Math 357
Short quiz 09

2024–02–23 (F)

Your name: ____________________________________

Let $C_4 = \langle g \,|\, g^4 = 1 \rangle$ be the cyclic group of order 4 with generator g. For each of the following maps, state whether it is a matrix representation of C_4, and if so, whether it is faithful.

$$\begin{array}{ccc} \rho_1 : C_4 \to GL_1(\mathbf{C}) & \rho_2 : C_4 \to GL_2(\mathbf{C}) & \rho_3 : C_4 \to GL_3(\mathbf{C}) \\ g^j \mapsto \begin{pmatrix} 1 \end{pmatrix} & g^j \mapsto \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^j & g^j \mapsto \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & i \end{pmatrix}^j \end{array}$$

Solution: By definition, a representation is a group homomorphism. For our present examples, it suffices to check that $\rho(g^4)$ is the identity matrix (why?). It is straightforward to check that this is true for all three maps. By definition, a representation is faithful if it is injective. The representation ρ_1 is not faithful, whereas ρ_2 and ρ_3 are.