

3 Propositional Logic

3.1 Declarative Sentences Or Propositions

Sentences that are in principle, either true or false (but not both).

Examples:

1. This is a class on logic.
2. The sun orbits the earth.
3. The sum of 2 and 2 is 5.
4. Every even natural number greater than 2 is the sum of 2 prime numbers.
(Goldbach's conjecture. We are not sure whether this statement is true or false.)

3.2 Non-Declarative Sentences

1. Pay Attention!
2. Would you accompany me to class?
3. May fortune come your way.

Non-Declarative Sentences are not propositions.

3.3 Twin Prime Conjecture

- For all the numbers n , there exists an $n_0 > n$, such that n_0 and $n_0 + 2$ are prime numbers.
- Twin prime conjecture has not been proved yet.

3.4 Bounded Gap Theorem

- For all natural numbers n , there exists a constant c , for all natural numbers $n_0 > n$, so that there are two prime numbers in interval $[n_0, n_0 + c]$
- This has been proved by Yitang Zhang.

3.5 Truth Tables

Given these atomic propositions p, q, r, \dots or p_1, p_2, p_3, \dots we can combine them in compositional ways to form other propositions using logical connections.

p	$\neg p$
T	F
F	T

Table 1: Negation (\neg)

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Table 2: Conjunction (\wedge)

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Table 3: Disjunction (\vee)

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Table 4: Conditional (Implication)

$$p \leftrightarrow q \implies (p \rightarrow q) \wedge (q \rightarrow p)$$