BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 19

Lemma: *A disjunction of literals $L_1 \vee L_2 \vee .... \vee L_m$ is valid iff there are i,j such that $1 \leq i, j \leq m$, so that L_i is $\neg L_j$*

Proof: Consider i,j so that L_i is $\neg L_j$.

Now,

$$L_1 \vee L_2 \vee .. \vee L_m \equiv (L_1 \vee ...) \vee (L_i \vee \neg L_i) \quad (1)$$

Now, $(L_i \vee \neg L_i)$ is always true.
Suppose L_i is an atom P. In any valuations, p is either true or false.

Case 1: p is true
Then L_i is true.
$\therefore (L_i \vee \neg L_i)$ is true.
$\therefore$ (1) is true.

Case 2: p is false.
L_i is false, $\neg L_i$ is true.
$\therefore (L_i \vee \neg L_i)$ is true.
$\therefore$ (1) is true.
Suppose L_i is $\neg p$, the formula is valid for similar reasons.

To prove the converse, suppose for all i,j $1 \leq i, j \leq m$. L_i is not $\neg L_j$ then

The formula is $L_1 \vee L_2 \vee ... \vee L_m$.
Now, consider a valuation where each literal is made false. Suppose a literal L_i is P_k. Then set P_k false in the valuation.
Suppose L_i is $\neg P_l$. Then set P_l to true in the valuation.
This procedure will not make any literal evaluate to false because that would imply that such a literal is a negation of a literal that had previously been made false.

Definition: *Given a formula ϕ in propositional logic, we say that ϕ is satisfiable if it has a validation in which it evaluates to true.*

e.g.

$$(1) p \vee q \to p \text{ is satisfiable}$$
$$(2) (p \vee q) \wedge (\neg p) \wedge (\neg q)$$