TEMPORAL LOGIC

(Models are abstractions!)

- Linear Time Logics (LTL)

  think of time as a set of paths where a path is a sequence of time instances

- Branching time Logics (CTL)

  represent time as a tree rooted at the present moment & branch out into the future.

Linear-time Temporal Logic (LTL)

SEMANTICS of LTL

or Model

NOT conditional!

Definition 3.4 A transition system $\mathcal{M} = (S, \rightarrow, L)$ is a set of states S endowed with a transition relation $\rightarrow$ (a binary relation on S), such that every $s \in S$ has some $s' \in S$ with $s \rightarrow s'$, and a labelling function $L \colon S \rightarrow \mathcal{P}(\texttt{Atoms})$.

$\rightarrow$ (transition relation) $\subseteq S \times S$

Powerset of the set of Atoms

Set

Atoms : p, q, r or p_1, p_2, p_3

p : Printer 21 is offline.

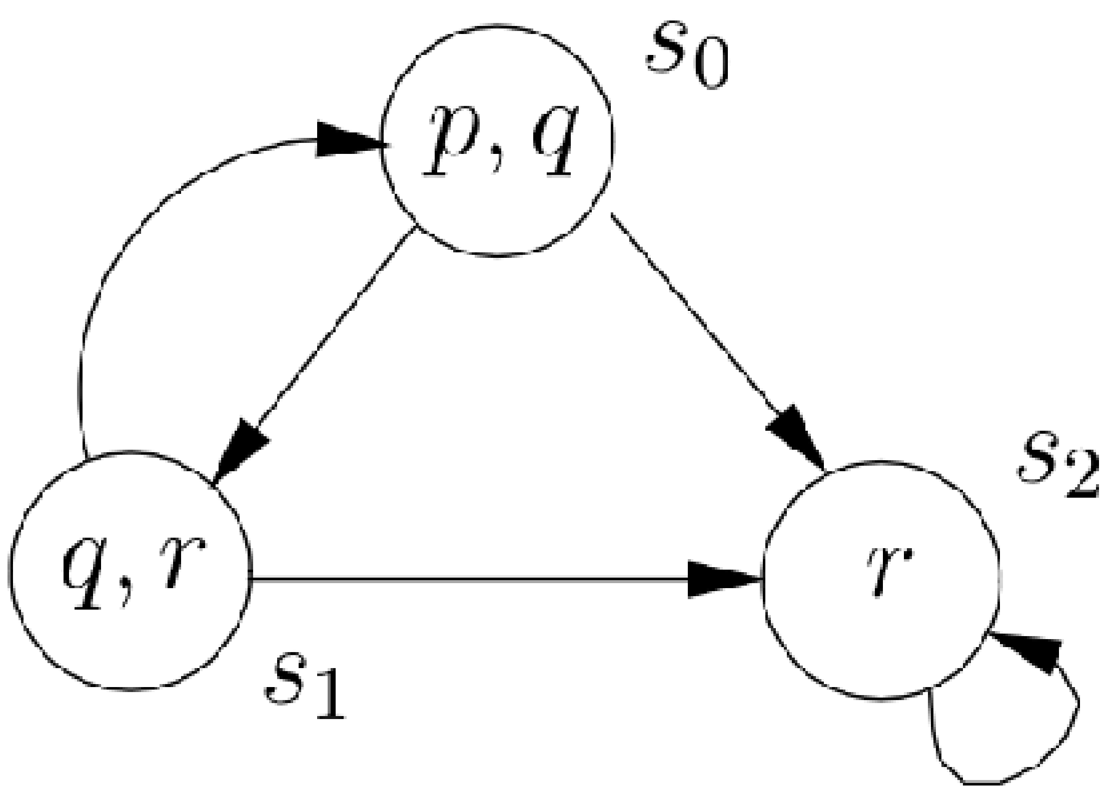
q : Process 1375 is not responding.

Set of Atoms is specified beforehand

our system has only three states s_0, s_1 and s_2; if the only possible transitions between states are $s_0 \to s_1$, $s_0 \to s_2$, $s_1 \to s_0$, $s_1 \to s_2$ and $s_2 \to s_2$; and if $L(s_0) = \{p, q\}$, $L(s_1) = \{q, r\}$ and $L(s_2) = \{r\}$, then we can condense all this information into Figure 3.3. We prefer to present models by means of such pictures whenever that is feasible.

$S = \{s_0, s_1, s_2\}$

Atoms $= \{p, q, r\}$



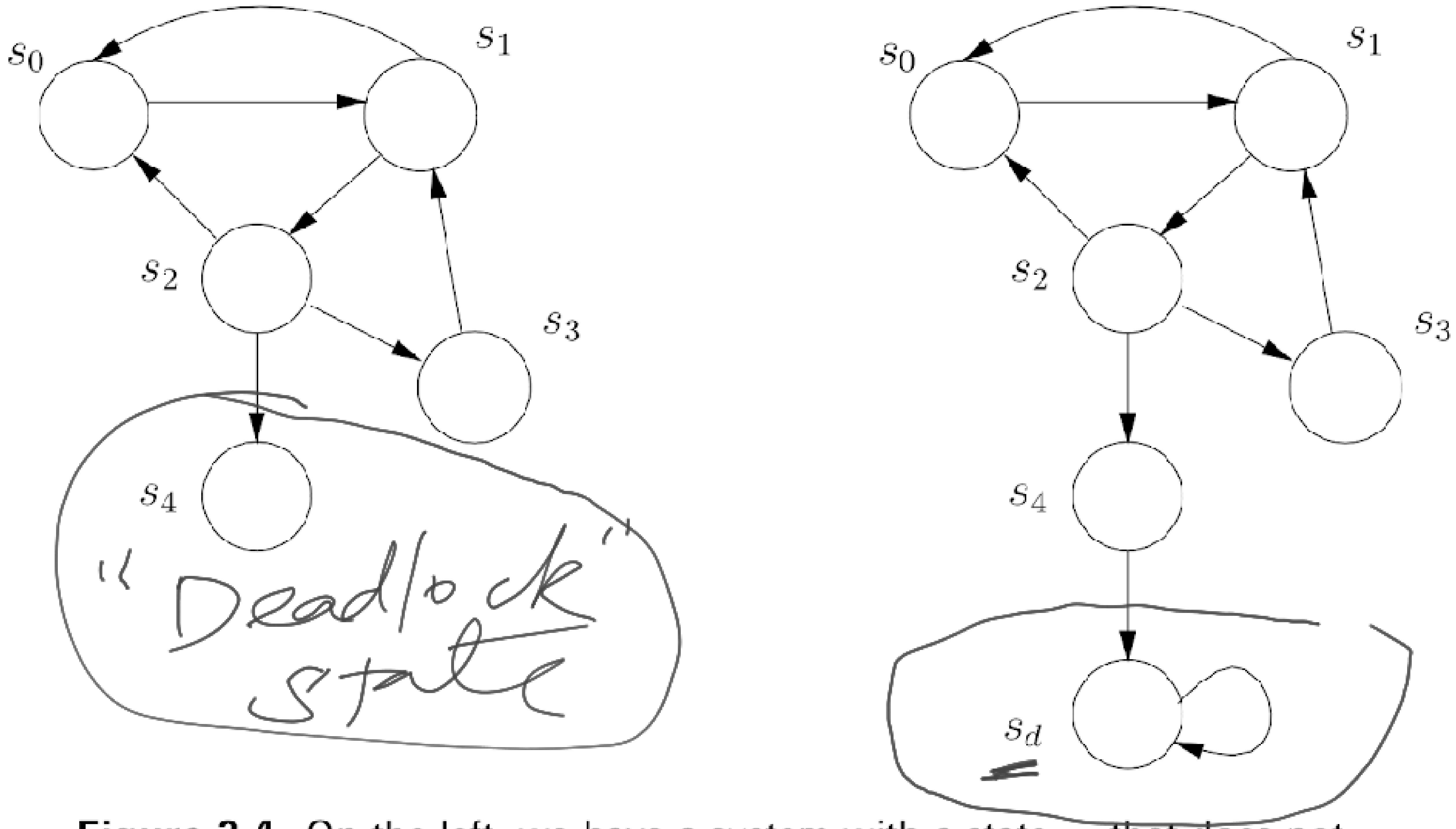


Figure 3.4. On the left, we have a system with a state s_4 that does not have any further transitions. On the right, we expand that system with a 'deadlock' state s_d such that no state can deadlock; of course, it is then our understanding that reaching the 'deadlock' state s_d corresponds to deadlock in the original system.

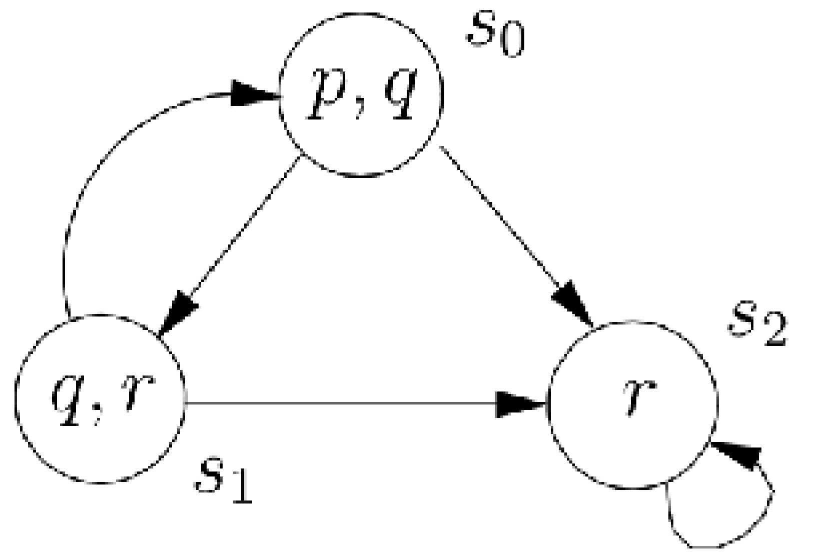
Definition 3.5 A path in a model $\mathcal{M} = (S, \rightarrow, L)$ is an infinite sequence of states $s_1, s_2, s_3, \ldots$ in S such that, for each $i \geq 1$, $s_i \rightarrow s_{i+1}$. We write the path as $s_1 \rightarrow s_2 \rightarrow \ldots$.

$\pi = s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_4 \rightarrow \ldots$

π^i is suffix of π starting at i,

$\pi^3 = s_3 \rightarrow s_4 \rightarrow \ldots$

$\pi^i = s_i \rightarrow s_{i+1} \rightarrow \ldots$



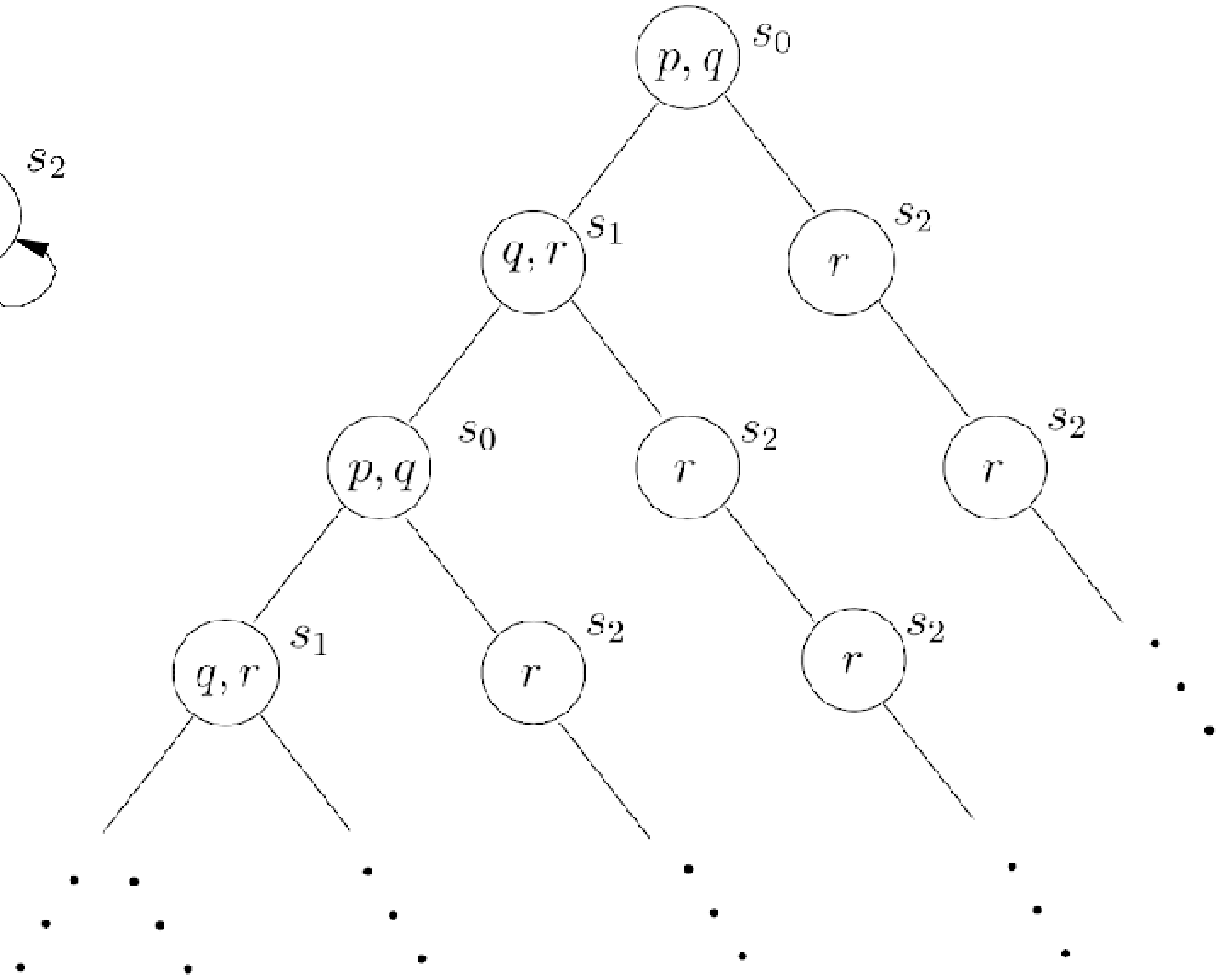


Figure 3.5. Unwinding the system of Figure 3.3 as an infinite tree of all computation ~~paths beginning in a particular~~ state.

SYNTAX

Definition 3.1 Linear-time temporal logic (LTL) has the following syntax given in Backus Naur form:

$$\begin{aligned}\phi ::= \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \to \phi) \\ \mid (\mathrm{X}\,\phi) \mid (\mathrm{F}\,\phi) \mid (\mathrm{G}\,\phi) \mid (\phi\ \mathrm{U}\ \phi) \mid (\phi\ \mathrm{W}\ \phi) \mid (\phi\ \mathrm{R}\ \phi)\end{aligned} \tag{3.1}$$

where p is any propositional atom from some set `Atoms`.

Temporal Connectives

X : neXt state

F : some Future state

G : all future states (Globally)

U : Until

R : Release

W : Weak-until

Definition 3.6 Let $\mathcal{M} = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow \ldots$ be a path in $\mathcal{M}$. Whether π satisfies an LTL formula is defined by the satisfaction relation $\vDash$ as follows:

$\phi R \psi \equiv \neg(\neg \phi \, U \, \neg \psi)$

1. $\pi \vDash \top$
2. $\pi \nvDash \bot$
3. $\pi \vDash p$ iff $p \in L(s_1)$
4. $\pi \vDash \neg\phi$ iff $\pi \nvDash \phi$
5. $\pi \vDash \phi_1 \wedge \phi_2$ iff $\pi \vDash \phi_1$ and $\pi \vDash \phi_2$
6. $\pi \vDash \phi_1 \vee \phi_2$ iff $\pi \vDash \phi_1$ or $\pi \vDash \phi_2$
7. $\pi \vDash \phi_1 \rightarrow \phi_2$ iff $\pi \vDash \phi_2$ whenever $\pi \vDash \phi_1$
8. $\pi \vDash \mathrm{X}\,\phi$ iff $\pi^2 \vDash \phi$
9. $\pi \vDash \mathrm{G}\,\phi$ iff, for all $i \geq 1$, $\pi^i \vDash \phi$
10. $\pi \vDash \mathrm{F}\,\phi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \phi$
11. $\pi \vDash \phi \,\mathrm{U}\, \psi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$
12. $\pi \vDash \phi \,\mathrm{W}\, \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$; or for all $k \geq 1$ we have $\pi^k \vDash \phi$
13. $\pi \vDash \phi \,\mathrm{R}\, \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \phi$ and for all $j = 1, \ldots, i$ we have $\pi^j \vDash \psi$, or for all $k \geq 1$ we have $\pi^k \vDash \psi$.

π

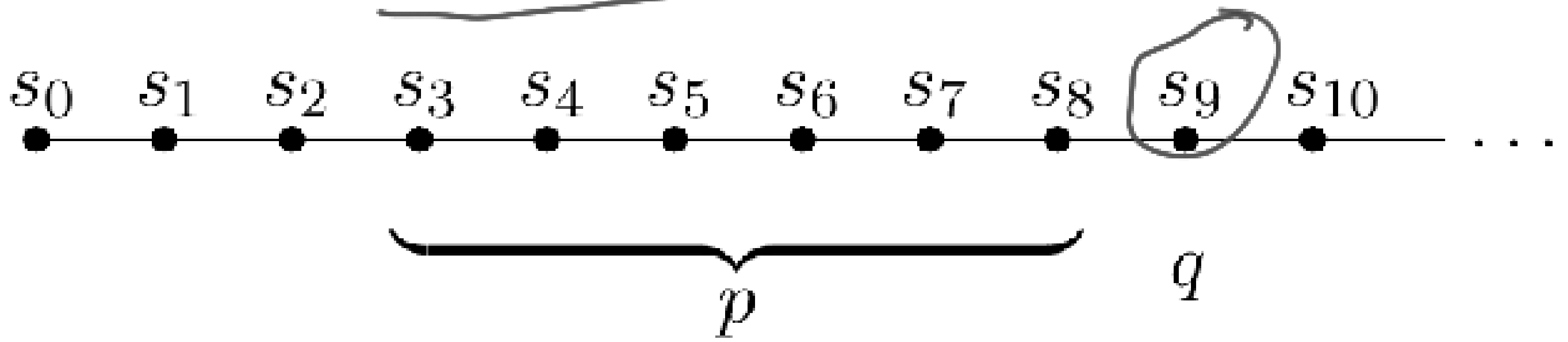


Figure 3.6. An illustration of the meaning of Until in the semantics of LTL. Suppose p is satisfied at (and only at) s_3, s_4, s_5, s_6, s_7, s_8 and q is satisfied at (and only at) s_9. Only the states s_3 to s_9 each satisfy $p \,\mathrm{U}\, q$ along the path shown.

$\pi^4 : s_3 \to s_9 \to \ldots$

$\pi^4 \vDash p \,\mathrm{U}\, q$ ✓