Problem 1



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# CE 541 A - SYMBOLIC COMPUTATION ASSIGNMENT - 1

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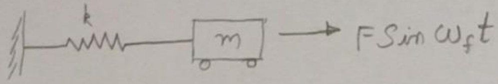


DE of the system $m\ddot{x} + kx = F \sin \omega_f t \longrightarrow (1)$

$$\ddot{x} + \frac{k}{m}x = \frac{F}{m}\sin\omega_f t$$

$$\ddot{x} + \omega^2 x = \frac{F}{m}\sin\omega_f t.$$

The impulse response of undamped SDOF is given by $h(t) = \frac{\omega_n}{k}\sin\omega_n t$

where $\omega_n$ is the natural frequency of the system

to determine the response of the above system we use duhamels integral. in duhamels integral we have

$$x(t) = \int_0^t f(u)\, g(t-u)\,du$$

where $f(u)$ is the forcing function and $g(t-u) = h(t-u)$ the impulse response

$$\therefore \quad f(u) = F\sin\omega_f u \qquad g(t-u) = \frac{\omega_n}{k}\sin\omega_n(t-u)$$

$$x(t) = \int_0^t F\sin\omega_f u \times \frac{\omega_n}{k}\sin\omega_n(t-u)$$

impulse response.

$$\ddot{x} + \omega^2 x = \frac{\delta(t)}{m}$$

laplace transform

$$s^2 X(s) + \omega^2 X(s) = \frac{1}{m}$$

$$X(s) = \frac{1}{m} \times \frac{1}{s^2+\omega^2} \times \frac{\omega^2}{\omega^2}$$

$$X(s) = \frac{\omega}{m\omega^2} \times \frac{\omega}{s^2+\omega^2}$$

$$x(t) = \frac{\omega}{m \times \frac{k}{m}} \times \sin\omega t$$

$$x(t) = \frac{\omega}{k}\sin\omega t$$

The solution derived in class is

$$x(t) = \frac{F/k}{1-\Omega^2}\left(\sin\omega_f t - \Omega\sin\omega_n t\right)$$

where $\Omega$ is $\frac{\omega_f}{\omega_n}$

The solution derived from Computer algebra is

$$x(t) = \frac{F\omega_n}{k}\,\frac{(\omega_f \sin\omega_n t - \omega_n\sin\omega_f t)}{\omega_f^2 - \omega_n^2}$$

Solution from class

$$x(t) = \frac{F/k}{1-\Omega^2}\left(\sin\omega_f t - \Omega \sin\omega_n t\right)$$

$$= \frac{F/k}{1-\frac{\omega_f^2}{\omega_n^2}}\left(\sin\omega_f t - \frac{\omega_f}{\omega_n}\sin\omega_n t\right)$$

$$= \frac{F/k}{\omega_n^2-\omega_f^2}\times\omega_n^2\left(\sin\omega_f t - \frac{\omega_f}{\omega_n}\sin\omega_n t\right)$$

$$= \frac{F/k \times \omega_n}{\omega_n^2-\omega_f^2}\left(\omega_n\sin\omega_f t - \omega_f\sin\omega_n t\right) \times \frac{-1}{-1}$$

$$= \frac{F\omega_n}{k}\times\frac{\left(\omega_f\sin\omega_n t - \omega_n \sin\omega_f t\right)}{\omega_f^2-\omega_n^2}$$

Which is same as computer algebra solution. Hence the solution is correct.

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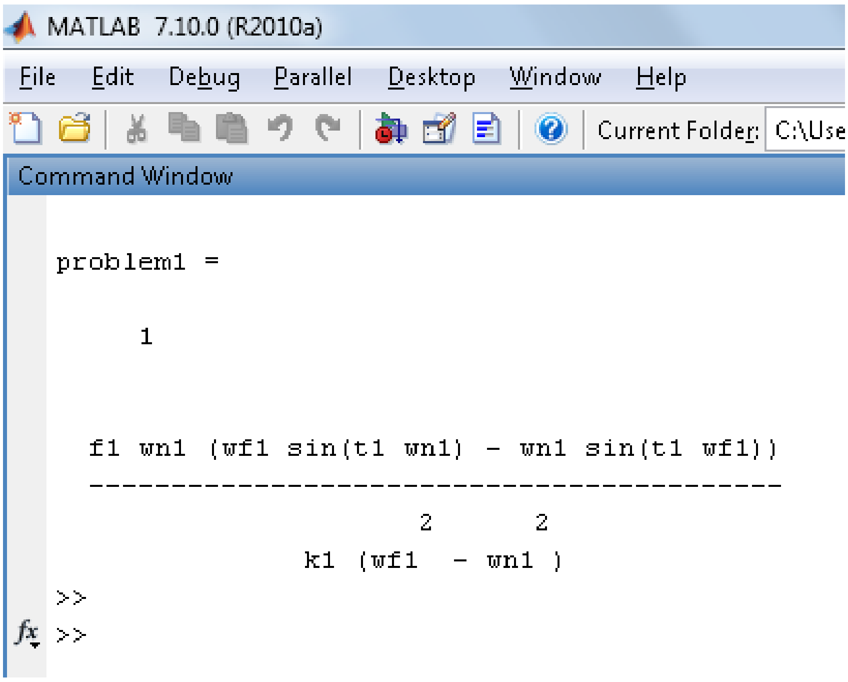
Symbolic Solution



Problem 2A

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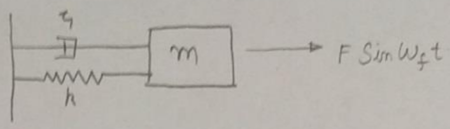
2 a)




The differential eqn of the system is given by

$$m\ddot{x} + c\dot{x} + kx = F \sin \omega_f t$$

The solution derived in class is given by

$$x(t) = e^{-\xi\omega t}\left(A \sin \omega_d t + B \cos \omega_d t\right) + D \sin \omega_f t + E \cos \omega_f t$$

where A & B come from initial conditions and we solve for the terms D, E. by particular integral

Since we use quiescent initial condition $A = 0$ and $B = 0$

we have

$$D = \frac{F}{k} \frac{1 - \Omega^2}{[1-\Omega^2]^2 + (2\xi\Omega)^2}$$ [Solution from first principles]

$$E = -\frac{F}{k} \frac{2\xi\Omega}{[1-\Omega^2]^2 + [2\xi\Omega]^2}$$

we compute the duhamel integral as follows [For computing using MATLAB]

$$x(t) = \int_0^t f(\tau)\, h(t-\tau)\, d\tau$$

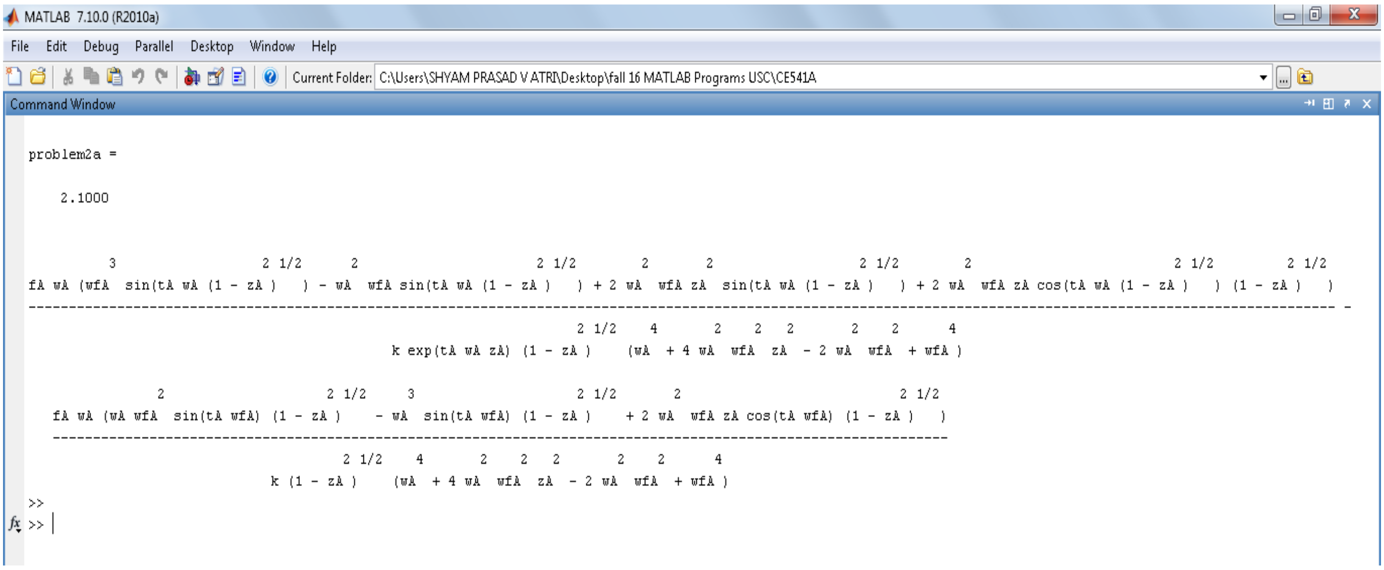
where $h(t)$ is the impulse response. and $h(t) = \frac{1}{\omega_d} e^{-\xi\omega_n t} \sin \omega_d t$ replace t with $t-\tau$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$. and $f(t) = F \sin \omega_f t$ [Forcing function]

$$\therefore \quad x(t) = \int_0^t F \sin \omega_f \tau * \frac{1}{\omega_d} e^{-\omega_n(t-\tau)} \sin \omega_d(t-\tau)\, d\tau$$

This is computed from Matlab and plotted with the solution derived from first principles we find that both the solutions are similar hence our solution from MATLAB is correct. (Symbolic Solution is shown in the end)

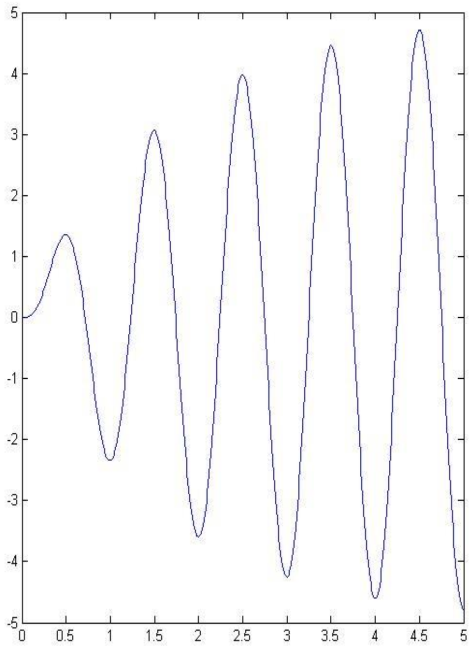
Symbolic Solution

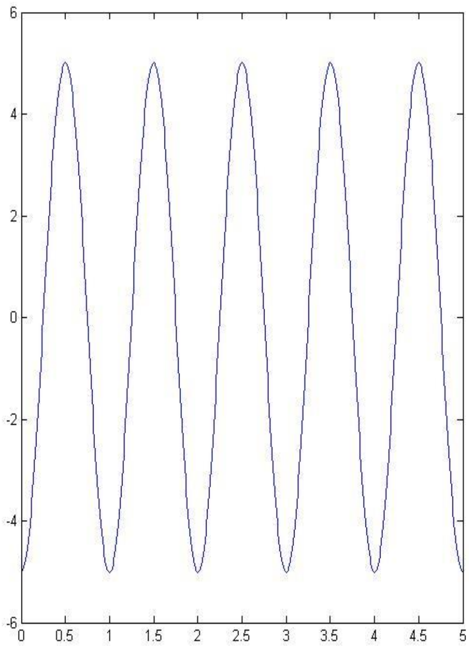



For Plot on LHS:- Plot using Duhamel integral.

For Plot on RHS:- Plot using solution derived in class.

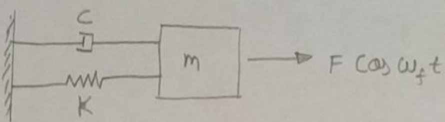
We observe that the transient part dies out as the system reaches steady state.



Problem 2b

b.




The DE of the system is

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega_f t \longrightarrow DE$$

The following System will have 2 parts of solution.

$x(t)$ = Complementary function (CF) + Particular Integral (PI)

$$CF = e^{-\zeta\omega_n t}\left(A\sin\omega_d t + B\cos\omega_d t\right)$$

$$PI = D\sin\omega_f t + E\cos\omega_f t$$

$$A = 0 \qquad B = 0$$

plugging the PI into the DE

$$-m\omega_f^2(D\sin\omega_f t + E\cos\omega_f t)$$
$$+c\omega_f(D\cos\omega_f t - E\sin\omega_f t)$$
$$+k(D\sin\omega_f t + E\cos\omega_f t)$$
$$= F\cos\omega_f t$$

$$\sin\omega_f t(-m\omega_f^2 D - c\omega_f E + kD)$$
$$+\cos\omega_f t(-m\omega_f^2 E + c\omega_f D + kE) = F\cos\omega_f t$$

Comparing the coefficients of Sin & Cos.

$\div m$ $\quad -m\omega_f^2 D - c\omega_f E + kD = 0 \Rightarrow D(\omega_n^2 - \omega_f^2) - 2\zeta\omega_n\omega_f E = 0$ $\quad \div\omega_n^2$

$\div m$ $\quad -m\omega_f^2 E + c\omega_f D + kE = F \Rightarrow D(2\zeta\omega_n\omega_f) + E(\omega_n^2 - \omega_f^2) = \frac{F}{m\omega_n^2}$ $\quad \div\omega_n^2$

$$D(1-\Omega^2) - 2\zeta\Omega E = 0$$

solving for D & E we get

$$D(2\zeta\Omega) + E(1-\Omega^2) = F/m\omega_n^2 \qquad m\omega_n^2 = k$$

$$D = \frac{F}{k}\,\frac{2\zeta\Omega}{[1-\Omega^2]^2 + [2\zeta\Omega]^2}, \quad E = \frac{F}{k}\,\frac{[1-\Omega^2]}{[1-\Omega^2]^2 + [2\zeta\Omega]^2}$$

where $\Omega = \frac{\omega_f}{\omega_n}$.

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m}\cos\omega_f t$$

Let $\frac{c}{m} = 2\zeta\omega_n \qquad \frac{k}{m} = \omega_n^2$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega^2 = \frac{F}{m}\cos\omega_f t$$

for CF

$$[D^2 + 2\zeta\omega_n D + \omega^2]x = \frac{F}{m}\cos\omega_f t$$

$$D = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$D = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2}\,i$$

Let $\omega_n\sqrt{1-\zeta^2} = \omega_d$.

$$\therefore CF = e^{-\zeta\omega_n t}(A\sin\omega_d t + B\cos\omega_d t)$$

we know $x(0) = 0$

$$\therefore 0 = e^{-\zeta\omega_n(0)}(A\cdot 0 + B\cdot 1)$$

$$\dot{x}(0) = 0 \qquad \Rightarrow B = 0$$

$$\dot{c}_f = e^{-\zeta\omega_n t}(A\omega_d\cos\omega_d t - B\omega_d\sin\omega_d t)$$
$$-\zeta\omega_n e^{-\zeta\omega_n t}(A\sin\omega_d t + B\cos\omega_d t)$$

$$0 = (A) + B \Rightarrow A = 0$$

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use the values of A, B, D, & E to get the plot from first principals.

now when we use the duhamels integral our solution looks like.

$$x(t) = \int_0^t f(\tau)\, h(t-\tau)\, d\tau$$

where $f(t) = F\cos\omega_f t$ $\quad h(t) = \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t$ $\quad$ replace $t$ with $t-\tau$

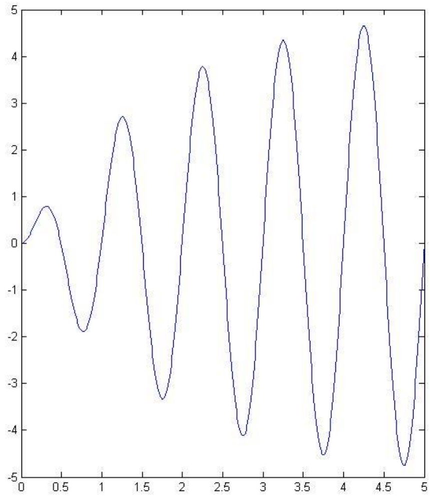
$$x(t) = \int_0^t F\cos\omega_f\tau \times \frac{1}{\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau$$

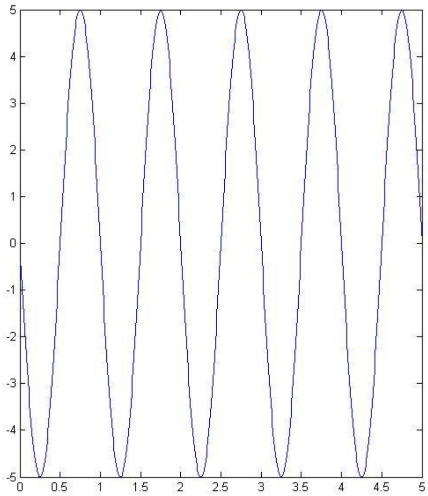
This is computed in MATLAB and plotted with the solution derived from first principals we find both the solutions are similar hence our solution from MATLAB is correct. (symbolic solution is shown in the end.

Symbolic Solution

```
MATLAB 7.10.0 (R2010a)
File  Edit  Debug  Parallel  Desktop  Window  Help
Current Folder: C:\Users\SHYAM PRASAD V ATRI\Desktop\fall 16 MATLAB Programs USC\CE541A
Command Window

problem2b =

    2.2000


       3                     2 1/2         2                         2 1/2       2                             2 1/2
fB wB (wB  cos(tB wfB) (1 - zB )     - wB wfB  cos(tB wfB) (1 - zB )     + 2 wB  wfB zB sin(tB wfB) (1 - zB )    )
--------------------------------------------------------------------------------------------------------------- -
                                      2 1/2    4       2    2   2       2    2      4
                            k (1 - zB )    (wB  + 4 wB  wfB  zB  - 2 wB  wfB  + wfB )

           3                     2 1/2      3                    2 1/2         2 1/2         2                  2 1/2         2 1/2
   (fB wB (wB  zB sin(tB wB (1 - zB )   ) + wB  cos(tB wB (1 - zB )    ) (1 - zB )    - wB wfB  cos(tB wB (1 - zB )    ) (1 - zB )    +

       2                     2 1/2                                  2 1/2    4       2    2   2       2    2      4
   wB wfB  zB sin(tB wB (1 - zB )   ))) / (k exp(tB wB zB) (1 - zB )    (wB  + 4 wB  wfB  zB  - 2 wB  wfB  + wfB ))
>>
>>
>>
```

Plots



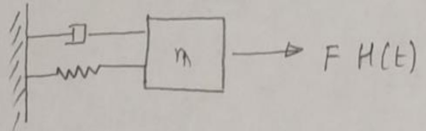

For Plot on LHS:- Plot using Duhamel integral.

For Plot on RHS:- Plot using solution derived in class.

We observe that the transient part dies out as the system reaches steady state.

Problem 2.3

2 c




$$F * H(t) = \begin{cases} F & t > 0 \\ 0 & t < 0 \end{cases}$$

The differential eq$^n$ of our system is given by

$$m\ddot{x} + c\dot{x} + kx = F\,H(t)$$

for solution in class we took $m\ddot{x} + c\dot{x} + kx = H(t)$. hence our solution in class must be multiplied by F as in our given question F is multiplied hence the result is multiplyed by a factor of F

$$x(t) = F \times X \text{ derived in class.}$$

Page 3.1

$$x(t) = \frac{F}{k}\left[1 - e^{-\zeta\omega_n t}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t + \cos\omega_d t\right)\right]$$

where $\omega_n = \sqrt{\frac{k}{m}}$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$

This needs to be compared with duhamels integral.
using duhamels integral we have.

$$x(t) = \int_0^t f(\tau)\, h(t-\tau)\, d\tau$$

where $f(\tau) = F\,H(\tau) = \begin{cases} F & \tau > 0 \\ 0 & \tau < 0 \end{cases}$

$$h(t) = \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t \qquad \text{replace } t \text{ with } t-\tau$$

$$x(t) = \int_0^t F\, H(\tau) \cdot \frac{1}{\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau$$

we can clearly see that when both the plots are superimposed they are colinear hence MATLAB solution is equal to the one derived from first principles. (symbolic solution shown in the end.)

Symbolic Solution

MATLAB 7.10.0 (R2010a)

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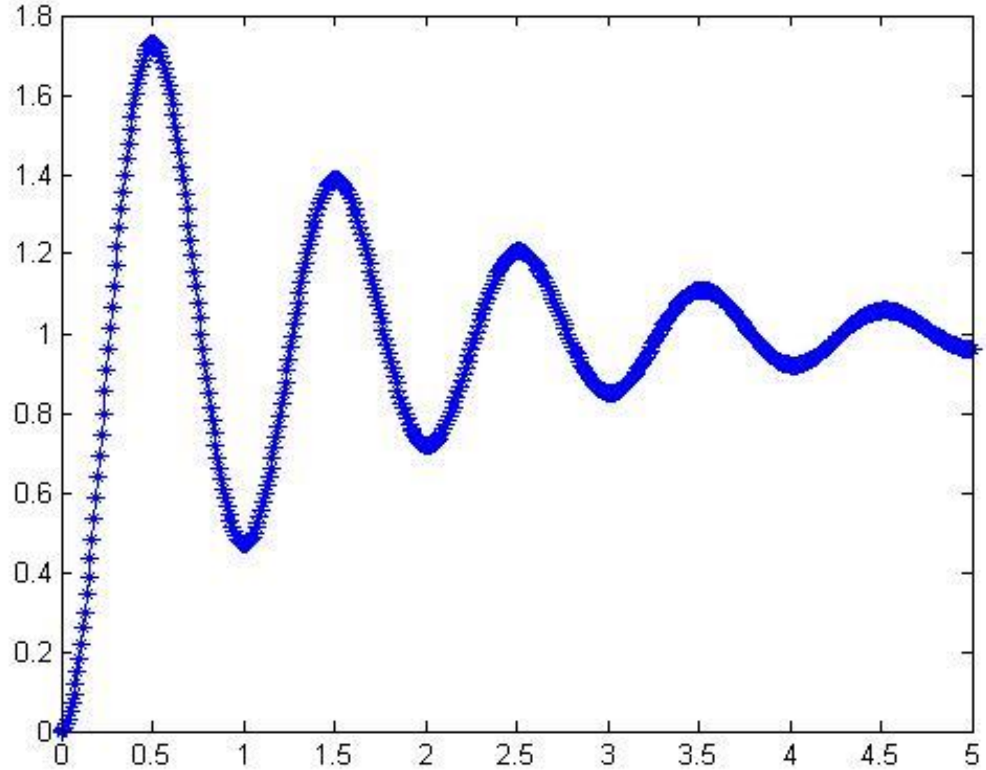
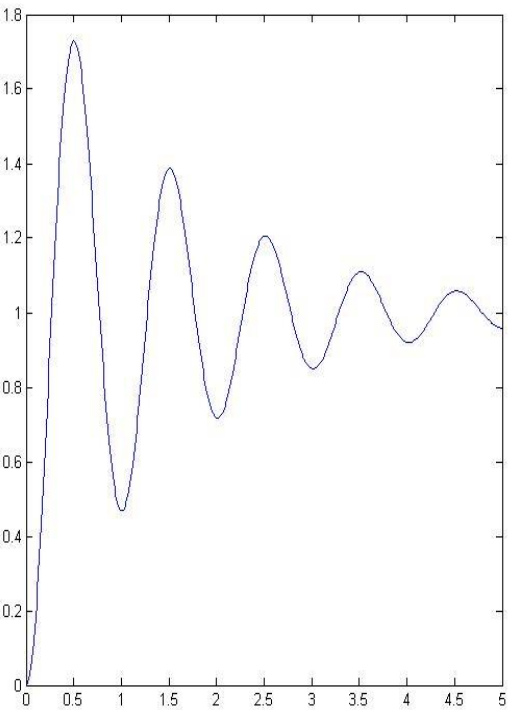
Command Window

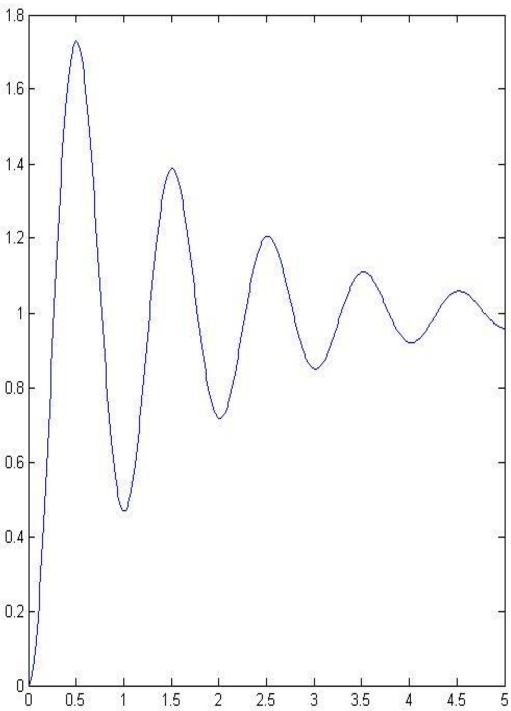
```
problem2c =

    2.3000


                                 2 1/2                             2 1/2
  fC     fC cos(tC wC (1 - zC )    )    fC zC sin(tC wC (1 - zC )    )
  --- - -------------------------- - ------------------------------
    2          2                         2                       2 1/2
  wC         wC   exp(tC wC zC)        wC   exp(tC wC zC) (1 - zC )
>>
>>
>>
```

Super imposed plot of Duhamel integral and solution from class




Individual plots





Problem 2d

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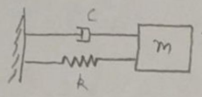
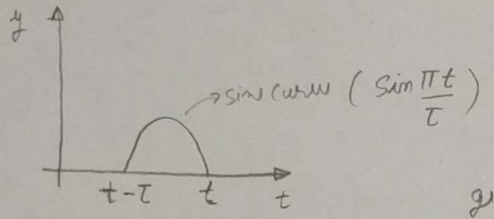
2 d) Half sine Pulse




$\longrightarrow F\left[H(t) - H(t-\tau)\right] \sin\frac{\pi t}{\tau}$ .




given $\tau = 1$

$$x(t) = \int_0^t f(\tau)\, h(t-\tau)\, d\tau$$

$$f(\tau) = F \sin\frac{\pi\tau}{1} \left\{ H(\tau) - H(\tau - 1) \right\}$$

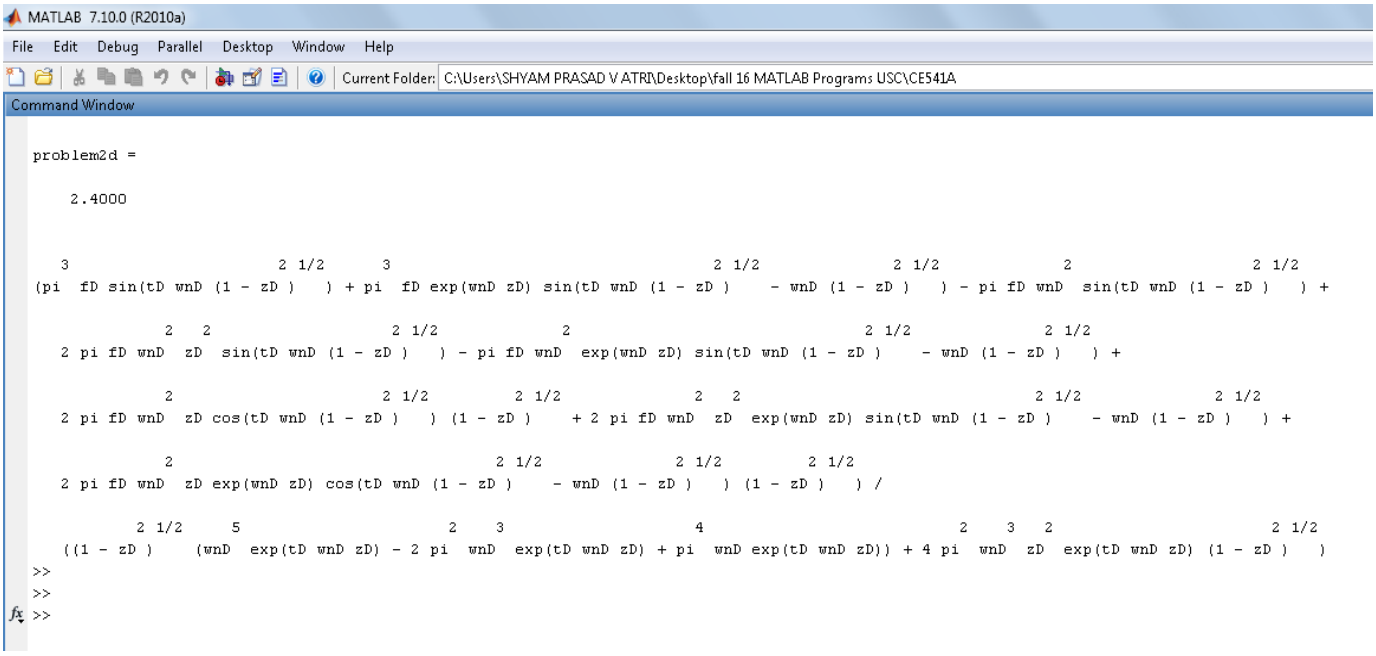
$$h(t-\tau) = \frac{1}{\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)$$

$$x(t) = \int_0^t F \sin(\pi\tau) \left\{ H(\tau) - H(\tau-1) \right\} \frac{1}{\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau$$

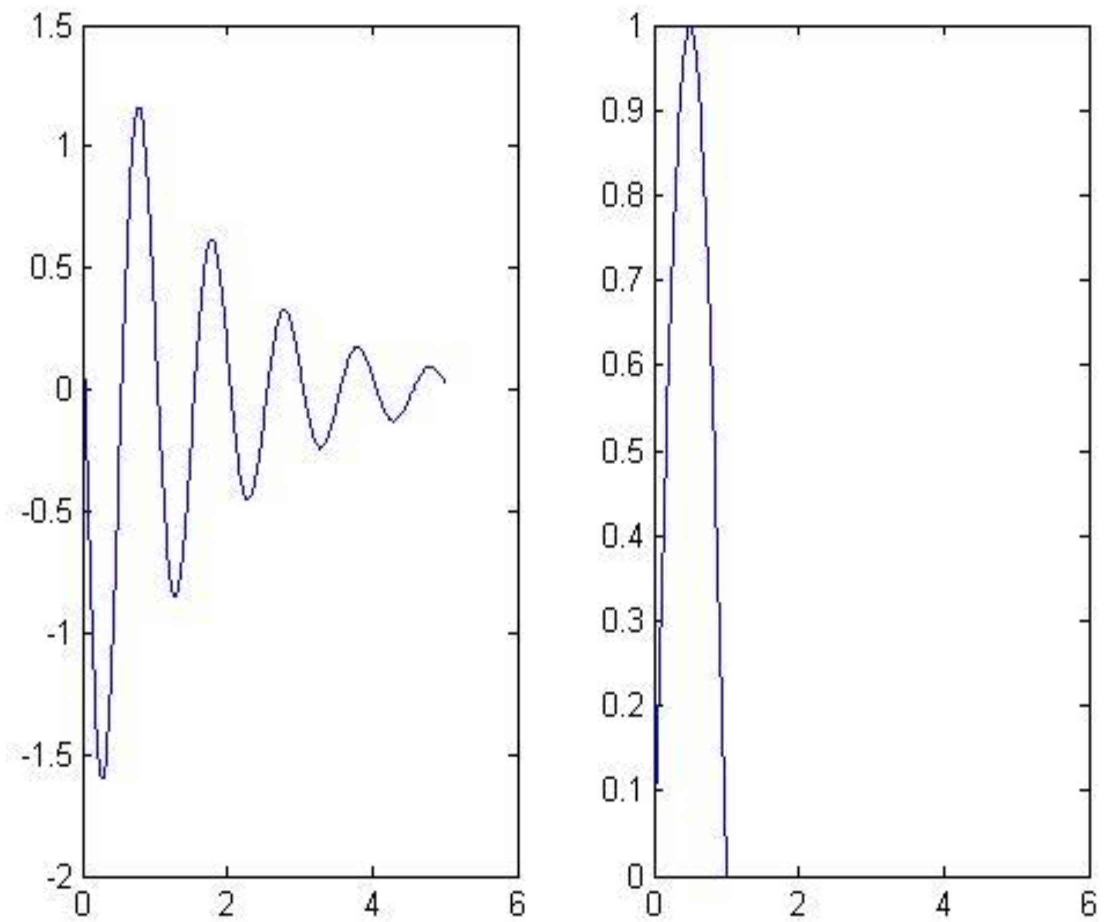
The symbolic expression is given in the later pages
Since we are doing the integration only upto 1

$$x(t) = \int_0^1 F \sin(\pi\tau) \left\{ H(\tau) - H(\tau-1) \right\} \frac{1}{\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin(t-\tau)\, d\tau$$

Symbolic Solution




Plot



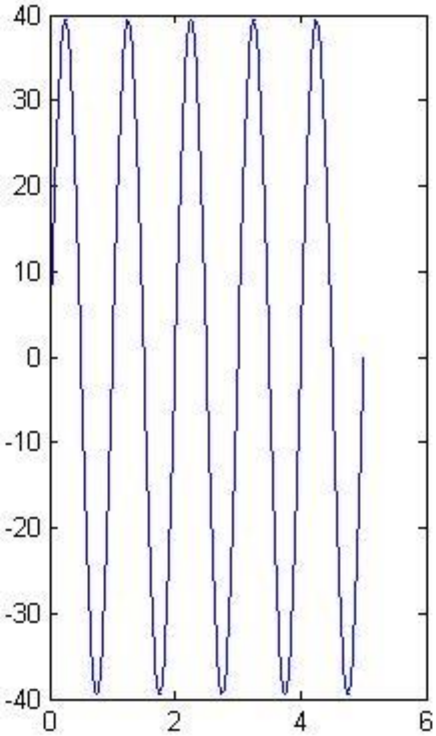

For Plot on LHS:- Plot using Duhamel integral.

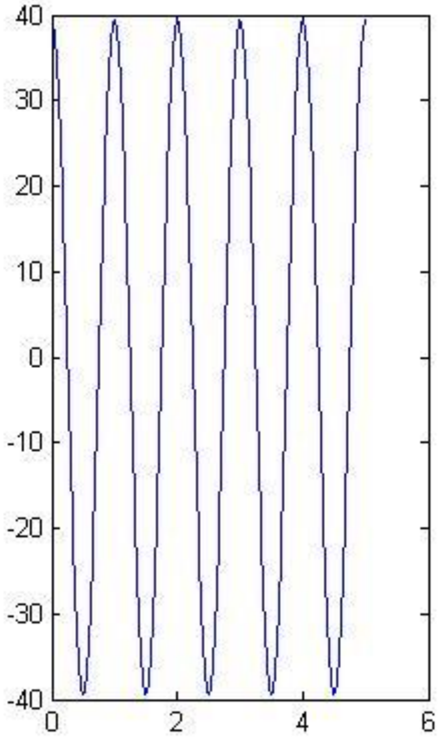
For Plot on RHS:- half sine wave

Plot of all excitation functions

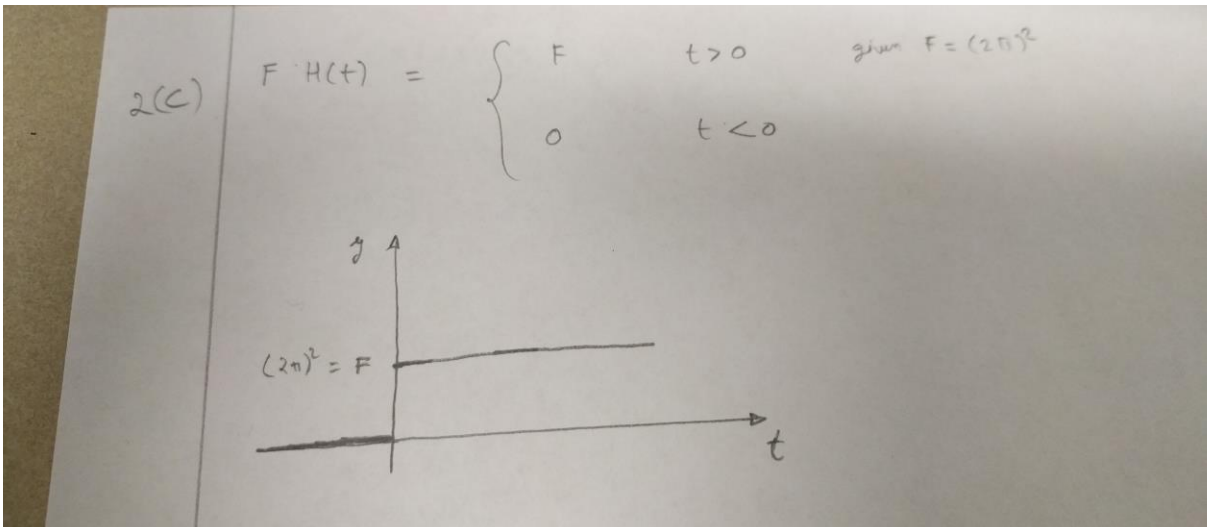
Image to LHS for problem 2a

Image to LHS for problem 2b




Excitation plot problem 2c



All Codes

```
clear all
clc

%---------------------------------------------


problem1=1
syms f1 wf1 t1 wn1 tau1 k1
v1=int((f1*sin(wf1*tau1)*(wn1/k1)*sin(wn1*(t1-tau1))),tau1,0,t1);
simplify(v1);
pretty(v1)
%



clear all
clc
%----------------------------------------------------------
problem2a=2.1
syms fA wfA tA wA zA tauA k
vA=int((fA*sin(wfA*tauA)*(wA/(k*(1-zA^2)^0.5))*(exp(-zA*wA*(tA-
tauA)))*(sin(wA*(tA-tauA)*(1-zA^2)^0.5))),tauA,0,tA);

k=4*3.14*3.14;
fA=4*3.14*3.14;
wA=2*3.14;
zA=0.1;
wfA=2*3.14;
tA=0:0.01:5;
res=eval(vA);
res;

t=0:0.01:5;
figure
subplot(1,2,1)
plot(t,res)

simplify(vA);
pretty(vA)



f=4*pi*pi;

omg=2*pi; %omg -- wA
z=0.1; %z -- zA
omd=omg*(1-z^2)^0.5;
omf=2*pi; % omf-- wfA
mag=omf/omg;
t=0:0.01:5;
y=ones(1,size(t,2));
c=ones(1,size(t,2));
d=ones(1,size(t,2));
for i=1:size(t,2)
    c(i)=(f/k)*((1-mag^2)/((1-mag^2)^2+(2*z*mag)^2));
    d(i)=(f/k)*((-2*z*mag)/((1-mag^2)^2+(2*z*mag)^2));
    y(i)=(c(i)*sin(omf*t(i))+d(i)*cos(omf*t(i)));

end



subplot(1,2,2)
plot(t,y)


clear all
```

```
clc

%---------------------------------------------

problem2b=2.2
syms fB wfB tB wB zB tauB k

vB=int((fB*cos(wfB*tauB)*(wB/(k*(1-zB^2)^0.5))*(exp(-zB*wB*(tB-
tauB)))*(sin(wB*(tB-tauB)*(1-zB^2)^0.5))),tauB,0,tB);


k=4*pi*pi;
fB=4*pi*pi;
wB=2*pi;
zB=0.1;
wfB=2*pi;
tB=0:0.01:5;
res=eval(vB);
res;

t=0:0.01:5;
figure
subplot(1,2,1)
plot(t,res)


simplify(vB);
pretty(vB)

%
%
%
f=4*3.14*3.14;

omg=2*pi; %omg -- wA
z=0.1; %z -- zA
omd=omg*(1-z^2)^0.5;
omf=2*pi; % omf-- wfA
mag=omf/omg;
t=0:0.01:5;
y=ones(1,size(t,2));
c=ones(1,size(t,2));
d=ones(1,size(t,2));
for i=1:size(t,2)
    d(i)=(f/k)*((1-mag^2)/((1-mag^2)^2+(2*z*mag)^2));
    c(i)=(f/k)*((-2*z*mag)/((1-mag^2)^2+(2*z*mag)^2));
    y(i)=(c(i)*sin(omf*t(i))+d(i)*cos(omf*t(i)));

end


subplot(1,2,2)
plot(t,y)


clear all
clc

%----------------------------------------------

problem2c=2.3
syms fC tC wC zC tauC k
vC=int((fC*(1/(wC*(1-zC^2)^0.5))*(exp(-zC*wC*(tC-tauC)))*(sin(wC*(tC-
tauC)*(1-zC^2)^0.5))),tauC,0,tC);
```

```
k=4*pi^2;
fC=4*pi*pi;

wC=2*pi;
zC=0.1;

tC=0:0.01:5;
res=eval(vC);
% to get individual plot please uncomment this
figure
subplot(1,2,1)

plot(tC,res)
%hold on % when you want individual subplot please comment out this

simplify(vC);
pretty(vC)

f=4*pi*pi;
omg=2*pi;
z=0.1;
omd=omg*(1-z^2)^0.5;
t=0:0.01:5;
y=ones(1,size(t,2));
for i=1:size(t,2)
    y(i)=(f/k)*(1-exp(-z*omg*t(i))*(sin(omd*t(i))*((z)/(1-
z^2)^0.5)+cos(omd*t(i))));
    %y(i)=(1-exp(-z*omg*t(i))*(sin(omd*t(i))*((z)/(1-
z^2)^0.5)+cos(omd*t(i))));
end


% to get individual plot please uncomment this
subplot(1,2,2)
plot(t,y)
%plot(t,y, '*') % when you want individual subplot please comment out this



clear all
clc

%-----------------------------------------------------


problem2d=2.4
syms fD tD wnD zD tauD

%we set the limit as 5 as we are doing it for half sine wave and we need
%only 5 amplitudes
vD=int((fD*(heaviside(tauD)-heaviside(tauD-1))*(sin(pi*tauD))*(1/(wnD*(1-
zD^2)^0.5))*(exp(-zD*wnD*(tD-tauD)))*(sin(wnD*(tD-tauD)*(1-
zD^2)^0.5))),tauD,0,5);

%original expression
%vD=int((fD*(heaviside(tauD)-heaviside(tauD-1))*(sin(pi*tauD))*(1/(wD*(1-
zD^2)^0.5))*(exp(-zD*wD*(tD-tauD)))*(sin(wD*(tD-tauD)*(1-
zD^2)^0.5))),tauD,0,tD);
%original expression

k=4*pi*pi;
fD=4*pi*pi;
wnD=2*pi;
zD=0.1;
%wfD=2*3.14;
tD=0:0.01:5;
```

```
%tD=0.5;
res=eval(vD);


t=0:0.01:5;
figure
subplot(1,2,1)
plot(t,res)

simplify(vD);
pretty(vD)

p=0:0.01:5;
qw=ones(1,size(p,2));
for i=1:size(p,2)

    qw(i)=(heaviside(p(i))-heaviside(p(i)-1))*(sin(pi*p(i)));

end
subplot(1,2,2)
plot(p,qw)



%plotting all excitation functions

clear all
clc

t=0:0.01:5;
f=4*pi*pi;
p2a=ones(1,size(f,2));
p2b=ones(1,size(f,2));

for i=1:size(t,2)
p2a(i)=f*sin(2*pi*t(i));
p2b(i)=f*cos(2*pi*t(i));
end
figure
subplot(1,2,1)
plot(t,p2a)

subplot(1,2,2)
plot(t,p2b)
```