0/1 Knapsack Problem

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$M = 8$ $P = \{1, 2, 5, 6\}$ - Profit

$n = 4$ $w = \{2, 3, 4, 5\}$ - weight

capacity

Q fill the bag with object so that its under the weight m and profit is maximized.

→ The objects are not divisible or breakable.

$\max \sum P_i x_i$ & $\sum w_i x_i \le m$

$x_i = 0/1$ for n objects there will be 2^n scenarios and time will be 2^n, instead use the tabulation ~~method~~.

capacity →

P_i	w_i		0	1	2	3	4	5	6	7	8	
		0	0	0	0	0	0	0	0	0	0	
1	2	1	0	0	1	1	1	1	1	1	1	→ consider 1st object
2	3	2	0	0	1	2	2	3	3	3	3	→ " " & 2nd "
5	4	3	0	0	1	2	5	5	6	7	7	→ " " " & 3rd
6	5	4	0	0	1	2	5	6	6	7	8	→ " " " " 4th

$$V[\underset{\text{Row}}{i}, \underset{\text{column}}{w}] = \max\{V[i-1, w],\ V[i-1, w - w[i]] + P[i]\}$$

Ex:- $V[4, 1] = \max\{\underbrace{V[3, 1]}, V[3, 1-5] + 6\}$

$3, -4 \rightarrow X$

$\Rightarrow 0$ ✓

x_1 x_2 x_3 x_4

$\{0 \quad 1 \quad 0 \quad 1\}$ → max profit with 4th

as 2
is there
in 2nd
object

object so that is
included.

$8-6=2$

$2-2=0$

Set Method :

$S(P, W)$ → $S^0 = \{(0,0)\}$

Profit   weigh

$S^0_1 = \{(1,2)\}$

$S^1 = \{(0,0), (1,2)\}$

$S^1_1 = \{(2,3), (3,5)\}$

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S^2_1 = \{(5,4), (6,6), (7,7), \cancel{(8,9)}\}$

exceeding the capacity
so don't include.

$S^3 = \{(0,0), (1,2), (2,3), \cancel{(3,5)}, (5,4), (6,6), (7,7)\}$

This order
pair is
wrong

$w \downarrow$ but $P \uparrow$

$S^3_1 = \{(6,5), (7,7), (8,8), \cancel{(11,9)}, \cancel{(12,11)}, \cancel{(13,12)}\}$

$S^4 = \{(0,0), (1,2), (2,3), (5,4), \cancel{(6,6)}, (6,5), (7,7),$
$(8,8)\}$

Max.

① $(8,8) \in S^4$

but $(8,8) \notin S^3 \quad \therefore x_4 = 1$

$$(\underbrace{8-6}_{\text{Profit}},\ \underbrace{8-5}_{\text{weight}}) = (2,3)$$

② $(2,3) \notin S^3 \quad \therefore x_3 = 0$

$(2,3) \in S^2$

Ans $(0,1,0,1)$

③ $(2,3) \in S^2$

$(2,3) \notin S^1 \quad \therefore x_2 = 1$

$$(2-2,\ 3-3) = (0,0)$$

④ $(0,0) \notin S_1 \quad \therefore x_1 = 0$

$(0,0) \in S_0$