

**BITS Pilani Hyderabad Campus**  
**CS F214 Logic in Computer Science,**  
**I Semester 2021-2022**  
**Lecture Notes**  
**Lecture 21**

Please read sec 1.5.2 from textbook regarding the conversion of any formula in CNF (Page 57).

The logical constants ('bottom')  $\perp$  and ('top')  $\top$  denote respectively unsatisfiable formula and tautology.

## 15 Horn Formula

A Horn formula is a formula  $\phi$  in propositional logic, if it can be generated as instance of the following grammar.

$$P ::= \perp \mid \top \mid P$$

$$A ::= P \mid P \wedge A$$

$$\text{Horn Clause, } C ::= A \rightarrow P$$

$$\text{Horn Formula, } H ::= C \mid C \wedge H$$

Formula	Explanation
1. $(p \wedge q \wedge r \rightarrow p) \wedge (q \wedge s \rightarrow p) \wedge (\top \rightarrow s) \wedge (r \wedge s \rightarrow \perp) \wedge (\perp \wedge p \rightarrow r)$	Horn Formula
2. $(p \wedge q \wedge r \rightarrow \neg p) \wedge (q \wedge r \rightarrow q)$	Not Horn Formula due to $\neg p$
3. $(p \wedge r \wedge r \rightarrow \perp) \wedge (\neg q \wedge r \rightarrow p)$	Not Horn Formula due to $\neg q$
4. $(p_1 \wedge p_2 \wedge p_3 \rightarrow (p_4 \wedge p_5)) \wedge (\top \rightarrow p_5)$	Not Horn Formula due to $p_4 \wedge p_5$
5. $(p \wedge q \rightarrow r) \wedge (p \wedge q) \wedge (r \vee s \rightarrow p)$	Not Horn Formula due to $\vee$ and $p \wedge q$

### 15.1 Deciding Satisfiability of Horn Formula

- Maintain a list of all occurrences of type P in your formula.
  1. It marks  $\top$ , if it occurs in that list.
  2. If there is a conjunct  $p_1 \wedge p_2 \wedge \dots \wedge p_k \rightarrow p'$  of  $\phi$  such that all  $p_j$  with  $1 \leq j < k$  is marked, then mark  $p'$  as well and go to step 2. Otherwise if there is no such conjunct go to step 3.
  3. If  $\perp$  is marked, print 'Unsatisfiable' and Stop. Otherwise go to step 4.
  4. Print 'Satisfiable'.

Example

$$1. (p \wedge q \wedge w \rightarrow \perp) \wedge (t \rightarrow \perp) \wedge (r \rightarrow p) \wedge (\top \rightarrow r) \wedge (\top \rightarrow q) \wedge (u \rightarrow s)$$

Solution:

- Mark all occurrences of  $\top$ .
- Mark  $r$ ,  $q$ ,  $u$ .
- Mark  $p$ .
- Mark  $S$ .
- Print Satisfiable.