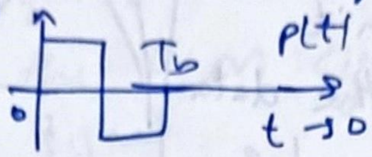
4th November, 2025 Tut-11 comm sys

Q1. Derive $S_y(\omega)$, the PSD of the manchester (split phase) signal, assuming 1, 0 are equally likely. Sketch the PSD

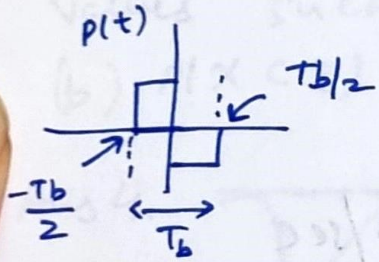


Ans 1. $S_y(\omega) = \frac{|P(\omega)|^2}{T_b}\left(R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b\right)$

Polar Code: "1" is transmitted by p(t) and "0" is transmitted by -p(t). Hence,

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k^2 = \lim_{N\to\infty}\frac{1}{N}(N) = 1$$

$$R_n = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} a_k a_{k+n} = \lim_{N\to\infty}\frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0$$



$$p(t) = \text{rect}\left(\frac{t + \frac{T_b}{4}}{\frac{T_b}{2}}\right) - \text{rect}\left(\frac{t - T_b/4}{T_b/2}\right)$$

we know that $\boxed{\text{rect}\frac{t}{\tau} \longleftrightarrow \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)}$

Here, $\tau = 0.5T_b$

$\Rightarrow \text{rect}\left(\frac{t}{0.5T_b}\right) \longleftrightarrow 0.5T_b\,\text{sinc}(0.25\omega T_b)$

We also know that $x(t-t_0) \longleftrightarrow X(\omega)e^{-j\omega t_0}$

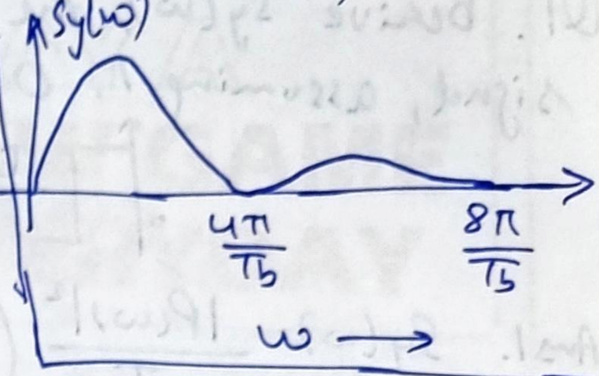
$x(t+0.25T_b) \longleftrightarrow X(\omega)e^{-j\omega(0.25(-T_b))} = X(\omega)e^{j\omega(0.25T_b)}$

$\therefore \text{rect}\left(\frac{t+0.25T_b}{0.5T_b}\right) \longleftrightarrow 0.5T_b\,\text{sinc}(0.25\omega T_b)\,e^{0.25j\omega T_b}$

$\text{rect}\left(\frac{t-0.25T_b}{0.5T_b}\right) \longleftrightarrow 0.5T_b\,\text{sinc}(0.25\omega T_b)\,e^{-0.25\omega j T_b}$

$\therefore P(\omega) = j T_b\,\text{sinc}\left(\frac{\omega T_b}{4}\right)\sin\left(\frac{\omega T_b}{4}\right)$

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = T_b \,\text{sinc}^2\left(\frac{\omega T_b}{4}\right) \sin^2\left(\frac{\omega T_b}{4}\right)$$



Q2. A leased telephone of bandwidth 3kHz is used to transmit binary data. Calculate the data rate (in bps) that can be transmitted if we use:

(a) Polar signal with rectangular half-width pulses.
(b) Polar signal with rectangular full-width pulses.
(c) Polar signal ~~with~~ using Nyquist criterion pulses of $r = 0.25$.
(d) Bipolar signal with rectangular half-width pulses.
(e) Bipolar signal with rectangular fullwidth pulses.

Ans 2. (a) For polar signal: $S_y(\omega) = \frac{|P(\omega)|^2}{T_b}$

where $p(t) = \text{rect}(2t/T_b)$

$P(\omega) = \left(\frac{T_b}{2}\right) \text{sinc}\left(\frac{\omega T_b}{4}\right)$

Hence, $\boxed{B_T = 2R_b} \Rightarrow R_b = \frac{B_T}{2} = 1.5$ k bits/sec

(b) $p(t) = \text{rect}(t/T_b)$; $P(\omega) = T_b \,\text{sinc}\left(\frac{\omega T_b}{2}\right)$

Essential $\boxed{BW(B_T) = R_b}$, hence transmission bit rate $= 3$ k bits/se

(c) $B_T = (1+r)\frac{R_b}{2} = 3000$, hence $R_b = \frac{6000}{1.25} = 4800 \frac{\text{bit}}{\text{sec}}$

(d) $S_y(\omega) = \frac{T_b}{4} \text{sinc}^2\left(\frac{\omega T_b}{4}\right) \text{sinc}^2\left(\frac{\omega T_b}{2}\right)$

Essential $BW = R_b$: Bit rate possible is 3 kbits

(e) Bipolar $\Rightarrow S_y(\omega) = \frac{|P(\omega)|^2}{T_b} \sin^2\left(\frac{\omega T_b}{2}\right)$

$P(\omega) = T_b \,\text{sinc}\left(\frac{\omega T_b}{2}\right)$

$$S_y(\omega) = T_b \,\text{sinc}^2\left(\frac{\omega T_b}{2}\right) \sin^2\left(\frac{\omega T_b}{2}\right)$$

Q3. A 64 kbps binary PCM Bipolar NRZ signal is passed through a communication system with a raised cosine filter with $r = 0.25$. Determine the bandwidth of the PCM signal and the bandwidth of the filtered PCM signal

Ans 3. $BW = (1+r)\,\boxed{BW_{Nyquist}} \rightarrow \frac{R_b}{2}$ = Bandwidth of PCM signal

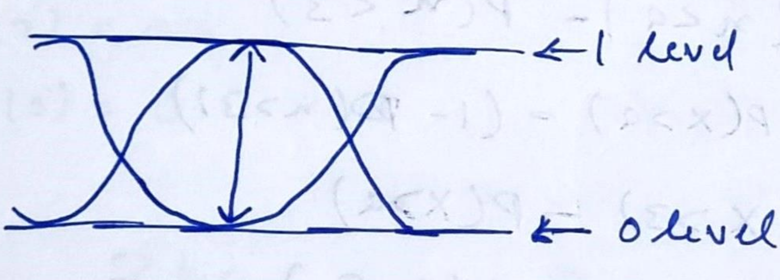
Filtered signals

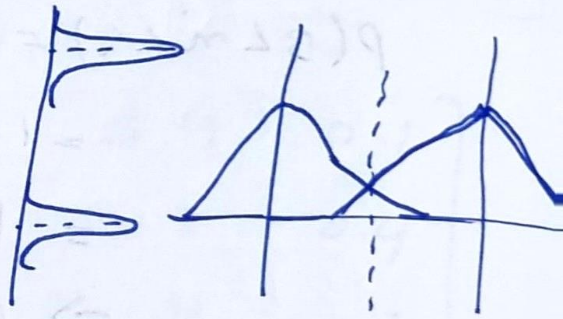
$$= (1+r)\frac{R_b}{2}$$

$$= \left((1+0.25)\frac{64}{2}\right) kHz = 40 kHz$$

Q4. For the Gaussian random variable, X with mean and standard deviation values are given as $\mu = 5$ and $\sigma = 2$, respectively. Find the probability values such as (a) $P(X > 8)$ (b) $P(X < 8)$ and (c) $P(3 < X < 8)$

Ans 4.





(a) $P(X > 8) = Q\left(\frac{X-\mu}{\sigma}\right)$

$$= Q\left(\frac{8-5}{2}\right) = Q(1.5) = 0.0668$$ (From the Q table)

(b) $P(X < 8) = 1 - P(X > 8)$

$$= 1 - 0.0668 = 0.9332$$

$\boxed{Q(-x) = 1 - Q(x)}$

(c) $P(3 < X < 8) = P(X < 8) - P(X < 3)$

$$= P(X < 8) - (1 - P(X > 3))$$

$$= 0.9332 - 1 + Q\left(\frac{3-5}{2}\right) \rightarrow -1$$

$$= 0.774533$$

Q5. For the Gaussian random variable, X with mean and standard deviation values are given as $\mu = 5$ and $\sigma = 3$, respectively. Find the factor a such that $P(X<a) = 0.9$

Ans 5. $P(X<a) = 0.9$

$$P(X<a) = 1 - P(X>a) = 1 - Q\left(\frac{X-\mu}{\sigma}\right) = 1 - Q\left(\frac{a-5}{3}\right) = 0.9$$

$$\Rightarrow Q\left(\frac{a-5}{3}\right) = 0.1$$

$$\Rightarrow \frac{a-5}{3} = 1.3 \Rightarrow \boxed{a = 5 + 3(1.3) = 8.9}$$

↳ take either 1.25 or 1.3

Q6. For the Gaussian random variable, X with mean and standard deviation values are given as $\mu = 5$, $\sigma = 3$ respectively. Find the factor such that $P(3 < X < a) = 0.7$.

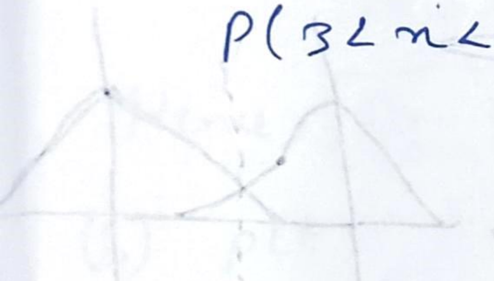
Ans 6. $P(3<x<a) = 0.7$

$$P(3<x<a) = P(x<a) - P(x<3)$$

$$= 1 - P(x>a) - (1 - P(x>3))$$

$$= P(x>3) - P(X>a)$$

$$\Rightarrow Q\left(\frac{3-5}{3}\right) - Q\left(\frac{a-5}{3}\right) = 0.7$$



$$\boxed{a = 9.65}$$

11th November, 2025

Tutorial-12

Q1. The received signal input to the zero forcing equaliser (ZFE) is given as $x(n) = \{0, 0.1, 0.4, 0.8, 0.4, 0.1, 0\}$. The centre value $x(0) = 0.8$. Design a 3-tap ZFE and find the equaliser coefficients. Find the equaliser output. Check for the reduction of ISI in the filtered signal.

Ans1. $x(n) = \{0, 0.1, 0.4, \underset{\uparrow}{0.8}, 0.4, 0.1, 0\}$ $\Rightarrow$ ~~$x(0) = 0.8$~~

$\Rightarrow$ ~~$x(-1) = 0.4$~~ ~~and~~ ~~$x(1) = 0.4$.~~

$x(-3) = 0$
$x(-2) = 0.1$
$x(-1) = 0.4$
$x(0) = 0.8$
$x(1) = 0.4$
$x(2) = 0.1$
$x(3) = 0$

3 Tap ZFE

$$X = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(+2) & x(+1) & x(0) \end{bmatrix}$$

3 tap equaliser matrix

$$X = \begin{bmatrix} 0.8 & 0.4 & 0.1 \\ 0.4 & 0.8 & 0.4 \\ 0.1 & 0.4 & 0.8 \end{bmatrix}$$

coefficient matrix $= C = \boxed{X^{-1} Z}$ Now, ~~$X^{-1} =$~~ ~~$[0.8 \quad 0.4 \quad 0.1]$~~

$$X^{-1} = \left[\begin{array}{ccc|ccc} 0.8 & 0.4 & 0.1 & 1 & 0 & 0 \\ 0.4 & 0.8 & 0.4 & 0 & 1 & 0 \\ 0.1 & 0.4 & 0.8 & 0 & 0 & 1 \end{array}\right]$$

$$C = \begin{bmatrix} -1 \\ 2.25 \\ -1 \end{bmatrix} \qquad Z = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$ for zero forcing equaliser

Equaliser output = $x^2(n) = x(n) * c(n)$

$\boxed{y(n) = (-1)x(n+1) + 2.25\,x(n) + (-1)(x(n-1))}$ convolution.

$x^2(n) = (0, -0.01, -0.0175, \underbrace{0, 1, 0}, -0.0175, -0.01, 0)$

ISI reduction required for $x(\mp 1)$, $x(\mp 2)$ sample values

Q2 The sampled values of unequalised pulses are given by 0.4, 0.6 and 0.8 for k=1, 0 and -1 respectively. Design the zero forcing equaliser. What will be equaliser output for k = -3, -2, +2 and +3 respectively?

Ans 2. $P_r[+1] = 0.4$

$P_r[0] = 0.6$

$P_r[-1] = 0.8$

$P_r[2] = 0$

$P_r[-2] = 0$

Assume since not given.

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} P_r(0) & P_r(-1) & P_r(-2) \\ P_r(+1) & P_r(0) & P_r(-1) \\ P_r(+2) & P_r(+1) & P_r(0) \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_{+1} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 & 0 \\ 0.4 & 0.6 & 0.8 \\ 0 & 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} C_1 \\ C_0 \\ C_1 \end{bmatrix}$$

$C_{-1} = \frac{20}{7} = 2.85$ $\quad C_0 = \frac{-15}{7} = -2.142$ $\quad C_1 = \frac{10}{7} = 1.4$

By using $\boxed{\begin{aligned} y(n) &= \frac{10}{7} x(n-1) + \left(-\frac{15}{7}\right) x(n) + \frac{20}{7}(x(n+1)) \\ y(-3) &= \frac{20}{7}(x(-4)) + \left(-\frac{15}{7}\right)(x(-3)) + \frac{10}{7} x(-2) = 0 \end{aligned}}$

$$\begin{bmatrix} P_o(-3) \\ P_o(-2) \\ 0 \\ 1 \\ 0 \\ P_o(2) \\ P_o(3) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & \\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0.8 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 20/7 \\ -15/7 \\ 10/7 \\ 0 \\ 0 \end{bmatrix}$$

For $k = -3$, $P_o(k) = 0$

$k = -2$, $P_o(k) = 0.8 \times \frac{20}{7} = \frac{16}{7} = 2.28$

$k = 2$, $P_o(k) = 0.4 \times \frac{10}{7} = \frac{4}{7} = 0.571$

$k = 3$, $P_o(k) = 0$

Q3. Given $N_0 = 10^{-10} \frac{W}{Hz}$, $A = 50mV$, and the bit rate is $R_b = \frac{1}{T} = 5Mbps$, find P_E.

Ans 3. $P_E = \frac{1}{2} erfc(\sqrt{\gamma})$ where $\boxed{\gamma = \frac{E_b}{N_0}} = \boxed{\frac{A^2 T}{N_0}}$

$\therefore P_E = \frac{1}{2} erfc(\sqrt{5})$ $\qquad \gamma = \frac{(50 \times 10^{-3})^2 (1/5)(10^{-6})}{10^{-10}}$

1. Get the value from the error function table $\qquad \gamma = 5$

2. $erfc(x) \cong \frac{1}{x\sqrt{\pi}} e^{-x^2}$ $\Rightarrow$ true only for $x > 3$

$erfc(x) \simeq \frac{2x}{\sqrt{\pi}}$ $\Rightarrow$ true for $|x| << 1$

We can use the first formula despite $\sqrt{5} < 3$

$\Rightarrow erfc(x) = \frac{1}{x\sqrt{\pi}} e^{-x^2} = 1.7 \times 10^{-3}$

$\therefore P_E = \frac{1}{2} erfc(\sqrt{5}) = 0.768 \times 10^{-3} = Q\left(\sqrt{\frac{2E_B}{N_0}}\right) = Q\sqrt{2\gamma}$

Q4. Prove that the maximum signal to noise ratio of the matched filter is

$$\left(\frac{S}{N}\right)_{o\,max} = \frac{2E}{N_0}.$$

Ans 4. $\left(\frac{S}{N}\right)_o = \int_{-\infty}^{\infty} \frac{|X(f)|^2 \, df}{S_N(f)} = \int_{-\infty}^{+\infty} \frac{|X(f)|^2 \, df}{N_0/2}$

For system assume AWGN $= \frac{2}{N_0} \int_{-\infty}^{\infty} |X(f)|^2 df = \frac{2E}{N_0}$

Noise PSD $= \frac{N_0}{2} = S_{N_i}(f)$

Rayleigh's Theorem, $E = \int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |X(f)|^2 df$

Q5. In binary transmission, one of the messages is represented by a rectangular pulse $x(t)$. Another message is transmitted by the absence of the pulse. Evaluate the signal to noise ratio $t = T$. Assume the PSD of Noise is $N_0/2$. Sketch the impulse response and output of match filter.

Ans 5.

$$x(t) = \begin{cases} A \;; & 0 \le t \le T \rightarrow \text{bit } 1 \\ 0 \;; & \text{otherwise} \rightarrow \text{bit } 0 \end{cases}$$

Noise is AWGN as PSD $= \frac{N_0}{2}$

Impulse

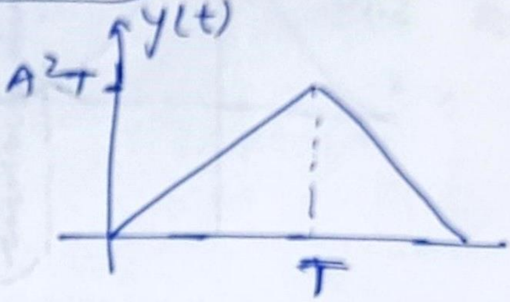
$h(t) = \cancel{x(t)}\; x(T-t)\,\frac{2k}{N_0}$ as I assume

$$\boxed{y(t) = x(t) * h(t)}$$

Now, $x(-t) = \begin{cases} A\ ; & -T \le t \le 0 \\ 0\ ; & \text{otherwise} \end{cases}$

$$\Rightarrow \boxed{x(T-t) = \begin{cases} A\ ; & 0 \le t \le T \\ 0\ ; & \text{otherwise} \end{cases}} = h(t)$$

$$SNR = \frac{2E_b}{N_0} = \frac{2A^2T}{N_0}$$



18th November, 2025

Tut-B

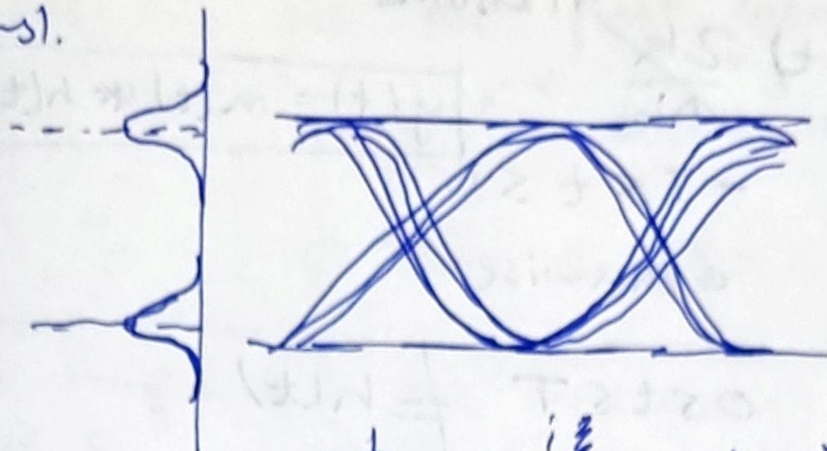
Q1. A binary communication system transmits signals $s_i(t)$ ($i = 1, 2$). The receiver test statistic $z(T) = a_i + n_0$, where the signal component a_i is ~~the~~ either $a_1 = +1$ or $a_2 = -1$ and the noise component n_0 is uniformly distributed, yielding the conditional density functions $p(z/s_i)$ given by:

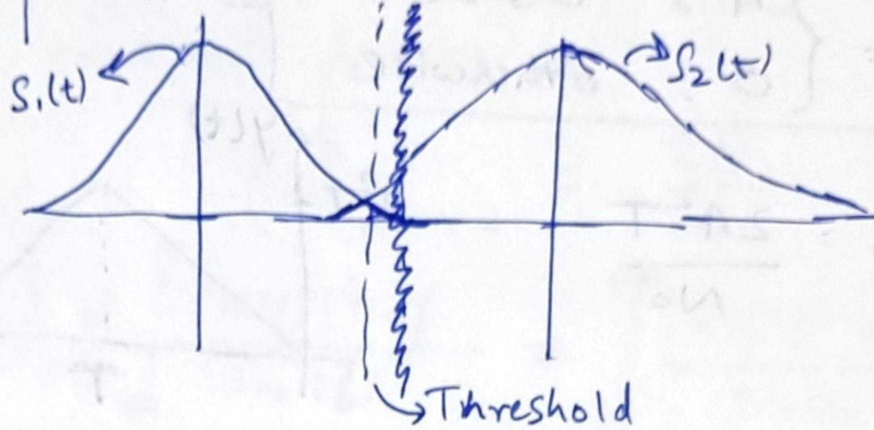
$$p(z/s_1) = \begin{cases} 1/2 & \text{for } -0.2 \le z \le 1.8 \\ 0 & \text{otherwise} \end{cases}$$ and

$$p(z/s_2) = \begin{cases} 1/2 & \text{for } -1.8 \le z \le 0.2 \\ 0 & \text{otherwise} \end{cases}$$

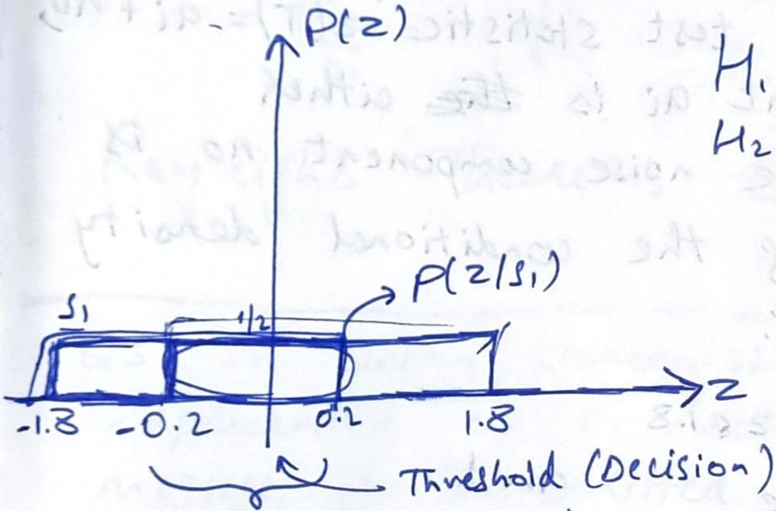
Find the probability of a bit error, P_B, for the case of equally likely signalling and the use of an optimum decision threshold.

Ans 1.





$$P_e = P[1_D, 0_T] + P[0_D, 1_T]$$
$$= P[1_D / 0_T]\, P[0_T] + P[0_D / 1_T]\, P[1_T]$$



H_1: $S_1(t)$ is detected / received

H_2: $S_2(t)$ is detected / received

$$\frac{P(z/a_2)}{P(z/a_1)} \underset{H_1}{\overset{H_2}{\gtrless}} \frac{P(S_1(t))}{P(S_2(t))}$$

The error will be in this range.

$$\therefore \quad \frac{P(z/a_2)}{P(z/a_1)} \underset{H_1}{\overset{H_2}{\gtrless}} 1$$

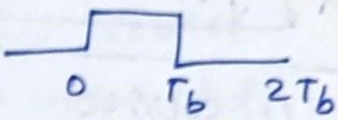
Probability of error $= P_B = 0.5 \int_{-0.2}^{0} 0.5\, dz + 0.5 \int_{0}^{0.2} 0.5\, dz$

$$= 0.25 \left[\int_{-0.2}^{0} dz + \int_{0}^{0.2} dz \right] = \frac{1}{4} \times 0.4 = \boxed{0.1}$$

Q2. Consider that NRZ binary pulses are transmitted along a cable that attenuates the signal power by 3dB (from transmitter to receiver). The pulses are coherently detected at the receiver, and the data rate is 56 kbits/sec. Assume Gaussian noise with $N_0 = 10^{-6}$ W/Hz. What is the ~~probability~~ minimum amount of power needed at the transmitter in order to maintain a bit error probability of $P_B = 10^{-3}$?

{ Q table }

Ans 2. Binary transmission $\rightarrow$ NRZ



Attenuation = 3dB

Data rate = R_b = 56 kbps

$$P_B = \underline{10^{-3}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \underline{0.001} \Rightarrow \sqrt{\frac{2E_b}{N_0}} = \underline{3.08}$$

$N_0 = 10^{-6}$ W/Hz $\Rightarrow \boxed{E_b = 4.74 \times 10^{-6}}$

$\boxed{E_b = A^2 T_b} \longrightarrow A^2 T_b = 4.74 \times 10^{-6}$

$\Rightarrow A^2 = (4.74 \times 10^{-6} \times 56 \times 10^3)\,W = \boxed{265\,mW}$

Now, the required power at Tx $= 2 P_{Rx}$ (on 3dB loss)

$= (2 \times 265)$ mW

$\approx \boxed{0.531\,W}$

Q3. A binary digital communication system employs the signal

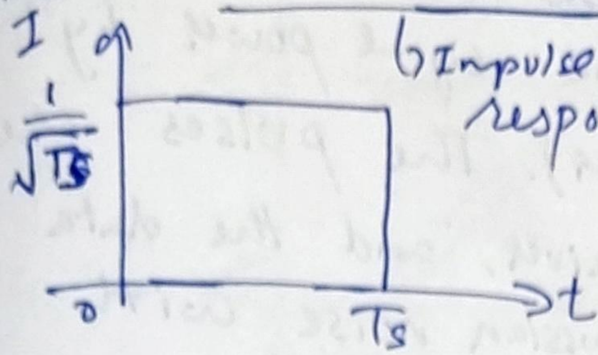
$$s(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.

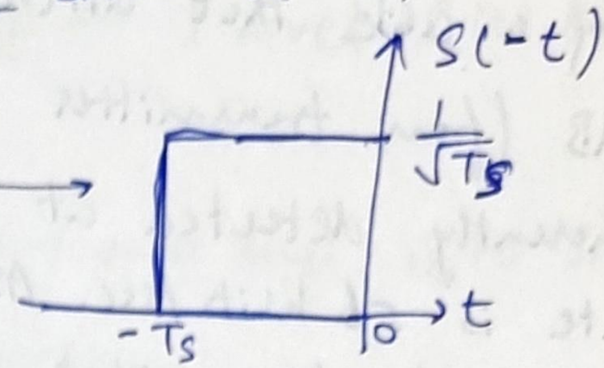
(b) Plot the matched filter output as a function of time.

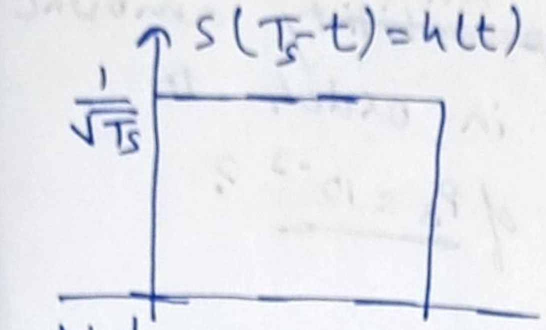
Ans 3. $h(t) = s(T-t)$ ↳ Impulse response

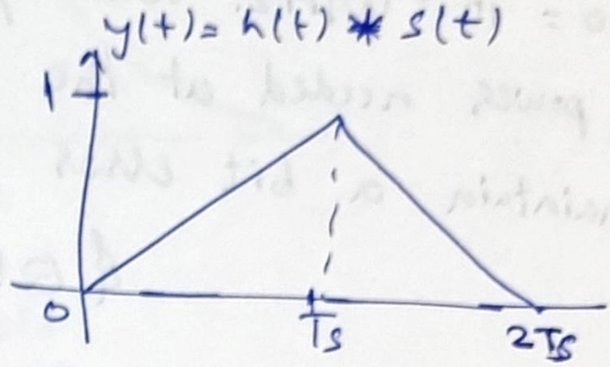
I



II Draw $s(-t)$







matched filter $\boxed{y(t) = s(t) * h(T-t)}$

Q4. A source emits seven messages with probabilities 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, and 1/64 respectively. Find the entropy of the source.

Ans 4. $H = \sum_i P_i \log_2\left(\frac{1}{P_i}\right) = \frac{1}{2}\log_2\left(\frac{1}{1/2}\right) + \frac{1}{4}\log_2\left(\frac{1}{1/4}\right) + \dots$

$\cong 1.96$ bits/msg.

Q5. Assume that normal speech rate is 250 words/minute. Also assume that the average word contains six characters and the characters within a word are statistically independent. Based on these assumptions, compute the information rate considering the English alphabet.

(Note: Information rate $= (R_s \times H(m)$ per sec)

Ans 5. Per min words = 250

per msg characters = 6

total ch. per min = 1500

Total ch. per sec $= \frac{1500}{60} = 25$ ch/msg/sec

$$H = \left[\frac{1}{26}\log_2\left(\frac{1}{1/26}\right)\right] \times 26 = 4.7 \text{ bits/ch}$$

$$I = 4.7 \text{ bits/ch} \times 25$$
$$= (4.7 \times 25) \text{ bps} = 117.5 \text{ bps.}$$

25th November, 2025 — Tut-14

Q5. An analog signal having 4kHz bandwidth is sampled at 1.25 times the Nyquist rate, and each sample is Quantised into one of equally likely levels. Assume that the successive samples are statistically independent.

(i) what is the information rate of this source?

(ii) Can the output of this source be transmitted without error over an AWGN channel with a bandwidth of 10kHz and an S/N ratio of 20dB?

(iii) Find the S/N ratio required for error-free transmission for part (i).

(iv) Find the bandwidth required for an AWGN channel for error free transmission of the output of this source if the S/N ratio is 20dB?

No. of symbols = 256

Ans 5 (i) $f_m = 4\text{kHz}$, $f_s = 2f_m = 8\text{kHz}$

$f_s^{\text{actual}} = 1.25 f_s = (1.25 \times 8) = 10\text{kHz}$

No. of symbols = 256, each symbol probability = $\frac{1}{256}$

$$H(x) = \sum_{i=1}^{m} P(x_i)\log_2\left(\frac{1}{P(x_i)}\right) \longrightarrow \text{entropy}$$

$$= \log_2(256) = 8 \text{ bits/sample.}$$

Information Rate $R = H(x)\, f_s^{actual}$

$= 8 \frac{\text{bits}}{\text{sample}} \times 10^4\ \frac{\text{samples}}{\text{second}}$

$\boxed{R = 80\text{ kbps}}$

(ii) $C = B\log_2(1+SNR)$

$B = 10\text{kHz}$

$SNR = 20\text{dB} = 10^{20/10} = 100$

$C = (10\times10^3)\times\log_2(1+100) = 66.58\text{ kbits/s}$

$\Rightarrow R > C \Rightarrow$ no error free Transmission

$R < C \Rightarrow$ error free Transmission.

(iii) $R = 80\text{ kb/s}$

for error free Transmission, the Required correlatio[n]

is $C \geq R$

$B\log_2(1+SNR) \geq R$

$\Rightarrow (10\times10^3)(\log_2(1+SNR_{req})) \geq (80\times10^3)$

$\Rightarrow SNR_{req} \geq 2^8 - 1 = \boxed{24.06\text{ dB}} = 255$

(iv) $B\log_2(1+SNR) \geq R$

$\Rightarrow B_{req}\log_2(1+100) \geq 80\times10^3$

$\Rightarrow \boxed{B_{req} \geq 12.01\text{ kHz}}$

Q5. Given an AWGN channel with 4 kHz bandw[idth] and the noise power spectral density $\eta/2 = 10^{-1[2]}$

The signal power required at the receiver $= 0.1$ m[W]

Calculate the capacity of this channel.

Ans 5. $P_{Tx} = 0.1\text{ mW}$, $\eta/2 = 10^{-12}\ \frac{W}{Hz}$, $B = 4\text{kHz}$

$C = B\log_2(1+SNR)$ $\Rightarrow SNR = \dfrac{\text{Signal Power}}{\text{Noise Power}} = \dfrac{0.1\times10^{-}}{2\times10^{-1}\cdot 4\times10^{3}} = 12500$

$\Rightarrow C = B\log_2(1+12500)$

$= (4\times10^3)(\log_2(12501))$

$\boxed{C = 54.439 \text{ kbps}}$

Q4. A DMS X has five symbols x_1, x_2, x_3, x_4, and x_5 with the $P(x_1) = 0.4$, $P(x_2) = 0.19$, $P(x_3) = 0.16$, $P(x_4) = 0.15$, $P(x_5) = 0.1$.

(i) Construct a Shannon-Fano code for X, and calculate the efficiency of the code.

(ii) Repeat for the Huffman code and compare the results.

write in descending order only

Ans 4 (i)

x_i	$P(x_i)$		Step1	Step2	Step3
x_1	0.4	$\rightarrow 0$	0	$\rightarrow$ 0	
x_2	0.19		0	$\rightarrow$ 1	
x_3	0.16	$\rightarrow 1$	1	$\rightarrow$ 0	
x_4	0.15		1	$\rightarrow$ 1	$\rightarrow$ 0
x_5	0.1		1	$\rightarrow$ 1	$\rightarrow$ 1

Find Code

	Code
x_1	0 0
x_2	0 1
x_3	1 0
x_4	1 1 0
x_5	1 1 1

$L = (0.4)(2) + (0.19\times2) + (0.16\times2) + (0.15\times3) + 0.1\times3$

$\boxed{L = 2.25}$

$$H(x) = -\sum_{i=1}^{5} P(x_i) \log_2 [P(x_i)] = 2.15$$

$$\eta = \frac{H(x)}{L} = \text{Coding Efficiency} = \frac{2.15}{2.25} = 95.55\%$$

(ii) Huffman Coding (Not unique)

x_i	$P(x_i)$	Step 1	Step 2	Step 3	Final
x_1	0.4	0.4	0.4	0.6 (0)	1
x_2	0.19	0.25	0.35 (0)	0.4 (1)	
x_3	0.16	0.19 (0)	0.25 (1)		000
x_4	0.15 (0)	0.16 (1)			001
x_5	0.1 (1)				010
					011

Since $0.25 \geq 0.19$, we have to interchange, else keep it there.

$$L = (0.4 \times 1) + (0.19 \times 3) + (0.16 \times 3) + (0.15 \times 3) + (0.1 \times 3)$$

$$= 2.2$$

$$H(x) = 2.15 \Rightarrow \eta = \frac{H(x)}{L} = \frac{2.15}{2.2} = 97.73\%$$

Q3. A DMS X has 5 equally likely symbols.

(i) Construct a Shannon-Fano code for X, and calculate the efficiency of the code.

(ii) Construct another Shannon-Fano code and compare the results.

(iii) Repeat for the Huffman code and compare the results.

Ans 3. $P(X_i) = \frac{1}{5} = 0.2$

(i)

x_i	$P(x_i)$	Step 1	Step 2	Step 3	Step 4 (Final)
x_1	0.2	0	0		00
x_2	0.2	0	1	0	010
x_3	0.2	0	1	1	011
x_4	0.2	1	0		10
x_5	0.2	1	1		11

(ii)

x_i	$P(x_i)$	Step 1	Step 2	Step 3	Final Code
x_1	0.2	0	0		00
x_2	0.2	0	1		01
x_3	0.2	1	0		10
x_4	0.2	1	1	0	110
x_5	0.2	1	1	1	111

$$L = (0.2)(2) + (0.2)(2) + (0.2)(2) + (0.2)(3) + (0.2 \times 3)$$

$$= \cancel{0.12}\ 1.2 + 1.2 = \boxed{2.4}$$

(iii) Huffman Coding

x_i	$P(x_i)$	Step 1	Step 2	Step 3	Final Code
x_1	0.2	0.4	0.4	0.6 (0)	01
x_2	0.2	0.2	0.4 (0)	0.4 (1)	10
x_3	0.2	0.2 (0)	0.2 (1)		11
x_4	0.2 (0)	0.2 (1)			000
x_5	0.2 (1)				001

$\rightarrow 1$

Q2. A DMS X has 4 symbols x_1, x_2, x_3 and x_4 with $P(x_1) = \frac{1}{2}$, $P(x_2) = \frac{1}{4}$ and $P(x_3) = P(x_4) = \frac{1}{8}$. Construct a Shannon-Fano code for X; show that this code has the optimum property that $n_i = I(x_i)$ and that the code efficiency is 100%.

Ans 2.

x_i	$P(x_i)$	Step1	Step2	Step3	Final Code
x_1	0.5	0			0
x_2	0.25	1	0		10
x_3	0.125	1	1	0	110
x_4	0.125	1	1	1	111

$L = (0.5 \times 1) + (0.25 \times 2) + (0.125 \times 3) \times 2 = \boxed{1.75}$

$H(x) = 1.75$ $\quad \therefore \eta = \frac{H(x)}{L} = 100\%$.

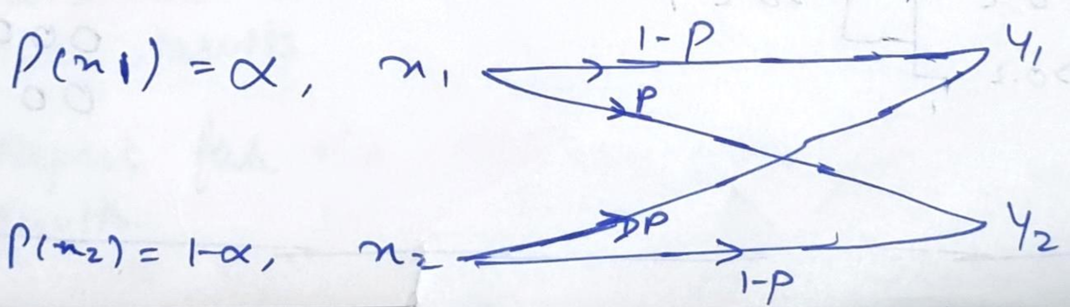
Q1. Given a binary symmetric channel (BSC) (figure1) with $P(x_1) = \alpha$,

(i) Show that the mutual information $I(x;y)$ is given by $I(x;y) = H(y) + p\log_2 p + (1-p)\log_2(1-p)$

(ii) Calculate $I(x;y)$ for $\alpha = 0.5$ and $p = 0.1$,

(iii) Repeat part (ii) for $\alpha = 0.5$ and $p = 0.5$, and comment on the results.

Figure1 shows the diagram of the binary symmetric channel (BSC) with associated input probabilities.



Ans1.

$$[P(x,y)] = \begin{bmatrix} \alpha & 0 \\ 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$

$$= \begin{bmatrix} \alpha(1-p) & \alpha p \\ p(1-\alpha) & (1-\alpha)(1-p) \end{bmatrix} = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) \\ P(x_2,y_1) & P(x_2,y_2) \end{bmatrix}$$

$$\Rightarrow H(Y/x) = -\sum_{j=1}^{n} \sum_{i=1}^{m} P(x_i,y_i) \log_2 (y_j/x_i)$$

$$= -p\log_2 p - (1-p)\log_2(1-p)$$

$$I(x;Y) = H(Y) - H(Y/x)$$

$$= H(Y) + p\log_2 p + (1-p)\log_2(1-p)$$

$\alpha = 0.5, \quad p = 0.1$

$$H(x) = -\sum_{i=1}^{m} P(x_i)\log_2(P(x_i)) = 1$$

$$H(Y) = -\sum_{i=1}^{m} P(Y_i)\log_2(P(y_i)) = 1$$

$$H(Y/x) = 0.469$$

$$\therefore \boxed{I(X;Y) = 1 - 0.469 = 0.531}$$