Tut_7

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CS F222: Discrete Structures for Computer Science

Tutorial - 7 (Topological Sorting and Counting)

1. Find the topological sorting of the elements in the posets given in Figure 1.

Figure 1: Posets

a, b, c, d, e, g, f, h

a, b, c, e, g, d, f, h, i, j

$O(m+n)$

2. A multiple-choice test contains 10 questions. There are four possible answers for each question.
   (a) In how many ways can a student answer the questions on the test if the student answers every question?
   (b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

$\underbrace{4 \cdot 4 \cdot 4 \cdot \cdots \cdot 4}_{10 \text{ times}} = 4^{10}$

5^{10}

3. A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

3^{10} ways

4. There are 345 students at a college who have taken a course in calculus, 212 who have taken a course in discrete mathematics, and 188 who have taken courses in both calculus and discrete mathematics. How many students have taken a course in either calculus or discrete mathematics?

$C \to$ students who have taken calculus

$D \to$ " DM

$C \cap D \to$ both calculus & DM.

$|C \cup D| = |C| + |D| - |C \cap D|$

$= 345 + 212 - 188 = 369$

5. There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C programming (CP). Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and CP, and 290 have taken courses in both Java and CP. If 189 of these students have taken courses in Linux, Java, and CP. How many of these 2504 students have not taken a course in any of these three programming languages?

1

6. Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11. Is the 7th pick is necessary to achieve the goal.
7. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
   (a) How many balls must she select to be sure of having at least three balls of the same color?
   (b) How many balls must she select to be sure of having at least three blue balls?
8. Show that every sequence of n^2+1 distinct real numbers contains a subsequence of length $n+1$ that is either strictly increasing or strictly decreasing.

2

Q5.

$|J| = 1876$

$|L| = 999$

$|C| = 345$

$|J \cap L| = 876$

$|L \cap C| = 231$

$|J \cap C| = 290$

$|J \cap L \cap C| = 189$

$|L \cup J \cup C| = |L| + |J| + |C| - |L \cap J| - |J \cap C| - |C \cap L| + |L \cap J \cap C|$

$|L \cup J \cup C| = 999 + 1876 + 345 - 876 - 290 - 231 + 189 = 2012$

$2504 - 2012 = 492$ no of student who did take take any course.

Q7. (a) 5 (b) 13

Q8. A sequence of n^2+1 distinct real numbers

1, 2, 8, 3, 4, 10

2, 3, 10, 2, 8, 3

strictly increasing

Q6. 1, 2, ..., 10.

(1, 10), (2, 9), (3, 8), (4, 7) and (5, 6) are five pairs such that their sum is 11.

By PHP, we say that when you pick the 6th integer, there is a pair whose sum is 11.

7th pick is not necessary.

Let $a_1, a_2, \dots a_{n^2+1}$ be the distinct real numbers.

with term a_k associate the pair (i_k, d_k) where i_k is the length of the longest increasing subsequence starting a_k and d_k is the length of the longest decreasing sequence starting at a_k.

Assume there are no decreasing or increasing subsequence of length $n+1$, Then i_k and d_k are both +ve integers $\le n$.

for $k = 1, 2, \ldots, n^2+1$.

By the product rule, we have n^2 possible ordered pairs (i_k, d_k)

By PHP, two of these pairs must be equal.

i.e. for some a_s and a_t with $s < t$, we have $i_s = i_t$ and $d_s = d_t$.

If $a_s < a_t$, then since $i_s = i_t$, an increasing subsequence of length $i_t + 1$ can be built starting at a_s and following it with the increasing subsequence of length i_t which begins at a_t.

Therefore $i_s = i_t + 1 \neq i_t$, a contradiction

If $a_s > a_t$, a contradiction in this case also.

i_t

a_t (t, d_t) a_s (i_s, d_s) $i_t = i_s$ and $d_t = d_s$

Assume $a_s < a_t$

$t < s$

Consider the decreasing subsequence starting at a_s, length is d_s

append a_t to the beginning of the above sequence

length is $d_s + 1$

$d_t = d_s + 1 \neq d_s$, a contradiction.

Q. Give a combinatorial proof for the following

$$\sum_{k=1}^{n} k \binom{n}{k} = n 2^{n-1}$$

# of ways to form a committee with a leader

$$\boxed{\sum_{k=1}^{n} \binom{n}{k} k = n 2^{n-1}}$$