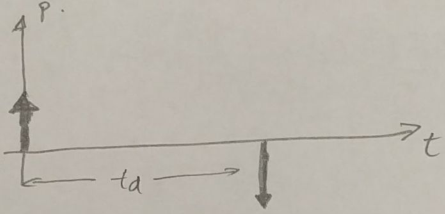
NAME:- SHYAM PRASAD. V. ATRI
USC ID:- 7708538799.

CE 541A HOME WORK :- 04

4.3. An SDF undamped system is subject to a force $P(t)$ consisting of a sequence of two impulses, each of magnitude I as in fig.

a) Plot the displacement response of the system for $t_d/T_n = \frac{1}{8}, \frac{1}{4}$ and 1. For each case show the response to individual impulses and the combined response.

b) Plot $u_o \div (I/m\omega_n)$ as a function of t_d/T_n indicate separately the maximum occurring at $t \le t_d$ and $t \geq t_d$. Such a plot is called the response spectrum for this excitation.



We know that response to SDF system with unit impulse at start is given by.

$$u_1(t) = I\left[\frac{1}{m\omega_n}\sin\omega_n t\right] = \frac{I}{m\omega_n}\sin\frac{2\pi t}{T_n}$$

The response to second impulse is given by

$$u_2(t) = -I\left[\frac{1}{m\omega_n}\sin\omega_n(t-t_d)\right] = \frac{-I}{m\omega_n}\sin 2\pi\left(\frac{t}{T_n}-\frac{t_d}{T_n}\right) \qquad t > t_d$$

Combined response of both the inputs is given by

For $t < t_d$.

$$u(t) = u_1(t) = \frac{I}{m\omega_n}\sin\frac{2\pi t}{T_n} \longrightarrow ①$$

For $t > t_d$

$$u(t) = u_1(t) + u_2(t)$$

$$u(t) = \frac{I}{m\omega_n} \sin \frac{2\pi t}{T_n} - \frac{I}{m\omega_n} \sin \frac{2\pi (t - t_d)}{T_n}$$

$$u(t) = \frac{I}{m\omega_n} \, 2 \sin \frac{2\pi t_d}{2T_n} \cos \left(\frac{2\pi (2t - t_d)}{2T_n} \right)$$

$$u(t) = \frac{2I}{m\omega_n} \sin \frac{\pi t_d}{T_n} \cos 2\pi \left(\frac{t}{T_n} - \frac{t_d}{2T_n} \right) \longrightarrow ②$$

$$u(t) = \begin{cases} \dfrac{I}{m\omega_n} \sin \dfrac{2\pi t}{T_n} & \text{when } t < t_d \\[2ex] \dfrac{2I}{m\omega_n} \underbrace{\sin \dfrac{\pi t_d}{T_n}}_{\beta} \cos 2\pi \left(\dfrac{t}{T_n} - \dfrac{t_d}{2T_n} \right) & t \geq t_d \end{cases} \rightarrow ③$$

For $t_d / T_n = \frac{1}{8}$

Page 1.1.

** First Impulse. $= \frac{I}{m\omega_n} \sin \frac{2\pi t}{T_n}$, Second Impulse $= \frac{-I}{\omega_n m} \sin 2\pi \left(\frac{t}{T_n} - \frac{1}{8} \right) \longrightarrow \alpha$

* combined. as in f^n ③ where T_d/T_n changes with 1/8

For $t_d/T_n = \frac{1}{4}$.

** First Impulse $= \frac{I}{m\omega_n} \sin \frac{2\pi t}{T_n}$, second Impulse $= \frac{-I}{m\omega_n} \sin 2\pi \left(\frac{t}{T_n} - \frac{1}{4} \right)$

* combined plot as in f^n ③ where t_d/T_n is replaced with 1/4.

For $t_d/T_n = 1$

** First Impulse $= \frac{I}{m\omega_n} \sin \frac{2\pi t}{T_n}$, Second Impulse $= \frac{-I}{m\omega_n} \sin 2\pi \left(\frac{t}{T_n} - 1 \right)$

* combined plot as in f^n 3 where t_d/T_n is replaced with 1

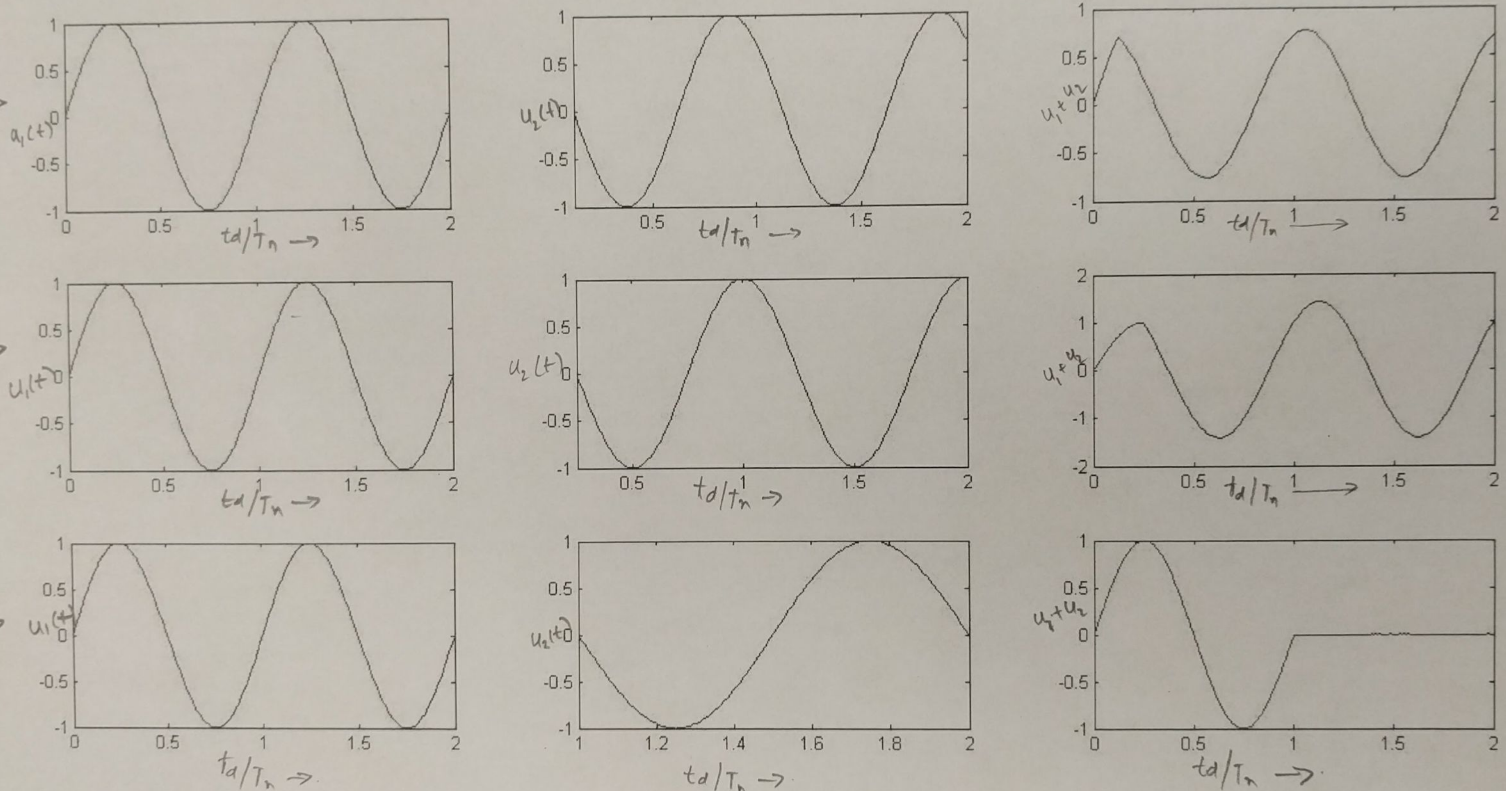
Q 4.3

First Impulse ↓ Second Impulse ↓ Both impulses ↓

$\frac{t_d}{T_n} = \frac{1}{8}$ →

$\frac{t_d}{T_n} = \frac{1}{4}$ →

$\frac{t_d}{T_n} = 1$ →

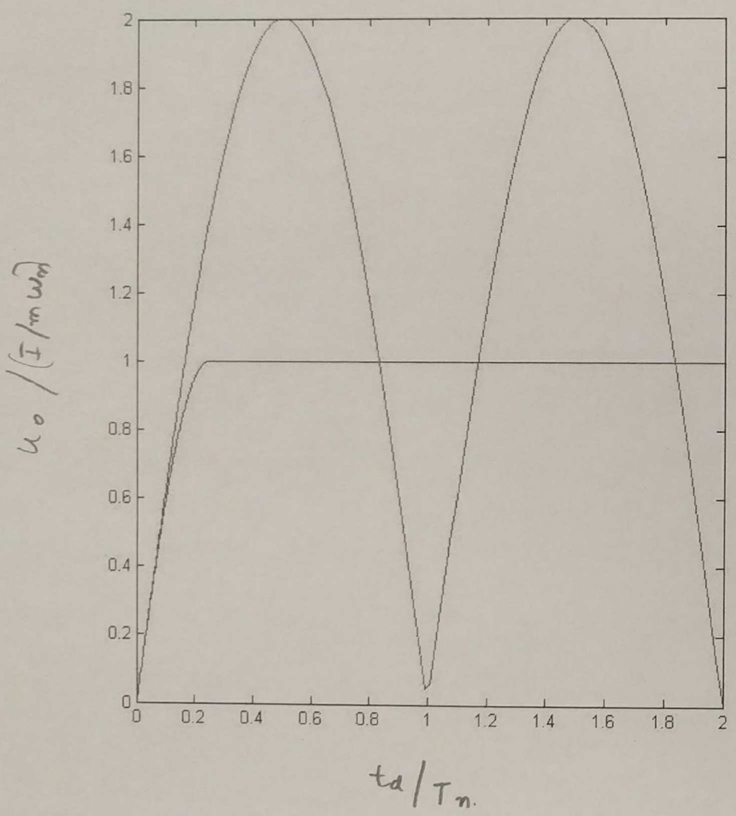


page 1.2

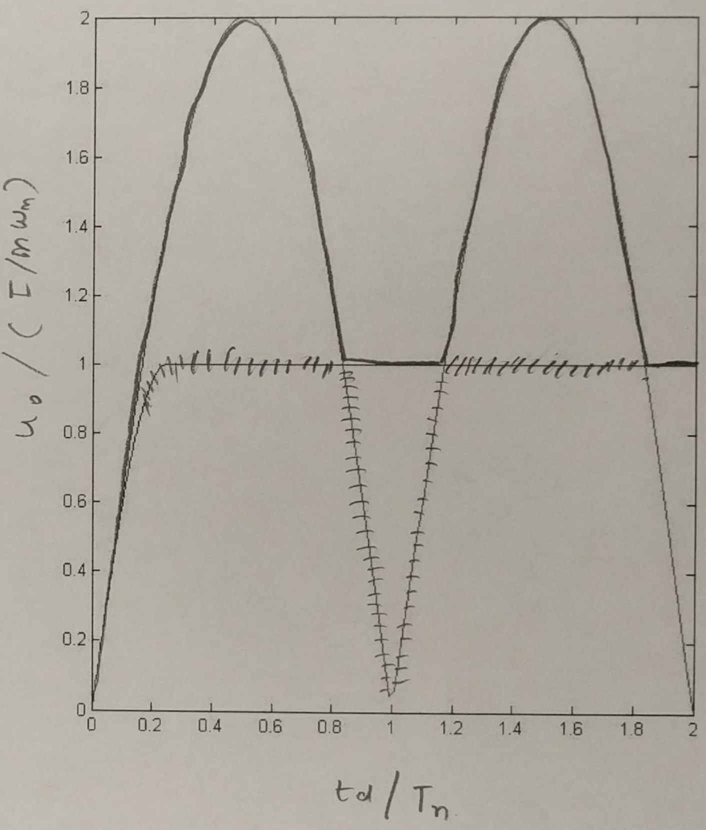
Q 4.3

Combined Plot of maxima.

All combined ~~maxima~~ maxima



Only the desired values.



Page 1.3

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To determine the maxima when $t \leq t_d$

The first peak occurs for the system when $t_0 = \frac{T_n}{4}$ hence $t_d > \frac{T_n}{4}$ to allow the first peak to form. whose value is given by

$$\frac{u_0}{I/m\omega_n} = 1$$

If $t_d < \frac{T_n}{4}$ no peak will develop hence the response will be same as $\sin\frac{2\pi t}{T_n}$

$$\frac{u_0}{I/m\omega_n} = \sin\frac{2\pi t}{T_n}$$

$\therefore$ for $0 \leq t \leq t_d$

$$\frac{u_0}{I/m\omega_n} = \begin{cases} \sin\frac{2\pi t}{T_n} & t_d \leq T_n/4 \\ 1 & t_d \geq T_n/4 \end{cases}$$ *

To determine the maxima when $t > t_d$

we can see from eq^n 3 the maxima will be.

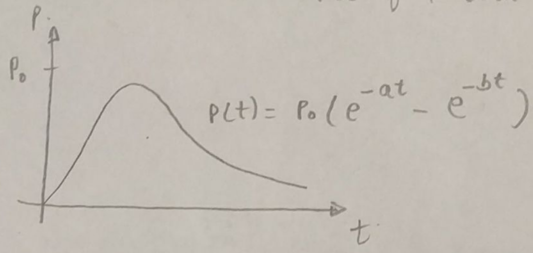
$$\frac{u_0}{I/m\omega_n} = 2\left|\sin\frac{\pi t_d}{T_n}\right|$$

Hence overall. we can say that

$$\frac{u_0}{I/m\omega_n} = \begin{cases} 2\left|\sin(\pi t_d/T_n)\right| & \text{when} \quad t_d \leq 1/4 \\ \text{max. of} \begin{cases} 1 \\ 2\left|\sin(\pi t_d/T_n)\right| \end{cases} & t_d \geq 1/4 \end{cases}$$ *

4.6 a) Determine the motion of an undamped system starting from rest due to the force $P(t)$ shown in fig. $b > a$

b) Plot the motion for $b = 2a$ for three values of $a/\omega_n = 0.05, 0.1,$ and 0.5



The governing equation for the following system is

$$m\ddot{u} + ku = P_0\left(e^{-at} - e^{-bt}\right). \longrightarrow (1)$$

We know that for a system whose equation is

$$m\ddot{u} + ku = P_0 e^{-at}$$ The solution is given by

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+\frac{a^2}{\omega_n^2}}\left[\frac{a}{\omega_n}\sin\omega_n t - \cos\omega_n t + e^{-at}\right]$$

Page 2.1

Hence for the following system the solution for eq^n (1) will be given by.

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+\frac{a^2}{\omega_n^2}}\left[\frac{a}{\omega_n}\sin\omega_n t - \cos\omega_n t + e^{-at}\right] \longrightarrow (2)$$

$$-\left\{\frac{1}{1+\frac{b^2}{\omega_n^2}}\left[\frac{b}{\omega_n}\sin\omega_n t - \cos\omega_n t + e^{-bt}\right]\right\}$$

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7708538799.

b). given that $b = 2a$.

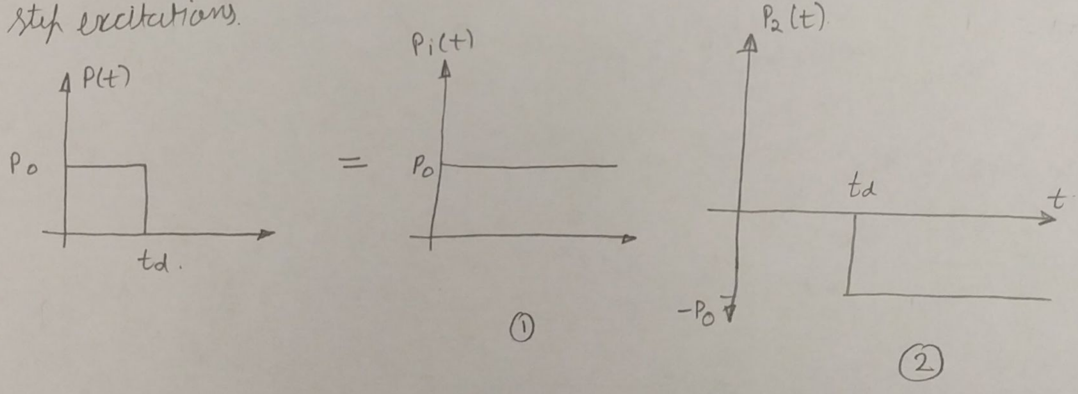
∴ Solution will be.

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1+\frac{a^2}{\omega_n^2}}\left[\frac{a}{\omega_n}\sin\omega_n t - \cos\omega_n t + e^{-at}\right]$$

*** $$- \left\{\frac{1}{1+\frac{4a^2}{\omega_n^2}}\left[\frac{2a}{\omega_n}\sin\omega_n t - \cos\omega_n t + e^{-2at}\right]\right\}$$

we will have 3 plots for $\frac{a}{\omega_n} = 0.05,\ 0.1$ and 0.5

4.14 Determine the response of an undamped system to a rectangular pulse force of amplitude P_0 and duration t_d by considering the pulse as the superposition of two step excitations.



we know the response to step input is.

$$u_1(t) = \frac{P_0}{k}(1 - \cos\omega_n t) \qquad t \geq 0$$

∴ This is the response to the part in fig ①

For fig ②. the response is given by

$$u_2(t) = -\frac{P_0}{k}(1 - \cos\omega_n(t - t_d)) \qquad t \geq t_d$$

But due to superposition principle the response before t_d is zero

$P_2(t) = 0$ for $0 \le t \le t_d$.

Hence the total response of the system is given by

$$u(t) = \begin{cases} \frac{P_0}{k}(1 - \cos \omega_n t) & 0 \le t \le t_d. \\ \frac{P_0}{k}\left[\cos \omega_n (t - t_d) - \cos \omega_n t\right] & t \ge t_d. \end{cases}$$

4.15 Using Duhamel's integral, determine the response of an undamped system to a rectangular pulse force of amplitude P_0 and duration t_d.

$$P(t) = \begin{cases} P_0 & 0 \le t \le t_d. \\ 0 & t \ge t_d. \end{cases}$$

$$u(t) = \frac{1}{m\omega_n} \int_0^t P(\tau) \sin \omega_n (t - \tau)\, d\tau$$

Page 3.1

for $0 \le t \le t_d$. we have

$$u(t) = \frac{1}{m\omega_n} \int_0^t P_0 \sin \omega_n (t - \tau) \cdot d\tau$$

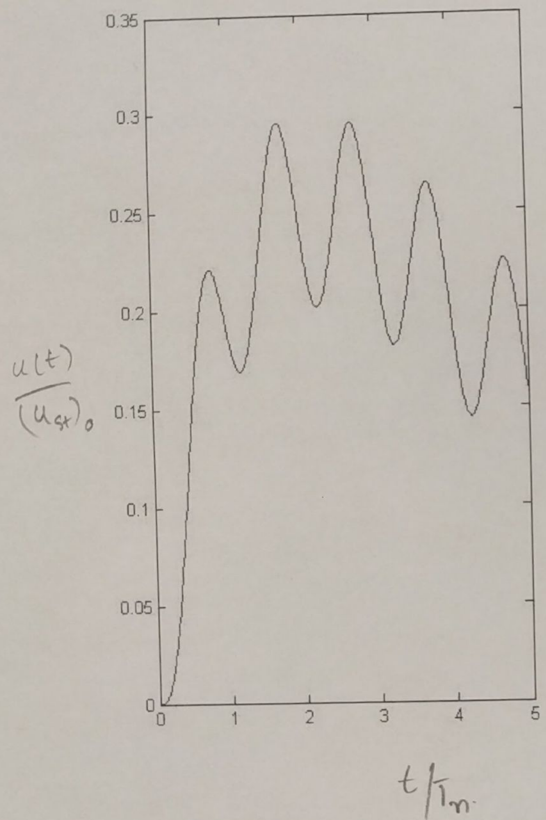
$$= \frac{P_0}{m\omega_n} \left[\frac{\cos (t - \tau)\omega_n}{\omega_n} \right]_0^t$$

$$= \frac{P_0}{m\omega_n^2} \left[1 - \cos \omega_n t \right] = \frac{P_0}{k} \left[1 - \cos \omega_n t \right]$$

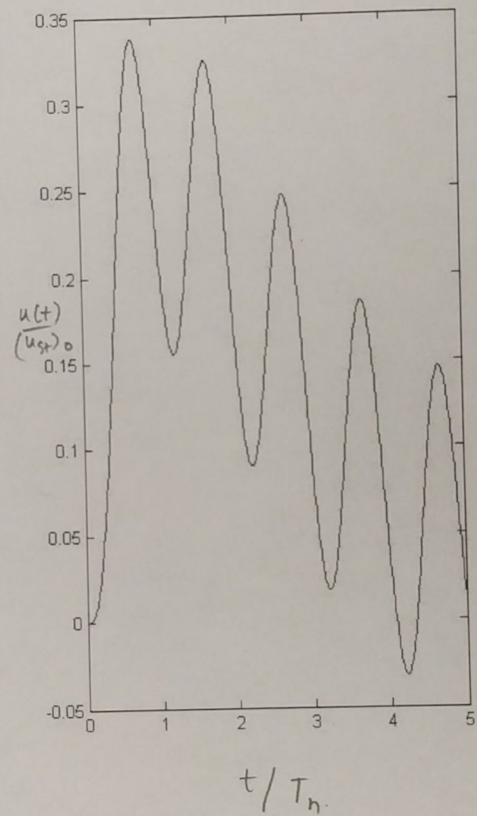
when $t > t_d$ PTO

Q 4.6

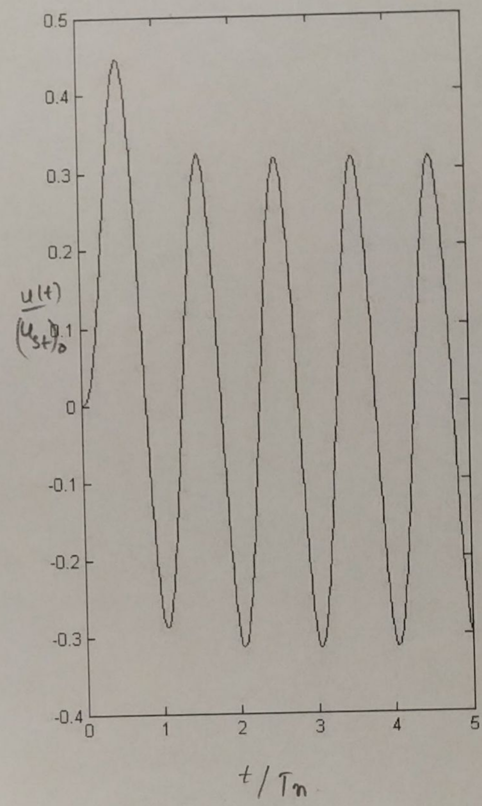
$a/\omega_n = 0.05$



$\frac{a}{\omega_n} = 0.1$



$\frac{a}{\omega_n} = 0.5$



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when $t > t_d$

$$u(t) = \frac{1}{m\omega_n} \int_0^{t_d} P_0 \sin \omega_n (t-\tau)\, d\tau + \int_{t_d}^{t} 0 \cdot \sin \omega_n (t-\tau)\, d\tau$$

$$= \frac{P_0}{m\omega_n} \left[\frac{1}{\omega_n} \cos \omega_n (t-\tau) \right]_0^{t_d}$$

$$= \frac{P_0}{k} \left[\cos \omega_n (t - t_d) - \cos \omega_n t \right]$$

$$\therefore \quad u(t) = \begin{cases} \frac{P_0}{k} \left[1 - \cos \omega_n t \right] & 0 \le t \le t_d \\ \frac{P_0}{k} \left[\cos \omega_n (t - t_d) - \cos \omega_n t \right] & t \geq t_d \end{cases}$$

4.17. The one story building of Example 4.1 is modified so that columns are clamped at the base instead of hinged. For the same excitation determine the maximum displacement at the top of the frame and maximum bending stress at the columns. Comment on the effect of base fixity.

From example 4.1

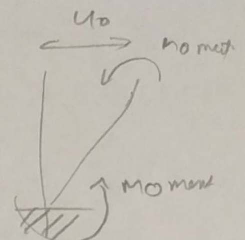
$k = 3.73$ kips in^{-1} Since the base is fixed the stiffness increases by 4 times

So $k = 4 \times 3.73 = 14.92$ kips in^{-1}

$$\frac{t_d}{T_n} = R_d = \frac{0.2}{0.25} = 0.8 > \frac{1}{2}$$

So. $$R_d = \frac{u_0}{(u_{st})_0} = 2$$

$$(u_{st})_0 = \frac{P_0}{k} = \frac{4}{14.92} = 0.268 \text{ in.}$$



$$u_0 = (u_{st})_0 R_d = 0.268 \times 2 = \underline{0.536} \text{ in}$$

$$M = \frac{6EI}{L^2} u_0 = \left[\frac{6 \times 30000 \times 61.9}{(12 \times 12)^2} \right] 0.536$$

$$= 287.9 \text{ kip in}^{-1}$$

The bending stress is usualy largest at the outside of the flanges at the top & bottom of columns.

$$\sigma = \frac{M}{S} = \frac{287.9}{15.2} = 18.9 \text{ ksi}$$

Page 4-1.

In this case due to Clamping the deformation & bending is reduced at the base.

4.26. The 80 ft high water tank of examples 2.6 and 2.7. is subjected to a force P(t) as in fig. The maximum response of the structure with tank full (weight = 100.33 kips) was determined in Example 4.2.

a) If the tank is empty (weight = 20.03 kips) Calculate the maximum base shear and bending moment at the base of the tower supporting the tank

b) By Comparing these results with those of the full tank (Example 4.2) Comment on the effect of mass on the response to impulse forces. Explain.

Shyam
7708538799

4.26 Soln

From the example.

$W = 20.03$ kips , $T_n = 0.5\,s$ $\zeta = 2.75\,\%$

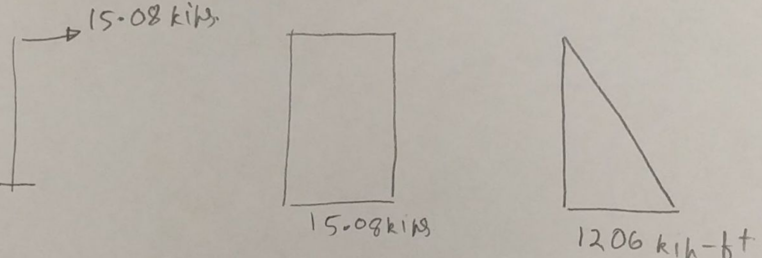
$$\frac{T_d}{T_n} = \frac{0.08}{0.5} = 0.16.$$

as $T_d/T_n < 0.25$ force can be considered as a impulse force with magnitude $I = 1.2$ kip sec

$$u_o = \frac{I}{k}\frac{2\pi}{T_n} = \frac{1.2}{8.2} \times \frac{2\pi}{0.5} = 1.86.$$

The equivalent static force is

$$f_{so} = k\,u_o = 8.2 \times 1.83 = 15.08 \text{ kips.}$$



	u_o (in)	V_b (kips)	M_b kips-ft
Empty	1.84	15.08	1206
Full	0.821	6.73	538

Thus we can conclude increasing the mass reduces the dynamic response. which can also be mathematically explained as.

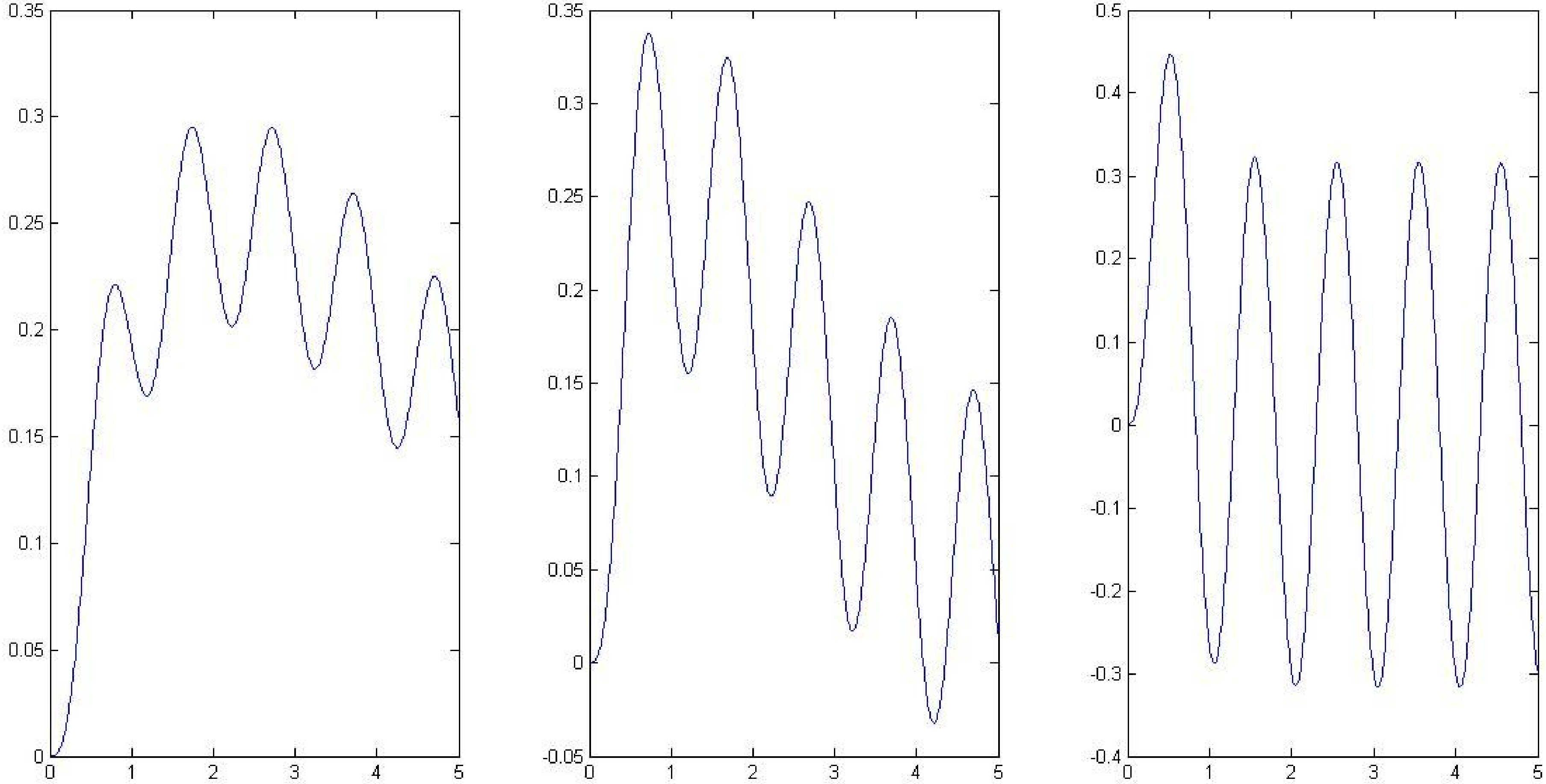
$\dot{u}(0) = \frac{I}{m}$ → impulse imparts velocity This velocity is inversely proportional to mass hence increasing mass reduces velocity.

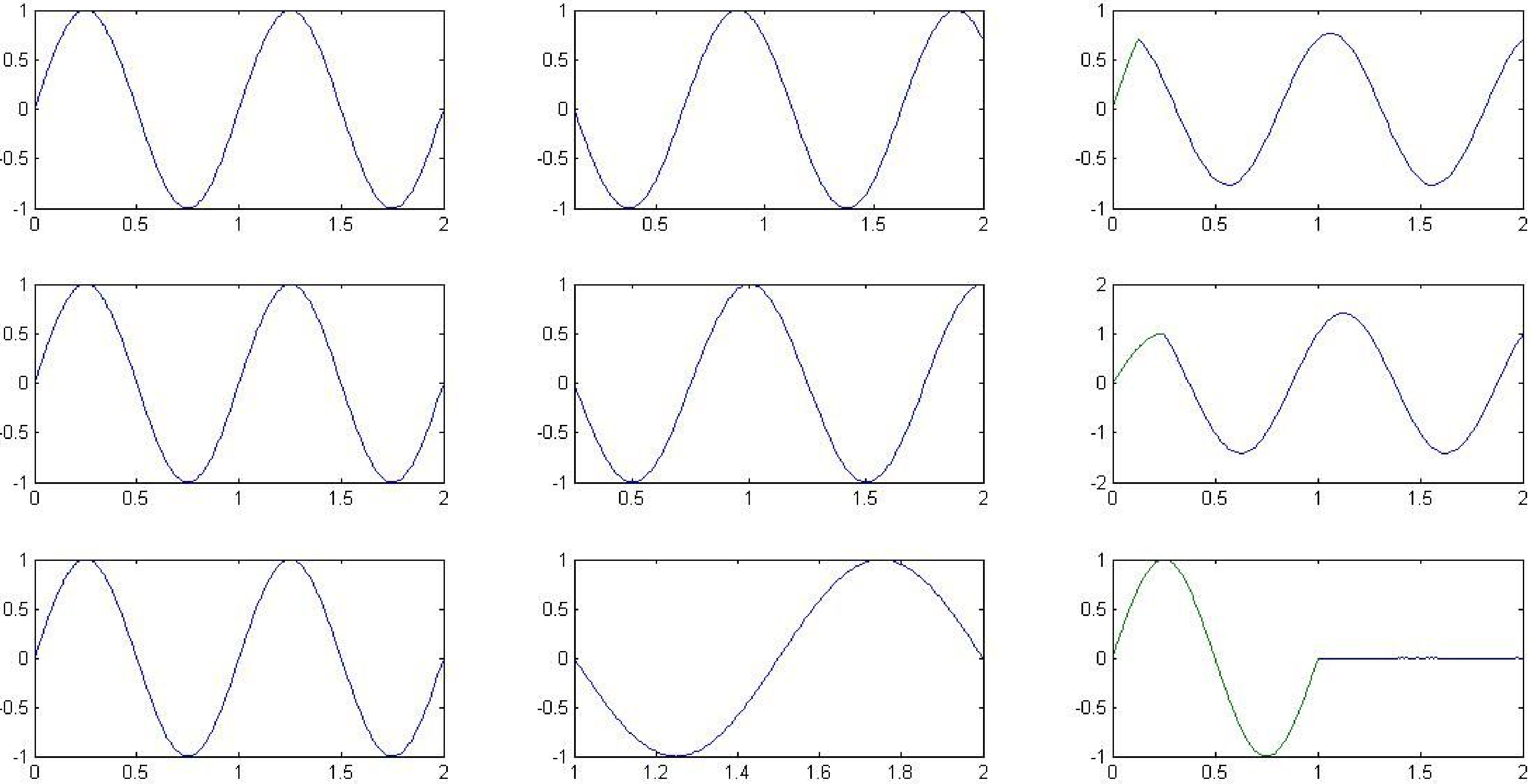
```
% problem 4.3 over all plot


figure

subplot(1,2,1)

x=linspace(0,0.25,100);

y=sin(2*pi*x);

ya=2*abs(sin(pi*x));

xa=linspace(0.25,2,100);

z=ones(1,size(xa,2));

za=2*abs(sin(pi*xa));

plot(x,y,x,ya,xa,z,xa,za)


subplot(1,2,2)

plot(x,y,x,ya,xa,z,xa,za)



%Problem 4.6


aByOmegaN=[0.05;0.1;0.5];

tByTn=[0:0.01:5];

lhs=zeros(size(aByOmegaN,1),size(tByTn,2));

term1=zeros(size(aByOmegaN,1),size(tByTn,2));


term2=zeros(size(aByOmegaN,1),size(tByTn,2));


for i=1:size(aByOmegaN,1)
```

```
   for j=1:size(tByTn,2)


      term1(i,j)=(1/(1+(aByOmegaN(i,1))^2))*(aByOmegaN(i,1)*sin(2*pi*tByTn(1,j))-cos(2*pi*tByTn(1,j)) + exp(-2*pi*aByOmegaN(i,1)*tByTn(1,j)));

      term2(i,j)=(1/(1+4*(aByOmegaN(i,1))^2))*(2*aByOmegaN(i,1)*sin(2*pi*tByTn(1,j))-cos(2*pi*tByTn(1,j)) + exp(-2*pi*2*aByOmegaN(i,1)*tByTn(1,j)));

      lhs(i,j)=term1(i,j)-term2(i,j);

   end


end


figure

subplot(1,3,1)

plot(tByTn,lhs(1,:))

subplot(1,3,2)

plot(tByTn,lhs(2,:))

subplot(1,3,3)

plot(tByTn,lhs(3,:))


%problem 4.3

clear all

clc


%combined plot

%fplot(@(x) sin(2*pi*x), [0 1/8],'b')

%fplot(@(x) sin(2*pi*x-1/8), [1/8 2],'r')
```

```
figure

% when td/tn=1/8
subplot(3,3,1)
fplot(@(x) sin(2*pi*x),[0,2])

subplot(3,3,2)
fplot(@(x) -sin(2*pi*(x-0.125)),[1/8,2])

subplot(3,3,3)
a=linspace(0,0.125,100);
x=linspace(0.125,2,100);
z=sin(2*pi*a);
y=sin(2*pi*x)-sin(2*pi*(x-0.125));
plot(x,y,a,z)

% when td/tn=1/4
subplot(3,3,4)
fplot(@(x) sin(2*pi*x),[0,2])

subplot(3,3,5)
fplot(@(x) -sin(2*pi*(x-0.25)),[1/4,2])

subplot(3,3,6)
s=linspace(0,0.25,100);
xa=linspace(0.25,2,100);
d=sin(2*pi*s);
```

```
ya=sin(2*pi*xa)-sin(2*pi*(xa-0.25));

plot(xa,ya,s,d)



% when td/tn=1

subplot(3,3,7)

fplot(@(x) sin(2*pi*x),[0,2])


subplot(3,3,8)

fplot(@(x) -sin(2*pi*(x-1)),[1,2])


subplot(3,3,9)

sa=linspace(0,1,100);

xaa=linspace(1,2,100);

da=sin(2*pi*sa);

yaa=sin(2*pi*xaa)-sin(2*pi*(xaa-1));

plot(xaa,yaa,sa,da)
```


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0 1 2 3 4 5
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0 1 2 3 4 5


1
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0
-0.5
-1
0
0.5
1
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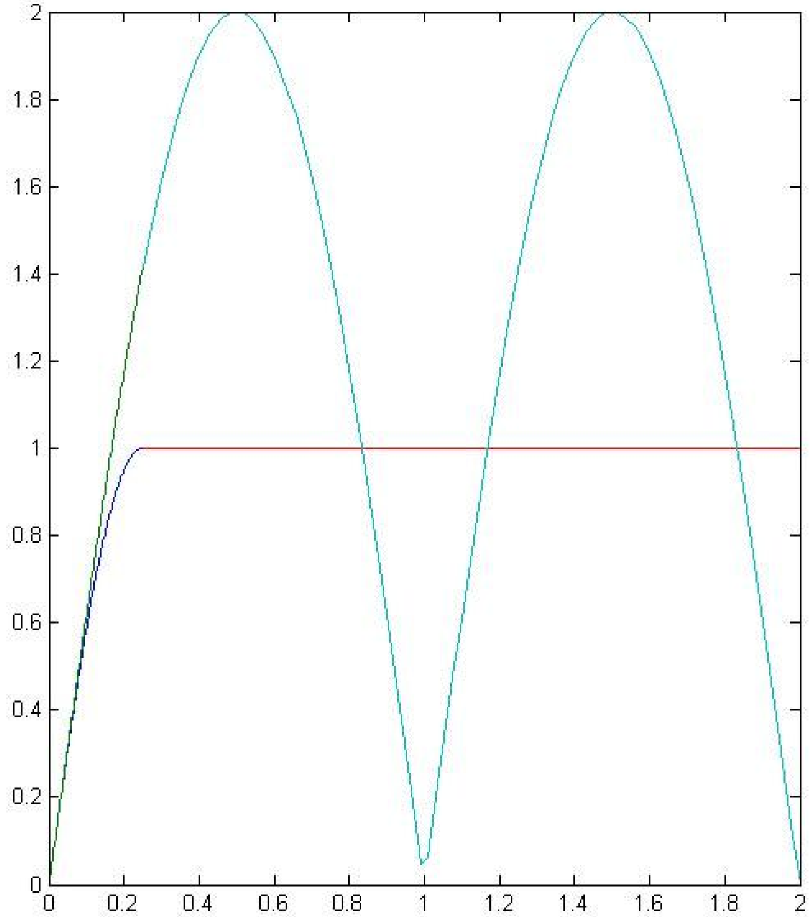
2
1.2
1.4
1.6
1.8
-2


2
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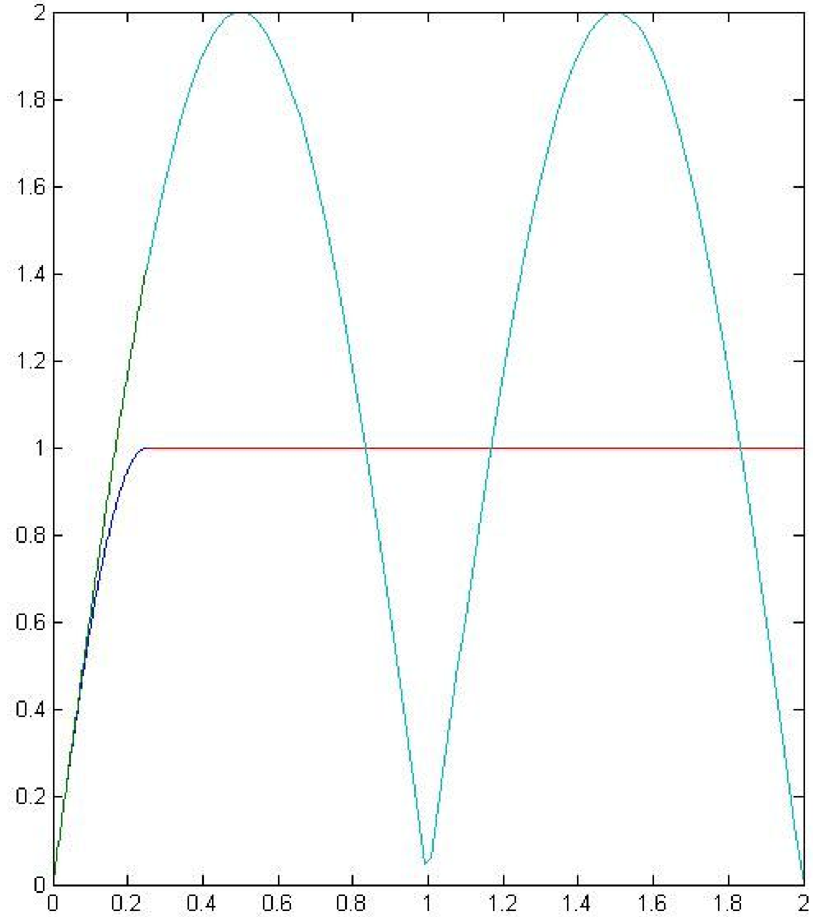
2
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0.2
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