CS-2349 Theory of Computation: Final Project

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1 Introduction

1.1 Cellular Automata

A cellular automaton is a model of a system of "cell" objects with the following characteristics:

- The cells live on a grid
- Each cell has a state. The number of states is usually finite, eg. binary states 0 or 1.
- Each cell has a neighbourhood i.e. it has adjacent cells which can be defined in a number of ways depending on the grid

Elementary or one-dimensional cellular automata can be visualised as a horizontal array of cells. For the purposes of our study, we will look at elementary cellular automata with two possible states per cell: 1 or 0. The rules determine how this arrangement of cells will be updated over time steps, also known as generations. In particular, we will focus on a nearest neighbor scheme for the updation of rules. This means that to determine the state of a cell in position p at time step $t+1$, we will look at the states of cells in position $p-1$, p, and $p+1$ all in time step t.

Guided by Stephen Wolfram's systematic exploration of elementary cellular automata in his book *A New Kind of Science* (1), we want to observe the wide range of qualitatively different behaviours exhibited by cellular automata in random initial conditions over a space-time evolution and perhaps demonstrate how valuable some of these systems can be in the simulation and recognition of patterns.

1.2 Non-Standard Computation and Applications in Physical Systems

Cellular automata are austere machines capable of showing complex emergent behavior based on small range interactions defined by local rules which may or may not be the same in all localities in a given space. This set up provides us with a way to model emergent behavior in nature, from natural pattern formation to computation in the absence of interconnecting entities such as a network of neurons. By virtue of its capability to do highly parallelized computation, it provides simple yet robust models to uncover the underlying rules of complex emergent phenomena, such as social network dynamics, collective behavior in animals, phantom vehicular congestion on roads, reaction-diffusion based pattern formation, and so on. One can use cellular automata to do parallel formal language recognition and parallel arithmetic, and based on defined rules of interaction between neighbors in both space and time, study the evolution of its behavior.

A problem of particular interest to us is the Majority Problem, also known as the Density Classification Problem, which is as follows: In the absence of global interaction rules, can we build an automaton that can compute the majority vote? To explain it with a $1-$dimensional example, consider a binary string of some arbitrary length; Given how a cellular automaton computes, can it detect whether the density of one symbol is greater than the other or equal to the other? This problem is pertinent because the automaton cannot make use of an external entity to count the number of different symbols (if we did, we would have a trivial solution). It must parallely compute its state in successive discrete time steps for all individual cells constituting it, without an external agent. There exist specific governing rules that can take an initial configuration as input and over a couple hundred time steps, compute the majority symbol by exhibiting specific dynamics. As mentioned earlier, such an automaton is capable of modelling traffic congestion as an emergent phenomenon. It has recently been shown to be an effective model of how patches of stomata in leaves are able to compute the overall optimal opening and closing to minimise water loss and maximise carbon-dioxide intake at a given time of day, with the multitude of environmental parameters acting in opposing directions. Stomatal patches are sparse in distribution, and lends itself to a distributed computing model such as the cellular automaton, which is also capable of showing such a large variety of behaviors, covering the whole range of ordered to chaotic, as is necessary for adaptive systems such as the one being discussed.

2 Project Goals

We have 2 main goals for this project:

- Simulation of all possible one dimensional cellular automata with 2 possible values ($k=2$) and range 1 ($r=1$) as in (2), *Statistical mechanics of cellular automata* and here: Tables of Cellular Automaton Properties and a detailed study of the rules that show interesting behaviour.
- Understanding the application of the density classification problem in (3), *Evidence for complex, collective dynamics and emergent, distributed computation in plants*

Computing a state based on certain neighbouring states is not a concept we were unfamiliar with (a simple example that comes to mind is blurring an image wherein the value of a pixel is the average of its neighbours) however, before we understood its applications in the real world, we want to be able to fully appreciate how such a set of simple rules could exhibit remarkably complex behaviour.

Wolfram proposes a heuristic classification of automaton behavior based on his observations. For our understanding, we wish to look at all possible configurations of a cell and its neighbours and define the state outcome for every possible configuration through the implementation of 256 rules as enlisted by Wolfram. We would also like to study the sensitivity of each rule to perturbations in the input configuration.

References

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