irreducible in $\mathbf{Q}[t]$.

(b) Prove that p has two distinct zeros in $\mathbf{R}$ and two distinct zeros in $\mathbf{C} - \mathbf{R}$. (You need not compute them.) You may use the following information in your proof:

t	-3	-2	-1	0	1	2	3
$p(t)$	45	-14	5	18	13	50	261

(c) Let $\alpha \in \mathbf{C}$ be a zero of p, and let $\beta \in \mathbf{C}$ such that the minimal polynomial of β over $\mathbf{Q}$ has degree 3. (Note that α or β may be real.) Can $\beta \in \mathbf{Q}(\alpha)$? in a splitting field of p? Justify your assertions.