# Matrix Chain Multiplication

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For multiplying two matrix the number of columns of first matrix should be equal to number of rows in second matrix.

The dimension of multiplied result will be no. of rows of first one x no. of column of second matrix.

$A_1 \times A_2 \times A_3$

2 3 3 4 4 2

$d_0\ d_1 \quad d_1\ d_2 \quad d_2\ d_3$

Which pair should we select such that the total cost of multiplication is minimum?

No. of multiplications required = rows × columns × columns

$^{n-1}C_{n-1} \rightarrow {}^{3}C_{3} = 5$

$n$: 1 2

$(A_1 \times A_2) \times A_3$ 2 3 3 4 4 2

$C[1,2]$ $2 \times 3 \times 4 = 24$

$C[1,3]$ $2 \times 4 \times 2 = 16$ ($d_0\ d_2\ d_3$)

$24 + 16 + 0 = 40$

OR

$A_1 \times (A_2 \times A_3)$ 2 3 3 4 4 2

$C[1,1]$ 0

$C[2,3]$ $3 \times 4 \times 2 = 24$

$C[1,3]$ $2 \times 3 \times 2 = 12$ ($d_0\ d_1\ d_3$)

$24 + 0 + 12 = 36$

# Formula :-

cost from $1 \rightarrow 3$

$$C[i,j] = \begin{cases} C[i,k] + C[k+1,j] + d_{i-1} \times d_k \times d_j \\ i \leq k < j \end{cases}$$

Ex: $A_1 \times A_2 \times A_3 \times A_4$

$d_0\ d_1\ d_1\ d_2\ d_2\ d_3\ d_3\ d_4$

3 2 2 4 4 2 2 5

$$C[1,4] = \min_{1 \le k < 4} \begin{cases} k=1: & C[1,1] + C[2,4] + d_0 \times d_1 \times d_4 \\ k=2: & C[1,2] + C[3,4] + d_0 \times d_2 \times d_4 \\ k=3: & C[1,3] + C[4,4] + d_0 \times d_3 \times d_4 \end{cases}$$

Apply same formula

$A_1 (A_2 A_3 A_4)$

$(A_1 A_2)(A_3 A_4)$

$(A_1 A_2 A_3) A_4$

| i \ j | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 24 | 28 | 58 |
| 2 | | 0 | 16 | 36 |
| 3 | | | 0 | 40 |
| 4 | | | | 0 |

from $1 \to 4$ $\to k=3$

$(A_1\ A_2\ A_3)(A_4)$

from $1 \to 3$

$k=1$

$((A_1)(A_2 A_3))A_4$

↳ value which gives above results.

| K \ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | 1 | 1 | 3 |
| 2 | | | 2 | 3 |
| 3 | | | | 3 |
| 4 | | | | |