1. Given cost: $C(q) = 120 + 2q^2$

Revenue $R(q) = 100q$ ie $p = 100$

Find profit maximising quantity (q^*) ?

$$MR = P = 100$$

$$MC = \frac{\partial C(q)}{\partial q} = 4q$$

At equilibrium $MR = MC$

$$\Rightarrow 100 = 4q$$

$$\Rightarrow q^* = \frac{100}{4} = 25$$

2. given cost : $C(q) = 420 + 3q + 4q^2$

Revenue : $R(q) = 100q - q^2$

Determine q^*.

$$MR = \frac{\partial R(q)}{\partial q} = 100 - 2q$$

$$MC = \frac{\partial C(q)}{\partial q} = 3 + 8q$$

At equilibrium $MR = MC$

$$\Rightarrow 100 - 2q = 3 + 8q$$

$$\Rightarrow 97 = 10q$$

$$\Rightarrow q^* = 9.7$$

3. Given the demand function $q = 80 - 2P$, calculate marginal revenue as function of q

If cost function is $C(q) = 120 + 2q^2$, find q^*

$$TR = P \cdot q$$

$$= \left(40 - \frac{q}{2}\right) \times q$$

$\because q = 80 - 2P$

$\Rightarrow 2P = 80 - q$

$\Rightarrow P = 40 - \frac{q}{2}$

$$\therefore TR \Rightarrow 40q - 0.5q^2$$

$$MR = 40 - q$$

$$MC = \frac{\partial C(q)}{\partial q} = 4q$$

At equilibrium : $MR = MC \Rightarrow \cancel{40q - 0.5q^2}\ 40 - q = 4q$

$\Rightarrow \cancel{36q - 0.5q^2 = 0}$

$\Rightarrow \cancel{q(36 - 0.5q) = 0} \Rightarrow 5q = 40$

$\Rightarrow q = 8$

$\because q \cancel{= 0,\ 36 - 0.5q = 0 \Rightarrow q = 72}$

4. Cost function of a firm : $TC = 500 + 10q + 5q^2$
Market demand is given by : $Q_D = 105 - \frac{1}{2}P$

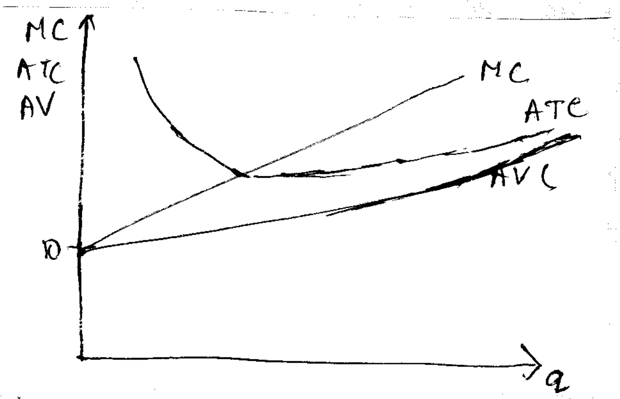
a) Find marginal cost (MC), average total cost (ATC), average variable cost (AVC) and average fixed cost (AFC). Show the firm's MC, ATC and AVC on one graph.

$$MC = \frac{\partial TC}{\partial q} = 10 + 10q$$

$$ATC = \frac{500 + 10q + 5q^2}{q} = \frac{500}{q} + 10 + 5q$$

$$AVC = 10 + 5q$$

$$AFC = \frac{500}{q}$$



b) Find break even price and quantity in the short-run?

Break even : $MC = ATC$

$$\Rightarrow 10 + 10q = \frac{500}{q} + 10 + 5q$$

$$\Rightarrow 5q = \frac{500}{q}$$

$$\Rightarrow 5q^2 = 500$$

$$\Rightarrow q^2 = 100$$

$$q = 10$$

Now, $q = 10$ and equilibrium is $P = MC$

$\therefore P = 10 + 10q = 10 + (10 \times 10)$
$= 110$

c) Find shut down price and quantity.

At shut down point, $q = 0$

At shut down $MC = AVC$

$\therefore P = 10$

d) If market price is Rs. 50, how many units will this firm produce?

If price is given as 50, & $MC = 10 + 10q$.

$\therefore \quad P = MC$

$50 = 10 + 10q$

$\Rightarrow 10q = 40$

$\Rightarrow q = 4$.

e) If market price is Rs 50, how many firms are there in this market.

Market demand curve: $Q_D = 105 - \frac{1}{2}P$

At $P = 50$, $Q_D = 105 - \frac{1}{2} \times 50$

$= 80$

If firms are identical, number of firms $= \frac{80}{4} = 20$

f) Assume the industry to be perfectly competitive, what output will be produced in a long run equilibrium? What is long run equilibrium price?

In long run equilibrium, there must be zero profit.

$\therefore$ Profit : π = Total revenue − Total cost

$= TR - TC$

$= Pq - ATC \cdot q$

$= (P - ATC)\, q$

$\because q \neq 0$, for long run equilibrium: $P = ATC$.

Normal equilibrium condition $P = MR = MC$.

Combining, $P = ATC = MC$

$\Rightarrow 10 + 10q = \frac{500}{q} + 10 + 5q$

$\Rightarrow 5q^2 = 500 \Rightarrow q = 10$

$P = 10 + 10q = 110.$

g) How many firms will be there in the industry in long run equilibrium?

$$Q_D = 105 - \frac{1}{2} \times 110$$
$$= 50$$
$$\therefore N = \frac{50}{10} = 5$$

5) Find the returns to scale for the following production functions :

a) $Q = 2K + 3L$ b) $Q = 0.5KL$

c) $Q = K^{0.3}L^{0.2}$

Increase capital and labour by same proportion 'm' to find the effect on output.

i) $Q = 2K + 3L$

$\therefore 2Km + 3mL = m(2K + 3L) = mQ$

$\therefore$ Constant returns to scale.

ii) $Q = 0.5KL$

$\therefore 0.5(mK)(mL) = 0.5m^2KL = m^2(0.5KL)$
$= m^2Q$

$\therefore$ ~~Interesting~~ Increasing returns to scale.

iii) $Q = K^{0.3}L^{0.2}$

$\therefore (mK)^{0.3}(mL)^{0.2}$
$= m^{0.3+0.2}K^{0.3}L^{0.2}$
$= m^{0.5}K^{0.3}L^{0.2} = m^{0.5}Q$

$\therefore$ Decreasing returns to scale.