Q// A continuous random variable X has the probability density function (PDF)

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) verify that $f_X(x)$ is a valid PDF.

(b) Find CDF, $F_X(x)$

(c) compute $P(0.25 \le X \le 0.75)$

(d) Find $P(X > 0.5)$

Ans (a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (validity condition)

Now $\int_0^1 2x\,dx = 2\left[\frac{x^2}{2}\right]_0^1 = 1^2 = 1$ (valid)

(b) $F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(x)\,dx$

$$= \begin{cases} 0, & x < 0 \\ \int_0^x 2x\,dx, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

$$= \begin{cases} 0, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

(c). $P(0.25 \le X \le 0.75) = F_X(0.75) - F_X(0.25)$

$= 0.75^2 - 0.25^2 = 0.5$

(d) $P(X > 0.5) = 1 - P(X \le 0.5) = 1 - F(0.5)$

$= 1 - 0.5^2 = 1 - 0.25$

$= 0.75$

Q/ Find CDF for $f_X(x) = \begin{cases} \dfrac{1}{b-a}, & \text{for } a < x < b \\ 0, & \text{elsewhere} \end{cases}$

Ans

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(x)\,dx = \int_{-\infty}^{a} f_X(x)\,dx + \int_{a}^{x} f_X(x)\,dx$$

$$= 0 + \int_{a}^{x} \frac{1}{b-a}\,dx$$

$$= \frac{x-a}{b-a}$$

for $x > b$ $\Rightarrow F_X(x) = \int_{a}^{b} f_X(x)\,dx = \frac{1}{b-a} \times b-a = 1$

$$F_X(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$$