CS 590 - Assignment 3

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Abstract:

A binary search tree is a data structure that can support multiple dynamic-set operations like search, insert, remove, etc. Such a binary search tree was implemented with few functionalities using C++ for different inputs. The binary tree works well with smaller heights but is significantly inefficient for larger heights. So, the red-black trees which are balanced are used to improve efficiency. The red-black tree was also implemented for different inputs. A random generator was used to create various keys up to the given input size. These keys were inserted in both trees in random, sorted, and inverse sorted. The tree implementation included insertion (for unique nodes), removal of nodes, traversing nodes, converting a tree to an array in ascending order, finding the black height, performing various insertion cases for RBT, and performing different rotations for RBT. Also, counter for various mentioned operations. The runtime performance for sorting and computing black height for these trees was observed for input sizes = = 50000; 100000; 250000; 500000; 1000000; 2500000; 5000000. A scatterplot was obtained for these performances. An analysis of both trees' performance was done.

Observations:

a. **Performance and counter for BST**

Binary Search Tree						
	Random		Sorted		Inverse Sorted	
n	Sorting Runtime (ms)	Duplicate Nodes	Sorting Runtime (ms)	Duplicate Nodes	Sorting Runtime (ms)	Duplicate Nodes
50,000	25.6	0.8	4348.2	0	4203.8	0
100,000	51.4	2	17164.2	0	18483.6	0
250,000	143	13	-	-	-	-
500,000	378.2	59.2	-	-	-	-
1,000,000	1047.4	233.2	-	-	-	-
2,500,000	3595	1446.2	-	-	-	-
5,000,000	8805.2	5780.8	-	-	-	-

b. Performance for random inputs for RBT

n	Random Input								
	Sorting Runtime	Duplicate Nodes	Case 1 Insertion	Case 2 Insertion	Case 3 Insertion	Left Rotation	Right Rotation	Runtime (Black Height)	Black Height
50000	25.8	0.6	25692.4	9699.6	19402.2	14526	14575.8	1.2	19
100000	51.8	2	51364.8	19435.8	38916.6	29213.4	81254.6	6	20.4
250000	158.6	14.2	128368.8	48747.2	97233	72949.6	73039.6	19.8	22
500000	386.4	54.8	256711	97027	543482	145634.2	406582	53.2	23
1000000	1057	235.8	513207.6	193873.4	1088205	291126.2	290952.2	120	24.6
2500000	3603.6	1475.8	1283300.2	484921.8	970801.2	727947.2	727729.8	337.4	26
5000000	7923	5794.6	2564106.4	969564.4	1939730.2	1454626.2	1454668.4	684.4	27

c. Performance for sorted inputs for RBT

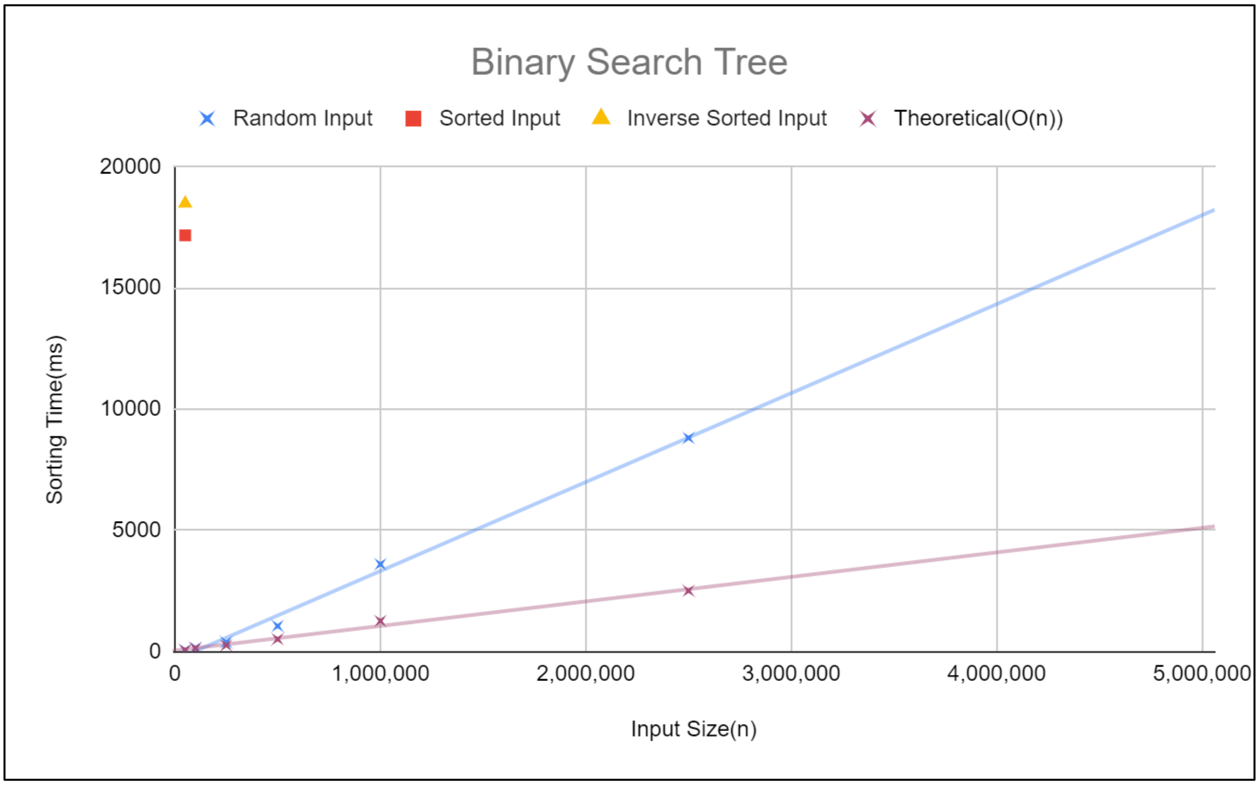
n	Sorted Input								
	Sorting Runtime	Duplicate Nodes	Case 1 Insertion	Case 2 Insertion	Case 3 Insertion	Left Rotation	Right Rotation	Runtime (Black Height)	Black Height
50000	20.6	0	49966	0	49971	49971	0	3	29
100000	37.4	0	99964	0	99969	99969	0	3.2	31
250000	81.8	0	249961	0	249967	249967	0	7.4	33
500000	156.6	0	499959	0	499965	499965	0	17.6	35
1000000	293.6	0	999957	0	999963	999963	0	35	37
2500000	779.2	0	2499952	0	2499960	2499960	0	90.6	40
5000000	1616.4	0	4999950	0	4999958	4999958	0	177	42

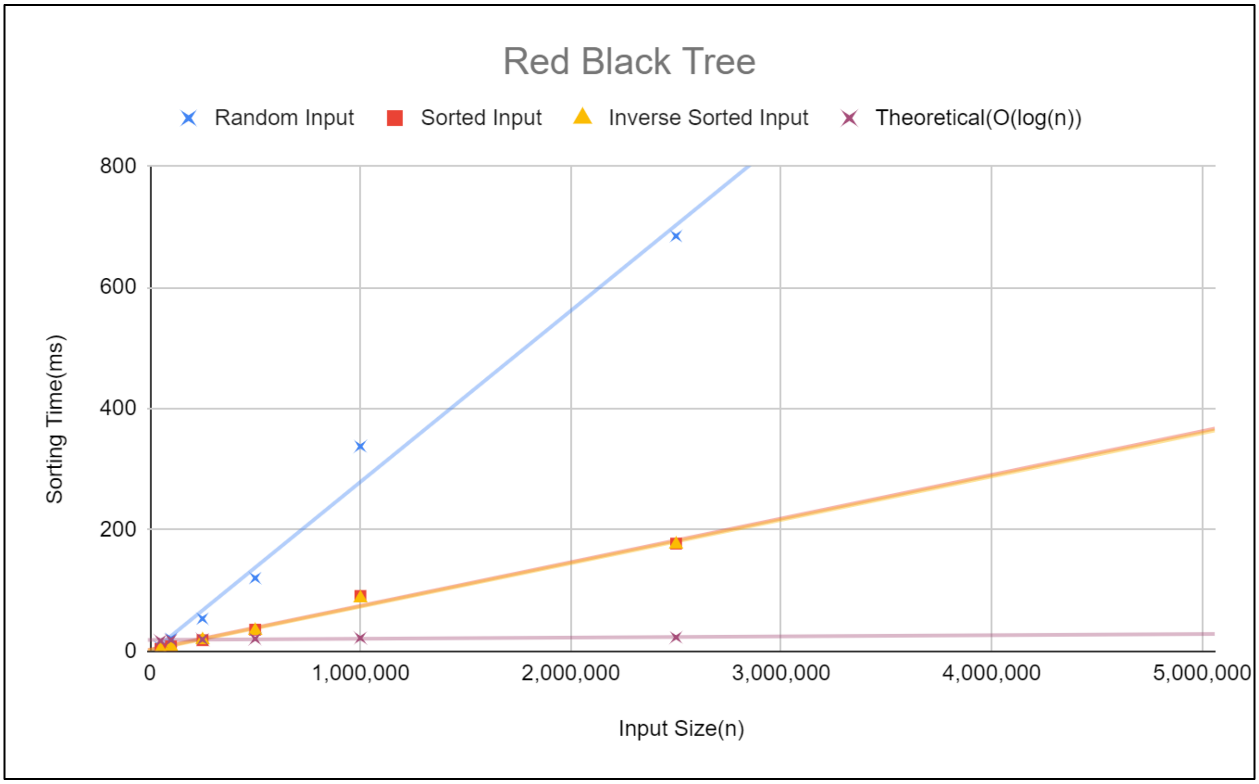
d. Performance for inverse sorted inputs for RBT

n	Random Input								
	Sorting Runtime	Duplicate Nodes	Case 1 Insertion	Case 2 Insertion	Case 3 Insertion	Left Rotation	Right Rotation	Runtime (Black Height)	Black Height
50000	22	0	49966	0	49971	0	49971	2.8	29
100000	44.2	0	99964	0	99969	0	99969	2.2	31

250000	81.2	0	249961	0	249967	0	249967	6.8	33
500000	150.4	0	499959	0	499965	0	499965	18	35
1000000	293.6	0	999957	0	999963	0	999963	33.4	37
2500000	781.8	0	2499952	0	2499960	0	2499960	87	40
5000000	1602.2	0	4999950	0	4999958	0	4999958	175.8	42

Discussion:





The plots of the runtime vs the input size parameters are shown above. From the plots above, the BST sorting performance becomes significantly worse with an increase in input size for the sorted and inverse sorted inputs. There is not a big difference in the BST and RBT performance for the random input, but this might change with increasing input size. The RBT performed significantly better than BST, especially for the sorted and inverse sorted inputs. This verifies that the RBT's performance is much better than BST for greater heights.

For the RBT, with an increase in input, the counters increase rapidly, the black height increases much more slowly. For sorted input, there is no insertion with case 2, nor is there a right rotation. Whereas, for the inverse sorted input, there is no case 2 insertion and there is no left rotation.

Analysis:

i. Sorting performance of BST and RBT is comparable for random inputs (RBT is slightly better)
ii. The sorting performance of RBT is extremely more efficient than BST for sorted and inverse sorted inputs.
iii. RBT is more efficient than BST, especially for increasing input size.
iv. The black height of RBT increases gradually with the rapid increase in the input size.
v. For sorted input, no case 2 insertions are done, and there is no right rotation.
vi. For inverse sorted input, no case 2 insertions are done, and there is no left rotation.

Conclusion:

i. After implementing BST and RBT with their set operations, the performance of each of these was observed.
ii. BST runtime is slightly worse than BST for random input.
iii. BST runtime is extremely bas than BST for sorted and inverse sorted input.
iv. RBT is significantly more efficient than BST.
v. BST has best case runtime of O(log(N)) and worst case of O(N), whereas, RBT has best and worst-case runtime of O(log(N))
vi. Black height of RBT increases gradually with increasing input size.