EEE/ECE F311
Communication Systems
Tutorial-2

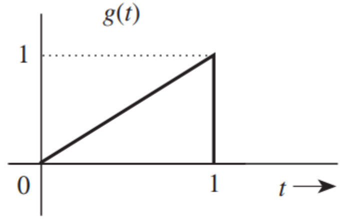
Date : 14/08/2025
19/08/2025

Tutorial-2

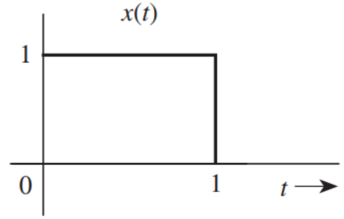
1. For the signals $g(t)$ and $x(t)$, find the component of the form $x(t)$ contained in $g(t)$. What is the resulting error signal energy?

Tutorial-2

Solution 1



(a)



(b)

(a) In this case $E_x = \int_0^1 dt = 1$, and

$$c = \frac{1}{E_x} \int_0^1 g(t)x(t)\, dt = \frac{1}{1} \int_0^1 t\, dt = 0.5$$

$$e(t) = g(t) - \tfrac{1}{2}x(t) = t - \tfrac{1}{2} \text{ on } [0, 1]$$

(b) Thus, $g(t) \approx 0.5x(t)$, and the error $e(t) = t - 0.5$ over $(0 \leq t \leq 1)$, and zero outside this interval. Also E_g and E_e (the energy of the error) are

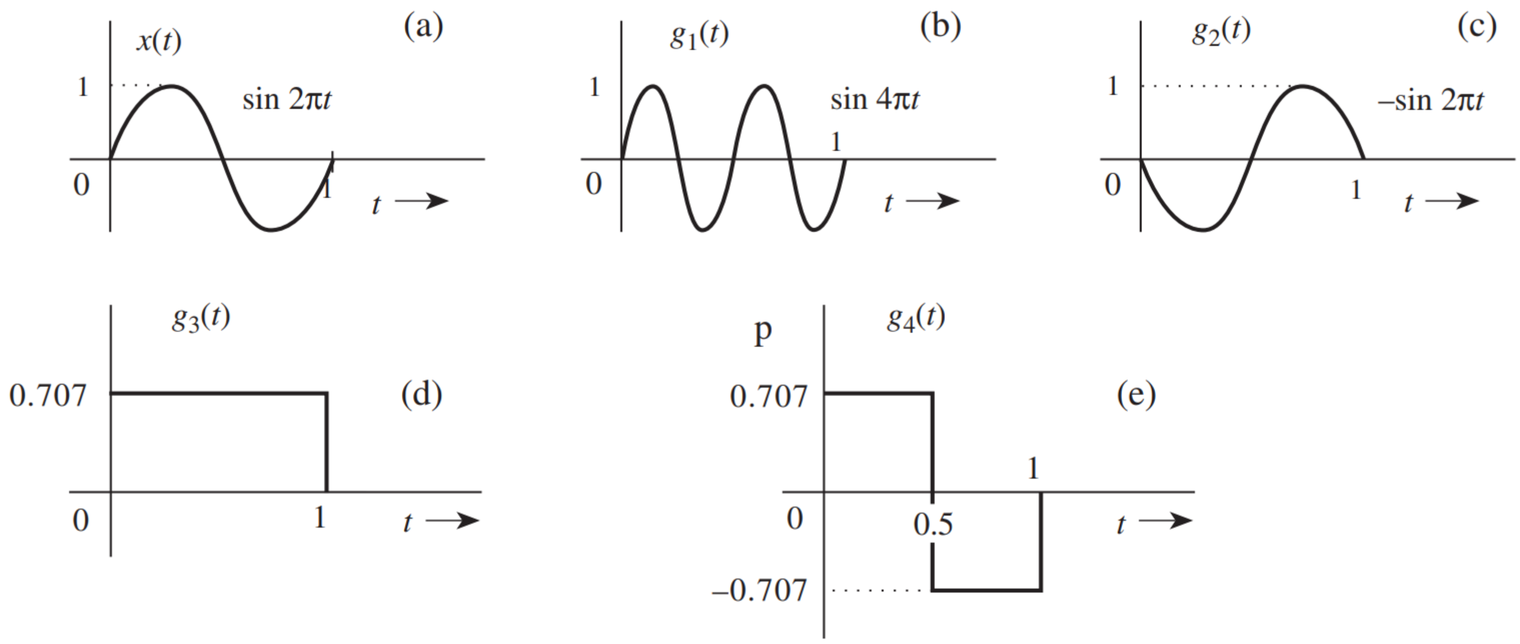
$$E_g = \int_0^1 g^2(t)\, dt = \int_0^1 t^2\, dt = 1/3 \quad \text{and} \quad E_e = \int_0^1 (t - 0.5)^2\, dt = 1/12$$

The error $(t - 0.5)$ is orthogonal to $x(t)$ because

$$\int_0^1 (t - 0.5)(1)\, dt = 0$$

Tutorial-2

2. Determine the correlation coefficient c_n of the signal x(t) and each of the four pulses. To provide maximum margin against the noise along the transmission path, which pair of pulses would you select for a binary communication?



Tutorial-2

Solution 2

The energy of the signal x(t): $$E_x = \int_0^1 \sin^2(2\pi t)\,dt = 0.5$$

Similarly, find the energy of the signals $g_1(t)$, $g_2(t)$, $g_3(t)$, and $g_4(t)$:

$$E_{g1} = E_{g2} = E_{g3} = E_{g4} = 0.5$$

(1) $$\rho_n = \frac{1}{\sqrt{0.5\times 0.5}}\int_0^1 \sin(2\pi t)\sin(4\pi t)\,dt = 0$$

(2) $$\rho_n = \frac{1}{\sqrt{0.5\times 0.5}}\int \sin(2\pi t)\sin(-2\pi t)\,dt = -1$$

(3) $$\rho_n = \frac{1}{\sqrt{0.5\times 0.5}}\int_0^1 0.707\sin(2\pi t)\,dt = 0$$

(4) $$\rho_n = \frac{1}{\sqrt{0.5\times 0.5}}\left[\int_0^{0.5} 0.707\sin(2\pi t)\,dt - \int_{0.5}^1 0.707\sin(2\pi t)\,dt\right] = 0.9$$

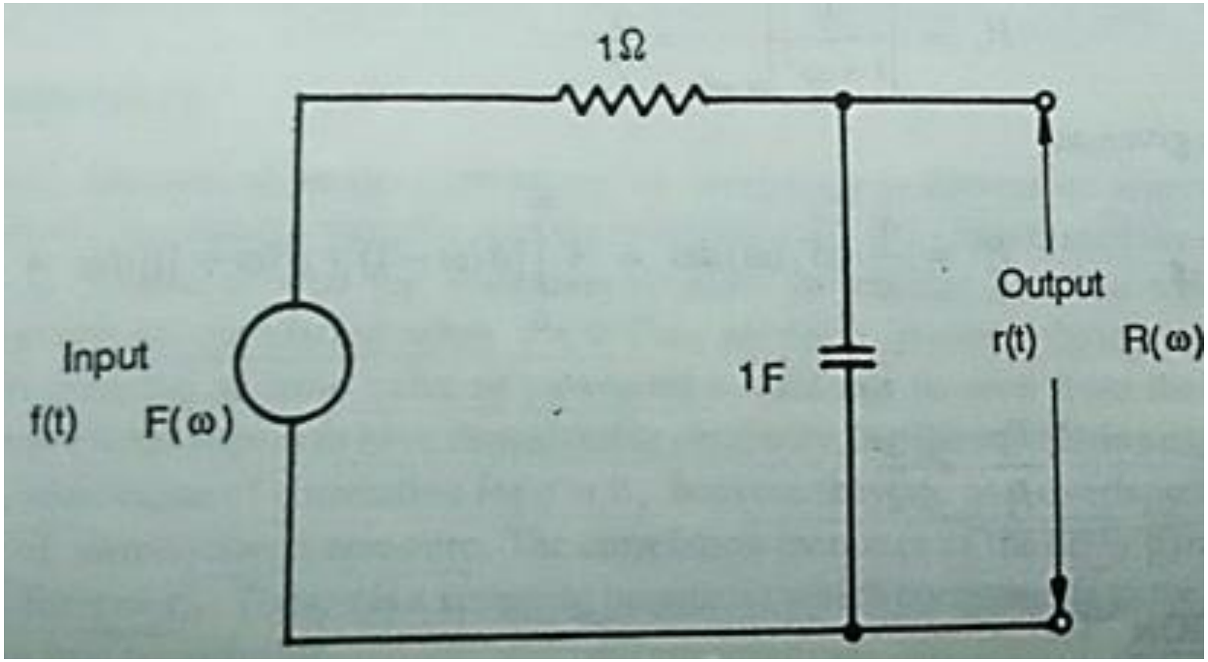
Chosen signal for binary transmission is x(t) and $g_2(t)$ as the correlation coeff. is -1

Tutorial-2

3. A signal with power spectral density

$$D_f(\omega) = A\pi\left[\delta(\omega-1)+\delta(\omega+1)\right]$$

is applied to the input of a linear system shown in figure given below . Find the ratio of average power of the response and excitation.



Solution

Transfer function $$H(j\omega) = \frac{Z_C}{R + Z_C} = \frac{1/(j\omega)}{1 + 1/(j\omega)} = \frac{1}{1 + j\omega}$$

Magnitude $$|H(j\omega)|^2 = \frac{1}{1 + \omega^2}$$

Input PSD $$S_f(\omega) = D_f(\omega) = A\pi\left[\delta(\omega - 1) + \delta(\omega + 1)\right]$$

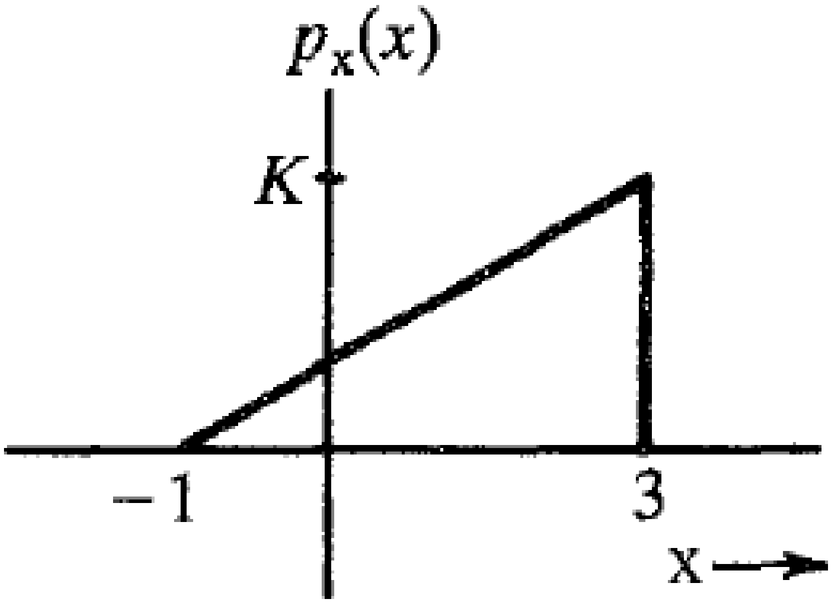
Average input power $$P_f = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)\, d\omega = \frac{1}{2\pi}(A\pi + A\pi) = A$$

Average output power $$P_r = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(j\omega)|^2 S_f(\omega)\, d\omega = \frac{1}{2\pi} A\pi\left(|H(j1)|^2 + |H(j{-1})|^2\right) = A|H(j1)|^2$$

Power ratio $$\frac{P_r}{P_f} = |H(j1)|^2 = \frac{1}{1 + 1^2} = \boxed{\frac{1}{2}}$$

Tutorial-2

4. Find the mean, the mean square, and the variance of the RVx whose PDF is shown in figure.



K=0.5
$P_X(x)=1/8(x+1)$
Mean=5/3
Mean square=11/3
Variance=8/9

Tutorial-2

5. Find the autocorrelation function of $x(t) = A\cos(2\pi f_0 t+\varphi)$ in terms of its period, $T_0 = 1/f_0$. find the average normalized power of x(t), using $P_x = R(0)$.

For a deterministic periodic signal, its autocorrelation over one period is

$$R_x(\tau) = \frac{1}{T_0}\int_0^{T_0} x(t)\,x(t+\tau)\,dt, \qquad T_0 = \frac{1}{f_0}.$$

With $x(t) = A\cos(2\pi f_0 t + \phi)$,

$$\begin{aligned}
R_x(\tau) &= \frac{A^2}{T_0}\int_0^{T_0} \cos(2\pi f_0 t + \phi)\cos(2\pi f_0(t+\tau) + \phi)\,dt \\
&= \frac{A^2}{2T_0}\int_0^{T_0} [\cos(2\pi f_0 \tau) + \cos(4\pi f_0 t + 2\phi + 2\pi f_0 \tau)]\,dt \\
&= \frac{A^2}{2}\cos(2\pi f_0 \tau) = \frac{A^2}{2}\cos\left(\frac{2\pi\tau}{T_0}\right),
\end{aligned}$$

$$P_x = R_x(0) = \frac{A^2}{2}$$

6. A random variable has exponential PDF given by $p_X(x) = a.e^{-b|x|}$, where a and b are constants.

Find:

(i) relationship between a and b

(ii) the CDF of X.

Tutorial-2

Sol.

$$\int_{-\infty}^{\infty} p_x(x)dx = 1$$

$$\int_{-\infty}^{\infty} ae^{-b|x|}dx = 1$$

(i) After solving the above equation, we will get b=2a

(ii) For x<0, CDF F(x)= $\int_{-\infty}^{x} f(u)du = \int_{-\infty}^{x} ae^{-bu}du$ =1/2 e^{bx}

For x>0, CDF F(x)= $\int_{-\infty}^{x} f(u)du = \int_{-\infty}^{0} ae^{bu}du + \int_{0}^{x} ae^{-bu}du$ =1- 1/2 e^{-bx}

$$F(x) = \begin{cases} \frac{1}{2}e^{bx}, & x < 0 \\ 1 - \frac{1}{2}e^{-bx}, & x > 0 \end{cases}$$