BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 8

Example: Prove $\neg p \ \vee q \vdash p \rightarrow q$ is valid.

$$
\begin{array}{|rl}
1. & \neg p \ \vee q \quad \text{premise} \\
\hline
\quad 2. & \neg p \quad \text{assumption} \\
\quad \hline
\quad\quad 3. & p \quad \text{assumption} \\
\quad\quad 4. & \bot \qquad \neg e3, 2 \\
\quad\quad 5. & q \qquad \bot e4 \\
\quad 6. & p \rightarrow q \ [\rightarrow i3 - 5] \\
\hline
\quad 7. & q \quad \text{assumption} \\
\quad \hline
\quad\quad 8. & p \quad \text{assumption} \\
\quad\quad 9. & q \qquad \text{from } 7 \\
\quad 10. & p \rightarrow q \ [\rightarrow i8 - 9] \\
\hline
11. & p \rightarrow q \vee \qquad 1, 2 - 6, 7 - 10
\end{array}
$$

6.4 Negation Introduction and Proof by Contradiction

6.4.1 Negation Introduction

$$
\dfrac{\begin{array}{|c} \phi \\ \vdots \\ \bot \end{array}}{\neg\phi}\neg i
$$

6.4.2 Proof By Contradiction (Reductio Ad Absurdum)

$$
\dfrac{\begin{array}{|c} \neg\phi \\ \vdots \\ \bot \end{array}}{\phi}PBC
$$