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Analyzing somitegenesis models using XPP

1. The Clock and Wavefront model

The Clock and Wavefront model is by far one of the most effective models describing somitegenesis process. The system contains forcing segments and diffusion segments, with diffusion segments vary with respect to x. In the XPP analysis, we want to ignore the diffusion segment and only look at the forcing segment, which is given by the below equations:

$$\frac{\partial u}{\partial t} = \frac{(u+\mu v)^2}{\gamma + \rho u^2}\chi_u$$
$$\frac{\partial v}{\partial t} = \frac{\kappa}{\epsilon + u}\chi_v - \lambda v$$

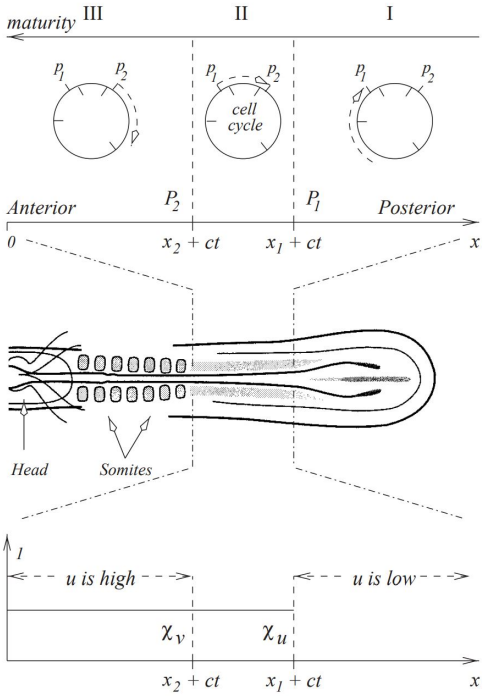
The equations are non-dimensionalized to allow an absolute measure of the quantities involved independent of units of measurement and to reduce the number of parameters. And it becomes:

$$\frac{\partial u}{\partial t} = \frac{(u+\mu v)^2}{\gamma + u^2}\chi_u$$
$$\frac{\partial v}{\partial t} = \kappa\left(\frac{\chi_v}{\epsilon + u} - v\right.$$

Where Xu, Xv are controlled by two Heaviside functions:

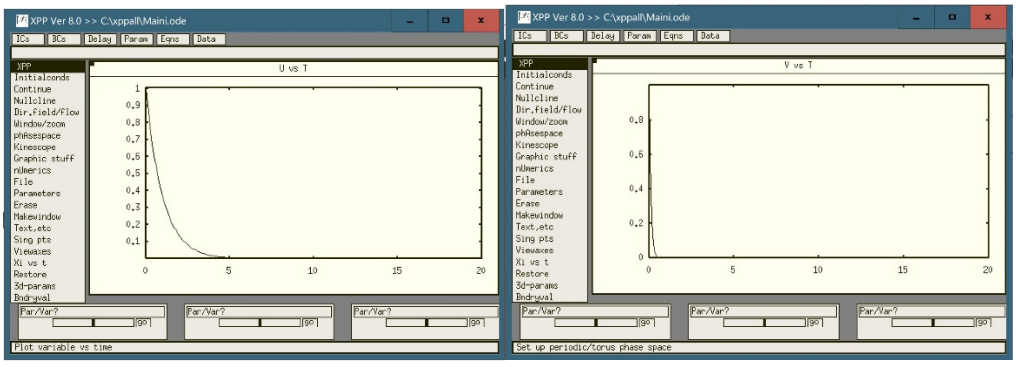
$$\chi_u(x,t) = H(ct - x + x_1), \qquad \chi_v(x,t) = H(ct - x + x_2).$$

U, V respectively represents somatic factor concentration and signaling molecule levels. Somatic factor is produced when the cell is about to approach the determination front. And after one clock oscillation, cells become able to produce the signaling molecule, and after responding to the molecule, a cell is specified as somatic and becomes refractory to FGF8 signaling.

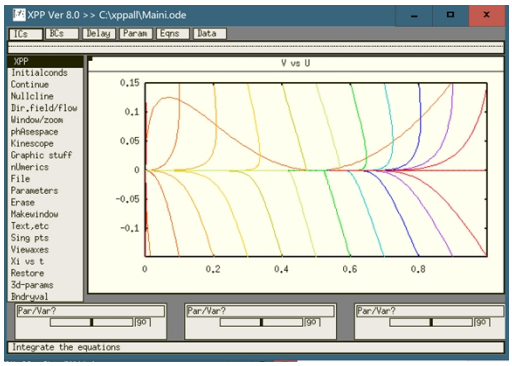


From a mathematical perspective, this process can be segmented into three main stages. The first stage is when the cell hasn't arrive at the determination front, and it can neither respond to signaling molecules nor emit them. The second is during the time when the cell is undergoing one circle of segmentation clock, where it can respond to molecules but can't emit them. The third is when after it finishes one circle and becomes somitic, and somitic cells can either response or emit signaling molecules. Another important factor in this process is the gradient of FGF8, which has a downregulation on the process. Yet, since it does not directly influence u and v, the equation about fgf8 can be omitted when analyzing the somitic factor and signaling molecule.

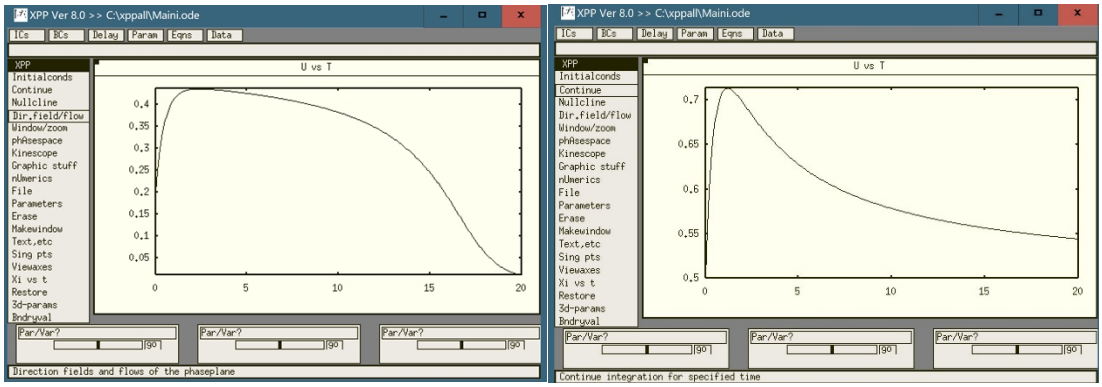
In the first stage, Xu=0, Xv=0. U and V decays with V decaying much faster than U if k>1. It's imaginable that in their phase plane, no matter where they start from, they will quickly go to the origin like a half-parabola, since there will be only one stable point, which is the origin.



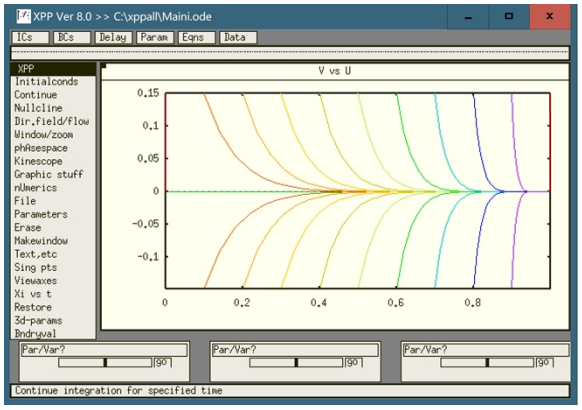
In the second stage. Xu=1, Xv=0. U decays in a lower speed while V in the same speed. The phase plane of u and v is the following, with v0=0.5, μ=0.5, κ=2, ϒ=0.25, which is in accordance with the reference 9 paper.



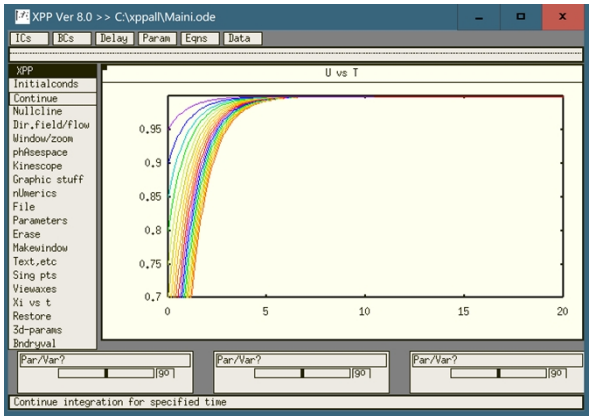
The nullclines of u and v have two intersections, both are stable equilibriums. If U0 is below a certain level, it will decrease to 0, otherwise, it will increase to 1. In the two figures below, the first one is with u0=0.2, and it quickly diminished to 0, while the second is with u0=0.5, and it didn't go to 0.



When I adjust the parameters into v0=0.5, μ=0.0001, κ=10, ϒ=0.001, in accordance with the Maini paper, the phase-plane changes:

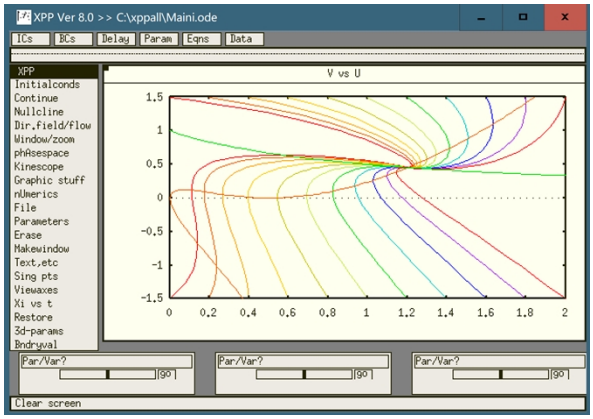


The nullcline of u is a vertical line at u=1 and it makes one intersection with the nullcline of v. It's still a stable equilibrium and no matter what's U's initial value, it will approach 1, whereas V always diminish to 0. This shows that in stage two, after reaching the determination front and before finishing one cycle of segmentation clock, cells have the ability to produce somitic factor but can not produce signaling molecule, which is why V always diminish to 0.

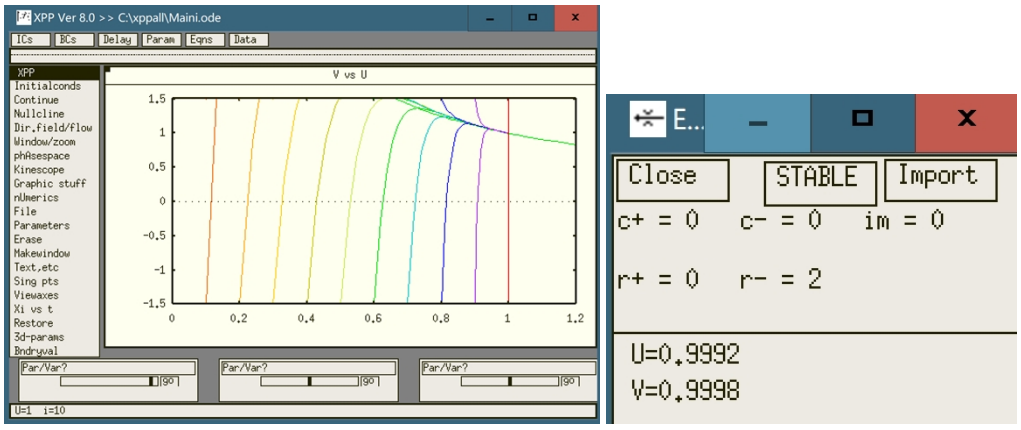


This comparison shows that this model undergoes a bifurcation when varying μ, κ, and ϒ. Since u and v in this case aren't oscillating, I assume it would be a saddle-node bifurcation.

In the third stage. Xu=1, Xv=1. When setting μ=0.5, κ=2, ϒ=0.25, ε=1, the nullclines of u and v have one intersection which is a stable equilibrium. And no matter what their initial values are, they all approach that equilibrium point.



For μ=0.0001, κ=10, ϒ=0.001, ε=0.001, the phase plane changes, as well as the equilibrium:



U and V approaches (1,1) regardless of their initial values. Which shows that cells gain the ability to produce somitic factor and signaling molecule after one cycle of segmentation clock is finished.