BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 6

Prove: $\neg q \to \neg p \vdash p \to q$

Solution:

1.$\neg q \to \neg p$	premise
2.p	assumption
3.$\neg\neg p$	$\neg\neg i$ 2
4.$\neg\neg q$	MT 3,1
5.q	$\neg\neg e$ 4
6.$p \to q$	$\to$ i 2-5

Consider,

1.p	assumption
2.$p \to p$	$\to$ i 1-1

Above is a poof of the statement $\vdash (p \to p)$

Def: A logical formula ϕ with a valid sequent $\vdash \phi$ is called a ***Theorem.***
Remarks: Any sequent $\phi \vdash \psi$ is equivalent to $\vdash \phi \to \psi$.

Proof of $\vdash \phi \to \psi$:

1.ϕ	assumption
2.	
.	
.	
$n.\psi$	$<>$
$\phi \to \psi$	$\to i$ 1-n (n+1)

Remark(b): $\phi_1, \phi_2, ..., \phi_n \vdash \psi$ is equivalent to $\vdash \phi_1 \to (\phi_2 \to (...(.. \to (\phi_n \to \psi))$

5 Rules for Disjunction

5.1 OR-Introduction

$$\frac{\phi}{\phi \vee \psi} \vee_{i1}$$

$$\frac{\phi}{\psi \vee \phi} \vee_{i2}$$

5.2 OR-Elimination

$$\frac{\phi \vee \psi \quad \boxed{\begin{array}{c} \phi \\ \vdots \\ \chi \end{array}} \quad \boxed{\begin{array}{c} \psi \\ \vdots \\ \chi \end{array}}}{\chi} \vee_e$$

Prove: $p \vee q \vdash q \vee p$

$$\begin{array}{ll} 1. p \vee q & \text{premise} \end{array}$$

$$\boxed{\begin{array}{ll} 2. q & \text{assumption} \\ 3. q \vee p & \vee_{i2}\ 2 \end{array}}$$

$$\boxed{\begin{array}{ll} 4. q & \text{assumption} \\ 5. q \vee p & \vee_{i}4 \end{array}}$$

$$\begin{array}{ll} 6. q \vee p & \vee_e\ 1,2\text{-}3,4\text{-}5 \end{array}$$