

Math 212
Quiz 32

F 18 Nov 2016

Your name: _____

Exercise

(5 pt) In this quiz we prove that the curl of any gradient is zero. More precisely, let $f : \mathbf{R}^3 \rightarrow \mathbf{R}$ be \mathcal{C}^2 (i.e. twice continuously differentiable). Then

$$\operatorname{curl}(\nabla f) = \mathbf{0}. \tag{1}$$

(a) (1 pt) What kind of object is the $\mathbf{0}$ in (1)?

(b) (3 pt) Prove (1), justifying your steps. *Hint:* Definitions, compute, Clairaut.

(c) (1 pt) Application: Let $\mathbf{F} : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be a vector field, and suppose that $\operatorname{curl} \mathbf{F} \neq \mathbf{0}$. An unenlightened colleague from Math 212 (in another section, of course) asks you to find a potential function for \mathbf{F} , i.e. some $f : \mathbf{R}^3 \rightarrow \mathbf{R}$ such that $\nabla f = \mathbf{F}$. State why you refuse, with logical justification. *Hint:* Use (1).