

# Electric Vehicles

Pratyush Chakraborty

# Vehicle Dynamics

- How to estimate the forces on the vehicle and the power that needs be delivered by the powertrain to control the vehicle speed?
- We will discuss the forces on the electric vehicle while it's driving, and how to use traction to control vehicle speed.



A good understanding of vehicle-dynamics (common for ICE and Electric Vehicles) **prepares ground** for EV subsystem Designs

- EV Drive-train Requirement comes from this
- Power, Torque, Speed and Energy Considerations

NDTFI

# Driving an ICE or Electric Vehicle

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How much **Power** is required to drive a vehicle?

How much **Energy** is required to carry out a road-trip?

- What is the composite mass of the vehicle (including passenger and goods): **Gross Vehicle Weight (GVW)**
- What is the condition of the roads (**rolling resistance**)
- What is the aerodynamics of the vehicle (**Aerodynamic drag**)
- What is the incline that it needs to traverse? (**Gradient Resistance**)
- What are the velocities and accelerations at different points of time (**Drive Cycle**)
- What is the **maximum speed** and **maximum acceleration** of the vehicle?

# Rolling resistance force

- The rolling resistance force occurs due to the friction between the tires and the driving surface.
- The rolling resistance force is zero at standstill.
- When the vehicle starts moving, the rolling resistance force acts in the direction opposite to the direction of motion and can be calculated by the rolling resistance coefficient  $\mu$  multiplied by the normal force between the vehicle and the road.

# Rolling resistance force

- For a flat surface, the normal force is the vehicle mass  $m$  times the standard gravity  $g$ .
- In the case of a road with an inclination angle, the normal force becomes the weight  $m.g$  multiplied by the cosine of the road angle.
- It is important to note that the rolling resistance force is independent of the vehicle speed, and it is always opposite the driving direction. The coefficient  $\mu$  should be low so as to keep the frictional losses low. For modern cars, it's typically around 0.01 to 0.02.

# Aerodynamic drag force

- As the vehicle speed increase, the aerodynamic drag force opposes the vehicle motion as the air is forced to flow around the moving vehicle.
- It can be calculated as the product of the aerodynamic drag coefficient  $C_d$ , the front area of the vehicle  $A$ , the air density and the square of the vehicle speed  $v$ , divided by 2.

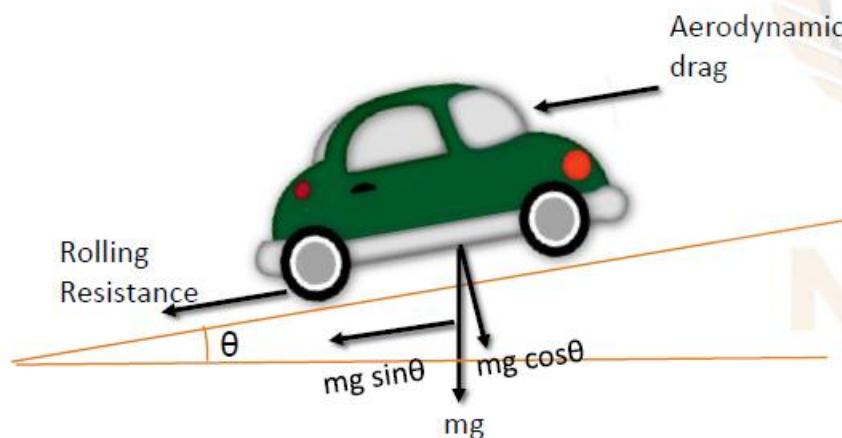
# Aerodynamic drag force

- It is hence important to note that the aerodynamic drag is independent of vehicle mass but has a strong dependence on the vehicle speed. That is why in a car, the aerodynamic drag force is higher than the rolling resistance force when the speed is above about 70 to 80 km/h.
- Secondly, the coefficient of drag is typically about 0.25 to 0.35 for a modern car. SUVs, with their typically boxy shapes, have coefficients in the range of 0.35 to 0.45.

# Gradient/Climbing Force

- The third force that acts on a vehicle is the gradient force, and it occurs when the vehicle is driving on an uphill or a downhill road.
- The gradient force is due to the longitudinal component of gravitational force, namely  $mg \sin \theta$  where  $\theta$  is the inclination angle of the road.
- The gradient force and the angle  $\theta$  are negative when driving downhill, and positive when driving uphill.
- Road gradients are expressed as a percentage in terms of tangent  $\theta$  and have a value typically between plus or minus 10%.

# What does tractive force overcomes?

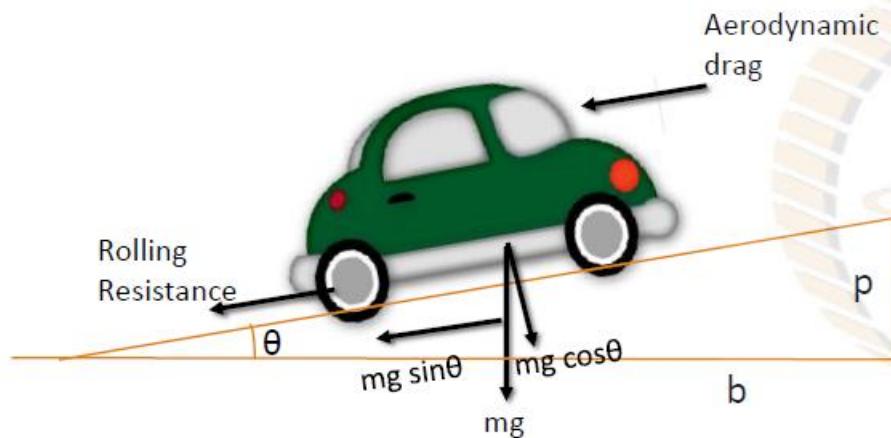


- Aerodynamic Drag
- Rolling Resistance
- Uphill Resistance
- Acceleration

$$\text{Aerodynamic Drag} = \frac{1}{2} * \rho * C_D * A * v^2$$

- $v$  = velocity (m/sec)
- Air density @ 27°C =  $\rho$  = 1.2 (kg/m<sup>3</sup>)
- Vehicle Frontal Area or Projected Area =  $A$  (sq. m)
- Drag coefficient =  $C_D$

# Forces acting on a vehicle in motion



$$\text{Rolling Resistance} = m * g * \mu * \cos\theta$$

- Permissible load =  $m$  (kg)
- Weight =  $mg$  (newton or  $\text{kg.m/s}^2$ ), where  $g = 9.80665 \text{ m/s}^2$
- $\mu$  = rolling coefficient

$$\text{Uphill Resistance or Climbing Force} = mg \sin\theta$$

- Maximum grade =  $\theta^\circ = \theta * \pi / 180$  radians

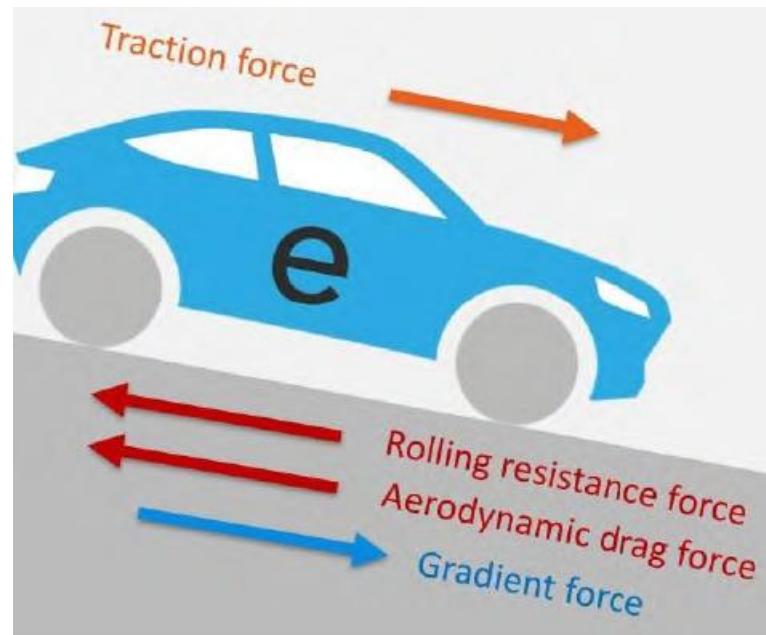
Grade/Inclination:

$$\text{Grade in \%} = \frac{\text{Height of grade}}{\text{Base of the grade}} * 100 \% = \frac{p}{b} * 100 \%$$

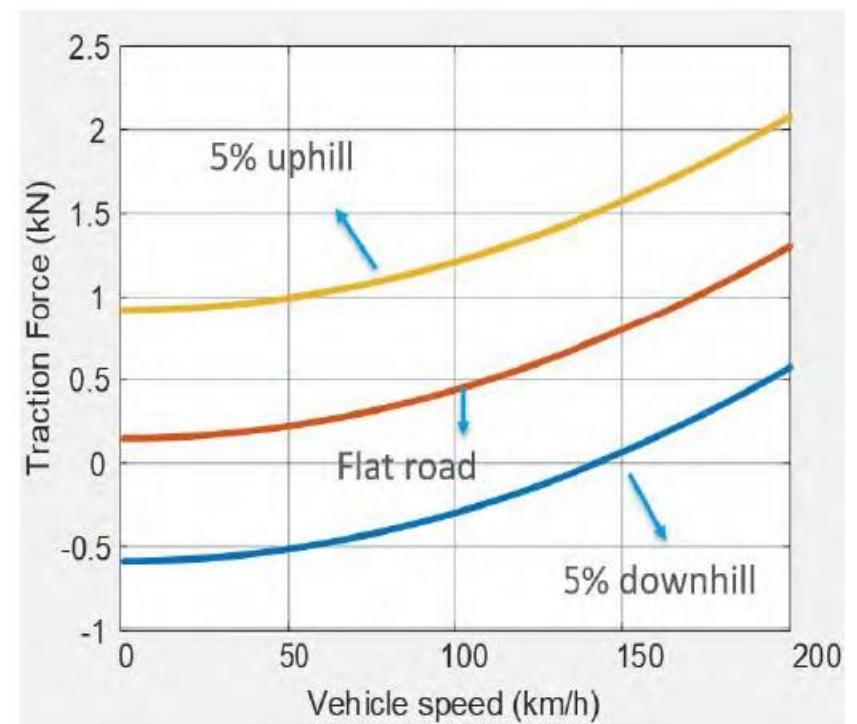
$$\text{Grade in Degree} = \tan^{-1} \frac{p}{b}$$

Tractive force created by power-train first overcomes these resistances and then provides acceleration

# Net force



We can see in the downhill condition, the traction force needed for low speeds is negative, as the gradient force is larger than combined rolling resistance and aerodynamic drag forces. On the other hand, the uphill gradient requires a significantly higher traction force for the same speed than the 0% or downhill gradient.



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Compute Forces due to drag, rolling resistance and gradient for the following vehicles assuming  $\rho = 1.2 \text{ (kg/m}^3\text{)}$  and  $\theta = 8^\circ$ . For the three vehicles given in the table, find Aerodynamic drag at velocity  $v_1$  and  $v_2$ ; also find rolling resistance at two velocities.

Vehicle	GVW (kg)	$C_D$	Area(sqm)	$\mu$	$v_1(\text{kmph})$	$v_2(\text{kmph})$	Tyre radius (m)
2-wheeler	200	0.9	0.6	0.015	30	80	0.28
3-wheeler	600	0.45	1.6	0.015	30	80	0.2
4-wheeler	1500	0.3	2.5	0.015	30	80	0.3

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For (a), Drag (N) =  $\frac{1}{2} * \rho * C_D * A * v^2 = 0.5 * 1.2 (\text{kg/m}^3) * A (\text{m}^2) * C_D * v^2 (\text{m/sec})^2$   
=  $0.5 * 1.2 * 0.6 * 0.9 * (30 * 1000 / 3600)^2 = 0.5 * 1.2 * 0.6 * 0.9 * (30 / 3.6)^2 = 22.5 \text{ N}$

Drag at 80 kmph =  $22.5 * (8/3)^2 = 160 \text{ N}$

Rolling Resistance =  $m * g * \mu * \cos\theta = 200(\text{kg}) * 9.81 (\text{m/s}^2) * 0.015 * \cos(8 * \pi / 180) = 29.4 \text{ N}$

Climbing force =  $m * g * \sin \theta = 200 * 9.81 * \sin(8 * \pi / 180) = 200 * 9.81 * \sin(0.139 \text{ rad}) = 273 \text{ N}$

$0.99$   
 $0.139$

Note

- Units of Drag =  $\text{Kg} * \text{m/sec}^2$  = Newton
- $1 \text{ kmph} = (1000 / 3600) \text{ m/sec} = (1 / 3.6) \text{ m/sec}$

## $P_{\text{trac}}$ applied to vehicle to move

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Vehicle needs Traction Power,  $P_{\text{trac}}$  (in Watts), applied for it to move and accelerate

- Traction power in Internal Combustion Engine (ICE) comes from petrol / diesel engine
- Traction power in EVs comes from Battery through motors and its controllers

The traction power creates a Force  $F_{\text{trac}}$  on the vehicle to move forward

- $P_{\text{trac}} = F_{\text{trac}} * v$ , where  $v$  is velocity (in m/sec) of the vehicle

The resulting Torque  $T$  (in Nm) on the vehicle wheel created by the force is

- $T = F_{\text{trac}} * r_{\text{wheel}}$ , where  $r_{\text{wheel}}$  is radius of the vehicle in meters

**Torque and Speed** (referred to as **rpm**) are the fundamental parameters of a motor or an engine, and vehicle rpm is obtained from  $v = \text{rpm} * 2 * \pi * r_{\text{wheel}} / 60$

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$$v \text{ (m/sec)} = rpm * 2*\pi* r_{\text{wheel}} / 60$$

$$v \text{ in kmph} = v(\text{m/s}) * 3600/1000 = (3.6*2 * \pi / 60) * rpm * r_{\text{wheel}}$$

- Or  $v \text{ in kmph} = 0.377 \text{ rpm} * r_{\text{wheel}}$
- For example, for a 2-wheeler, if rpm is 7500 and  $r_{\text{wheel}}$  is 0.28 m, Velocity (kmph) is 790 km
  - If gear ratio is 9, the vehicle can go to about 89 kmph
  - The Torque will now get multiplied by 9

## Thus Traction Force is given by

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- $F_{trac}$  = Acceleration Force + Aerodynamic Drag + Rolling Resistance + Climbing Force  
 $= m * a + \frac{1}{2} * \rho * C_D * A * v^2 + m * g * \mu + mg \sin\theta$ , where  $a$  is the acceleration and is  $dv/dt$

The energy consumed by vehicle in motion is the integration of Traction Power

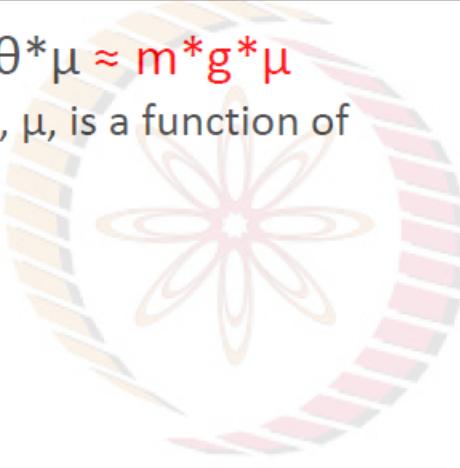
- $Energy = \int P_{trac} dt$  in Watt-sec and is converted to kWh by dividing by 3.6
- Vehicle may have regeneration, which converts deceleration of vehicle while climbing down or otherwise applying brakes (using Regenerative Braking) into Regenerative Energy
- Thus net energy consumed is  $R * Energy$ , where  $R$  is regeneration efficiency
  - As Regeneration factor is typically 15% to 30%,  $R$  is  $(1-RegenFactor)$  or typically 0.85 to 0.70

# Rolling Resistance

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$$\text{Rolling Resistance} = m \cdot g \cdot \cos\theta \cdot \mu \approx m \cdot g \cdot \mu$$

- The rolling resistance coefficient,  $\mu$ , is a function of
  - tyre material
  - tyre structure
  - tyre temperature
  - tyre inflation pressure
  - tread geometry
  - road roughness
  - road material and presence of absence of liquids on the road
- Also  $\mu$  is a function of velocity:  $\mu \approx \mu_0 (1 + v/160)$  for cars on concrete up to 120 kmph
  - And  $\mu_0$  is rolling resistance at zero velocity

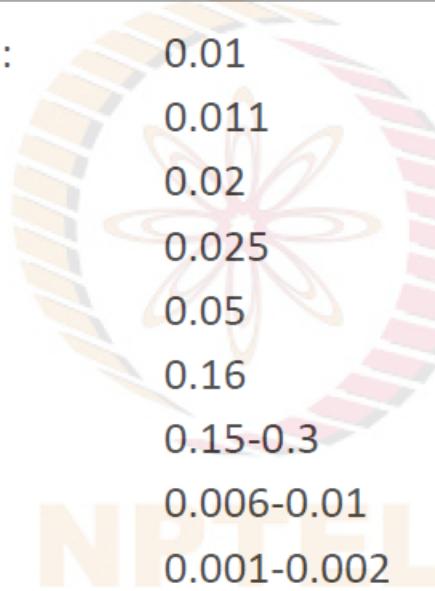


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# Typical values of Rolling Resistance

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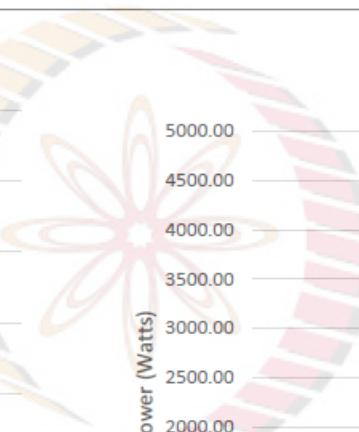
- Car tyre on smooth tarmac road: 0.01
- Car tyre on concrete road : 0.011
- Car tyre on a rolled gravel road: 0.02
- Tar macadam road 0.025
- Unpaved road 0.05
- Bad earth tracks 0.16
- Loose sand 0.15-0.3
- Truck tyre (concrete/ asphalt) 0.006-0.01
- Wheel on iron rail 0.001-0.002



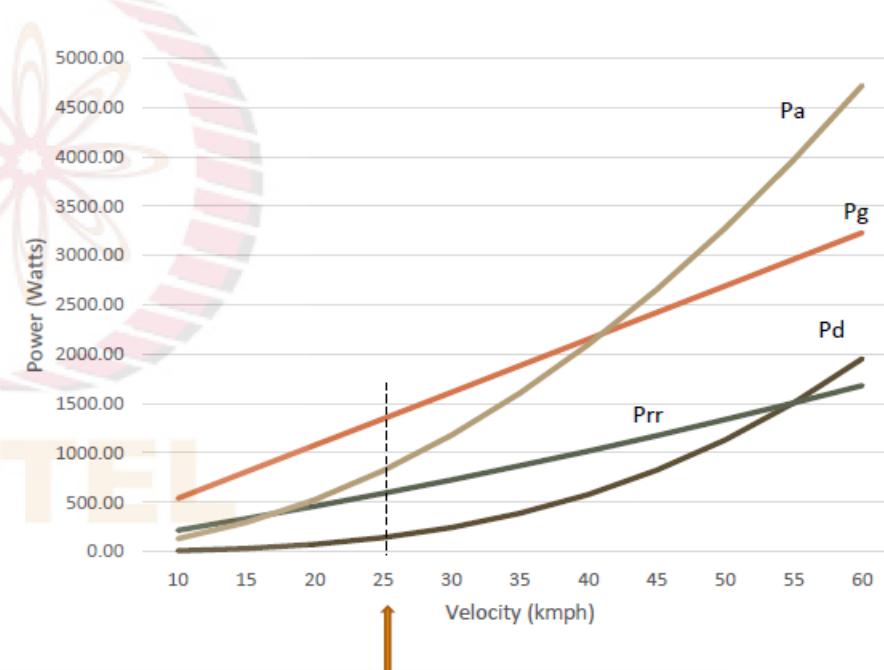
Force due to rolling resistance is a function of velocity **only at high speed**

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# E-rickshaw: Power/ Force components



Power for e-rickshaw



$$\rho = 1.2 \text{ kg/m}^3, C_D = 0.44, A = 1.6 \text{ sqm}, \mu = 0.013, \text{ weight} = 680 \text{ Kg, Gradient of } 5^\circ$$

# Prob 1

- A EV battery has a capacity of 15 kWh. Assuming 0.9 DoD and 75% end of life, what is the range that the vehicle (using 80 Wh/km) can support, when the battery is new. What range will it support at the end of life? The efficiency of motor and controller is 85% each.
- Case 1: Assume, no auxiliary power is used.
- Case 2: Assume, Auxiliary power used is 500 W continuously. The average speed of the car is 40 km/hr

Depth of discharge is defined as the maximum allowable discharging energy below which the lifetime of a battery energy storage (BES) device would be degraded, associated with a critical or cutoff voltage that indicates the lowest potential value for complete battery discharge.

## Prob 2

- What is the Force required to reach maximum speed  $v_f$  in  $T$  seconds if the acceleration is “ $a$ ”?
- in  $T$  seconds, if a vehicle accelerates at a rate “ $a_1$ ” for first  $T/2$  time and at a rate “ $a_1/2$ ” from  $T/2$  to  $T$ . Find the average power during acceleration and the power required at the end of time  $T$ . Compare it if there was linear acceleration  $a$ .

