Math 212
Quiz 16

W 05 Oct 2016

Your name: ________________________________

Exercise

(2 pt) Our goal is to find the minimum and maximum of the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by

$$f(x, y) = 1 - x + y$$

among the points (x, y) that lie on the unit circle $x^2 + y^2 = 1$.

(a) (1 pt) Write the Lagrangian (a.k.a. auxiliary function) L associated with this optimization problem. *Hint:* Identify the constraint function $g = 0$. L is a function of three variables.

Solution: The constraint that the point (x, y) lie on the unit circle corresponds to the condition

$$g(x, y) = x^2 + y^2 - 1 = 0.$$

Thus the Lagrangian is

$$L(x, y, \lambda) = f - \lambda g = 1 - x + y - \lambda(x^2 + y^2 - 1).$$

(b) (1 pt) Write the first-order condition(s) that you could solve to find the minimum and maximum. *Hint:* You can write as few as four symbols.

Solution: The candidate points are given by the vector equation

$$\nabla L = \mathbf{0},$$

where the gradient of L includes the three partial derivatives of L with respect to x, y, λ.