Math 112
ReQuiz 13A

2022–04–27 (W)

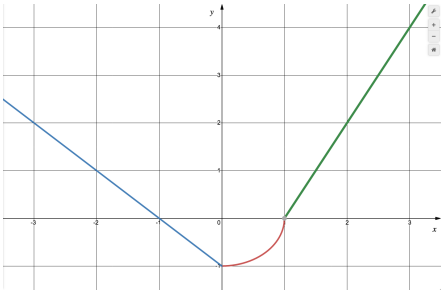
Your name: _____

Exercise

(4 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be the piecewise function, graphed below, whose rule of assignment is

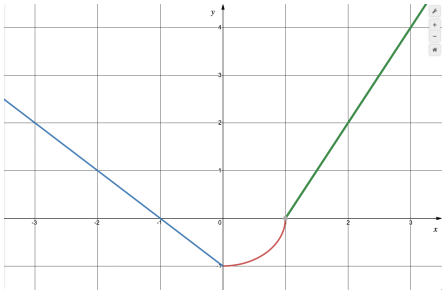
$$f(x) = \begin{cases} -x - 1 & \text{if } x \leqslant 0 \\ -\sqrt{1 - x^2} & \text{if } 0 \leqslant x \leqslant 1 \\ 2x - 2 & \text{if } x \geqslant 1 \end{cases}$$

This exercise explores the signed area under the graph of f from $x = -3$ to $x = 3$.

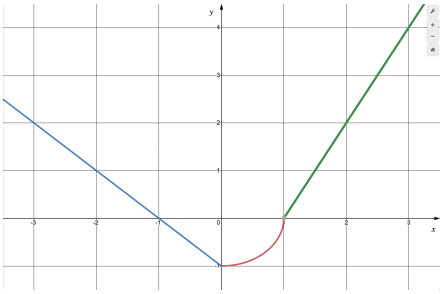


(a) (1 pt) Use geometry to show that $\int_{-3}^{3} f(x)\,dx = \frac{22-\pi}{4} \approx 4.71$.

(b) (1 pt) On separate graphs below, draw a lower sum and an upper sum, each with six intervals of width 1, to estimate $\int_{-3}^{3} f(x)\,dx$. Show that $L \leqslant \int_{-3}^{3} f(x)\,dx \leqslant U$.



Lower sum (L)



Upper sum (U)

For reference, $f : \mathbf{R} \to \mathbf{R}$ is the piecewise function whose rule of assignment is

$$f(x) = \begin{cases} -x - 1 & \text{if } x \leqslant 0 \\ -\sqrt{1 - x^2} & \text{if } 0 \leqslant x \leqslant 1 \\ 2x - 2 & \text{if } x \geqslant 1 \end{cases}$$

(c) (2 pt) Find an antiderivative $F_i(x)$ for each "piece" $f_i(x)$ of $f(x)$. You may take as given that

$$\int -\sqrt{1 - x^2}\, dx = -\frac{1}{2}\sin^{-1}(x) - \frac{1}{2}x\sqrt{1 - x^2} + C$$

Use these antiderivatives $F_i(x)$ and the fundamental theorem of calculus to compute the value of each integral on the right side of the following equation:

$$\int_{-3}^{3} f(x)\, dx = \int_{-3}^{0} f(x)\, dx + \int_{0}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$$

Add your values for the three integrals on the right, and compare the result to part (a).