

# Proof Rules QUANTIFICATION FOR EXISTENTIAL

EXISTENTIAL  
QUANTIFIER  
INTRODUCTION

$$\frac{\phi[t/x]}{\exists x \phi} \exists x i.$$

# EXISTENTIAL QUANTIFIER ELIMINATION

$$\frac{\exists x \phi \quad \boxed{\begin{array}{c} x_0 \ \phi[x_0/x] \\ \vdots \\ \chi \end{array}}}{\chi} \quad \exists e.$$

has to  
be indep.  
of  $x_0$ !

$\chi$  cannot depend on  $x_0$ .

—  $\forall x \phi \vdash \exists x \phi.$  '

1	$\forall x \phi$	premise
2	$\phi[x/x]$	$\forall x$ e 1
3	$\exists x \phi$	$\exists x$ i 2

$\forall x (P(x) \rightarrow Q(x)), \exists x P(x) \vdash \exists x Q(x)$

1	$\forall x (P(x) \rightarrow Q(x))$	premise
2	$\exists x P(x)$	premise
3	$x_0 \quad P(x_0)$	assumption
4	$P(x_0) \rightarrow Q(x_0)$	$\forall x$ e 1
5	$Q(x_0)$	$\rightarrow$ e 4, 3
6	$\exists x Q(x)$	$\exists x$ i 5
7	$\exists x Q(x)$	$\exists x$ e 2, 3–6

WRONG!

1	$\forall x (P(x) \rightarrow Q(x))$	premise
2	$\exists x P(x)$	premise
3	$x_0 \quad P(x_0)$	assumption
4	$P(x_0) \rightarrow Q(x_0)$	$\forall x \text{ e } 1$
5	$Q(x_0)$	$\rightarrow \text{e } 4, 3$
6	$Q(x_0)$	$\exists x \text{ e } 2, 3-5$
7	$\exists x Q(x)$	$\exists x \text{ i } 6$

not  
independ-  
g  $x_0$ !

$$\forall x (Q(x) \rightarrow R(x)), \exists x (P(x) \wedge Q(x)) \vdash \exists x (P(x) \wedge R(x))$$

$\exists x Q(x)$

1	$\forall x (Q(x) \rightarrow R(x))$	premise
2	$\exists x (P(x) \wedge Q(x))$	premise
3	$x_0 \quad P(x_0) \wedge Q(x_0)$	assumption
4	$Q(x_0) \rightarrow R(x_0)$	$\forall x$ e 1
5	$Q(x_0)$	$\wedge$ e <sub>2</sub> 3
6	$R(x_0)$	$\rightarrow$ e 4, 5
7	$P(x_0)$	$\wedge$ e <sub>1</sub> 3
8	$P(x_0) \wedge R(x_0)$	$\wedge$ i 7, 6
9	$\exists x (P(x) \wedge R(x))$	$\exists x$ i 8
10	$\exists x (P(x) \wedge R(x))$	$\exists x$ e 2, 3–9