

CS F214 Logic in Computer Science I Semester 2021-2022

Tutorial 3 Solutions

$$p \vdash q \rightarrow (p \wedge q)$$



1. p	Premise
<hr/>	
2. q	Assumption
3. $p \wedge q$	\wedge i 1,2
<hr/>	
4. $q \rightarrow (p \wedge q)$	\rightarrow i 2-3

$p \vdash (p \rightarrow q) \rightarrow q$



1. p

Premise

2. $p \rightarrow q$

Assumption

3. q

$\rightarrow e$ 2,1

4. $(p \rightarrow q) \rightarrow q$

$\rightarrow i$ 2-3

$$(p \rightarrow r) \wedge (q \rightarrow r) \vdash (p \wedge q) \rightarrow r$$



1. $(p \rightarrow r) \wedge (q \rightarrow r)$

2. $p \rightarrow r$

Premise

$\wedge e_1$ 1

3. $p \wedge q$

4. p

5. r

Assumption

$\wedge e_1$ 3

$\rightarrow e$ 4,2

6. $(p \wedge q) \rightarrow r$

$\rightarrow i$ 3-5

$$\neg p \rightarrow \neg q \vdash q \rightarrow p$$



1. $\neg p \rightarrow \neg q$

Premise

2. q

Assumption

3. $\neg p$

Assumption

4. $\neg q$

$\rightarrow e$ 3,1

5. \perp

$\neg e$ 2,4

6. p

PBC 3-5

7. $q \rightarrow p$

$\rightarrow i$ 2-7

$$\neg p \rightarrow p \vdash p$$



1. $\neg p \rightarrow p$

Premise

2. $\neg p$

Assumption

3. p

$\rightarrow e$ 2,1

4. \perp

$\neg e$ 2,3

5. p

PBC 2-4

$\neg p \vdash p \rightarrow q$



1. $\neg p$ Premise

2. p Assumption

3. \perp $\neg e$ 2,1

4. q $\perp e$ 3

5. $p \rightarrow q$ $\rightarrow i$ 2-4

$p \vee q, \neg q \vdash p$



1. $p \vee q$

Premise

2. $\neg q$

Premise

3. p

Assumption

4. p

Copy 3

5. q

Assumption

6. \perp

$\neg e$ 2,5

7. p

$\perp e$ 6

8. p

$\vee e$ 1, 3-4, 5-7

$$\vdash \neg p \rightarrow (p \rightarrow (p \rightarrow q))$$



1. $\neg p$ Assumption

2. p Assumption

3. \perp $\neg e$ 2,1

4. $p \rightarrow q$ $\perp e$ 3

5. $p \rightarrow (p \rightarrow q)$ $\rightarrow i$ 2-4

6. $\neg p \rightarrow (p \rightarrow (p \rightarrow q))$ $\rightarrow i$ 1-5

$$p \wedge \neg p \vdash \neg(r \rightarrow q) \wedge (r \rightarrow q)$$



1. $p \wedge \neg p$

Premise

2. p

$\wedge e_1 1$

3. $\neg p$

$\wedge e_2 1$

4. \perp

$\neg e 2,3$

5. $\neg(r \rightarrow q) \wedge (r \rightarrow q)$

$\perp e 4$

$\neg(p \rightarrow q) \vdash q \rightarrow p$



1. $\neg(p \rightarrow q)$

Premise

2. q

Assumption

3. p

Assumption

4. q

Copy 2

5. $p \rightarrow q$

\rightarrow i 3-4

6. \perp

\neg e 5,1

7. p

\perp e 6

8. $q \rightarrow p$

\rightarrow i 2-7

Thank
You