Math 112
ReQuiz 14A

2022–04–05 (T)

Your name: _____

Exercise

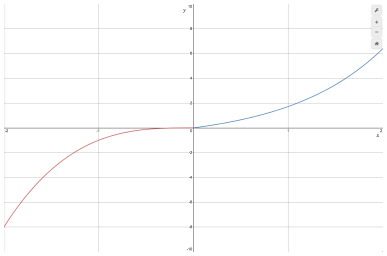
(4 pt) Let $f : \mathbf{R} \to \mathbf{R}$ be the function whose rule of assignment is

$$f(x) = \begin{cases} x^3 & \text{if } x \leqslant 0 \\ e^x - 1 & \text{if } x \geqslant 0 \end{cases}$$

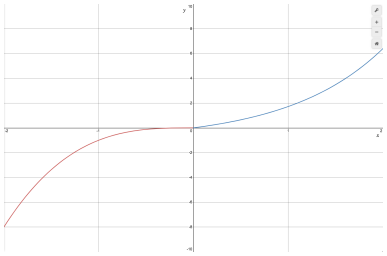
The function f is graphed below. This exercise explores the signed area under the graph of f from $x = -2$ to $x = 2$.

(a) (1 pt) Briefly (!) explain why we can't use finite geometry to find the exact value of $\int_{-2}^{2} f(x)\,dx$.

(b) (1 pt) On separate graphs below, draw a lower sum and an upper sum, each with four intervals of width 1. Use these to compute a lower and upper estimate for $\int_{-2}^{2} f(x)\,dx$. You may leave your answers in terms of e, or use the approximations $e \approx 2.72$ and $e^2 \approx 7.39$.



Lower sum (L)



Upper sum (U)

(c) (2 pt) Find an antiderivative $F_i(x)$ for each "piece" of $f(x)$. Use these antiderivatives and the fundamental theorem of calculus to compute the integrals on the right side of

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

Add your results to determine the integral on the left side. Show that $L \leqslant \int_{-2}^{2} f(x)\,dx \leqslant U$..