Math 211
Quiz 17

T 30 Jul 2019

Your name : _____

Exercise

(5 pt) For each of the following linear maps $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$, where $\mathbf{R}^n, \mathbf{R}^m$ are viewed as vector spaces over $\mathbf{R}$,

   (i) write a basis for the image $\mathrm{im}(T)$ and a basis for the kernel $\ker(T)$,

  (ii) find the dimensions $\dim(\mathrm{im}(T))$ and $\dim(\ker(T))$, and

 (iii) confirm the rank–nullity theorem:

$$\dim(\mathrm{domain}(T)) = \dim(\mathrm{im}(T)) + \dim(\ker(T)).$$

Hint: For each linear map T, write a corresponding matrix representation A, so that $T(x) = y$ corresponds to the matrix equation $Ax = y$. Then focus on the pivot or nonpivot columns of A.

  (a) (2.5 pt) The linear map T_1 given by

$$T_1 : \mathbf{R}^4 \rightarrow \mathbf{R}^3$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} x_1 & + & & & x_3 \\ & & x_2 & - & x_3 \\ & & 0 & & \end{bmatrix}.$$

  (b) (2.5 pt) The linear map T_2 given by

$$T_2 : \mathbf{R}^4 \rightarrow \mathbf{R}^3$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} x_1 & + & x_2 & + & 2x_3 & + & x_4 \\ 3x_1 & + & 2x_2 & + & 3x_3 & + & 2x_4 \\ x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 \end{bmatrix}.$$