Tutorial 10 Solutions

Question

Let $F(x, y)$ mean that x is the father of y; $M(x, y)$ denotes x is the mother of y. Similarly, $H(x, y)$, $S(x, y)$, and $B(x, y)$ say that x is the husband/sister/brother of y, respectively. You may also use constants to denote individuals, like 'Ed' and 'Patsy.' However, you are not allowed to use any predicate symbols other than the above to translate the following sentences into predicate logic:

(a) Everybody has a mother.
(b) Everybody has a father and a mother.
(c) Whoever has a mother has a father.
(d) Ed is a grandfather.
(e) All fathers are parents.
(f) All husbands are spouses.
(g) No uncle is an aunt.
(h) All brothers are siblings.
(i) Nobody's grandmother is anybody's father.

Solution

a) $\forall x \exists y\, M(y, x)$

b) $\forall x \exists y \exists z\, (F(y, x) \land M(z, x))$

c) $\forall x[(\exists y M(y, x)) \rightarrow (\exists z F(z, x))]$

d) $\exists x \exists y(F(Ed, y) \wedge (F(y, x) \vee M(y, x)))$

e) $\forall x[\exists y F(x, y) \rightarrow \exists z(F(x, z) \vee M(x, z))]$

f) $\forall x[\exists y H(x, y) \rightarrow \exists z(H(x, z) \vee H(z, x))]$

g)

an uncle is (i) the brother of one's father or mother or (ii) the husband of one's aunt; where an aunt is either the sister of one's father or mother or else the wife of one's uncle.

U(x,y): x is an uncle of y:

$\exists z[B(x, z) \wedge (F(z, y) \vee M(z, y))] \vee \exists z \exists w[H(x, z) \wedge S(z, w) \wedge (F(w, y) \vee M(w, y))]$

A(x,y): x is the aunt of y:

$\exists z[S(x, z) \wedge (F(z, y) \vee M(z, y))] \vee \exists z \exists w[H(z, x) \wedge B(z, w) \wedge (F(w, y) \vee M(w, y))]$

No uncle is an aunt: $\neg \exists x \exists y \exists z[A(x, y) \wedge U(x, z)]$

h) $\forall x \forall y(B(x, y) \wedge B(y, x) \rightarrow \exists z[(F(z, x) \wedge F(z, y)) \vee (M(z, x) \wedge M(z, y))])$

i) $\forall x \forall y[(\exists z(M(y, z) \wedge (M(z, x) \vee F(z, x)))) \rightarrow \neg \exists w F(y, w)]$

Prove that the following sequent is valid:

$$\forall x \, \forall y \, P(x, y) \vdash \forall u \, \forall v \, P(u, v)$$

Solution

1.	$\forall x \forall y P(x,y)$	premise
2. x_0	$\forall y P(x_0, y)$	$\forall$x e 1
3. y_0	$P(x_0, y_0)$	$\forall$y e 2
4.	$\forall v P(x_0, v)$	$\forall$v i 3
5.	$\forall u \forall v P(u,v)$	$\forall$u i 2-4

Prove that the following sequent is valid:

$$\exists x\, \exists y\, F(x, y) \vdash \exists u\, \exists v\, F(u, v)$$

Solution

1. ∃x∃yF(x,y)	premise
2. x_0 ∃yF(x_0,y)	assumption
3. y_0 F(x_0, y_0)	assumption
4. ∃v F(x_0, v)	∃v i 3
5. ∃v F(x_0, v)	∃y e 3-4
6. ∃u ∃v F(u, v)	∃u i 5
7. ∃u ∃v F(u, v)	∃x e 2-6

Prove that the following sequent is valid:

$$\forall x\,(\neg P(x) \wedge Q(x)) \vdash \forall x\,(P(x) \rightarrow Q(x))$$

Solution

1. $\forall x(\neg P(x) \wedge Q(x))$	premise
2. x_0 $(\neg P(x_0) \wedge Q(x_0))$	$\forall x$ e 1
3. $Q(x_0)$	$\wedge$e 2
4. $P(x_0)$	assumption
5. $Q(x_0)$	copy 3
6. $P(x_0) \rightarrow Q(x_0)$	$\rightarrow$i 4-5
7. $\forall x(P(x) \rightarrow Q(x))$	$\forall x$ i 2-6

Prove that the following sequent is valid:

$$\forall x\, P(x) \vee \forall x\, Q(x) \vdash \forall x\, (P(x) \vee Q(x)).$$

Solution

1. $\forall x P(x) \vee \forall x Q(x)$	premise
2. $\forall x P(x)$	assumption
3. x_0 $P(x_0)$	$\forall x$ e 2
4. $P(x_0) \vee Q(x_0)$	$\vee$i1 3
5. $\forall x(P(x) \vee Q(x))$	$\forall x$ i 3-4
6. $\forall x Q(x)$	assumption
7. x_0 $Q(x_0)$	$\forall x$ e 6
8. $P(x_0) \vee Q(x_0)$	$\vee$i2 7
9. $\forall x(P(x) \vee Q(x))$	$\forall x$ i 7-8
10. $\forall x(P(x) \vee Q(x))$	$\vee$e 1,2-5,6-9