Submitted by

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MATLAB version 7.10.0 R2010a

5.3 Solve the problem in example 5.1 using central difference method implemented by a computer program in language of your choice using delta t=0.1sec. Note that this problem was presented in example 5.2 and the results were presented in table 5.2

Solution

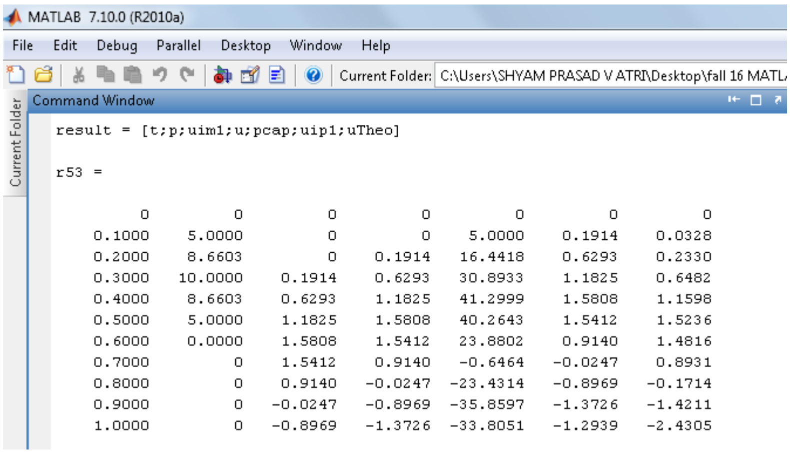
Use the general program written for implementing central difference method

And make the following call

```
% result = [t;p;uim1;u;pcap;uip1;uTheo]= CentralDifference(dt,z,tn)
r53=CentralDifference(0.1,0.05,1)
```

The Theoretical value computed is in close comparison to the numerical values u computed using central difference method.

Here is the result Image




```
result = [t;p;uim1;u;pcap;uip1;uTheo]

r53 =
```

0	0	0	0	0	0	0
0.1000	5.0000	0	0	5.0000	0.1914	0.0328
0.2000	8.6603	0	0.1914	16.4418	0.6293	0.2330
0.3000	10.0000	0.1914	0.6293	30.8933	1.1825	0.6482
0.4000	8.6603	0.6293	1.1825	41.2999	1.5808	1.1598
0.5000	5.0000	1.1825	1.5808	40.2643	1.5412	1.5236
0.6000	0.0000	1.5808	1.5412	23.8802	0.9140	1.4816
0.7000	0	1.5412	0.9140	-0.6464	-0.0247	0.8931
0.8000	0	0.9140	-0.0247	-23.4314	-0.8969	-0.1714
0.9000	0	-0.0247	-0.8969	-35.8597	-1.3726	-1.4211
1.0000	0	-0.8969	-1.3726	-33.8051	-1.2939	-2.4305

5.4 Repeat the problem 5.3 using delta t=0.05sec. How does the time step affect the accuracy of the solution?

The Same function is used with minor small change to time step of 0.05.

```
clear all
clc
result = [t;p;uim1;u;pcap;uip1;uTheo]= CentralDifference(dt,z,tn)
r54=CentralDifference(0.05,0.05,1)
```

MATLAB 7.10.0 (R2010a)

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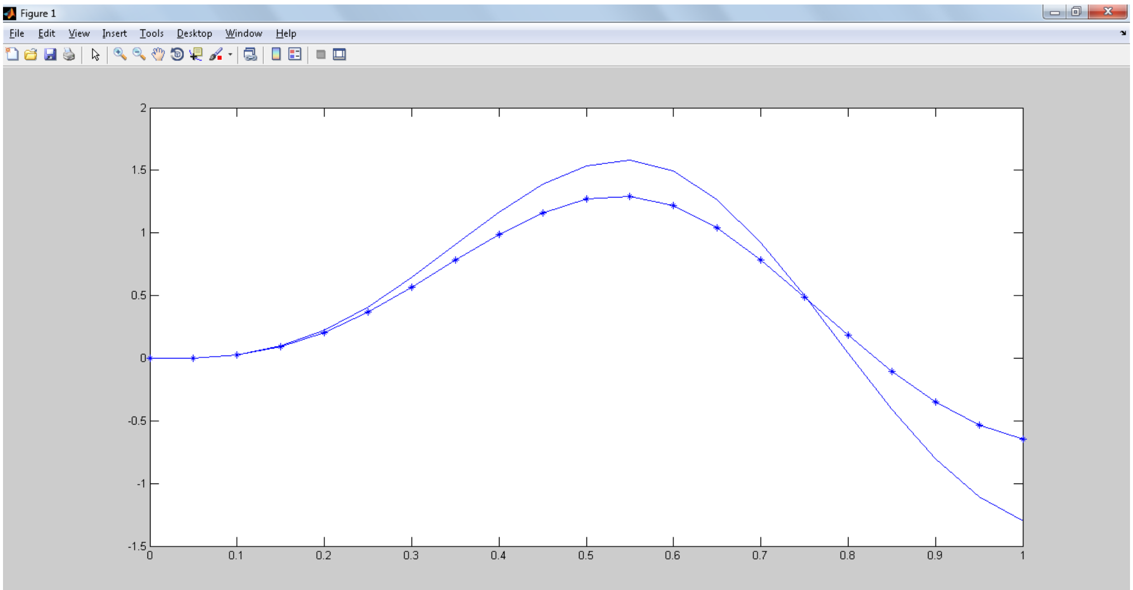
Command Window

```
result = [t;p;uim1;u;pcap;uip1;uTheo]

r54 =

         0         0         0         0         0         0         0
    0.0500    2.5882         0         0    2.5882    0.0251    0.0042
    0.1000    5.0000         0    0.0251    9.8448    0.0957    0.0328
    0.1500    7.0711    0.0251    0.0957   22.9915    0.2234    0.1052
    0.2000    8.6603    0.0957    0.2234   42.1576    0.4096    0.2330
    0.2500    9.6593    0.2234    0.4096   66.2937    0.6442    0.4173
    0.3000   10.0000    0.4096    0.6442   93.2415    0.9060    0.6482
    0.3500    9.6593    0.6442    0.9060  119.9546    1.1656    0.9053
    0.4000    8.6603    0.9060    1.1656  142.8457    1.3880    1.1598
    0.4500    7.0711    1.1656    1.3880  158.2194    1.5374    1.3776
    0.5000    5.0000    1.3880    1.5374  162.7433    1.5814    1.5236
    0.5500    2.5882    1.5374    1.5814  153.9015    1.4955    1.5663
    0.6000    0.0000    1.5814    1.4955  130.3785    1.2669    1.4816
    0.6500         0    1.4955    1.2669   94.9140    0.9223    1.2568
    0.7000         0    1.2669    0.9223   51.3236    0.4987    0.8931
    0.7500         0    0.9223    0.4987    4.0943    0.0398    0.4068
    0.8000         0    0.4987    0.0398  -42.0720   -0.4088   -0.1714
    0.8500         0    0.0398   -0.4088  -82.7222   -0.8038   -0.7981
    0.9000         0   -0.4088   -0.8038 -114.0769   -1.1085   -1.4211
    0.9500         0   -0.8038   -1.1085 -133.3769   -1.2960   -1.9838
    1.0000         0   -1.1085   -1.2960 -139.1195   -1.3518   -2.4305
```

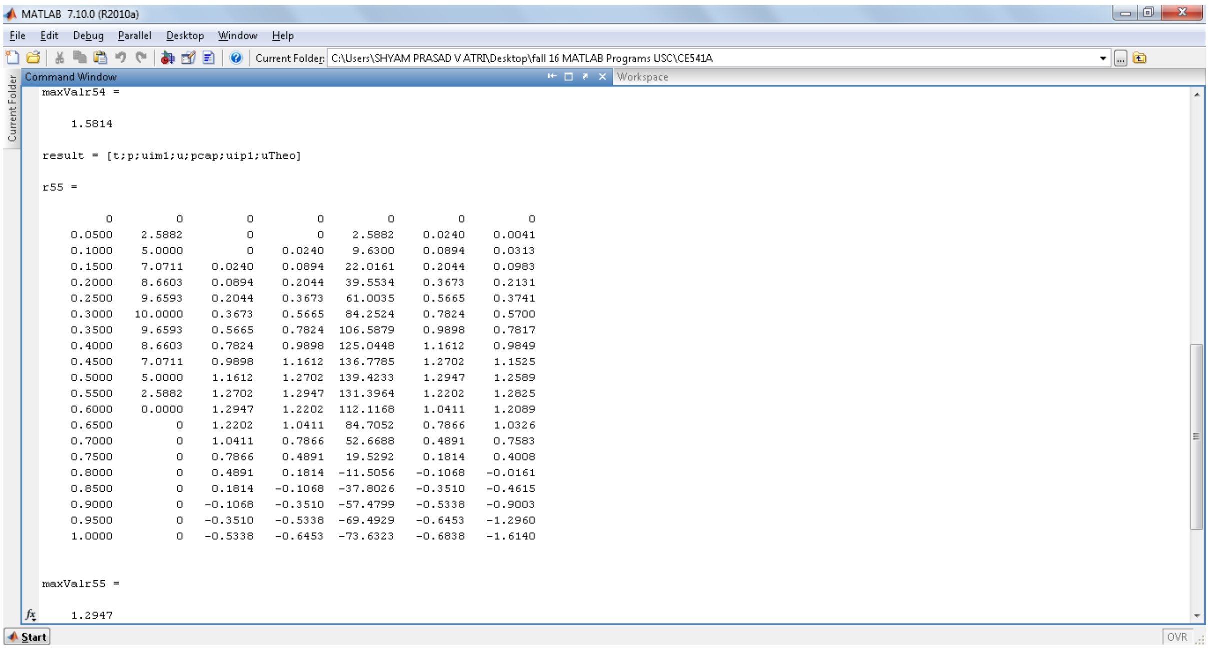
We observe that as the time step is reduced the solution becomes more accurate which is clearly seen from the above table that is as the time step reduces the solution is more close to the theoretical solution.

5.5 An SDF system has the same mass and stiffness as in example 5.1, But the damping ratio is zeta=20%. Determine the response to this system to the excitation as in example 5.1, by the central difference method using delta t=0.05. Plot the response as a function of time, compare the solution with problem 5.3 and comment how the damping affects the peak response.

```
clear all
clc
%result = [t;p;uim1;u;pcap;uip1;uTheo]= CentralDifference(dt,z,tn)
r54=CentralDifference(0.05,0.05,1)
maxValr54=max(r54(:,4))
r55=CentralDifference(0.05,0.2,1)
maxValr55=max(r55(:,4))
plot(r54(:,1),r54(:,4))
hold on
plot(r55(:,1),r55(:,4),'-*')
```



The solution with '*' depicts the solution for z=0.2



```
maxValr54 =

    1.5814

result = [t;p;uim1;u;pcap;uip1;uTheo]

r55 =

         0         0         0         0         0         0         0
    0.0500    2.5882         0         0    2.5882    0.0240    0.0041
    0.1000    5.0000         0    0.0240    9.6300    0.0894    0.0313
    0.1500    7.0711    0.0240    0.0894   22.0161    0.2044    0.0983
    0.2000    8.6603    0.0894    0.2044   39.5534    0.3673    0.2131
    0.2500    9.6593    0.2044    0.3673   61.0035    0.5665    0.3741
    0.3000   10.0000    0.3673    0.5665   84.2524    0.7824    0.5700
    0.3500    9.6593    0.5665    0.7824  106.5879    0.9898    0.7817
    0.4000    8.6603    0.7824    0.9898  125.0448    1.1612    0.9849
    0.4500    7.0711    0.9898    1.1612  136.7785    1.2702    1.1525
    0.5000    5.0000    1.1612    1.2702  139.4233    1.2947    1.2589
    0.5500    2.5882    1.2702    1.2947  131.3964    1.2202    1.2825
    0.6000    0.0000    1.2947    1.2202  112.1168    1.0411    1.2089
    0.6500         0    1.2202    1.0411   84.7052    0.7866    1.0326
    0.7000         0    1.0411    0.7866   52.6688    0.4891    0.7583
    0.7500         0    0.7866    0.4891   19.5292    0.1814    0.4008
    0.8000         0    0.4891    0.1814  -11.5056   -0.1068   -0.0161
    0.8500         0    0.1814   -0.1068  -37.8026   -0.3510   -0.4615
    0.9000         0   -0.1068   -0.3510  -57.4799   -0.5338   -0.9003
    0.9500         0   -0.3510   -0.5338  -69.4929   -0.6453   -1.2960
    1.0000         0   -0.5338   -0.6453  -73.6323   -0.6838   -1.6140


maxValr55 =

    1.2947
```

5.6 Solve the problem in example 5.1 by central difference method using delta t=1/3 sec. Carry out the solution to 2 sec, and comment on what happens to the solution and why.

```
clear all
clc
%result = [t;p;uim1;u;pcap;uip1;uTheo]= CentralDifference(dt,z,tn)
r56=CentralDifference(1/3,0.05,2)
```

```
r56 =

         0         0         0         0         0         0         0
    0.3333    9.8481         0         0    9.8481    3.9104    0.8184
    0.6667         0         0    3.9104  -21.2749   -8.4477    1.1505
    1.0000         0    3.9104   -8.4477   37.9795   15.0806   -2.4305
    1.3333         0   -8.4477   15.0806  -64.8062  -25.7328   -0.3851
    1.6667         0   15.0806  -25.7328  109.2225   43.3693    3.6182
    2.0000         0  -25.7328   43.3693 -183.4351  -72.8371   -1.7509
```

Here the central difference gives meaning less results as deltaT/Tn=1/3 which exceeds the stability limit 1/pi.

The program for all the problems

```
function [result] = CentralDifference(dt,z,tn)
m=0.2533; %kip s2 in-1
k=10; %kips in-1
%tn=1; %s
wn=(k/m)^0.5; %rd s-1
%z=0.05;% zeta
c=2*m*wn*z; % damp coef
%dt=0.1;%s

t=0:dt:tn;


p=10*sin(pi*t/0.6);

h=1;

while(h<=size(t,2) )
  if t(h)>0.6
     p(h)=0;
  end
  h=h+1;
end

u=zeros(1,size(t,2));
% uTheo=zeros(1,size(t,2));
pcap=zeros(1,size(t,2));


u0=0; % initial displacement
u(1)=u0;
ud0=0; % initial velocity
udd0=(p(1)-c*u0-k*u0)/m;
um1=(u0-dt*ud0+dt^2*0.5*udd0);

kcap=m/dt^2+c/(2*dt);
a=m/dt^2-c/(2*dt);
b=k-2*m/(dt)^2;

pcap(1)=p(1)-a*um1-b*u(1);

for i=2:size(t,2)

    u(i)=pcap(i-1)/kcap;
```

```
    pcap(i)=p(i)-a*u(i-1)-b*u(i);

end

uim1=[um1 u(1:(size(t,2)-1))];
up1=pcap(size(t,2))/kcap;
uip1=[u(2:(size(t,2))) up1];

%Theoritical values computed using first principles
% w=pi/0.6;
% omg=w/wn;
% C=(10/k)*((1-omg^2)/((1-omg^2)^2+(2*z*omg)^2));
% D=(10/k)*((-2*z*omg)/((1-omg^2)^2+(2*z*omg)^2));
% for k=1:size(t,2)
%
%     uTheo(1,k)=C*sin(w*t(k))+D*cos(w*t(k));
% end

% The theoritical Value is calculated
% usin Duhamels integral
syms fA wfA tA wA zA tauA k
%vA=int((fA*sin(wfA*tauA)*(wA/(k*(1-zA^2)^0.5))*(exp(-zA*wA*(tA-tauA)))*(sin(wA*(tA-tauA)*(1-zA)^0.5))),tauA,0,tA);
%vA=int((fA*sin(wfA*tauA)*(wA/(k*(1-zA^2)^0.5))*(exp(-zA*wA*(tA-tauA)))*(sin(wA*(tA-tauA)*(1-zA)^0.5))),tauA,0,tA);

% differential equation is
% mxdd+cxd+kx= 10 sin(pi*t/0.6)
vA=int((fA*sin(wfA*tauA/0.6)*(wA/(k*(1-zA^2)^0.5))*(exp(-zA*wA*(tA-tauA)))*(sin(wA*(tA-tauA)*(1-zA^2)^0.5))),tauA,0,tA);


k=10;
fA=10;
wA=2*3.14;
zA=z;
wfA=pi;
tA=0:dt:tn;
res=eval(vA);

t=0:dt:tn;
disp('result = [t;p;uim1;u;pcap;uip1;uTheo]')

result=[t;p;uim1;u;pcap;uip1;res]';
%plot(t,u,t,res)

end
%%%%% calling the function
clear all
clc
%result = [t;p;uim1;u;pcap;uip1;uTheo]= CentralDifference(dt,z,tn)
r53=CentralDifference(0.1,0.05,1)
r54=CentralDifference(0.05,0.05,1)
maxValr54=max(r54(:,4))
r55=CentralDifference(0.05,0.2,1)
```

```
maxValr55=max(r55(:,4))
r56=CentralDifference(1/3,0.05,2)
plot(r54(:,1),r54(:,4))
hold on
plot(r55(:,1),r55(:,4),'-*')
```

6.11 a. A full water tank is supported on a 80ft cantilever tower. It is idealized as an SDF system with weight w=100kips, lateral stiffness k= 4kips/in, the damping ratio is zeta=5%. The tower supporting the tank is to be designed for ground motion characterised by the design spectrum of fig 6.9.5 scaled to 0.5g peak ground acceleration. Determine the design values of lateral deformation and base shear.

b. The deformation computed for the system in part a seemed excessive to the structural designer, who decided to stiffen the structure by increasing the size. Determine the design values of deformation and base shear for the modified system if its lateral stiffness is 8kips/in. Assume the damping ratio is still 5%. Comment on how stiffening of the system has affected the design requirements. What is the disadvantage of stiffening of the system?

Sec. 6.9 Elastic Design Spectrum 235

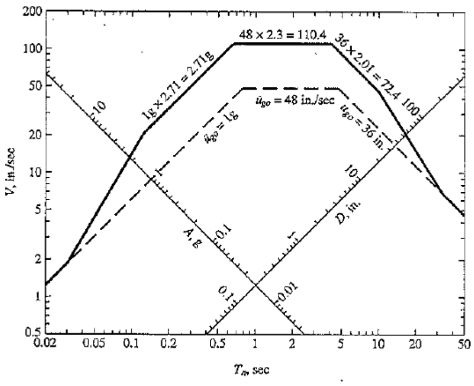


Figure 6.9.4 Construction of elastic design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1g$, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

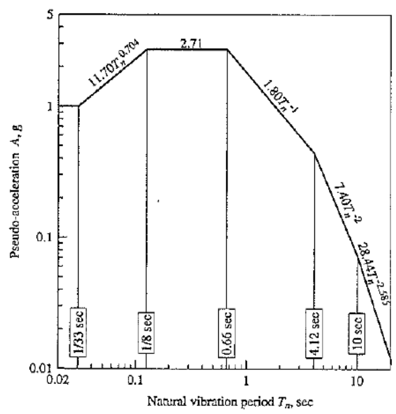
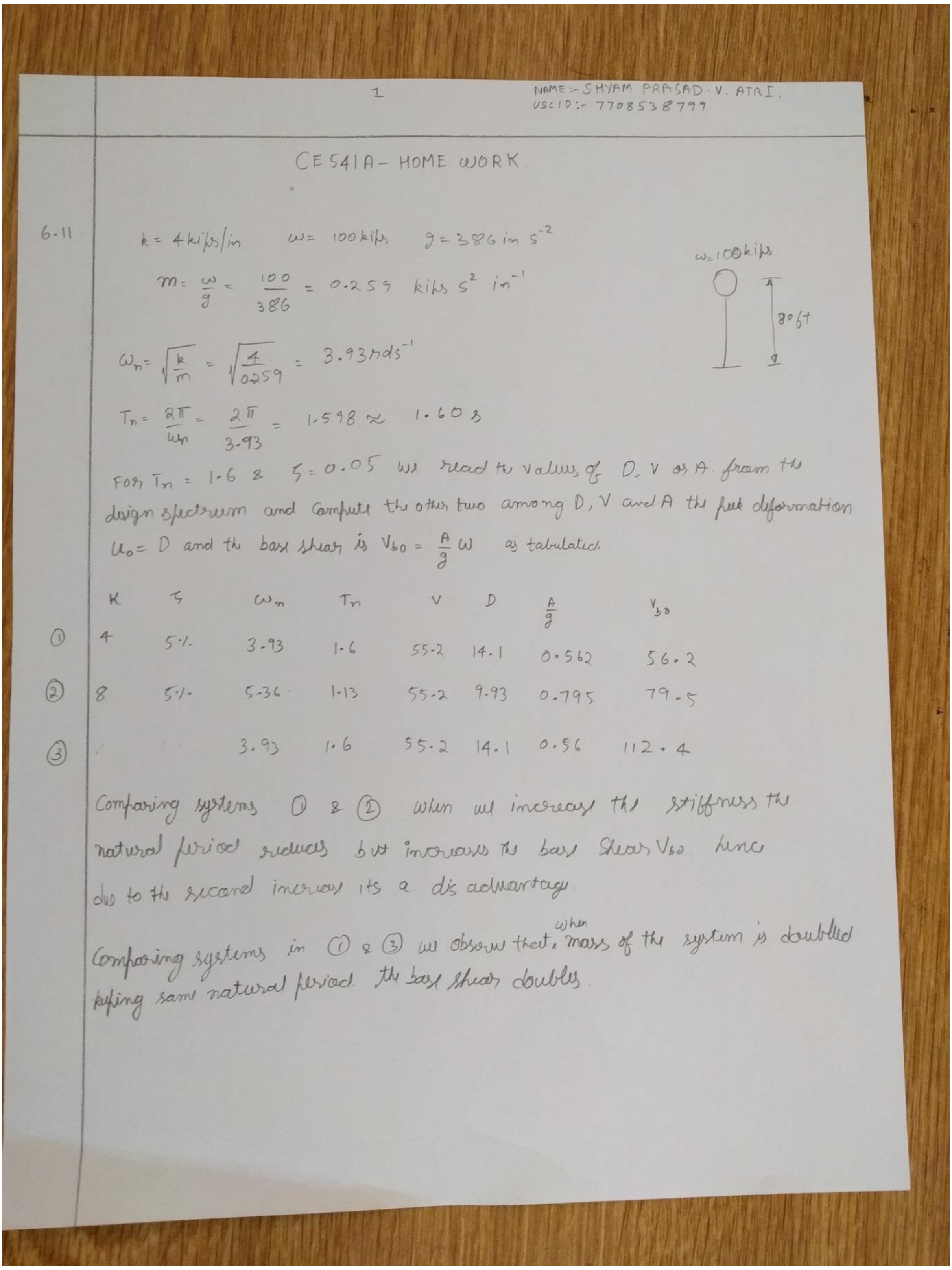


Figure 6.9.5 Elastic pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1g$, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

Using these diagrams to get values of V,D,A/g



1

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6.11

$k = 4\,kips/in \quad w = 100\,kips \quad g = 386\,in\,s^{-2}$

$m = \frac{w}{g} = \frac{100}{386} = 0.259\ kips\ s^2\ in^{-1}$

$w = 100\,kips$

80 ft

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{0.259}} = 3.93\,rads^{-1}$

$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.93} = 1.598 \approx 1.60\,s$

For $T_n = 1.6$ & $\zeta = 0.05$ we read the values of D, V or A from the design spectrum and compute the other two among D, V and A the peak deformation $u_0 = D$ and the base shear is $V_{b0} = \frac{A}{g} w$ as tabulated.

	K	ζ	ω_n	T_n	V	D	$\frac{A}{g}$	V_{b0}
1	4	5%	3.93	1.6	55.2	14.1	0.562	56.2
2	8	5%	5.36	1.13	55.2	9.93	0.795	79.5
3			3.93	1.6	55.2	14.1	0.56	112.4

Comparing systems 1 & 2 when we increase the stiffness the natural period reduces but increases the base shear V_{b0} hence due to the second increase its a disadvantage.

Comparing systems in 1 & 3 we observe that, when mass of the system is doubled keeping same natural period the base shear doubles.

6.18 A one story steel frame of 24ft span and 12 ft height has following properties: The second moment of cross sectional area for beam and columns are Ib=160in^4 and Ic=320 in^4 respectively;

the elastic modulus for steel is 30E3 ksi. For purposes of dynamic analysis the frame is considered as massless with a weight of 100kips lumped at the beam level; the columns are clamped at the base; the damping ratio is estimated at 5%. Determine the peek values of lateral displacement at the beam level and bending moment throughout the frame due to the design spectrum of fig 6.9.5 scaled to the peak ground acceleration of 0.5g.

Using this chart to get A

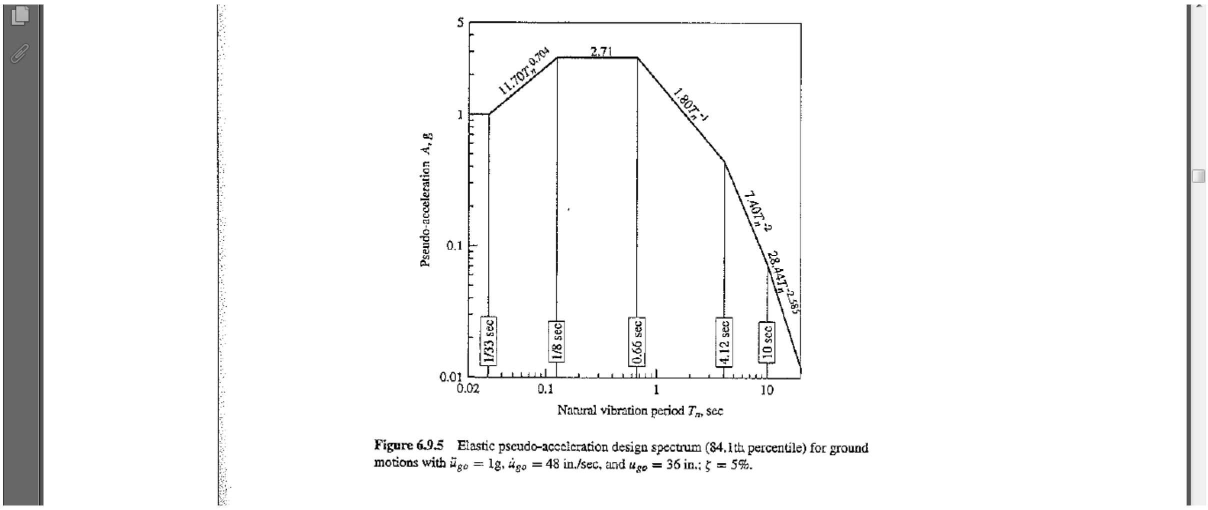

Figure 6.9.5 Elastic pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1g$, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

Check the images below.!

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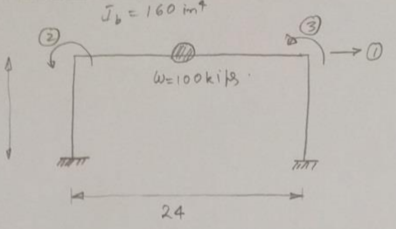
6.18

$I_c = 320\ in^4$ 12



$E = 30 \times 10^3$ ksi

$\zeta = 5\%$

Scaled peak ground acceleration = 0.5g.

Using Static Condensation method we determine the lateral stiffness of the structure as in Chapter 1.

$$k = \frac{EI_c}{h^3}\left[\begin{array}{c|cc} 24 & 6h & 6h \\ \hline 6h & 5h^2 & h^2/2 \\ 6h & \frac{h^2}{2} & 5h^2 \end{array}\right] \qquad \begin{bmatrix} k_{tt} & k_{to} \\ k_{to}^T & k_{oo} \end{bmatrix}$$

$$\frac{EI_c}{h^3}\left[\begin{array}{c|cc} 24 & 6h & 6h \\ \hline 6h & 5h^2 & h^2/2 \\ 6h & h^2/2 & 5h^2 \end{array}\right]\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_s \\ 0 \\ 0 \end{Bmatrix}$$

applying Static condensation.

$$k = k_{tt} - k_{to}\, k_{oo}^{-1}\, k_{to}^T$$

$$k = 24\frac{EI_c}{h^3} - \frac{EI_c}{h^3}\begin{bmatrix} 6h & 6h \end{bmatrix}\begin{bmatrix} 5h^2 & h^2/2 \\ h^2/2 & 5h^2 \end{bmatrix}^{-1}\begin{bmatrix} 6h \\ 6h \end{bmatrix}$$

$$k = \frac{120}{11}\frac{EI}{h^3} = \frac{120}{11} \times \frac{(30 \times 10^3)(320)}{(12 \times 12)^3} = 35.07 \text{ kips in}^{-1}$$

$$m = \frac{w}{g} = \frac{100}{386} = 0.2591 \text{ kip } s^2 \text{ in}^{-1}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{35.09}{0.2591}} = 11.64 \text{ rad s}^{-1}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{11.64} = 0.540\ s$$

using the design spectrum chart

$A = 0.5 \times 2.71\, g = 1.355\, g.$

$$D = \frac{A}{\omega_n^2} = \frac{1.355}{\omega_n^2} g = \frac{1.355 \times 386}{11.64^2} = 3.86 \text{ in}$$

The peak displacement is

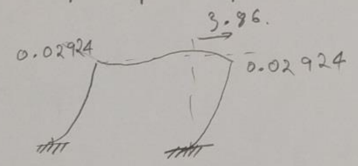
$u_{10} = 3.86$ in

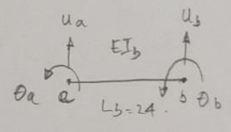
To determine the bending moments we recover the joint rotations

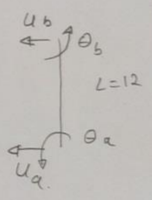
$$\begin{Bmatrix} u_{20} \\ u_{30} \end{Bmatrix} = \frac{12}{-11h} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} u_0 = -\frac{12}{11 \times 12 \times 12} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \times 386$$

$$= -0.02924 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

The deformed shape at peak response is







For column

$$M_a = \frac{4EI_c}{L_c}\theta_a + \frac{2EI_c}{L_c}\theta_b + \frac{6EI_c}{L_c^2}u_a - \frac{6EI_c}{L_c^2}u_b$$

$$M_b = \frac{2EI_c}{L_c}\theta_a + \frac{4EI_c}{L_c}\theta_b + \frac{6EI_c}{L_c^2}u_a - \frac{6EI_c}{L_c^2}u_b.$$

using $\theta_a = 0$, $\theta_b = -0.02924$, $u_a = 0$ and $u_b = -3.86$

and substituting values for E, I_c and L_c gives.

$M_a = 6824$ kip-in $\quad M_b = 2925$ kip in.

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For beam

$$M_a = \frac{4EI_b}{L_b}\theta_a + \frac{2EI_b}{L_b}\theta_b + \frac{6EI_b}{L_b^2}u_a - \frac{6EI_b}{L_b^2}u_b$$

$$M_b = \frac{2EI_b}{L_b}\theta_a + \frac{4EI_b}{L_b}\theta_b + \frac{6EI_b}{L_b}u_a - \frac{6EI_b}{L_b^2}u_b$$

using $\theta_a = \theta_b = -0.02924$ $u_a = u_b = 0$ and using E I_b & L_b

gives $M_a = M_b = -2925$ kip in.