Math 212
Quiz 20

M 17 Oct 2016

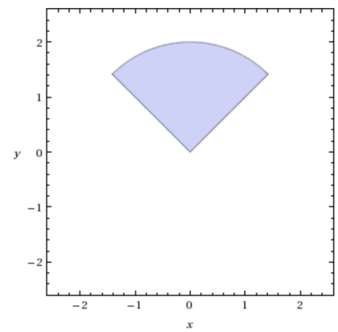
Your name: ______________________________

Exercise

(2 pt) Let $D \subseteq \mathbf{R}^2$ be the region in the upper half plane (i.e. $y \geqslant 0$) bounded by the circle $C : x^2 + y^2 = 4$ and the lines $y = x$ and $y = -x$.

(a) (0.5 pt) Sketch and shade the region D. *Hint:* The point $(0, 1) \in D$.

Solution: The region D is shown below.



(b) (1.5 pt) Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function

$$f(x, y) = 2xy.$$

Set up (but do NOT evaluate) an iterated (!) integral for $\iint_D f(x, y)\, dA$ **using polar coordinates**. *Hint:* Describe the region D algebraically using polar coordinates. When writing the iterated integral, remember to translate (x, y) to (r, θ), and mind your integration factor.

Solution: Because $f(x, y)$ is continuous on the region D of integration, Fubini's theorem allows us to write the double integral as an iterated integral. In polar coordinates, the region D can be described as

$$D = \left\{ (r, \theta) \,|\, 0 \leqslant r \leqslant 2, \frac{\pi}{4} \leqslant \theta \leqslant \frac{3\pi}{4} \right\},$$

the function $f(x, y)$ writes as

$$f(x, y) = 2xy = 2(r\cos\theta)(r\sin\theta) = r^2 2\sin\theta\cos\theta = r^2\sin(2\theta),$$

where in the final equality we have used the trigonometric identity

$$\sin(2\theta) = 2\sin\theta\cos\theta,$$

and

$$dA = r\, dr\, d\theta$$

(note the integration factor of r). Thus

$$\iint_D f(x, y)\, dA = \int_{\theta=\frac{\pi}{4}}^{\theta=\frac{3\pi}{4}} \int_{r=0}^{r=2} r^2 \sin(2\theta)\, r\, dr\, d\theta$$
$$= \int_{\theta=\frac{\pi}{4}}^{\theta=\frac{3\pi}{4}} \sin(2\theta)\, d\theta \int_{r=0}^{r=2} r^3\, dr.$$

Note that this iterated integral would be easy to compute, if we so desired.