Math 112
LQuiz 05

2022–01–25 (T)

Your name: ________________________________

Exercise

Consider the function

$$f : \mathbf{R} \to \mathbf{R} \qquad \text{given by} \qquad f(x) = x^3 + 3x^2 - 9x + 4$$

(a) (3 pt) Compute the linearization of (aka linear approximation to) f at $x = 2$.

Solution: By definition, the linearization of f at $x = 2$ is the function $L : \mathbf{R} \to \mathbf{R}$ given by

$$L(x) = f(2) + f'(2)(x - 2) \tag{1}$$

We compute

$$f(2) = 6 \qquad f'(x) = 3x^2 + 6x - 9 \qquad f'(2) = 15$$

Substituting these results into (1), we conclude that the rule of assignment for L is

$$L(x) = 6 + 15(x - 2) = 15x - 24 \tag{2}$$

(b) (1 pt) Use your linearization from part (a) to approximate the value $f(2.1)$. Given that the actual value is $f(2.1) = 7.591$, find the error in this approximation.

Solution: Using (2),[1] we compute

$$L(2.1) = 7.5$$

The error ε in this approximation is[2]

$$\varepsilon = L(2.1) - f(2.1) = 7.5 - 7.591 = -0.091$$

(Question: Can we predict, with accuracy, the direction of the approximation error? I claim that in this exercise, we can use certain features of f to deduce with certainty that our approximation will be too small, i.e. that $L(2.1) < f(2.1)$. How?)

[1] It may be easier to compute with the first expression in (2), rather than the second, "simplified" one.

[2] The negative sign simply indicates that our estimate $L(2.1)$ is below the actual value $f(2.1)$. Taking the absolute value gives the magnitude (size) of the error, without its direction (too small or too large).