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Problem 1 : Full Joint Probability Distributions

1) P (A ^ !B ^ C ^ D) = 1 – (0.002 + 0.018 + 0.013 +…. +0.080+0.070+0.060+0.090) = 1 – 0.86 **= 0.14**
Ans) Missing probability is 0.14

2) P (!B ^ !C ^ D) = 0.019 + 0.018 **= 0.037**

3) P (A = true or B = false) = P(A) + P(!B) – P(A ^ !B)
= 0.64 + 0.5 – 0.34 **= 0.8**

4) P (A= true | B=false and C =false and D=true)
P (B = false ^ C =false ^ D=true) [from full joint] **= 0.037**
From probability laws : P(A= true | B=false and C =false and D=true) = P(A= true and B=false and C =false and D=true) / P(B = false ^ C =false ^ D=true)
= 0.019 / 0.037
= 0.5135

5) P(B = false and C =false and D = true | A = true) =
P(A= true | B=false and C =false and D=true) * P(B = false and C =false and D = true) / P(A= true)
- from Bayes Law
= (0.5135 * 0.037) / 0.64 = **0.029686**

Problem 2 : Bayesian Networks

Note : We can also write some of them as partial world states, but for understanding and clarity, I have included the complete world states and showed work wherever possible.

1) P(A= true and B=true and C=true and D=true)
= P(A) * P(B | A) * P(C | A ^ B) * P(D | B ^ C)
= 0.7 * 0.8 * 0.6 * 0.2
= 0.0672

2) P(B=false and C=false and D=false)
= P(A) * P(!B | A) * P(!C | A ^ !B) * P(!D | !B ^ !C) + P(!A) * P(!B | !A) * P(!C | !A ^ !B) * P(!D | !B ^ !C)
= (0.7 * 0.2 * 0.5 * 0.1) + (0.3 * 0.9 * 0.7 * 0.1)
= **0.0259**

3) P(A=true | B=true and C=true and D=false) =
P(A= true and B=true and C=true and D=false) / P(B=true and C=true and D=false)
Numerator : P(A) * P(B | A) * P(C | A ^ B)* P(!D | B^C) = 0.7 * 0.8 * 0.6 * 0.8 = 0.2688
Denominator: P(!A) * P(B | !A) * P(C | !A ^ B) * P(!D | B ^ C) + P(A) * P(B | A) * P(C | A ^ B) * P(!D | B ^ C) = (0.3 * 0.1 * 0.4 * 0.8) + (0.7 * 0.8 * 0.6 * 0.8) = 0.2784
= 0.2688 / 0.2784
= **0.965517**

4) P(!D | A ^ B ^ C) = P(!D ^ A ^ B ^ C) / P(A ^ B ^ C)

Numerator: P(A) * P(B | A) * P(C | A^B) * P(!D | B ^ C) = 0.7 * 0.8 * 0.6 * 0.8 = **0.2688**
Denom: P(A) * P(B |A) * P(C | A ^ B) * P(D | B ^ C) + P(A) * P(B |A) * P(C | A ^ B) * P(!D | B ^ C)
= 0.7 * 0.8 * 0.6 * 0.2 + 0.7 * 0.8 * 0.6 * 0.8 = **0.336**
= 0.2688/0.336 = **0.8**
Additionally, we need to show numerically that A=true doesn't really matter :
Evaluating P(!D | B ^ C)
= P(!D ^ B ^ C) / P(B^ C)
= P(A ^ B ^ C ^ !D) + P(!A ^ B ^ C ^ !D) / P(A^B^C) + P(!A^B^C)
= 0.8

5) P(A ^ D or B ^ C) = P(A ^ D) + P(B^ C) – P(A ^ B ^ C ^ D)
P(A ^ D) = P(A * B * C * D) + P (A * !B * C * D) + P(A * !B * !C * D) + P(A * B * !C * D)
= 0.0672 + [0.7 * 0.2 * 0.5 * 0.3] + [0.7 * 0.2 * 0.5 * 0.9] + [0.7 * 0.8 * 0.4 * 0.4]
= 0.0672 + 0.021 + 0.063 + 0.0896 **= 0.2408**
P(B ^ C) = P(A * B * C * D) + P (!A * B * C * D) + P(!A * B * C * !D) + P(A * B * C * !D)
= 0.0672 + [0.3 * 0.1 * 0.4 * 0.2] + [0.3* 0.1 *0.4 * 0.8] + [0.7 * 0.8 * 0.6 * 0.8]
= 0.0672 + 0.012 + 0.2688 = **0.348**
P(A ^ D or B ^ C) = 0.2408 + 0.348 – 0.0672 = **0.5216**

Problem 3: BNs and Full Join Probability Table (From Bayes Net Problem 2)

A	B	C	D	Prob
F	F	F	F	[redacted]
F	F	F	T	[redacted]
F	F	T	F	[redacted]
F	F	T	T	[redacted]
F	T	F	F	[redacted]
F	T	F	T	[redacted]
F	T	T	F	[redacted]
F	T	T	T	[redacted]
T	F	F	F	[redacted]
T	F	F	T	[redacted]
T	F	T	F	[redacted]
T	F	T	T	[redacted]
T	**T**	**F**	**F**	**0.1344**
T	**T**	**F**	**T**	**0.0896**
T	**T**	**T**	**F**	**0.2688**
T	**T**	**T**	**T**	**0.0672**

- P (A ^ B ^ !C ^ !D) = P(A) * P(B | A) * P(!C | A ^B) *
  P(!D | B^!C) = 0.7 * 0.8 * 0.4 * 0.6 = **0.1344**
- P (A ^ B ^ !C ^ D) = P(A) * P(B | A) * P(!C | A ^B) * P(D | B^!C) = 0.7 * 0.8 * 0.4 * 0.4 = **0.0896**
- P(A ^ B ^ C ^ !D) = **0.2688** (Problem 2 part 3)
- P(A ^ B ^ C ^ D) = 0.0672 (From problem 2 part 1)
  = P(A) * P(B | A) * P(C | A ^ B) * P(D | B ^ C)
  = 0.7 * 0.8 * 0.6 * 0.2
  = 0.0672

Problem 4: Bayes' Rule

We are given P(shot), P(flu) and P(flu | shot) in the problem, and we use the Bayes' Rule in order to derive for the asked conditional probabilities. I calculated some intermediate conditional probabilities

i) P (flu | shot=false) = P(shot=false | flu) * P(flu) / P (shot=false)
Firstly lets calculate:
P(flu | shot) = P(shot|flu) * P(flu) / P(shot) => **P(shot|flu) = 0.35**
P(flu|shot=false) = [(1-0.35) * 0.2] / 0.3 = **0.43333**

ii) P(shot | flu=false) = P(flu=false | shot) * P(shot) / P(flu=false)
We know that P(flu|shot) = 0.1 from previous problem derivation
P(shot | flu=false) = [(1-0.1) * 0.7] / 0.8 = **0.7875**

Part III

P(DE) = 57/1000000
P(T4 | DE) = 0.95
P(T4 | !DE) = 0.05

to compute :P(DE | T4)
P(T4) = P(T4 ^ DE) + P(T4 ^ !DE)
= P(T4 | DE) * P(DE) + P(T4 | !DE) * P(!DE)
= 0.95 * 57/1000000 + 0.05 * (1-57/1000000) = 0.05005
from Bayes' rule : P(T4 | DE) = P(DE | T4) * P(T4) / P(DE)
P(DE|D4) = 1.08e-3
