BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 11

10 Well Formed Formulas

10.1 What is allowable formula ϕ?

- Formulas are string over the alphabet $\{p, q, r..\} \cup \{p_1, p_2, ...\} \cup \{\neg, \wedge, \vee, \rightarrow, (,)\}$.
- But not all strings are admissible.
  e.g $(\neg)() \wedge pqr \rightarrow$.

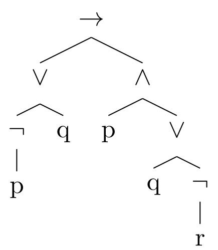
Definition : The well formed formulas of propositional logic are those which we obtain by using the construction rules below and only those by applying them finitely many times.

1. atom: Every propositional atom $p, q, r, ..$ or $p_1, p_2, p_3, ..$ is a well formed formula.
2. $\neg$: If ϕ is a well formed formula then so is $\neg\phi$.
3. $\wedge$: If ϕ and ψ are well formed formulas so is $\phi \wedge \psi$.
4. $\vee$: If ϕ and ψ are well formed formulas so is $\phi \vee \psi$.
5. $\rightarrow$: If ϕ and ψ are well formed formulas so is $\phi \rightarrow \psi$.

10.2 How do we show that a formula is well formed?

Construct in a top down manner, a parse tree where its leaves are atoms. e.g.

$$(((\neg p) \vee q) \rightarrow (p \wedge (q \vee (\neg r))))$$



- Parse trees of well formed formulas are either an atom as root or the root contains $\neg, \wedge, \vee$ or $\rightarrow$.

- In case of $\neg$, there is only one sub-tree coming out of the root. In case of $\vee, \wedge, \rightarrow$ these are forms.
- A Sub-formula of a formula corresponds to a sub-tree of the parse tree.
  - You can obtain the formula back by using In-order traversal of the parse tree.
  - In-order traversal can be obtained by recursively printing left sub-tree, printing root, printing right sub-tree.
  - Pre-order traversal can be obtained by recursively printing root, printing left sub-tree, printing right sub-tree.
  - Post-order traversal can be obtained by recursively printing left sub-tree, printing right sub-tree, printing root.