BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 10

Example: Using LEM, show that $p \to q \vdash \neg p \vee q$ as valid.

1.$p \to q$	premise
2.$p \vee \neg p$	LEM
3.p	assumption
4.q	$\to e$ 3,1
5.$\neg p \vee q$	$\vee_{i2}$ 4
6.$\neg p$	assumption
7.$\neg p \vee q$	$\vee_i$ 6
8.$\neg p \vee q$	$\vee_e$ 2,3-5,6-7

9 Provable Equivalence

- ϕ and ψ are provable equivalent if and only if the sequents $\phi \vdash \psi$ and $\psi \vdash \phi$ are valid.
- It is denoted by $\phi \dashv\vdash \psi$.
- Ultimately we could define the $\phi \dashv\vdash \psi$ as $\vdash (\phi \to \psi) \wedge (\psi \to \phi)$.

$$\neg(p \to q) \vdash p \wedge \neg q$$

1.$\neg(p \to q)$	premise
2.$\neg(p \wedge \neg q)$	assumption
3.p	assumption(3)
4.$\neg q$	assumption
5.$p \wedge \neg q$	$\wedge_i$ 3,4
6.$\perp$	$\neg e$ 2,5
7.q	PBC 4-6
8.$p \to q$	$\to i$ 3-7
9.$\perp$	$\neg e$ 8,1
10.$p \wedge \neg q$	PBC 2-9