READING: Sec 3.2.3

Examples of things LTL cannot do:

- From ~~any~~ a state s_0 *it is possible* to get to a **restart** state (i.e., there is a path from all states to a state satisfying **restart**).
- The lift *can* remain idle on the third floor with its doors closed (i.e., from the state in which it is on the third floor, there is a path along which it stays there).

LTL Equivalence (contd..)

It's also the case that F distributes over $\vee$ and G over $\wedge$, i.e.,

$$\mathrm{F}\,(\phi \vee \psi) \equiv \mathrm{F}\,\phi \vee \mathrm{F}\,\psi$$

$$\mathrm{G}\,(\phi \wedge \psi) \equiv \mathrm{G}\,\phi \wedge \mathrm{G}\,\psi.$$

F does not distribute over $\wedge$!

$$\mathrm{F}\,\phi \equiv \top\ \mathrm{U}\ \phi$$

$$\phi\ \mathrm{U}\ \psi \equiv (\phi\ \mathrm{W}\ \psi) \wedge (\mathrm{F}\,\psi)\,. \tag{3.2}$$

PROOF (OF ONE SIDE)

To prove equivalence (3.2), suppose first that a path satisfies $\phi\ \mathrm{U}\ \psi$. Then, from clause 11, we have $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$. From clause 12, this proves $\phi\ \mathrm{W}\ \psi$, and from clause 10 it proves $\mathrm{F}\,\psi$. Thus for all paths π, if $\pi \vDash \phi\ \mathrm{U}\ \psi$ then $\pi \vDash \phi\ \mathrm{W}\ \psi \wedge \mathrm{F}\,\psi$. As an exercise, the reader can prove it the other way around.

Proof that if $\phi\ \mathrm{U}\ \psi$ is true, then $\phi\ \mathrm{W}\ \psi \wedge \mathrm{F}\,\psi$ is true.

ADEQUATE SETS OF CONNECTIVES in LTL

Small adequate sets of connectives also exist in LTL. Here is a summary of the situation.

- X is completely orthogonal to the other connectives. That is to say, its presence doesn't help in defining any of the other ones in terms of each other. Moreover, X cannot be derived from any combination of the others.

- Each of the sets $\{U, X\}$, $\{R, X\}$, $\{W, X\}$ is adequate.

Recall: in Prop Logic $\{\bot, \wedge, \neg\}$ is an adequate set of connectives since $\vee$, $\rightarrow$, $\top$ can been written using them.