MSCA 37016 - HOMEWORK 3

1] $\vec{u}_1 = \vec{V}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

2] $\vec{u}_2 = \vec{V}_2 - \text{proj}_{\vec{V}_1} \vec{V}_2$

$$\text{proj}_{\vec{V}_1} \vec{V}_2 = (\|\vec{V}_2\| \cos\theta)\,\hat{v}_1$$

$$= (\|\vec{V}_2\| \cos\theta)\,\frac{\vec{V}_1}{\|\vec{V}_1\|}$$

$$= \left(\frac{\vec{V}_2 \cdot \vec{V}_1}{\|\vec{V}_1\|^2}\right)\vec{V}_1$$

$$= \left(\frac{\vec{V}_2 \cdot \vec{V}_1}{\vec{V}_1 \cdot \vec{V}_1}\right)\vec{V}_1$$

$$= \left(\frac{2(1) + 3(1) + 0(0)}{1(1) + 1(1) + 0(0)}\right)\vec{V}_1$$

$$= \frac{5}{2}\vec{V}_1$$

$$= 2.5\,\vec{V}_1$$

$$= 2.5\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}$$

$\vec{u}_2 = \vec{V}_2 - \text{proj}_{\vec{V}_1} \vec{V}_2$

(contd...)

$$\vec{u_2} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}$$

Ans $\therefore$ $\boxed{\vec{u_2} = \begin{pmatrix} -0.5 \\ 0.5 \\ 0 \end{pmatrix}}$

3] To verify $\vec{u_1} \perp \vec{u_2}$; i.e, $\cos\theta = 0$ where $\theta = 90^\circ$

$$\cos\theta = \frac{\vec{u_1} \cdot \vec{u_2}}{\|\vec{u_1}\| \, \|\vec{u_2}\|}$$

$$\|\vec{u_1}\| = \sqrt{1^2 + 1^2 + 0} = \sqrt{2}$$

$$\|\vec{u_2}\| = \sqrt{(-0.5)^2 + (0.5)^2 + 0} = \sqrt{0.5}$$

$$\|\vec{u_1}\| \, \|\vec{u_2}\| = (\sqrt{2})(\sqrt{0.5}) = 1$$

$$\vec{u_1} \cdot \vec{u_2} = 1(-0.5) + 1(0.5) + 0(0)$$

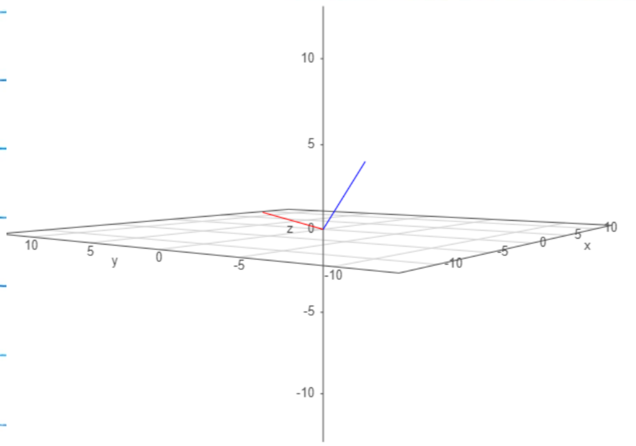
$$= -0.5 + 0.5 + 0$$

$$= 0$$

Ans $\therefore$ $\cos\theta = 0$

thereby proving that the angle between vectors $\vec{u_1}$ and $\vec{u_2}$ is 90° and are perpendicular to each other.

4] Yes, the orthogonal set of vectors $\{\vec{u}_1, \vec{u}_2\}$ span the xy-plane as their z-coordinate is 0. If represented in a 3D space it can be seen that they span only the xy-plane as per below.



5] $\hat{u}_1 = \dfrac{\vec{u}_1}{\|\vec{u}_1\|}$

$\|\vec{u}_1\| = \sqrt{1^2 + 1^2 + 0} = \sqrt{2}$

$\hat{u}_1 = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0.7071\\0.7071\\0\end{pmatrix}$

$\hat{u}_2 = \dfrac{\vec{u}_2}{\|\vec{u}_2\|}$

$\|\vec{u}_2\| = \sqrt{(-0.5)^2 + (0.5)^2 + 0} = \sqrt{0.5}$

$\hat{u}_2 = \dfrac{1}{\sqrt{0.5}}\begin{pmatrix}-0.5\\0.5\\0\end{pmatrix} = \begin{pmatrix}-0.7071\\0.7071\\0\end{pmatrix}$

$\boxed{\text{Ans. } \hat{u}_1 = \begin{pmatrix}0.7071\\0.7071\\0\end{pmatrix}, \ \hat{u}_2 = \begin{pmatrix}-0.7071\\0.7071\\0\end{pmatrix}}$

Proved: xy-plane $= \text{span}(\vec{v}_1, v_2) = \text{span}(\vec{u}_1, \vec{u}_2)$
$= \text{span}(\hat{u}_1, \hat{u}_2)$