Utility Maximization

Let the consumer has a utility function of the form $U = f(x, y)$ and the total income available to her is m while the prices are P_x and P_y. The consumer's problem is choose the affordable bundle that maximizes her utility.

The feasible set: The consumer cannot spend more than the total income m. Thus,

$$P_x * x + P_y * y \leq m \tag{1}$$

Since *monotonicity*, i.e., *more is better* holds for the consumer's choice of bundle, so the inequality of eq.2 becomes equality

$$P_x * x + P_y * y = m \tag{2}$$

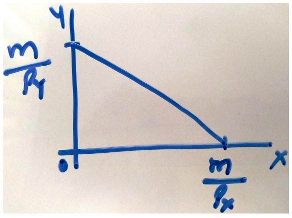

Figure 1: The Budget Constraint (Feasible Set)

The slope of the budget constraint is given as:

$$slope = -\frac{P_x}{P_y} \tag{3}$$

We know that the indifference curve exhibit *diminishing marginal rate of substitution* and this is reflected in the slope of the indifference curves. The slope is given by

$$MRSyx = -\frac{dy}{dx}|du = 0 \tag{4}$$

i.e., the marginal rate of substitution of y for x is the negative of the slope of the indifference curve in figure 2.

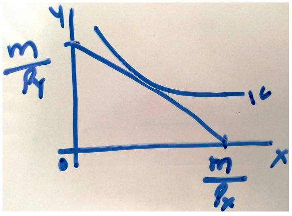

Figure 2: Consumer's Equilibrium

Also, we know that:

$$MRSyx = \frac{MU_x}{MU_y} \tag{5}$$

The optimal bundle for the consumer is one where the following two conditions are fulfilled:
a. The bundle must lie on the budget line;
b. The indifference curve must be tangent to the budget line at the point of optimal bundle.

From (3), (4), and (5), we get that at point of consumer's equilibrium (ignoring the negative signs):

$$MRSyx = \frac{P_x}{P_y} = \frac{MU_x}{MU_y} \tag{6}$$

Using Lagrange multiplier to find consumer's equilibrium:

Let the Utility function of the consumer be given by $U = f(x, y)$ and the feasible set by $P_x * x + P_y * y = m$.
We set the Lagrange function as:

$$\mathcal{L} = U(x, y) + \lambda(m - P_x * x - P_y * y) \tag{7}$$

getting the first order conditions for the eq.7 and setting it equal to zero shall give us values of x and y that are in the optimal bundle.

Example: Let $U = x_1^{0.25} x_2^{0.75}$, $m = Rs100$; $Px_1 = Rs2$ and $Px_2 = Rs4$. Find the optimal bundle of x_1 and x_2 for the consumer.
Solution:

Set the Lagrangian function as:

$$\mathcal{L} = x_1^{0.25} x_2^{0.75} + \lambda(m - Px_1 * x_1 - Px_2 * x_2) \tag{8}$$

Setting the *first order conditions* equal to zero, we get:

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0.25 x_1^{-0.75} x_2^{0.75} - 2\lambda = 0 \tag{9}$$

and,

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0.75 x_1^{0.25} x_2^{-0.25} - 4\lambda = 0 \tag{10}$$

and,

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 100 - 2x_1 - 4x_2 = 0 \tag{11}$$

Solving eq. 9 and 10 gives $x_2 = \frac{3}{2} x_1$.
Substituting value of x_2 in eq. 11 gives $x_1^* = 12.5$ and $x_2^* = 18.75$.