Math 211
Quiz 05

F 12 Jul 2019

Your name: _____

Exercise

(2 pt) Consider the first-order IVP

$$y' = t\sqrt[3]{y} + \cos t^2, \qquad y(1) = 0.$$

(a) (1 pt) What, if anything, can we conclude about existence of solutions to this IVP? Justify briefly.

Solution: The expression for y' is continuous, so Picard's theorem implies that there exists a solution (i.e. at least one solution) to this IVP on some t-interval around $t = 1$.

(b) (1 pt) What, if anything, can we conclude about uniqueness of solutions to this IVP? Justify briefly.

Solution: We compute

$$\frac{\partial(y')}{\partial y} = \frac{\partial}{\partial y}\left(t\sqrt[3]{y} + \cos t^2\right) = \frac{t}{3\sqrt[3]{y^2}}.$$

This function is not defined at $y = 0$, therefore it cannot be continuous at the point $(t, y) = (1, 0)$. The uniqueness statement of Picard's theorem requires that this partial derivative be continuous, so we can't use Picard's theorem to conclude anything about uniqueness.