

Linear Programming Notes - Summaries and Concepts

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1 Linear Programming

1.1 Linear Programs (LP)

A linear program (LP) is an optimization problem with objective linear and linear constraint functions. Any linear program can be expressed in the form

$$\begin{aligned} p^* = \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax \preceq b, \\ & Dx = f, \end{aligned} \tag{1}$$

Above is in matrix form, where A and D are matrices , b and f are vectors. LPs are subset of convex programs used in several applications - Combintarics, Graph theory etc.

Geometrical interpretation - intersection of half spaces / polyhedron.

$$(g, h) \doteq \{x : g^T x \leq h\},$$

Definitions:

- Feasible - Point is feasible , if it satisfies all constraints
- Optimal - its optimal if value objective is minimal
- Infeasible LP - Point doesn't satisfy given linear constraints
- Unbounded - Objective tends to go to negative infinity

For LP's optima always occur at extreme points.

Extreme point Definition 1 : It should not be convex combination of any other two points in polytope.

$$\exists x \in z, \exists y, z \in z, \lambda \in [0, 1], y(\lambda) + z(1 - \lambda) \neq x$$

Extreme point Definition 2 : It should be unique vector of C

$$C^T x < C^T y$$

Extreme point Definition 3 : X is extreme point if its Basic feasible solution. (X is Basic feasible solution if its active set A has n linearly independent vectors)