BITS Pilani Hyderabad Campus
CS F214 Logic in Computer Science,
I Semester 2021-2022
Lecture Notes
Lecture 17

We wish to show $\phi_1, \phi_2, ..\phi_n \vDash \psi$ holds i.e. $\phi_1, \phi_2, ..\phi_n \vDash \psi_1 \wedge \psi_2$ holds.

From (1) and (2), we have that for each valuation for which $\phi_1, \phi_2, ..., \phi_n$ is true ψ_1 is true also ψ_2 is true.

$\therefore$ for each valuation $\psi_1 \wedge \psi_2$ is true.
$\therefore$ we have,

$$\phi_1, \phi_2, ..\phi_n \vDash \psi$$

Case 2: When last rule applied is $\vee_e$. We must have proved on have as a premise $\eta_1 \vee \eta_2$.
This means that the sequent $\phi_1, \phi_2, ..\phi_n \vdash \eta_1 \vee \eta_2$ is valid.

The first 'box' $\boxed{\begin{array}{c} \eta_1 \\ . \\ . \\ . \\ \psi \end{array}}$ gives us a proof of the sequent

$$\phi_1, \phi_2, ..\phi_n, \eta_1 \vdash \psi \qquad 4$$

likewise, the second 'boxes' gives us

$$\phi_1, \phi_2, ..\phi_n, \eta_2 \vdash \psi \qquad 5$$

By induction hypothesis, the corresponding semantic entailment relations for (3),(4) and (5) holds. We shall call semantic entailment relations as (6),(7) and (8).
Consider an arbitrary evaluation for which $\phi_1, ...\phi_n$ are true.
By (6), we have that $\eta_1 \vee \eta_2$ is true for this valuation. This means that for this valuation at least η_1, η_2 is true.

Case(2a) η_1 is true.
By (7) we have that ψ is true.

Case (2b) η_2 is true.
By (8), we have that ψ is true.
Therefore in both the cases ψ is true.
$\therefore we have \phi_1, \phi_2, ...\phi_n \vDash \psi$ holds.

Rest of the cases can be shown similarly.

13.2 Contrapositive of The Soundness Theorem

If $\phi_1, ..\phi_n \vDash \psi$ does not hold then,$\phi_1, ..\phi_n \vdash \psi$ is not valid.

This implies that in order to show that a sequent is not valid, it suffices to find a valuation for which $\phi_1, ..\phi_n$ are true but ψ is false.