Tutorial-4 Solutions

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Q1.Establish the provable equivalence: $(p \rightarrow q) \wedge (p \wedge \neg q) \dashv\vdash r \wedge \neg r$

(i) $(p \rightarrow q) \wedge (p \wedge \neg q) \vdash r \wedge \neg r$

Proof:

1.$(p \rightarrow q) \wedge (p \wedge \neg q)$	premise
2.$p \rightarrow q$	$\wedge$e1 1
3.$p \wedge \neg q$	$\wedge$e2 1
4.p	$\wedge$e1 3
5.q	$\rightarrow$e 2,4
6.$\neg q$	$\wedge$e2 3
7.$\perp$	$\neg$e 5,6
8.$r \wedge \neg r$	$\perp$e 7

(ii) $r \wedge \neg r \vdash (p \rightarrow q) \wedge (p \wedge \neg q)$

Proof:

1. $r \wedge \neg r$	premise
2. r	$\wedge$e1 1
3. $\neg r$	$\wedge$e2 1
4. $\perp$	$\neg$e 2,3
5. $(p \rightarrow q) \wedge (p \wedge \neg q)$	$\perp$e 4

From (i) and (ii) $(p \rightarrow q) \wedge (p \wedge \neg q) \dashv\vdash r \wedge \neg r$

Q2.Establish the provable equivalence: $p \wedge q \rightarrow p \dashv\vdash r \vee \neg r$

(i) $p \wedge q \rightarrow p \vdash r \vee \neg r$

Proof:

1.$p \wedge q \rightarrow p$	premise
2. $r \vee \neg r$	LEM

(ii) $r \vee \neg r \vdash p \wedge q \rightarrow p$

Proof:

1. $r \vee \neg r$	premise
2.$p \wedge q$	assumption
3.p	$\wedge$e1 2
4.$p \wedge q \rightarrow p$	$\rightarrow$i 2-3

From (i) and (ii) , $p \wedge q \rightarrow p \dashv\vdash r \vee \neg r$

Q3.Establish the provable equivalence: $p \wedge q \rightarrow r \dashv\vdash p \rightarrow (q \rightarrow r)$

(i) $p \wedge q \rightarrow r \vdash p \rightarrow (q \rightarrow r)$

Proof:

1.$p \wedge q \rightarrow r$	premise
2.p	assumption
3.q	assumption
4.$p \wedge q$	$\wedge$i 2,3
5.r	$\rightarrow$e 1,4
6.$q \rightarrow r$	$\rightarrow$i 3-5
7.$p \rightarrow (q \rightarrow r)$	$\rightarrow$i 2-6

(ii) $p \rightarrow (q \rightarrow r) \vdash p \wedge q \rightarrow r$

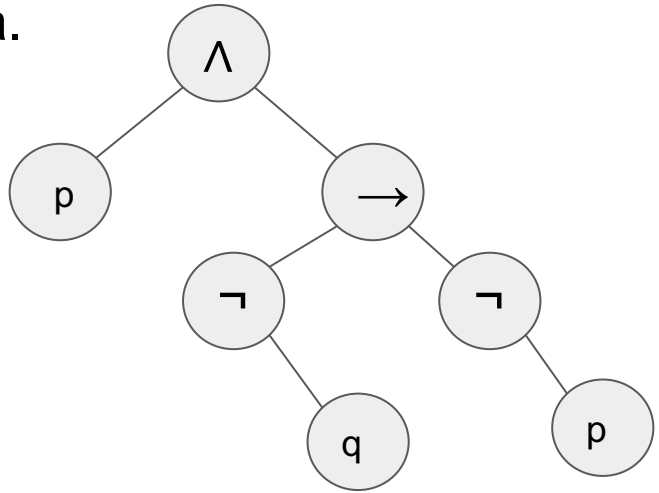
Proof:

1.$p \rightarrow (q \rightarrow r)$	premise
2.$p \wedge q$	assumption
3.p	$\wedge$e1 2
4.q	$\wedge$e2 2
5.$q \rightarrow r$	$\rightarrow$e 1,3
6.r	$\rightarrow$e 5,4
7.$p \wedge q \rightarrow r$	$\rightarrow$i 2-6

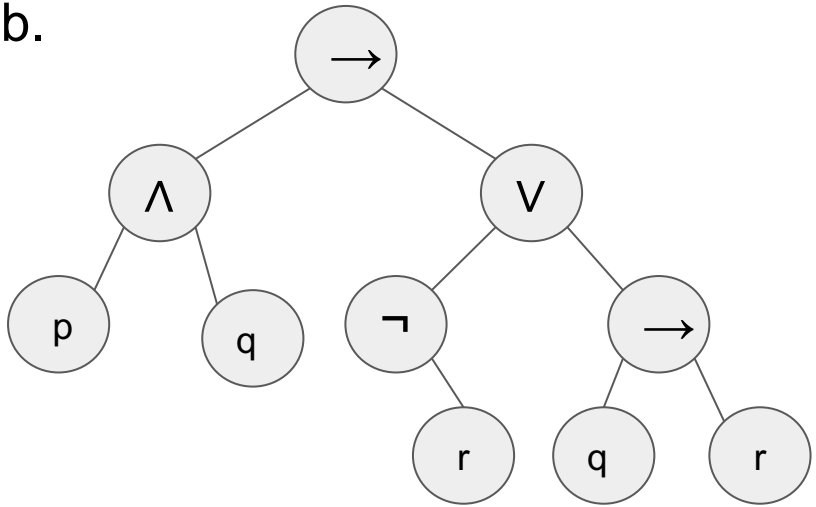
From (i) and (ii), $p \wedge q \rightarrow r \dashv\vdash p \rightarrow (q \rightarrow r)$

Q4.Draw the parse tree of
a.p∧(¬q→¬p)
b.(p∧q)→(¬r∨(q→r))

a.



b.

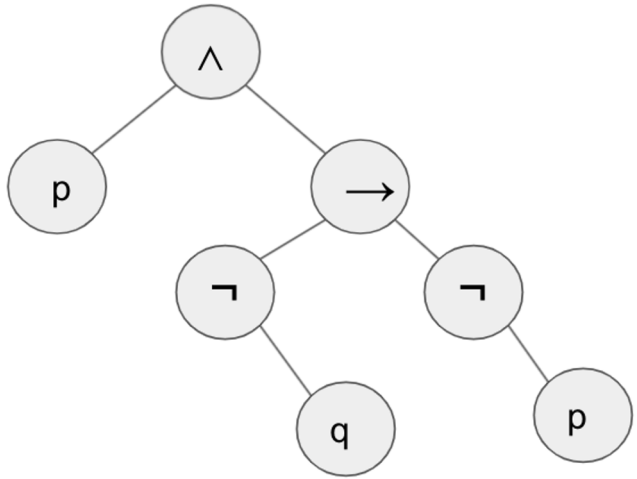


Q5.Write the preorder,postorder and inorder traversal of the parse trees

a.Preorder: ∧ p → ¬ q ¬ p

Postorder: p q ¬ p ¬ → ∧

Inorder: p ∧ ¬ q → ¬ p



b.Preorder: → ∧ p q ∨ ¬ r → q r

Postorder: p q ∧ r ¬ q r → ∨ →

Inorder: p ∧ q → ¬ r ∨ q → r