

Recognition of Handwritten Mathematical Expressions: A Survey

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Abstract— Handwritten Mathematical Expressions (HME) play a very quintessential part in different domains, starting from engineering to education. Now, with the COVID-19 pandemic, the utility of handwritten interaction has gained impetus, with online education becoming the new normal. Also, the digital revolution has made digital documentation preferable over the traditional one. The field of HME recognition is flourishing now, more than ever before. Advancements in multiple fields, has led to developments in solutions to overcome the challenges of extant technologies for HME recognition. With this survey paper, we try to provide brief data about the existing techniques and methodologies which are used for HME recognition in some past works. Classification on the basis of techniques for different stages of HME recognition is also done. Merits and demerits of the approaches surveyed are also discussed. Metrics of the past works are also briefly mentioned. Lastly, the paper presents a prospective improvement for past works, proposing future works in handwritten mathematical expression recognition systems.

Keywords- Handwritten mathematical expressions recognition, Classifiers, Neural networks, CNN, LSTM, structural analysis.

I. INTRODUCTION

For the centennial, handwriting had been the most common way of communication. Even though modern ways of communication rely on digital devices like mobile phones, tablets, PDAs or PCs. Still, use of handwritten communication is prevalent in today's world. Development of all these devices was accompanied by initiation of recognition systems. These recognition systems are capable of translating texts from our natural handwriting, into machine readable format that could be explicitly understood by computers ^[1].

For the recognition of English characters in electronic books, Optical Character Recognition (OCR) can be used to attain higher recognition

accuracy^[2]. Mathematics is broadly used in various domains like in scientific papers, engineering, medicine, economics etc. Due to difficulties like two-dimensional nesting assembly, variation of different individuals, the correction rate of symbol segmentation and recognition which makes recognition of Handwritten mathematical equation the most challenging jobs to do in the area of computer vision and still cannot achieve its actual requirements

Mathematics is broadly used in various domains like in scientific papers, engineering, medicine, economics etc. However, handwritten mathematical expression recognition is still a challenging job especially in the field of computer vision. Some difficulties include, two dimensional nesting assembly, variation of different individuals, the correction rate of symbol segmentation and recognition still cannot achieve its actual requirements.

For formatting mathematical expressions, mathematical typesetters and editors are available. These systems produce a well-formatted expression, and can be professionally used, they are time inefficient compared to traditional handwriting. Obscurities in symbol location or layout, usage of many different mathematical symbols can prove to be unyielding.

In the computing world there are currently many production level math analysis tools available. However, greater the number of features in a

mathematical tool, the higher is the difficulty of leveraging its advantages. Several methods are being explored to enable computers to classify and identify handwritten Mathematical equations^[4].

II. NEED FOR HME RECOGNITION SYSTEMS

Mathematical Expressions are used in all domains and thus its correct recognition is very crucial. HME written manually using pen and paper method is inflexible and cannot be used beyond that. These HME cannot be repurposed, have issues regarding storage and longevity and are temporary in nature. When storage of past scientific work is done manually, changing and updating these written records is difficult, as well as labor is required for proper organization of the data. All these problems have resulted in severe loss of data^[9]. Though extensive recognition systems for typical handwritings are available, recognition of HME is much more difficult and challenging because of the geometric structures, enormous ambiguities in handwritten input and the strong dependency on contextual information^[32].

III. RECOGNITION SYSTEMS

Handwritten Mathematical equation recognition systems primarily consist of Input data and datasets, segmentation, classification and structural analysis. When these steps are followed, the recognition engine will classify the recognized symbols and best interpretation will be generated.

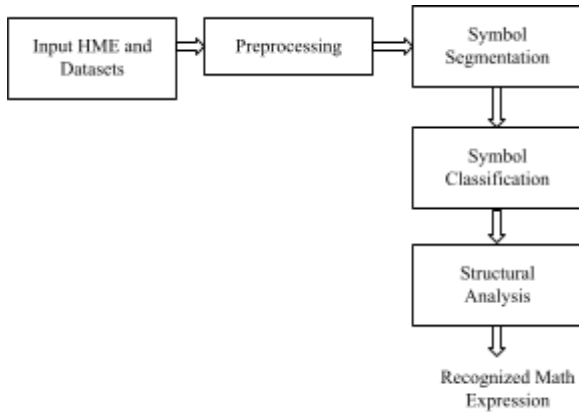


FIGURE I. BLOCK DIAGRAM FOR HME RECOGNITION SYSTEM

A. Input HME and Datasets

HME can be given as input through two subclasses viz. Offline and Online. In the offline mode, an image of input i.e handwritten data is considered and is then provided to the recognition system. Whereas, online mode makes use of digital devices like tablets and mobile phones. The input to these devices is stored as recorded stroke data and then is provided as input to the recognition system.

Datasets are a collection of data samples which are used to train and test a model. In this system, datasets consisting of data like numerals, alphabets, characters, etc are used. Different input type includes alphabets of lower and uppercase, greek alphabets like alpha, beta, gamma and many more, numerals from 0 to 9 and mathematical symbol including $\{+ - * | , < > , * , \% , / , \sqrt{} , \alpha , \beta , \delta , \Delta , , \gamma , > , - , \infty , R , [, (, < , \mu , \Omega , \Pi ,] , \rho ,) , \sigma , \Sigma , \sqrt{} , \theta , \xi \text{ and } \zeta \}$.

In HME Recognition system, use of open source datasets like MNIST(Modified National Institute of Standards and Technology database), CROHME(Competition on Recognition of Online Handwritten Mathematical Expressions) as well as personal datasets was done. MNIST, which is an open source dataset, is a database consisting of images of 28x28 size. These images include handwritten digits from 0-9 which are labelled. CROHME is another open source dataset which consists of more than 10,000 handwritten expressions. These expressions are available in a variety of handwriting from different authors from different countries.

TABLE I
HME DATASETS

Dataset	Peculiarity
MNIST	Set of images consisting handwritten digits, with 1000 handwritten character symbols ^[7]
	2000 data items of handwritten digits ^[2]

CROHME	75 different mathematical symbols ^[3]
CROHME 2013 and 2014	101 symbol classes including similar symbols ^[19]
CROHME 2016	10,000 labelled handwritten formulae ^[6]
Personal Dataset	
Winkler et al. ^[34]	153 expressions
Zanibbi et al. ^[25]	93 classes of symbols including numeric symbols, alphabets, Greek alphabets, mathematical symbols.
Shinde and Waghulade ^[13]	690 characters, 420 digits, 660 mathematical symbols
D'Souza and Mascarenhas ^[11]	Alphabets, numbers, Greek letters, special characters

B. Preprocessing

From the previous step, the system now has input HME in the form of an image. The next step in recognition is preprocessing. This step in a recognition system is used to reduce imperfections, providing clear and smooth data to the system. Preprocessing can be done with different approaches like filtering, size normalization, smoothing, resampling, binarization. In filtering, noise and false points are filtered while size normalization is used to reduce variation of size. Smoothing is another technique which is used for eliminating noise, resampling for reducing the effect of writing velocity. Whereas, binarization for converting a grayscale image into a binary image. For binarization, a self-adjusting thresholding method was used. Bharadwaj et al. used binarization, global and adaptive thresholding^[4]. Skew detection and correction can also be used in pre-processing for analysing deviation of text from horizontal and vertical axis. Another technique used was Laplacian and Gradient technique. In Laplacian, zero crossings in the second derivative of the image were used while for Gradient technique edge detection was done by looking for the maximum and minimum in

the first derivative of the image. These both techniques were used by Attigeri in her work^[21].

C. Symbol Segmentation

After pre-processing, the output image is fed to the system for segmentation. In segmentation, segregation of lines, characters, symbols, words of the image are done. Image is segmented into individual characters and each character is fed as a separate complete image to the system. Different methods are used for segmentation purposes like Equation line segmentation, Character line Segmentation, Character geometry technique. In equation line segmentation, split-up of the different lines of characters is done. Whereas in Character line Segmentation, segmentation series of characters into sub images of individual symbols is carried out. Hossain et al. used compact horizontal projection for equation line segmentation while connected component analysis method was used for character line segmentation^[2]. Character geometry technique is a method in which the image is divided into equal size. Gupta et al. used Character geometry technique in which zoning, character traversal, distinguishing line segment was done^[12]. Lu et al. intersected traces of the symbol and added steps to find symbols with non-intersecting strokes^[3]. In the DFS algorithm, each symbol is considered to be a connected component. Bharadwaj et al. condensed the input image into a two-dimensional matrix after pre-processing and used the Depth First Search (DFS) algorithm for pre-processing^[4]. Zhelezniakov et al. used Baseline detection, size normalization, slant/slope correction, and resampling for segmentation^[33]. Shinde et al. used Structural and Functional analysis^[13]. In this method, input image is segmented into individual characters, each character is resized into 5x7 pixels and are isolated by assigning a number to each character using the labelling process. Labelling process provides information regarding the number of components in the image.

D. Symbol Classification

Classification is a technique with which the symbols, which were recognized in the prior step, are divided into different groups or classes using trained and tested datasets. This step is essential as it helps to categorize each character as either digit, alphabet, symbol, etc. Symbol classification is done using a variety of techniques like Convolutional Neural Networks(CNN), Multi Layer Perceptron(MLP), K Nearest Neighbor(KNN), Genetic Algorithm, Hidden Markov Model(HMM), Hopfield Neural Network, Support Vector Machine(SVM), and Recurrent Neural Network(RNN). Most works used the CNN technique for classification, CNN models involve passing input layers through a series of trained and tested convolutional layers with filters. This is followed by pooling of the fully connected layer and application of an activation function to obtain a numeric value; this value then classifies the input image. The CNN technique was used by Ramadhan et al.^[14], Bender et al.^[8], Bilbeisi et al.^[17], Tran et al.^[10], Bharadwaj^[4], Lu and Mohan^[3], Nazemi et al.^[7], Bodnia and Kozulia^[15], Hossain et al.^[2], Fang and Zhang^[24], D'Souza and Mascarenhas^[11]. Bender et al. used CNN along with a fine-grained feature extractor and an attention based encoder-decoder module^[8]. G. Bilbeisi also paired CNN with a fine-grained feature extractor but used an LSTM(Long Short Term Memory) encoder-decoder module^[17]. Tran et al. used the CNN with four single shot detectors(SSD)^[10]. Hossain et al. used the CNN with no predefined features and used smaller layers to reduce training time^[2]. D'Souza and Mascarenhas used a Sp-Net CNN for classification^[11]. Fang and Zhang used Squeeze-extracted multi-feature CNN for the classification purpose^[24]. Nazemi made use of the LeNet and SqueezeNet CNN models for the same^[7]. Next technique used was a MLP; it is a multi-layer fully connected neural network which makes use of the perceptron algorithm. This MLP technique was used for classification in works of Montaser et al.^[1], Attigeri^[21], Shinde and Waghulade^[13], and Karpagavalli and Padmapriya^[26]. Another classification technique is the KNN; it is a supervised learning algorithm

which uses a set of input values to predict output values, it classifies the new data points based on the similarity measure of the previous points. This method of classification was incorporated for symbol classification by Gupta^[12]. Genetic algorithm was also used for classification purposes, this algorithm is based on the genetic structure and behavior of the chromosome population. This algorithm classifies the symbol once its termination criterion is reached. It was used by Bodnia and Kozulia^[15] in their work. HMM which is another classification technique, works on the basis of data and initial parameters provided like transition probability, number of states. HMM was used as a classifier in the works by Hu and Zanibbi^[25], Kosmala et. al^[27]. Hopfield Neural Network is another classification technique which has a single layer of connected recurrent neurons and performs auto association and optimization tasks. This technique was implemented in the work of Bodnia and Kozulia^[15]. SVM was also used for classification of symbols; it makes use of a hyperplane which is a decision boundary to categorize future values. This SVM classifier was used as a classifier in the works of Le and Nakagawa^[19], Shinde et al.^[5], Gupta^[12], Bharambe^[22], and Karpagavalli and Padmapriya^[26]. Shinde et al. used complex features viz. Entropy, mean, variance, standard deviation, skewness, kurtosis, correlation and contrast as parameters for the SVM^[5]. Whereas, Bharambe made use of parameters like normalized chain codes, moment variant features, density features and histogram in the SVM classifier^[22]. Lastly, RNN was also used for classification of the symbols. RNN saves the output from a particular layer and then feeds this back to the input layer to predict the output of the layer; this principle of RNN was used to classify the symbols. RNN as a classifier was implemented in the works of Zhang et al.^[32], Zhelenzniakov et al.^[33], Graves et al.^[20] and Nguyen et al.^[6]. Zhang had proposed a model which used the RNN based Gated Recurrent Unit with an attention-based encoder-decoder module^[32].

TABLE II
BRIEF ANALYSIS OF CLASSIFICATION METHODS PROPOSED

Approach	Authors	Claimed Accuracy
CNN (Convolutional Neural Network)	Bender et al. ^[8]	84.3%
	Ramadhan et al. ^[14]	87.7%
	G. Bilbeisi et al. ^[17]	92.35%
	Tran et al. ^[10]	65%
	Bharadwaj ^[4]	99.2%
	Lu and Mohan ^[3]	90%
	Nazemi et al. ^[7]	90%
	Bodnia and Kozulia ^[15]	-
	Hossain et al. ^[2]	39.11%
	Fang and Zhang ^[24]	92.96%
	D'Souza and Mascarenhas ^[11]	-
MLP (Multi Layer Perceptron)	Montaser et al. ^[1]	37.1%
	Shinde and Waghulade ^[13]	94.11%
	Padmapriya and Karpagavalli ^[26]	93.8%
	Attigeri ^[21]	-
KNN (K Nearest Neighbors)	Gupta ^[12]	-
Genetic Algorithm	Bodnia and Kozulia ^[15]	-
Hidden Markov Models (HMM)	Hu and Zanibbi ^[25]	86.8%
	Kosmala et al. ^[27]	-
Hopfield Neural Network	Bodnia and Kozulia ^[15]	-
SVM (Support Vector Machine)	Le and Nakagawa ^[19]	60.36%
	Shinde et al. ^[5]	88.5%
	Gupta ^[12]	-
	Bharambe ^[22]	93.8%
	Padmapriya and Karpagavalli ^[26]	85%

RNN (Recurrent Neural Network)	Zhang et al. ^[32]	52.43%
	Zhelezniakov et al. ^[33]	65.76%
	Graves et al. ^[20]	74.1%
	Nguyen et al. ^[6]	92.3%

E. Structural Analysis

Structural analysis is the final step in the classic HME recognition process. In this step, relation between the spatial relationship of previously recognized symbols is determined^[16]. Use of different techniques like a Baseline Structure Tree(BST), Long Short Term Memory(LSTM), Connectionist Temporal Classifier, and Pattern Recognition was done in some works for the purpose of structural analysis. BST method involves constructing a tree, this tree describes a 2D arrangement of the input symbols. This technique was used by Tran et al. with help of DRACULAE parser for structural analysis; here two passes of the tree viz. Layout pass and Lexical pass were used for efficient recognition of layout of the equation^[31]. A LSTM is a variant of RNN which is designed to capture long range dependencies for modelling sequential data. This LSTM model was used in the work by Bender et al.^[8]. Use of LSTM was done row-wise on the feature map and concatenation of output rows was done for final local representation^[8]. Another structural analysis model is the Connectionist Temporal Classification(CTC). CTC is a kind of neural network output as well as an associated scoring function, this is used for tackling sequence problems which involve variable time. This CTC model was used in the work of Zhelezniakov et al.^[33]. In system proposed by Zhelezniakov et al., CTC was used in order to learn the alignment as well as for classification of the sequence of symbols in an HME^[33]. Pattern recognition is used to recognise patterns or any kind of regularities in data; it was used in the system by Bodnia and Kozulia.^[15]. In the work of Bodnia and Kozulia, pattern recognition index was used to calculate an equivalent value which is used for displaying the

output text^[15]. System proposed by Zhang et al. unified the structural analysis in one framework with symbol segmentation and recognition^[32]. Whereas, Bharadwaj proposed a custom algorithm which strings the characters together^[4].

TABLE III
COMPARATIVE ANALYSIS OF HME RECOGNITION SYSTEMS

Authors	Methodology	Merits	Limitations
Ramadhan et al. ^[14]	CNN for feature maps and fully-connected perceptrons	Tuning of performance easily by changing parameters.	Misclassification due to structural similarity
Tran et al. ^[10]	Deep CNN with SSD and DRACULAE ^[31] parser	LaTeX formatted output with higher accuracy from parser, useful on larger input.	Incorrect classification of similar symbols. Parser unable to work with all cases.
Hossain et al. ^[2]	Horizontal compact projection analysis with combined component analysis methods and CNN.	Solutions for recognized quadratic equations obtained with no predefined features.	Inefficient for multiple mathematical formulas.
Bender et al. ^[8]	CNN with fine-grained structures and attention-based encoder decoder modules trained jointly in an end-to-end manner with LSTM.	Useful for variable-sized symbols and on large scale dataset. Minimized cross entropy loss.	Errors on very long formulas.
G. Bilbeisi et al. ^[17]	Attentional encoder-decoder architecture, with CNN to and Max-Blur-Pooling with Fine-Grained feature map.	Reduced number of filters with extraction of finer features.	Difficulty in parsing of repeating symbols, errors for small sized symbols. Inefficient for dense equations.
Nguyen et al. ^[6]	Bidirectional RNN with deep BLSTM and Temporal recognition.	Two way context, so less ambiguity, recognition of junk symbols.	Missing context from strokes could not handle ambiguous symbols.

Bodnia and Kozulia ^[15]	Neural networks. Genetic algorithm, CNN and Hopfield network	Web application was used for implementation.	Limited to recognition of Ukrainian handwritten texts only.
D'Souza and Mascarenhas ^[11]	Preprocessing followed by CNN classifier, use of SpNet-CNN.	Retaining of order and output in latex.	Variation in writing style causes difficulty in recognizing characters, inefficient for complex or connected symbols.
Shinde et al. ^[5]	Morphological operations and Support Vector Machine(SVM)	Accuracy varies with extracted features and classifiers used.	Efficiency decreases with larger labels. Use of the correct classifier required for appropriate recognition rate.

V. CONCLUSION

This survey illustrates the growing interest in HME recognition over the past decades. With advances in pen-based computing, optical scanning technologies, availability of required hardware for inputting mathematical expressions into computers based on either online or offline data is easier. Persistent errors are observed due to similarity in symbols, difficulty for recognition of multiple expressions. Consistent higher accuracy is obtained with the help of CNN. Classifiers, even if offering high accuracy are inefficient on larger labels.

VI. FUTURE WORK

Issues regarding recognition due to similar symbols can be alleviated with use of a larger dataset. Use of deeper or more layers in the CNN architecture can also further improve the results. Preprocessing of the data can decrease a greater amount of errors. Combining the structural analysis with semantic analysis enables better interpretation of data.

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