BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 9

Example: Prove the following sequent is valid.

$$p \to q, p \to \neg q \perp \neg p$$

Solution:

$$\begin{array}{ll}
1. p \to q & \text{premise} \\
2. p \to \neg q & \text{premise} \\
\end{array}$$

$$\boxed{\begin{array}{ll}
3. p & \text{assumption(3)} \\
4. q & \to e \text{ 3,1} \\
5. \neg q & \to e \text{ 3,2} \\
6. \perp & \neg e \text{ 4,5}
\end{array}}$$

$$\begin{array}{ll}
7. \neg p & \neg i \text{ 3-6}
\end{array}$$

Example: Prove $p \to (q \to r), p, \neg r \vdash \neg q$ is valid, without using MT.
Solution:

$$\begin{array}{ll}
1. p \to (q \to r) & \text{premise} \\
2. p & \text{premise} \\
3. \neg r & \text{premise}
\end{array}$$

$$\boxed{\begin{array}{ll}
4. q & \text{assumption} \\
5. q \to r & \to e \text{ 2,1} \\
6. r & \to e \text{ 4,5} \\
7. \perp & \neg e \text{ 3,6}
\end{array}}$$

$$\begin{array}{ll}
8. \neg q & \neg_i \text{ 4-7}
\end{array}$$

7 Derived Rules

MT can be derived from $\to e, \neg e$ and $\neg_i$.

$$\begin{array}{ll}
1.\phi \rightarrow \psi & \text{premise} \\
2.\neg\psi & \text{premise}
\end{array}$$

$$\boxed{\begin{array}{ll}
3.\neg\phi & \text{assumption)} \\
4.\neg\psi & \rightarrow e \text{ 3,1} \\
5.\bot & \neg e \text{ 2,4}
\end{array}}$$

$$\begin{array}{ll}
6.\neg\phi & \neg_i 3-5
\end{array}$$

8 Law of the Excluded Middle (LEM)

- It is also called as Tertium non datur. (There is no third possibility)
- Says that $\phi \vee \neg\phi$ is always true.

$$\vdash \phi \vee \neg\phi$$

$$\boxed{\begin{array}{ll}
1.\neg(\phi \vee \neg\phi) & \text{assumption} \\
\multicolumn{2}{c}{\boxed{\begin{array}{ll}
2.\phi & \text{assumption} \\
3.\phi \vee \neg\phi & \vee_i \text{ 2} \\
4.\bot & \neg e \text{ 3,1}
\end{array}}} \\
5.\neg\phi & \neg_i \text{ 2-4} \\
6.\phi \vee \neg\phi & \vee_{i2} \text{ 5} \\
7.\bot & \neg e \text{ 6}
\end{array}}$$

$$\begin{array}{ll}
8.\phi \vee \neg\phi & \text{PBC 1-7}
\end{array}$$