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EEE/ECE F311
Communication Systems
Tutorial-1

Date : 07/08/2025
12/08/25

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1. Compute the energy and power of the following signals and find out whether the signal is an energy signal or power signal, or neither.

   a) $x(t) = 4\sin\left(2\pi t\right); \ -\infty < t < \infty$

   b) $x(t) = 2e^{-2|t|}; \ -\infty < t < \infty.$

   c) $x(t) = \begin{cases} 2/\sqrt{t}; & t > 1 \\ 0; & t \leq 1 \end{cases}$

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Solution 1(a)

$$E=\int_{-\infty}^{\infty}\left|x(t)\right|^2dt=\int_{-\infty}^{\infty}\left|4\sin(2\pi t)\right|^2dt=16\int_{-\infty}^{\infty}\left[\frac{1-\cos(4\pi t)}{2}\right]dt=8\int_{-\infty}^{\infty}dt-8\int_{-\infty}^{\infty}\cos(4\pi t)dt=\infty$$

$$P=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}x^2(t)dt=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}16\sin^2(2\pi t)dt=8\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}dt-16\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\frac{\cos(4\pi t)}{2}dt=8$$

As the energy of x(t) is infinite, and average power is finite. Thus x(t) is a <u>power signal</u>.

Solution 1(b)

$$E=\int_{-\infty}^{\infty}\left|x(t)\right|^2dt=\int_{-\infty}^{\infty}\left|2e^{-2|t|}\right|^2dt=4\int_{-\infty}^{0}e^{4t}dt+4\int_{0}^{\infty}e^{-4t}dt=2$$

$$P=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}x^2(t)dt=4\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{0}e^{4t}dt+4\lim_{T\to\infty}\frac{1}{T}\int_{0}^{T/2}e^{-4t}dt=0$$

The energy of x(t) is finite, and average power is zero. Thus x(t) is an <u>energy signal.</u>

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Solution 1(c)

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\mathrm{dt} = \lim_{T\to\infty} \frac{1}{T} \int_{1}^{T/2} \frac{4}{t}\,dt$$

$$= 4 \lim_{T\to\infty} \left(\frac{1}{T} \ln\left[t\right]^{T/2}_{1} \right) = 4 \lim_{T\to\infty} \left(\frac{1}{T} \ln\left[\frac{T}{2}\right] - \frac{1}{T} \ln\left[1\right] \right)$$

$$= 4 \lim_{T\to\infty} \left(\frac{1}{T} \ln\left[\frac{T}{2}\right] \right)$$

$$= 4 \lim_{T\to\infty} \left(\frac{\ln\left[\frac{T}{2}\right]}{T} \right)$$

Using L'Hospital's rule we have,

$$P = 4 \lim_{T\to\infty} \left(\frac{\ln\left[\frac{T}{2}\right]}{T} \right) = 4 \lim_{T\to\infty} \left(\frac{\frac{2}{T}}{1} \right) = 0$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{1}^{\infty} \frac{4}{t}\,dt$$

$$= 4 \ln\left[t\right]_1^{\infty}$$

$$= \infty$$

The energy of the signal is infinite & its average power is zero; x(t) is <u>neither energy signal nor power signal</u>.

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2. Determine the power and RMS value of the following signal.

$$x(t) = e^{j2t}\cos\left(10t\right)$$

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Solution 2

$$x(t) = e^{j2t}\cos(10t)$$

$$= (\cos 2t + j\sin 2t)\cos(10t)$$

$$= \frac{(\cos 12t + \cos 8t)}{2} + j\frac{(\sin 12t - \sin 8t)}{2}$$

Power of the signal $= \dfrac{\left(\frac{1}{2}\right)^2}{2} + \dfrac{\left(\frac{1}{2}\right)^2}{2} + \dfrac{\left(\frac{1}{2}\right)^2}{2} + \dfrac{\left(\frac{1}{2}\right)^2}{2} = \dfrac{1}{2}$

RMS value of the signal $= \sqrt{\dfrac{1}{2}}$

$$\cos(10t) = \frac{e^{j10t} + e^{-j10t}}{2}$$

$$x(t) = e^{j2t} \cdot \frac{e^{j10t} + e^{-j10t}}{2} = \frac{1}{2}\left(e^{j(12t)} + e^{-j8t}\right)$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$|x(t)|^2 = \left|\frac{1}{2}\left(e^{j12t} + e^{-j8t}\right)\right|^2 = \frac{1}{4}\left|e^{j12t} + e^{-j8t}\right|^2$$

$$|a+b|^2 = |a|^2 + |b|^2 + 2\mathrm{Re}(ab^*)$$

$$a = e^{j12t}, b = e^{-j8t}$$

$$|x(t)|^2 = \frac{1}{4}(1 + 1 + 2\cos(20t)) = \frac{1}{4}(2 + 2\cos(20t)) = \frac{1}{2}(1 + \cos(20t))$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2}(1 + \cos(20t))dt$$

$$= \frac{1}{2} \lim_{T\to\infty} \left[\frac{1}{2T}\int_{-T}^{T} 1\, dt + \frac{1}{2T}\int_{-T}^{T} \cos(20t)\, dt\right]$$

$$P = \frac{1}{2}\left(\frac{2T}{2T} + 0\right) = \frac{1}{2}$$

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3. Find the odd and even components of the following signal.

$$x(t) = \sin 2t + \sin 2t \cos 2t + \cos 2t$$

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Solution 3

$$x(t) = \sin 2t + \sin 2t \cos 2t + \cos 2t$$

$$x(-t) = \sin\left(-2t\right) + \sin\left(-2t\right)\cos\left(-2t\right) + \cos\left(-2t\right)$$

$$= -\sin 2t - \sin 2t \cos 2t + \cos 2t$$

Therefore the odd and even components are calculated as,

$$x_e(t) = \frac{1}{2}\left[x\left(t\right) + x\left(-t\right)\right] = \cos 2t$$

$$x_o\left(t\right) = \frac{1}{2}\left[x\left(t\right) - x\left(-t\right)\right] = \sin 2t + \sin 2t \cos 2t$$

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4. The energies of the two energy signals x(t) and y(t) are E_x and E_y, respectively.
   a) If x(t) and y(t) are orthogonal, then show that the energy of the signal $x(t) \pm y(t)$ is given by E_x+E_y.

   b) The cross-energy of the two signals is defined as, $$E_{xy} = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt$$

   If $z(t) = x(t) \pm y(t)$, then show that, $E_z = E_x + E_y \pm (E_{xy} + E_{yx})$

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Solution 4(a)

$$\int_{-\infty}^{+\infty}\left|x(t)\pm y(t)\right|^2 dt=\int_{-\infty}^{+\infty}\left|x(t)\right|^2 dt+\int_{-\infty}^{+\infty}\left|y(t)\right|^2 dt\pm\underbrace{\int_{-\infty}^{+\infty}x(t)\,y^*(t)\,dt}_{=0}\pm\underbrace{\int_{-\infty}^{+\infty}x^*(t)\,y(t)\,dt}_{=0}$$

$$=\int_{-\infty}^{+\infty}\left|x(t)\right|^2 dt+\int_{-\infty}^{+\infty}\left|y(t)\right|^2 dt$$

$$=E_x+E_y$$

Solution 4(b)

$$\int_{-\infty}^{+\infty}\left|x(t)\pm y(t)\right|^2 dt=\int_{-\infty}^{+\infty}\left|x(t)\right|^2 dt+\int_{-\infty}^{+\infty}\left|y(t)\right|^2 dt\pm\underbrace{\int_{-\infty}^{+\infty}x(t)\,y^*(t)\,dt}_{\text{Cross Energy Component}}\pm\underbrace{\int_{-\infty}^{+\infty}x^*(t)\,y(t)\,dt}_{\text{Cross Energy Component}}$$

$$=E_x+E_y\pm\left(E_{xy}+E_{yx}\right)$$

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5. Show that an exponential signal e^{-at} starting at $-\infty$ is neither energy nor a power signal for any real value of ‘a’. However, if imaginary, it is a power signal with unity power, regardless of the value of ‘a’.

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Solution 5

For real 'a',

$$E_g = \int_{-\infty}^{+\infty} (e^{-at})^2 dt = \int_{-\infty}^{+\infty} e^{-2at} dt = \infty$$

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} (e^{-at})^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-2at} dt = \infty$$

Hence for real value of 'a', the signal e^{-at} starting at $-\infty$ is neither energy nor a power signal.

Consider, a is imaginary and given by $a = jx$.

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} (e^{jxt})(e^{-jxt})\, dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt = \underline{1} \text{ (Ans)}$$

Hence the statement is proved.

In case of imaginary

$$|x(t)|^2 = x(t)\, x^*(t)$$
$$= e^{j(t)}\, e^{-jt} = e^0 = 1$$