Math 357
Long quiz 01B

2024–03–22 (F)

Your name: ______________________________

(a) Let R be a ring with a multiplicative identity 1_R. R is **boolean** if, for all $a \in R$, $a^2 = a$. Prove that a boolean ring is commutative.

(b) Let R be a ring; and let I_1, I_2 be ideals of R. Recall that

$$I_1 + I_2 = \{a_1 + a_2 \mid a_i \in I_i\} \qquad I_1 I_2 = \left\{ \sum_{j=1}^{n} a_{1,j} a_{2,j} \mid n \in \mathbf{Z}_{>0}; \forall j, a_{i,j} \in I_i \right\}$$

are ideals. (In particular, note that $I_1 I_2$ comprises all finite sums of terms of the form $a_1 a_2$ with $a_i \in I_i$.) Prove that if R is a commutative ring with a multiplicative identity, and if $I_1 + I_2 = R$, then $I_1 \cap I_2 = I_1 I_2$.