EEE/ECE F311
Communication Systems
Tutorial-12

Date : 06/11/2025

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Tutorial-12

1. **The received signal input to the zero forcing equalizer (ZFE) is given as x(n)=(0, 0.1, 0.4, 0.8, 0.4, , 0.1, 0). The center value x(0)=0.8. Design a 3-tap ZFE and find the equalizer coefficients. Find the equalizer output. Check for the reduction of ISI in the filtered signal.**

Tutorial-12

Solution 1

$$c = x^{-1} z = \begin{pmatrix} -1 \\ 2.25 \\ -1 \end{pmatrix}$$

$$\tilde{x}(n) = x(n) * c(n)$$

$$= (0, -0.1, -0.175, 0, 1, 0, -0.175, -0.1, 0)$$

ISI reduced for $x(\mp 1)$, $x(\mp 2)$ sample values.

Tutorial-12

2. The sampled values of unequalized pulses are given by 0.4, 0.6 and 0.8 for k=1, 0, and -1 respectively. Design the zero forcing equalizer. What will be equalizer output for k= -3, -2, +2 and +3 respectively?

Tutorial-12

Solution 2

Hence for a three tap [N=1] equalizer,

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 & 0 \\ 0.4 & 0.6 & 0.8 \\ 0 & 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}$$

Solving the above,

$$C_{-1} = \frac{20}{7} = 2.85$$

$$C_0 = \frac{-15}{7} = -2.14$$

$$C_1 = \frac{10}{7} = 1.42$$

Tutorial-12

Solution 2

By using

$$\begin{bmatrix} P_o[-3] \\ P_o[-2] \\ 0 \\ 1 \\ 0 \\ P_o[2] \\ P_o[3] \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ \vdots \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0.8 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 20/7 \\ -15/7 \\ 10/7 \\ 0 \\ 0 \end{bmatrix}$$

For $K=-3$, $P_o[K] = 0$

$K=-2$, $P_o[K] = 0.8 \times \frac{20}{7} = \frac{16}{7} = 2.28$

$K=2$, $P_o[K] = 0.4 \times \frac{10}{7} = \frac{4}{7} = 0.571$

$K=3$, $P_o[K] = 0$

Tutorial-12

3. Given $N_0 = 10^{-10}$ W/Hz, A= 50mV, and the bit rate is R_b= 1/T= 5 Mbps, find P_E.

Tutorial-12

Solution 3

$$P_E = Q\left(\sqrt{2\text{X}5}\right) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{10}{2}}\right) = 7.827 \times 10^{-4}$$

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Tutorial-12

4. Proof that the maximum signal to noise ratio of the matched filter is

$$\left(\frac{S}{N}\right)_{0\,\max} = \frac{2E}{N_0}$$

Tutorial-12

Solution 4

$$\left(\frac{S}{N}\right)_{0\,\max} = \frac{2E}{N_0}$$

Tutorial-12

5. **In binary transmission, one of the messages is represented by a rectangular pulse x(t). An another message is transmitted by the absence of the pulse. Evaluate the signal to noise ratio at t=T. Assuming the PSD of noise is No/2. Sketch the impulse response and output of match filter.**

Tutorial-12

Solution 5

The rectangular pulse can be represented as

$$x(t) = \begin{cases} A & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \qquad ...(i)$$

Further, we know that impulse response of the matched filter is given by,

$$h(t) = \frac{2k}{N_0} x(T-t) \qquad ...(ii)$$

From equation (i), we can write,

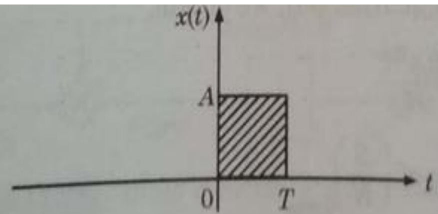
$$x(-t) = \begin{cases} A & \text{for } -T \le t \le 0 \\ 0 & \text{otherwise} \end{cases} \qquad ...(iii)$$

Let us delay this signal by T to get

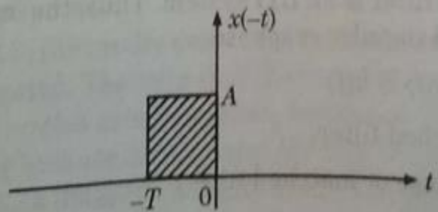
$$x(T-t) = \begin{cases} A & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \qquad ...(iv)$$

Substituting this value in equation (ii), we obtain

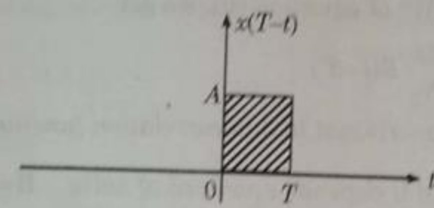
$$h(t) = \begin{cases} \frac{2k}{N_0} A & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \qquad ...(v)$$



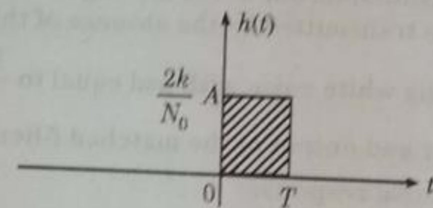
(a) Input signal $x(t)$. It is rectangular pulse given by equation (i)



(b) Input signal pulse folded in time. This signal is represented by equation (iii)



(c) Folded signal of Fig. (b) is delayed by T. This signal is represented by equation (iv)



(d) Impulse response of the matched filter to the input signal pulse in figure (a).

We know that the maximum signal to noise ratio of the matched filter $\left(\frac{S}{N}\right)_{0\,max}$ is given by,

$$\left(\frac{S}{N}\right)_{0\,\max} = \frac{2E}{N_0} = \frac{E}{N_0/2} \qquad \text{...(vi)}$$

Here, $\frac{N_0}{2}$ is given as psd of white noise.

Further, we can evaluate energy of signal $x(t)$ by,

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

Substituting, $x(t) = A$ and changing the limits from 0 to T as described in equation (*i*), the last equation becomes,

$$E = \int_0^T A^2\,dt = A^2[t]_0^T = A^2T$$

Substituting this value in equation (*vi*), we obtain,

$$\left(\frac{S}{N}\right)_{0\,\max} = \frac{2A^2T}{N_0}$$

Output of the Matched Filter:

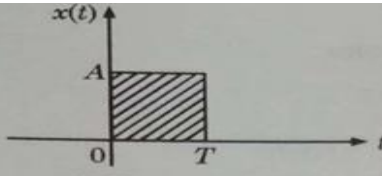
We know that the matched filter is an LTI system. Thus, the output of the matched filter will be the convolution of input and impulse response.

Thus, we have $y(t) = x(t) \otimes h(t)$

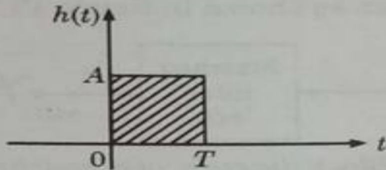
where $y(t)$ = output of matched filter,

$h(t)$ = impulse response of matched filter,

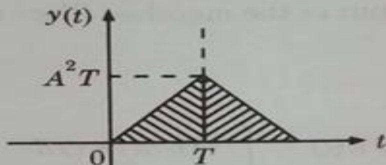
and $x(t)$ = input signal.



(a) Input signal $x(t)$



(b) Impulse response $h(t)$ assuming that $2k/N_0 = 1$



(c) Output of the matched filter for rectangular pulse input. It may be observed that output is maximum at $t = T$

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Thank You !