Family Preferences and Horizontal Differentiation in Urban School Choice Markets

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Abstract: Many urban public school systems in the United States allow families to pick among schools differentiated by academic theme. For example, New York City students can choose to attend high schools focused on topics as varied as health sciences, journalism, and performing arts. While some of these themed programs were introduced in order to promote racial integration, we do not have evidence on whether curricular differentiation achieves this goal. In this paper, I investigate the impact of curricular themes on cross-school segregation and student outcomes in New York City high schools. I estimate a structural model using data on student applications to determine how families trade off curricular themes and other school characteristics in the application process. I find that all demographic groups, but particularly white and Asian applicants, tend to prefer Humanities and Interdisciplinary programs, the most general curricular theme, to more specialized themes. Using the model to compare the baseline assignment to a simulated counterfactual assignment in which all programs are Humanities and Interdisciplinary, I find that curricular differentiation does not reduce segregation or white flight, and if anything, slightly increases them. I also find that while the average applicant prefers their counterfactual assignment, a substantial minority of applicants, including half of all Black applicants, prefer their status quo assignment, suggesting that the optimal distribution of high school capacity across themes involves more general theme seats, and fewer, but still some, specialized theme seats. Finally, to provide a more complete picture of the trade-offs involved in offering curricular differentiation, I use random and quasi-random variation in the school assignment process to identify whether being assigned to one's preferred curricular theme improves high school outcomes.[1]

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1 Introduction

In recent decades, urban school systems in the United States have expanded school choice. Rather than defaulting to a neighborhood school, families are increasingly encouraged to actively choose among a broad set of schools with different attributes, express their preferences through an application process, and receive a school assignment. Proponents of school choice have highlighted its potential to equalize access to high-quality schools across income and racial gaps.

While schools differ on test scores, graduation rates, and other "vertical" indicators of quality, they are also horizontally differentiated. Families can choose schools in different locations, with different teaching philosophies, and with different curricula, dimensions along which preferences are more likely to vary.

Urban school systems in the United States often feature a wide degree of curricular heterogeneity. Students can choose high schools focused on themes as diverse as performing arts, environmental science, and teaching. But while a vast literature has examined the implications of school choice for student sorting along demographic lines into higher or lower-quality schools, little is known about family preferences with respect to curricular themes.

This paper explores the implications of curricular differentiation for segregation and student outcomes in New York City high schools. First, I provide novel descriptive evidence on how school application and enrollment behavior with respect to curricular theme varies by race, income, and achievement. I then estimate a structural model of school program choice in which curricular theme enters applicant program utility, enabling me to quantify the extent to which applicants care about theme relative to other school characteristics. I use this model to measure the extent to which theme preferences contribute to cross-school segregation. Finally, I use random variation in school program assignment generated by the assignment mechanism to identify whether assignment to an applicant's preferred theme affects their high school outcomes.

The question of whether applicants sort along demographic lines into different curricular themes is especially important in light of the historic role of curricular differentiation as desegregation policy in the decades after *Brown vs. Board of Education.* In the 1970s, cities like New York City, Boston, Minneapolis, and Los Angeles, began establishing schools with specialized curricula with the explicit goal of racial integration. In 1975, *New York Times* reported on this development: "The magnets, theme-centered schools or institutes, emphasizing such areas as oceanography or communications, grew out of attempts in recent years to meet mounting demands for greater relevance in education and to find ways to attract white students to heavily black schools" (Buder 1975). Appealing to preferences in this way to prevent white flight took on a special importance in the wake of the *Milliken v Bradley* in 1974, which limited the ability of courts to mandate cross-district desegregation plans (Frankenberg and Le 2008). Appealing to family preferences was the only remaining policy instrument for cross-district desegregation policy. Themed schools still play a role in desegregation policy today. In the past couple of decades, themed inter-district magnet schools have been at the center of desegregation policy in Connecticut. Through the Magnet School Assistance Program, the federal government provides grants to schools "with special curricula, to attract a diverse group of students and desegregate public schools" (Office of Elementary and Secondary Education 2021).

The potential of curricular differentiation for desegregation is twofold. First, if families care about themes, and preferences for themes are distributed more similarly across demographic groups than preferences for other characteristics, they could, on net, lead families from different backgrounds to choose more similar schools than they would otherwise. The second is that by improving match quality, curricular differentiation could increase student-school match quality for families on the margin of leaving the urban public school system. Families with a better outside option in a vertical sense may opt to stay in city public schools if their child finds a school that interests them. Because of the relatively dense placement of schools in urban school districts, they may have a comparative advantage for providing this kind of curricular heterogeneity as well, relative to suburban or rural districts.

In spite of the historical motivation behind curricular themes, their implications for segregation are ambiguous. Following the justification for their introduction, they could on net, decrease segregation. However, if theme preferences differ on average across demographic groups, or if advantaged families on the margin of exiting the district actively dislike themed schools, curricular differentiation could exacerbate segregation.

My paper resolves this ambiguity by using administrative data from the New York City high school application match to measure theme preferences by race, income, and achievement, and assess their contribution to segregation in New York City high schools. Prospective New York City high school students must apply via a centralized application system, submitting up to 12 schools ranked in order of preference, and receiving a single assignment from a Deferred Acceptance mechanism. In this setting, all schools are themed, though the largest theme category is a general, "Humanities and Interdisciplinary" theme in contrast with the more specialized themes. For each eighth grade applicant to high school, I observe demographic characteristics, rank-ordered list, high school assignment, high school enrollment, and high school outcomes.

I begin by presenting novel descriptive evidence on application and enrollment behavior with respect to theme by race, income and achievement. I find that white and Asian students, non-low-income students, and high baseline achievement students are more likely to enroll at Humanities and Interdisciplinary programs than others.

I then estimate a structural model of strategic application choice, allowing for rich heterogeneity by observable and unobservable applicant characteristics. I decompose program preferences to isolate the theme component, allowing me to measure how much applicants care about theme relative to other school characteristics. My model estimates are consistent with my descriptive evidence: while all applicants prefer Humanities and Interdisciplinary programs, relative to more specialized themes, white and Asian applicants have the strongest relative preference for Humanities and Interdisciplinary programs.

I simulate a counterfactual assignment without curricular differentiation, and measure segregation in the counterfactual relative to the baseline. I find that removing the contribution of specialized theme preference to assignment, by making all programs Humanities and Interdisciplinary programs, slightly reduces segregation along all dimensions and district exit by white and non-low-income applicants. These results suggest that offering specialized theme programs is unlikely to promote integration, and may even encourage segregation. I also investigate who is worse off and better off in this counterfactual based on how much they like their assigned program. I find that while applicants on average are better off in the counterfactual without specialized themes, a substantial minority of applicants are worse off, including half of all Black applicants.

Finally, to get a fuller picture of the trade-offs involved in offering curricular differentiation, I propose a future analysis to determine whether students benefit from being assigned to a specialized themed program, relative to a Humanities and Interdisciplinary program. Using estimates of individual theme preference from my model, I examine whether students sort on expected outcomes into specialized theme programs.

1.1 Related Literature

1.1.1 Family preferences for schools

My paper contributes to a vast literature that examines what families care about when choosing schools and how families make these decisions. Abdulkadiroglu, Pathak, Schellenberg, and Walters 2020 find that parents care about proximity and baseline peer achievement, rather than school effectiveness. Using house prices, Black 1999 measures parents' willingness to pay for schools with higher test scores. Even in settings where all parents prefer high-performing schools, differences in distance to these schools or peer preferences may result in differences in how school performance must be traded off against these other characteristics, by income and race (Hastings, Kane, and Staiger 2008; Burgess, Greaves, Vignoles, and Wilson 2014).

Other features of applicants' choice environment can influence their application and enrollment decisions. Corcoran, Jennings, Cohodes, and Sattin-Bajaj 2018 suggests information may be a barrier in the choice of high-performing schools, as low-income New York City middle school students responded in application and enrollment decisions to an information intervention in which they received a list of nearby, high-performing schools. In New York City, Hahm and Park 2022 finds that middle school assignment can affect students' high school enrollment, through its effect on high school application choices.

1.1.2 Horizontal school differentiation and student outcomes

My paper also contributes to a literature on horizontal differentiation and student outcomes. Career and Technical Education (CTE) is one form of curricular differentiation in schools. Previous work has documented heterogeneity in selection into CTE, as well as heterogeneity in its effect on high school and postsecondary outcomes (Brunner, Dougherty, and Ross 2021; Ecton and Dougherty 2021; Jacob and Ricks 2023). In my setting, I consider CTE offerings to be part of the curricular differentiation captured by theme categorization. School social climate and environment are also valued by parents and have been shown to be correlated with effectiveness (Crespin 2023; Dobbie and Fryer 2013).

Student-school match quality can be seen as a form of horizontal differentiation, since it takes on different values for different students. In some contexts, students do sort on gains; Norweigian college applicants sort into postsecondary fields in which they have a comparative advantege (Kirkeboen, Leuven, and Mogstad 2016). On the other hand, Walters 2018 finds negative selection on gains into the charter middle school sector in Boston.

1.1.3 Magnet schools and desegregation

Previous work has examined whether themed magnet schools have worked to reduce segregation. Based on a descriptive analysis comparing districts from 1968 to 1991 that adopted desegregation plans with magnet schools versus without, Rossell 2003 concludes that magnet schools do not produce more interracial exposure. Bifulco, Cobb, and Bell 2009 found that themed, inter-district magnet schools in an inner-ring suburb of Hartford, Connecticut reduced the racial and economic isolation for city students who enrolled there.

While some of the themed school in my setting, New York City, started out as magnet schools, they are now part of the same public high school centralized application and enrollment process as all other high schools. Students do not need to specially opt into specialized themed schools in my setting, as all schools are part of the same application process. Unlike the studies of magnet schools, my setting in which all schools are themed enables me to focus specifically on theme preferences, and the contribution of these preferences to segregation.

1.1.4 Structural determinants of school segregation

My paper also contributes to a literature on contributors to present-day school segregation. Monarrez 2023 finds that residential segregation explains more than 100 percent of school segregation, with local government policy promoting integration having relatively little effect. Even in school choice contexts meant to equalize access, selective admissions and differential distance to high-quality schools contribute to segregation (Idoux 2021; Laverde 2022). Monarrez, Kisada, and Chingos 2022 find that the proliferation of charter schools may have increased cross-school segregation, it may have also decreased cross-district segregation in the largest metropolitan areas. Caetano and Macartney 2021 finds that much of school segregation is driven by neighborhood factors in residential choice, rather than school choice itself. Oosterbeek, Sovago, and Klaauw 2021 find that in Amsterdam, 40% of school segregation by ethnicity and 25% of school segregation by household income can be attributed to preference heterogeneity.

2 Setting

The setting of my study is the New York City high school application and assignment system. In this section, I list the themes available to high schoolers in New York City, and describe the application and assignment process for high school.

2.1 Themes

I define nine academic themes for high school programs in New York CIty: Arts and Design; Business, Hospitality, and Services; Computing and Engineering; Humanities and Interdisciplinary; Media; Military and Law Enforcement; Multicultural; Public Service, Law, and Social Justice; and Science. Based on consultation with the New York City Department of Education, I defined the themes largely based on the academic interest areas listed in the high school directory, as well as the name of the program. I describe more about how I defined themes in Appendix A.

Themes are self-reported to the Department of Education by schools, and there are no accountability measures or guidelines governing theme implementation. Regardless of how the themes are implemented, the theme labeling itself can have implications for the sorting of students. In Section 10, I shed more light on how themed curricula differ in practice by looking at how the distribution of credits by subject area differs by theme.

2.2 Application

I study themes in New York City high schools from 2011-2019. In New York CIty, since 2004, every eighth grader applies to high school through a centralized application and assignment system. Rather than applying to schools, students apply to "programs" at schools, which are categorized by theme. About one in four schools have multiple programs, and of these, the median school has three programs. Students are given a printed with information on each program, organized by borough [2]. They must submit a rank-ordered application list of up to 12 programs, in order of preference [3]. The application may be submitted online, through the applicant's middle school counselor, or through one of several Family Welcome Centers throughout the city.

Over the time period of my study, New York City students do not necessarily have a "zoned" neighborhood high school, so there is not necessarily a "default" option for students choosing high schools. Even students who do live in an area with a zoned high school need to apply to it through the same centralized application system. There are relatively few high school eligibility restrictions; while there exist a few single-sex or zoned programs, applicants are able to apply to the vast majority of high school programs.

There are a variety of admissions methods by which high schools determine who is assigned to their programs. Broadly, these methods either choose applicants via Lottery, or by ranked score based on various criteria (Score). These criteria can include middle school grades, attendence, and test scores, or additional application materials like an audition or essay. Lottery admission methods include Zoned, Unscreened, and Limited Unscreened. Score admission methods include

[2] During my time period of study, this booklet was also available as a PDF online, in several languages. In the middle of my time period, an online search tool called MySchools was introduced that enables students to search for high school programs by characteristic of interest.

[3] On its website, the Department of Education instructs students to "Be sure to place programs in your true order of preference, with your first choice as number one" (NYC Department of Education 2023).

Audition, Screened, and Screened for Language. The Education Option admissions method uses a combination of lottery and score ranks to admit students. Programs employing any of these methods group students by "priority" levels, applying the admit rule within priority group for students in descending levels of priority.

In short, the New York City high school application process involves applicants choosing and ranking programs, and programs ranking applicants. In the following subsection, I describe how these ranks are transformed into a single program assignment for each applicant.

2.2.1 Deferred Acceptance Algorithm

The centralized assignment system is an implementation of the Deferred Acceptance algorithm. The main round of the assignment works as follows. Until every student is assigned, or has exhausted their ranked list, it repeats the following two steps:

1. Each student "proposes" to their best choice of the programs that have not yet rejected them. High school programs tentatively assign all of their seats in order of applicant priority. Ties within priority group are broken by lottery number or score rank. If more students propose than the program capacity, remaining proposals are rejected.

2. Students rejected in the current iteration propose to the next program on their list in the next iteration. Students tentatively assigned to a program seat propose to the same program in the next iteration.

After the algorithm terminates, some students may not have an assignment, if they were rejected from every program they listed. All students, including unassigned students, have the option of participating in a supplementary round, in which students rank programs with capacity still remaining, and are assigned according to the same algorithm. Students who are unassigned after the supplementary round are administratively placed into programs remaining capacity.

2.3 Assignment and Enrollment

Students receive a single assignment from the centralized assignment process. The New York City public school system also includes eight specialized exam schools, and one specialized audition school. I refer to these schools throughout as "Exam / Audition" schools[4]. Admission to these schools does not operate through the centalized assignment mechanism described above. Instead, students are invited to apply to the Exam / Audition schools in a parallel process. If they receive an offer from an Exam / Audition school, and the main high school assignment system, they can choose where they want to enroll. Alternatively, students can enroll at a New York City charter school, a private school, or a public school outside of New York City.

[4] Exam / Audition includes the following schools: The Bronx High School of Science, The Brooklyn Latin School, Brooklyn Technical High School, Fiorello H. LaGuardia High School of Music & Art and Performing Arts, High School for Mathematics, Science and Engineering at City College of New York, High School of American Studies at Lehman College, Queens High School for the Sciences at York College, Staten Island Technical High School, and Stuyvesant High School.

3 Data

I use administrative data from the New York City Department of Education. My population of interest is New York City 8th grade applicant cohorts from 2011-12 to 2019-20. I receive demographic information on these 8th grade students, including gender, race, eligibility for free or reduced price lunch, receipt of special education services, and residential census tract. I also observe the middle school at which they are enrolled, and their 7th grade standardized exam scores.

High school and program directories are publicly available online. These directories contain several program characteristics, including the program name and interest area that I use to categorize programs into themes, and the admissions method. I supplement these program-level characteristics with additional public data on school-level demographics, outcomes, and report cards.

I observe the rank-ordered application lists that each 8th grader in my sample submitted to the centralized high school assignment system for the main and supplementary rounds. I observe their first round and second round offers, final assignments, and final 9th grade enrollments. I observe enrollment at all New York City public high schools, including Exam / Audition schools, and excluding charter schools.

Finally, I observe high school outcomes, including attendance, test scores, graduation, and postsecondary outcomes.

My school choice model relies on computing public transit travel times between the centroid of each applicant's census tract, and each program in their choice set. I do so using OpenTripPlanner, an open source software built on publicly available street maps and timetables from public transit agencies (Appendix B).

There are 604,793 unique New York City 8th grade applicants to New York City public high schools from 2011-12 to 2019-20. This population excludes some students wtih disabililties who are manually placed into high school programs with highly specialized instructional support. Dropping students with missing race indicator, poverty indicator, 7th grade test scores, or residential census tract, drops the sample down to $N = 550,537$. Since my study is motivated largely by racial segregation, and I present my results by a rich set of interactions of race, income and achievement, I restrict my sample to focus on the four racial groups that make up the vast majority of my sample: Black, Asian, Hispanic, and white students. My final sample has size $N = 539,890$. Descriptive statistics for the full sample of high school applicants, the subsample who eventually enroll at a New York City high school, and the rates of various enrollment outcomes are in Table 2.

Over the time period of the study there are 1075 unique programs in 474 schools. Table 1 shows how programs are distributed across theme, and Figure 1 shows how the themed programs are distributed throughout the city. The Humanities and Interdisciplinary theme connotes a general high school, and can alternatively be thought of as "unthemed".

3.1 Temporary restrictions on sample for model estimation

For now, my model estimation sample currently has two additional sample restrictions, compared to the full sample described above. These are temporary; I will eventually update the model estimates to include the full sample.

1. I currently exclude the years 2018-19 and 2019-20 because of a couple of additional data cleaning steps I have not yet done for these years.
2. I exclude applicants who are students with disabilities. The Department of Education reserves seats for these applicants at each program, with separate capacities from the rest of the applicant pool. I need to add additional logic to my data cleaning and model estimation code to include these students. They account for 18% of the applicant pool.

My sample for the counterfactual simulation is the same as for the model estimation. However, I report applicant demographics and measures of simulation that take all students into account, whether or not they are in the model estimation sample. Applicants who are not in the model estimation sample simply retain their baseline assignment and enrollment in the counterfactual simulation.

4 Descriptive Evidence

4.1 General application descriptives

I begin by presenting a picture of what student applications look like, in general. My sample includes 539,890 New York City public 8th grade applicants to New York City public high schools, from 2011-12 through 2019-20. Their median application list length is 7, with 80% ranking fewer than 12 programs. The median application has 3 distinct themes represented, and a theme concentration (proportion of ranked programs within the most-ranked theme) of 0.5. 47 % of students are assigned their first choice in the main round. 75% of students ultimately enroll in their main round match. 6% enroll at an Exam / Audition school, 10% are assigned to another New York City public high school through the supplementary or administrative round , 1% remain unassigned, and 8% received an assignment but did not enroll at a New York City non-charter public high school. Finally, the majority of programs are observed rejecting people, meaning they are filled to capacity.

4.2 Application and enrollment behavior with respect to theme, by demographic group

Figure 2 shows differences in application and enrollment behavior by race, with respect to theme. Panels (a) and (b) plot differences in application behavior by race, with respect to theme. Panel (a) plots, by race, the proportion of applicants ranking each theme first. Panel (b) plots, by race, the proportion of applicants ranking each theme more than any other theme. Both application panels suggest that white (and to a lesser extent, Asian) applicants are more likely to rank Humanities and

Interdisciplinary programs first, and frequently, than other groups. Black and Hispanic students, on the other hand, are more likely to rank the more specialized themes. These differences in ranking behavior are somewhat attenuated by enrollment decisions, as shown in panel (c). Panel (c) shows the themes at which students ultimately enroll. It also indicates whether applicants remain unassigned, are reassigned from their main round assignment, enroll at an Exam / Audition school, or Opt Out (do not enroll at a New York City public high school). This last option is indicated by the white space at the top of each bar. While white students are more likely to enroll at Humanities and Interdisciplinary programs than non-white students, the difference is smaller than it is for application.

Figures 3 and 4 show application and enrollment behavior by income and baseline achievement of applicants, rather than by race. Non-low-income students, and high-achieving students, are more likely to rank Humanities and Interdisciplinary programs, relative to more specialized themes. High-achievement students are also more likely to rank Science schools. However, much of the application gap by achievement with respect to Humanities and Interdisciplinary and Science goes away in enrollment. This suggests that the high-achieving students interested in Humanities and Interdisciplinary and Science schools are especially likely to be the ones in this group who end up at an Exam / Audition school, or Opt Out.

Table 3 presents mean school-level characteristics of each high school program, by theme. While high schools containing multicultural programs stand out as having smaller total enrollment, more English language learners, and more students with low test scores at baseline, for the most part, the differences in characteristics across theme are not large. Table 4 presents mean school-level graduation outcomes of each high school program. While Computing and Engineering stands out as enabling many students to graduate with a Career and Technical Education endorsement on their high school diploma, in general, differences in outcomes across theme are not large.

4.3 Segregation in New York City high schools

Since I will eventually explore the extent to which themes contribute to segregation in the New York City public high schools, I provide descriptive evidence on the baseline level of segregation by race, income, and achievement. To measure segregation, I use the variance ratio index of segregation, a widely-used measure of same-group isolation (Massey and Denton 1988). Let applicant i's same-group exposure in a school or city, $SGE_{i,school}$ or $SGE_{i,city}$, equal the proportion of high school applicants in the same race, income, or achievement group as i in i's school or city. Then, the school segregration index is defined as follows:

$$SI = \frac{1}{N}\sum_{i}^{N}\frac{SGE_{i,school} - SGE_{i,city}}{1 - SGE_{i,city}} \tag{1}$$

The numerator of the variance ratio equals the average deviation in same-group exposure experienced by applicant i at their school, from their same-group exposure in the city. In this way, the index captures school segregation relative to the demographics of the city. The denominator adjustment enables us to measure this deviation in same-group exposure relative to the maximum deviation in same-group exposure, $1 - SGE_{i,city}$, that would even be possible in the reference population. The adjustment makes it easier to compare the index values across subgroups with different levels of representation in the reference population.

The overall segregation index in Equation can be computed by summing over all students, or a group-specific segregation index can be computed by summing over only the students in one racial, income, or achievement group.

I define my segregation index to use the reference population of same-cohort[5] 8th grade applicants. As a result, students who opt out of district public schools are part of the reference population, even when their same-group exposure is not captured in the index. If all schools in the city perfectly mirrored the demographics of 8th grade high school applicants in the city, the index would equal 0.

The segregation index for various residential subdivisions, as well as the school segregation indices for each demographic subgroup are in Table 5. The first column shows the proportion of the applicant sample in each race, income, and baseline achievement group.The next three columns provide a picture of residential segregation in New York City, with city demographics as the reference population. While boroughs are fairly representative of the population of New York City, subdistricts and schools are more segregated. Census tracts are the most highly segregated; schools are integrated relative to census tracts. The last two columns contain assigned school segregation index, and the enrolled school segregation index, by race, income, and achievement. These values can be interpreted as an individual in each group's mean experience of racial isolation, as it reflects the distance between their same-group exposure at school and same-group exposure in their city.

5 Model

In previous section, I have provided descriptive evidence on differences in application and enrollment behavior with respect to themed schools by race, income, and achievement. I now attempt to quantify the extent to which theme preferences drive school choice, relative to other school characteristics. To do so, I estimate a structural model of high school choice that allows for strategic behavior (Agarwal and Somaini 2018; Idoux 2021).

5.1 Definitions

There are N applicants $i \in \{1, ..., N\}$ to New York City high schools during the years $t \in \{1, ..., T\}$. There are J programs indexed $j \in \{1, ..., J\}$ that are available in at least one year in this time period. Each student has a program choice set $CS(i) \subset J$. Let $\mathcal{CS}_i$ equal the set containing all ordered subsets of CS_i from size 1-12, which is i's application choice set. Each student i submits a rank-ordered list $R_i \in \mathcal{CS}_i$.

[5] Year subscripts are omitted from 4.3 for simplicity, but all exposure measures are computed within-cohort.

For all students, the choice set includes the outside option 0. This option represents enrolling in an assignment from the supplementary or administrative round of the match, enrolling at an Exam / Audition school, attending a charter school, attending a private school, or attending a public school outside of New York City. Note that students are free to pick the outside option whether or not they receive an assignment from the main round of the match.

5.2 Subjective assignment probabilities

When applicants add programs to their list, they take into account their subjective probability of assignment to each program. They do not add a program to their list if their expected utility from that program (the indirect utility of that program weighted by assignment probability) is less than their marginal application cost. In this section, I explain my model and estimation of applicant beliefs.

5.2.1 Assignment probabilities

I model applicant beliefs as consistent with the equilibrium outcome. The probability that applicant i is assigned to program j if they rank it in slot k is $p_{ijk}(R_{i,k-1}, \mathbf{q_i})$. $R_{i,k-1}$ refers to the programs that applicant i ranked in slots 1 through $k-1$ of their list. Note that applicant i cannot be assigned to a program they rank in slot k without being rejected from the programs they rank in slots 1 through $k-1$. $\mathbf{q_i} := (q_{i1}, ...q_{iJ})$ is the full vector of subjective probabilities q_{ij} that applicant i will be admitted to program j. In the context of the centralized assignment mechanism, where each applicant only gets one offer from the assignment mechanism, i's "admission" to j refers to whether i's lottery tiebreaker clears j's priority and lottery cutoff, if j is a lottery program, or i's score clears j's priority and score cutoff, if j is a score program. I describe below how $\mathbf{q_i}$ is computed to be consistent with the equilibrium outcome, conditional on applicant i's information set.

For applicant i ranking program j in slot k, the true assignment probability $p_{ijk}(R_i)$ is their probability of being rejected at programs they rank in slots 1 through $k-1$, and assigned to program j. Note that each applicant is assigned a single lottery tiebreaker that is used for them at any lottery they enter over the course of the iterations of the Deferred Acceptance algorithm. For this reason, admissions at lottery programs are dependent events. The formulas for computing assignment probabilities as a function of ranked list and subjective admit probabilities take these dependencies into account (Appendix C).

5.2.2 Model of coarse beliefs

I model applicant beliefs as consistent with the equilibrium outcome, but coarse. For applicant i and program j, let $priority_{ij}$ be i's priority at j, $rank_{ij}$ be i's rank at j, if j is a score program, and $tiebreaker_i$ be i's lottery tiebreaker. Applicant i's admission at program j is a function of these three variables. I allow applicant i to know $priority_{ij}$, but not the other two variables. Instead of their exact rank, $rank_{ij}$, applicant i only knows $rankcat_{ij}$ a coarser category based on their characteristics [6]. $rankcat_{ij}$ is uninformative for lottery programs, so I set it to 0 if j is a lottery program. Then, applicant i's information set is $\mathbf{\Omega_i} = ((priority_{i1}, rankcat_{i1}), ..., (priority_{iJ}, rankcat_{iJ}))$. The information set reflects what an applicant would know in reality: rules for individual priority can be determined from a program's directory entry, but applicants would not know their exact rank at a score program or the tiebreaker that will be drawn for them at lottery programs.

One small deviation from consistent beliefs is that I do not allow perceived assignment probability to be 0 or 1. I truncate perceived assignment probabilities p_{ijk} to be within $[.01, .99]$. I do this to rationalize, given positive marginal application costs, applicant decisions to rank programs that they do not have a chance of getting into in equilibrium, and to continue adding programs to their list even if their probability of rejection from all higher-ranked programs is zero in equilibrium.

5.2.3 Admit probabilities

As described above, the assignment probabilities p_{ijk} depend on applicant i's choice of R_{k-1} and vector of admit probabilities, $\mathbf{q_i}$. In this section, I discuss how I assign subjective admit probabilities q_{ij} to all applicants i for all programs j in their choice sets. The assigned admit probabilities q_{ij} are consistent with the equilibrium assignment, conditional on i's information set $\mathbf{\Omega_i}$.

Let q_{ij} equal the subjective probability i is admitted to j. The marginal priority for program j refers to the priority group within which the lottery or score determines admission. For any program j, q_{ij} equals 1 if $priority_{ij}$ is above than the marginal priority for j, 0 if $priority_{ij}$ is below than the marginal priority for j, and the frequency at which applicants with characteristics $\mathbf{\Omega_i}$ are admitted to j if $priority_{ij}$ is the marginal priority for j. In the Deferred Acceptance mechanism, q_{ij} does not depend on the slot in which i ranks j.

The uncertainty around admission arises from the fact that the applicant does not know their lottery number (for a lottery program) or exact relative rank (for a score or audition program) within rank category [7].

5.3 Application cost

Since the vast majority of applicants do not fill out all of their rank slots, I include application costs in my model. Without application costs, I would need to assume that these students prefer or are indifferent between the outside option and the programs they did not rank. If this assumption is untrue, and some of the unranked programs are very similar to ranked programs on observables, estimating a model with application costs could inflate the taste shock variance relative to the other coefficient estimates.

[6] For screened programs, the rank category is based on baseline achievement category. For audition programs, the rank category is based on application and admission to an Audition program

[7] My approach differs from the rational expectations baseline in Agarwal and Somaini 2018, in which the applicant knows her rank within the population distribution of applicants, but uncertainty arises instead the from the applicant pool changing year-to-year.

Application cost is parameterized as linear in application length. Applicant i's marginal application cost c_i is distributed $TruncatedNormal(0, \infty, c, \sigma_{zeta})$.

5.4 Model of application choice

An individual's application R_i induces a lottery over assignments to each program they list. Agarwal and Somaini 2018 models application choice as the choice of the optimal lottery. In my setting, the number of possible applications for each applicant is intractable. Even limiting each applicant's choice set to $|CS(i)| = 30$ programs, each applicant chooses among $\sum_{i=0}^{11} \frac{30!}{(12-i)!}$ application portfolios. Estimation of a model in which applicants choose among all portfolios would involve repeatedly computing the expected utility of all of these portfolios for all applicants, which is not feasible.

Previous studies have found theoretical results allowing limitation of the portfolio choice set in order to make estimation tractable (Larroucau and Rios 2020; Calsamiglia, Fu, and Guell 2020). These results do not apply or do not sufficiently simplify the problem in my setting, as assignment probabilities are not independent, and there are application costs.

Following Idoux 2021, I simplify the problem by invoking an assumption of Limited Rationality that governs how students choose their applications.

Assumption 1. *(Limited Rationality of Applicants, Assumption 1 in Idoux 2021).*
v_{ij} is the utility applicant i receives from being assigned to program j. p_{ijk} is the probability i is assigned to j. c_i is the per-unit cost of application for applicant i. $R_{i,k-1}$ refers to the order-preserving subset of R_i containing the first $k-1$ elements of R_i. Program $j = 0$ represents the outside option.
For each position k in the list, an applicant i chooses $j \in \{0\} \cup \{1, ..., J\} \setminus R_{i,k-1}$ to maximize:

$$v_{ij} p_{ijk}(R_{i,k-1}) - I(j \neq 0)c_i + \tilde{V}_k(\{j\}) \tag{2}$$

where the continuation value for position k is defined for each ordered set of choices C as:

$$\tilde{V}_k(C) := \max_{j' \in \{0\} \cup \{S_k \setminus C\}} v_{ij'} p_{ij',k+|C|}(R_{i,k-1} \cup C) + \tilde{V}_k(C \cup \{j'\}) \tag{3}$$

where $S_k = \{j' \in \{0\} \cup \{1, ..., J\} \setminus R_{i,k-1} : v_{ij'} p_{ij'k}(R_{i,k-1}) - I(j' \neq 0)c_i \geq 0\}$ i.e. S_k is the "consideration set" consisting of remaining programs which would clear the per-unit cost c_i if they were added in position k.

The applicant maximizes this objective function sequentially by picking the highest utility program in her consideration set, step by step, until $\{0\}$, the outside option, is the only element left in the consideration set. The applicant's "mistake" is in the specification of her continuation value, in which the consideration set is the same as it is in the current step. The applicant acts as if her consideration set will not change in future steps.

This strategy deviates from full rationality when ranking a different program at k instead would increase the expected utility of the application, but the other program never gets ranked because after rank k, it is no longer part of the consideration set. The limited rationality applicant does not consider this; she acts as if she will eventually rank every program in her current consideration set, not anticipating how the consideration set will evolve further down the list. There is no reason for her to consider ranking a non-maximum utility program, because she believes she can rank it later without hurting her conditional probability of assignment, thanks to the properties of Deferred Acceptance.

Under full rationality, the continuation value is correctly specified, so the applicant would choose a non-maximum-utility program at spot k , if doing so would increase the total expected utility of the application.

5.5 Parametrization of Indirect Utility

The indirect utility for applicant i of assignment to program j is v_{ij}.

$$v_{ij} := \delta_{cell(i),j} + \theta_{boro(i),boro(j)} + \mathbf{X_{j,t(i)-1}}\boldsymbol{\beta_{cell(i)}} - d_{ij} + \gamma_{cell(i),ms(i),t(i),theme(j)} + \epsilon_{ij} \tag{4}$$

$$v_{i0} = \epsilon_{i0} \tag{5}$$

$\delta_{cell(i),j}$ is a demographic cell-by-program fixed effect. $\theta_{boro(i),boro(j)}$ is a residential borough-by-program borough fixed effect. $\mathbf{X_{j,t(i)-1}}$ includes time-varying school characteristics (lagged proportion of high-achieving students, and lagged proportion of low-income students). d_{ij} is the public transit travel time between i's census tract and j's location. $\gamma_{\mathbf{cell,(i),ms(i),t(i)}}$ is a random vector of unobserved preference for each theme at the demographic group, middle school cohort level, distributed $MultivariateNormal(\mathbf{0}, \mathbf{\Sigma}_\gamma)$. This term captures correlation of preferences with respect to theme within same demographic cell, middle school cohort, which would pick up similar application behavior with respect to themes due to applicants wanting to go to the same high school as their middle school friends, or applicants receiving similar advice from their middle school guidance counselor. I allow these unobserved preferences to be correlated across themes; a positive covariance in Σ_γ tells us that groups of same demographic, same middle school cohort students that prefer one theme prefer the other as well.

ϵ_{ij} is a taste shock, distributed $Normal(0, \sigma_\epsilon)$. I set the coefficient on travel time to -1 as a scale normalization, and the expected indirect utility of the outside option to 0 as the location normalization. The unit of travel time is 1000 seconds, which is approximately 15 minutes. Therefore, the scale of coefficient estimates can be interepreted in relation to the disutility from 15 additional minutes of public transit travel time. The location normalization means that all v_{ij}s can be interpreted as an individual's value of a program relative to their value of being unassigned after the main application round.

5.6 Identifying Theme Preference

So far, I have not mentioned how theme preferences enter the model. In the parametrization of indirect utililty, theme preferences are absorbed by the program-by-year fixed effect. If we wanted to examine how preferences over programs of each theme differ, on average, we could compare the average program-by-year FE for each theme, by demographic group of interest.

However, this comparison would not allow us to disentangle theme preference from preference over other time-invariant program characteristics. Thus, I implement a second step after estimation of the structural model. I regress program-by-year fixed effects $\delta_{cell(i),j}$ on non-Humanities and Interdisciplinary theme-by-demographic cell dummies, time-invariant program characteristics $\mathbf{X_j}$, and demographic cell dummies.

$$\delta_{cell(i),j} = \alpha_{cell(i),theme(j)} + \boldsymbol{\eta}\mathbf{X_j} + \rho_{cell(i)} + \upsilon_{cell(i),j} \tag{6}$$

The coefficients on theme-by-cell dummies, $\alpha_{cell,theme}$, capture each demographic cell's preference for programs of each theme, on average, after controlling for time-invariant program characteristics, relative to the their preference for Humanities and Interdisciplinary programs.

While estimating $\alpha_{cell(i)}$ is necessary for implementing the counterfactual, and for learning about preferences over all themes, I also estimate an alternative specification aimed at learning how much applicants like any themed program on average, relative to Humanities and Interdisciplinary. I regress program-by-year fixed effects on a dummy for non-Humanities and Interdisciplinary.

$$\delta_{cell,j} = \phi_{cell}[theme(j) \neq Hum.] + \boldsymbol{\eta}\mathbf{X_j} + \rho_{cell(i)} + \xi_{cell,j} \tag{7}$$

The non-humanities coefficients, ϕ_{cell}, capture each demographic cell's preference for themed programs, on average, after controlling for time-invariant program characteristics, relative to the their preference for Humanities and Interdisciplinary programs.

My goal is to eventually include a rich set of time-invariant program characteristics in $\mathbf{X_j}$, so that $\alpha_{cell,theme}$ and ϕ_{cell} more credibly capture theme preference, rather than other program characteristics. So far, $\mathbf{X_j}$ only contains school size and 4-year graduation rate. Note that I do not want $\mathbf{X_j}$ to contain characteristics that are inextricably linked with theme, since I want to be able to identify preferences over theme. For example, I do not necessarily want to control for availability of Career and Technical Education or language courses, I consider these to be part of how themes are implemented, and want them to be reflected in the theme coefficients.

6 Identification

In settings with strategic reporting of preferences, Agarwal and Somaini 2018 details the conditions for non-parametric identification of the distribution of indirect utilities. One sufficient condition is the existence of a "special regressor", which is additively separable in the indirect utility function and independent of the unobservable component of utility . In my setting, travel time serves this role, requiring the assumption that conditional on observed student and school characteristics, the unobservable components of utility $(\epsilon_{ij}, \gamma_{\mathbf{cell(i),ms(i),t(i)}})$ are independent of travel time. If students systematically tend to live near schools they prefer, after controlling for observables, my estimates would understate preferences for other characteristics relative to travel time. Note that the inclusion of residential borough-by-program borough fixed effects means that only within-borough correlation of travel time and unobserved preference is a threat to identification. The distribution of indirect utilities is nonparametrically identified by variation in travel time within student demographic group and program characteristics.

The distribution of marginal application cost is identified by cross-sectional and cross-year variation in admission probabilities (Idoux 2021). Costs are identified by the degree of response in ranking behavior to shifts in admission probabilities. Applicants who want to respond to a downward shift in assignment probabilities by ranking more programs may be inhibited from doing so by their marginal application costs.

The theme correlation term is identified by within-same demographic cell, middle school cohort ranking behavior with respect to each pair of themes, that is, whether a demographic cell-by-middle school cohort group that ranks a lot of one theme is likely to rank a lot of the other theme.

7 Estimation

I compute program assignment probabilities in equilibrium, and set each applicant's subjective program assignment probabilities to these. Then, I estimate the choice model. Finally, I decompose the estimated demographic cell-by-program fixed effects to estimate theme preference by demographic cell.

7.1 Subjective probabilities of assignment

The assignment probabilities p_{ijk} depend on the admit probabilities q_{ij}, so I compute those first. Following the assumption of consistent beliefs, I set q_{ij} equal to the proportion of applicants in priority group $priority_{ij}$ and rank category $rankcat_{ij}$ who clear the priority cutoff or are marginal priority and clear the score cutoff, for a score program, and clear the marginal priority or are marginal priority and clear the lottery cutoff, for a lottery program. One challenge is that I do not observe i's priority or rank at j in the data if i did not apply to j. Thus, I back out the rules for assigning priority from the characteristics and assigned priorities of applicants to each program, and use these to impute the priorities for non-applicants.

I compute the probability of assignment for each applicant i to each program j in slot k, which is a function of $\mathbf{q_i}$ and choice of higher-ranked programs $R_{i,k-1}$. p_{ijk} equals the probability of being rejected from all programs in $R_{i,k-1}$, and admitted to program j. Appendix C contains the formulas for computing assignment probabilities from admit probabilities in a way that accounts for interdependencies in admission events.

7.2 Choice model

Since there is no closed-form solution for the likelihood function, and large choice sets raise challenges of simulation error and tractability with simulated maximum likelihood, I follow the previous literature and instead use a Gibbs sampler adapted from McCulloch, Rossi, and Allenby 1996 to estimate my model (Abdulkadiroglu, Agarwal, and Pathak 2017; Agarwal and Somaini 2018; Idoux 2021).

The Gibbs sampler yields estimates asymptotically equivalent to the maximum likelihood estimator. The mean of the posterior distribution of the parameters, given the prior distribution and the data, is asymptotically equivalent to the maximum of the likelihood function. The posterior distribution is simulated by repeatedly taking draws from the posterior one parameter at a time, conditional on the values of the other parameters. After enough iterations, the sampler converges to draws from the joint posterior of all of the parameters. The means of the parameter draws post-convergence are their point estimates, and the standard deviations are their standard errors.

Because of the bounds on indirect utilities and costs (Appendix D) derived in Idoux 2021 as a result of Assumption 1, the number of constraints on utilities is linear rather than exponential in program choice set size. Further estimation details are in Appendix E.

7.3 Identifying cell-by-theme preference from cell-by-program fixed effects

I store the cell-by-program fixed effects draws $\delta_{cell(i),j}$ for each iteration of the Gibbs sampler. Post-estimation, for each post-convergence iteration, I run the regression specifications in equations 6 and 7. I get point estimates and standard errors from the coefficients from each iteration by taking the mean and standard deviation over iterations, respectively.

8 Results

8.1 Preference for themed schools, relative to Humanities and Interdisciplinary

Figure 5 plots 95 % confidence intervals for the theme preference coefficients, $\phi_{cell(i)}$. These coefficients represent the mean preference for the themed programs, relative to Humanities and Interdisciplinary programs, for each demographic cell. All of the point estimates are negative, indicating that most subgroups prefer Humanities and Interdisciplinary themed programs to the other themes.

Preferences vary by race. Consistent with the descriptive evidence on application behavior, white and Asian applicants have higher relative preferences for Humanities and Interdisciplinary programs than Black and Hispanic applicants.

In general, keeping other demographic characteristics fixed, low-income and non-high baseline achievement applicants have stronger relative preferences for themed programs, compared to non-low-income and high baseline achievement applicants, respectively. However, the differences are not always significant or consistent across all demographic cells, making them less conclusive than differences in preference by race. Within Asian applicants, baseline achievement appears to be an especially important driver of theme preference heterogeneity.

Figure 6 plots estimates of theme preference, relative to Humanities and Interdisciplinary and Science, rather than just Humanities and Interdisciplinary. Here, I observe clearer patterns of heterogeneity by achievement, consistent with descriptive evidence on application and enrollment. High-achievement applicants have stronger preferences for Humanities and Interdisciplinary and Science programs than non-high achievement applicants.

Both Figures 5 and 6 mask the considerable heterogeneity of preferences over the nine themes. Appendix Table 7 contains estimates for the entirety of relative theme preference coefficients, $\alpha_{cell,theme}$. Table 8 displays the top three preferred themes for each demographic cell. It provides an idea of the extent of theme preference heterogeneity across demographic groups.

8.2 Other parameter estimates

Table 6 displays estimates of taste shock variance, mean marginal application cost, and application cost variance. I estimate a taste shock variance of 7.51, which is quite large relative to other coefficient estimates, indicating substantial unobserved heterogeneity in preferences. Magnitude-wise, this estimate is in line with previous estimates (Abdulkadiroglu, Agarwal, et al. 2017; Idoux 2021). For the marginal cost distribution, I estimate a mean of 0.0086 and a variance of 0.00004.

My estimates for within-middle-school-cohort-cell correlation of unobserved theme preferences are in Table 9. One notable result that unobserved preferences for Military and Law Enforcement programs, within students in the same demographic group, middle school, and cohort, are highly correlated, with a covariance of 14.8, indicating that even after controlling for mean preferences by observable characteristics, some groups of applicants strongly like these programs, and some strongly dislike them. Unobserved preferences for Military and Law Enforcement programs and Multicultural programs, are also in general negatively correlated with unobserved preferences for other themes, indicating that applicants who like these themes tend to only like these themes, and dislike other themes. Finally, unobserved preferences for Humanities and Interdisciplinary programs and Science programs are postively correlated.

8.3 To do

1. Perform various model robustness checks, such as estimating the model with different specifications, priors, or starting values.
2. Assess model convergence using typical diagnostics, such as coefficient trace plots and the potential scale reduction factor.

9 Counterfactual

I use the model estimates to simulate the counterfactual assignment of students that would occur if programs were not differentiated by theme (that is, if all programs were Humanities and Interdisciplinary). I interpret the results of this exercise as measuring the contribution of theme preference to racial isolation in the current system.

From my estimates, I can compute program indirect utility relative to the outside option, both in the baseline and the simulation. I model the decision to exit the New York City public school district as a function of demographic cell, assigned program utility, Exam school offer, and Audition school offer. I predict whether each applicant exits the district in the simulation as a function of their assigned program utility in the simulation.

9.1 Simulation

For a variety of reasons, I cannot perfectly replicate the assignment process that would be used by the New York City Department of Education for a counterfactual set of applications [8]. Therefore, I compare the results of my counterfactual simulation to the results of my baseline simulation, rather than the real data. That way, any simulation differences are netted out in the comparison.

I take the following steps to simulate the baseline and counterfactual assignments.

1. I set model parameters equal to their point estimates.

2. For the counterfactual, I set the theme preference coefficient $\alpha_{cell,theme}$ to 0 for all themes, updating the program-by-cell fixed effects $\delta_{cell,theme}$ accordingly. Since $\alpha_{cell,theme}$ was estimated relative to Humanities and Interdisciplinary, 0 is the coefficient for Humanities and Interdisciplinary. For the baseline, I keep $\delta_{cell,theme}$ the same.

3. I compute the non-random component of indirect utility for each program in each applicant's choice set.

4. For K iterations:

   a) Get random variable draws. Use the draw of random variable values ϵ_{ij}, $\gamma_{cell(i),ms(i),t(i),theme(j)}$, and v_{ij} from one iteration of the Gibbs sampler. For the counterfactual, I set $\gamma_{cell(i),ms(i),t(i),theme(j)}$ to $\gamma_{cell(i),ms(i),t(i),\text{Humanities and Interdisciplinary}}$. Using draws from the Gibbs sampler incorporates additional information from the data about likely values for these variables.

   b) Compute indirect utilities for each program in each applicant's choice set.

   c) Until a fixed point is reached:

      i. Compute assignment probabilities for each program in each applicant's choice set, using the formulas in C.

[8] One reason is that I do not observe program ranks or priorities for applicants at a program unless they actually applied to it.

ii. Given the assignment probabilities to each program and utilities, select applications for each student according to their objective function 1

iii. Given student applications, run the Deferred Acceptance algorithm to get the updated program assignments. Since I do not observe $rank_{ij}$ I impute rank as $rankcat_{ij}$ with ties broken randomly for Audition programs, and 8th grade math and ela scores for other programs.

iv. Compute admit probabilities consistent with the new assignments.

v. Compute last year's school peer characteristics consistent with the new assignment.

vi. Update program utilities to reflect the updated last year peer characteristics for each program's school.

d) Estimate a probit model of district exit using the real data, as a function of demographic cell, Exam / Audition school offer, and indirect utility of the main round assignment, for students assigned in the main round. Repeat for students unassigned in the main round, with the model depending only on demographic cell and Exam / Audition school offer. Use the estimated probit model to predict who will exit the district in the baseline and counterfactual simulations, using their simulated main round outcomes. If I predict a student stays in the district, and I observe them enrolling at an Exam / Audition school in the real data, I assume they enroll at the same Exam / Audition school in the counterfactual. If I predict an unassigned student stays in the district, and I observe them enrolling at any school in the real data, I assume they enroll at the same school in the counterfactual.

e) Compute segregation indices, district exit rates, and other parameters of interest based on the simulated assignments, and simulated enrollments post-district exit.

5. Using the mean and standard deviation over all iterations, report point estimates and standard errors for segregation indices, district exit rates, and other parameters of interest.

9.2 Results

I present results of comparing the simulated Humanities and Interdisciplinary-only counterfactual to the simulated baseline. My main comparison parameters of interest are the segregation indices, district exit rates, and assigned program utilities. I begin by discussing general changes in application patterns from the baseline to the counterfactual, which helps explain the mechanisms behind some of the changes in parameters I care about.

9.2.1 General changes in application behavior

For each applicant, I compute the difference in the simulated counterfactual program utility and the simulated baseline program utility, for each program in their choice set. Choice sets are fixed across simulations. 9a shows the mean change in program choice set utililty for each demographic cell.

In the Humanities and Interdisciplinary-only counterfactual, the utilities of the programs in each applicant's program choice set are higher. This is consistent with applicants in all demographic cells preferring the Humanities and Interdisciplinary theme to others, on average. The demographic groups with the strongest relative Humanities and Interdisciplinary preferences see the largest mean increase in choice set program utility. As expected with an increase in program utilities, applicants rank more programs on average (Figure 9b), as more programs are "worth" ranking relative to the marginal application cost. White and Asian applicants, who have the largest increases in list length in the counterfactual, also see a lower rate of not receiving a main round offer in the counterfactual, relative to the baseline (Figure 9c).

In the counterfactual, applicants act as if all programs are Humanities and Interdisciplinary programs. Since programs are not differentiated by theme in this counterfactual, I expect to see applicants applying to a broader set of programs across baseline theme. The median applicant applies to 3 distinct themes in the baseline and the simulated baseline, and 4 distinct baseline themes in the counterfactual (where the "theme" of a program in the counterfactual refers to its former theme in the baseline).

Application behavior overall is fairly similar in the counterfactual simulation to the baseline simulation. Of the programs ranked by each applicant in the counterfactual simulation, 87 percent were also ranked by that applicant in the baseline simulation, and 47 percent were ranked by that applicant in the baseline simulation in the same slot. 74 percent ultimately enrolled at the same program in the baseline and counterfactual simulation.

9.2.2 Segregation and District Exit

Table 10 displays district exit rates for each demographic subgroup in the model estimation sample. Relative to the baseline with themes, white and non-low-income applicants in the all-Humanities and Interdisciplinary counterfactual see a slight reduction in district exit. This suggests that curricular differentiation in this context could be contributing to white flight, but I need to run more iterations of the counterfactual simulation to see whether this small change is significant.

Table 11 compares the segregation indices in the counterfactual simulation to the baseline simulation. The first two columns compare the simulated baseline and simulated counterfactual segregation indices of school assignment. The second two columns compare the simulated baseline and simulated counterfactual segregation indices of school enrollment. I find that relative to the baseline, the all-Humanities and Interdisciplinary counterfactual regime modestly decreases segregation by race. I find even smaller decreases in segregation by income and achievement. Conditional on assignment, enrollment decisions do not appear to differentially increase segregation in the counterfactual, relative to the baseline.

The segregation indices are computed for the full set of applicants, whether or not they are in my model estimation sample. The students for whom I have not yet estimated the model amount to 18 percent of all applicants. Of these, students with disabililties, for whom I will eventually estimate the model, make up 17 percent of all applicants, and students who are not Asian, Black, Hispanic, or White, for whom I will not estimate the model, make up 1 percent of all applicants. I keep their school assignment and district exit decisions the same in the counterfactual as they are in the baseline. Assuming they have similar preferences to my model estimation sample, these observations attenuate the difference between the counterfactual and baseline simulation in my current results.

9.2.3 Welfare

Figure 11 presents the change in mean assigned program utility, from the baseline to the counterfactual simulation. Except for Black applicants, who on average have relatively strong preferences for themed programs relative to Humanities and Interdisciplinary, all demographic groups are better off, with substantial heterogeneity in magnitude. White and Asian applicants benefit the most. The results are driven by applicants within those demographic subgroups who, due to supply constraints, were assigned to non-Humanities and Interdisciplinary programs in the baseline in spite of strong relative preferences for Humanities and Interdisciplinary schools.

Not all applicants are better in the Humanities and Interdisciplinary-only counterfactual, even within demographic groups who are better off on average (Figure 12). Over half of low-income Black students are worse off in the counterfactual. Substantial minorities of other demographic groups are also worse off in the counterfactual. This finding highlights the importance of unobserved preference heterogeneity in this setting; and is consistent with the relatively large magnitdues of σ_ϵ and Σ_γ. It also suggests that increasing the capacity of Humanities and Interdisciplinary seats, while preserving specialized program seats, would leave more students better off than replacing all specialized program seats with Humanities and Interdisciplinary.

The mean utility changes in Figure 11 can be converted to minutes of travel time. From the scale normalization, one unit of utility is worth 1000 seconds of travel time. White, non-low income, high baseline achievement students are better off by 5.8 minutes of travel time on average in the all-Humanities and Interdisciplinary counterfactual. Black, low income, non-high baseline achievement students are worse off by 3.0 minutes of travel time.

Note that these welfare changes are driven both by all applicants, whether they enroll at a different program in the counterfactual or stay at the same program. Applicants who enroll at the same program in the simulated counterfactual may have a different enrollment utility, because the program in the counterfactual is now a Humanities and Interdisciplinary program, versus whichever theme it was in the baseline.

9.3 To do

1. Run for several iterations to report point estimates and standard errors of segregation indices, district exit rates, and utility changes. Currently, the segregation and district exit results presented here are an average from 5 iterations, and all other results in this section are from only one iteration.

10 Causal impact of assignment to theme of interest

In this section, I propose an analysis that will better illuminate the full set of trade-offs involved with offering curricular differentiation

10.1 Individual theme preference

I obtain individual theme preference coefficients for each applicant i by taking the average value of $\alpha_{cell(i),theme} + \gamma_{cell(i),ms(i),t(i),theme}$ over the post-convergence Gibbs sampler draws.

10.2 Identifying the effect of being assigned to one's preferred theme

Bloom and Unterman 2014 and Abdulkadiroglu, Angrist, Narita, and Pathak 2022 both propose methods of identifying the effect of being assigned to a particular set of schools in my setting. Bloom and Unterman 2014 look at the effect of attending a small school of choice (SSC), a particular set of high schools in my sample, on high school outcomes. They use outcome of first SSC lottery encountered in the algorithm as an instrument for SSC attendance, identifying the average effect of small school of choice attendance for students who would not have enrolled in a SSC had they lost their first lottery, but would have enrolled had they won the lottery. This identification strategy does not fully exploit the random variation in assignment created by lotteries, or use the quasi-random variation in assignment created by score cutoffs.

Abdulkadiroglu, Angrist, et al. 2022 outlines a method for exploiting the full extent of variation from lottery and score cutoff assignment. They isolate the variation in assignment probability coming from lotteries and score cutoff discontinuities by controlling for assignment propensity scores, estimated from a large number of simulated assignments, that absorb all non-random determinants of assignment.

Stratifying on propensity score, I can estimate the effect of assignment to Computing and Engineering programs on outcomes by two stage least squares (Abdulkadiroglu, Angrist, et al. 2022):

$$\underbrace{D_i}_{\substack{\text{enrolls at}\\ \text{CompEng}\\ \text{school}}} = \delta \underbrace{Z_i}_{\substack{\text{assigned}\\ \text{to CompEng}\\ \text{school}}} + \underbrace{\sum_{s \in schools} \gamma_{score(is)}}_{\substack{\text{local propensity score}\\ \text{FE for each school } s}} + \underbrace{g_1(\mathcal{R}_i; \delta_N)}_{\substack{\text{local linear control}\\ \text{for screened school}\\ \text{tiebreakers}}} + \nu_i \tag{8}$$

$$\underbrace{Y_i}_{\substack{\text{high school}\\ \text{outcome}}} = \beta \underbrace{D_i}_{\substack{\text{enrolls at}\\ \text{CompEng}\\ \text{school}}} + \underbrace{\sum_{s \in schools} \alpha_{score(is)}}_{\substack{\text{local propensity score}\\ \text{FE for each school } s}} + \underbrace{g_2(\mathcal{R}_i; \delta_N)}_{\substack{\text{local linear control}\\ \text{for screened school}\\ \text{tiebreakers}}} + \epsilon_i \tag{9}$$

If I restrict the sample of this two stage least squares regression to applicants who prefer Computing and Engineering to Humanities and Interdisciplinary, I can identify the effect no outcomes specifically for them, and compare it to the same effect for the group of students who prefer

Humanities and Interdisciplinary to Computing and Engineering. From these comparisons, I can determine whether students sort on gains into themes.

If any subgroups benefit from enrollment at a specialized theme, relative to Humanities and Interdisciplinary, or I observe that students sort on gains into themes, these would be important considerations in assessing the trade-offs of curricular differentiation.

11 Conclusion

Previous studies have used administrative data on school choice to look at how students sort along demographic lines into schools of varying quality, and the implications of this sorting for student outcomes. Most of this literature focuses on vertical dimensions of school quality, such as graduation rates or test scores, or on unobserved match quality. Relatively little work has been done to explore the implications of horizontal differentiation, in this case an explicit choice by districts. This paper fills this gap in the literature.

I also provide novel evidence on the extent to which specialized themed schools are valued by students, and how this varies by race, income, and achievement. I find that even if students on average prefer general schools, a substantial minority of students, including half of all black applicants in my setting, prefer specialized themed schools, and would lose out in a school system without curricular differentiation.

Offering curricular heterogeneity and labeling school programs by curricular theme is a policy choice with important implications. My findings suggest that theme preferences are unlikely to integrate schools, and may potentially increase district exit and segregation. At the same time, I find that a substantial minority of students prefer programs with specialized themes, and getting rid of themes would leave them with less favored program assignments. Knowing the implications of curricular heterogeneity for student outcomes would provide a clearer picture of the tradeoffs involved in offering themed schools, and can inform whether themes should be emphasized in the application process. Previous work has documented the barriers faced by disadvantaged applicants in the application process (Sattin-Bajaj, Jennings, Corcoran, and Baker-Smith 2018). Given the complexity of the application process, and the inequities embedded in navigating such a complex process, the question of whether the benefit of an additional dimension of school differentiation outweighs the cost is an important one.

Segregation of students by income and race remains a challenge for school districts in the United States. Present-day school segregation reflects and continues the historic injustice of legally enforced school segregation. It continues to have negative implications for student and societal outcomes (Reardon, Kalogrides, and Shores 2019; Card and Rothstein 2007; Billings, Demings, and Rockoff 2014), in contrast to the positive effects of integration (Billings, Chyn, and Haggag 2021; Burgess and Platt 2021; Rao 2019; Domina, Carlson, Carter, Lenard, McEachin, and Perera 2021). Differentiated curricula also increase the stakes of segregation, since they could determine whether different groups of students have access to the same opportunities. For example, the existence of STEM-themed schools could widen the gap in STEM preparation between students who are aware of their preference for STEM at the time of high school application, and those who are not.

My results have policy implications not just for New York City schools, but for many school districts in the United States that offer themed schools. They would also have implications for federal policy, as themed programs are federally funded by the Magnet School Assistance program. Since desegregation is a stated goal of this policy, the question of whether curricular differentiation increases or decreases racial isolation is an important one.

Tables

Table 1: Number of programs offered for the 2012-13 through 2020-21 school years by theme

Theme	**N**
Arts and Design	179
Business, Hospitality, and Services	68
Computing and Engineering	135
Humanities and Interdisciplinary	271
Media	42
Military and Law Enforcement	16
Multicultural	33
Public Service, Law, and Social Justice	75
Science	172
All	1075

Table 2: New York City High School Applicant and Enrollee Descriptives

Group	City Share HS Applicants	City Share HS Enrollees	Group rate of high school enrollment at... Public NYC	Exam / Audition	Opt Out
Asian	0.18	0.19	0.96	0.22	0.04
Black	0.26	0.26	0.91	0.01	0.09
Hispanic	0.41	0.41	0.91	0.01	0.08
White	0.16	0.15	0.85	0.11	0.12
Not Poor	0.27	0.26	0.87	0.12	0.11
Poor	0.73	0.74	0.92	0.05	0.07
Low Ach.	0.33	0.33	0.92	0.00	0.08
Middle Ach.	0.34	0.33	0.90	0.00	0.09
High Ach.	0.34	0.34	0.91	0.19	0.08
Brooklyn	0.30	0.31	0.93	0.08	0.06
Manhattan	0.10	0.10	0.90	0.11	0.09
Queens	0.31	0.31	0.92	0.07	0.07
Staten Island	0.07	0.06	0.88	0.06	0.10
Bronx	0.22	0.22	0.87	0.02	0.12
N	539,890	490,618			

Notes: Contains all applicants in the sample from 2011-12 to 2019-20. Public NYC refers to enrollment at a non-charter public high school (including Exam / Audition). Opt out refers to applicants who do not enroll at any non-charter NYC public high school. Enrollment is measured only in the year following application.

Table 3: School-level demographic characteristics of New York City high school programs, by theme

Theme	Total Enrollment	Prop. Poor	Prop. ELL	Prop. Low Baseline Scores	Prop. High Baseline Scores
Arts and Design	1209	0.74	0.09	0.38	0.26
Business, Hospitality, and Services	1408	0.78	0.13	0.45	0.17
Computing and Engineering	1046	0.78	0.11	0.42	0.22
Humanities and Interdisciplinary	984	0.76	0.12	0.39	0.25
Media	1324	0.72	0.12	0.39	0.26
Military and Law Enforcement	1559	0.75	0.08	0.46	0.19
Multicultural	592	0.86	0.52	0.62	0.11
Public Service, Law, and Social Justice	1330	0.78	0.10	0.41	0.22
Science	1312	0.77	0.11	0.35	0.29
All	1150	0.76	0.12	0.40	0.25

Notes: Program descriptives for all 1075 programs offered from 2012-13 through 2019-20. School-level data from the first year a program appears is used.

Table 4: School-level graduation outcomes of New York City high school programs, by theme

Theme	Prop. 6-year HS Grad	Prop. CTE Endorsement	Prop. 3+/4+ on AP/IB	Prop. Any College	Prop. In-State 4-year Public
Arts and Design	0.78	0.06	0.11	0.61	0.10
Business, Hospitality, and Services	0.73	0.07	0.08	0.57	0.08
Computing and Engineering	0.73	0.18	0.09	0.57	0.08
Humanities and Interdisciplinary	0.78	0.00	0.14	0.66	0.11
Media	0.81	0.06	0.12	0.63	0.09
Military and Law Enforcement	0.75	0.02	0.11	0.57	0.08
Multicultural	0.76	0.00	0.15	0.60	0.07
Public Service, Law, and Social Justice	0.77	0.00	0.11	0.65	0.10
Science	0.79	0.02	0.14	0.66	0.10
All	0.78	0.05	0.13	0.63	0.10

Notes: Program descriptives for all 1075 programs offered from 2012-13 through 2019-20. School-level data from the first year a program appears is used. For graduation and postsecondary outcomes, the denominator is equal to the cohort of students who entered high school 6 years earlier.

Table 5: Residential and school segregation in New York City high schools

Group	City Share HS Applicants	Segregation Indices, Relative to City Population				
		Borough SI	Subdistrict SI	Census Tract SI	Assigned School SI	School SI
Asian	0.17	0.07	0.17	0.25	0.21	0.25
Black	0.27	0.04	0.27	0.32	0.23	0.25
Hispanic	0.41	0.08	0.21	0.22	0.21	0.22
White	0.15	0.11	0.18	0.30	0.26	0.23
Not Poor	0.27	0.03	0.06	0.11	0.10	0.08
Poor	0.73	0.02	0.04	0.06	0.06	0.07
High Ach.	0.34	0.03	0.07	0.08	0.27	0.29
Non-High Ach.	0.66	0.02	0.06	0.06	0.17	0.19

Notes: The first column presents the share of the model estimation applicant sample in each demographic group. The next three columns show residential segregation indices for the high school applicant population. The last two columns show school segregation indices for the high school applicant population, for assigned school and enrolled school. In all cases, the reference population is the city applicant population. Residential geographic unit and school segregation indices are computed according to 4.3, with same-group exposure computed within-cohort, and the mean taken over students in all years of the sample.

Table 6: Other parameters: taste shock variance, marginal cost mean, and marginal cost variance

Taste shock variance σ_ϵ	Marginal cost mean c	Marginal cost variance σ_ζ
7.514676	0.008628	0.000040
(0.012803)	(0.000019)	(0.000000)

Notes: Estimates for taste shock variance, marginal cost mean, and marginal cost variance. Standard errors in parentheses.

Table 7: All coefficient estimates $\alpha_{cell,theme}$

	Arts and Design	Business, Hospitality, and Services	Computing and Engineering	Media	Military and Law Enforcement	Multi-cultural	Public Service, Law, and Social Justice	Science
Asian, non-low-income, high achievement	-2.86	-1.32	-0.09	-1.46	-5.53	-1.31	-1.75	0.04
	(0.18)	(0.30)	(0.17)	(0.30)	(0.43)	(0.46)	(0.19)	(0.11)
Asian, non-low-income, non-high achievement	-2.30	-0.63	-0.21	-0.93	-3.00	1.08	-1.06	-0.02
	(0.24)	(0.25)	(0.17)	(0.38)	(0.60)	(0.22)	(0.27)	(0.22)
Asian, low-income, high achievement	-3.19	-1.07	-0.02	-2.07	-4.66	-0.38	-1.09	0.26
	(0.17)	(0.13)	(0.15)	(0.22)	(0.23)	(0.19)	(0.16)	(0.08)
Asian, low-income, non-high achievement	-2.66	-0.56	-0.07	-1.13	-2.81	0.84	-0.90	0.01
	(0.12)	(0.08)	(0.10)	(0.19)	(0.18)	(0.10)	(0.15)	(0.08)
Black, non-low-income, high achievement	-1.29	-1.23	-0.40	-1.06	-3.75	-0.81	-0.06	0.50
	(0.20)	(0.27)	(0.19)	(0.30)	(0.49)	(0.32)	(0.19)	(0.13)
Black, non-low-income, non-high achievement	-0.89	-0.55	-0.24	-0.78	-1.60	-0.74	-0.00	0.06
	(0.07)	(0.09)	(0.07)	(0.15)	(0.15)	(0.12)	(0.08)	(0.08)
Black, low-income, high achievement	-1.08	-0.92	-0.13	-0.74	-2.21	-1.00	0.08	0.64
	(0.11)	(0.14)	(0.09)	(0.17)	(0.17)	(0.19)	(0.11)	(0.08)
Black, low-income, non-high achievement	-0.79	-0.37	-0.23	-0.65	-0.99	-0.73	0.17	0.19
	(0.04)	(0.04)	(0.04)	(0.06)	(0.10)	(0.07)	(0.04)	(0.03)
Hispanic, non-low-income, high achievement	-1.03	-0.83	-0.35	-0.85	-3.43	-0.84	-0.38	0.01
	(0.17)	(0.27)	(0.15)	(0.33)	(0.32)	(0.33)	(0.24)	(0.18)
Hispanic, non-low-income, non-high achievement	-1.02	-0.50	-0.35	-0.44	-2.29	-0.02	-0.15	-0.43
	(0.08)	(0.11)	(0.09)	(0.11)	(0.18)	(0.10)	(0.09)	(0.10)
Hispanic, low-income, high achievement	-1.45	-0.81	-0.33	-1.07	-2.69	-0.56	-0.36	-0.01
	(0.09)	(0.15)	(0.08)	(0.15)	(0.13)	(0.10)	(0.08)	(0.08)
Hispanic, low-income, non-high achievement	-1.22	-0.51	-0.44	-0.74	-1.56	-0.16	-0.18	-0.28
	(0.05)	(0.04)	(0.03)	(0.04)	(0.10)	(0.06)	(0.03)	(0.03)
White, non-low-income, high achievement	-1.71	-1.74	-0.57	-1.33	-5.34	-1.69	-1.57	-0.64
	(0.22)	(0.23)	(0.18)	(0.48)	(0.54)	(0.28)	(0.17)	(0.18)
White, non-low-income, non-high achievement	-2.35	-0.80	-0.60	-0.64	-4.19	-0.87	-0.72	-0.82
	(0.18)	(0.22)	(0.19)	(0.31)	(0.45)	(0.39)	(0.23)	(0.15)
White, low-income, high achievement	-1.93	-1.29	-0.56	-0.54	-4.61	-0.70	-0.52	0.39
	(0.23)	(0.31)	(0.30)	(0.28)	(0.59)	(0.34)	(0.17)	(0.17)
White, low-income, non-high achievement	-2.01	-0.75	-0.87	-0.53	-3.31	0.26	-0.55	-0.53
	(0.12)	(0.13)	(0.18)	(0.11)	(0.32)	(0.15)	(0.13)	(0.09)

Notes: Estimates for all theme preference coefficients for each demographic cell, relative to Humanities and Interdisciplinary. Standard errors in parentheses.

Table 8: Top three favorite themes

Cell	1st favorite theme	2nd favorite theme	3rd favorite theme
Asian, non-low-income, high achievement	Science	Humanities and Interdisciplinary	Computing and Engineering
Asian, non-low-income, non-high achievement	Multicultural	Humanities and Interdisciplinary	Science
Asian, low-income, high achievement	Science	Humanities and Interdisciplinary	Computing and Engineering
Asian, low-income, non-high achievement	Multicultural	Science	Humanities and Interdisciplinary
Black, non-low-income, high achievement	Science	Humanities and Interdisciplinary	Public Service, Law, and Social Justice
Black, non-low-income, non-high achievement	Science	Humanities and Interdisciplinary	Public Service, Law, and Social Justice
Black, low-income, high achievement	Science	Public Service, Law, and Social Justice	Humanities and Interdisciplinary
Black, low-income, non-high achievement	Science	Public Service, Law, and Social Justice	Humanities and Interdisciplinary
Hispanic, non-low-income, high achievement	Science	Humanities and Interdisciplinary	Computing and Engineering
Hispanic, non-low-income, non-high achievement	Humanities and Interdisciplinary	Multicultural	Public Service, Law, and Social Justice
Hispanic, low-income, high achievement	Humanities and Interdisciplinary	Science	Computing and Engineering
Hispanic, low-income, non-high achievement	Humanities and Interdisciplinary	Multicultural	Public Service, Law, and Social Justice
White, non-low-income, high achievement	Humanities and Interdisciplinary	Computing and Engineering	Science
White, non-low-income, non-high achievement	Humanities and Interdisciplinary	Computing and Engineering	Media
White, low-income, high achievement	Science	Humanities and Interdisciplinary	Public Service, Law, and Social Justice
White, low-income, non-high achievement	Multicultural	Humanities and Interdisciplinary	Media

Notes: For each demographic cell, the top three preferred themes based on theme coefficient estimates.

Table 9: Individual unobserved theme preference covariance matrix $\mathbf{\Sigma}_\gamma$

	Arts and Design	Business, Hospitality, and Services	Computing and Engineering	Humanities and Interdisciplinary	Media	Military and Law Enforcement	Multi-cultural	Public Service, Law, and Social Justice	Science
Arts and Design	2.37	0.09	0.09	0.50	0.30	-2.72	-0.60	-0.10	0.19
	(0.06)	(0.02)	(0.03)	(0.03)	(0.02)	(0.14)	(0.04)	(0.02)	(0.03)
Business, Hospitality, and Services	0.09	0.61	0.20	0.35	0.02	-1.71	-0.02	0.23	0.28
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.07)	(0.02)	(0.01)	(0.02)
Computing and Engineering	0.09	0.20	1.10	0.44	0.11	-2.37	-0.35	0.28	0.58
	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.09)	(0.02)	(0.01)	(0.02)
Humanities and Interdisciplinary	0.50	0.35	0.44	1.25	0.06	-3.45	-0.10	0.16	0.90
	(0.03)	(0.02)	(0.02)	(0.03)	(0.02)	(0.11)	(0.02)	(0.02)	(0.02)
Media	0.30	0.02	0.11	0.06	0.53	-0.95	-0.11	0.04	0.02
	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.07)	(0.02)	(0.01)	(0.02)
Military and Law Enforcement	-2.72	-1.71	-2.37	-3.45	-0.95	14.79	0.21	-1.09	-3.07
	(0.14)	(0.07)	(0.09)	(0.11)	(0.07)	(0.54)	(0.09)	(0.07)	(0.10)
Multicultural	-0.60	-0.02	-0.35	-0.10	-0.11	0.21	1.33	-0.24	-0.15
	(0.04)	(0.02)	(0.02)	(0.02)	(0.02)	(0.09)	(0.04)	(0.02)	(0.02)
Public Service, Law, and Social Justice	-0.10	0.23	0.28	0.16	0.04	-1.09	-0.24	0.50	0.27
	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)	(0.07)	(0.02)	(0.01)	(0.01)
Science	0.19	0.28	0.58	0.90	0.02	-3.07	-0.15	0.27	1.09
	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.10)	(0.02)	(0.01)	(0.02)

Notes: Contains point estimates and standard errors for the covariance matrix, Σ_γ, of the unobserved mean-zero theme preference vector, $\boldsymbol{\gamma_{cell(i),ms(i),t(i)}}$.

Table 10: District exit in all-Humanities and Interdisciplinary Counterfactual

Group	Baseline District Exit	Sim. Baseline District Exit	Sim. Counterfactual District Exit
Asian	0.039	0.044	0.044
Black	0.093	0.101	0.103
Hispanic	0.090	0.101	0.101
White	0.154	0.176	0.171
Not Poor	0.130	0.147	0.143
Poor	0.076	0.084	0.085
High Ach.	0.093	0.103	0.101
Non-High Ach.	0.091	0.102	0.103

Notes: District exit refers to enrollment at a school at a school outside of the NYC centralized high school assignment system, and outside of the Exam / Audition category, at which I observe enrollment. The three columns show the district exit rate of each group, at baseline, in the baseline simulation, and the counterfactual simulation. These results are averaged over 5 iterations of simulation.

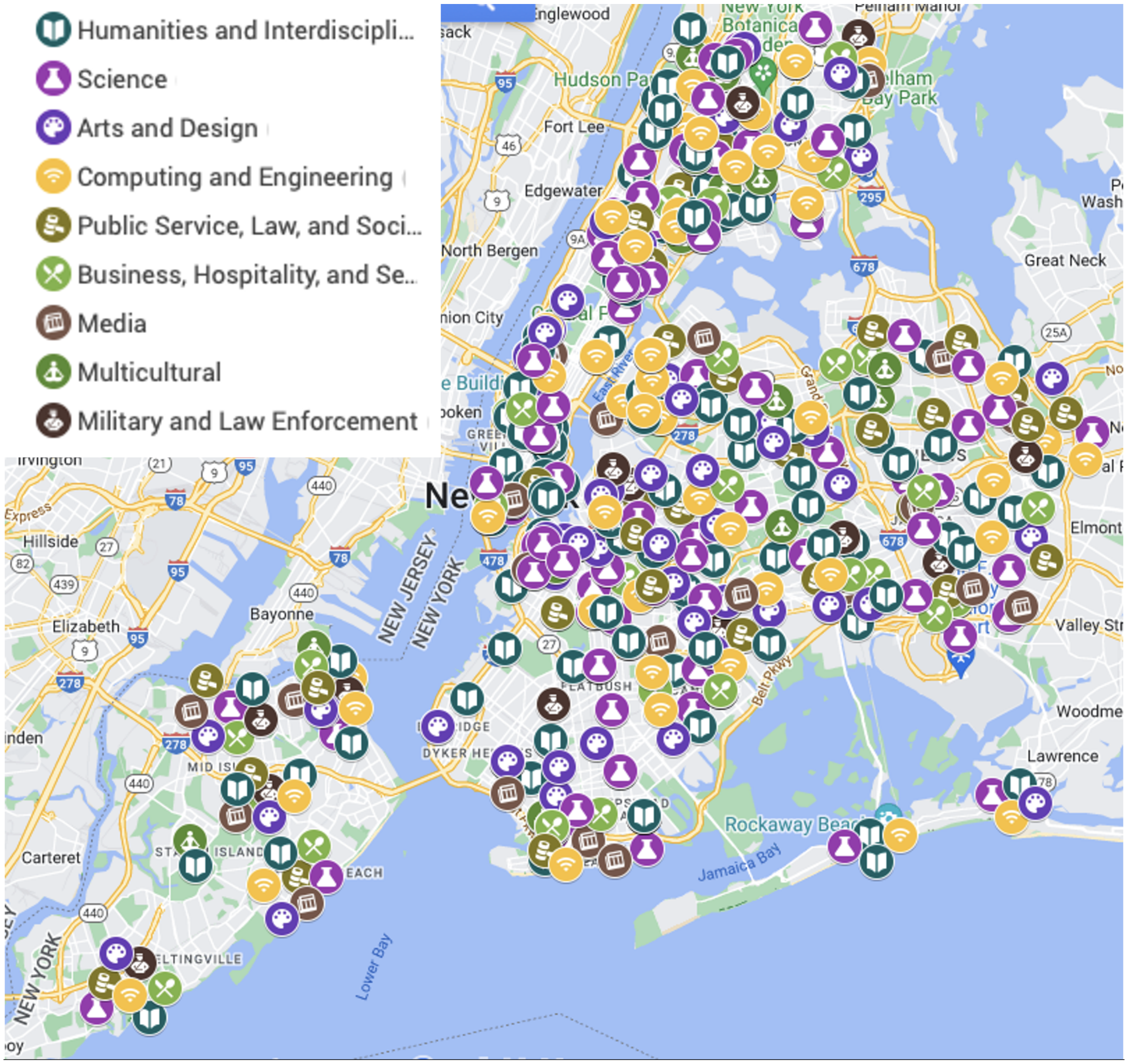
Table 11: School Segregation in all-Humanities and Interdisciplinary Counterfactual

Group	City-Relative Assigned HS SI		City-Relative Enrolled HS SI	
	Baseline	Counterfactual	Baseline	Counterfactual
Asian	0.216	0.210	0.250	0.245
Black	0.235	0.225	0.232	0.225
Hispanic	0.204	0.198	0.212	0.208
White	0.253	0.247	0.224	0.218
Not Poor	0.096	0.097	0.077	0.076
Poor	0.058	0.054	0.067	0.065
High Ach.	0.275	0.272	0.290	0.287
Non-High Ach.	0.168	0.160	0.185	0.182

Notes: The first two columns compare the assigned high school segregation in the simulated baseline and counterfactual. The last two columns compare enrolled high school segregation in the simulated baseline and counterfactual. Residential geographic unit and school segregation indices are computed according to 4.3, with same-group exposure computed within-cohort, and the mean taken over students in all years of the sample. These results are averaged over 5 iterations of simulation.

Figures

Figure 1: Distribution of themed programs across New York City



Notes: Locations of each program program offering from 2010-2019, plotted in Google Maps. Some programs are placed at a slight offset from their exact coordinates, so that multiple programs in the same location are visible.

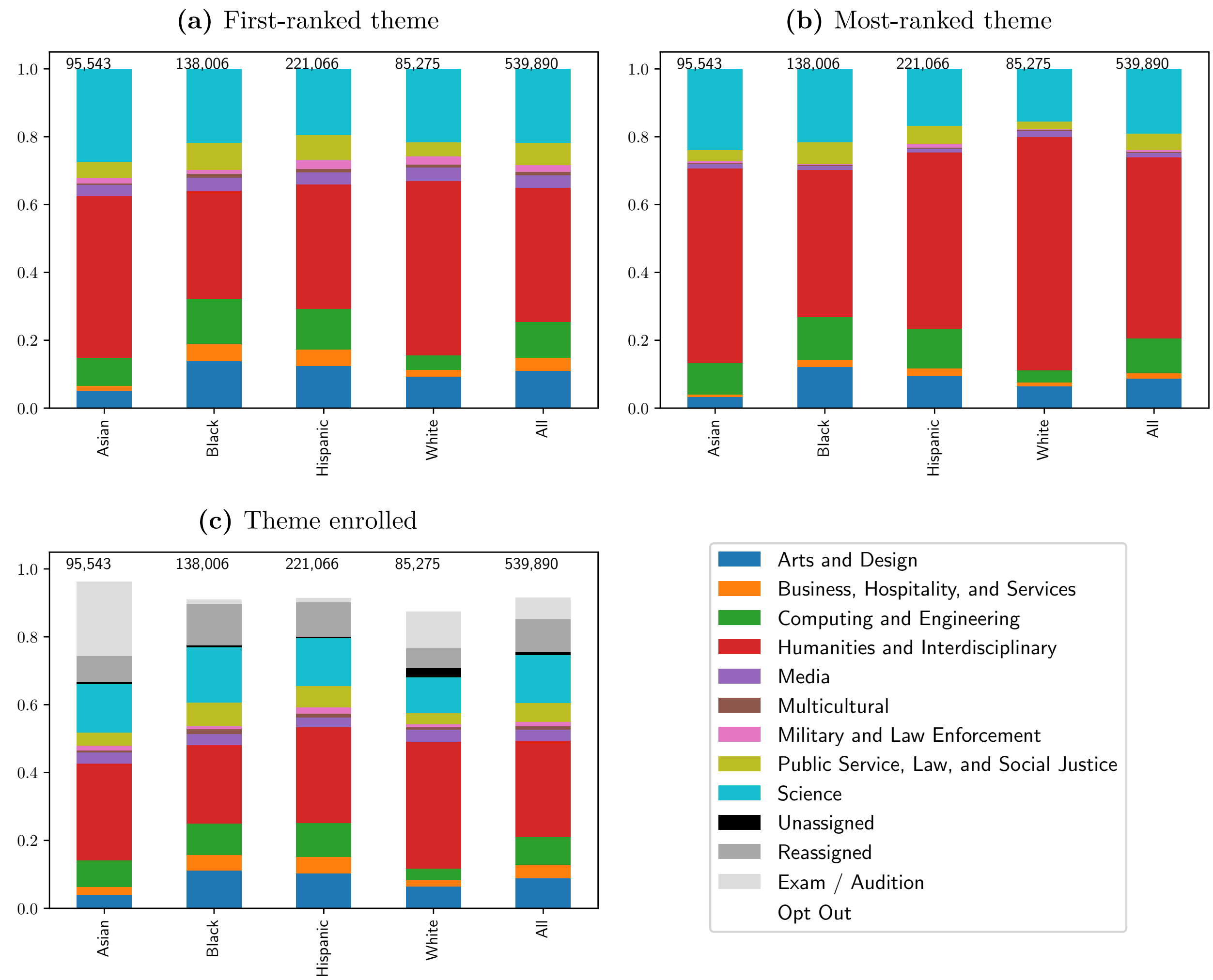


Figure 2: Application and enrollment behavior with respect to themes, by race

Notes: For each applicant in the sample from 2011-2019, I observe the theme they rank first, the theme they rank more than others (with ties broken by whichever theme was ranked earliest), and the theme they enrolled. I then plot the distribution of first ranked theme, most ranked theme, and theme enrolled, by race. All figures represent the same number of students; the empty space at the top of the bars in panel (c) represent the students who received a main round assignment but do not enroll at a New York City public high school. "Unassigned" refers to students who did not receive a main round assignment and do not enroll at a New York City public high school. "Reassigned" refers to students who enroll at a New York City public school that was not their main round assignment, and is not an Exam / Audition school.

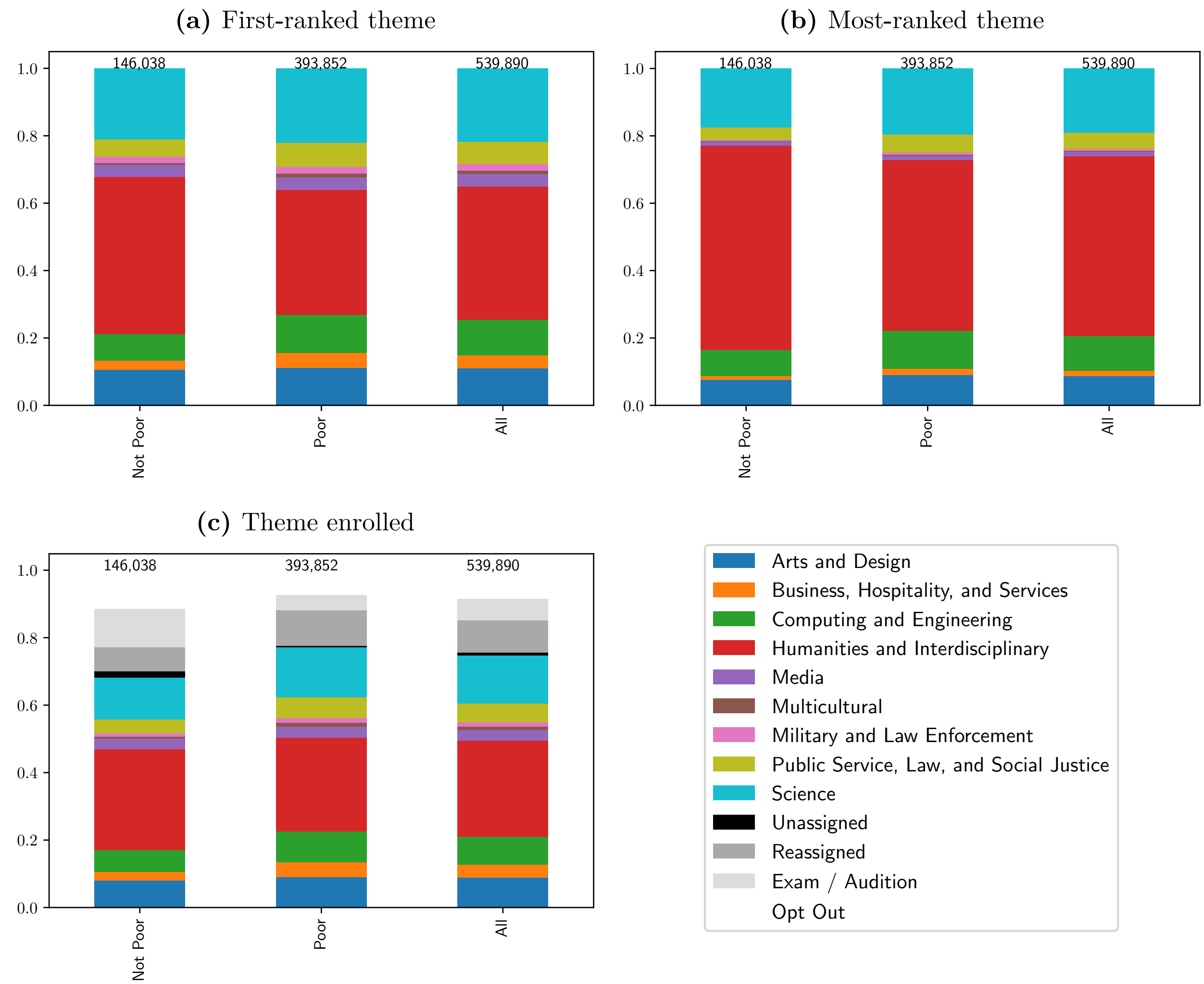


Figure 3: Application and enrollment behavior with respect to themes, by income

Notes: For each applicant in the sample from 2011-2019, I observe the theme they rank first, the theme they rank more than others (with ties broken by whichever theme was ranked earliest), and the theme they enrolled. I then plot the distribution of first ranked theme, most ranked theme, and theme enrolled, by income (measured by free- or reduced- price lunch eligibility). All figures represent the same number of students; the empty space at the top of the bars in panel (c) represent the students who received a main round assignment but do not enroll at a New York City public high school. "Unassigned" refers to students who did not receive a main round assignment and do not enroll at a New York City public high school. "Reassigned" refers to students who enroll at a New York City public school that was not their main round assignment, and is not an Exam / Audition school.

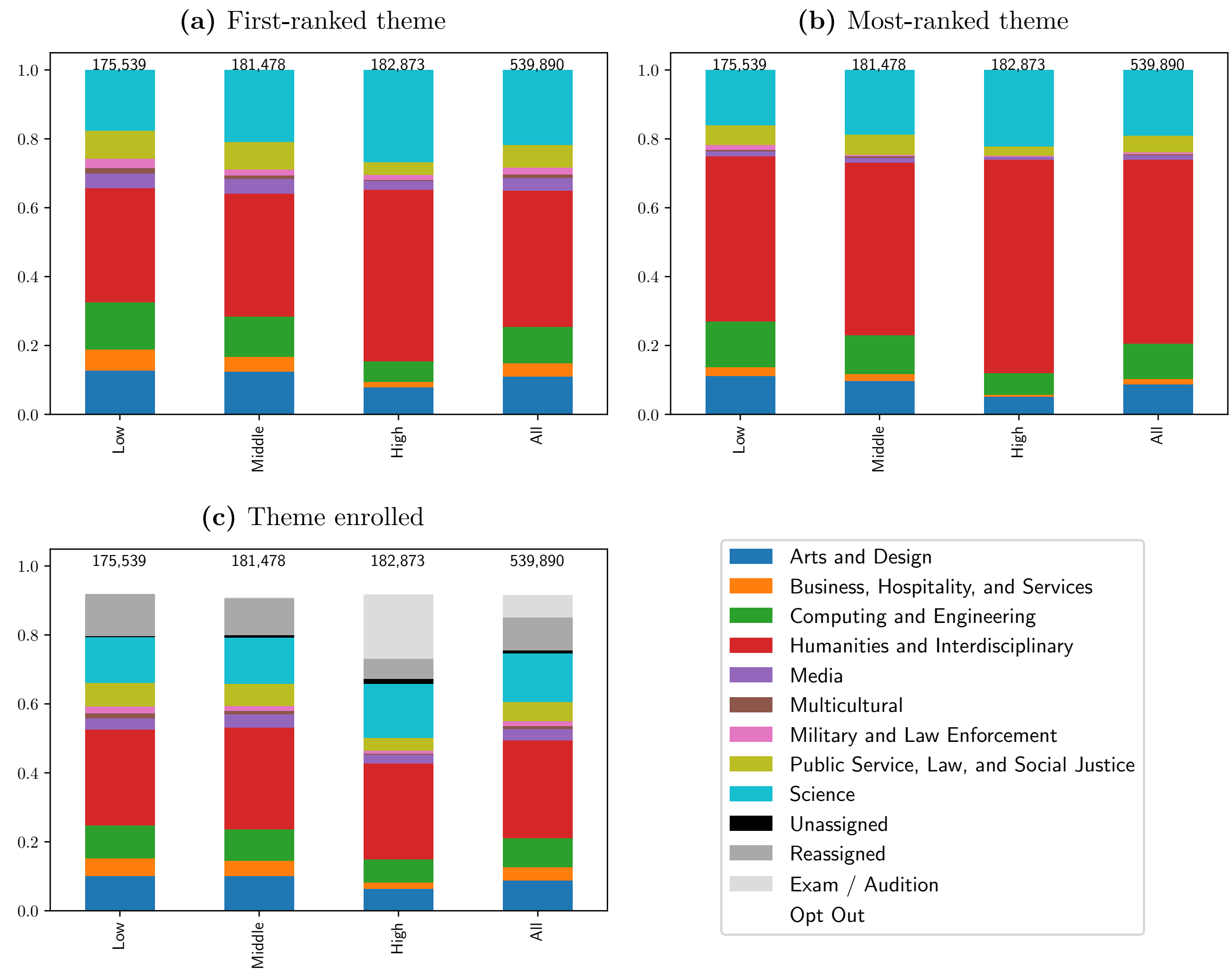
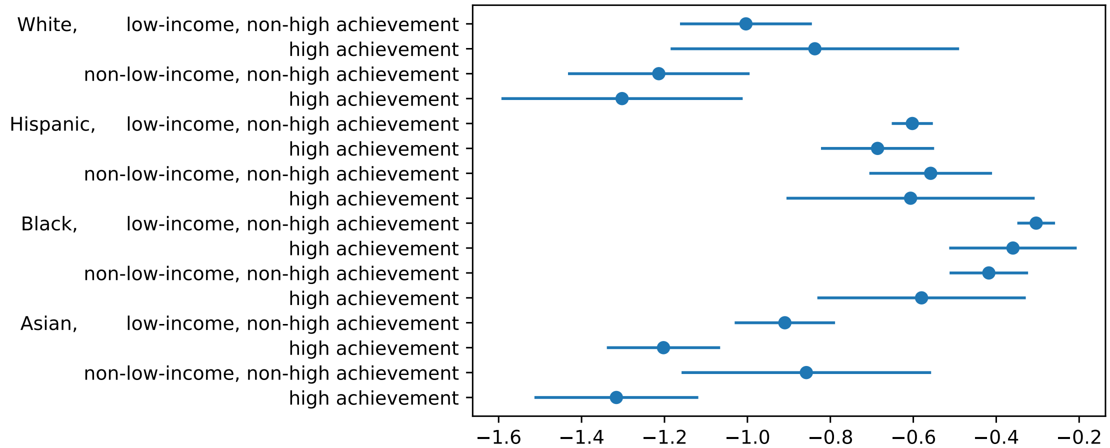


Figure 4: Application and enrollment behavior with respect to themes, by achievement

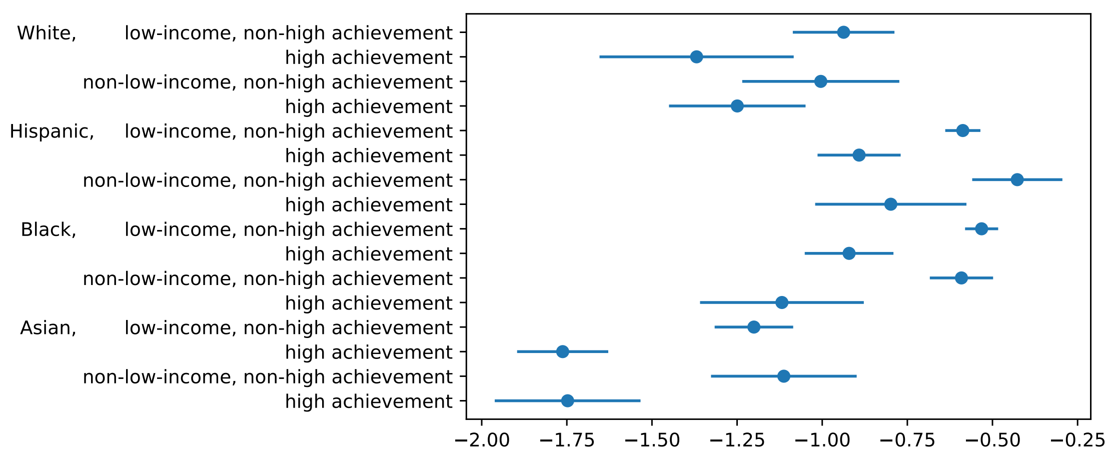
Notes: For each applicant in the sample from 2011-2019, I observe the theme they rank first, the theme they rank more than others (with ties broken by whichever theme was ranked earliest), and the theme they enrolled. I then plot the distribution of first ranked theme, most ranked theme, and theme enrolled, by baseline achievement. Baseline achievement is measured by percentile of 7th grade ELA and math test score, equally weighted, and split into the lowest third, middle third, and highest third. All figures represent the same number of students; the empty space at the top of the bars in panel (c) represent the students who received a main round assignment but do not enroll at a New York City public high school. "Unassigned" refers to students who did not receive a main round assignment and do not enroll at a New York City public high school. "Reassigned" refers to students who enroll at a New York City public school that was not their main round assignment, and is not an Exam / Audition school.

Figure 5: Preference for themed programs, relative to Humanities and Interdisciplinary ϕ_{cell}



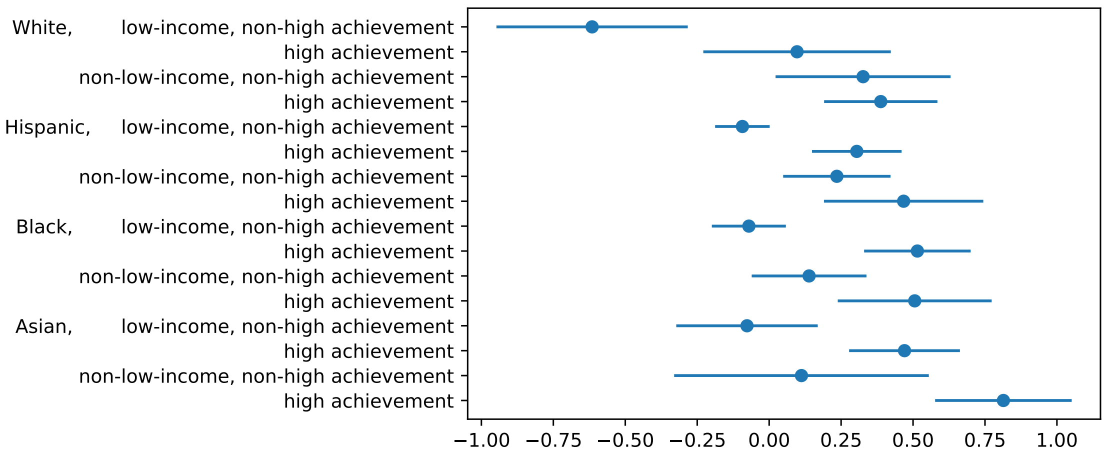
Notes: This figure shows 95% confidence intervals for estimated theme preference coefficients, ϕ_{cell}. The theme coefficient can be interpreted as the demographic cell's average preference for other themed programs, relative to the Humanities and Interdisciplinary theme.

Figure 6: Preference for themed programs, relative to Humanities and Interdisciplinary and Science ϕ_{cell}



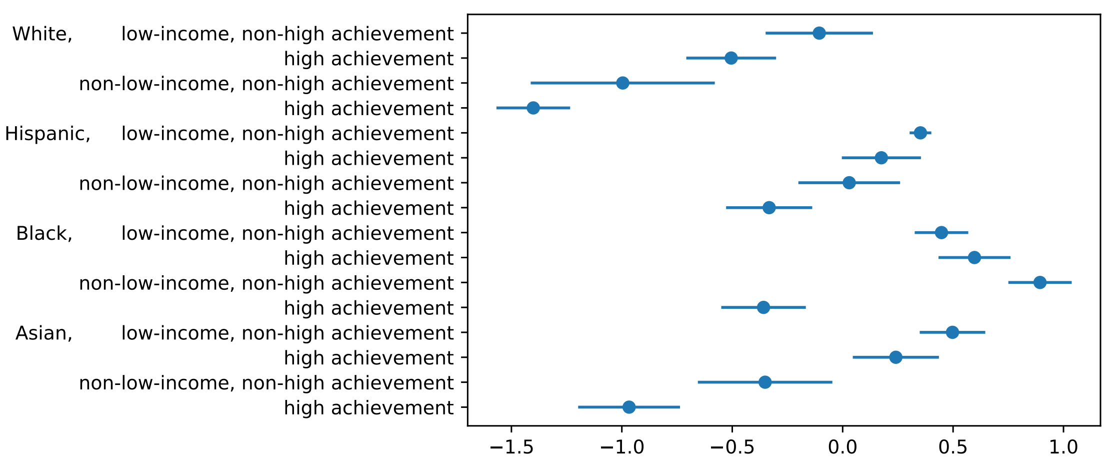
Notes: This figure shows 95% confidence intervals for estimated theme preference coefficients, ϕ_{cell}. The theme coefficient can be interpreted as the demographic cell's average preference for other themed programs, relative to Humanities and Interdisciplinary or Science themes.

Figure 7: Preference coefficients on lagged peer characteristics $\beta_{cell,\text{ prop. high-achieving}}$



Notes: This figure shows shows 95% confidence intervals for the coefficient on proportion of high baseline achievement peers in a program's last year enrolled cohort. If a program did not exist in the previous year, this value is imputed by the mean value for the program subdistrict.

Figure 8: Preference coefficients on lagged peer characteristics $\beta_{cell,\text{ prop. low-income}}$



Notes: This figure shows 95% confidence intervals for the coefficient on proportion of low-income peers in a program's last year enrolled cohort. If a program did not exist in the previous year, this value is imputed by the mean value for the program subdistrict.

Figure 9: General application patterns for real baseline, simulated baseline, and simulated counterfactual

(a) Mean change in applicant program choice set utilities, from simulated baseline to simulated counterfactual

(b) Mean list length

(c) Unassignment rate

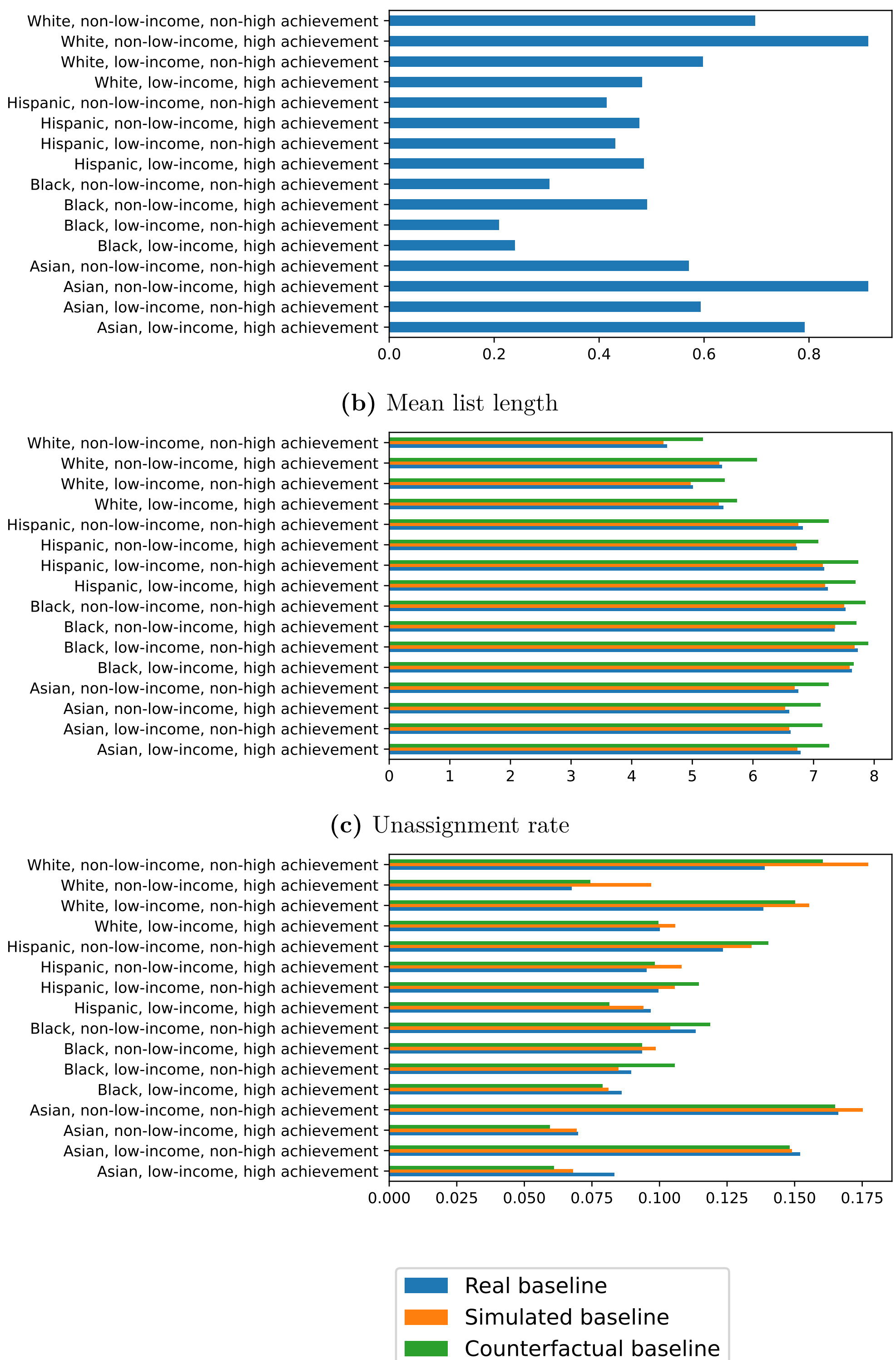
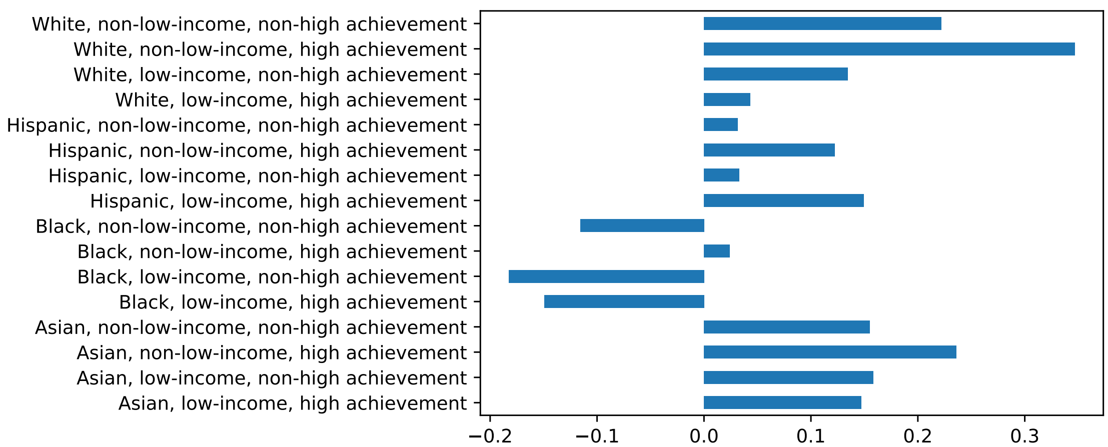
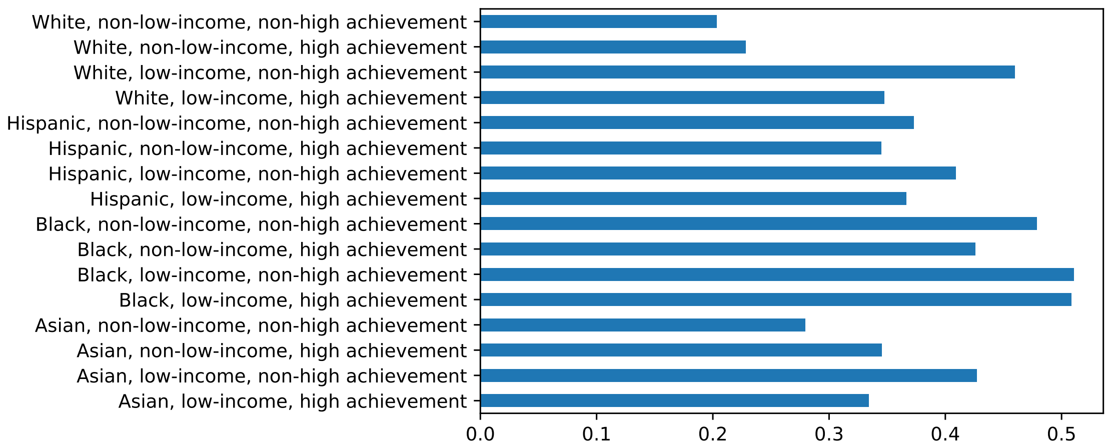


Figure 11: Mean change in assigned program utility from baseline simulation to counterfactual simulation



Notes: For each applicant, I subtract the utility of their assigned program in the simulated baseline from the utility of their assigned program in the simulated counterfactual. I plot the mean for each demographic group. These results are from only one simulation iteration, and will be updated after I run more iterations.

Figure 12: Proportion of applicants worse off in the counterfactual simulation than in the baseline simulation



Notes: For each applicant, I subtract the utility of their assigned program in the simulated baseline from the utility of their assigned program in the simulated counterfactual. I plot the proportion for whom the change is negative for each demographic group. These results are from only one simulation iteration, and will be updated after I run more iterations.

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APPENDIX

A Definition of program themes

After proposing this study to the New York City Department of Education, they suggested I propose and refine a categorization of high school programs to be used. I was guided by their advice to base the theme categories not solely on academic interest area, as assigned in the program directories, but also on the program name, which contains additional information on theme.

I began with the New York City Department of Education high school program directories for the years 2014-2020, from the NYC Open Data website. I divided these programs by curricular theme into nine categories:

- Arts and Design
- Business, Hospitality, and Services
- Computing and Engineering
- Humanities and Interdisciplinary
- Media
- Military and Law Enforecement
- Multicultural
- Public Service, Law, and Social Justice
- Science

My categorization is based on the interest area categorization in the directory, as well as the words contained in the program name.

A.1 Methodology

I began thinking about the program categorization with the DOE interest areas in mind. By looking at the programs within each interest area, I decided which interest areas my categorization should keep intact, and which interest areas my categorization should split. I kept the following interest areas intact, ultimately assigning all their programs to the same category:

Interest area in directory	**Category**
Culinary Arts	Business, Hospitality, and Services
Cosmetology	
Hospitality, Travel, and Tourism	
Computer Science & Technology	Computing and Engineering
Computer Science, Math & Technology	
Zoned	Humanities, Interdisciplinary, and General
Project-Based Learning	
JROTC	Military and Law Enforcement
Communications	Media
Film/Video	
Teaching	Public Service, Law, and Social Justice
Animal Science	Science
Environmental Science	Science
Health Professions	Science
Architecture	Arts and Design
Performing Arts	
Performing Arts/Visual Art & Design	
Visual Art & Design, and Performing Arts	
Visual Art & Design	

I decided to split the interest areas listed below, based on divisions between their programs that I observed. For each of these interest areas, I explain below why I thought it made sense to split up the programs into different categories, and the categorization rules:

- Humanities and Interdisciplinary
  - Within this interest area, I saw some programs that would seem to fit better under other already-defined categories, or under their own new category.
  - Program names including the words “International” (not “International Baccalaurate” or “International Marketing”), “Multicultural”, “Global”, “Diversity”, “Diplomacy”, or “World Cultures” were placed in Multicultural.
  - Program names including the words “Civil Rights”, “Leadership”, “Social”, “Public Service”, “Community Service”, “Human Rights”, “Civic”, or “Peace” were placed in Public Service, Law, and Social Justice.
  - Program names including the words “Enforcement”, “Safety”, “Crim”, or “Forensics” were placed in Military and Law Enforcement.
  - Program names including the words “Media” or “Communication” were placed in Media.
  - Program names including the words “Marketing” or “Sports Management” were placed in Business, Hospitality, and Services.

  - Program names including the word “Environmental” were placed in Science.
  - The remaining programs ended up in Humanities, Interdisciplinary, and General.

- Science and Math
  - Within this area, there were some programs that were related to computing or IT.
  - Program names including “Computer” or “Information Technology” were placed in Computing and Engineering
  - The remaining programs ended up in Science.

- Business
  - Within this area, there were some programs that were related to computing or IT.
  - Program names including “Computer” or “Information Technology” were placed in Computing and Engineering
  - The remaining programs ended up in Business, Hospitality, and Services.

- Law & Government
  - Within this area, some programs seemed more related to public safety, while others to social justice, activism, law, or government.
  - Program names including the words “Enforcement”, “Safety”, “Crim”, or “Forensics” were placed in Military and Law Enforcement.
  - The remaining programs ended up in Public Service, Law, and Social Justice.

B Computing public transit travel times

I compute the travel distance from the population-weighted centroid of each NYC residential census tract to each NYC high school [9].

I use OpenTripPlanner, an open-source project that enables analysis of transport networks (*OpenTripPlanner* 2023). Given publicly available formatted source files representing New York and New Jersey street maps and New York City public transit (including subway, bus, PATH, ferry, and more), it sets up a server on my computer that can handle walk and public transit routing requests.

For each census tract centroid – high school pair, I request the latest walk and public transit route from the centroid coordinates that arrives at the high school coordinates by 8 AM on Thursday, March 23 2023. The duration of this route is my travel time measure.

[9] New York City does not offer school buses for high school students. High school students who live more than half a mile away from their school are eligible for a free MetroCard allowing three trips a day on school days (NYC Department of Education 2023)

First, I use OTP 1.5 to get the times in batches for each school (batch travel times are not implemented for 2.3). Then, I fill in missing travel times with one-to-one calls to the OTP 2.3 server. OTP 2.3 uses an improved routing algorithm, but both 1.5 and 2.3 find the best route.

C Computing assignment probabililties

I follow the formulas in Idoux 2021 for computing assignment probabilities from admit probabilities in a way that captures interdependencies from the shared tiebreaker. I assume the applicant treats admissions events as independent across score programs, conditional on their information set. While this assumption may deviate slightly from belief consistency, screened programs place different weights on admit criteria, and can be subjective, limiting the extent to which these admit outcomes are actually interdependent.

Transform all applicant tiebreakers to within $[0, 1]$ The winners of the lottery to be those below the cutoff value.The probability that applicant i is assigned to score program j if ranked in slot k after R_{k-1} equals:

$$p_{ijk}(R_{k-1}) = q_{ij} \times \underbrace{(1 - \max_{sL^l:\{r_{is}<k\}} q_{is})}_{\substack{\mathbb{P}[\text{rejected from higher-ranked} \\ \text{lottery programs}]}} \underbrace{\prod_{sL^s:r_{is}<k} (1 - q_{is})}_{\substack{\mathbb{P}[\text{rejected from higher-ranked} \\ \text{score programs}]}} \tag{10}$$

The probability that applicant i is assigned to lottery program j if ranked in slot k after R_{k-1} equals:

$$p_{ijk}(R_{k-1}) = \underbrace{\max\left[0, q_{ij} - \max_{sL^l:r_{is}<k} q_{is}\right]}_{\substack{\mathbb{P}[\text{rejected from higher-ranked} \\ \text{lottery programs and} \\ \text{accepted to } j]}} \underbrace{\prod_{sL^s:\{r_{is}<k\}} (1 - q_{is})}_{\substack{\mathbb{P}[\text{rejected from higher-ranked} \\ \text{score programs}]}} \tag{11}$$

D Bounds on costs and utilities implied by Assumption 1 (Proposition 1, Idoux 2021)

I list the bounds on costs and utilities, derived in Idoux 2021 as implications of Assumption 1. I use these bounds for estimation.

1. For all applicants i, for any program j listed k^{th} in their list R_i, $p_{ijk}v_{ij} \geq c_i$. That is, applicant i's expected utility of ranking j in slot k is weakly higher than their marginal application cost.

2. For all applicants i who ranked $listlength$ programs, for any unlisted program j, $p_{ij,listlength}v_{ij} < c_i$. That is, applicant i's expected utility of adding j at the end of their list is less than their marginal application cost.

3. Each program j in R_i has indirect utility (v_{ij}) lower than all programs listed above it and higher than all those listed below it.

4. For all applicants i, for any program j listed in slot k, and any unlisted program j' such that $p_{ij'k}v_{ij} \geq c_i$, $v_{ij} \geq v_{ij'}$. That is, the program that applicant i chose for slot k must have indirect utility weakly higher than the indirect utilities of all other programs in their consideration set for slot k.

E Gibbs Sampler

The indirect utility for applicant i of assignment to program j is v_{ij}, where $\boldsymbol{\gamma_{cell(i),ms(i),t(i)}} \sim MultivariateNormal(\mathbf{0}, \boldsymbol{\Sigma_\gamma})$, and $\epsilon_{ij} \sim Normal(0, \sigma_\epsilon)$.

$$v_{ij} := \delta_{cell(i),j} + \theta_{boro(i),boro(j)} + \mathbf{X_{j,t(i)-1}}\boldsymbol{\beta_{cell(i)}} - d_{ij} + \gamma_{cell(i),ms(i),t(i),theme(j)} + \epsilon_{ij} \tag{12}$$

and the per-program application cost is parameterized as c_i, where $\zeta_i \sim TruncatedNormal(-c, \infty, 0, \sigma_\zeta)$

$$c_i = c + \zeta_i \tag{13}$$

My parameters of interest include program-by-cell fixed effects $\{\delta_{cell,j}\}$, residential borough-by-school borough fixed effects $\{\theta_{boro,boro}\}$, coefficients on time-varying lagged peer characteristics $\{\boldsymbol{\beta_{cell}}\}$, unobserved theme preference covariance matrix $\boldsymbol{\Sigma_\gamma}$, taste shock variance σ_ϵ, mean marginal cost c, and marginal cost variance σ_ζ. I estimate these parameters with data augmentation, drawing values of $\{v_{ij}\}$, $\{c_i\}$ and $\{\boldsymbol{\gamma_{cell,ms,t}}\}$ from their conditional posteriors at each step, because it is easier to sample my parameters of interest conditional on them. I normalize scale by setting the coefficient on travel time to -1, and I normalize location by setting the outside option indirect utility for each individual to ϵ_{i0}.

I initialize $\{\delta_{cell,j}\}$, $\{\theta_{boro,boro}\}$, and $\{\boldsymbol{\beta_{cell}}\}$ to 0, and $\boldsymbol{\Sigma_\gamma}$, σ_ϵ, and σ_ζ to 100. I initialize $\{c_i\}$, and $\{\gamma_{cell,ms,t,theme}\}$ to 0. I initialize indirect utilities $\{v_{ij}\}$ to values consistent with the bounds implied by the limited rationality assumption.

The parameter priors are $\delta_{cell,j} \sim N(\mu^0_{\delta_{cell,j}}, V^0_{\delta_{cell,j}})$, $\theta_{boro,boro} \sim N(\mu^0_{\theta_{boro,boro}}, V^0_{\theta_{boro,boro}})$, $\boldsymbol{\beta_{cell}} \sim N(\boldsymbol{\mu^0_{\beta_{cell}}}, \mathbf{V^0_{\beta_{cell}}})$, $c \sim TruncatedNormal(0, \infty, \mu^0_c, V^0_c)$, $\boldsymbol{\Sigma_\gamma} \sim InverseWishart(v^0_{\Sigma_\gamma}, \mathbf{V^0_{\Sigma_\gamma}})$, $\sigma_\epsilon \sim InverseWishart(v^0_{\sigma_\epsilon}, V^0_{\sigma_\epsilon})$, and $\sigma_\zeta \sim InverseWishart(v^0_{\sigma_\zeta}, V^0_{\sigma_\zeta})$. I choose fairly diffuse priors, to minimize their contribution to the results: $\mu^0_{\delta_{cell,j}} = 0$, $V^0_{\delta_{cell,j}} = 100$, $\mu^0_{\theta_{boro,boro}} = 0$, $V^0_{\theta_{boro,boro}} = 100$, $\boldsymbol{\mu^0_{\beta_{cell}}} = \left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right)$, $\mathbf{V^0_{\beta_{cell}}} = \left(\begin{smallmatrix} 100 & 0 \\ 0 & 100 \end{smallmatrix}\right)$, $\mu^0_c = 0$, $V^0_c = 100$, $v^0_{\Sigma_\gamma} = 12$, $\mathbf{V^0_{\Sigma_\gamma}} = 12 \times \boldsymbol{I_9}$, $v^0_{\sigma_\epsilon} = 3$, $V^0_{\sigma_\epsilon} = 3$, $v^0_{\sigma_\zeta} = 3$, and $V^0_{\sigma_\zeta} = 3$.

On each iteration, I perform the following steps in order:

1. Sample $\{\delta_{cell,j}\}$ from $N(\mu^1_{\delta_{cell,j}}, V^1_{\delta_{cell,j}})$.
    - $R_{ij} = v_{ij} + d_{ij} - \theta_{boro(i),boro(j)} - \mathbf{X_{j,t(i)-1}}\boldsymbol{\beta_{cell(i)}} - \gamma_{cell(i),ms(i),t(i),theme(j)}$

- $V^1_{\delta_{cell,j}} = \frac{\sum_{\text{i,j}} \mathbb{I}[cell(\text{i})=cell,\text{j}=j]}{\sigma_\epsilon} + \frac{1}{V^0_{\delta_{cell,j}}}$
- $\mu^1_{\delta_{cell,j}} = V^1_{\delta_{cell,j}} \times (\frac{\sum_{\text{i,j}} R_{ij}\mathbb{I}[cell(\text{i})=cell,\text{j}=j]}{\sigma_\epsilon} + \frac{\mu^0_{\delta_{cell,j}}}{V^0_{\delta_{cell,j}}})$

2. Sample $\{\boldsymbol{\beta_{cell}}\}$ from $N(\boldsymbol{\mu}^{\mathbf{1}}_{\boldsymbol{\beta}_{\mathbf{cell}}}, \mathbf{V}^{\mathbf{1}}_{\boldsymbol{\beta}_{\mathbf{cell}}})$.
   - $R_{ij} = v_{ij} + d_{ij} - \delta_{cell(i),j} - \theta_{boro(i),boro(j)} - \gamma_{cell(i),ms(i),t(i),theme(j)}$
   - For each cell, let $\mathbf{X}(\mathbf{cell})$ and $\mathbf{R}(\mathbf{cell})$ equal the stacked matrix of observations of $\mathbf{X}_{\mathbf{j},\mathbf{t(i)-1}}$ and R_{ij} for applicants in the cell, respectively.
   - $\mathbf{V}^{\mathbf{1}}_{\beta_{\mathbf{cell}}} = (\frac{\mathbf{(X(cell)'X(cell)}}{\sigma_\epsilon} + (\mathbf{V}^{\mathbf{0}}_{\beta_{\mathbf{cell}}})^{-1})^{-1}$
   - $\boldsymbol{\mu}^{\mathbf{1}}_{\boldsymbol{\beta_{cell}}} = \mathbf{V}^{\mathbf{1}}_{\beta_{\mathbf{cell(i)}}}(\frac{\mathbf{(X(cell)'R(cell)}}{\sigma_\epsilon} + (\mathbf{V}^{\mathbf{0}}_{\beta_{\mathbf{cell}}})^{-1}\boldsymbol{\mu}^{\mathbf{0}}_{\boldsymbol{\beta_{cell}}}$

3. Sample $\{\boldsymbol{\gamma_{cell,ms,t}}\}$ from $N(\boldsymbol{\mu}^{\mathbf{1}}_{\boldsymbol{\gamma_{cell,ms,t,theme}}}, \mathbf{V}^{\mathbf{1}}_{\gamma_{\mathbf{cell,ms,t}}})$.
   - $R_{ij} = v_{ij} + d_{ij} - \delta_{cell(i),j} - \theta_{boro(i),boro(j)} - \mathbf{X}_{\mathbf{j},\mathbf{t(i)-1}}\boldsymbol{\beta_{cell(i)}}$. Only the differences between preference for each theme are identified, so normalize R_{ij} so that mean residuals by theme add up to 0 within-cell-by-middle school cohort.
   - For each cell, middle school, and year combination, let $\boldsymbol{N}_{cell,ms,t}$ equal a vector where each element equals the number of observations within each theme. Let $\boldsymbol{R}_{cell,ms,t}$ equal a vector where each element equals the sum of residuals R_{ij} within each theme.
   - $\mathbf{V}^{\mathbf{1}}_{\gamma_{\mathbf{cell,ms,t}}} = (\frac{\mathbf{diag}(\boldsymbol{N}_{cell,ms,t})}{\sigma_\epsilon} + \boldsymbol{\Sigma}_\gamma{}^{-1})^{-1}$
   - $\mu^{\mathbf{1}}_{\gamma_{\mathbf{cell,ms,t}}} = \mathbf{V}^{\mathbf{1}}_{\gamma_{\mathbf{cell,ms,t}}} \times \frac{\boldsymbol{R}_{cell,ms,t}}{\sigma_\epsilon}$

4. Sample $\boldsymbol{\Sigma}_\gamma$ from $InverseWishart(\boldsymbol{N}_{cell,ms,t} + v^0_{\Sigma_\gamma}, \boldsymbol{\gamma'\gamma} + V_{\Sigma^0_\gamma})$, where $\boldsymbol{\gamma}$ is a stacked matrix of all $\boldsymbol{\gamma_{cell,ms,t}}$.

5. Sample $\{\theta_{boro,boro}\}$ from $N(\mu^1_{\theta_{boro,boro}}, V^1_{\theta_{boro,boro}})$.
   - $R_{ij} = v_{ij} + d_{ij} - \delta_{cell(i),j} - \mathbf{X}_{\mathbf{j},\mathbf{t(i)-1}}\boldsymbol{\beta_{cell(i)}} - \gamma_{cell(i),ms(i),t(i),theme(j)}$. Only the differences between preferences for each high school borough are identified, so normalize R_{ij} so that mean residuals by high school borough add up to 0 within residential borough.
   - $V^1_{\theta_{rb,hb}} = \frac{\sum_{\text{i,j}} \mathbb{I}[\text{residential boro(i)}=rb, \text{ high school boro(j)}=hb]}{\sigma_\epsilon} + \frac{1}{V^0_{\theta_{rb,hb}}}$
   - $\mu^1_{\theta_{rb,hb}} = V^1_{\theta_{rb,hb}} \times (\frac{\sum_{\text{i,j}} \mathbb{I}[\text{residential boro(i)}=rb, \text{ high school boro(j)}=hb]R_{ij}}{\sigma_\epsilon} + \frac{\mu^0_{\theta_{rb,hb}}}{V^0_{\theta_{rb,hb}}})$

6. Sample σ_ϵ from $InverseWishart(N \times |CS(i)| + v^0_{\sigma_\epsilon}, \epsilon'\epsilon + V^0_{\sigma_\epsilon})$
   - N is the number of applicants, $|CS(i)| = 30$ is number of programs in each applicant's program choice set
   - $\epsilon_{ij} = v_{ij} + d_{ij} - \delta_{cell(i),j} - \theta_{boro(i),boro(j)} - \mathbf{X}_{\mathbf{j},\mathbf{t(i)-1}}\boldsymbol{\beta_{cell(i)}} - \gamma_{cell(i),ms(i),t(i),theme(j)}$

7. Sample $\{v_{ij}\}$ from $TN(lower_{ij}, upper_{ij}, mean_{v_{ij}}, \sigma_\epsilon)$, where $lower_{ij}$ and $upper_{ij}$ are based on the bounds in Appendix D.

- $mean_{v_{ij}} = \delta_{cell(i),j} + \theta_{boro(i),boro(j)} + \mathbf{X_{j,t(i)-1}}\boldsymbol{\beta_{cell(i)}} - d_{ij} + \gamma_{cell(i),ms(i),t(i),theme(j)}$

8. Sample c from $TN(\mu_c^1, \sigma_c^1, 0, \infty)$.
   - $\sigma_c^1 = (N/\sigma_\zeta + 1/\sigma_c^0)^{-1}$
   - $\mu_c^1 = \sigma_c^1((\sum_i c_i)/\sigma_\zeta + \mu_c^0/\sigma_c^0)$
9. Sample σ_ζ from $IW(N + v_{\sigma_\zeta}^0, \sum_i (c_i^0 - c^1)^2 + V_{\sigma_\zeta}^0)$
10. Sample $\{c_i\}$ from $TN(lower_{ij}, upper_{ij}, c, \sigma_\zeta)$, where $lower_{ij}$ and $upper_{ij}$ are based on the bounds in Appendix D.

After discarding the first 2000 iterations, I used the subsequent 3100 iterations for my estimates.