

DELFT UNIVERSITY OF TECHNOLOGY

MODELLING OF THERMO- AND HYDRODYNAMIC SYSTEMS
ME45155

Assignment 1

Modelling of Solar Panel Heating of Swimming Pool

Authors

Darshan Raju (5068800)
Shyam Sundar Hemamalini (5071984)

May 15, 2020



Problem Description

An employee of the TU Delft has in his garden a small swimming pool and wants to add a heating system, see the Figure above. The heating system consists of a conventional electric heater with a power of 4kW and a solar collector with an area 10 m^2 . The pool has an average depth of 1.5 meter and an area of 40 square meters. On the 1st of April the temperature of the water in the pool is 11.07°C . On the 1st of May the temperature in the pool should be 18°C .

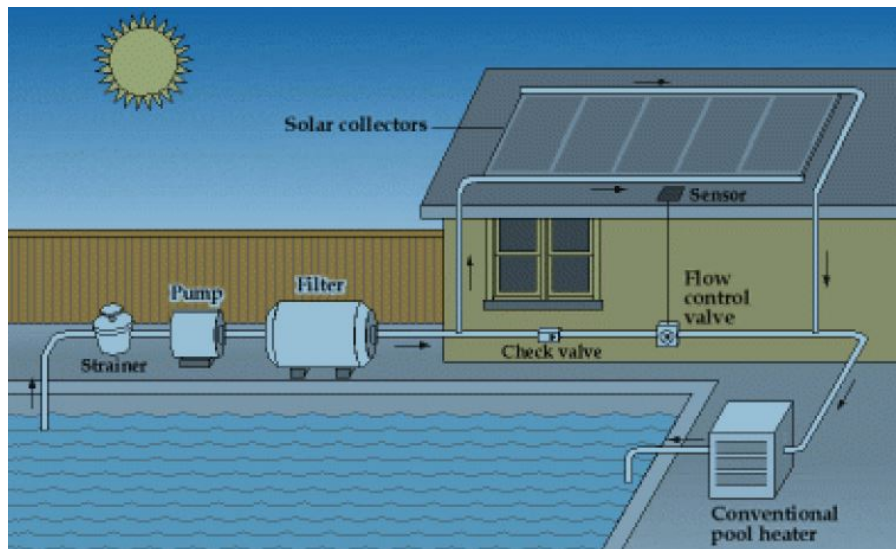


Figure 1: Solar Panel Heating Arrangement

To reduce the costs the solar heating system will be used. To estimate the reduction in costs weather data from Rotterdam airport over the month of April 2019 will be used. You may assume that the heat transfer coefficient to the surroundings as $2 \text{ W/m}^2\text{K}$ and that on a daily basis 1 liter water evaporates from the swimming pool. For simplicity you may assume that the water in the pool itself is not heated by the sun.

Objectives

- How much heat has to be added to the system to reach the required temperature, ignoring losses due to convection and evaporation? If only the electric heater is used and the kWh price is 0.25 EUR what are the costs?.
- Formulate the mass and energy balance for the system.
- Calculate the temperature in the swimming pool as a function of time (without the electric heater).
- To reach the required temperature the electric heater probably has to be used. Give an estimate on the savings in electricity.

Assumptions

1. In the entirety of the analysis, it is assumed that any transient effects of the pool water are absent. *The bulk of the water heats up or cools down simultaneously.*
2. *Convection, evaporation and heating are the only modes of heat or mass transfer to or from the pool water.* Any other heat transfer mechanisms like conduction with the pool surface, radiation influx and outflux etc., are ignored. Any other mass transfer mechanism other than evaporation like precipitation are also neglected. The pool water is also not refilled.
3. There are no energy gains or losses during the pumping and filtration operations. *Pipe losses and flow losses are neglected.*
4. *The volumetric rate of evaporation specified is dependent on the wind speed.* It has been assumed that the evaporation of the pool water depends on the prevailing weather conditions. The suggested value for the volumetric rate of evaporation is 1 liter per day. However, evaporation has been cited as one of the key factors affecting loss of pool water with losses upto 300 liters per day depending on the temperature and wind speed [1]. For this analysis, it has been assumed that the rate of evaporation is linear with respect to the wind speed. However, different values are compared at the end.
5. *The solar panel operates at an efficiency of 20%.* Although it is suggested to assume a 100% efficiency for the panels, this is unrealistic as solar panels barely exceed 20% during domestic operation. However, during the compilation of results, both the cases were considered.
6. *Convective heat transfer coefficient is dependent on wind speed.* It has been suggested that the convective heat transfer coefficient is dependent on the wind speed quite linearly until a wind speed of 10 m/s [2]. For this analysis, both the suggested heat transfer coefficient of 2 W/m²K and the linear correlation has been taken into account and compared.

Given Data

The pool dimensions have been given as

$$\begin{aligned}\text{Area} &= 40 \text{ m}^2 \\ \text{Depth} &= 1.5 \text{ m} \\ \implies V_0 &= 60 \text{ m}^3\end{aligned}$$

The initial temperature of the pool water is specified as

$$T_0 = 11.07^\circ\text{C}$$

The suggested convective heat transfer coefficient is

$$h = 2 \text{ W/m}^2\text{K}$$

The suggested volumetric rate of evaporation is

$$\Delta V = 1.0 \text{ liters/day}$$

The area of the solar panels is given as

$$A_s = 10 \text{ m}^2$$

Assumed Data

The suggested convective heat transfer coefficient is lower than realistic values by a considerable margin. Hence, apart from the suggested value, a more realistic constant value as given below is considered.

$$h_2 = 20 \text{ W/m}^2\text{K}$$

The following linear trend is also considered for the convective heat transfer coefficient.

$$h_3 = 10 + \frac{30}{9}(v_w - 1) \quad (1)$$

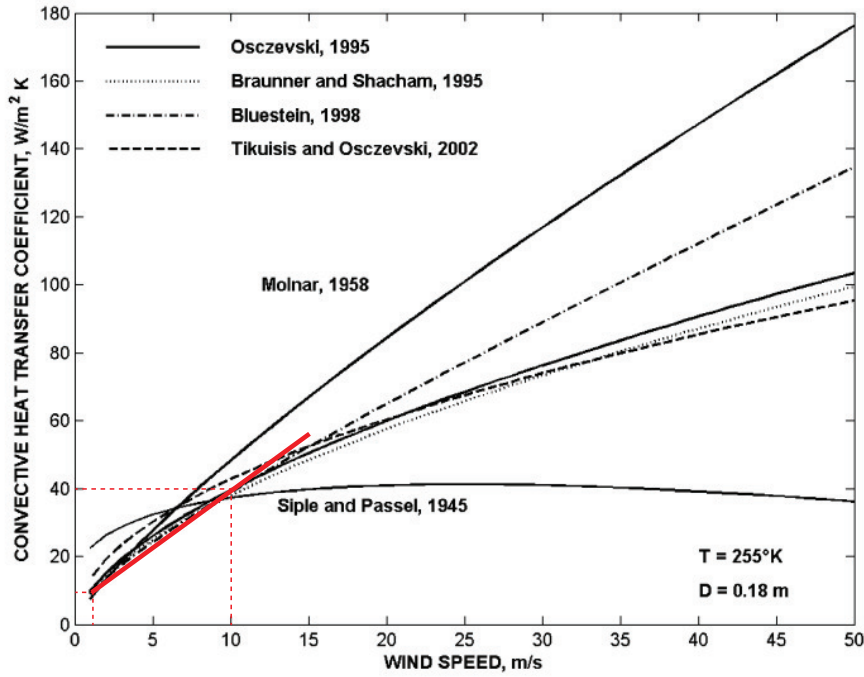


Figure 2: Linear trend for convective heat transfer coefficient [2] with h_3 as overlay

Also, the suggested volumetric evaporation rate is also considerably lower than realistic values. Hence, apart from the suggested value, the below realistic value is also assumed.

$$\Delta V_2 = 10 \text{ liters/day}$$

The following linear trend is also considered for the volumetric rate of evaporation.

$$\Delta V_3 = (0.1 + 0.1v_w) \times \frac{A}{\rho} \text{ m}^3/\text{hr} \quad (2)$$

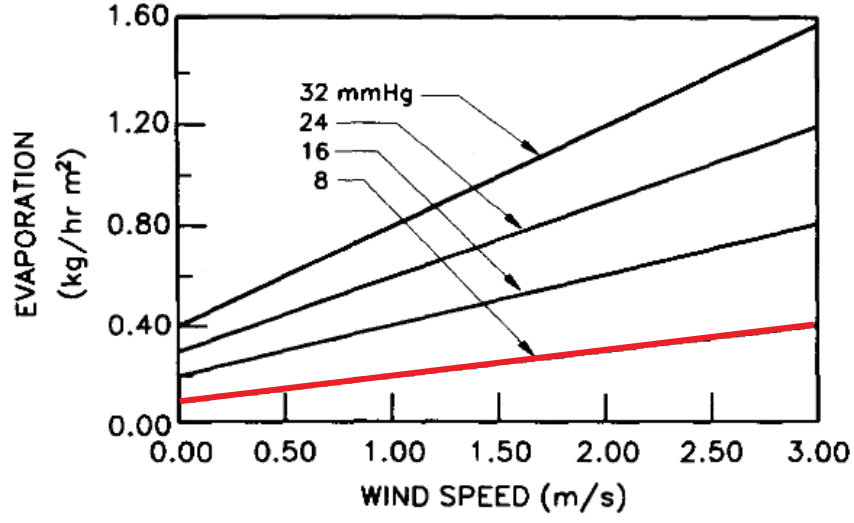


Figure 3: Linear trend for volumetric rate of evaporation [1] with ΔV_3 as overlay

KNMI Data

Meteorology data like the temperature, wind speed and solar irradiation for the timeframe between 1st April 2019 till 30th April 2019 has been downloaded from the KNMI downloads portal [3] for the location Rotterdam.

The data includes hourly values for the required parameters. Hence, the analysis is done in steps of an hour.

The temperature variation in the month of April 2019 as per the dataset from KNMI is as below.

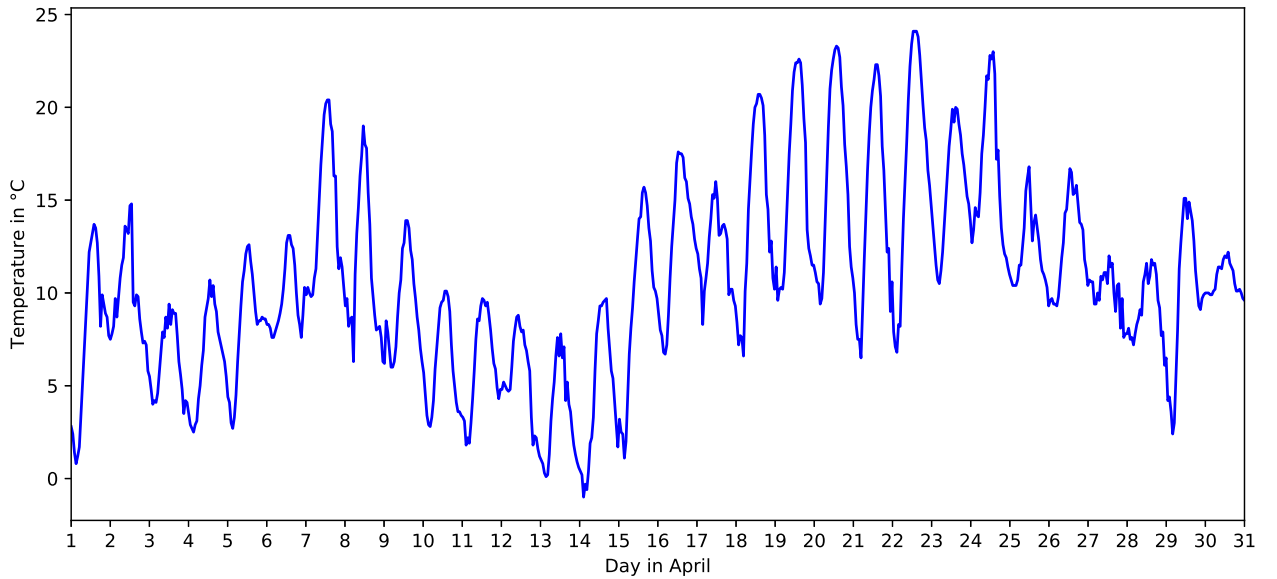


Figure 4: Air temperature variation - April 2019 [3]

The variation in the solar irradiation is given by the figure below.

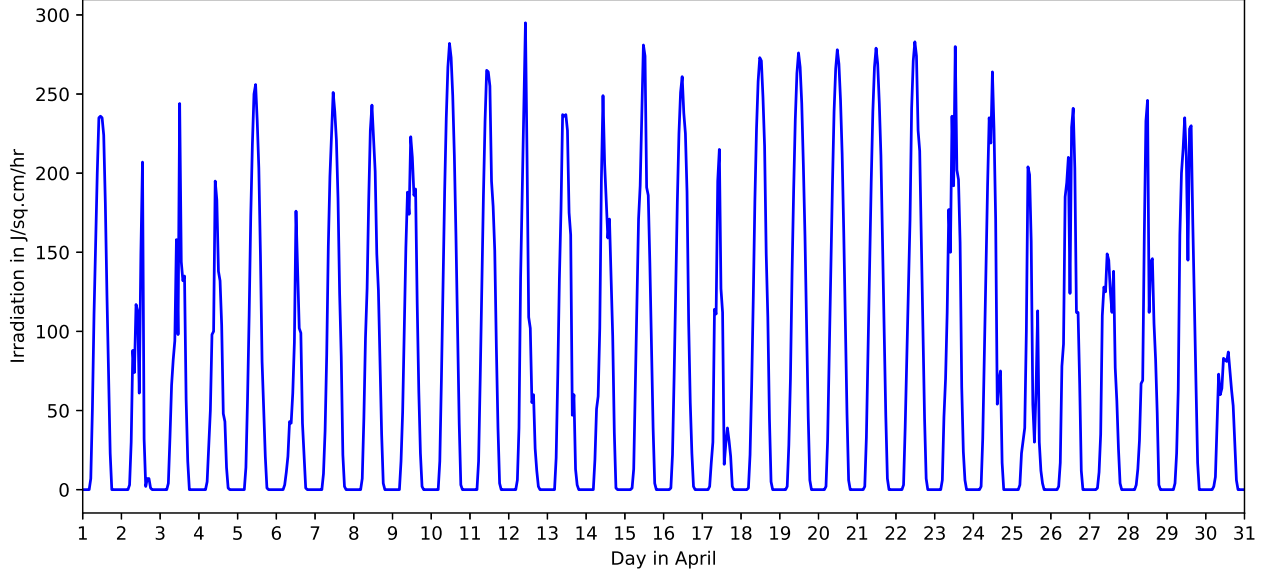


Figure 5: Solar irradiation variation - April 2019 [3]

The variation in the wind speed is shown in the figure below.

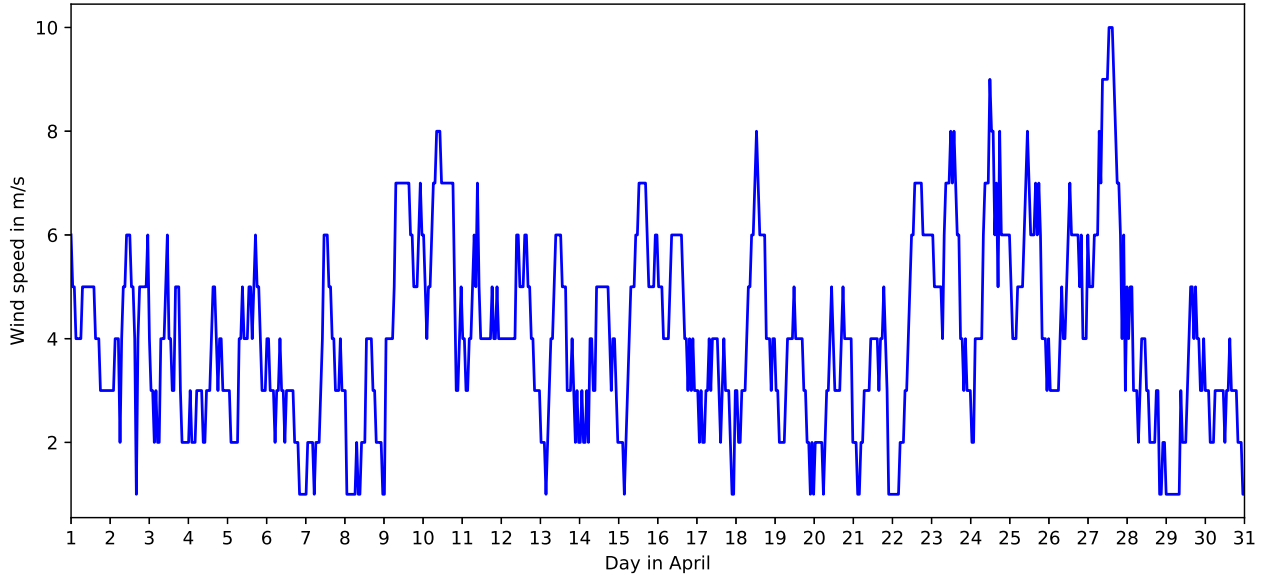


Figure 6: Wind variation - April 2019 [3]

Formulation of Equations

Mass Balance

Since our problem involves evaporation, there is a definite mass flux out of the pool water. Hence, every hour,

$$m_2 = m_1 - \rho \Delta V \Delta t \quad (3)$$

Energy Balance

The energy balance of the swimming pool is done by considering the heat flux from the solar panel, heat transfer by evaporation and convection and the conventional electric heater. It has been assumed that any other modes of heat transfer are neglected. Hence, the energy balance can be derived as below.

$$\frac{dH}{dt} = \dot{Q}_{evap} + \dot{Q}_{conv} + \dot{Q}_{solar} \quad (4)$$

In the above equation, the individual terms can be computed separately as below.

The first term of the RHS corresponds to the heat flux through evaporation. It is given by the following expression.

$$\dot{Q}_{evap} = -\Delta h_V \rho \Delta V \quad (5)$$

where Δh_V is the latent heat of vaporization given in J/kg. The above expression has a negative sign because evaporation generally takes away the heat from the liquid.

The second of the RHS of Eq.(4) corresponds to the heat flux through convection. It is given by the following expression.

$$\dot{Q}_{conv} = h A (T_a - T) \quad (6)$$

In the above expression, if $T > T_a$, the heat flux would be negative corresponding to flow of energy from the water to the air and vice versa.

The third term of Eq.(4) corresponds to the heat input from the solar panel which is given by the expression below.

$$\dot{Q}_{solar} = \eta G A_s \quad (7)$$

where η is the efficiency of the solar panel and G is the solar irradiation in J/m²/hr.

The LHS of Eq.(4) can be discretised in time which will result in the following expression.

$$\frac{dH}{dt} = \frac{\Delta H}{\Delta t} = \frac{m C_p \Delta T}{\Delta t} \quad (8)$$

Therefore, substituting Eq.(8) in Eq.(4),

$$\Delta T = \frac{-\Delta h_V \rho \Delta V + h A (T_a - T) + \eta G A_s + P}{m C_p} \times \Delta t \quad (9)$$

In the above equation, the mass of the system considered is assumed as

$$m = \frac{m_1 + m_2}{2}$$

Hence, expanding Eq.(9) with $\Delta T = T_2 - T_1$, we have

$$T_2 = T_1 + \frac{-\Delta h_V \rho \Delta V + hA(T_a - T) + \eta G A_s + P}{(m_1 + m_2)C_p} \times 2\Delta t \quad (10)$$

Therefore, we have the system of equations as

$m_2 = m_1 - \rho \Delta V \Delta t$	(11a)
$T_2 = T_1 + \frac{-\Delta h_V \rho \Delta V + hA(T_a - T) + \eta G A_s + P}{(m_1 + m_2)C_p} \times 2\Delta t$	(11b)

Once the system of equations are iterated until the end of April, the required heat input for the heater can be determined as below.

$$Q_{heater} = mC_p(T_{user} - T_{end}) \quad (12)$$

In the above equation, T_{user} is the required temperature of the pool and T_{end} is the temperature at the end of April as determined from iterations. From the above equation, the total energy consumption can be determined as

$$\text{Number of units} = \frac{Q_{heater}}{3600 \times 1000} \text{ kWh}$$

Therefore, the total heating costs can be determined from the given price per unit as

$$\text{Heating costs} = \text{Number of units} \times \text{Price per unit} \quad (13)$$

The above system is coded in Python using **CoolProp** for the values of density, specific heat capacity and the latent heat of vaporisation. The code can be found [in this Python Notebook](#).

Compilation of results

The system of equations are executed for the following cases:

1. Solar panel turned off, no convection and evaporation.
2. Solar panel turned off, convection and evaporation at recommended values.
3. Solar panel turned on at 100% efficiency, convection and evaporation at recommended values.
4. Solar panel turned on at 100% efficiency, convection and evaporation at recommended values, control valve driven by temperature sensor
5. Solar panel turned off, convection and evaporation at realistic but constant values.
6. Solar panel turned on at 20% efficiency, convection and evaporation at realistic but constant values.

7. Solar panel turned off, convection and evaporation correlated to wind speed.
8. Solar panel turned on at 20% efficiency, convection and evaporation correlated to wind speed.

The cases are iterated and the results are tabulated as below.

Case	Solar Panel	Convection	Evaporation	η	h	ΔV	Heating Costs
				%	W/m ² K	liters/day	EUR
1	Off	Off	Off	-	-	-	121.04
2	Off	On	On	-	2	1	124.26
3	On	On	On	100	2	1	-
4	Controlled	On	On	100	2	1	0.97
5	Off	On	On	-	20	10	120.03
6	On	On	On	20	20	10	112.50
7	Off	On	On	-	Eq.(1)	Eq.(2)	167.96
8	On	On	On	20	Eq.(1)	Eq.(2)	160.67

Table 1: Compilation of results for the aforementioned cases

Case 1 When the solar panel is turned off and there is no convection or evaporation, there would be no change in the temperature of the pool and hence, the pool would remain at the initial temperature of 11.07°C until the end of April. Hence, the total heating costs correspond to heating the pool from 11.07°C to 18°C.

Case 2 Now, the effect of convection and evaporation are considered and the heat transfer coefficient and the volumetric rate of evaporation are kept at suggested values. The temperature variation with time and the heat flux variation are seen below.

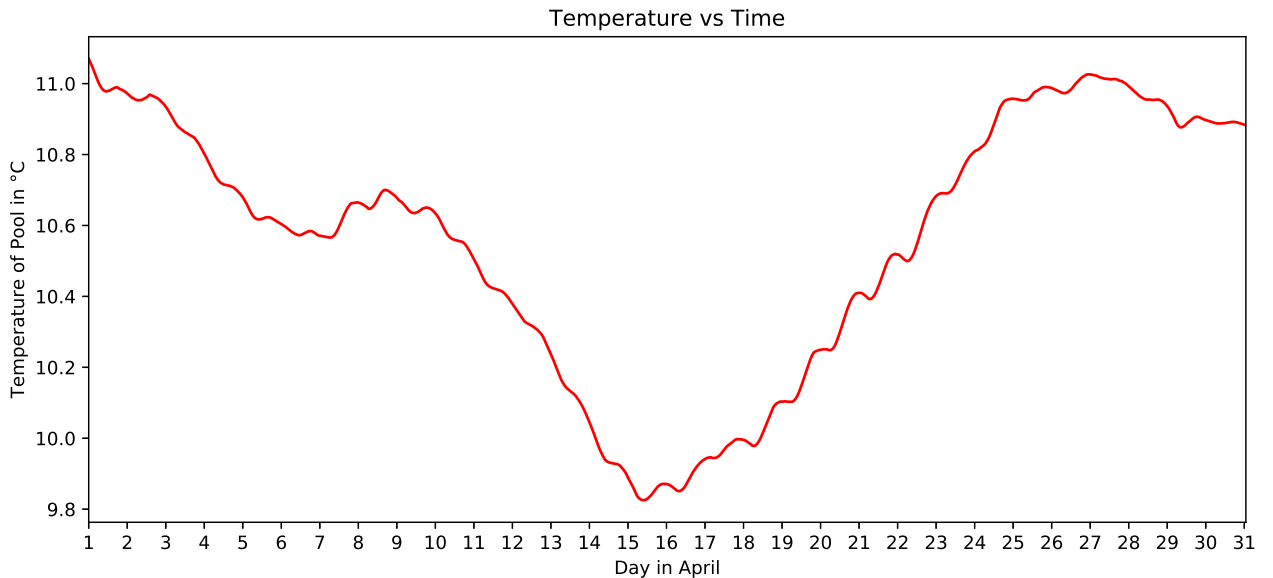


Figure 7: Temperature Variation for Case 2

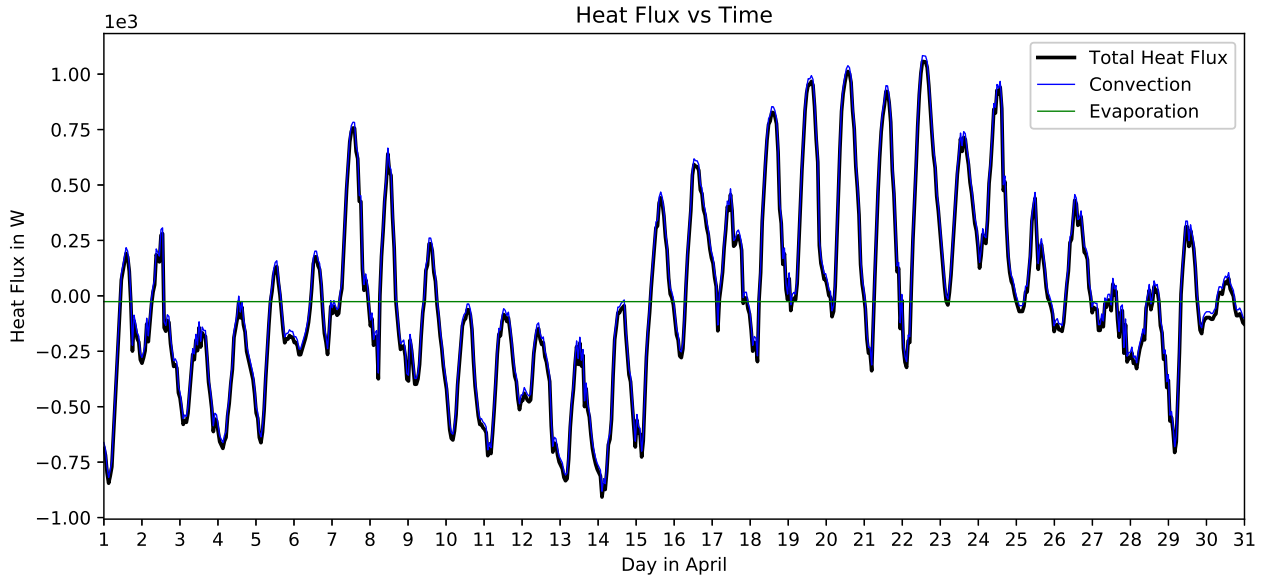


Figure 8: Heat Flux Variation for Case 2

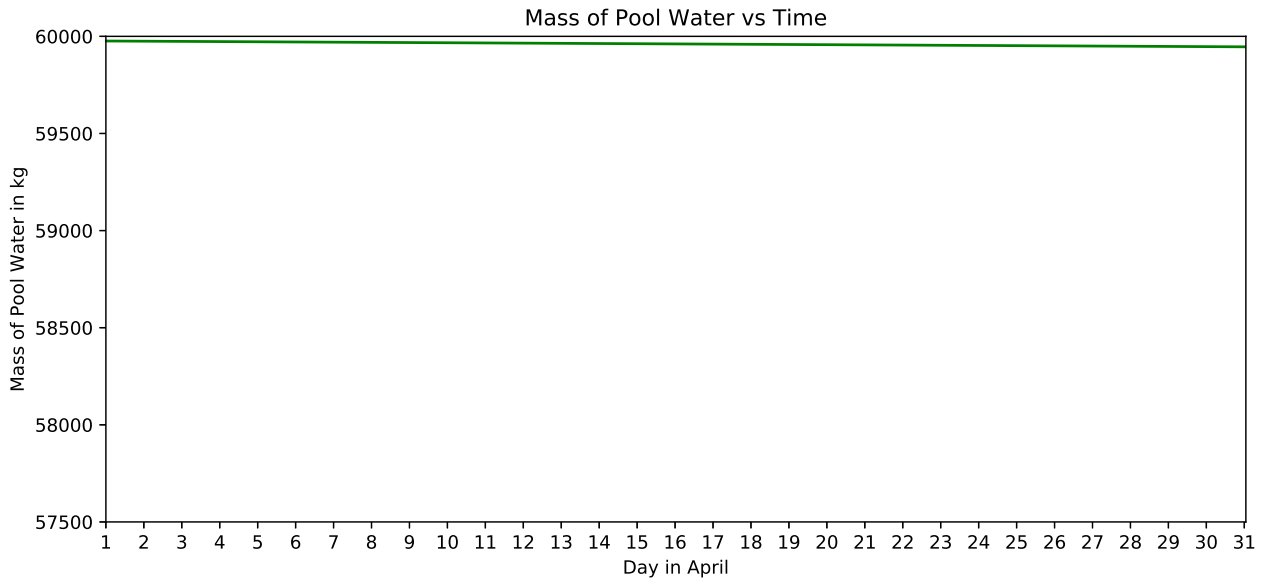


Figure 9: Mass Variation for $\Delta V = 1$ liters/day

Case 3 Now, the solar panel aided heating is turned on at 100% efficiency without the use of a control valve. Now, the temperature rises well above the 18°C mark as can be seen in the plots below. The solar panel heat flux clearly dominates and increases the temperature by a huge margin. The shootup of the temperature calls for the use of a control valve dependent on the temperature of the pool.

It is to be noted that the mass variation is the same as seen in Figure 9.

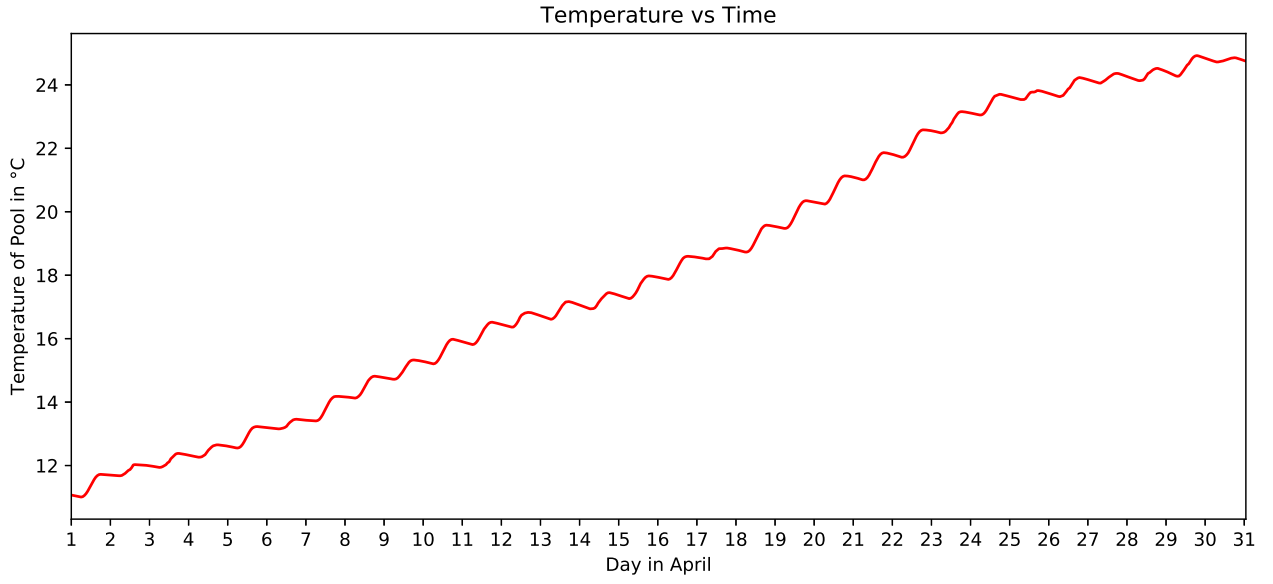


Figure 10: Temperature Variation for Case 3

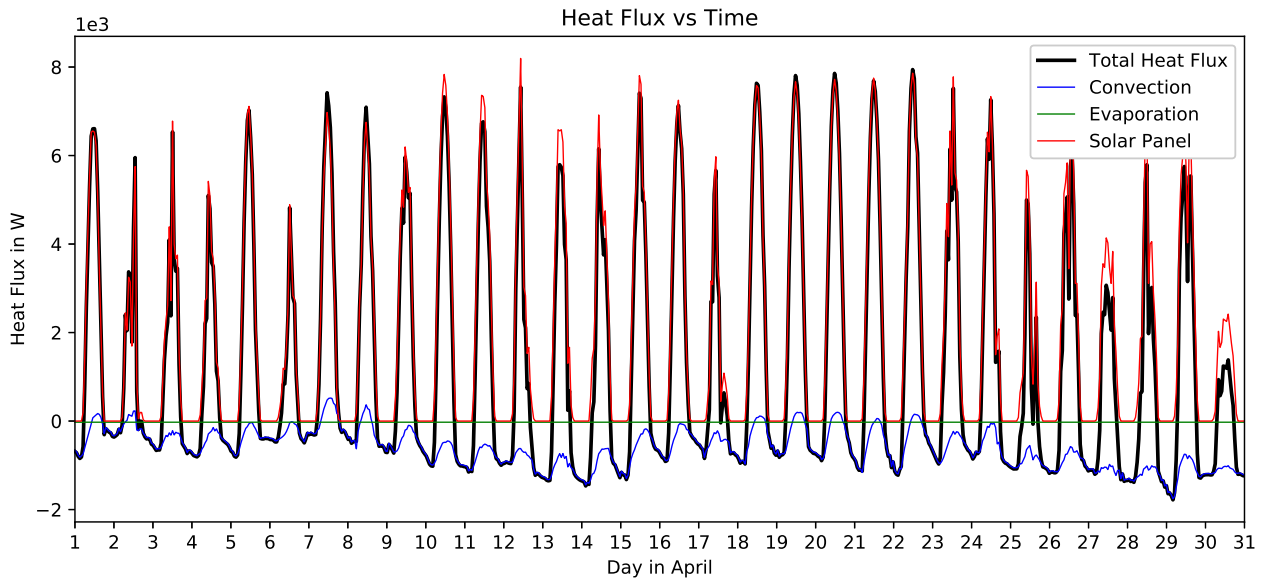


Figure 11: Heat Flux Variation for Case 3

Case 4 The control valve is activated, which is dependent on the temperature of the pool. Now the solar panel heating is turned off if the temperature of the pool reaches the user-required temperature of 18°C. This can be seen in the temperature variation plot and the heat flux variation plot below.

It is to be noted that in this case again, the mass variation is the same as seen in Figure 9.

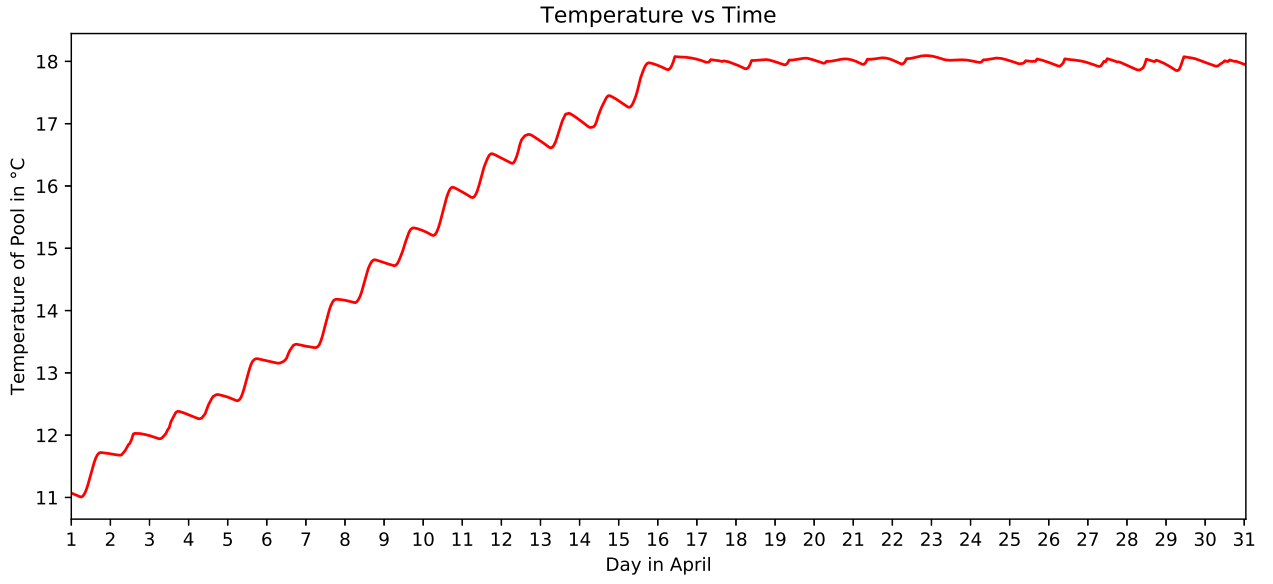


Figure 12: Temperature Variation for Case 4

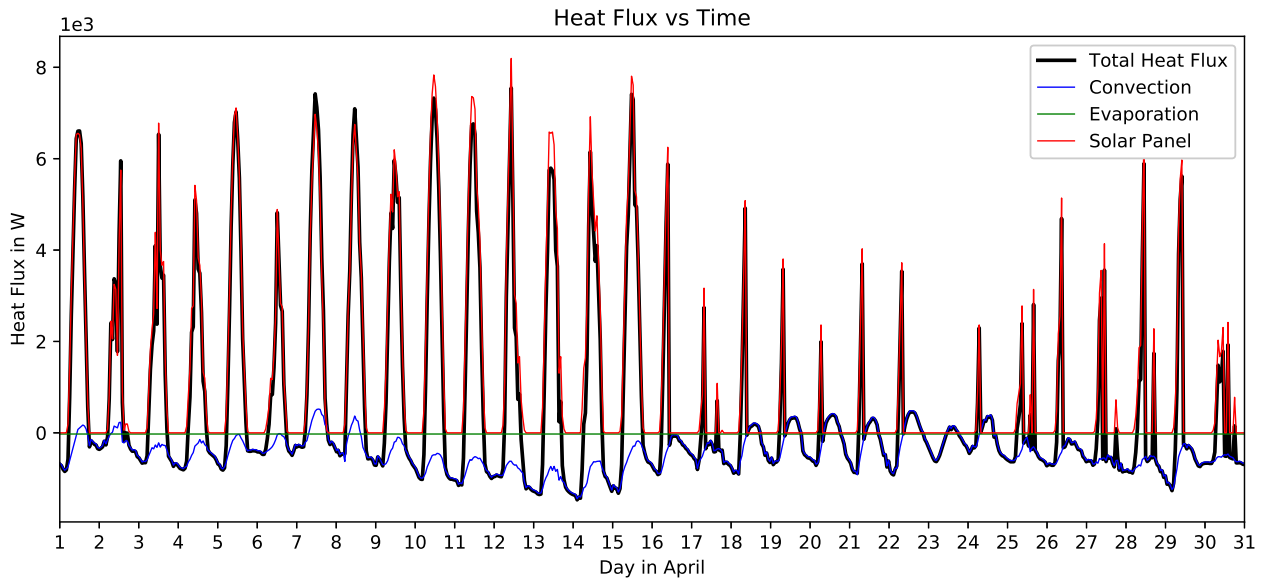


Figure 13: Heat Flux Variation for Case 4

Case 5 Now, the convective heat transfer coefficient is altered to a more realistic but constant value of $h = 20 \text{ W/m}^2\text{K}$. The volumetric rate of evaporation is also increased to $\Delta V = 10 \text{ liters/day}$. For this case, the temperature variation without the heat from the solar panel is plotted.

It can be seen that the mass variation in the pool is higher than the previous cases but still, not considerably high.

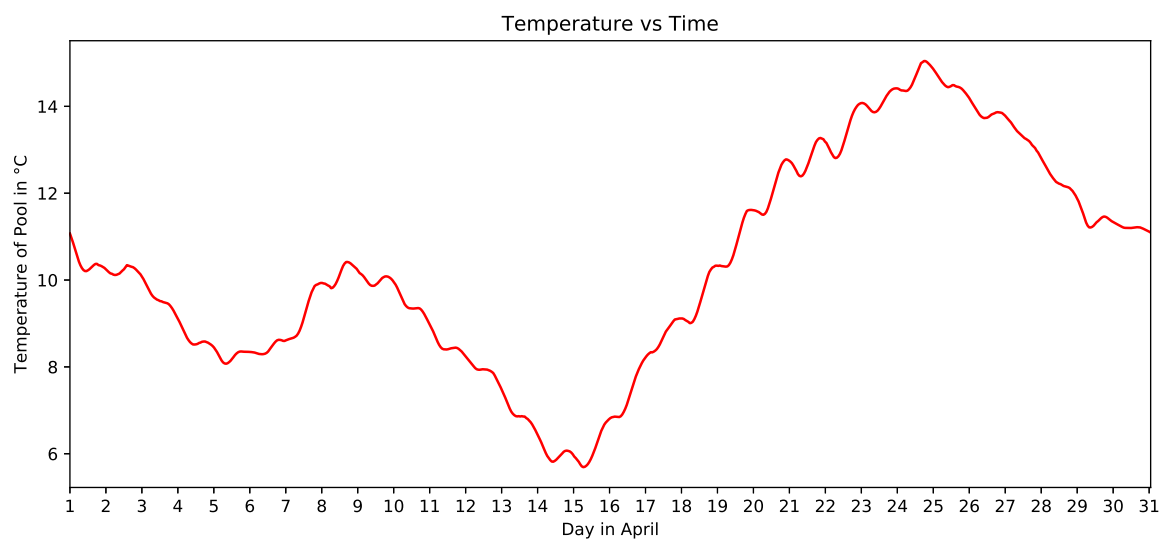


Figure 14: Temperature Variation for Case 5

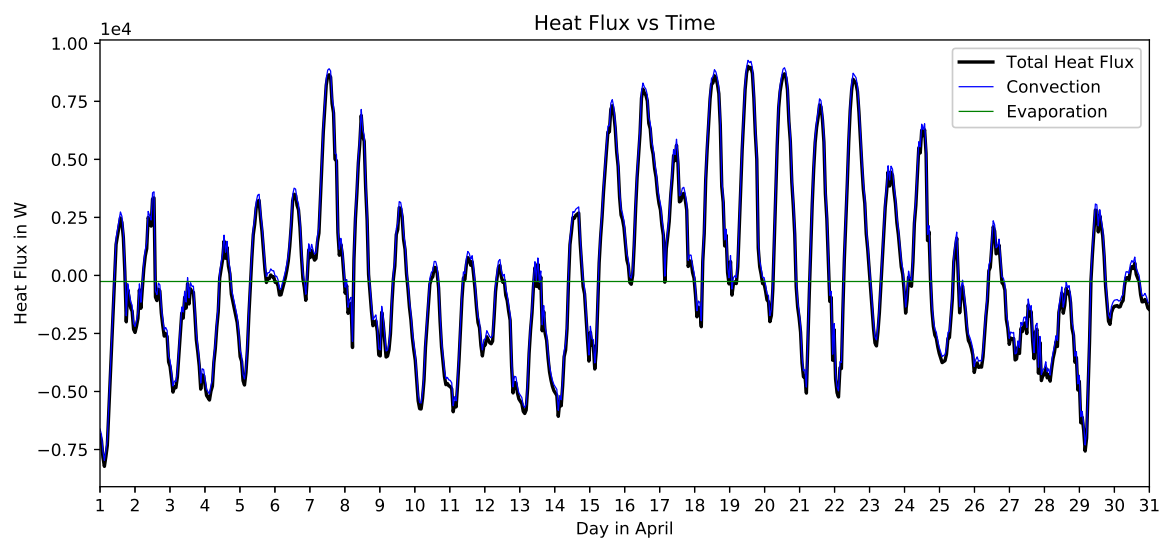


Figure 15: Heat Flux Variation for Case 5

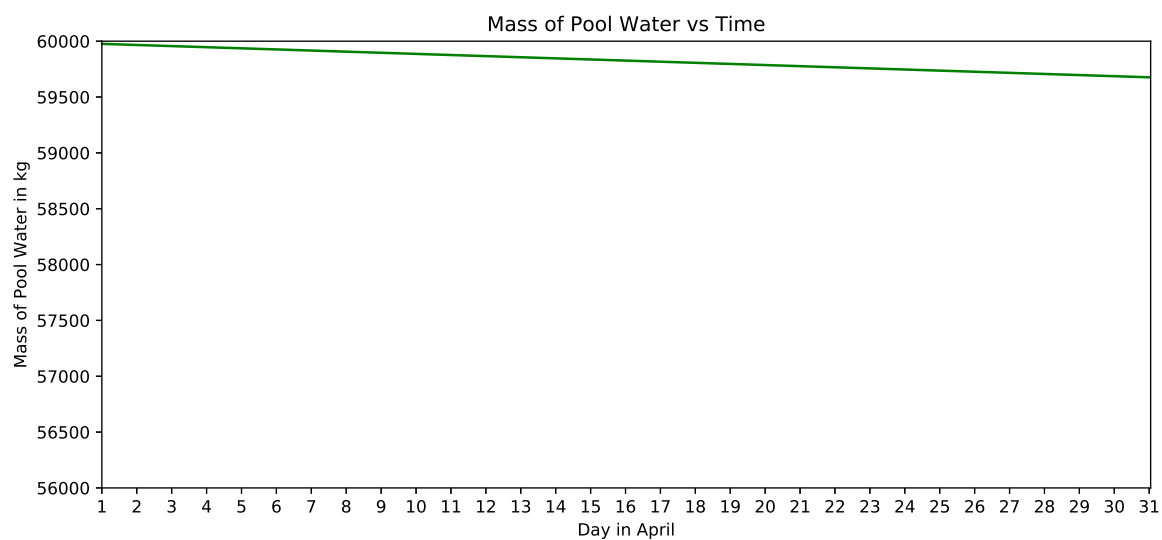


Figure 16: Mass Variation for $\Delta V = 10$ liters/day

Case 6 Now, the heat flux from the solar panel is considered albeit only at 20% efficiency as this value is much more realistic than the 100% efficiency. The convective heat transfer coefficient and the volumetric rate of evaporation are kept the same as in Case 5 and hence, show no change in the heat flux and mass variation. The temperature variation and the heat flux variation are as below.

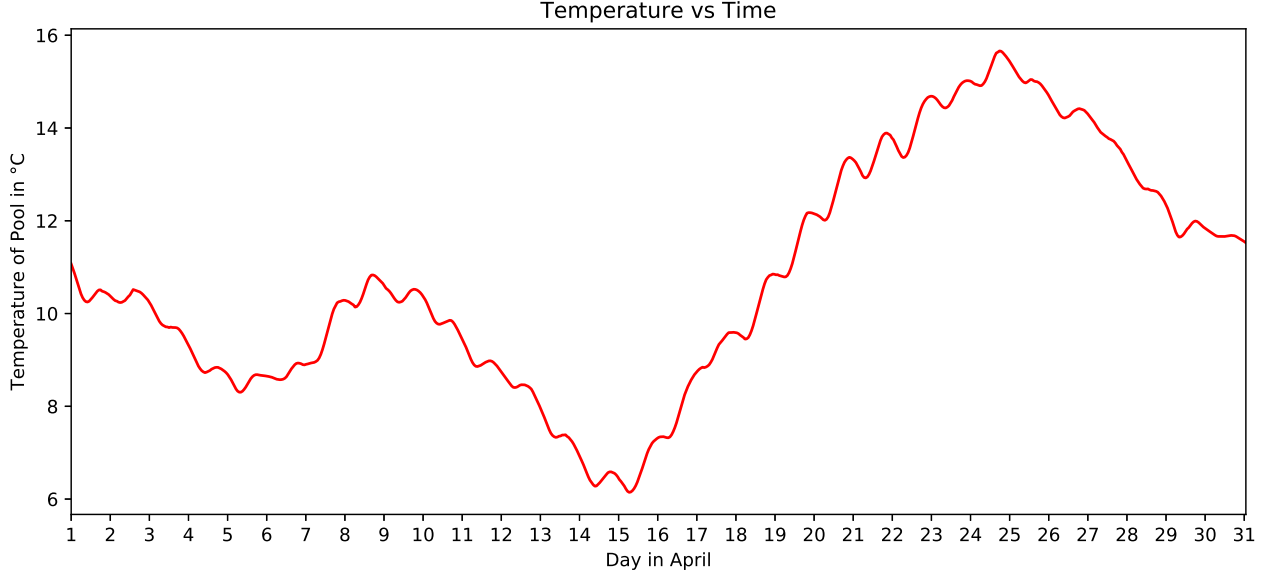


Figure 17: Temperature Variation for Case 6

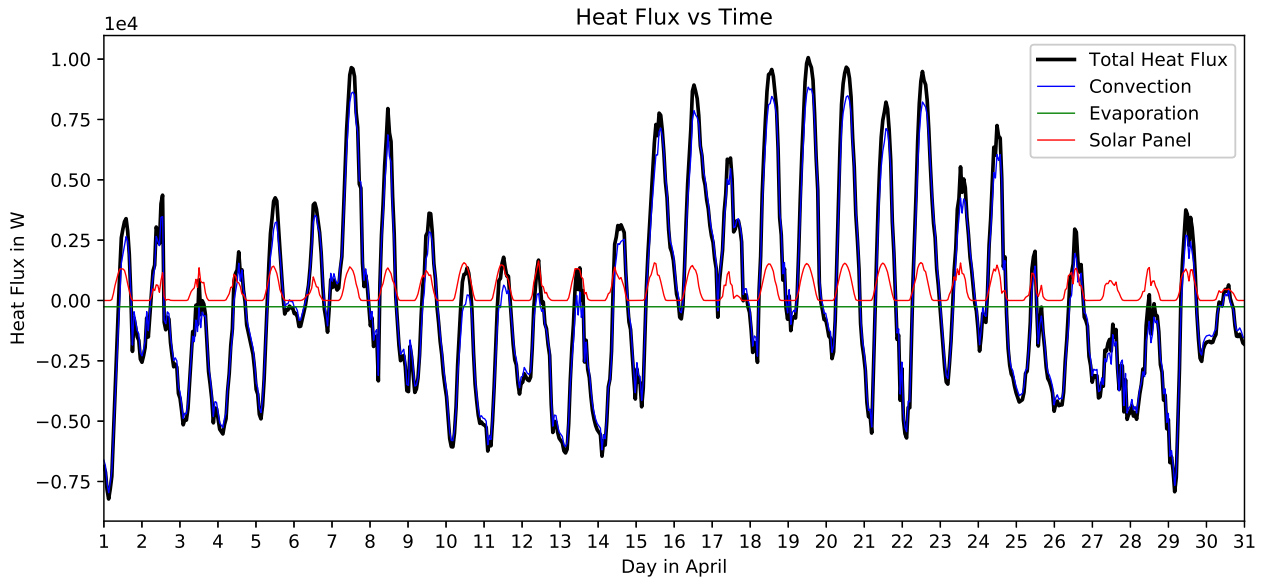


Figure 18: Heat Flux Variation for Case 6

Case 7 The values for the convective heat transfer coefficient and the volumetric rate of evaporation are still not realistic as in they are not dependent on the prevalent climatic conditions. Hence, known correlations are used for relating the convective heat transfer coefficient and the volumetric rate of evaporation to the wind speed [1][2]. For this case, the solar panel heating is not considered to evaluate the effect of convection and evaporation. The temperature, heat flux and the mass variations are as below.

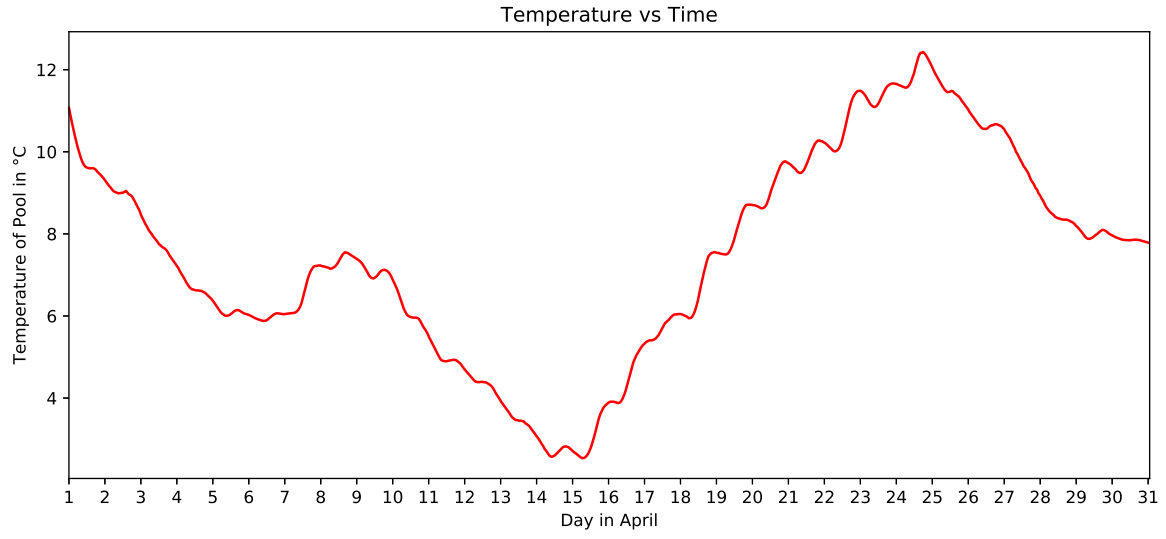


Figure 19: Temperature Variation for Case 7

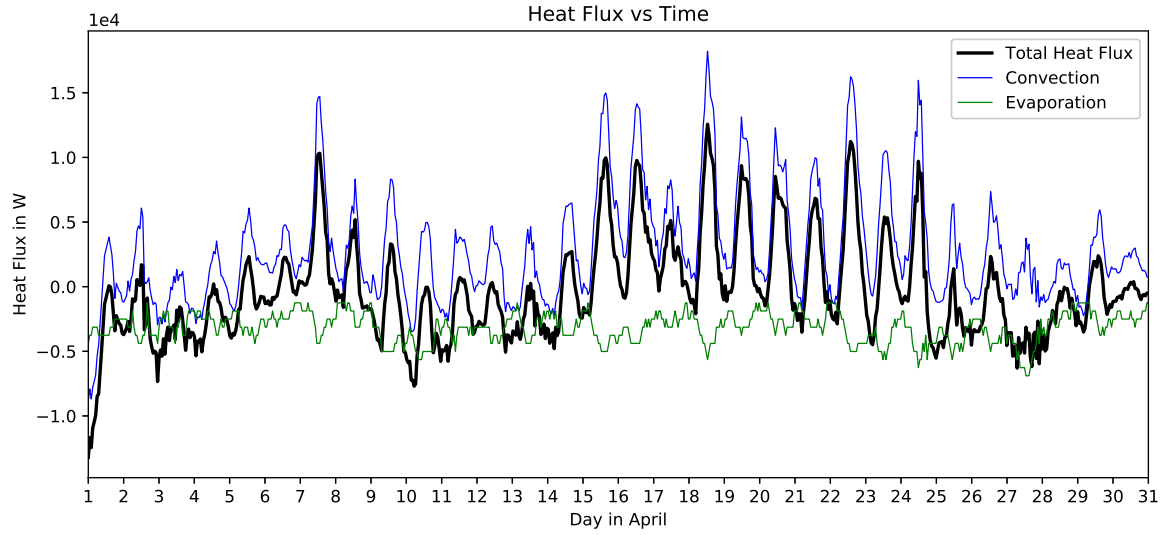


Figure 20: Heat Flux Variation for Case 7

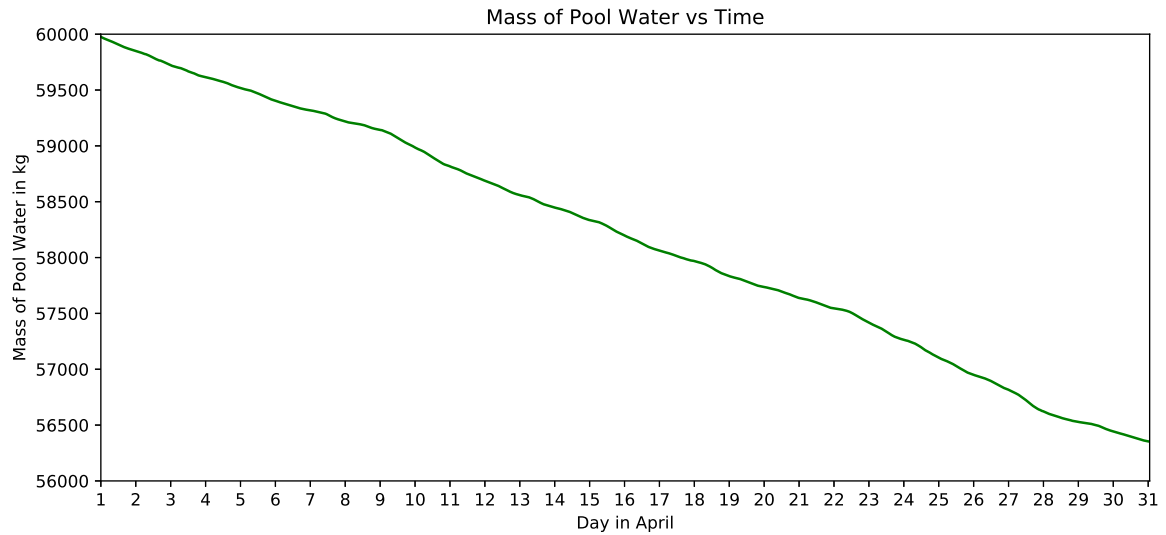


Figure 21: Mass Variation for ΔV prescribed by *Löf et al.* [1]

Case 8 When the solar panel heating is now taken into account, at an efficiency of 20%, the resulting variation in the temperature and the heat flux is as seen in the plots below. The variation in mass of the pool water is the same as seen in Figure 21.

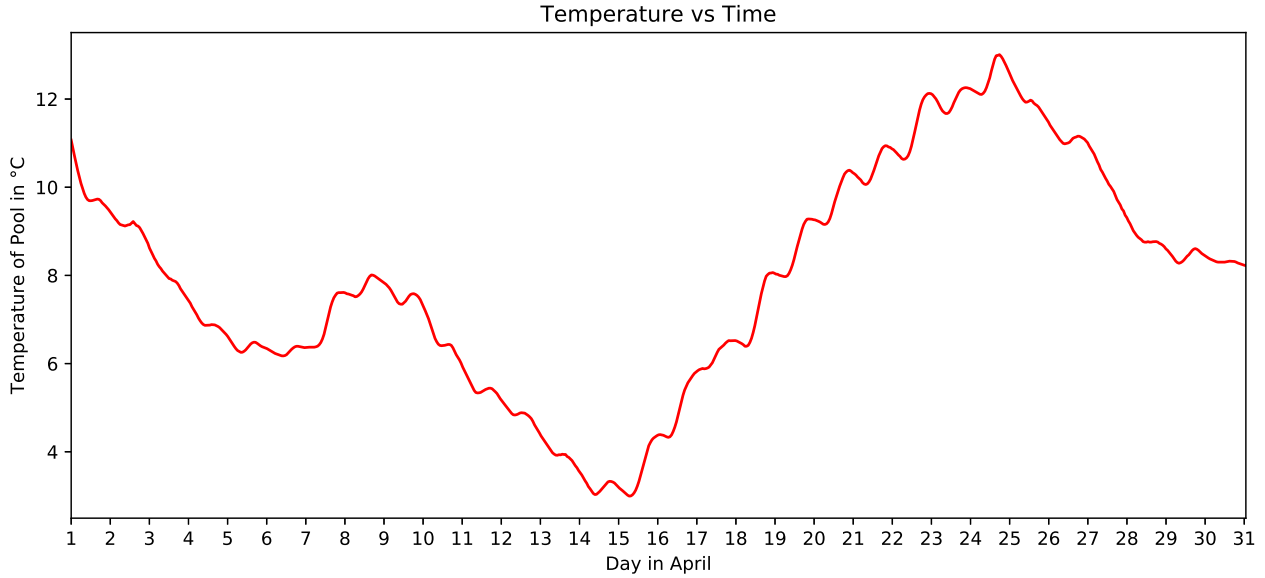


Figure 22: Temperature Variation for Case 8

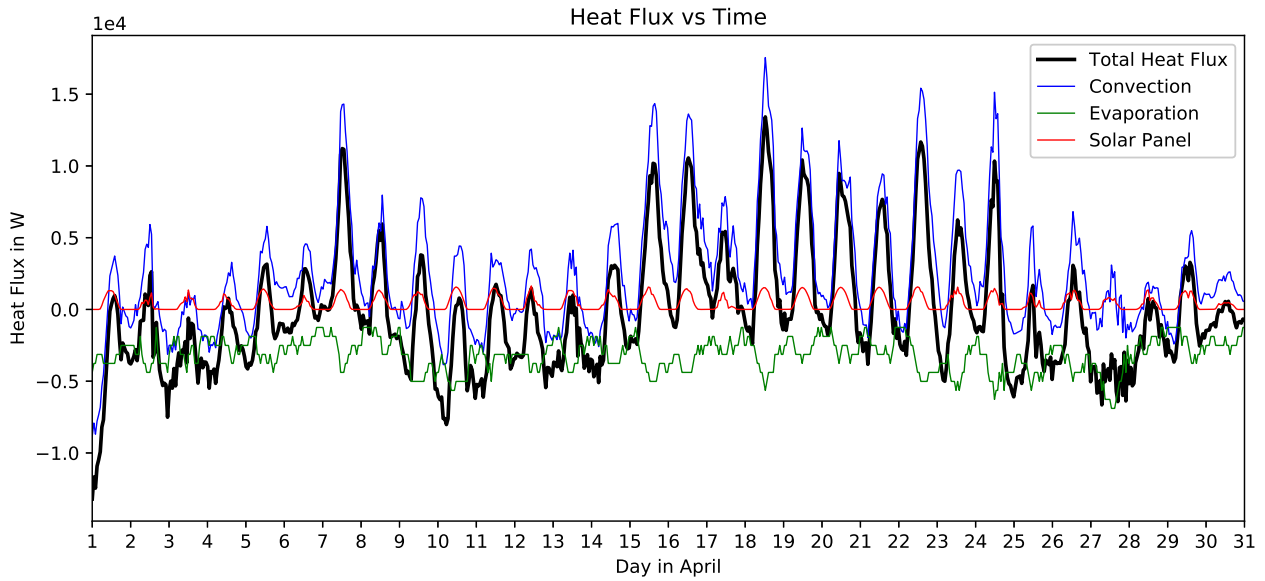


Figure 23: Heat Flux Variation for Case 8

Interpretation of Results

Effect of Solar Panel

For the suggested magnitudes of convection and evaporation, combined with the solar panel being ideal, the panel is more than adequate to heat up the pool water to the required temperature of 18°C as seen in Figure 10. Under these conditions, the control valve is essential

as it ensures that the pool water is maintained at the user-required temperature. The basic algorithm behind the working of the sensor-actuated control valve is also tested and is proven to work as seen in Figures 12, 13.

However, considering that most solar panels that are domestically used possess a maximum of 20% efficiency, an area of 10 m² might not be sufficient to heat up the pool to 18°C as seen in Cases 5,6,7 and 8. Under realistic conditions, the solar panel is dominated by the effect of convection and evaporation as seen in Figures 18 and 23.

Effect of Convection

Convection essentially acts as a term that ties the temperature of the pool water to the air temperature. Higher the convective heat transfer coefficient, the more closely coupled is the temperature of the pool water to that of air. This is clearly seen when the coefficient is increased from the suggested value in Cases 2, 3 and 4 as seen in Figures 8, 11 and 13 to more realistic values in Cases 5 and 6, seen in Figures 15 and 18.

When the heat transfer coefficient is correlated to the wind speed, convection becomes the dominant of the three modes of heat transfer. It has a higher effect on the increase in the temperature of the pool water than solar panel aided heating as seen in Figure 23.

Effect of Evaporation

Evaporation is a mode of heat transfer that always cools down the pool water and reduces the total mass. For the suggested value, the variation in mass is generally negligible as seen in Figure 9. However, the suggested value is very low when compared to realistic values and a higher value is considered. Even then, the effect of evaporation on the heat flux is still absent and is shadowed by the effect of convection and the heat flux from the solar panel as seen in Figure 16.

Correlating the rate of evaporation to the wind speed as found in literature, the effect can be noticed, as seen in Figure 21. In this case, evaporation reduces the mass considerably and also cools down the pool water.

Savings on Heating Costs

For the suggested magnitudes of convection, evaporation and for the suggested solar panel efficiency, the solar panel helps in cutting down the costs for electric heater driven heating altogether as seen in Table 1. The control valve is also essential in maintaining the temperature to the required value, thereby eliminating the need for a heater or a cooler.

However, considering most solar panels have a maximum efficiency of 20%, it can be seen that there are only minimal savings, approximately 7 EUR, on heating costs for more realistic values of convection and evaporation. This might be in part due to the dominance of convection and evaporation over the heat flux from the solar panel. This can be overcome by increasing the area of the solar panels or by building an enclosure for the pool.

Considering the correlations, it can be seen that both convection and evaporation are dependent on the wind speed. They exhibit a linear trend for the given range of wind speeds. This

essentially means that convection and evaporation will become dominant only in the presence of strong winds. If the pool was enclosed, the effect of convection and evaporation would reduce drastically, giving the control back to the heat flux from the solar panel. Hence, a simple remedy to bring about savings in heating costs even under realistic conditions is building an enclosure to prevent winds.

Conclusion

To answer the given objectives,

- In the absence of convection or evaporation and without the use of solar panel heating, the costs incurred from using the electric heater would be approximately 124 EUR.
- The temperature of the pool water is affected by the three modes of heat transfer, with the heat flux from the solar panel trying to increase the temperature, evaporation trying to reduce the temperature and convection making it equal to the ambient temperature.
- The temperature variation in time is dependent on the magnitudes of these three modes. Several cases involving different magnitudes for the three modes were studied upon. The temperature variation for each of the cases were plotted along with the heat flux variation and the variation in mass of the pool water for the cases.
- Considering a solar panel efficiency of 100%, the heating costs are drastically reduced with the expected savings being approximately 124 EUR. Usage of control valve actuated with a temperature sensor helps in maintaining the water at the required temperature.
- However, considering realistic values for the solar panel efficiency, the solar panel falls short in heating the pool with savings of approximately 7 EUR.

To further develop the modelled system,

- The effect of secondary and tertiary modes of heat transfer like conduction and radiation can be considered.
- Pipe flow losses and splashing and turbulent losses can be accounted for.
- Transient effects can be taken into consideration and the heating modes can be localized to particular surfaces or nodes in the pool.

References

- [1] George Löf Smith, Charles C. and Randy Jones. Measurement and analysis of evaporation from an inactive outdoor swimming pool. *Solar Energy*, 53(1):3–7, 1994.
- [2] Avraham. Shitzer. Wind-chill-equivalent temperatures: regarding the impact due to the variability of the environmental convective heat transfer coefficient. *International Journal of Biometeorology*, 50(4):224–232, 2006.
- [3] Ministry of Infrastructure and Water Management. Hourly weather data in the netherlands, 2019. URL <https://projects.knmi.nl/klimatologie/uurgegevens/selectie.cgi>.