Tut_6

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CS F222: Discrete Structures for Computer Science

Tutorial - 6 (Equivalence and Partial Order Relations)

1. Suppose that A is a nonempty set, and f be a function that has A as its domain. Let R be a relation on A consisting of all order pairs (x, y) such that $f(x) = f(y)$.
   (a) Show that R is an equivalence relation on A.
   (b) Find all the equivalence classes of R.
2. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.
3. Let R be a transitive and reflexive relation on A. Let T be a relation on A, such that $(a, b) \in T$ if and only if $(a, b) \in R$ and $(b, a) \in R$. Show that T is an equivalence relation.
4. Is (S, R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if
   (a) a is taller than b?
   (b) $a = b$ or a is an ancestor of b?
   (c) a and b have a common friend?
5. Draw the Hasse diagram for divisibility on the set
   (a) $\{2, 3, 4, 9, 12, 18\}$.
   (b) $\{1, 2, 3, 4, 6, 9, 36\}$
   (c) $\{2, 3, 5, 30, 60, 120, 180, 360\}$.
   (d) $\{1, 3, 9, 27, 81, 243\}$.
6. Answer the following questions for the poset $(\{3, 5, 9, 15, 24, 45\}, |)$.
   (a) Find the maximal elements.
   (b) Find the minimal elements.
   (c) Is there a greatest element?
   (d) Is there a least element?
   (e) Find all upper bounds of $\{3, 5\}$.
   (f) Find the least upper bound of $\{3, 5\}$, if it exists.
   (g) Find all lower bounds of $\{15, 45\}$.
   (h) Find the greatest lower bound of $\{15, 45\}$, if it exists.
7. Is there any lattice in the posets given in question 5 above.

1

Q1.

① R is reflexive
② R is symmetric
③ R is transitive

① For any (x, x), where $x \in A$, $f(x) = f(x)$, $(x, x) \in R$ $\forall x \in A$.
R is reflexive

② Let $(x, y) \in R$, $f(x) = f(y)$ which holds iff $f(y) = f(x)$,
thus $(y, x) \in R$, R is symmetric.

③ Let $(x, y) \in R$ and $(y, z) \in R$, then $f(x) = f(y)$ and $f(y) = f(z)$
$\Rightarrow$ $f(x) = f(z)$. thus $(x, z) \in R$, It follows that R is transitive.

(b) Let B be the range of the function f, Then for each $b \in B$, $[b]_R = \{a \mid b = f(a) \text{ where } a \in A\}$

Q2. ① reflexivity :

$((a, b)(a, b)) \in R$ iff $a + b = b + a$
Hence, R is reflexive

② Let $((a, b)(c, d)) \in R$, then $a + d = b + c$, so $c + b = d + a$,
so $((c, d)(a, b)) \in R$. Hence R is symmetric.

③ Transitivity ; suppose that $((a, b), (c, d)) \in R$ and $((c, d), (e, f)) \in R$

then $a + d = b + c$ and $c + e = d + f$,
by adding both equations

$a + d + c + e = b + c + d + f$

$a + e = b + f$, so $((a, b)(e, f)) \in R$.

Hence R is transitive.

Q3. S.T. T satisfies the three properties, reflexive symmetric and transitive.

Given that $(a, b) \in T$ iff $(a, b) \in R$ and $(b, a) \in R$,
where R is reflexive and transitive.

Since R is reflexive $(a, a) \in R$ and hence $(a, a) \in T$
Thus, T is reflexive.

Let $(a, b) \in T$, This implies $(a, b) \in R$ and $(b, a) \in R$.

Let $(a,b) \in T$, This implies $(a,b) \in R$ and $(b,a) \in R$.
Thus $(b,a) \in T$. Hence T is symmetric.

Suppose $(a,b) \in T$ and $(b,c) \in T$. $\Rightarrow$ $(a,b) \in R$, $(b,a) \in R$, $(b,c) \in R$ and $(c,b) \in R$,

Since R is transitive,
we have (i) $(a,c) \in R$
(ii) $(c,a) \in R$

$\Rightarrow$ $(a,c) \in T$,
Hence, T is transitive

Q4. To show some relation is partially ordering
(1) reflexive
(2) antisymmetric
(3) transitive

(a) $(a,b) \in R$ if a is taller than b,
since nobody is taller than himself,
R is not reflexive , $(a,a) \notin R$.

(S,R) is not a poset.

(b) $(a,b) \in R$ if $a=b$ or a is an ancestor of b.

clearly, R is reflexive
R is antisymmetric

$$\left.\begin{array}{l}(a,b) \in R \\ (b,c) \in R\end{array}\right\} \Rightarrow (a,c) \in R$$ R is transitive.

(S,R) is a poset.

(c) a and b have a common friend.

$(a,a) \in R$, reflexive

, not antisymmetric
$(a,b) \in R$
$\Rightarrow (b,a) \in R$.

(S,R) is not poset.

(d) a is not taller than b?

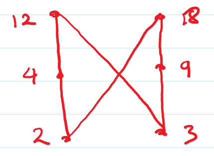
$(a,b) \in R$, $(a,a) \in R$. reflexive

pick two distinct people x and y who have the same height.

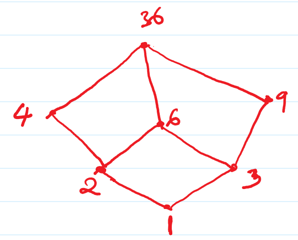
$(x,y) \in R$, $(y,x) \in R$ $\Rightarrow$ R is not antisymmetric.

(S,R) is not a poset.

Q5. (a) $(\{2,3,4,9,12,18\}, |)$



(b) $(\{1,2,3,4,6,9,36\}, |)$



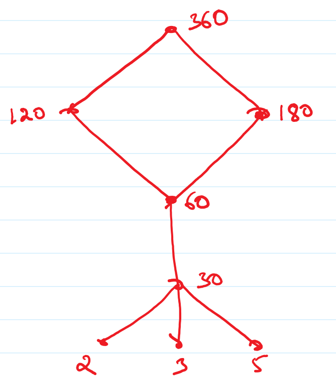
lattice

$\{2,6,3\}$

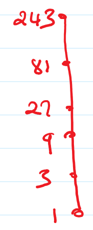
$(6, 36)$ upper bounds

least upper bound (lub) $= 6$

(c) $(\{2,3,5,30,60,120,180,360\}, |)$

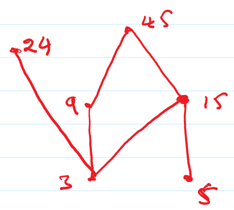


(d) $(\{1,3,9,27,81,243\}, |)$



lattice

Q6. $(\{3,5,9,15,24,45\}, |)$



(a) maximal elements

24, 45

(b) minimal elements

3, 5

(c) Is there a greatest element?

NO

(d) Is there a least element?

NO

(e) all upper bounds of $\{3,5\}$

$\{15, 45\}$

(f) lub of $\{3,5\}$ if it exists?

15

(h) glb of $\{15, 45\}$

15

Homework:

Q1. Let R_1 and R_2 be the congruent modulo 3 and the congruent modulo 4 relations, resp. on the set of integers.

Q1. Let R_1 and R_2 be the congruent modulo 3 and the congruent modulo 4 relations, resp. on the set of integers.

i.e., $R_1 = \{(a,b) \mid a \equiv b \pmod 3\}$

$R_2 = \{(a,b) \mid a \equiv b \pmod 4\}$

Find a) $R_1 \cup R_2$

b) $R_1 \cap R_2$

c) $R_1 - R_2$

d) $R_2 - R_1$

e) $R_1 \oplus R_2$

Q2 How many relations are there on a set with n elements that are

a) symmetric ?

b) antisymmetric ?

c) asymmetric ?

d) irreflexive ?

e) reflexive and symmetric

f) neither reflexive nor irreflexive

Q3. S.T the closure with respect to the property P of the relation $R = \{(0,0), (0,1), (1,1), (2,2)\}$ on $\{0,1,2\}$ does not exist if P is the property

a) " is not reflexive "

b) " has an odd number of elements"