BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 4-5

4 Rules for Natural Deduction

4.1 And Introduction

- Given premises ϕ, ψ, we can conclude $\phi \wedge \psi$.

$$\frac{\phi, \psi}{\phi \wedge \psi} \wedge_i$$

4.2 And Elimination

- Given premises $\phi \wedge \psi$, we can conclude ϕ.

$$\frac{\phi \wedge \psi}{\phi} \wedge_{e_1}$$

$$\frac{\phi \wedge \psi}{\psi} \wedge_{e2}$$

Example Prove that the following statement is valid p, $p \wedge q$, $r \vdash q \wedge r$

1.$p \wedge q$	premise
2.r	premise
3.q	e_2 1
4.$q \wedge r$	$\wedge_i 3, 2$

4.3 Double Negation

Double Negation Elimination

$$\frac{\neg\neg\phi}{\phi} \neg\neg e$$

Double Negation Introduction

$$\frac{\neg\neg\phi}{\phi} \neg\neg i$$

4.4 Eliminating Implication (Modus Ponens)

$$\frac{\phi, \phi \rightarrow \psi}{\psi} \rightarrow e$$

Example: p : It rained. q : The street is wet. $p \rightarrow q$: If it rained then the street is wet

4.5 Modus Tollens

$$\frac{\phi \rightarrow \psi, \neg\psi}{\neg\phi} \rightarrow MT$$

e.g. $p \rightarrow q$: If it rained then the street is wet. (premise) $\neg q$: The street is not wet. (premise) $\therefore$ it did not rain ($\neg p$) (conclusion)

Prove the following statement $p \rightarrow (q \rightarrow r), p, \neg r \vdash \neg q$.

1.$p \rightarrow (q \rightarrow r)$	premise
2.p	premise
3.$\neg r$	premise
4.$q \rightarrow r$	$\rightarrow$e 2,1
5.$\neg q$	MT 4,3

By using Modus Tollens Elimination,

$$\frac{p, p \rightarrow (q \rightarrow r)}{q \rightarrow r}$$

By using Modus Tollens,

$$\frac{q \rightarrow r, \neg r}{\neg q}$$

4.6 Implies Introduction

- We wish to build implications that do not already appear as premises (or parts theorem) in our proof.

Example Prove if following condition is valid: $p \rightarrow q \vdash \neg q \rightarrow \neg p$

1.$p \rightarrow q$	premise
2.$\neg q$	assumption
3.$\neg p$	Modus Tollens 1,2
4.$\neg q \rightarrow \neg p$	[$\rightarrow$i 2,3]

2,3 $\implies$ Scope of the assumption.
* Implies introduction is formulated as :

$$\frac{\phi, \psi}{\phi \rightarrow \psi} \rightarrow i$$