BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 13

Theorem: *The sum of the first n natural numbers in equal to* $\frac{n(n+1)}{2}$

Proof: Let $M(n)$: the sum of the first n natural numbers not equal to $\frac{n(n+1)}{2}$.

- Base Case: To show that $M(1)$ is true.

  The sum of first one natural number is 1.
  Furthermore $M(1)$ states that the sum of first natural number equals $\frac{1(1+1)}{2} = 1$.

  $\therefore M(1)$ is true.

- Inductive Step: Suppose $M(n)$ is true. Consider $M(n+1)$: The sum of the first $(n+1)$ natural numbers is $\frac{(n+1)(n+2)}{2}$.

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$$

By the induction hypothesis, $M(n)$ is true.

$$\sum_{i=1}^{n+1} = \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$$
$$\sum_{i=1}^{n+1} = \frac{(n+1)(n+2)}{2}$$

$\therefore M(n+1)$ is true.

12 Course of Values Induction or Strong Induction

1. **Base Case**: Natural number 1 has the property i.e. we have a proof of $M(1)$.

2. **Inductive Step**: We assume that $M(1) \wedge M(2) \wedge M(3).. \wedge M(n)$ is true and show that $M(n+1)$ is true.

Statements on parse trees are often shown by strong induction on height. It is also called as structural Induction.

Theorem: *For every well-formed proposition logic formula, the number of left brackets equal the number of right brackets.*

Proof: Course of values induction on the height og the parse tree corresponding to the well-formed formula.
Let $M(n)$: All formulas of height n, have the same number of left and right brackets.

- **Base Case**: We will show that $M(1)$ is true.
  A parse tree of height 1 has only an atom and no bracket. Hence $M(1)$ is true.

- **Inductive Step**: Suppose $M(1), M(2), ...M(n)$ is true, we will show that $M(n+1)$ is true.
  A parse tree of height ≥ 2 has as its root either of $\neg, \vee, \wedge, \rightarrow$. Suppose the root as $\neg$. Then the sub-tree rooted at $\neg$ is of height n.
  By the induction hypothesis property as true for the formula ϕ corresponding to that of sub-tree. The formula corresponding to the full tree is $(\neg\phi)$, which has equal number of left and right brackets. Since we added one left bracket and one right bracket.