Logic Tutorial 1 Solutions

1. p : the sun shines today, q : the sun shines tomorrow, $p \to \neg q$.
2. p : Robert was Jealous of Yvonne, q : Robert was in a good mood, $p \vee \neg q$.
3. p : barometer falls. q : it will rain, r : t will snow, $p \to (q \vee r)$.
4. p : a request occurs, q : the request is eventually acknowledged, r : the requesting process makes progress, $p \to (q \vee \neg r)$
5. p : the cause of cancer is determined, q : a new cancer drug is found, r : cancer is cured, $\neg(p \wedge q) \to \neg r$.
6. q : You can ride the roller coaster,r : You are under 4 feet tall,s : You are older than 16 years old.$(r \wedge \neg s) \to \neg q$.
7. p : You can access the internet from campus, q : you are a Computer Science major, r : You are a freshman. $p \to (q \vee \neg r)$.

*Q: When we have a logical statement of the form $p \rightarrow q$, where p and q are propositions corresponding to real world events, does the truth of the statement $p \rightarrow q$ necessarily imply that the event p *caused* the event q? If not, could you provide an example where the statement $p \rightarrow q$ can be true without p necessarily causing q.*

It could simply be the case, for example that a common cause (what is called a confounder) causes both p and q to be true, so that whenever p is true q is also true, without necessarily p causing q to be true.

For example,

p: This week, the weekly ice-cream sales are 3x the average.

q: This week, the heat-strokes are 5x the weekly average.

Both p and q could be true during peak summer, yet without one causing the other.

p	q	$(p \to q)$
F	F	**T**
F	T	**T**
T	F	**F**
T	T	**T**

p	q	$(\neg p \vee q)$
F	F	**T**
F	T	**T**
T	F	**F**
T	T	**T**

- Both the truth tables are identical in each row, and what this means is that the two propositions are *equivalent*, in that one can substitute one for the other in any larger compound proposition without altering its truth value.

p	q	(p → q)
F	F	**T**
F	T	**T**
T	F	**F**
T	T	**T**

p	q	(¬q → ¬p)
F	F	**T**
F	T	**T**
T	F	**F**
T	T	**T**

- Here again ¬q→¬p is equivalent to p→q.
- In fact, ¬q→¬p is called the *contrapositive* of p→q.