BITS Pilani Hyderabad Campus CS F214 Logic in Computer Science, I Semester 2021-2022 Lecture Notes Lecture 6

Prove: $\neg q \rightarrow \neg p \vdash p \rightarrow q$ Solution:

$$\begin{array}{ccc}
1. \neg q \rightarrow \neg p & \text{premise} \\
2.p & \text{assumption} \\
3. \neg \neg p & \neg \neg i \ 2 \\
4. \neg \neg q & \text{MT } 3,1 \\
5.q & \neg \neg e \ 4
\end{array}$$

$$\begin{array}{ccc}
6.p \rightarrow q & \rightarrow \text{i } 2\text{-5}
\end{array}$$

Consider,

1.p assumption
$$2.p \to p \to i 1-1$$

Above is a poof of the statement $\vdash (p \rightarrow p)$

Def: A logical formula ϕ with a valid sequent $\vdash \phi$ is called a **Theorem**. Remarks: Any sequent $\phi \vdash \psi$ is equivalent to $\vdash \phi \rightarrow \psi$.

Proof of $\vdash \phi \rightarrow \psi$:

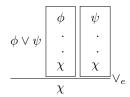
Remark(b): $\phi_1, \phi_2, ..., \phi_n \vdash \psi$ is equivalent to $\vdash \phi_1 \rightarrow (\phi_2 \rightarrow (...(.. \rightarrow (\phi_n \rightarrow \psi)))$

5 Rules for Disjunction

5.1 OR-Introduction

$$\frac{\phi}{\phi \vee \psi} \vee_{i1}$$
$$\frac{\phi}{\psi \vee \phi} \vee_{i2}$$

5.2 OR-Elimination



Prove: $p \lor q \vdash q \lor p$

$$1.p \lor q$$
 premise

$$\begin{array}{ccc} 2.q & \text{assumption} \\ 3.q \lor p & \lor_{i2} \end{array} 2$$

$$\begin{array}{ccc} 4.q & \text{assumption} \\ 5.q \lor p & \lor_i 4 \end{array}$$

$$6.q \lor p \quad \lor_e 1,2\text{-}3,4\text{-}5$$