MSCA 37016 - HOMEWORK 1

1] L_2-norm $\|\vec{u}\|_2 = \sqrt{(u_1)^2 + \ldots + (u_d)^2}$

$$= \sqrt{(0)^2 + (4)^2 + (-1)^2}$$

$$= \sqrt{17}$$

$$\approx 4.1231056256 17661$$

Ans: L_2-norm $\|\vec{u}\|_2 = 4.123105625617661$

2] Unit vector of $\vec{u}$ in opposite direction $= \frac{-1}{\|\vec{u}\|} (\vec{u})$

$$= \frac{-1}{\sqrt{0^2 + 4^2 + (-1)^2}} \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$$

$$= \frac{-1}{\sqrt{17}} \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -4/\sqrt{17} \\ 1/\sqrt{17} \end{pmatrix}$$

$$\approx \begin{pmatrix} 0 \\ -0.9701425001453318 \\ 0.24253562503633297 \end{pmatrix}$$

3] $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$

$\|\vec{u}\| = \sqrt{0^2+4^2+(-1)^2} = \sqrt{17}$

$\|\vec{v}\| = \sqrt{(-2)^2+3^2+7^2} = \sqrt{62}$

$\vec{u}\cdot\vec{v} = (0\times -2) + (4\times 3) + (-1\times 7)$

$= 5$

$\therefore \cos\theta = \dfrac{5}{\sqrt{17}\sqrt{62}}$

Ans. $\cos\theta = 0.15401027275908 4243$

4] For two vectors to be orthogonal, the angle between them (θ) should be $90°$ thereby giving $\cos(\theta) = 0$.
Based on this, the vectors $\vec{u}$ and $\vec{v}$ are not orthogonal as the cosine of the angle between them does not compute to 0 based on the answer/workings in Answer 3 above.

5] L_1-norm $\|\vec{u}\|_1 = |u_1| + \ldots + |u_d|$

$= |0| + |4| + |-1|$

$= 0 + 4 + 1$

$= \underline{5}$

Ans. L_1-norm $\|\vec{u}\|_1 = 5$

6] $$AB = \begin{bmatrix} 2 & 0 & 1 \\ 3 & -1 & 4 \\ -2 & 5 & 0 \end{bmatrix} \begin{bmatrix} 4 & -2 & 7 \\ 0 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2(4)+0(0)+1(4) & 2(-2)+0(2)+1(0) & 2(7)+0(-1)+1(3) \\ 3(4)+(-1)(0)+4(4) & 3(-2)+(-1)(2)+4(0) & 3(7)+(-1)(-1)+4(3) \\ -2(4)+5(0)+0(4) & -2(-2)+5(2)+0(0) & -2(7)+5(-1)+0(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 8+0+4 & -4+0+0 & 14+0+3 \\ 12+0+16 & -6-2+0 & 21+1+12 \\ 8+0+0 & 4+10+0 & -14+(-5)+0 \end{bmatrix}$$

$$\boxed{AB = \begin{bmatrix} 12 & -4 & 17 \\ 28 & -8 & 34 \\ 8 & 14 & -19 \end{bmatrix}}$$

$$BA = \begin{bmatrix} 4 & -2 & 7 \\ 0 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 3 & -1 & 4 \\ -2 & 5 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4(2)+(-2)(3)+7(-2) & 4(0)+(-2)(-1)+7(5) & 4(1)+(-2)(4)+7(0) \\ 0(2)+2(3)+(-1)(-2) & 0(0)+2(-1)+(-1)(5) & 0(1)+2(4)+(-1)(0) \\ 4(2)+0(3)+3(-2) & 4(0)+0(-1)+3(5) & 4(1)+0(4)+3(0) \end{bmatrix}$$

$$BA = \begin{bmatrix} 8-6-14 & 0+2+35 & 4-8+0 \\ 0+6+2 & 0-2-5 & 0+8-0 \\ 8+0-6 & 0+0+15 & 4+0+0 \end{bmatrix}$$

$$\boxed{BA = \begin{bmatrix} -12 & 37 & -4 \\ 8 & -7 & 8 \\ 2 & 15 & 4 \end{bmatrix}}$$

Ans: AB is not equal to BA.

7] To verify $tr(AB) = tr(BA)$

$$AB = \begin{bmatrix} 12 & -4 & 17 \\ 28 & -8 & 34 \\ -8 & 14 & -19 \end{bmatrix} \qquad BA = \begin{bmatrix} -12 & 37 & -4 \\ 8 & -7 & 8 \\ 2 & 15 & 4 \end{bmatrix}$$

trace of Matrix = sum of diagnal elements.

$$tr(AB) = 12 + (-8) + (-19) \qquad tr(BA) = -12 - 7 + 4$$
$$= -15 \qquad\qquad = -15$$

$$\boxed{\therefore\ tr(AB) = tr(BA)}$$

8] $$\begin{pmatrix}2\\3\\-2\end{pmatrix}\begin{pmatrix}4 & -2 & 7\end{pmatrix} + \begin{pmatrix}0\\-1\\5\end{pmatrix}\begin{pmatrix}0 & 2 & -1\end{pmatrix} + \begin{pmatrix}1\\4\\0\end{pmatrix}\begin{pmatrix}4 & 0 & 3\end{pmatrix}$$

$$= \begin{pmatrix}2(4) & 2(-2) & 2(7)\\3(4) & 3(-2) & 3(7)\\-2(4) & -2(-2) & -2(7)\end{pmatrix} + \begin{pmatrix}0(0) & 0(2) & 0(-1)\\-1(0) & -1(2) & -1(-1)\\5(0) & 5(2) & 5(-1)\end{pmatrix} + \begin{pmatrix}1(4) & 1(0) & 1(3)\\4(4) & 4(0) & 4(3)\\0(4) & 0(0) & 0(3)\end{pmatrix}$$

$$= \begin{pmatrix}8 & -4 & 14\\12 & -6 & 21\\-8 & +4 & -14\end{pmatrix} + \begin{pmatrix}0 & 0 & 0\\0 & -2 & 1\\0 & 10 & -5\end{pmatrix} + \begin{pmatrix}4 & 0 & 3\\16 & 0 & 12\\0 & 0 & 0\end{pmatrix}$$

$$= \begin{pmatrix}8+0+4 & -4+0+0 & 14+0+3\\12+0+16 & -6-2+0 & 21+1+12\\-8+0+0 & +4+10+0 & -14-5+0\end{pmatrix}$$

Ans $= \begin{pmatrix}12 & -4 & 17\\28 & -8 & 34\\-8 & 14 & -19\end{pmatrix}$

9] The answer in question 8 is resulting in a matrix which is equal to AB. The outer product in question 8 is the same as matrix multiplication of matrices A and B as it shows the same steps as well.