Tutorial 9 Solutions

In this question, assume the following predicate and constant symbols:

W(x,y) : x wrote y
L(x,y) : x is longer than y
N(x) : x is a novel
h : Hardy
a : Austen
j : Jude the Obscure
p : Pride and Predjudice

Given these specifications, which of the predicate logic formulas below represent the sentence, *'Hardy wrote a novel which is longer than any of Austen's'* in predicate logic?

1. $\forall x\ (W(h,x) \rightarrow L(x,a)))$
2. $\forall x\ \exists y\ (L(x,y) \rightarrow W(h,y) \wedge W(a,x))$
3. $\forall x\ \forall y\ (W(h,x) \wedge W(a,y) \rightarrow L(x,y)))$
4. $\exists x\ (N(x) \wedge W(h,x) \wedge \forall y\ (N(y) \wedge W(a,y) \rightarrow L(x,y)))$
5. $\exists x\ \forall y\ (W(h,x) \rightarrow W(a,y) \wedge L(x,y))$

Answer:
Option 4.

Prove the following sequents where F, G, P, and Q have arity 1, and S has arity 0 (a ‘propositional atom’):

a) $\exists x\,(S \rightarrow Q(x)) \vdash S \rightarrow \exists x\, Q(x)$
b) $\forall x\, P(x) \rightarrow S \vdash \exists x\,(P(x) \rightarrow S)$

Solution:

a)

1. $\exists x(S\rightarrow Q(x))$	premise
2. S	assumption
3. x_0	
4. $S\rightarrow Q(x_0)$	assumption
5. $Q(x_0)$	$\rightarrow$e 4,2
6. $\exists x\, Q(x)$	$\exists$x i 5
7. $\exists x\, Q(x)$	$\exists$x e 1,3-6
8. $S\rightarrow \exists x\, Q(x)$	$\rightarrow$i 2-7

b)

1	$\forall x\, P(x) \to S$	prem
2	$\neg\exists x\, (P(x) \to S)$	assum
3	x_0	
4	$\neg P(x_0)$	assum
5	$P(x_0)$	assum
6	$\bot$	$\neg$e 5, 4
7	S	$\bot$e 6
8	$P(x_0) \to S$	$\to$i 5–7
9	$\exists x\, (P(x) \to S)$	$\exists x$ i 8
10	$\bot$	$\neg$e 9, 2
11	$\neg\neg P(x_0)$	$\neg$i 4–10
12	$P(x_0)$	$\neg\neg$e 11
13	$\forall x\, P(x)$	$\forall x$ i 3–12
14	S	$\to$e 1, 13
15	$P(t)$	assum
16	S	copy 14
17	$P(t) \to S$	$\to$i 15–16
18	$\exists x\, (P(x) \to S)$	$\exists x$ i 17
19	$\bot$	$\neg$e 18, 2
20	$\neg\neg\exists x\, (P(x) \to S)$	$\neg$i 2–19
21	$\exists x\, (P(x) \to S)$	$\neg\neg$e 20

(b) $S \to \exists x\, Q(x) \vdash \exists x\,(S \to Q(x))$

(c) $\exists x\, P(x) \to S \vdash \forall x\,(P(x) \to S)$

Solution:

a) No (a)

b)

1. $S \to \exists x\, Q(x)$	premise
2. $S \lor \neg S$	LEM
3. S	assumption
4. $\exists x Q(x)$	$\to$e 1,3
5. x_0 $Q(x_0)$	assumption
6. S	assumption
7. $Q(x_0)$	copy 5
8. $S \to Q(x_0)$	$\to$i 6-7
9. $\exists x(S \to Q(x))$	$\exists x$ i 8
10. $\exists x(S \to Q(x))$	$\exists x$ e 4,5-9
11. $\neg S$	assumption
12. S	assumption
13. $\bot$	$\neg$e 11,12
14. $Q(x_0)$	$\bot$ e 13
15. $S \to Q(x_0)$	$\to$i 12-14
16. $\exists x(S \to Q(x))$	$\exists x$ i 15
17. $\exists x(S \to Q(x))$	$\lor$e 2,3-10,11-16

Solution c:

1. $\exists x P(x) \rightarrow S$	premise
x_0	
2. $P(x_0)$	assumption
3. $\exists x P(x)$	$\exists$x i 2
4. S	$\rightarrow$e 3,1
5. $P(x_0) \rightarrow S$	$\rightarrow$i 2-4
6. $\forall x(P(x) \rightarrow S)$	$\forall$x i 2-5

Let φ be ∃x (P(y, z) ∧ (∀y (¬Q(y, x) ∨ P(y, z)))), where P and Q are predicate

symbols with two arguments.

(a) Draw the parse tree of φ.

(b) Identify all bound and free variable leaves in φ.