1. Consider a monopolist who is facing the inverse market demand function $P(q) = a - bq$ & has a cost function $C(q) = cq$

a) what is monopolist's profit maximising quantity q^m & p^m?

$$P = a - bq$$

$$TR = Pq = aq - bq^2$$

$$MR = \frac{\partial TR}{\partial q} = a - 2bq$$

$$MC = \frac{\partial C}{\partial q} = c$$

$$MR = MC \Rightarrow a - 2bq = c$$

$$\Rightarrow q^m = \frac{a-c}{2b}$$

$$\therefore p^m = a - b\left(\frac{a-c}{2b}\right)$$

$$= a - \frac{ab - bc}{2}$$

$$= \frac{2a - a + c}{2} = \frac{a+c}{2}$$

b) what is equilibrium profit for the monopolist?

$$\text{Profit } \pi = P(q)q - C(q)$$

$$= aq - bq^2 - cq$$

$$= (a - bq - c)q$$

$$= \left(a - b\frac{a-c}{2b} - c\right)\left(\frac{a-c}{2b}\right)$$

$$= \left(\frac{2a - a + c - 2c}{2}\right)\left(\frac{a-c}{2b}\right)$$

$$= \frac{(a-c)^2}{4b}$$

c) What is the profit maximising quantity q^* & p^* in case of perfect competition?

Perfect competition solution is:

$$P = MC$$

$$\Rightarrow a - bq = c$$

$$\Rightarrow q^* = \frac{a-c}{b}$$

& $p^* = MC = c$

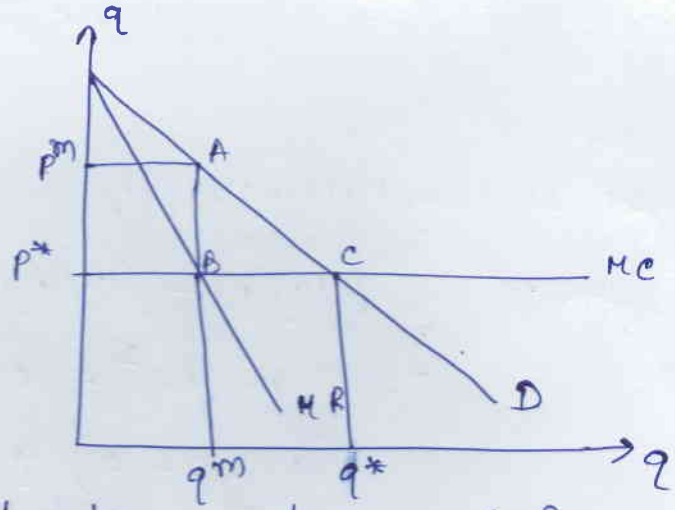
d) Compare the solution of both the cases & show them graphically.

$$q^* = \frac{a-c}{b}, \quad p^* = c$$

$$q^m = \frac{a-c}{2b}, \quad p^m = \frac{a+c}{2}$$

$$q^* > q^m$$

$$p^* < p^m$$



e) What is the dead weight loss because of monopoly?

Dead weight loss = Area ΔABC

$$= \frac{1}{2}(q^* - q^m)(p^m - p^*)$$

$$= \frac{1}{2}\left(\frac{a-c}{b} - \frac{a-c}{2b}\right)\left(\frac{a+c}{2} - c\right)$$

$$= \frac{1}{2}\left(\frac{2a-2c-a+c}{2b}\right)\left(\frac{a+c-2c}{2}\right)$$

$$= \frac{1}{8b}(a-c)(a-c) = \frac{(a-c)^2}{8b}$$

Q2. A monopoly faces the market demand $q = 30 - p$ & has a cost function $C(q) = \frac{1}{2}q^2$

a) what is monopolist's profit maximising quantity q^m, p^m & profit?

Equilibrium $\Rightarrow$ MR = MC

$q = 30 - p$

$\Rightarrow p = 30 - q$

$\therefore$ $TR = p \times q = 30q - q^2$

$MR = \frac{\partial TR}{\partial q} = 30 - 2q$

$MC = \frac{\partial C}{\partial q} = q$

$MR = MC$

$\Rightarrow 30 - 2q = q$

$\Rightarrow 3q = 30$

$\Rightarrow q^m = 10$

$p^m = 30 - 10 = 20$

$\pi = 10 \times 20 - \frac{1}{2}10^2$

$= 200 - 50 = 150$

b) What is the profit maximising quantity (q^*) and price (p^*) in case of perfect competition?

We need to equate $p = MC$

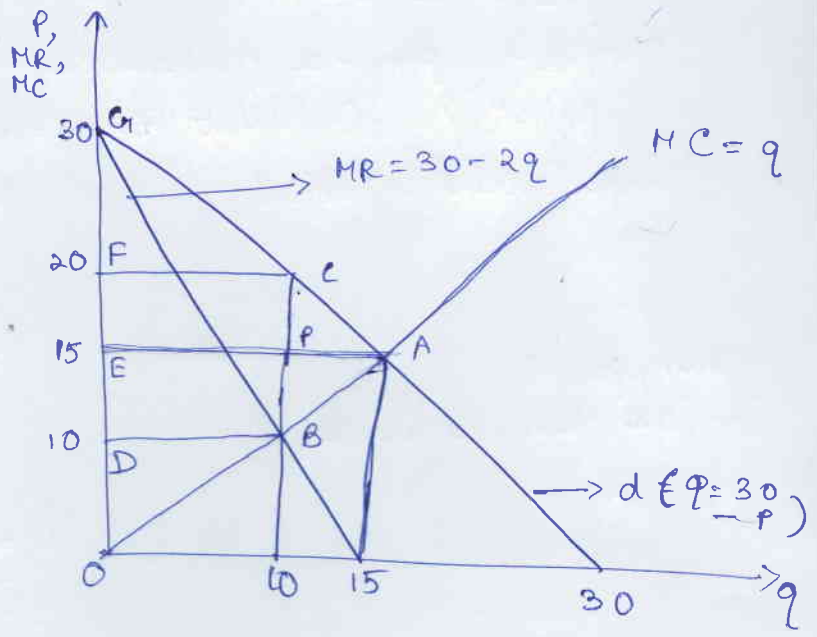
$\Rightarrow 30 - q = q$

$\Rightarrow q^* = 15$

$p^* = 30 - 15 = 15$

This is also known as socially optimal solution.

c) Calculate the dead-weight loss (DL) due to the monopolist behaviour of this firm.



① Point of intersection of demand curve & MC curve

dd : $P = 30 - q$

MC : $MC = q$

$\therefore 30 - q = q$

$\Rightarrow q = 15$

$P = 30 - 15 = 15$

② Again point of intersection between MC & MR curve

$30 - 2q = q$

$\Rightarrow q = 10$

& $MR = MC = 10$

Monopolists price at $q = 10$,

$q = 30 - P$

$\Rightarrow P = 30 - 10 = 20$

$\therefore$ Dead weight loss = Area of ΔABC

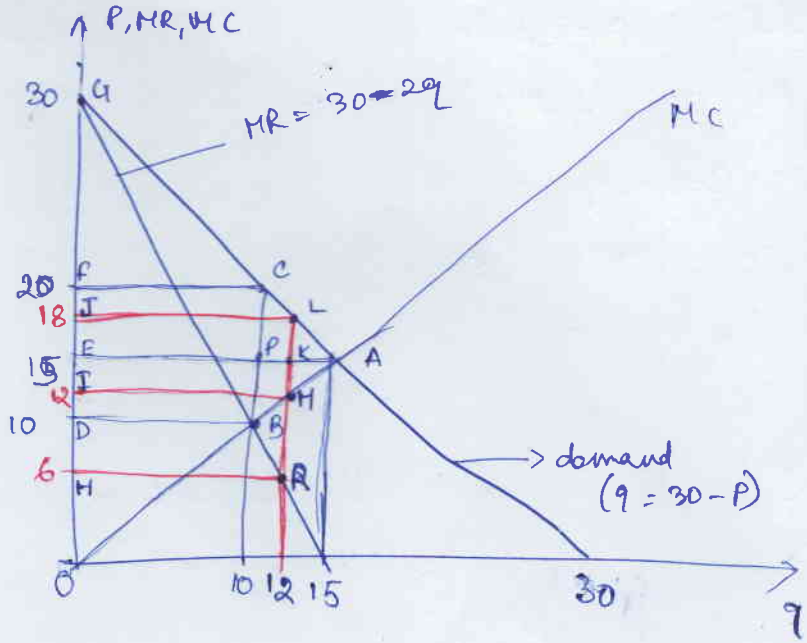
$= \frac{1}{2}(15 - 10) \times (20 - 10) = 25$

d) Calculate consumer surplus (CS)

Consumer surplus : Area of $\Delta GFC = \frac{1}{2}(10 - 0)(30 - 20)$

$= 50$

e) Assume that the government puts a price ceiling on the monopolist at P = 18. How much output will the monopolist produce? Calculate Consumers' surplus, producers' surplus and dead weight loss.



When, P = 18,
From the demand curve we get

$q = 30 - P$
$= 30 - 18$
$= 12$

When $q = 12$, $MC = q$
$= 12$

When $q = 12$, $MR = 30 - 2q$
$= 30 - 2 \times 12$
$= 6$

Consumer surplus : Area of Δ GJL $= \frac{1}{2}(30 - 18)(12 - 0)$
$= 6 \times 12 = 72$

Producer surplus : Area of Δ OIM + area of rectangle IJLM
$= \frac{1}{2} \times 12 \times 12 + 6 \times 12$
$= 72 + 72 = 144$

Dead weight loss $= \frac{1}{2} \times (18 - 12) \times (15 - 12)$
$= \frac{1}{2} \times 6 \times 3 = 9$

f) Assume that the government puts a price ceiling on the monopolist in order to maximise (i.e consumer plus producer) surplus. What price ceiling should he it choose? How much output the monopolist produce at this price ceiling? What will the profit of the monopolist be? What is dead weight loss?

We already know that the socially optimal solution (i.e perfect competion) is $p = 15$ & $q = 15$.
The government will fix the price $= 15$ in order to maximise the social surplus. In this case, the dead weight loss is zero.