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UCS654 Predictive Analysis using Statistics

Assignment: Parameter Estimations

- 1) Let (X1, X2,...) be a random sample of size n
 taken from a Normal Population with parameters:
 mean = 01 and variance = 02. Find the Maximum
 Likelihood Estimates of these two parameters.
- -> In finding the estimators, write the pdf as a function of 01 and 02.

$$f(x; \theta_1, \theta_2) = \frac{1}{\sqrt{\theta_2/2\pi}} exp \left[-\frac{(x_1 - \theta_1)^2}{2 \cdot \theta_2} \right]$$

for -∞ < 01 < ∞ -∞ < 02 < ∞

that makes the likelihood function as:

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2) = \theta_2^{-n|2} (2\pi)^{-n|2} \exp \left[-\frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_i)^2 \right]$$

and

therefore the log of the likelihood function:

$$log L(\theta_1, \theta_2) = -n log \theta_2 - n log (2\pi) - \sum (\pi_1 - \theta_1)^2$$

Upon taking the partial derivative of the log likelihood with respect to BI, and setting to D, we have:

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_1} = \frac{\sum (x_1 - \theta_1)}{\theta_2} \stackrel{\text{set}}{=} 0$$

Now, multiplying through by 02 and distributing the summation, we get:

On solving,
$$\widehat{\rho_1} = \underline{\Sigma} x_1^2 = \overline{x}.$$

Now for 02

Taking the partial derivatives of the log likelihood with respect to 02, and setting to 0, we get:

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{\sum (x_1 - \theta_1)^2}{2\theta_2^2} \stackrel{\text{set}}{=} 0$$

Multiplying through by
$$2\theta 2^2$$
 $\frac{3\log L(\theta_1, \theta_2)}{3\theta 2} = \frac{-n}{2\theta 2} + \frac{5(\pi i - \theta_1)^2}{2\theta 2^2} \stackrel{\text{def}}{=} 0 \times 2\theta 2^2$

we get:
$$-n\theta 2 + \sum (x_i - \theta 1)^2 = 0$$

On solving
$$\theta \hat{2} = \frac{\sum (x_i - \bar{x})^2}{n}$$

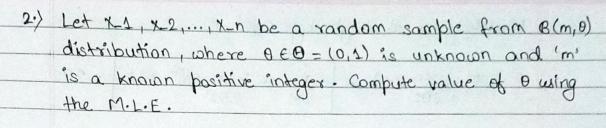
Thus,

Maximum Likelihood Estimate for mean: 0, is:

$$\hat{\theta_i} = \frac{\sum x_i^2}{n} = \overline{x}.$$

Maximum Likelihood Estimate for variance = 02 is:

$$\hat{\theta}_2 = \frac{\sum (x_1^2 - \overline{x})^2}{n}.$$



and we have observed x1:x,1 x2:x2,..., t-n:xn.

The maximum likelihood estimate of θ is the value that maximizes the likelihood function $L(x_1, x_2, ..., x_n; \theta)$.

Rembember that when we have a random sample, we can obtain the joint PMF and PDF by multiplying the marginal (individual) PMFs and PDFs.

Thus,

$$= \prod_{i=1}^{n} P_{X_i^n}(x_i^n; \theta)$$

$$= \frac{1}{n} \binom{x_i}{m} \theta_{x_i} (1-\theta)_{m-x_i}$$

Note that the first term does not depend on B

so we can write L(M, M2, ... Mn; 8) as L(x,1x2,...,xn; 0) = c 8s(1-8)mn-s where, c does not depend on 8, and $s = \sum_{k=1}^{n} N_{k}$ By differentiating and setting the derivative to 0 êm = 1 2 x: Thus, M.L.E. can be written as mn = 1 = x;