CHAPTER 5

1. **(a) What is the probability of rolling a pair of dice and obtaining a total score of 9 or more?**

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

The table below shows the outcomes of rolling a pair of dice.

Die 1	Die 2	Total	Die 1	Die 2	Total	Die 1	Die 2	Total
1	1	2	3	1	4	5	1	6
1	2	3	3	2	5	5	2	7
1	3	4	3	3	6	5	3	8
1	4	5	3	4	7	5	4	9
1	5	6	3	5	8	5	5	10
1	6	7	3	6	9	5	6	11
2	1	3	4	1	5	6	1	7
2	2	4	4	2	6	6	2	8
2	3	5	4	3	7	6	3	9
2	4	6	4	4	8	6	4	10
2	5	7	4	5	9	6	5	11
2	6	8	4	6	10	6	6	12

$$Probability\ of\ score\ 9\ or\ more = P(score \geq 9)$$

The maximum total score greater than 9 is 12 as seen in the below table.

$$P(score \geq 9) = P(score = 9) + P(score = 10) + P(score = 11) + P(score = 12)$$

$$P(score = 9) = \frac{4}{36}$$

$$P(score = 10) = \frac{3}{36}$$

$$P(score = 11) = \frac{2}{36}$$

$$P(score = 12) = \frac{1}{36}$$

$$P(score \geq 9) = \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36}$$

$$\boxed{P(score \geq 9) = \frac{10}{36} = \frac{5}{18} = \mathbf{0.2778}}$$

(b) What is the probability of rolling a pair of dice and obtaining a total score of 7?

$$\boxed{P(score = 7) = \frac{6}{36} = \frac{1}{6} = \mathbf{0.16667}}$$

3. **A card is drawn at random from a deck.**

There are 52 cards in a deck.

(a) What is the probability that it is an ace or a king?

There are 4 aces and 4 kings in a deck of cards.

$$P(ace\ or\ king) = P(ace) + P(king)$$

$$P(ace) = \frac{4}{52} = \frac{1}{13}$$

$$P(king) = \frac{4}{52} = \frac{1}{13}$$

$$\boxed{P(ace\ or\ king) = \left(\frac{1}{13}\right) + \left(\frac{1}{13}\right) = \frac{\mathbf{2}}{\mathbf{13}} = \mathbf{0.15384615384}}$$

(b) What is the probability that it is either a red card or a black card?

There are 26 red cards and 26 black cards in a deck

$$P(red\ or\ black) = P(red) + P(black)$$

$$P(red) = \frac{26}{52} = \frac{1}{2}$$

$$P(black) = \frac{26}{52} = \frac{1}{2}$$

$$\boxed{P(red\ or\ black) = \frac{1}{2} + \frac{1}{2} = \mathbf{1}}$$

5. A fair coin is flipped 9 times. What is the probability of getting exactly 6 heads?

Using the binomial distribution formula,

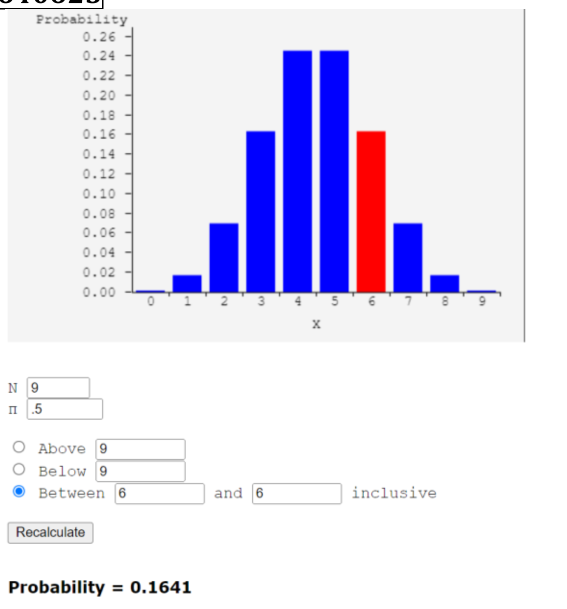
$$P(x) = \frac{N!}{(x!)(N-x)!}\Pi^x(1-\Pi)^{N-x}$$

$$where\ N = number\ of\ trials, \Pi = probability\ of\ event\ occurring, x = number\ of\ successes$$

$$N = 9, \quad x = 6, \quad \Pi = 0.5$$

$$P(6) = \frac{9!}{(6!)(9-6)!}(0.5)^6(1-0.5)^{9-6}$$

$$\boxed{\boldsymbol{P(6) = 0.1640625}}$$



7. You flip a coin three times. (a) What is the probability of getting heads on only one of your flips?

There are totally 8 outcomes as seen below

HHH	HHT	HTT	HTH
TTT	TTH	THH	THT

$$\boxed{\boldsymbol{P(only\ one\ head) = \frac{3}{8} = 0.375}}$$

(b) What is the probability of getting heads on at least one flip?

$$P(atleast\ 1\ head) = 1 - P(no\ heads)$$

$$\boxed{\boldsymbol{P(atleast\ 1\ head) = 1 - \frac{1}{8} = \frac{7}{8} = 0.875}}$$

9. **A jar contains 10 blue marbles, 5 red marbles, 4 green marbles, and 1 yellow marble. Two marbles are chosen (without replacement). (a) What is the probability that one will be green and the other red?**

$$P(green\ and\ red) = P(green)P(red|green) + P(red)(green|red)$$

$$P(green\ and\ red) = \frac{4}{20}\text{x}\left(\frac{5}{19}\right) + \frac{5}{20}\text{x}\frac{4}{19} = \frac{2}{19} = \mathbf{0.1053}$$

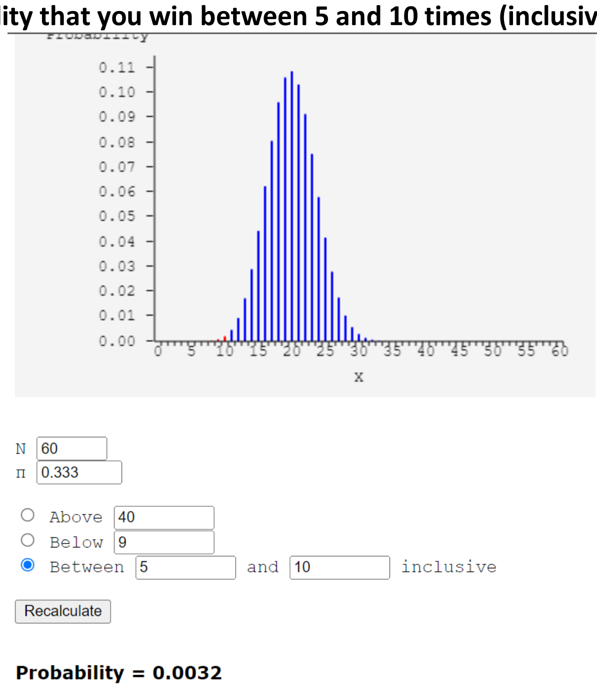
(b) What is the probability that one will be blue and the other yellow?

$$P(blue\ and\ yellow) = P(blue)P(yellow|blue) + P(yellow)P(blue|yellow)$$

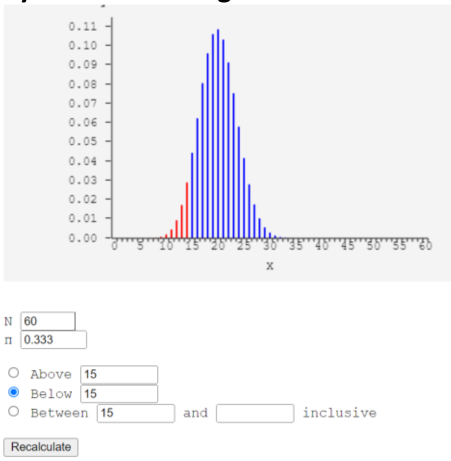
$$P(blue\ and\ yellow) = \frac{10}{20}\text{x}\frac{1}{19} + \frac{1}{20}\text{x}\frac{10}{19} = \frac{2}{38} = \frac{1}{19} = \mathbf{0.0526}$$

11. **You win a game if you roll a die and get a 2 or a 5. You play this game 60 times.**

a) **What is the probability that you win between 5 and 10 times (inclusive)?** <u>*0.0032*</u>

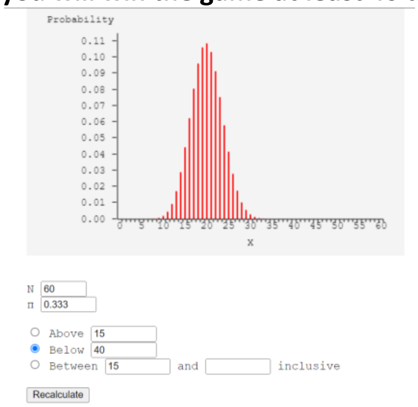


b) What is the probability that you will win the game at least 15 times? 1 - 0.0636 = *0.9364*



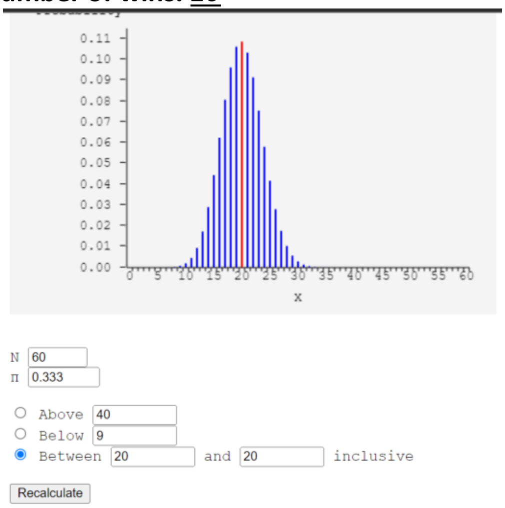
Probability = 0.0636

c) What is the probability that you will win the game at least 40 times? 1-1 = *0*



Probability = 1

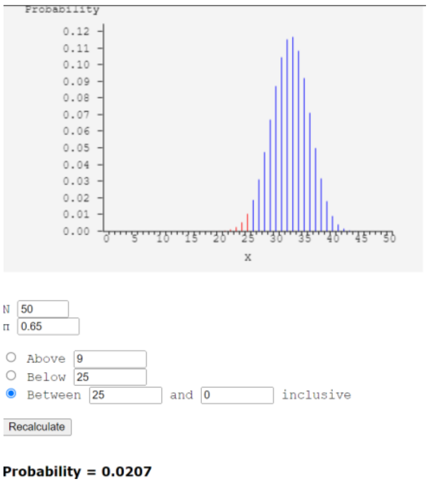
d) What is the most likely number of wins. *20*



Probability = 0.1087

e) What is the probability of obtaining the number of wins in d? *0.1087*

13. An unfair coin has a probability of coming up heads of 0.65. The coin is flipped 50 times. What is the probability it will come up heads 25 or fewer times? (Give answer to at least 3 decimal places). *0.021*



15. True/False: You are more likely to get a pattern of HTHHHTHTTH than HHHHHHHHTT when you flip a coin 10 times. *False*

CHAPTER 7

2. **(a) What are the mean and standard deviation of the standard normal distribution?**
   For a standard normal distribution, the mean = 0 and standard deviation = 1.

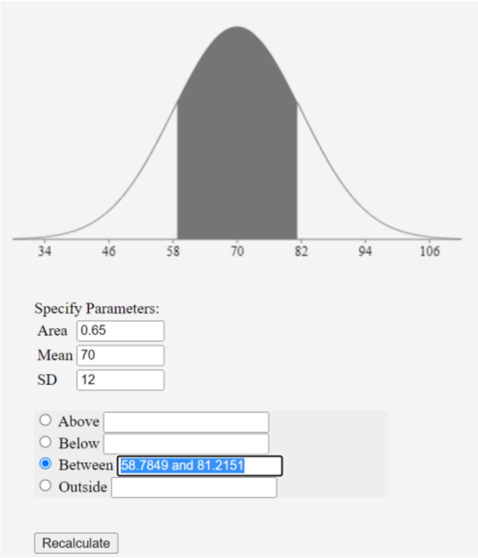
   (b) What would be the mean and standard deviation of a distribution created by multiplying the standard normal distribution by 8 and then adding 75?
   The mean would be 75 and the standard deviation would be 8.

4. **(a) What proportion of a normal distribution is within one standard deviation of the mean?**
   0.68 or ~68%

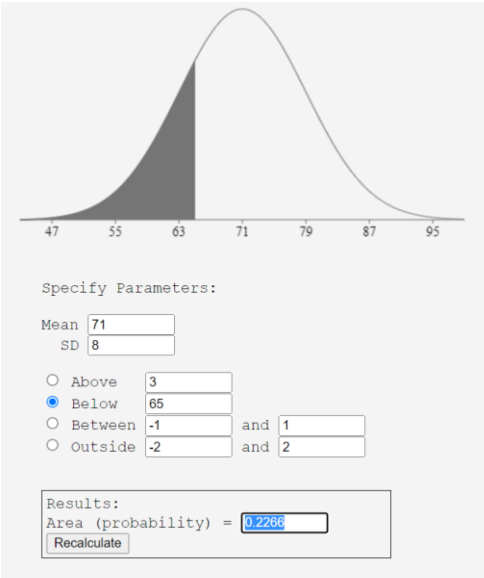
   (b) What proportion is more than 2.0 standard deviations from the mean?
   2.5%

   (c) What proportion is between 1.25 and 2.1 standard deviations above the mean?
   0.0878

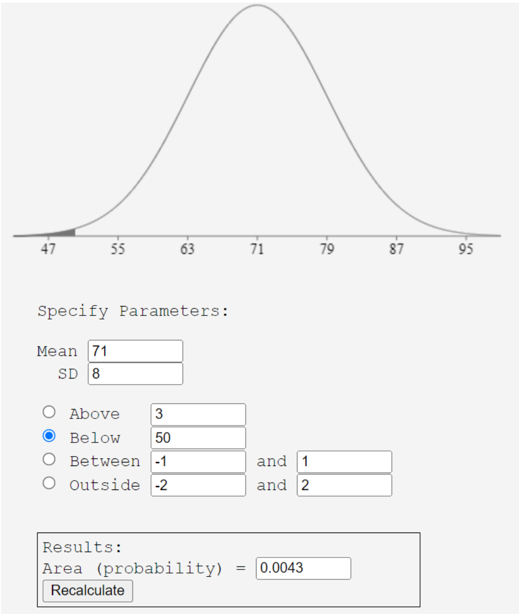
6. **Assume a normal distribution with a mean of 70 and a standard deviation of 12. What limits would include the middle 65% of the cases?**
   58.7849 and 81.2151



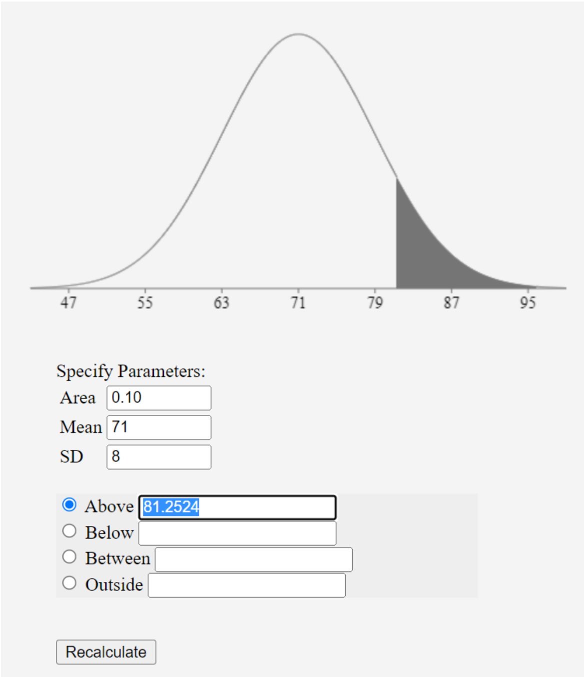
8. **Assume the speed of vehicles along a stretch of I-10 has an approximately normal distribution with a mean of 71 mph and a standard deviation of 8 mph.**
    a. **The current speed limit is 65 mph. What is the proportion of vehicles less than or equal to the speed limit?**
    0.2266 or 22.66%



    b. **What proportion of the vehicles would be going less than 50 mph?**
    0.0043 or 0.43%



c. **A new speed limit will be initiated such that approximately 10% of vehicles will be over the speed limit. What is the new speed limit based on this criterion?**
81.2524

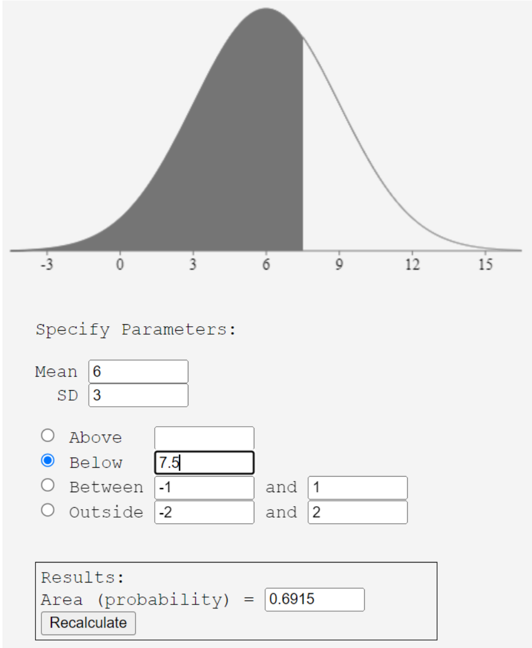


d. **In what way do you think the actual distribution of speeds differs from a normal distribution?**
The shape of the normal distribution differs as it is symmetric with more values at the center.

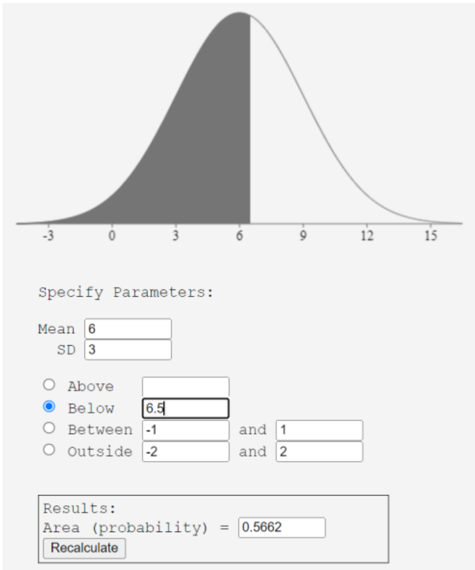
10. **You want to use the normal distribution to approximate the binomial distribution. Explain what you need to do to find the probability of obtaining exactly 7 heads out of 12 flips.**
    i) The binomial distribution has a mean $\mu = N\Pi$ where N = 12 and $\Pi = 0.5$, thus $\mu = 12*0.5 = 6$
    ii) The binomial distribution has a variance of $\sigma^2 = N\Pi(1-\Pi) = 12*0.5(1-0.5) = 3$
    iii) The binomial distribution thus has a standard deviation of square root of variance = 1.732.

iv) We round off values and find the area between 6.5 and 7.5 under the normal curve.

v) Find area of curve below 7.5 = 0.6915



vi) Find area of curve below 6.5 = 0.5662



vii) The difference between the two above areas is the probability of obtaining exactly 7 heads out of 12 flips = 0.6915 – 0.5662 = ***0.1295***

12. Use the normal distribution to approximate the binomial distribution and find the probability of getting 15 to 18 heads out of 25 flips. Compare this to what you get when you calculate the probability using the binomial distribution. Write your answers out to four decimal places.

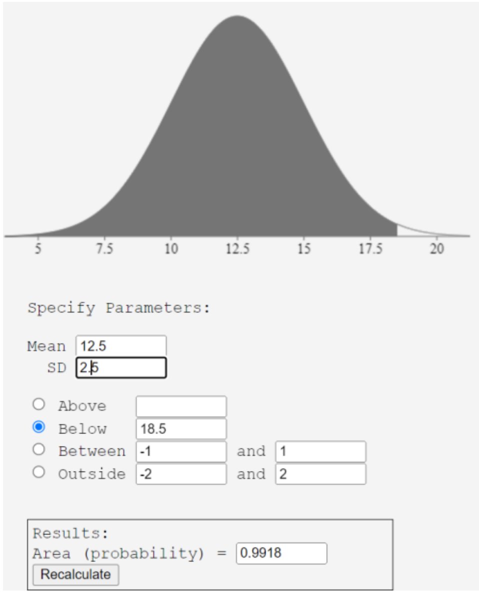
i) The binomial distribution has a mean $\mu = N\Pi$ where N = 25 and $\Pi = 0.5$, thus $\mu = 25*0.5 = 12.5$

ii) The binomial distribution has a variance of $\sigma^2 = N\Pi(1-\Pi) = 25*0.5(1-0.5) = 6.25$

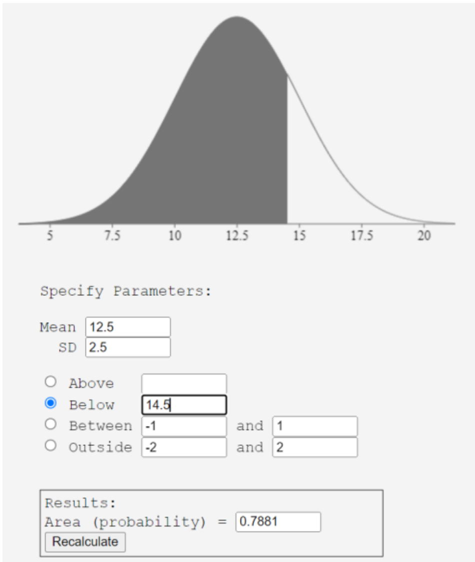
iii) The binomial distribution thus has a standard deviation of square root of variance = 2.5.

iv) We round off values and find the area between 14.5 and 18.5 under the normal curve.

v) Find area of curve below 18.5 = 0.9918



vi) Find the area of curve below 14.5 = 0.7881



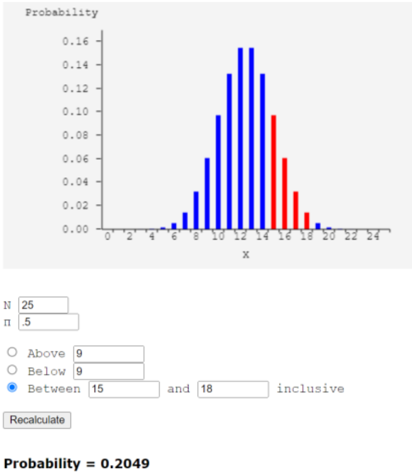
vii) The difference between the two above areas is the probability of obtaining exactly 15 to 18 heads out of 25 flips = 0.9918 – 0.7881 = ***<u>0.2037</u>***

Using the binomial distribution formula:

$$P(x) = \frac{N!}{(x!)(N-x)!}\Pi^x(1-\Pi)^{N-x}$$

$$where\ N = number\ of\ trials, \Pi = probability\ of\ event\ occurring, x = number\ of\ successes$$

$$N = 25, \quad x = 15\ to\ 18, \quad \Pi = 0.5$$



Probability = 0.2049

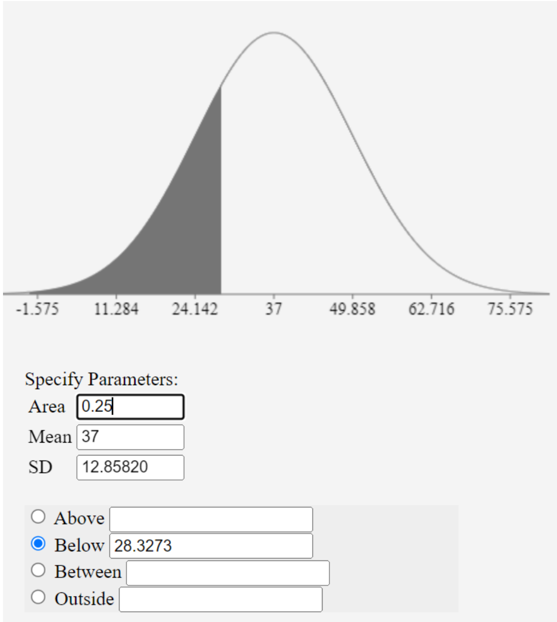
14. **True/false: In a normal distribution, 11.5% of scores are greater than Z = 1.2.** *True*
16. **True/false: The larger the π, the better the normal distribution approximates the binomial distribution.** *True*
18. **True/false: Abraham de Moivre, a consultant to gamblers, discovered the normal distribution when trying to approximate the binomial distribution to make his computations easier.** *True*
20. **True/false: In the figure below, the red distribution has a larger standard deviation than the blue distribution.** *True*
22. **The following question uses data from the Angry Moods (AM) case study. For this problem, use the Anger Expression (AE) scores. (a) Compute the mean and standard deviation.**

$$Mean = \frac{sum\ of\ scores}{total\ \ number\ of\ obserations}$$
$$Mean = 37$$

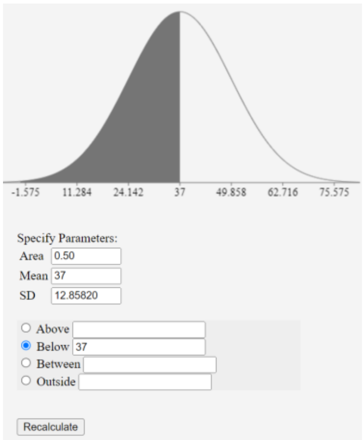
$$Standard\ deviation = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$
$$Standard\ deviation = \sqrt{\frac{12896}{78}} = 12.85820101$$

(b) Then, compute what the 25th, 50th and 75th percentiles would be if the distribution were normal.

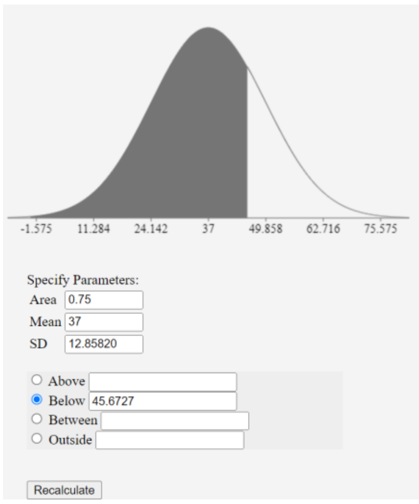
25^{th} percentile = ***28.3273***



50th percentile = ***37***



*75th percentile = **45.6727***



(c) Compare the estimates to the actual 25th, 50th, and 75th percentiles.

$$For\ 25th\ percentile, R = \frac{25}{100}(78+1) = 19.75$$
$$IR = 19, FR = 0.75$$
$$25th\ percentile = (0.75)(27-26) + (26) = 26.75$$

$$For\ 50th\ percentile, R = \frac{50}{100}(78 + 1) = 39.5$$
$$50th\ percentile = (0.50)(36 - 36) + 36 = 36$$
$$For\ 75th\ percentile, R = \frac{75}{100}(78 + 1) = 59.25$$
$$75th\ percentile = (0.25)(45 - 45) + 45 = 45$$

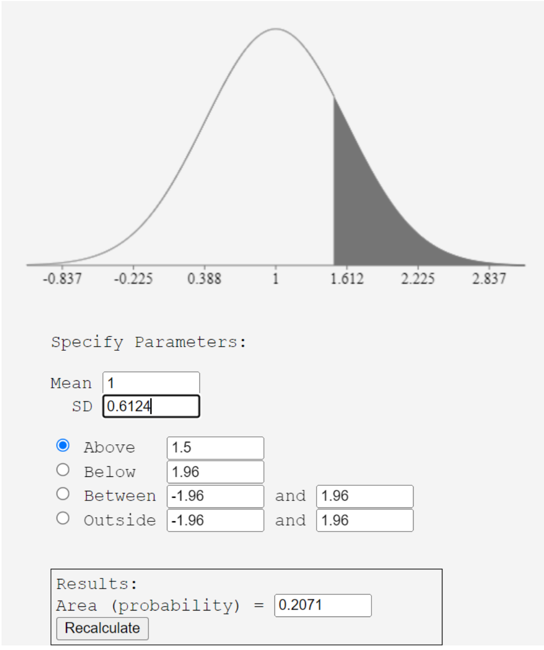
Estimates	Actual	Difference
28.3273	26.75	1.5773
37	36	1
45.6727	45	0.6727

CHAPTER 9

1. **A population has a mean of 50 and a standard deviation of 6. (a) What are the mean and standard deviation of the sampling distribution of the mean for N = 16?**
   Mean of sampling distribution = mean of population = 50
   $$Standard\ deviation = \frac{6}{\sqrt{16}} = \frac{3}{2} = 1.5$$
   (b) What are the mean and standard deviation of the sampling distribution of the mean for N = 20?
   Mean of sampling distribution = mean of population = 50
   $$Standard\ deviation = \frac{6}{\sqrt{20}} = 1.3416$$

3. **What term refers to the standard deviation of the sampling distribution?**
   Standard error

5. **A questionnaire is developed to assess women's and men's attitudes toward using animals in research. One question asks whether animal research is wrong and is answered on a 7-point scale. Assume that in the population, the mean for women is 5, the mean for men is 4, and the standard deviation for both groups is 1.5. Assume the scores are normally distributed. If 12 women and 12 men are selected randomly, what is the probability that the mean of the women will be more than 1.5 points higher than the mean of the men?**
   Mean of the sampling distribution = 5 – 4 = 1
   $$\text{Standard deviation of the sampling distribution } = \sqrt{\frac{2.25}{12} + \frac{2.25}{12}} = 0.6124$$
   The probability that the mean of the women will be 1.5 points higher than the mean of the men = ***<u>0.2071</u>***



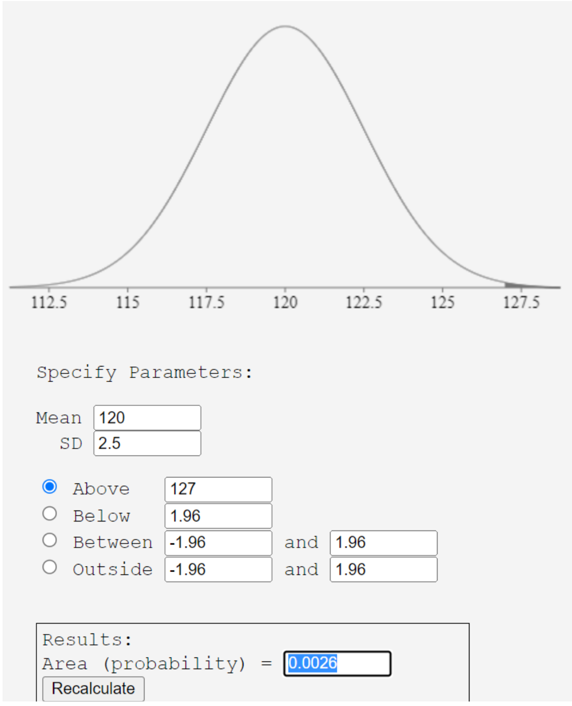
7. **If numerous samples of N = 15 are taken from a uniform distribution and a relative frequency distribution of the means is drawn, what would be the shape of the frequency distribution?**
   Central limit theorem predicts sample distributions are always *normally distributed* thus the shape of the curve would be similar to a normal distribution.

9. **What is the shape of the sampling distribution of r? In what way does the shape depend on the size of the population correlation?**
   The shape of the sampling distribution of r is *Skewed.* The greater the standard error of sampling distribution of r, the shape is more skewed.

11. **A variable is normally distributed with a mean of 120 and a standard deviation of 5. Four scores are randomly sampled. What is the probability that the mean of the four scores is above 127?**
   Mean of the sampling distribution = 120

$$\text{Standard deviation of the sampling distribution } = \sqrt{\frac{25}{4}} = 2.5$$

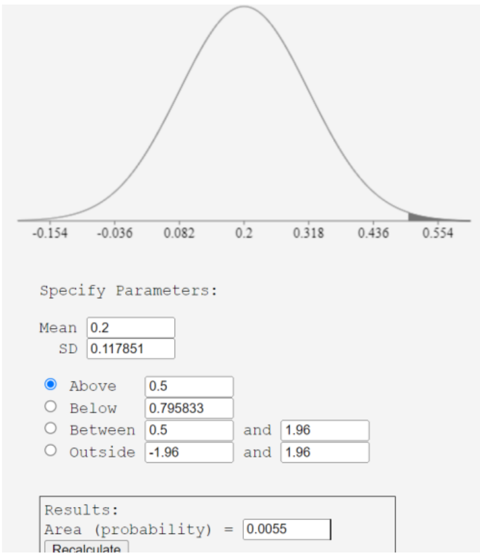
   The probability that the mean of the 4 scores is higher than 127 = ***0.0026***



13. **The mean GPA for students in School A is 3.0; the mean GPA for students in School B is 2.8. The standard deviation in both schools is 0.25. The GPAs of both schools are normally distributed. If 9 students are randomly sampled from each school, what is the probability that:**
(a) the sample mean for School A will exceed that of School B by 0.5 or more?
Mean of the sampling distribution = 3 – 2.8 = 0.2

$$\text{Standard deviation of the sampling distribution } = \sqrt{\frac{0.0625}{9}+\frac{0.0625}{9}} = 0.117851$$

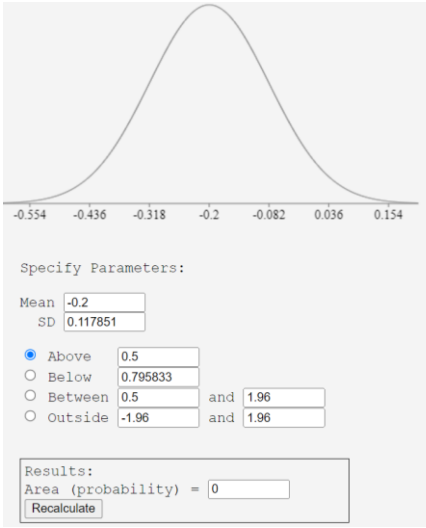
The probability that sample mean for school A will exceed School B by 0.5 or more = ***<u>0.0055</u>***



(b) the sample mean for School B will be greater than the sample mean for School A?
Mean of the sampling distribution = 2.8 - 3 = -0.2

$$\text{Standard deviation of the sampling distribution } = \sqrt{\frac{0.0625}{9}+\frac{0.0625}{9}} = 0.117851$$

The probability that sample mean for school B will be greater than school A = ***<u>0</u>***



15. **When solving problems where you need the sampling distribution of r, what is the reason for converting from r to z'?**
The shape of the sampling distribution of r is not normal, making it difficult to find the probability of a value of r for specific samples. To overcome this, the statistician Fisher developed a way to convert r to z' which is normally distributed with a standard error.

17. **True/false: The standard error of the mean is smaller when N = 20 than when N = 10.** _True_
19. **True/false: You choose 20 students from the population and calculate the mean of their test scores. You repeat this process 100 times and plot the distribution of the means. In this case, the sample size is 100.** _False_
21. **True/false: The median has a sampling distribution.** _True_
23. **(a) How many men were sampled?** _30_
(b) How many women were sampled? _48_

24. **What is the mean difference between men and women on the Anger-Out scores?**
Mean of men = 16.56666667
Mean of women = 15.77083333
Mean difference between men and women on the Anger-Out scores = 16.56666667 - 15.77083333 = ***0.795833333***

Gender	Sports	Anger-Out		Gender	Sports	Anger-Out
1	1	16		2	1	18
1	1	16		2	1	14
1	1	12		2	1	13
1	1	12		2	1	17
1	1	17		2	1	13
1	1	18		2	1	16
1	2	27		2	1	12
1	2	9		2	2	18
1	2	12		2	2	13
1	2	15		2	2	20
1	2	24		2	2	16
1	2	12		2	2	23
1	2	15		2	2	26
1	2	20		2	2	17
1	2	16		2	2	20
1	2	17		2	2	9
1	2	18		2	2	23
1	2	16		2	2	14
1	2	11		2	2	23
1	2	22		2	2	13
1	2	26		2	2	18
1	2	11		2	2	11
1	2	19		2	2	11
1	2	15		2	2	11
1	2	18		2	2	15
1	1	14		2	2	14
1	1	15		2	2	18
1	1	21		2	2	12
1	1	18		2	2	18
1	1	15		2	2	21
	Mean of men	16.56667		2	2	22
				2	2	17
				2	2	16
				2	2	9
				2	2	11
				2	2	17
				2	2	13
				2	2	18
				2	2	15
				2	2	12
				2	2	15
				2	1	14
				2	1	17
				2	1	24
				2	1	14
				2	1	10
				2	1	11
				2	1	15
					Mean of women	15.77083

25. Suppose in the population, the Anger-Out score for men is two points higher than it is for women. The population variances for men and women are both 20. Assume the Anger-Out scores for both genders are normally distributed. Given this information about the population parameters:

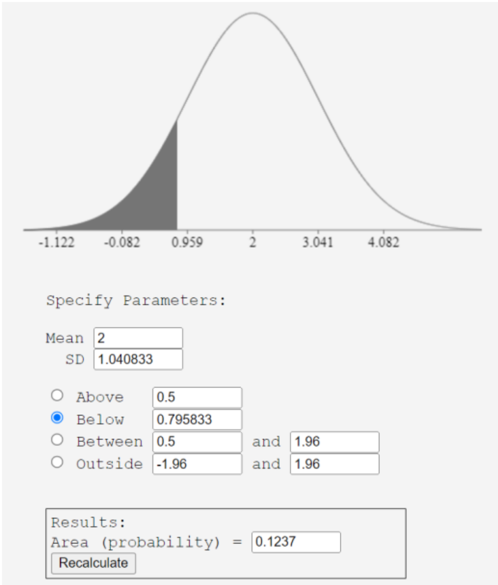
(a) What is the mean of the sampling distribution of the difference between means? *2, since the mean of women is increased by 2 for men.*

(b) What is the standard error of the difference between means?

$$\begin{aligned} &Standard\ error\ of\ the\ difference\ between\ means \\ &\qquad = Standard\ deviation\ of\ sampling\ of\ means \end{aligned}$$

$$SD = \sqrt{\frac{20}{30} + \frac{20}{48}} = 1.040833$$

(c) What is the probability that you would have gotten this mean difference (see #24) or less in your sample? *0.1237*


-1.122
-0.082
0.959
2
3.041
4.082
Specify Parameters:
Mean 2
SD 1.040833
Above 0.5
Below 0.795833
Between 0.5 and 1.96
Outside -1.96 and 1.96
Results:
Area (probability) = 0.1237
Recalculate