

## Part VII

# Index Structures for Data Warehouses

# Index Structures for Data Warehouses

- 1 Classification of Index Structures
- 2 B-Trees
- 3 Bitmap Indexes
- 4 Multidimensional Index Structures

# Motivation

- Fact tables in data warehouses can become very large, such that a full table scan becomes unadvantageous
- Example: Scan over 10 GB table at 10 MB/s = ca. 17 minutes
- Queries just affect a relatively small part of the available data:
  - ▶ Depending on the restrictions on individual dimensions the result set of a query may affect a few percent or per mille (or even less) of the data
- Use of index structures to minimize the number of required page accesses

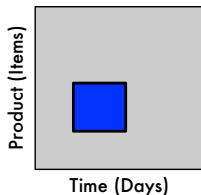
# Classification of Index Structures

# Classification of index structures

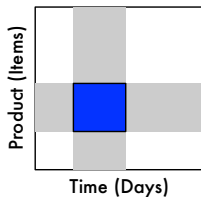
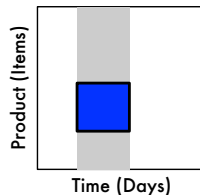
- **Clustering**: data that are likely to often processed together will also be stored physically close to each other
  - ▶ *Tuple Clustering*: Storing of tuples on the same physical page
  - ▶ *Page Clustering*: storing related pages close together in secondary storage (allows *prefetching*)
- **Dimensionality**: specify how many attributes (dimensions) of the underlying relation for calculation of the index key can be used
- **Symmetry**: If the performance is independent of the order of the index attributes, we have a symmetrical index structure, otherwise asymmetrical
- **Tuple references**: Type of tuple references within the index structure
- **Dynamic behavior**: Effort to update the index structure for insert, update and delete; (and possibly problem of "degeneration")

# Comparison of Index Structures

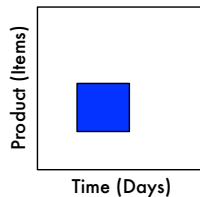
Full Table Scan



Clustered Primary Index



Several Secondary Indexes,  
Bitmap Indexes

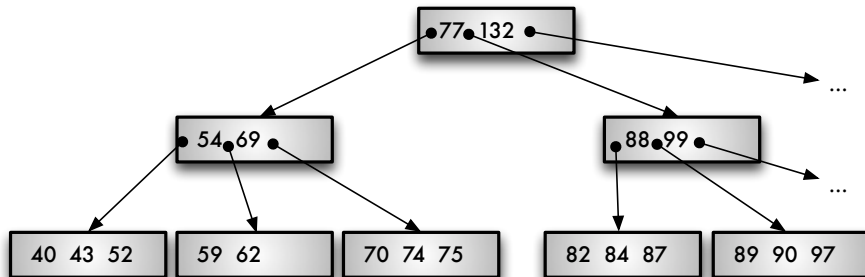


Multi-dimensional Index

# B-Trees

# One dimensional tree structures

- B-Tree [Bayer/McCreight 1972]





# B-Tree

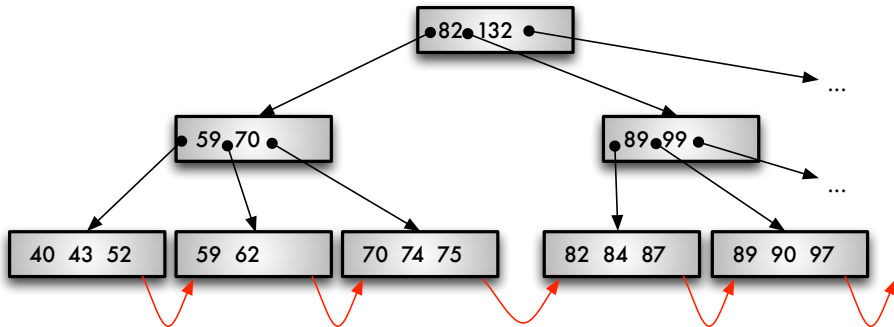
- Order of a B Tree: min. number of entries on the index pages (except for the root page)
- Definition: Index tree is a B Tree of order  $m$ , if
  - ▶ Each page contains at most  $2m$  elements
  - ▶ Each page except for the root page contains at least  $m$  elements
  - ▶ Each page is either a leaf side without successors or it has  $i + 1$  successors ( $i$ : number of elements)
  - ▶ All leaf pages are on the same level

# B-Tree: Properties

- $n$  records in the main file
  - $\log_m(n)$  page accesses from the root to the leaf
- Balance criterion leads to almost complete evenness
- Insert, delete, search with  $O(\log_m(n))$
- Memory space utilization: at least mindestens 50% (except for root)

# $B^+$ -Tree

- $B^+$  Tree (variant of the  $B$ -Tree): Tuples/TIDs only in the leaves; leaves are chained for a sequential (range) traversal



# Properties of $B$ - and $B^+$ -Trees

- One-dimensional structure (index on an attribute)
- As a primary index tuples can be stored directly in the tree (allowing simple clustering, especially in the  $B^+$ -Tree)
- As a secondary index only TIDs are stored in the tree
- Balanced tree (Path from root to leaf has everywhere the same length); balancing requires more effort in reorganization in case of updates on the data
  - ▶ For data warehouses is of minor importance
- $B^+$ -tree especially suited for range queries (by concatenation on leaf-level)

# Use of $B$ -/ $B^+$ -Trees

- $B$ - and  $B^+$ -Trees: one-dimensional structures:
  - ▶ Only insufficient support of multidimensional queries
- Possible applications with multi-dimensional queries
  - ▶ For each attribute involved, there is a  $B$ - or  $B^+$ -Tree as a secondary index
  - ▶ Then for each attribute the set of TIDs is determined depending on whether they fulfill the query restriction
  - ▶ Now the intersection of the independently from each other determined TID sets is taken, the corresponding tuples form the query result

# $B^+$ -Tree: Conclusion

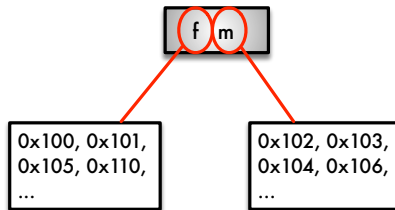
- Robust, generic data structure
- Independent of data type (only order required)
- Efficient update algorithms
- Compact
- "Working Horse" of all RDBMS
- Problems
  - ▶ Attributes of low cardinality → **degenerated trees**
  - ▶ Composed Indexes → **order sensitive**

# Degenerated B-Trees

- Example:

- ▶ Table `Customer`
- ▶ Attributes: among others `Gender` (m, f)
- ▶ Index

```
CREATE INDEX s_idx ON Customer(Gender)
```



# Order Dependency

- **Composed Index**
  - ▶ Indexing of concatenated attribute values
  - ▶ Problem: Adjust the order of the query predicates
- **Example:**
  - ▶ Table: Customer
  - ▶ Attributes: cclass, gender, profession
  - ▶ Index:

```
CREATE INDEX csp_idx  
  ON Customer(cclass, gender, profession)
```

- ▶ Queries:

```
SELECT ... FROM ...  
  WHERE cclass=1 AND  
        gender='m' AND profession='Lecturer'
```



# $B^+$ -Tree-Tricks: Oversized Index

- Query:

```
SELECT AVG (Age)
FROM Customer
WHERE cclass=1 AND gender='m'
      AND profession='Lecturer'
```

- Index usage

- ▶ Searching the value "1||m||Lecturer"
- ▶ Access to a block of the relation `Customer` over TID for the value of Age

- Better:

```
CREATE INDEX csp_idx
ON Customer(cclass, gender, profession, age)
```

## $B^+$ -Tree-Tricks: Calculated Indexes

- Calculation of indexed values by using a function
- Example: Index over `Customer(name)`

- ▶ Query:

```
SELECT * FROM Customer
WHERE name="Müller" OR name="mueller"
      OR name="Mueller" ...
```

- ▶ Index usage not possible

- Better

```
CREATE INDEX n_idx ON Customer(upper(name))
CREATE INDEX n_idx ON Customer(soundex(name))
```

# Oracle9i: Special Features

- **Index Skip Scan**: uses composed indexes also when the is not in the condition
- Example:
  - ▶ Table Customer, Index on (status, registration#)
  - ▶ Index usable for a query with

```
... WHERE registration# = 4245
```

- Searching the secondary index for all **DISTINCT** values of the first attribute
- Only useful if the first attribute has a low cardinality
- Infos: <http://otn.oracle.com/products/oracle9i/daily/apr22.html>

## Oracle9i: Special Features (2)

- Index-organized tables
  - ▶ Tuples are additionally stored in a  $B^+$ -Tree
  - ▶ No indirection via TID necessary
- User-defined indexes
  - ▶ Implementation of own index structures for user-defined data types
  - ▶ Transparent use
  - ▶ Specify own cost estimates

# Bitmap Indexes

# Bitmap Indexes

- Idea: **Bit-Array** to encode the tuple attribute value mapping
- Comparison with tree-based index structures:
  - ▶ Avoids degenerated B-trees
  - ▶ Insensitive towards higher number of dimensions
  - ▶ Easier support of queries, where only some (the indexed) dimensions are restricted
  - ▶ But for generally higher update costs
    - ★ In data warehouses mostly unproblematic due to the majority of read-only accesses unproblematisch

# Bitmap Index: Implementation

- Principle: Replacement of TIDs (rowid) for a key value in the  $B^+$ -Tree by bit list
- Node structure:

B: 010010...01	F: 101000...10	O: 000101...00
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- Advantage: lower storage consumption
  - ▶ Example: 150.000 Tuples, 3 different key values, 4 Bytes for a TID
    - ★  $B^+$ -Tree: 600 KB
    - ★ Bitmap:  $3 \cdot 18750 \text{ Byte} = 56\text{KB}$
- Disadvantage: higher update effort

# Bitmap Index: Implementation (2)

- Definition in Oracle

```
CREATE BITMAP INDEX orderstatus_idx  
ON Order(status);
```

- Use particularly for Star-Query transformation (Join between dimension- and fact table)
- Storage in compressed form
- Furthermore: Bitmap-based join indexes



# Standard Bitmap Index

- Each Dimension is stored separately
- For each attribute value, an individual bitmap vector is created:
  - ▶ For each tuple, there is a corresponding bit that is set to 1 when the indexed attribute in the tuple contains the reference value of this bitmap vector
  - ▶ The number of resulting bitmap vectors per dimension corresponds to the number of different values that occur for the attribute

## Standard Bitmap Index (2)

- Example: Attribute Gender
  - ▶ 2 Feature values (m/f)
  - ▶ 2 Bitmap vektors

<b>PersId</b>	<b>Name</b>	<b>Gender</b>	<b>Bitmap-f</b>	<b>Bitmap-m</b>
007	James Bond	M	0	1
008	Amelie Lux	F	1	0
010	Harald Schmidt	M	0	1
011	Heike Drechsler	F	1	0

## Standard Bitmap Index (3)

- Selection of tuples can be achieved by linking bitmap vectors
- Example: Bitmap index over the attributes gender and month of birth
  - ▶ 2 Bitmap vectors B-f and B-m for gender
  - ▶ 12 Bitmap vectors B-1, ..., B-12 for the months, if all months occur
- Query: "all women born in march"
  - ▶ Calculation:  $B-f \wedge B-3$  (bitwise conjunctively linked)
  - ▶ Result: all tuples, at whose position in the bitmap vectors a 1 is given in the result

# Multicomponent Bitmap Index

- For the Standard Bitmap Indexes many bitmap vectors are created for attributes with many feature values
- $\langle n, m \rangle$  Multicomponent Bitmap Index allow to index  $n \cdot m$  possible feature values by using  $n + m$  bitmap vectors
- Each value  $x (0 \leq x \leq n \cdot m - 1)$  can be represented by  $y$  and  $z$ :

$$x = n \cdot y + z \text{ mit } 0 \leq y \leq m - 1 \text{ und } 0 \leq z \leq n - 1$$

- ▶ At maximum  $m$  bitmap vektors for  $y$  and  $n$  bitmap vektors for  $z$
- ▶ Storage overhead is reduced from  $n \cdot m$  to  $n + m$  vektors
- ▶ However, point queries require reads of 2 bitmap vectors

## Multicomponent Bitmap Index (2)

- Example: Two Component Bitmap Index
- For  $M = 0..11$  for instance  $x = 4 \cdot y + z$
- y values: B-2-1, B-1-1, B-0-1
- z values: B-3-0, B-2-0, B-1-0, B-0-0

x	y			z			
M	B-2-1	B-1-1	B-0-1	B-3-0	B-2-0	B-1-0	B-0-0
5	0	1	0	0	0	1	0
3	0	0	1	1	0	0	0
0	0	0	1	0	0	0	1
11	1	0	0	1	0	0	0

## Example: ZIP-Codes

- Encoding of ZIP-Codes
- Values from 00000 to 99999
- Direct Implementation: 100.000 columns
- Two Component Bitmap Index (first 2 digits + 3 last digits): 1.100 columns
- Five Components: **50 columns**
  - ▶ Suitable for range queries "ZIP 39\*\*\*"
- Binary encoded (to  $2^{17}$ ): 34 columns
  - ▶ Only for point queries!
- *Note: Encoding to the base 3 results in 33 columns....*

# Range Encoded Bitmap Index

- Standard and Multicomponent Bitmap Indexes
  - ▶ Well suited for point queries
  - ▶ Inefficient for large ranges, because many bitmap vectors need to be linked
- Idea of range encoded bitmap indexes:

*In the bitmap vector a bit is set to 1 if the value of the attribute is smaller or equal to the given value.*
- Range query  $2 \leq attr \leq 7$  requires only 2 bitmap vectors: B-1 and B-7.
- Resulting bitmap vektor is  $((\neg B-1) \wedge B-7)$ .
  - ▶ For range queries at maximim 2 bitmap vectors need to be read (only one for one-side restricted ranges)
  - ▶ For point queries exactly 2 bitmap vectors need to be read

# Range Encoded Bitmap Index

Month M	Dec B-11	Nov B-10	Oct B-9	Sep B-8	Aug B-7	Jul B-6	Jun B-5	May B-4	Apr B-3	Mar B-2	Feb B-1	Jan B-0
Junq - 5	1	1	1	1	1	1	1	0	0	0	0	0
April - 3	1	1	1	1	1	1	1	1	1	0	0	0
Jan. - 0	1	1	1	1	1	1	1	1	1	1	1	1
Feb. - 1	1	1	1	1	1	1	1	1	1	1	1	0
April - 3	1	1	1	1	1	1	1	1	1	0	0	0
Dec. - 11	1	0	0	0	0	0	0	0	0	0	0	0
Aug. - 7	1	1	1	1	1	0	0	0	0	0	0	0
Sept. - 8	1	1	1	1	0	0	0	0	0	0	0	0

- Range query  $February \leq Date \leq August$  requires B-0 and B-7.
- Resulting bitmap vector is  $((\neg B-0) \wedge B-7)$



# Multicomponent Range Encoded Bitmap Index

- Combination of both techniques
- First, a multicomponent bitmap index is created
- On each built group of bitmap vectors the range encoding is applied
  - ▶ Due to the multicomponent technique less storage consumption (because of the smaller number of bitmap vectors)
  - ▶ The range encoding allows for efficient support of range queries
  - ▶ Because of the range encoding there is always one bitmap vector (component) dispensable in each group (representing the value  $n - 1$  and  $m - 1$ , respectively, since there always all bits have to be set to 1 da dort immer alle Bits auf 1); hence only  $n + m - 2$  bitmap vectors are needed

# Example MCREBMI

## ● Example Multicomponent Range Encoded Bitmap Index

- ▶  $B-0-1' = B-0-1$
- ▶  $B-1-1' = B-1-1 \vee B-0-1'$
- ▶  $B-2-1' = B-2-1 \vee B-1-1' = B-2-1 \vee B-1-1 \vee B-0-1 = 1$

M	B-1-1'	B-0-1'	B-2-0'	B-1-0'	B-0-0'
5	1	0	1	1	0
3	1	1	0	0	0
0	1	1	1	1	1
11	0	0	0	0	0

# Interval Encoded Indexing

- Principle: each Bitmap vector represents a defined interval
- Example: Intervals  $I-0 = [0, 5]$ ,  $I-1 = [1, 6]$ ,  $I-2 = [2, 7]$ ,  $I-3 = [3, 8]$ ,  $I-4 = [4, 9]$ ,  $I-5 = [5, 10]$

M	I-5	I-4	I-3	I-2	I-1	I-0
5	1	1	1	1	1	1
3	0	0	1	1	1	1
0	0	0	0	0	0	1
11	0	0	0	0	0	0
10	1	0	0	0	0	0

- Query:  $(2 \leq M \leq 8) \rightsquigarrow$  Evaluation of  $I-2 \vee I-3$

# Selection of Index Structures

- In Data Warehouses normally multidimensional range queries
- Selection of index structure dependent on query profile
  - ▶ Is a specific attribute predominantly restricted?
    - ⇒ For asymmetrical index structures, the order of the index attributes shall be selected according to their frequency in the query profile
  - ▶ When no attribute can be identified as particularly important or many ad-hoc queries occur, symmetrical structures (secondary indexes, multidimensional indexes) are advantageous

## Selection of Index Structures (2)

- Standard Bitmap Index
  - ▶ Fast, efficient implementation
  - ▶ Much storage space needed for a large number of feature values
- Multicomponent Bitmap Index
  - ▶ For point queries smallest number of read operations
- Range Encoded Bitmap Index
  - ▶ One-side restricted range queries
- Interval Encoded Bitmap Index
  - ▶ Two-side restricted range queries

# Join Index

- Accelerating join computations by indexing attributes of "foreign" relations
- Precomputation of the join and storing as an index structure
- Join/Grouping partially without access to foreign relation (e.g., dimension table) possible

# Join Index: Example

<b>Sales</b>	V_ROWID	GeoID	TimeID	Sales	...
	0x001	101	11	200	
	0x002	101	11	210	
	0x003	102	11	190	
	0x004	102	11	195	
	0x005	103	11	100	
	0x006	103	11	95	

<b>Geography</b>	G_ROWID	GeoID	Branch	City	...
	0x100	101	Allee-Center	Magdeburg	
	0x101	102	Bördepark	Magdeburg	
	0x102	103	Anger	Erfurt	
	0x103	104	Erfurter Str.	Ilmenau	

```
CREATE INDEX joinidx ON Sales (Geography.GeoID)
USING Sales.GeoID = Geography.GeoID
```

0x100: { 0x001, 0x002, ... }
0x101: { 0x003, 0x004, ... }
0x103: { ... }

# Bitmap Join Index

- So far:
  - ▶ Predicates for bitmap indexes not applied on foreign keys
  - ▶ Join has to be still executed
- Bitmap indexes only for Star Join optimization helpful
- Combination of
  - ▶ Bitmap Index and
  - ▶ Join Index



# Bitmap Join Index with Oracle

- Definition

```
CREATE BITMAP INDEX join_idx  
  ON Sales (Geography.GeoID)  
  FROM Sales, Geography  
  WHERE Sales.GeoID = Geography.GeoID
```

- Makes joins redundant (no access to `region` needed)
- Linking with other bitmap indexes on table `sales` possible

```
SELECT SUM(Sales.sales)  
FROM Sales, Geography  
WHERE Sales.GeoID = Geography.GeoID AND  
      Geography.Stadt = 'Magdeburg'
```

- Infos: Oracle Dokumentation 11g2 - Part E25789-01

# Indexed Views

- SQL Server 2008: Indexing Views
- Materialization of affected data
- Automated update for changes in the base data → materialized views

```
CREATE VIEW Sales2009 AS  
SELECT City, Sales, S.TimeID, S.GeographyID  
FROM Sales S, Time T, Geography G  
WHERE S.TimeID = T.TimeID AND T.Year = 2009  
      AND S.GeographyID = G.GeographyID;  
  
CREATE UNIQUE CLUSTERED INDEX V2009_IDX  
  ON Sales2009(TimeID, GeographyID);
```

# Multidimensional Index Structures

# Multidimensional Index Structures

- Hash-based Structures
  - ▶ Grid Files
  - ▶ Multidimensional Dynamic Hashing
- Tree-based Structures
  - ▶ kdB-Tree
  - ▶ R-Tree [Gutman 1984]
  - ▶ UB-Tree [Bayer 1996]

# Grid File

- Multidimensional form of data organization
  - ▶ Combination of key transformation elements (Hash approaches) and Index Files (Tree approaches)
- Idea
  - ▶ Dimension refinement: Equal distribution of the multidimensional space of the chosen dimension through a complete cut (insertion of a hyperplane)

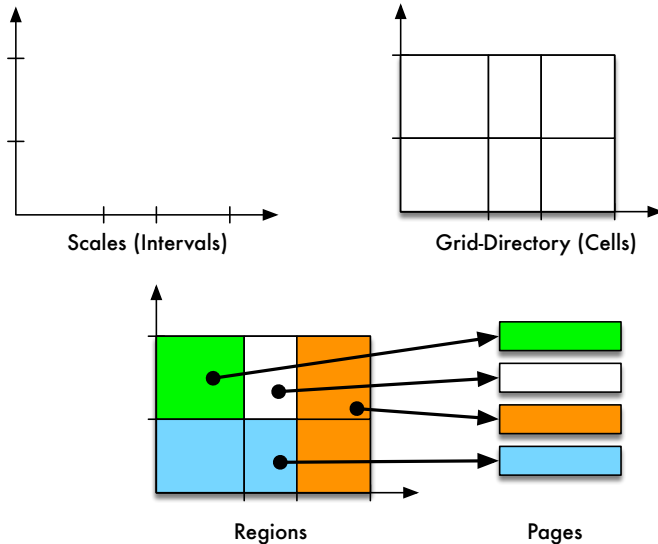
# Grid File: Principles

- Separation of the data space in squares (search region:  $k$ -dimensional cuboids)
- Neighborhood preservation: Storage of similar objects on the same page
- Symmetrical treatment of all space dimensions: partial match queries
- Dynamic adjustment of the structure during inserts and deletes
- **Principle of 2 disc accesses** for exact match queries

# Grid File: Structure

- Grid:  $k$  one dimensional fields (Scales), each scale represents an attribute
- Scales: consists of partitions from the mapped value space in intervals
- Grid Directory: consists of Grid cells, which dissect the data space in squares
- Grid Cells: form a Grid region, that is assigned to exactly one record page
- Grid Region:  $k$ -dimensional, convex construction (pairwise disjoint)

# Grid File: Structure (2)





# Grid File: Operations

- Beginning state: Cell = Region = one record page
- Page overflow
  - ▶ Dividing of pages
  - ▶ If there is just one cell in the region belonging to a page:  
Segmentation of the interval on a scale
  - ▶ If region consists of multiple cells: Division of those cells in separate regions
- Page underflow
  - ▶ Subsumption of two regions if the result is a convex region

# Multidimensional Hashing – MDH

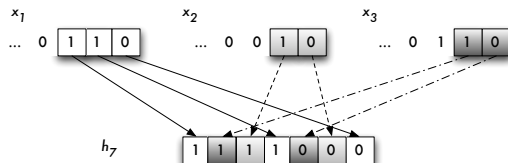
- Idea: Bit Interleaving
- Calculate in diverging order the address bits of the access attributes
- Example: two dimensions

	*0*0	*0*1	*1*0	*1*1
0*0*	0000	0001	0100	0101
0*1*	0010	0011	0110	0111
1*0*	1000	1001	1100	1101
1*1*	1010	1011	1110	1111

# Idea MDH

- MDH builds upon *linear Hashing*
  - ▶ Dynamic Hash Method
  - ▶ Bit sequence prefixes address storage blocks
- Hash values are bit sequences, each having a beginning section serving as a current hash value
  - ▶ For binary numbers: often inverting the bit representation before prefix computation
- Compute one bit string per involved attribute
- Traverse beginning sections according to the principle of bit interleaving in a cyclic manner
- Hash values are composed of the surrounding bits of the individual values
- Family of Hash functions  $h_i$  for bit sequences of length  $i$ 
  - ▶ Dynamic growth: go from  $i$  to  $i + 1$

# MDH Illustration



- Clarifies composition of the hash function  $h_i$  for three dimensions and the value  $i = 7$
- At  $i = 8$  a further bit of  $x_2$  is used (more specific: of  $h_{8_2}(x_2)$ )
- MDH Complexity
  - ▶ Exact Match Queries:  $O(1)$
  - ▶ Partial Match Queries,  $t$  of  $k$  attributes set, Effort  $O(n^{1-\frac{t}{k}})$
  - ▶ Follows from the number of pages when certain bits are "unknown"
  - ▶ Special cases:  $O(1)$  for  $t = k$  and  $O(n)$  for  $t = 0$

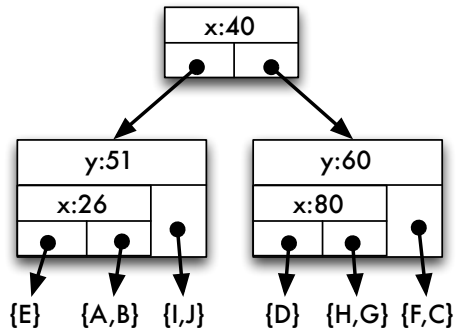
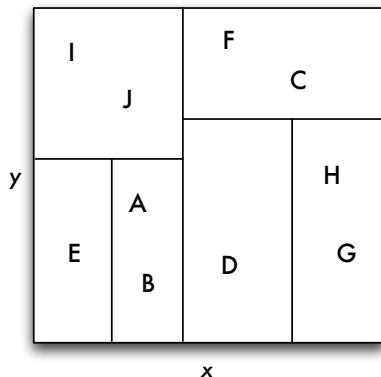
# kdB-Tree

- k-dimensional index trees
  - ▶ kd-Tree: binary Tree for multidimensional basic structure; Main memory storage algorithms [Bentley 1975]
  - ▶ kdB-Tree: Combination of kd-Tree and B-Tree (higher branching degree)
  - ▶ kdB-Tree: Improving the kd-Tree
- Idea of the kdB-Tree
  - ▶ On each index page a subtree is presented, which branches after multiple subsequent attributes
  - ▶ Effort: exact-match  $O(\log n)$ , partial match better than  $O(n)$

# kdB-Tree: Structure

- kdB-Tree of type  $(b, t)$
- Range pages (inner nodes): contain kd-Tree with max.  $b$  inner nodes
  - ▶ kd-Tree with with split elements and two pointers
  - ▶ Split element: Access attribute and value
  - ▶ Left pointer: smaller access attributes
  - ▶ Right pointer: larger access attributes
- Record pages (leaves): contain up to  $t$  tuples of the stored relation

# kdB-Tree: Example



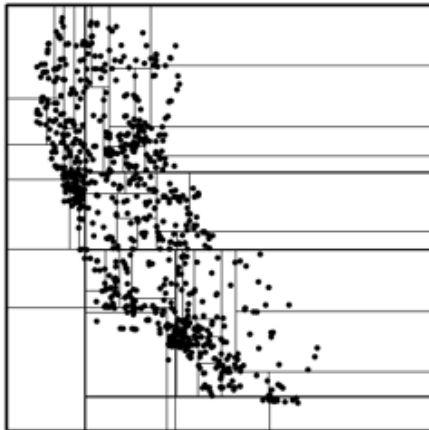
# kdB-Tree: Split attributes

- Order
  - ▶ Cyclic
  - ▶ Consideration of selectivity: Access attribute with high selectivity ideally early and used more often than split element einsetzen
- Split attribute value
  - ▶ Finding a suitable average of the value space given distribution information



# k[dD](B)-Tree: Conclusion

- Stores also bad distributed data
- Difficult to handle for more than three dimensions



# R-Tree

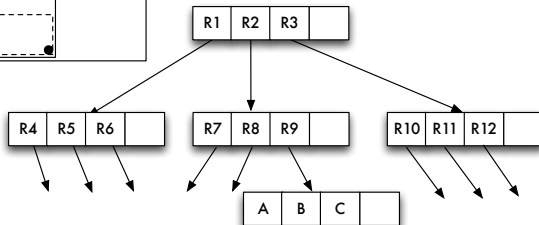
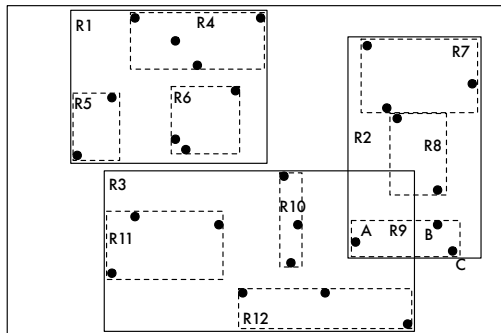
- Each node of the R-Tree stores max.  $m$  index entries
- Each node (except for the root) contains at least  $n$  entries
- $d$ -dimensional R-Tree uses  $d$ -dimensional intervals (rectangles, squares) for indexing the data space
- Entry on leaf level:  $(I, tid)$  with  $I$  a  $d$ -dimensional interval and  $tid$  tuple identifier, that references the respective tuple
- Entry in inner nodes:  $(I, cp)$  with  $I$  the  $d$ -dimensional interval, that contains all intervals of the child node entries (minimum bounding box) and  $cp$  pointers to this child node (child pointer)

## R-Tree (2)

### Special Features (in comparison to the $B^*$ -Tree)

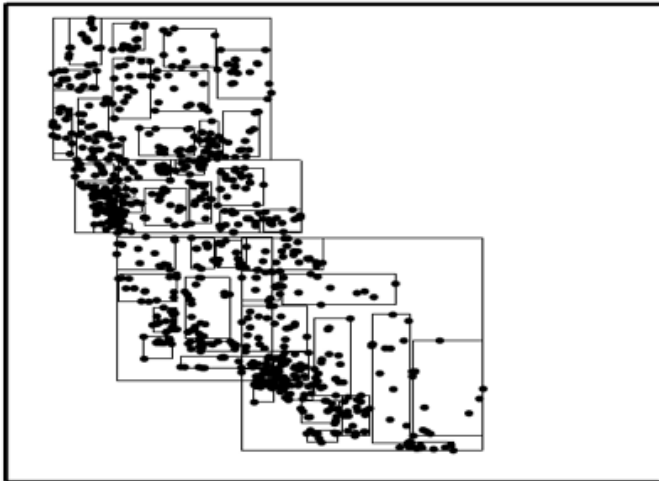
- Search: if different regions of the nodes on the same level overlap, multiple descendants may need to be traversed even for point queries
- Insert: attempt of finding an interval that does not need to be extended, otherwise the interval that has to be extended the least
- The deletion of data usually does not play a role in Data Warehouses; inserts only in big time intervals (but in turn often with many new tuples)
  - ▶ Efficient method important to built the R-Tree structure in a bottom-up manner

# R-Tree: Example



# R-Tree: Conclusion

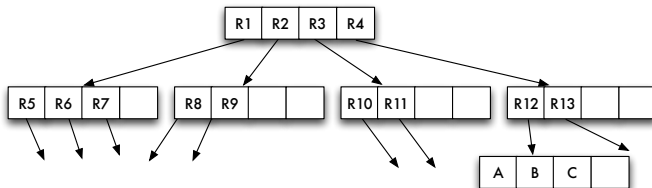
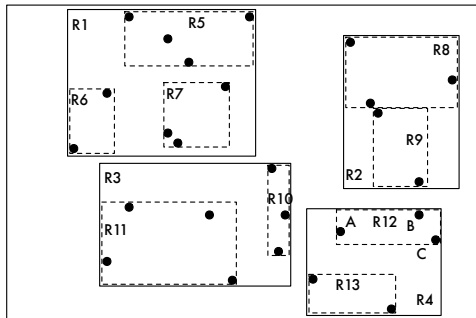
- Better adjustment of the regions to the data



# R<sup>+</sup>-Tree

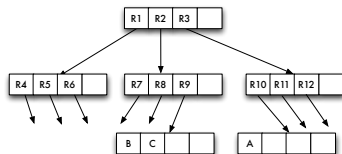
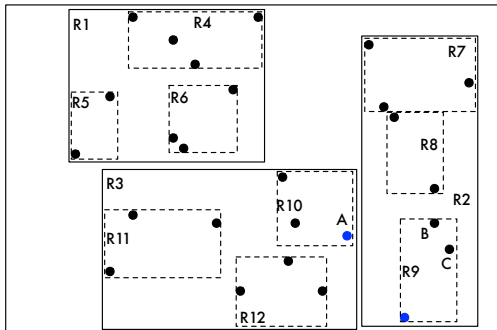
- Basic idea: **Forbid** overlaps
- Adjustment of multiple nodes
- Adjustment implies dissections in smaller MBRs without previous overflow → nodes with few tuples (unused capacity) → many nodes (Degeneration)
- Clipping for storing geometric objects

# R<sup>+</sup>-Baum: Example



# R\*-Tree

- Minimizing overlaps
- However, not forbidden!



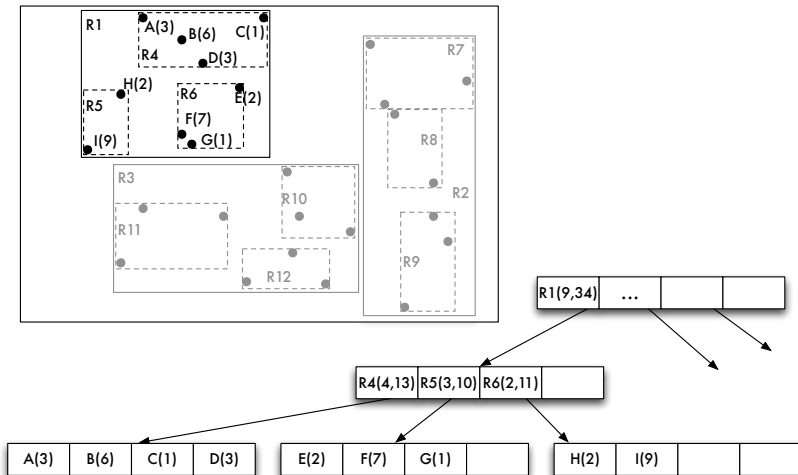


# $R_a^*$ -Tree

- Support for OLAP
  - Predominant for: Aggregation functions
- SUM, MAX, AVG, COUNT, ...
- $R_a^*$ -Tree stores selected aggregated values of lower nodes in each inner node (materialized views)
- *a* stands for *aggregated*

# $R_a^*$ -Tree II

## $R_a^*$ -Tree for COUNT and SUM

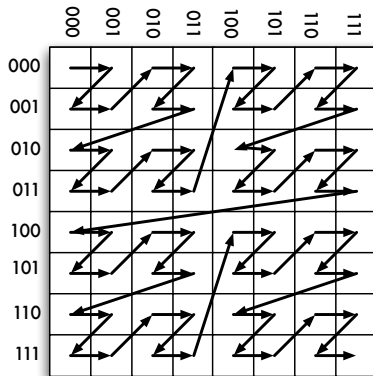


# UB-Tree

- Data space is dissected in disjoint subspaces using a space-filling curve (often the so-called Z-Curve)
- Each point of the attributes to be indexed within the multidimensional space is projected to a scalar value. the Z-Value
- Z-Values are used as keys in a standard  $B^+$ -Tree

## UB-Tree (2)

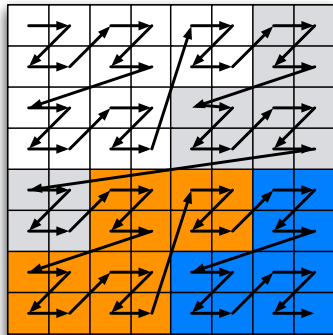
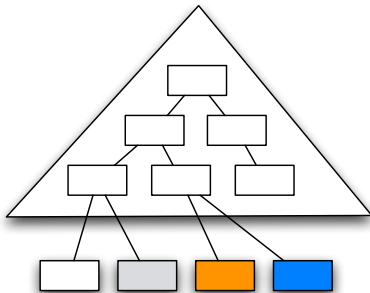
- Dissection of a 2-dimensional space with the Z-Curve:



## UB-Tree (3)

- **Z-Values** can be computed efficiently (in linear time):
  - ▶ Per dimension the basic intervals are binary numbered;
  - ▶ By interleaving the bits, the respective Z-Value is obtained.
- **Z-Region**: is determined by an interval  $[a, b]$  of Z-Values
  - ▶ Z-Regions of a UB-Tree are adjusted dynamically so that the objects within a Z-Region fit exactly within a page of a  $B^+$ -Tree
  - ▶ With that a  $B^+$ -Tree can be used as a basic structure

# UB-Tree (4)



# UB-Tree: Region Search (RQ-Algorithm)

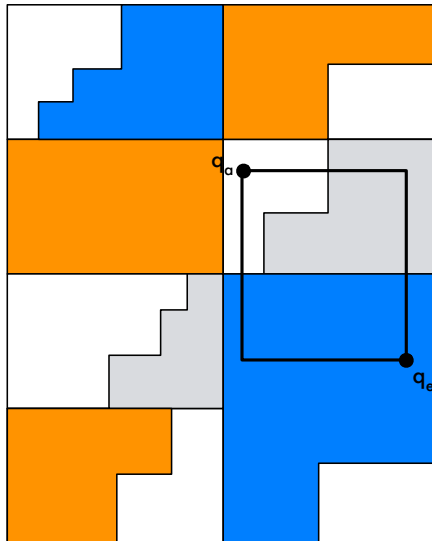
- Each range query is determined by 2 tuples  $q_a$  and  $q_e$  which (visually) specify the left upper and right lower edge of the query region
  - 1 Begin with  $q_a$  and compute the respective Z-Region
  - 2 Load the respective page and apply the query predicate to all tuples within the page
  - 3 Compute the next region of the Z-Kurze, that lies within the query region
  - 4 Repeat 2. and 3. until the end address of the edited Z-Region is bigger than  $q_e$  (and also contains the end point of the query region)

# UB-Tree: Region Search

- Step 3. (Computation of the intersection points of the Z-Curve with the query region)
  - ▶ Appears critical at first glance;
  - ▶ In fact efficiently solved by using "a few" bit operations (and without disk accesses) in linear time (linear in the length of the Z-Values)



# UB-Tree: Visual Region Search



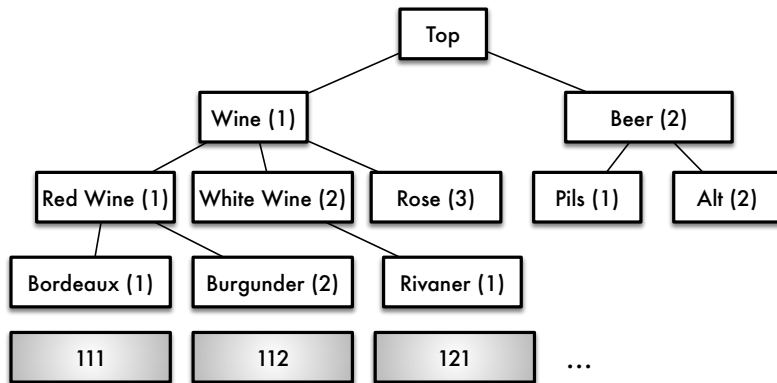
# UB-Tree and MHC

- Extensions of the UB-Tree for Data Warehouses:
  - ▶ **Multidimensional Hierarchical Clustering (MHC)**
  - ▶ Supports hierarchically organized dimensions, so that all advantages of the UB-Tree remain

# MHC: Principle

- Total order for hierarchy
- Assignment of a unique number to each leaf element of the hierarchy
- Elements of the same subtree contain grouped numbers (Clustering)
- Computation:
  - ▶ Each element of a hierarchy level contains a number (surrogate)
  - ▶ For leaf element: linking the surrogates together (binary representation) → multicomponent surrogate

# MHC Example



# MHC Usage

- Multicomponent surrogate as
  - ▶ Key for tuples in the fact table
  - ▶ Index attribute for UB-Tree
- Example: Range query
  - ▶ Minimum und maximum multicomponent surrogate as interval for restriction

# Summary

- Index structures allow improved multidimensional queries
- One dimensional index structures do not suffice
- B-Tree, Hash approaches and extensions are one dimensional
- Tree- and Hash approaches can be adjusted for multidimensionality
  - ▶ kdB-Tree, UB-Tree, R-Tree
  - ▶ Grid Files, MDH