Tutorial 12 Solutions

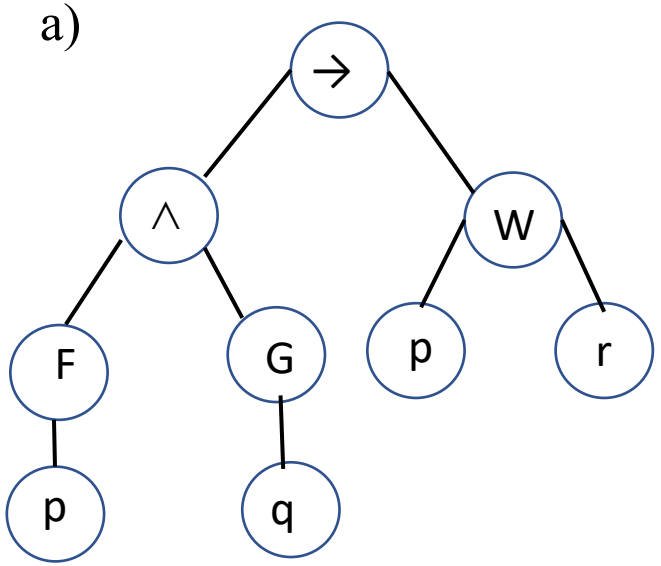
Draw parse trees for the LTL formulas:

(a) $\mathrm{F}\, p \wedge \mathrm{G}\, q \rightarrow p\, \mathrm{W}\, r$

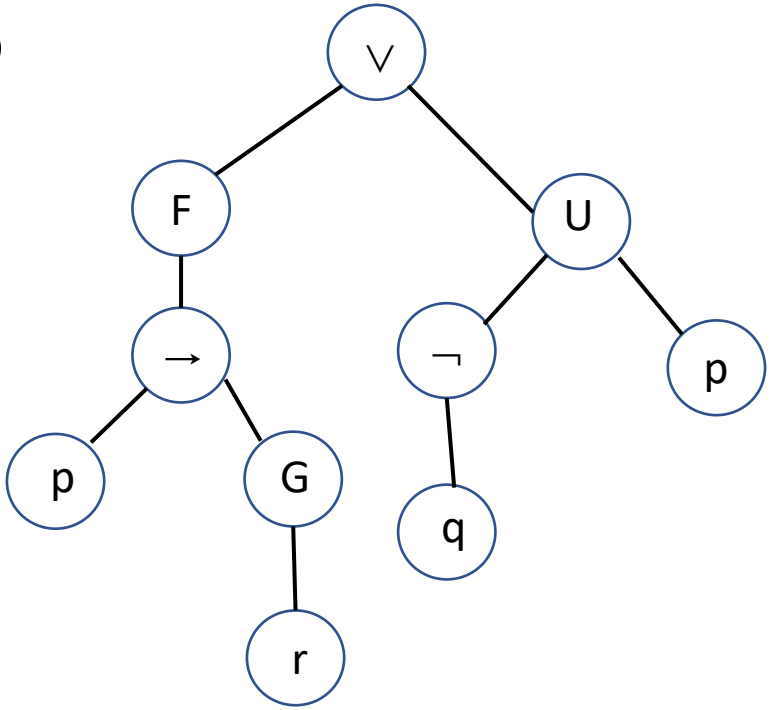
(b) $\mathrm{F}\, (p \rightarrow \mathrm{G}\, r) \vee \neg q\, \mathrm{U}\, p$

- **Solution**

a)



b)



Question

List all subformulas of the LTL formula $\neg p \,\mathrm{U}\, (\mathrm{F}\, r \vee G\, \neg q \rightarrow q \,\mathrm{W}\, \neg r)$.

Solution

The subformulas are

- p
- ¬p
- r
- Fr
- q
- ¬q
- G¬q
- ¬r
- qW¬r
- FrVG¬q
- FrVG¬q→qW¬r
- ¬pU(FrVG¬q→qW¬r)

Question

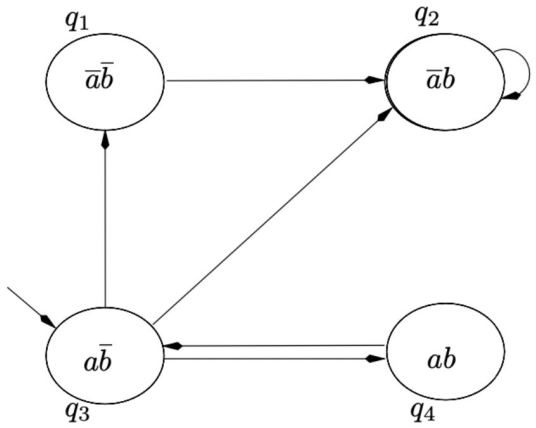


Figure 3.39. A model $\mathcal{M}$.

Consider the system of Figure 3.39. For each of the formulas ϕ:

(a) $\mathrm{G}\, a$

(b) $a\ \mathrm{U}\ b$

(c) $a\ \mathrm{U}\ \mathrm{X}\,(a \wedge \neg b)$

(d) $\mathrm{X}\,\neg b \wedge \mathrm{G}\,(\neg a \vee \neg b)$

(e) $\mathrm{X}\,(a \wedge b) \wedge \mathrm{F}\,(\neg a \wedge \neg b)$

(i) Find a path from the initial state q_3 which satisfies ϕ.

(ii) Determine whether $\mathcal{M}, q_3 \models \phi$.

- **Solution**

a)

i) π : q3->q4->q3->q4->.....

ii)No since π : q3->q2->q2->..... does not satisfy

b)

i) π : q3->q4->q3->q2->q2->.....

ii) No since π : q3->q1->q2->q2->..... does not satisfy

c)

i) π : q3->q4->q3->q4->.....

ii) No since π : q3->q2->q2->..... does not satisfy

d)

i) π :q3->q1->q2->q2->....

ii) No since π : q3->q4->q3->q3->..... does not satisfy

e)

i) π : q3->q4->q3->q1-> q2->q2->....

ii) No since π : q3->q1->q2->q2->..... does not satisfy

Question

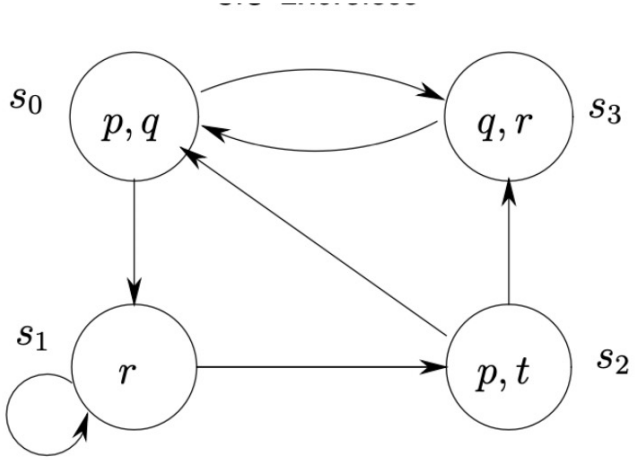


Figure 3.41. Another model with four states.

Consider the model $\mathcal{M}$ in Figure 3.41. Check whether $\mathcal{M}, s_0 \models \phi$ and $\mathcal{M}, s_2 \models \phi$ hold for the CTL formulas ϕ:

(a) AF q
(b) AG (EF $(p \vee r)$)
(c) EX (EX r)
(d) AG (AF q).

- **Solution**

a)

i)Yes since s0 itself has q

ii)Yes since every path will pass through s2 or s0 which has q

b)

i) Yes, since all the states have either p or r , thus for all paths there will always be a state reachable which satisfies p ∨r

ii) Yes, since all the states have either p or r , thus for all paths there will always be a state reachable which satisfies p ∨r

c)

i)Yes, consider the path s0->s1->s1->....

ii) Yes, consider s2->s0->s1->

d)

i) No, consider s0->s1->s1->s1…

ii) No, consider s2->s0->s1->s1…