Tutorial 8 Solutions

Q2 Let x and y be two persons and there are two predicates:

1. $P(x) = x$ is politician.
2. $S(y) = y$ is a sportsman

Convert the following predicate logic formula into English sentences.

1. $\neg P(x)$
2. $P(x) \rightarrow S(x)$
3. $\forall x(P(x) \rightarrow S(x))$
4. $\exists x(P(x) \rightarrow S(x))$

Solution

1. x is not a politician
2. If x is a politician, then x is also a sportsman
3. All politicians are sportsmen.
4. If all persons are politicians, then there is at least one who is a sportsman.

Q3 Let $P(x, y)$ denote x likes y. Convert the following English sentences into predicate logic formula.

1. Everybody likes everybody.
2. Somebody likes someone.
3. Everybody likes somebody.

4. There is one person whom everyone likes.

Solution

1. ∀x∀y(P(x,y))
2. ∃x∃y(P(x,y))
3. ∀x∃y(P(x,y))
4. ∃y ∀x(P(x,y))

Q4 Convert the following into English sentences

1. $\forall x(bought(Frank, x) \rightarrow bought(Susan, x))$.
2. $\forall x bought(Frank, x) \rightarrow \forall x bought(Susan, x)$.
3. $\forall x \exists y bought(x, y)$

Solution

1.Susan bought everything that Frank bought.

2. If Frank bought everything then Susan also bought everything.

3.Everyone bought something.

Q5 Let $\mathcal{F}$ be $\{d, f, g\}$, where d is a constant, f is a function symbol with two arguments, and g a function symbol with three arguments. Which of the following are terms over $\mathcal{F}$.

1. $g(d, d)$
2. $f(x, g(y, z), d)$
3. $g(x, f(y, z), d)$

Solution

1.Not a term

2.Not a term

3.Yes, it is a term

Q6 Which of the following strings are formulas in predicate logic? Let m be a constant, f a function symbol with one argument and S and B two predicate symbols, each with two arguments:

1. $S(m, x)$
2. $B(m, f(m))$
3. $f(m)$
4. $B(B(m, x), y)$

Solution

1.It is a formula

2.It is a formula

3.It is not a formula

4.It is not a formula

Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives. predicates:
C(x): x is a CSE student
L(x): x loves music
Universe of discourse for the variable x is all students.

(a) Every student loves music
(b) No student loves music
(c) Some students love music
(d) Every CSE student loves music.
(e) Some CSE students love music.

Solution

a) $\forall x L(x)$

b) $\forall x(\neg L(x))$

c) $\exists x(L(x))$

d) $\forall x(C(x) \rightarrow L(x))$

e) $\exists x(C(x) \wedge L(x))$

Let φ be $\exists x\,(P(y, z) \wedge (\forall y\,(\neg Q(y, x) \vee P(y, z))))$, where P and Q are predicate

symbols with two arguments.

(a) Draw the parse tree of φ.

Solution