Economic Dispatch with Elastic Demand

$$U(D) = \sum_{j=1}^{m} U_j(D_j) \qquad D = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix}$$

$U_j \rightarrow$ Utility function for a consumer (consider EVCS here along with other flexible consumers.)

$$\text{Social Welfare} = \sum_{j=1}^{m} U_j(D_j) - \sum_{i=1}^{n} F_i(G_i)$$ $\longrightarrow$ maximize this function

Subj. to $\sum_{j=1}^{m} D_j = \sum_{i=1}^{n} G_i$

$$\mathcal{L} = \sum_{j=1}^{m} U_j(D_j) - \sum_{i=1}^{n} F_i(G_i) - \lambda\left(\sum_{j=1}^{m} D_j - \sum_{i=1}^{n} G_i\right)$$

$$\frac{\partial \mathcal{L}}{\partial D_j} = \frac{dU_j(D_j)}{dD_j} - \lambda = 0 \qquad \forall j = 1, \ldots, m$$

$$\frac{\partial \mathcal{L}}{\partial G_i} = -\frac{dF_i(G_i)}{dG_i} + \lambda = 0 \qquad \forall i = 1, \ldots, n$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \sum_{i=1}^{n} G_i - \sum_{j=1}^{m} D_j = 0$$

The above result implies that all the generating units must operate at identical incremental costs and that all demands operate at identical marginal utilities. Moreover, the single incremental cost must be equal to the single incremental utility.

$$dF(G) = \sum_{i=1}^{n} dF_i(G_i)$$

$$= \sum_{i=1}^{n} \frac{dF_i(G_i)}{dG_i} dG_i = \sum_{i=1}^{n} \lambda \, dG_i = \lambda \, dD$$

$$\boxed{\lambda = \frac{dF(G)}{dD} = \frac{dF_i(G_i)}{dG_i} = \frac{dU_j(D_j)}{dD_j}}$$

Consider, j th EVCS,

It will maximiz $\left[U_j(D_j) - C_j\right]$

where $U_j(D_j) = \sum_{k=1}^{K} U_{jk}(D_{jk})$, k cars

C_j = Cost of buying D_j units of electricity

Each car's objective is to maximize $U_{jk}(D_{jk}) - P D_{jk}$

$$\left(\frac{dU_{jk}(D_{jk})}{dD_{jk}} = P \right) \rightarrow (1)$$

optimality conditions

$$\frac{\partial U_j(D_j)}{\partial D_{jk}} - \frac{dC_j}{dD_{jk}} = 0$$

or, $$\frac{dU_{jk}(D_{jk})}{dD_{jk}} = \frac{dC_j}{dD_{jk}} \quad (2)$$

$$\boxed{P = \frac{dU_{jk}(D_{jk})}{dD_{jk}} = \frac{dC_j}{dD_{jk}}}$$

The designed price is optimal lagrange parameter of the economic dispatch problem.