Newton - Raphson Method

Let x_0 be an approximate root of the equation $f(x)=0$.
If $x_1 = x_0 + h$ be the exact root, then $f(x_1)=0$

$\therefore$ Expanding $f(x_0+h)$ by Taylor's series

$$f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \cdots = 0$$

Since h is small, neglecting h^2 and higher powers of h, we get,

$$f(x_0) + h f'(x_0) = 0$$

$$\text{or, } h = -\frac{f(x_0)}{f'(x_0)}$$

$\therefore$ A closer approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly, starting with x_1, a still better approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general, $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$\rightarrow$ Geometric interpretation

Let x_0 be a point near the root α of the equation $f(x)=0$

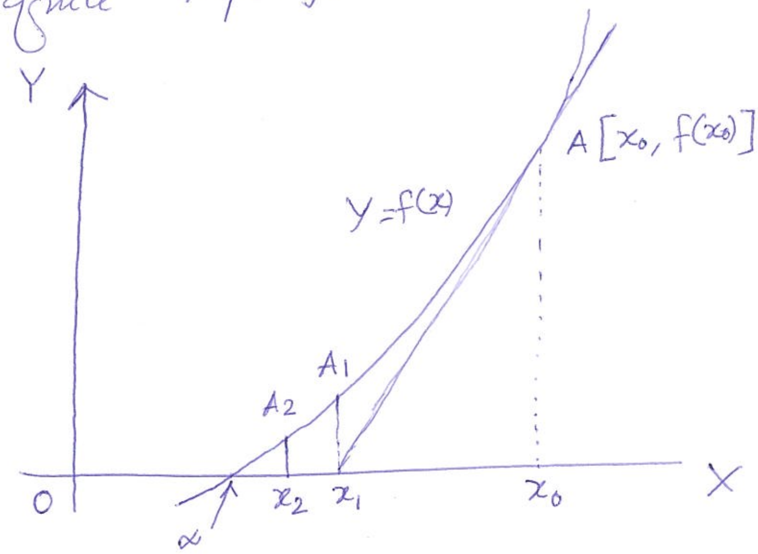
the eqnation of the tangent at $A_0\,[x_0, f(x_0)]$ is

$$y - f(x_0) = f'(x_0)\,(x - x_0)$$

It cuts the x-axis at $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

Which is a first approximation to the root α.

If A_1 is the point Corresponding to x_1 on the curve, then the tangent at A_1 will cut the x-axis at x_2 which is nearer to α and is, therefore, a second approximation to the root. Repeating this process, we approach the root α quite rapidly.



Find by Newton's method, the real root of the eqnation

$$3x = \cos x + 1$$

$$f(x) = 3x - \cos x - 1$$

$f(0) = -2 = -ve$; $f(1) = 1.4597 = +ve$

So, a root of $f(x) = 0$ lies between 0 and 1.

It is nearer to 1. Let us take $x_0 = 0.6$

$$f'(x) = 3 + \sin x$$

~~$f(x) = f(x_n) + f'(x_n)(x - x_n) +$~~

$\therefore$ Newton's iteration formula gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n}$$

$$= \frac{x_n \sin x_n + \cos x_n + 1}{3 + \sin x_n}$$

$n = 0$, $\quad x_1 = \dfrac{x_0 \sin x_0 + \cos x_0 + 1}{3 + \sin x_0} = 0.6071$

$n = 1$, $\quad x_2 = \dfrac{x_1 \sin x_1 + \cos x_1 + 1}{3 + \sin x_1} = 0.6071$

$x_1 = x_2$

The desired root is 0.6071.

Impact of Transmission Constraints on EV penetration

Wind plant cost function $C_1(x_1) = 70 + x_1^2$

Coal " " " $C_2(x_2) = 10 + 50x_2^2$

x_1, x_2 = power generated by wind and coal plants respectively

\# $$\min_{x_1, x_2} \; (70 + x_1^2) + (10 + 50x_2^2)$$

$$x_1 + x_2 = 51 \;\rightarrow\; \text{supply = demand}$$

$$\mathcal{L} = (70 + x_1^2) + (10 + 50x_2^2) + \lambda(51 - x_1 - x_2)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0 = 2x_1 - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0 = 100x_2 - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = 51 - x_1 - x_2$$

Solving, $\boxed{x_1^* = 50, \quad x_2^* = 1}$

\# $$\min_{x_1, x_2} \; (70 + x_1)^2 + (10 + 50x_2^2)$$

$$x_1 + x_2 = 51$$

$$x_1 \leq 10 \;\longrightarrow\; \text{transmission constraints}$$

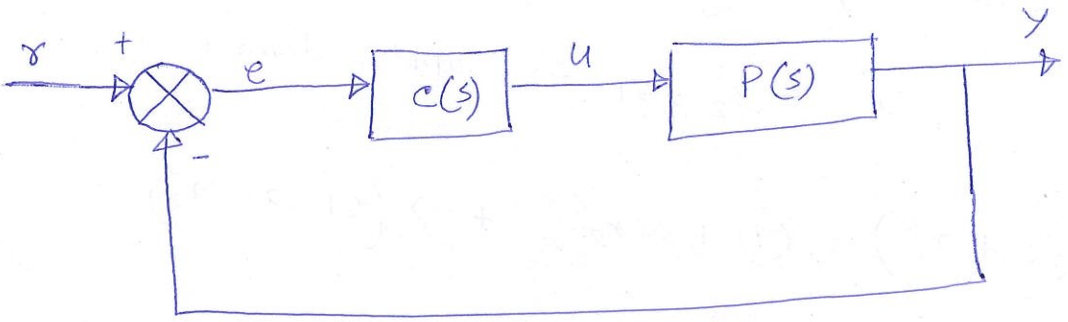
$$\mathcal{L} = (70 + x_1^2) + (10 + 50x_2^2) + \lambda(51 - x_1 - x_2) + \mu(x_1 - 10)$$

Optimality Conditions $\rightarrow$ $\begin{cases} 2x_1 - \lambda + \mu = 0, & 51 = x_1 + x_2, \\ 100x_2 - \lambda = 0, & \mu(x_1 - 10) = 0, \quad x \leq 10 \end{cases}$

Solution → $x_1^* = 10$, $x_2^* = \cancel{40}\ 41$

→ transmission Constraints has increased pollution.

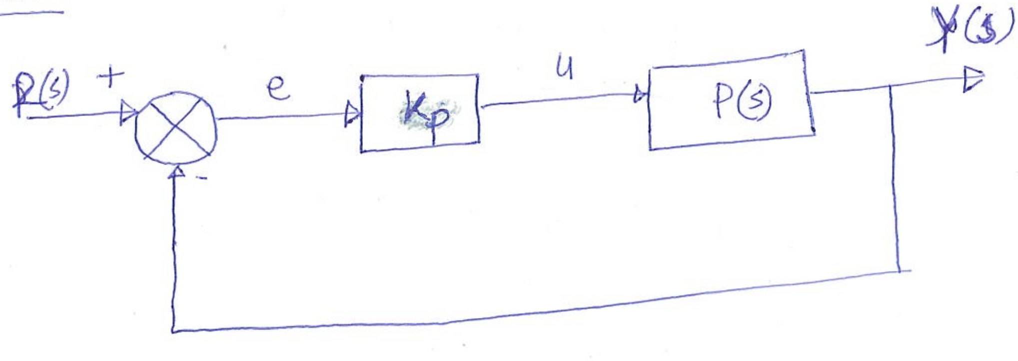
Design of a classical controller for meeting real time demand of an Eves with the help of a diesel generator



$C(s)$ = Transfer function of the controller

$P(s)$ = " " " " diesel generator $= \dfrac{1}{s^2+10s+20}$

* P- Control



$$Y(s) = \frac{K_p\, P(s)}{1 + K_p\, P(s)}\, R(s)$$

$$= \frac{K_p}{s^2+10s+(20+K_p)} \cdot \frac{A}{s}$$

$$E(s) = \frac{A}{s} - \frac{K_p}{s^2 + 10s + (20 + K_p)} \cdot \frac{A}{s}$$

$$= \frac{A}{s}\left[1 - \frac{K_p}{s^2 + 10s + (20 + K_p)}\right]$$

$$= \frac{A}{s}\left[\frac{s^2 + 10s + 20}{s^2 + 10s + 20 + K_p}\right]$$

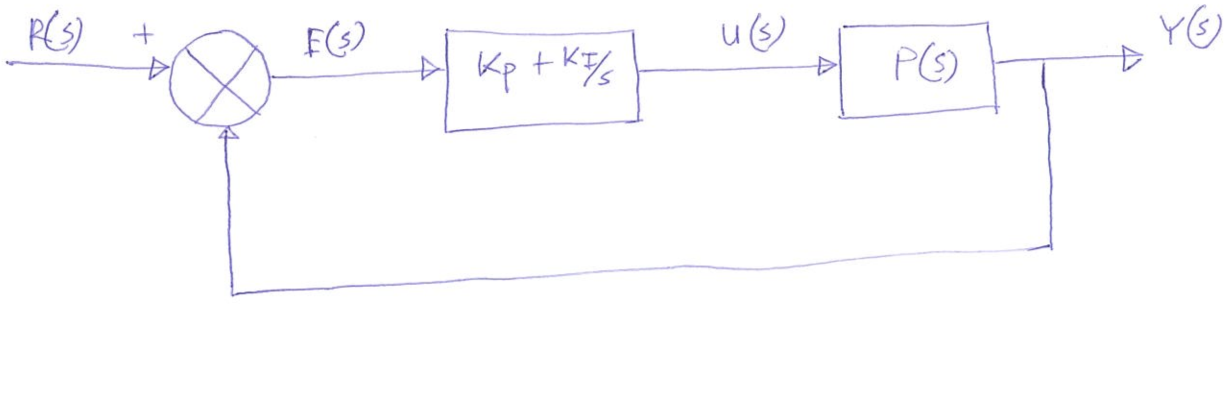
$$e(\infty) = \lim_{s \to 0} sE(s) = \frac{A \cdot 20}{(20 + K_p)}$$ (non-zero steady state error)

$$Y(s) = \frac{K_p}{s^2 + 10s + (20 + K_p)} \frac{A}{s}$$

$$= K_p A\left[\frac{1}{s} - \frac{s + 10}{s^2 + 10s + (20 + K_p)}\right]$$

$$y(t) = \frac{K_p A}{(20 + K_p)}\left[1 - \quad\right] u(t)$$

* PI Control



$$\cancel{\left(R(s) - Y(s)\right)\left(k_p + {}^{k_I}/_s\right) P(s) = Y(s)}$$

$$\text{or,}\quad \cancel{\left(k_p + {}^{k_I}/_s\right) R(s) P(s) - \left(k_p + {}^{k_I}/_s\right) Y(s) P(s) = Y(s)}$$

$$E(s) = R(s) - Y(s)$$

$$= \frac{A}{s} - \frac{\left(k_p + {}^{k_I}/_s\right) P(s)}{1 + \left(k_p + {}^{k_I}/_s\right) P(s)} \frac{A}{s}$$

$$= \frac{A}{s} - \frac{\left(k_p + {}^{k_I}/_s\right) \frac{1}{s^2+10s+20}}{1 + \left(k_p + {}^{k_I}/_s\right) \frac{1}{s^2+10s+20}} \frac{A}{s}$$

$$= \frac{A}{s} - \frac{\left(k_p + {}^{k_I}/_s\right)}{\left(s^2+10s+20\right) + \left(k_p + {}^{k_I}/_s\right)} \frac{A}{s}$$

$$= \frac{A}{s} - \frac{\left(k_p s + k_I\right)}{\left(s^3 + 10s^2 + 20s + k_p s + k_I\right)} \frac{A}{s}$$

$$= \frac{A}{s}\left[1 - \frac{\left(k_p s + k_I\right)}{\left(s^3 + 10s^2 + 20s + k_p s + k_I\right)}\right]$$

$$= \frac{A}{s}\left[\frac{\left(s^3 + 10s^2 + 20s + \cancel{k_p s} + \cancel{k_I}\right) - \cancel{k_p s} - \cancel{k_I}}{\left(s^3 + 10s^2 + 20s + k_p s + k_I\right)}\right]$$

$$s\,E(s) = \frac{A\left[s^3 + 10s^2 + 20s\right]}{\left(s^3 + 10s^2 + 20s + k_p s + k_I\right)}$$

$$e(\infty) = \underset{s \to 0}{Lt}\ sE(s) = \underset{s \to 0}{Lt} \frac{A\,(s^3 + 10s^2 + 20s)}{(s^3 + 10s^2 + 20s + K_P s + K_I)}$$

$$= 0$$

Steady state error is 0. But the response may be slow.

So, we need a PID Controller with proper choice of K_P, K_I, K_D.