Tutorial-2 Solutions

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Prove the following sequent.

1. $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s.$

Solution:

1.	(p∧q)∧r	premise
2.	s∧t	premise
3.	(p∧q)	∧e1 1
4.	q	∧e2 3
5.	s	∧e1 2
6.	q∧s	∧i 4,5

Question

2. $p \to (p \to q), p \vdash q.$

Solution:

1. p	premise
2. p →(p → q)	premise
3.p → q	→ e 2,1
4.q	→ e 3,1

Question

3. $p \wedge (q \wedge r), \neg s \rightarrow \neg r, p \rightarrow t \vdash t \wedge s$

Solution:

1.	p∧(q∧r)	premise
2.	¬s → ¬r	premise
3.	p→t	premise
4.	p	∧e1 1
5.	t	→e 3,4
6.	q∧r	∧e2 1
7.	r	∧e2 6
8.	¬ ¬r	¬ ¬i 7
9.	¬ ¬s	MT 2,8
10.	s	¬ ¬e 9
11.	t∧s	∧i 5,10

Question

4. $p \wedge (q \rightarrow r), \neg q \rightarrow s \vdash p \wedge (\neg r \rightarrow s)$

Solution:

1.	p∧(q→r)	premise
2.	¬q → s	premise
3.	p	∧e1 1
4.	q→r	∧e2 1
5.	¬r	assumption
6.	¬q	MT 4,5
7.	s	→e 2,6
8.	¬r → s	→i 5-7
9.	p /\ (~r ->s)	/\ i 3, 8

Question

5. $\vdash p \wedge q \rightarrow p$

Solution:

1.p∧q	assumption
2.p	∧e1 1

3.p∧q→p →i 1-2

Question

6. $p \rightarrow q, r \rightarrow s \vdash p \wedge r \rightarrow q \wedge s$

Solution:

1.	$p \rightarrow q$	premise
2.	$r \rightarrow s$	premise
3.	$p \wedge r$	assumption
4.	p	$\wedge e1$ 3
5.	q	$\rightarrow e$ 1,4
6.	r	$\wedge e2$ 3
7.	s	$\rightarrow e$ 2,6
8.	$q \wedge s$	$\wedge i$ 5,7
9.	$p \wedge r \rightarrow q \wedge s$	$\rightarrow i$ 3-8