MSCA 37016 - HOMEWORK 2

1] The given system of linear equations is a homogeneous system. Every homogeneous system has at least one trivial solution which is 0. The given system is also an underdetermined system with more variables than equations which means it has an infinite number of solutions other than zero.
Thus, it can be concluded that the given system of equations has more than one solution.

2] $$A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 2 & 0 & 4 & 1 \\ 2 & -1 & 3 & 0 \end{bmatrix}$$

$R_2 = R_2 - 2R_1$

$$A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & -6 & -6 & 3 \\ 2 & -1 & 3 & 0 \end{bmatrix}$$

$R_3 = R_3 - 2R_1$

$$A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & -6 & -6 & 3 \\ 0 & -7 & -7 & 2 \end{bmatrix}$$

$R_2 = -R_2/6$

$$A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & 1 & 1 & -1/2 \\ 0 & -7 & -7 & 2 \end{bmatrix}$$

$R_3 = R_3 + 7R_2$

$$A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & -3/2 \end{bmatrix}$$

$R_1 = R_1 - 3R_2$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1/2 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & -3/2 \end{bmatrix}$$

$R_3 = R_3\left(-2/3\right)$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1/2 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 = R_1 - R_3/2$

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 = R_2 - R_3/2$

Ans. $\boxed{A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

3] There are 3 pivots which are 1 in 1^{st} column, 1_2 in 2^{nd} column and 1 in 4^{th} column.

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4] The row echelon form of matrix A is $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\therefore \quad \begin{aligned} x_1 \quad + 2x_3 &= 0 \\ x_2 + x_3 &= 0 \\ x_4 &= 0 \end{aligned}$$

$$\therefore \quad \begin{aligned} x_1 &= -2x_3 \\ x_2 &= -x_3 \\ x_3 &= x_3 \\ x_4 &= 0 \end{aligned}$$

$$\boxed{\textbf{Ans. } \text{Solution set in parametric form: } \left\{ \begin{pmatrix} -2x_3 \\ -x_3 \\ x_3 \\ 0 \end{pmatrix} : x_3 \in R \right\} = \left\{ x_3 \begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix} : x_3 \in R \right\}}$$

5] Null space of A:

$$null(A) = \text{solution set of } A\vec{x} = \vec{0}$$

$$= \left\{ x_3 \begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix} : x_3 \in R \right\} = \left\{ \begin{matrix} -2 \\ -1 \\ 1 \\ 0 \end{matrix} \right\}$$

Its dimension → dim (A) = 1

6] The column space of $A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

<u>Rank(A)</u> = 3 which is number of pivots in A in row echelon form.

7] The Rank-nullity dimension theorem is:

$$\begin{pmatrix}\text{number of} \\ \text{variables } (d)\end{pmatrix} = \begin{pmatrix}\text{number of pivots/} \\ \text{leading variables}\end{pmatrix} + \begin{pmatrix}\text{number of free} \\ \text{variables}\end{pmatrix}$$

$$= \begin{pmatrix}\text{number of non-} \\ \text{redundant equations}\end{pmatrix} + \begin{pmatrix}\text{dimension of} \\ \text{solution set}\end{pmatrix}$$

number of variables $(d) = 4$

& number of pivots $= 3$

number of free variables $= 1$

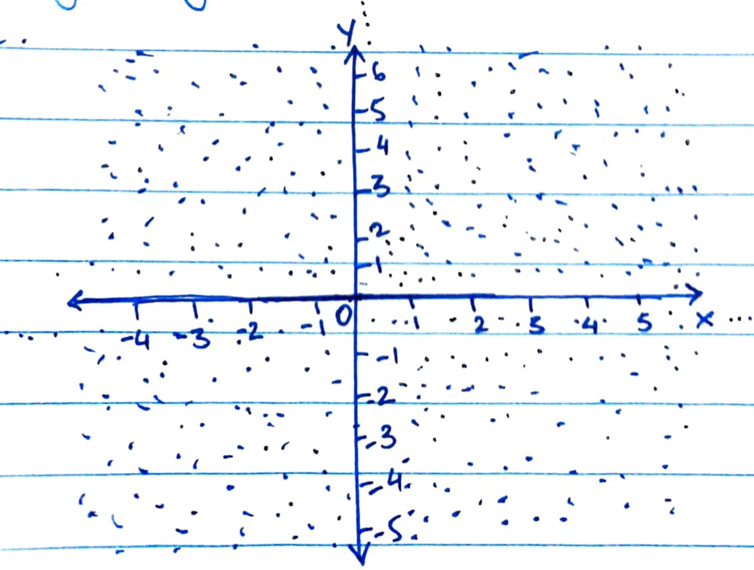
number of non-redundant equations $= 3$

dimension of solution set $= 1$

$$\boxed{\text{Thus, number of variables } (d) = 3 + 1 = 4}$$

Therefore, rank-nullity theorem holds for matrix A.

] Set $S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : xy = 0 \right\} \subset R^2$ covers the entire xy-plane.



(contd...)

Yes, S is a vector subspace of R^2 as it satisfies the vector space properties as follows:

* Must contain the zero vector $\vec{0} \in V$. — It contains the lines passing through the origin. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.9 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 12 \end{pmatrix},$

etc...

* Consider the vector $\vec{v}_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\vec{v}_3 = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ and $\vec{v}_4 = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$a_1 = 3, \; a_2 = 1, \; a_3 = 0, \; a_4 = 5$

then,

$$a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 + a_4\vec{v}_4$$

$$= 3\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 5 \end{pmatrix} + 5\begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ -10 \end{pmatrix}$$

this is part of vector space.