

## **1 Barber Paradox**

**There is a lone male barber who lives on an island. The barber shaves all those men who do not shave themselves and only those men.**

*Consider following statement and determine if it is true or false?* Statement: The barber shaves himself.

Such statements are called as propositions. (Formal definition will be discussed in later lectures).

This statement is paradoxical. The barber cannot shave himself as he only shaves those who do not shave themselves. Thus, if he shaves himself he ceases to be the barber (i.e. person who cannot shave). On the other hand, if barber does not shave himself, then he belongs to the group of people who would be shaved by the barber and hence barber should shave himself.

Hence, there is no clear answer to proposition.

## **2 Naive Set Theory**

- This has been proposed by *Georg Cantor*.
- A SET is a collection of objects or entities.
- Bertrand Russell proposed Russell's paradox for above definition

### **2.1 Russell's Paradox**

Let  $R$  be the set of all sets that are not members of themselves.

$$A = \{1, 2, 3, 4\}, \therefore A \in R$$
$$R = \{X | X \notin X\}$$

Is the statement  $R \in R$  true or false?

Similar to the Barber Paradox, if we assume that statement is true that  $R$  now belongs to itself. Now,  $R$  is a set which contains itself and hence should not be included in  $R$ . If we assume statement is false that  $R$  does not belong to itself. Then, as per the definition  $R$  is now a set which does not contain itself and hence should be included in the set  $R$ .

Hence, answering this results in paradox. There is no clear answer to the statement.