BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 22

Example

$$2.(p_5 \rightarrow p_{11}) \wedge (p_2 \wedge p_3 \wedge p_5 \rightarrow p_{13}) \wedge (\top \rightarrow p_5) \wedge (p_5 \wedge p_{11} \rightarrow \bot)$$

Solution:

- Mark all occurrences of $\top$.
- Mark $p_5, p_1 1, \bot$
- Print 'Unsatisfiable'.

Example: **1(a).Construct a proposition for the given truth table.**

p	q	ϕ
F	F	T
F	T	T
T	F	F
T	T	F

- The disjunctive normal form formula can be constructed as $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$
- In this example, we have considered all the cases for which the proposition is true and created set of clauses by anding them.
- Later all the clauses are combined using disjunction.

Example: **1(b).Prove the converse of the previous example.**

- The DNF clause consists of set of clauses combined using disjunction.
- All these clauses have set of atoms which are combined using conjunction.
- If the truth table is true then there is at least one clause with all atoms true.
- Now, for all the cases where the proposition evaluates to false there is at least one atom which differs to the evaluation where the formula is true.

- $\therefore$ the corresponding DNF clauses also evaluates to false as at least one of the atom in the clause evaluates to false.
- Thus, DNF becomes disjunction of all clauses which evaluates to false and thus formula evaluates to false.

Example: **2.Convert given truth table to CNF formula.**

Solution:

- Consider truth table for $\neg\phi$.
- Construct proposition in DNF for $\neg\phi$
- Take negation of DNF formula and apply Demorgan's Law to convert it into CNF.

p	q	$\neg\phi$
F	F	F
F	T	F
T	F	T
T	T	T

$$\neg\phi \equiv (p \wedge \neg q) \vee (p \wedge q)$$
$$\phi \equiv \neg(p \wedge \neg q) \wedge \neg(p \wedge q)$$
$$\boxed{\phi \equiv (\neg p \vee q) \wedge (\neg p \vee \neg q)}$$