

Math 211
Quiz 09

R 18 Jul 2019

Your name : _____

Exercise

(2 pt) Recall the three elementary row operations:

1. Interchange two rows.
2. Multiply a row by a nonzero scalar.
3. Add a scalar multiple of one row to another.

Using the elementary row operations, show that the augmented matrix

$$A = \left[\begin{array}{ccc|c} 5 & -5 & 17 & -2 \\ 1 & -1 & 3 & 0 \end{array} \right] \quad (1)$$

has reduced row echelon form (RREF)

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]. \quad (2)$$

Solution: As shown in (2), we want the upper left entry in an RREF matrix to be 1. We can achieve this (i) by multiplying row 1 of (1) by the nonzero scalar $\frac{1}{5}$; or even more simply, (ii) by interchanging row 1 and row 2:

$$\left[\begin{array}{ccc|c} 5 & -5 & 17 & -2 \\ 1 & -1 & 3 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 5 & -5 & 17 & -2 \end{array} \right].$$

Now we clear all other entries in column 1, identify and rescale to get the next pivot, and clear all other entries in that pivot's column:

$$\begin{aligned} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 5 & -5 & 17 & -2 \end{array} \right] &\xrightarrow{R_2 = R_2 - 5R_1} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 0 & 2 & -2 \end{array} \right] \\ &\xrightarrow{R_2 = \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 0 & 1 & -1 \end{array} \right] \\ &\xrightarrow{R_1 = R_1 - 3R_2} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]. \end{aligned}$$

The RREF matrix we obtain via row reduction is indeed (2).