Math 112
ReQuiz 11A

2022–04–08 (F)

Your name: _____

Exercise

(4 pt) Take as given the following "infinite polynomial" expression for $\ln(x+1)$:

$$\begin{aligned}\ln(x+1) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots \\ &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - O(x^4)\end{aligned} \tag{1}$$

(Recall that $O(x^n)$ means "terms involving x to powers n and higher".) Consider the limit

$$\lim_{x\to 0} \frac{\frac{1}{3}x^3}{\ln(x+1) - x + \frac{1}{2}x^2} \tag{2}$$

(a) (2 pt) Compute the limit in (2) by substituting (1) for $\ln(x+1)$, simplifying, and evaluating.

(b) (2 pt) Compute the limit in (2) by iteratively applying l'Hôpital's rule. (Briefly show it applies each time you use it!) Confirm you get the same result you got in part (a).