Analysis Of Velocity Profile For Laminar Flow In A Round Pipe

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***Abstract*— In this work, various parameters for laminar flow of a given incompressible Newtonian fluid in a round pipe have been analysed. Velocity profile, being the fundamental parameter, has been studied in particular. Starting from very basic, the development of laminar flow in a round pipe, its various parameters especially velocity profile and shear stress, and the flow-rate equation given by Hagen (1839) and Poiseuille (1840) as well as various seminal works compiled on the subject till-date have also been taken into account. Based on the available data, governing equations, and the analysis carried out there in, the algorithm for velocity profile was prepared, which was further utilized to calculate shear stress and the volume flow rate. In the process, the algorithm was generalized to plot velocity and shear stress profiles, and to calculate velocity, shear stress and the volume flow rate for laminar flow of given incompressible Newtonian fluid at different locations in round pipe for any given set of values of radius, length and pressure drop; and to depict effect of change in radius, length and pressure drop on these flow parameters by changing these control parameters one at a time and keeping the other two constant. The achievement is that the designed algorithm has automated calculation, plotting and analysis of these important parameters for variations in radius, length and pressure drop. Thus it would facilitate scientific users to perform calculations and analysis for various applications in engineering design and research involving laminar flow in round pipe / tube where values of velocity, shear stress and volume flow rate at different locations are required to be taken into account.**

Keywords— Laminar Flow; Incompressible Flow; Newtonian Fluid; Velocity Profile; Shear Stress; Volume Flow Rate

I. INTRODUCTION

Fluid Mechanics deals with study of all kind of fluids, liquid or gas, at rest (Fluid Statics) or in motion (Fluid Dynamics). There is an enormously large number of engineering applications of this subject in all walks of life including ships, sub-marines, swimming, aircraft, balloons, water-falls, windmills, respiratory systems, blood supply systems, irrigation and water supply systems, jets, rockets and spacecraft. In short, almost every object on earth is either a fluid or it moves through a fluid.

Fluid Dynamics (study of fluid flows) is an ancient subject. Sailing ships with oars and irrigation systems existed even in pre historic era. Romans built widespread aqueduct systems in 4th century BC. Archimedes postulated the law of buoyancy and applied it to floating and sub-merged objects in 3rd century BC. In earlier times, the designs of ships, canals and water supply systems etc were improved over the centuries but no works were recorded showing improvement in flow analysis. Later on, notable contributions were made to the subject from late 17th century through first half of 20th century by Newton, Bernoulli, Euler, d'Alembert, Lagrange, Rayleigh, Reynolds, Navier, Stokes and Prandtl which matured the physical understanding of the various fluid flow types and flow phenomena, and produced realistic mathematical solutions to the fluid flow problems. In later half of 20th century, advanced tools of Computational Fluid Dynamics (CFD) and Direct Numerical Simulation (DNS) became available for solving advanced flow problems. [1]

A. Fluid Flow Types

Fluid flows are classified into many categories based on the characteristic under study. The important types include continuum and free molecule flow; viscous and in-viscid flow; compressible and incompressible flow; and laminar and turbulent flows. Description of various flow types and terminologies has been given at Appendix 'A'.

B. Flow in a Pipe

We often come across situations where a fluid flows in a pipe connecting two reservoirs. The flow usually takes place due to pressure difference between the two reservoirs and it flows from one at higher pressure towards the one at lower pressure. The examples include gas cylinder charging, water supply and irrigation systems, blood transfusion etc.

The flow in pipes has been the subject of study for quite some time. The detailed analysis and mathematical equation for flow rate in round tubes was initially postulated by German engineer Hagen (1839) and French physician Poiseuille (1840) in parallel. The equation was derived by them after meticulous experimentation with flow of water in a set of round tubes of different sizes and lengths. [2] Analysis of laminar flow and laminar–turbulent transition in tube of rectangular cross-section was carried out by Davies and White (1928), followed by Cornish (1928). [3-4] A detailed account of the works of Hagen and Poiseuille was given by Prandtl and Tietjens (1934). Experimental data of their work was also given by Schlichting (1950). While conducting the experiments, Hagen and Poiseuille did not completely understand the entrance length effect, and when short tubes were used, the results deviated from the derived equation. Poiseuille was unable to justify this deviation, but Hagen gave an almost accurate explanation by attributing it to an additional pressure drop necessary for accelerating the fluid. [2] Numerical analysis of laminar flow for incompressible fluid in entrance region of round pipe was carried out by Hornbeck (1964). [5] Davey and Drazin (1969) numerically analysed the stability of Poiseuille flow in a round tube. [6] Fargie and Martin (1971) studied developing laminar flow in round tube. [7] Lyne (1971) analysed the unsteady viscous flow in a curved circular tube. [8] Ward-Smith (1980) analysed the dynamics of fluid flow inside round pipes and ducts in detail. [9] Enayet et al (1982) took Laser-Doppler readings of turbulent and laminar flow in bent tube. [10] Durst and Loy (1985) examined laminar flow in a round tube with abrupt cross sectional area changes. [11] Sutera and Skalak (1993) carried out a detailed review of Hagen and Poiseuille's works and studied the extension of Poiseuille's law to blood flow and its other practical applications. [12] Draad, Kuiken and Nieuwstadt (1998) studied laminar flow and relation between laminar–turbulent transition and Reynolds numbers for flow of non-Newtonian and Newtonian fluids in circular tubes. [13]

Lot of work has been done regarding the analyses of fluid flow in pipes. However, a software-based user-friendly algorithm for generalised solution of these problems is required for the facilitation of engineering users. As a first step towards fulfilment of this requirement, an algorithm for calculation of various parameters for laminar flow of incompressible Newtonian fluids in round pipes was proposed to be developed.

II. ANALYSIS AND PROPOSED APPROACH

A. Research Methodology

The proposed research methodology has been illustrated in Fig 1:

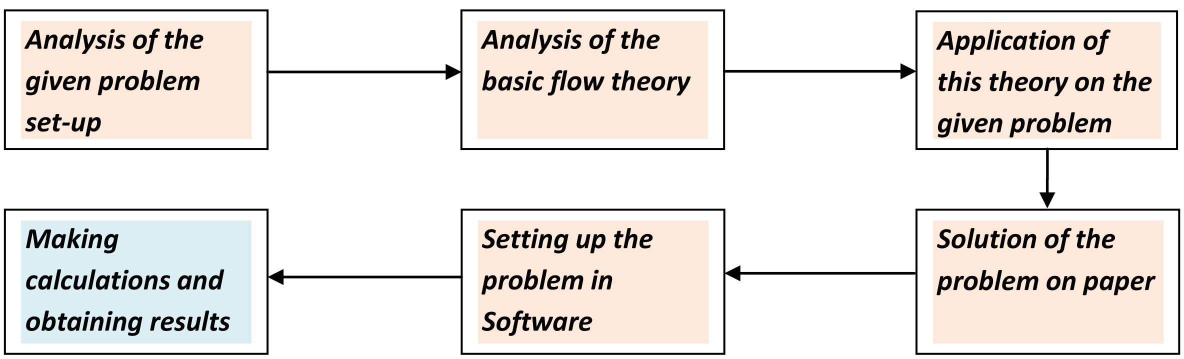

Fig 1. Research Methodology. Proposed way-forward for solution of problem.

B. Analysis

In order to develop the algorithm, various parameters for laminar flow of a given incompressible Newtonian fluid in round pipe were analysed. Basic description of the laminar flow in a round pipe, its various parameters especially velocity profile and shear stress, and the flow-rate equation given by Hagen (1839) and Poiseuille (1840) as well as various seminal works compiled on the subject till-date were also taken into account.

For the purpose of analysis, two reservoirs containing water (an incompressible Newtonian fluid) with surfaces at two different elevations were considered (Fig 2). A horizontal tube of length L with uniform round cross-section of radius R (diameter d = 2R) was used to connect the reservoirs such that water could flow among them. It was assumed that reservoirs were large enough such that changes in surface elevation would be very slow because of the flow into or out of the reservoirs. Therefore, the height of the fluid, and the associated hydrostatic pressure, in each reservoir was assumed to be constant for the purposes of analysis. It can safely be said that a constant pressure difference Δp was maintained through-out between both reservoirs. [2]

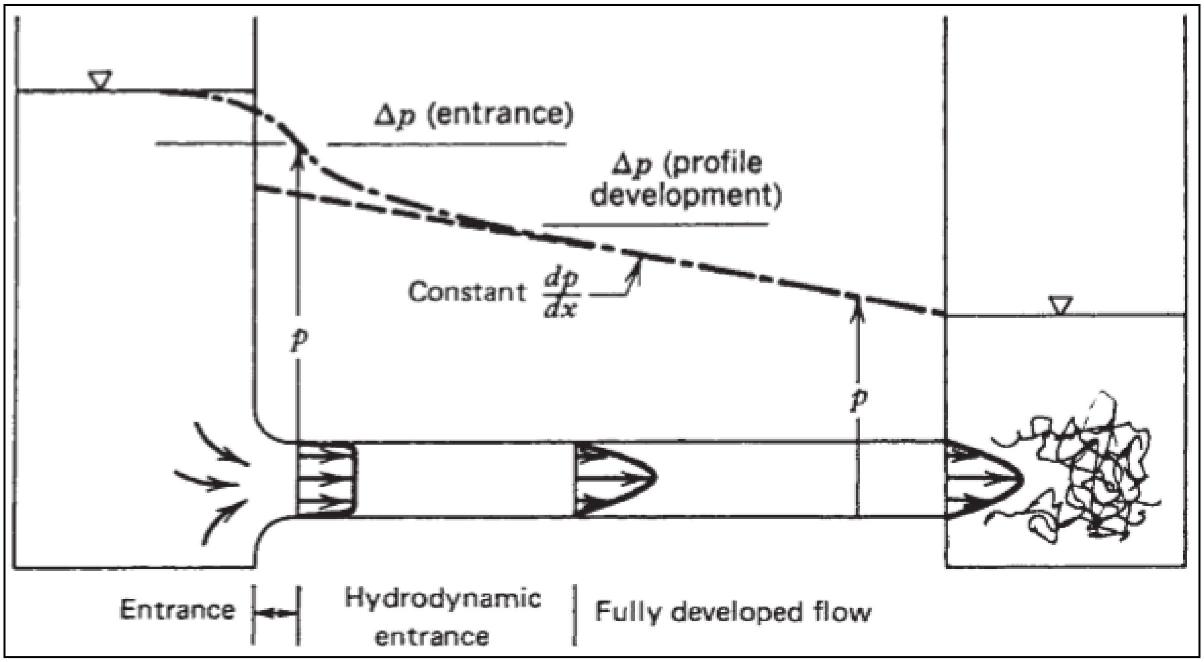

Fig 2. Problem Set-up. Pressure-driven flow in a round tube connecting two reservoirs containing an incompressible Newtonian fluid (water) at different (constant) pressures while the pressure difference between both reservoirs remains unchanged (Poiseuille's flow) (based on information given in Ref [2]).

The difference between hydrostatic pressures in vicinity of both ends of horizontal pipe was expected to cause development of a flow and reach a steady / quasi-steady state. The entrance of the pipe was smooth, so that fluid entered smoothly from reservoir with almost constant velocity through the pipe. It could be seen that the fluid accelerated from almost zero velocity in reservoir to an average velocity V_1 due to pressure forces, with pressure p_1 at pipe entrance being less than hydrostatic pressure p_0 at same level in reservoir. For a frictionless flow in entrance region, relation between pressure drop along central streamline and average velocity V_1 was given by Bernoulli's equation:

$$p_0 - p_1 = \frac{1}{2}\rho V_1{}^2 \tag{1}$$

A close analysis of the flow revealed that at the entrance region, the velocity profile v_x along the tube axis was not exactly flat; rather velocity was slightly lower on the centre line than V_1. This effect was caused due to the curved path (streamlines) followed by the particles while the flow entered the tube. The curved path warranted a normal pressure force that was generated due to a higher pressure on outer side of streamlines than inner side of streamlines. So as per Bernoulli's equation, a high pressure warrants a lower velocity, and vice versa. This curvature vanished downstream after about distance equal to one or two times the tube diameter. This phenomenon was very small for a smooth entrance and could be ignored for majority of the engineering applications.

As per analysis, major part of the flow conformed to the in-viscid flow theory. However, flow in immediate vicinity of wall was hindered due to friction. Since no-slip condition warranted zero velocity at wall; therefore, velocity profile at entrance region had a slim portion close to the wall where velocity dropped from V_1 to zero due to viscous effects (Fig 3). In totality, the entrance velocity profile was almost planar with sharp drop-offs adjacent to the wall. [2]

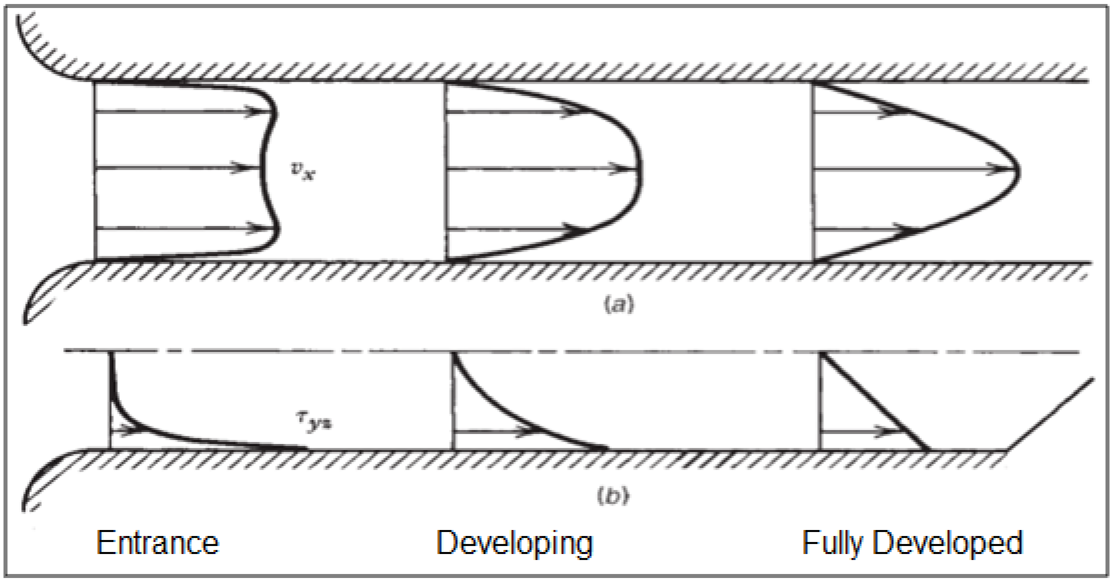

Fig 3. Flow Development. Profiles of (a) velocity and (b) shear stress based on information given in Ref [2].

While flow advanced through the tube, effects of viscous shear stress gradually spread further away from the wall towards centre line. Continuity of flow warranted that mass flow rate at each cross-section of tube remained the same; therefore, the deceleration of the particles near the wall had to be catered by the acceleration of particles in the centre. Since the acceleration of the flow was dependent on the pressure forces, acceleration of centre particles warranted that the pressure kept on decreasing in the flow direction. Finally, once pressure forces and shear forces balanced out each other, the velocity profile became constant as the flow advanced through the pipe. In this condition, profile was fully developed with maximum velocity along the centre line and zero velocity along the wall. The streamlines remained parallel and the flow was laminar. The region where velocity profile of flow was developing is known as entrance region or, more specifically, hydrodynamic entrance region. Hydrodynamic entrance is generally extended; about 50 - 100 times tube diameter is common in engineering applications. Entrance region is shortened only if flow speed is very less (in the sense that Reynolds number is low). [2] Completely developed region forms a major portion of the flow as it continues through the rest of the tube. Therefore, the analysis primarily focused upon the fully developed flow.

C. *Governing Equations*

The governing equations for steady laminar flow of incompressible Newtonian fluid (water for this case) in round pipe are well established. The velocity profile is given by: [14-20]

$$v_x = -\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1-\frac{r^2}{R^2}\right) \tag{2}$$

where

R is radius of round tube,

μ is shear viscosity of fluid,

$\frac{dp}{dx}$ is the pressure gradient along flow direction which is a –ve value indicating pressure drop along flow direction, and

r is perpendicular / radial distance from centre line of the tube.

Since the pressure drop / difference Δp maintained across the tube length L is a constant value, Eq (3) can be written as:

$$v_x = \frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\left(1-\frac{r^2}{R^2}\right) \tag{3}$$

where

$\frac{\Delta p}{L}$ is pressure gradient across length of tube which is a constant +ve value.

Shear stress τ is given by: [16-20]

$$\tau = -\mu\frac{dv_x}{dr} \tag{4}$$

where

$\frac{dv_x}{dr}$ is gradient of velocity component along shear direction, with respect to displacement along normal direction, which is a –ve value as velocity decreases away from centre line towards wall.

Substituting value of velocity from Eq (3) in Eq (4) we get:

$$\tau = \frac{r}{2}\frac{\Delta p}{L} \tag{5}$$

Volume flow rate Q is given by: [12, 15-21]

$$Q = \frac{\pi R^4}{8\mu}\frac{\Delta p}{L} \tag{6}$$

III. IMPLEMENTATION

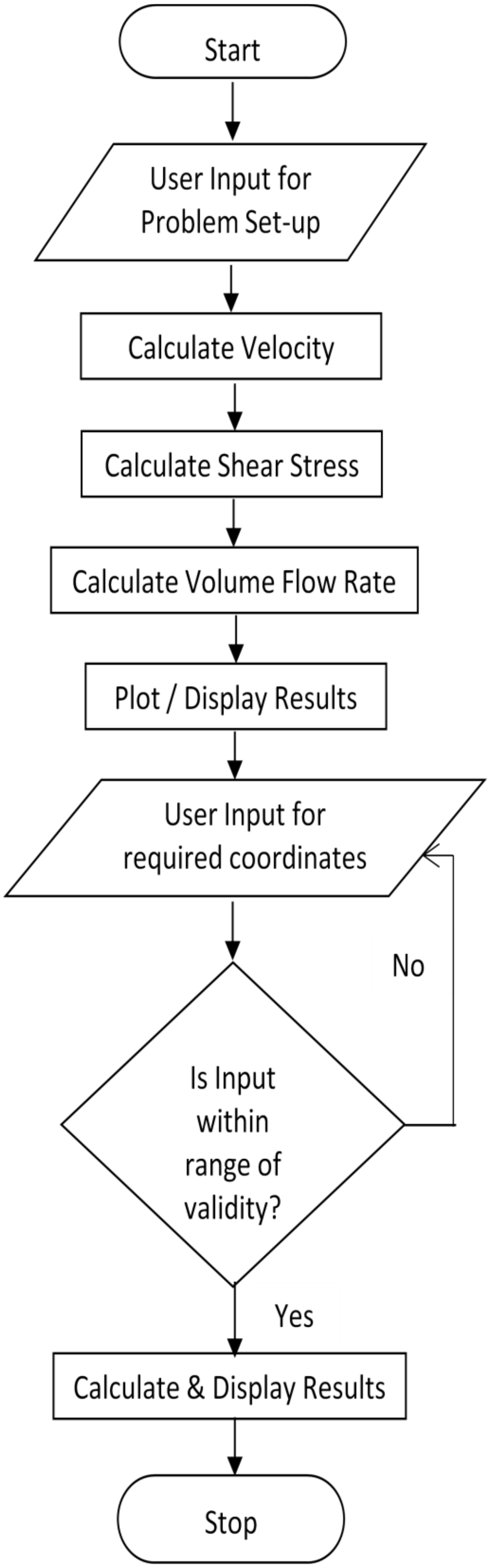

Fig 4. Algorithm. The Algorithm adopted for problem solution.

The velocity profile, shear stress, and volume flow rate equations are easier to solve by manual calculations for a single value of r. But solving repeatedly for changing values of r is a time consuming and very lethargic task. Thus a code was developed using MATLAB ver 2014a.

Setting up MATLAB

Defining Input Requirements:

a) Radius of pipe / tube R
b) Viscosity of Fluid μ
c) Pressure drop Δp
d) Length of pipe / tube L
e) Requirement for results at specific locations

IV. CALCULATIONS

Calculations Involved:

a) Solution for velocity profile v_x using Eq (3).
b) Solution for shear stress τ using Eq (5).
c) Solution for volume flow rate Q using Eq (6).
d) Plot for velocity profile v_x and shear stress τ calculated above.
e) Additional calculations for velocity profile, shear stress and volume flow rate using Eqs (3), (5) and (6) by keeping two of the three control parameters namely radius, pressure drop and length, constant and varying the third to observe its effect on the flow parameters and plotting / displaying the results.
f) Validation of user input for results at specific locations.
g) Calculation of velocity v_x and / or shear stress τ at specified locations using Eqs (3) and (5).

V. RESULTS

For demonstration, following set of values were input to get the results:

a) $R = 0.5$m
b) $\mu = 0.89\ kg/(ms)$ (water at 25^{o}C)
c) $\Delta p = 10\ N/m^2$
d) $L = 1m$

The following results were obtained:

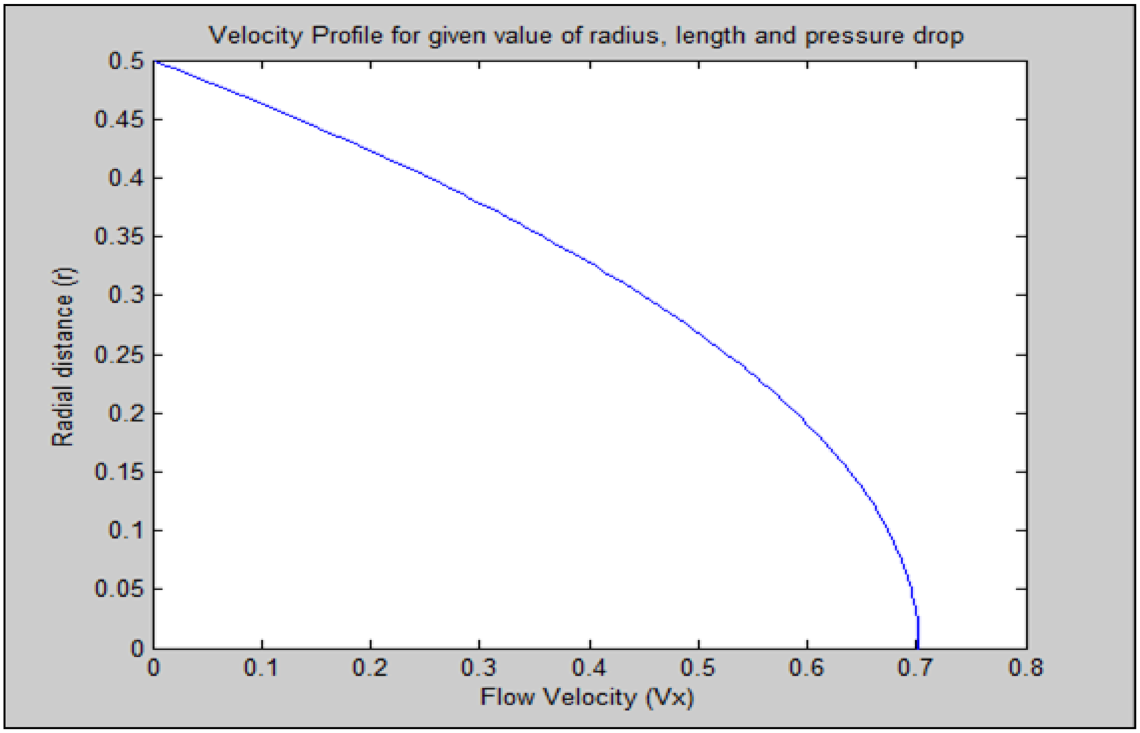

Fig 5. Velocity Profile. Velocity Profile is displayed for the values input by the user.

Fig 5 depicts that the velocity profile forms a parabola while velocity is maximum at the centre line and zero at the wall.

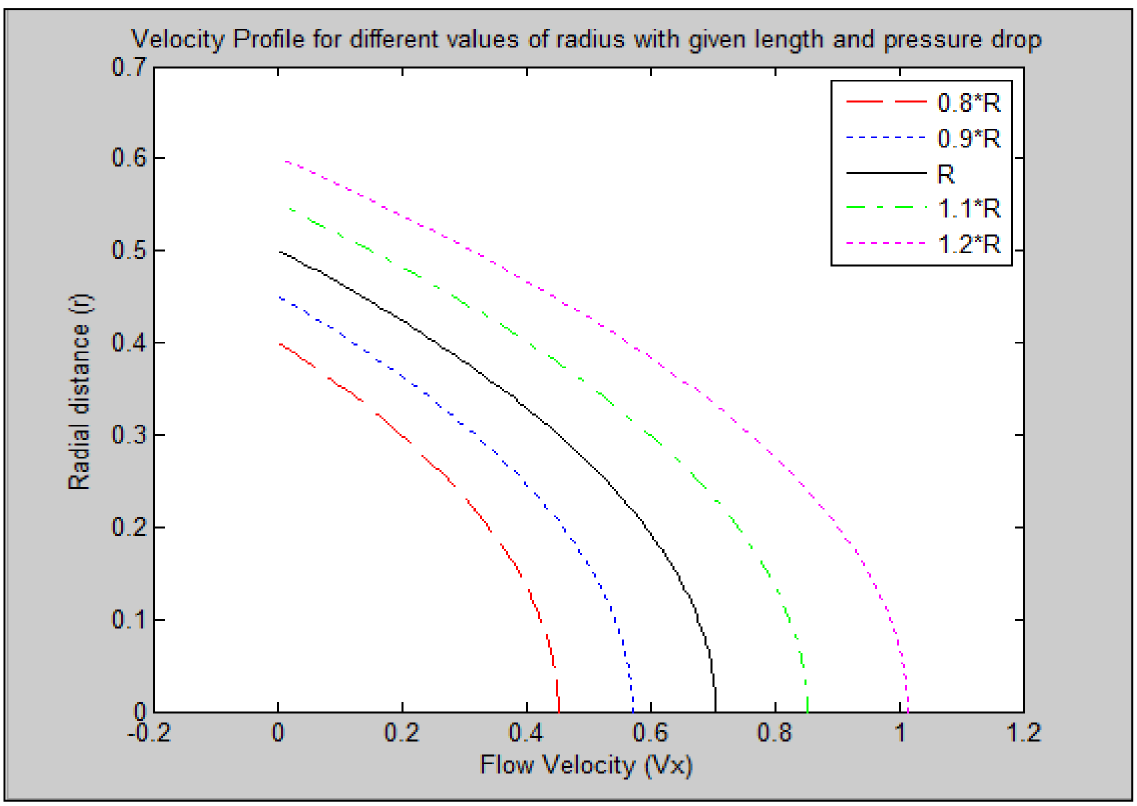

Fig 6. Variation of Velocity Profile with Radius of the Tube. Behaviour of Velocity Profile is analysed by changing values of tube radius while keeping length and pressure drop constant (values input by the user).

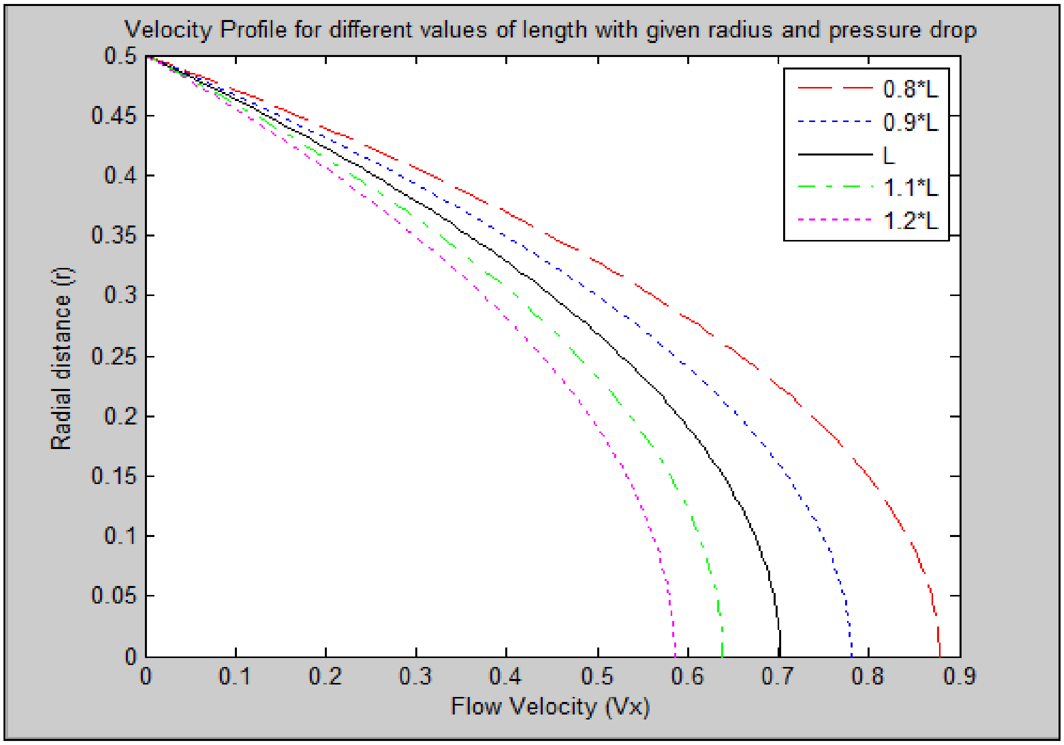

Fig 7. Variation of Velocity Profile with Length of the Tube. Behaviour of Velocity Profile is analysed by changing values of tube length while keeping radius and pressure drop constant (values input by the user).

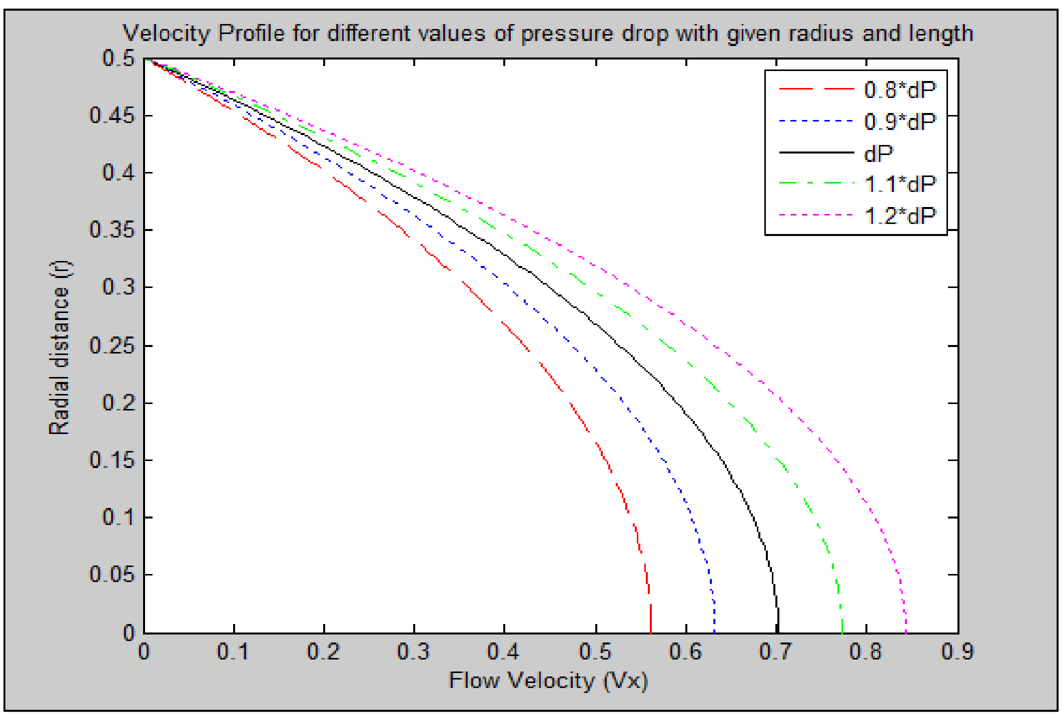

Fig 8. Variation of Velocity Profile with Pressure Drop. Behaviour of Velocity Profile is analysed by changing values of pressure drop while keeping radius and length of tube constant (values input by the user).

Fig 6, 7 and 8 depict that the magnitude of velocity grows as square of the tube radius, while it exhibits a linear reduction with increase in tube length and a linear growth with increase in pressure drop as shown by Eq (3):

$$v_x = \frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\left(1 - \frac{r^2}{R^2}\right) \tag{3}$$

but the behaviour of the velocity profile remains unchanged i.e. maximum at the centre and zero at the wall.

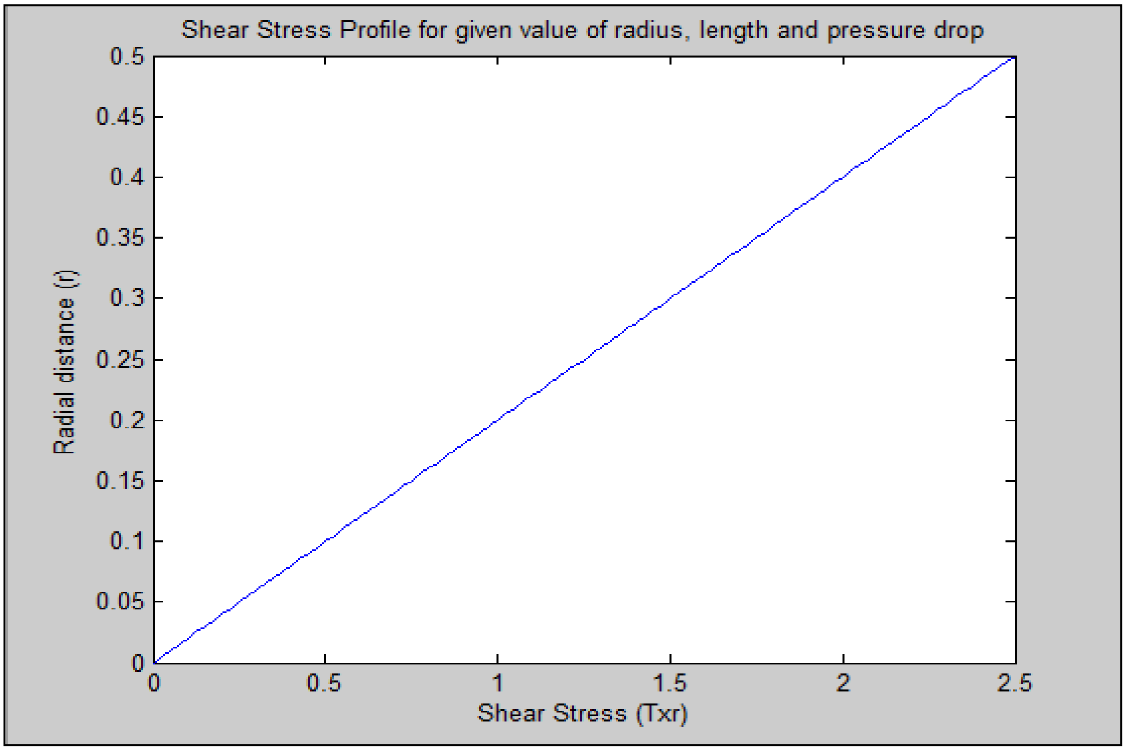

Fig 9. Shear Stress Profile. Shear Stress Profile is displayed for the values input by user.

Fig 9 depicts that the shear stress profile forms a straight line while shear stress is maximum at wall and zero at centre line.

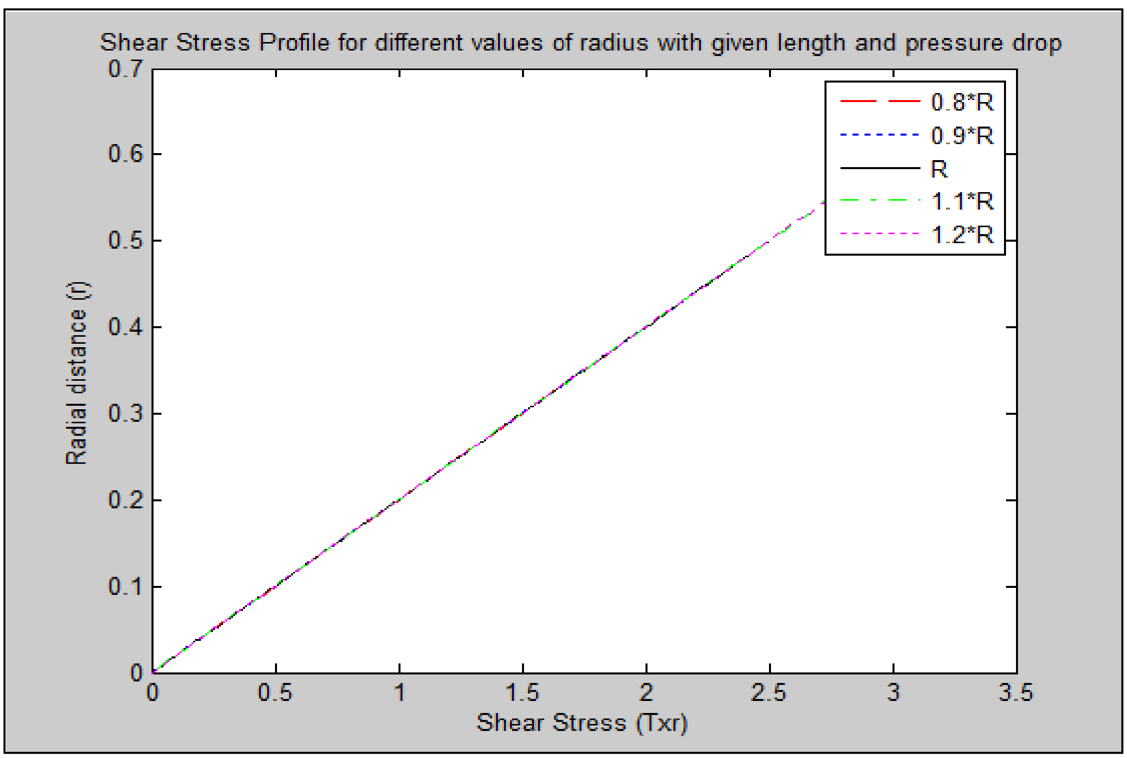

Fig 10. Variation of Shear Stress Profile with Radius of the Tube. Behaviour of Shear Stress Profile is analysed by changing values of tube radius while keeping length and pressure drop constant (values input by the user).

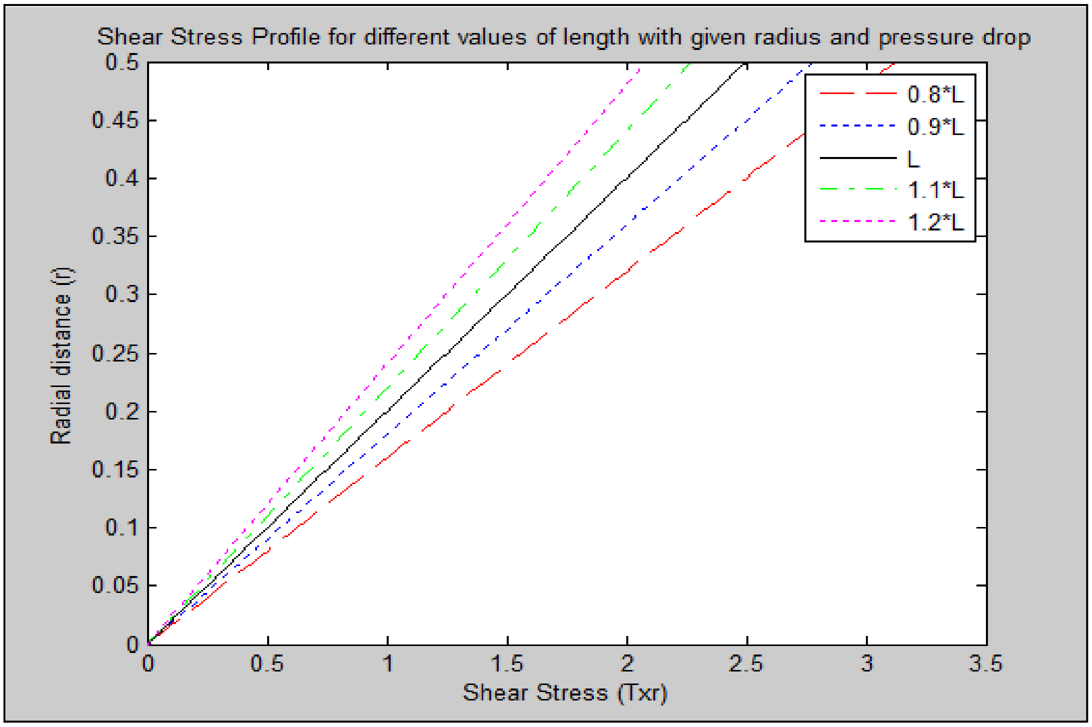

Fig 11. Variation of Shear Stress Profile with Length of the Tube. Behaviour of Shear Stress Profile is analysed by changing values of tube length while keeping radius and pressure drop constant (values input by the user).

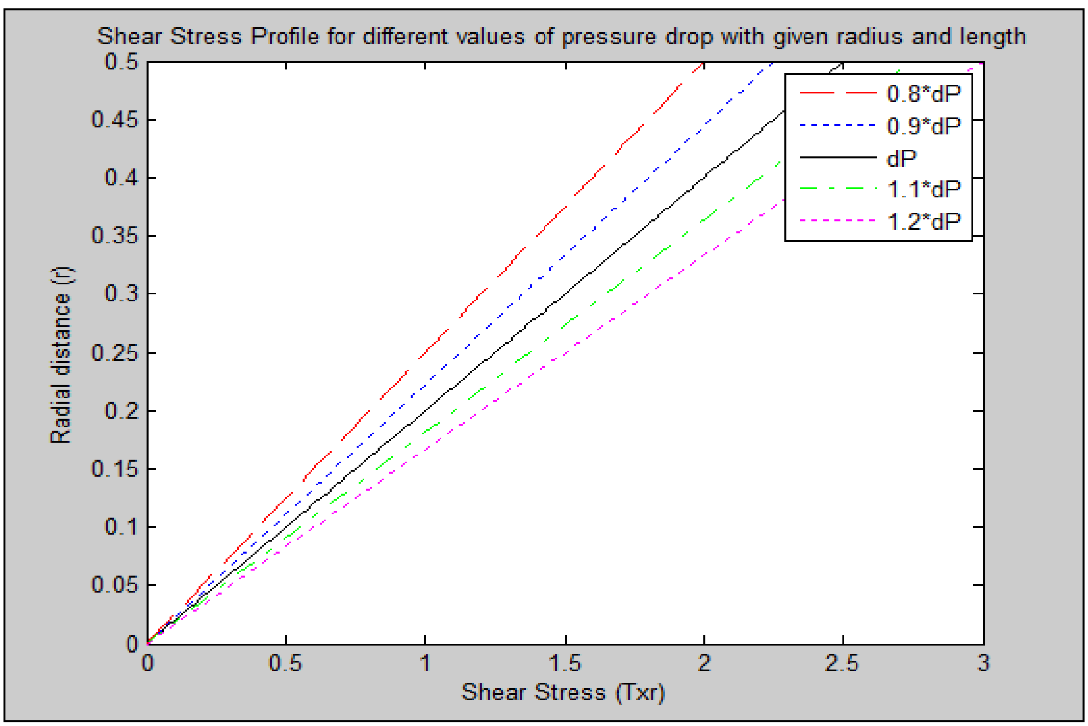

Fig 12. Variation of Velocity Profile with Pressure Drop. Behaviour of Velocity Profile is analysed by changing values of pressure drop while keeping radius and length of tube constant (values input by the user).

Fig 10, 11 and 12 depict that the magnitude of shear stress is unaffected by the tube radius, while it depicts a linear reduction with increase in tube length and a linear growth with increase in pressure drop as shown by Eq (5):

$$\tau = \frac{r}{2}\frac{\Delta p}{L} \quad (5)$$

but the behaviour of the shear stress profile remains unchanged i.e. maximum at the wall and zero at the centre line.

```
Please enter the radius of the pipe
0.5
Please enter the viscosity of the fluid
0.89
Please enter the pressure drop across the length
10
Please enter the length of pipe under study
1
The Volume Flow Rate for the given pipe is    0.2758
```

Fig 13. Volume Flow Rate. Volume Flow Rate for the given pipe is displayed along with input by the user.

The result for volume flow rate calculated for the given values of radius, length and pressure drop is shown in Fig 13, while results corresponding to variation of these control parameters have been shown in Table 1.

TABLE I. RADIUS, LENGTH AND PRESSURE DROP VS VOLUME FLOW RATE

For Constant *L* and Δ*p*		**For Constant *R* and Δ*p***		**For Constant *R* and *L***	
Tube Radius (R)	***Volume Flow Rate (Q)***	***Tube Length (L)***	***Volume Flow Rate (Q)***	***Pressure Drop (Δp)***	***Volume Flow Rate (Q)***
0.40	0.1130	0.80	0.3447	8	0.2206
0.45	0.1809	0.90	0.3064	9	0.2482
0.50	**0.2758**	**1.00**	**0.2758**	**10**	**0.2758**
0.55	0.4038	1.10	0.2507	11	0.3033
0.60	0.5718	1.20	0.2298	12	0.3309

(Behaviour of Volume Flow Rate (Q) with variation in either of Tube Radius (R), Tube Length (L) and Pressure Drop (Δ*p*) while keeping other two control parameters constant.)

Table 1 depicts that the magnitude of volume flow rate (Q) grows as fourth power of the tube radius (R^4), while it depicts a linear reduction with increase in tube length and a linear growth with increase in pressure drop as shown by Eq (6), while the third value in each case depicts nominal value of flow rate corresponding to the user input.

The additional calculations performed against user requirement for velocity and shear stress at a particular radial distance (r) from centre line are shown in Fig 14.

```
Do you want to calculate Velocity (Vx) or Shear stress (Txr)
for given value of radius, length and pressure drop at a
particular radial distance r from centre line
Please Enter 1 for Vx, 2 for Txr, 3 for both Vx and Txr,
or any other value to exit
3
Please enter value of r: .3
The Velocity (Vx) is    0.4494

The Shear Stress (Txr) is    1.5000
```

Fig 14. Results at Specific Locations. Velocity and Shear Stress are displayed at a specific radial distance r from the centre line against specific user requirement.

The results obtained for velocity (Fig 5-8) and shear stress profiles (Fig 9-12) were found in conformance with analysis done earlier (Fig 3). Moreover, the calculations done for velocity and shear stress profiles, and volume flow rate conform to the relations depicted by Eqs (3), (5) and (6). These facts clearly indicate correctness of the results generated by the algorithm.

VI. ADVANTAGES

Using a computer code not only enabled us to solve this problem, but also made our work dynamic enough to be applicable to any such setup having the same configuration:

- For analysing results for any pressure drop between the reservoirs, only the corresponding value of Δp has to be changed in the input.
- For any Newtonian fluid, only the corresponding value of viscosity (μ) has to be changed in the input.
- For uniform cross-section round tube of any size, only the corresponding radius R and length L have to be changed in the input.
- For velocity and shear stress at any location, only the perpendicular distance from the centre line of the tube r has to be input.
- In addition to the given values input by the user, the program also calculates and depicts the effect of changes in radius and length of tube, and pressure drop on behaviour of the velocity and shear stress profiles, and volume flow rate.

VII. LIMITATIONS

The scope of present work was limited by the nature of the problem in hand in the way that it only applies to:

- Constant pressure drop Δp between reservoirs.
- Constant diameter round pipes, that means it handles uniform cross-section round pipes only.
- Incompressible Newtonian fluids ie having constant density and viscosity.
- Laminar flows in pipes.
- Steady state ie fully developed flow.

VIII. CONCLUSIONS

An algorithm has been developed that helps in determining and understanding the complete velocity profile (v_x) and shear stress (τ) along with volume flow rate (Q) for laminar flow of a given incompressible Newtonian fluid in a round pipe. It also assists in analysing the effect of changes in radius and length of tube, and pressure drop on behaviour of the velocity and shear stress profiles, and volume flow rate. Moreover, it helps in determining velocity and shear stress at any point in the pipe.

The contribution of this work is that the proposed algorithm has automated calculation and plotting of important parameters for laminar flow of given incompressible Newtonian fluid in round pipe for use in engineering design and research involving such flows where these parameters are required to be taken into account.

Not only is this work applicable on this particular problem but also applicable to other flows having similar set-up (changing pressure drop Δp, viscosity μ, radius R and length L

Hence, this work can be applied on all such flow configurations, as discussed above, to calculate velocity profile (v_x) and shear stress (τ) along with volume flow rate (Q).

IX. RECOMMENDATIONS

This work can be further enhanced by future researchers by expanding the scope of work beyond present research by considering the following measures:

- Variable pressure drop between reservoirs.
- Variable diameter round pipes or non-circular cross-section pipes.
- Compressible or non-Newtonian fluids ie having variable density or viscosity.
- Turbulent or laminar–turbulent flows in pipes.
- Developing flow ie entrance or hydrodynamic entrance regions.

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APPENDIX A

Description of Various Flow Types and Terminologies

1) Continuum and Free Molecule Flow:

- This category is based on the inter-molecular distances of the fluid. The mean distance that a molecule of a fluid travels between collisions with the neighbouring molecules is called mean free path λ. Consider flow over a cylindrical body of diameter d. If λ is much smaller than body scale d, the body feels the flow as a continuous object. Then the flow is treated as continuum. Conversely, if λ is of the same order as d, the fluid molecules are felt by the body as distinct particles. Then the flow is called free molecular flow. Most of the flows on our planet are continuum whereas free molecular flow is rarely encountered eg for space vehicles at outer edge of atmosphere.
- The continuum assumption simply means that physical properties such as velocity, temperature, stress, and electric field strength are considered as spread all the way through space and these properties have definite values at every point in space. For all purposes, the general flows are treated as continuum.

2) Viscous and In-viscid Flow: Molecules in gas or liquid move freely and carry their mass (mass diffusion), viscosity (friction) and temperatures (thermal conduction). This is called transport phenomena. All real flows exhibit this phenomena and called viscous flows. In comparison, a flow hypothetically having no diffusion, friction and thermal conduction is called in-viscid flow. Purely in-viscid flows are unreal or ideal and do not really exist; however, practically (for very high Reynolds number flows), the effect of transport phenomena is restricted to only a thin layer around body surface called boundary layer and flow outside this boundary layer is essentially in-viscid flow.

3) Compressible and Incompressible Flow: Flow with constant density is called incompressible flow. In comparison, flow with variable density is called compressible flow. Ideally speaking, all fluid flows are compressible; however, all hydrodynamic flows are treated as incompressible. Also, gaseous flows at low Mach No (M < 0.3) behave as incompressible.

4) Laminar and Turbulent Flow:

- In fluid dynamics, a flow is called laminar (or streamline) flow when the fluid moves in layers which move parallel to each other, with no disturbance among the layers. The fluids have a tendency to flow without sideways assimilation at low speeds, and neighbouring layers tend to slip over each other in a smooth and frictionless manner. The fluid particles move in an orderly manner. In case of internal flows (flows completely bounded by solid surfaces such as a pipe), all fluid particles move along the pipe walls in straight line paths parallel to the walls. There are no currents normal to flow direction, nor eddies or fluid swirls. Technically, laminar flow is categorized by low momentum convection and high momentum diffusion.
- In contrast to the laminar flow, turbulent flow is regarded as comprising disorderliness and hectic variations in flow properties including low momentum diffusion, high momentum convection, and sudden changes in pressure and flow velocity across space and time. Turbulence is characterized by irregularity, diffusivity, rotationality and dissipation. While there is no mathematical formulation or theorem depicting the relation between flow type ie laminar or turbulent vis-à-vis the non-dimensional Reynolds number (Re), experimental observation reveals that flows at Reynolds number greater than 5000 are usually (not essentially) turbulent, while the small Reynolds numbers flows are typically laminar.

5) Newtonian Fluid: A fluid, for which the changes in viscous stresses generated from the flow with respect to local strain rate (fluid's rate of change of deformation over time) are linear at every point, is called Newtonian fluid. The relation between viscous stress for an incompressible Newtonian fluid and strain rate is given by a simple equation (called Newton's law of viscosity):

$$\tau = \mu \frac{du}{dy} \tag{7}$$

where

τ is (viscous) shear stress (drag) in fluid,

μ is scalar constant of proportionality called shear viscosity of fluid, and

$\frac{du}{dy}$ is gradient of velocity component along shear direction, with respect to displacement along normal direction.