Computational Economics **18:** 49–64, 2001.

Bilateral Trade and 'Small-World' Networks

ALLEN WILHITE
Department of Economics and Finance, University of Alabama in Huntsville, Huntsville, AL 35899, U.S.A.; e-mail: wilhitea@email.uah.edu

Abstract. Trade requires search, negotiation, and exchange, which are activities that absorb resources. This paper investigates how different trade networks attend to these activities. An artificial market is constructed in which autonomous agents endowed with a stock of goods seek out partners, negotiate a price, and then trade with the agent offering the best deal. Different trade networks are imposed on the system by restricting the set of individuals with whom an agent can communicate. We then compare the path to the eventual equilibrium as well as the equilibrium characteristics of each trade network to see how each system dealt with the tasks of search, negotiation, and exchange.

Initially, all agents are free to trade with any individual in the global market. In such a world, global resources are optimally allocated with few trades, but only after a tremendous amount of search and negotiation. If trade is restricted within disjoint local boundaries, search is simple but global efficiency elusive. However, a hybrid model in which most agents trade locally but a few agents trade globally results in an economy that quickly reaches a Pareto optimal equilibrium with significantly lower search and negotiation costs. Such 'small-world' networks occur in nature and may help explain the ease with which most of us acquire goods from around the world. We also show that there are private incentives for such a system to arise.

Key words: small-world networks, trade networks, agent-based computational economics, artificial economics

On a daily basis markets reconcile the self-interests of many individuals with system-wide resource constraints. Most of us have it easy. We need only shop at the corner grocery or the downtown mall to acquire goods from around the world. This paper explores this remarkable feat of global coordination by studying the dynamic behavior and equilibrium characteristics of trade under different types of exchange rules. Of particular interest are the micro-level and aggregate consequences of restricting trade to local neighborhoods. To capture the dynamics of trade and to compare those dynamics across different trade structures, we use an agent-based computational approach. Herein we observe an artificial world populated with autonomous agents who try to improve their situation by seeking out trading opportunities and exchanging.

Agent-based computational economic models are increasingly used to study dynamics (Tesfatsion, 1997) and questions involving pure exchange economies are well suited to this approach. For example, Albin and Foley (1992) model a world in which traders broadcast their interest in trade, potential partners respond, and agents trade if a mutually beneficial price can be negotiated. They find that this decentralized structure works quite well. That is, it reaches an equilibrium that possesses an across-the-board improvement in the agents' utility, although the final distribution of wealth and the presence of some local price dispersion distinguish it from a pure Walrasian world.

Epstein and Axtell (1996) construct a simulation that incorporates production and exchange. In their world, mobile individual agents collect (harvest) two different goods needed for their survival. And, if agents meet one another they can trade. Tracking the activities of these agents and comparing their activities to the aggregate stocks of both goods, Epstein and Axtell study the aggregate efficiency of this local trade. They conclude that these local markets differ in significant ways from what we might expect given neoclassical theory. For example, while the average price in their local market was usually close to the price that equated aggregate supply and aggregate demand, it did not remain at that price over time. Local price differences were persistent. Perhaps most importantly, the total quantity traded in these local markets was much less than the aggregate, market clearing quantity. Thus, in each period, some gains to trade went unrealized.

Bell (1998) investigates a variety of trading structures focusing on the speed of convergence to an equilibrium price as well as the persistence of neighborhood effects. In general she found that more centralized networks converge with fewer trades and have less residual price variation than more diverse networks. Relying solely on the number of trades as a measure of the speed of convergence, however, misses the cost imposed by search and negotiation. One of the results presented in this paper is that some trade networks may increase the numbers of times agents have to trade, but may also decrease the aggregate amount of search and negotiation costs in a system.

Axtell (1998) explores the relative complexity of bilateral exchange versus a Walrasian world. Viewing a market as a social computer, he compares the computational burden, measured by the number of calculations needed to find an equilibrium price, of each system. As the number of agents and commodities increases, he finds that the bilateral exchange process is less costly than a Walrasian system. Still, under certain conditions, the equilibria are Pareto optimal although they may not be in the core.

This study continues this line of research by investigating the performance of trade networks. In particular, we study the effects of local trade groups embedded in a larger population. Unlike Epstein and Axtell (1996), however, these local trade groups overlap to form a network that eventually connects the entire population. Thus, all trade is local but through repeated exchange goods can reach the entire population as they pass from one locality to another.

Of particular interest is the application of 'small-world networks' to a market setting. Watts and Strogatz (1998) define the small-world networks as highly clustered systems with small path lengths. Exploring some mathematical properties of such networks, they find that dynamical systems with small-world coupling display enhanced signal-propagation speed and computational power. We integrate these properties into a market. The result is an artificial world that has features commonly observed in everyday markets, such as the global distribution of resources even though most traders operate locally. We investigate the price dynamics and trading efficiency of these systems. In general, we find that even if economies have structural rigidities that interfere with the free flow of information and goods, markets can be organized in such a way to achieve efficient resource allocation. We also show that there are incentives for such networks to arise.

1. A Model Economy

We build a simple exchange economy with two goods.[1] These are durable goods in the sense that they survive and suffer no real depreciation (no degradation) during the experiment. There is no production and are no imports, thus the aggregate stock of goods at the beginning of the experiment is the stock at the end. These goods have value as an intrinsic source of pleasure and as a durable asset to be used in exchange. Examples would be toys (sturdy toys), dinnerware, collectibles such as baseball cards or works of art, and precious stones. One of the goods, g_2, is infinitely divisible but the other, g_1, must be traded in whole units.[2]

The simulations discussed in this paper come from a market containing 500 independent agents. All agents have the same symmetric Cobb–Douglas utility function they attempt to maximize. Agents are rational, non-strategic, and myopic in that they do not attempt to mislead potential trade partners, nor do they plan for future opportunities. They simply try to improve their current position in each period by engaging in voluntary trade. At the time of an exchange, agents are constrained by their existing wealth, which consists of their current stock of goods 1 and 2. Finally, each agent is initially endowed with a randomly determined amount of each good.[3] In this pure exchange economy these endowments define the entire resource base of society.

Formally, the utility of agent i, U^i, depends on the amount of the two goods, g_1 and g_2 he possesses.

$$U^i = g_1^i g_2^i, \quad i \in \{1, \ldots, 500\}. \tag{1}$$

Agents trade as long as the incremental exchange increases U^i and if they can afford the exchange. In this barter economy the income constraint in each period t depends on their stock of goods and the existing price. It can be written as

$$p_{i,j}(t) \cdot g_1^i(t) + g_2^i(t) = p_{i,j}(t) \cdot g_1^i(t-1) + g_2^i(t-1), \tag{2}$$

where $p_{i,j}(t)$ is the price of good g_1 between agents i and j in time period t.

An opportunity for mutually beneficial exchange exists if the marginal rates of substitution of two agents differ. With the utility function in (1), the mrs of agent i is

$$\mathrm{mrs}^i = \frac{U'(g_1^i)}{U'(g_2^i)} = \frac{g_2^i}{g_1^i}, \quad i \in \{1, \ldots, 500\}, \tag{3}$$

where $U'(\cdot)$ is the first derivative of U.

As a baseline measure, agents are allowed to trade with any other agent in the economy, an example we call the Global Network. Subsequent simulations place restrictions on the potential trade networks by limiting each trader to a subset of the population with whom he or she may bargain.

Regardless of the restrictions placed on the agents, trade proceeds sequentially. Traders must search for a partner, establish a price, initiate trade, and determine when to stop. The first agent to trade begins by calculating his marginal rate of substitution, as shown in (3), which reflects the amount of good 2 this agent would be willing to give up in exchange for another unit of good 1. The agent then searches for beneficial trade opportunities with other agents in the economy. We assume all agents truthfully reveal their reservation prices for the goods; that is, each agent reveals his or her mrs. Agent i then ranks all other agents according to their willingness to trade and selects the agent offering the best price. Notice, any agent can either buy good 1 (trade g_2 for g_1) or sell good 1 (trade g_1 for g_2). Indeed, in successive rounds a particular agent may buy and later sell the same good.

Once potential trading partners are selected, a price is negotiated. Throughout these experiments the trading price between agent i and agent j, $p_{i,j}$, was set according to the following rule.[4]

$$p_{i,j} = \frac{g_2^i + g_2^j}{g_1^i + g_1^j} \quad i, j \in \{1, \ldots, 500\}. \tag{4}$$

After the exchange price has been established, the two agents begin to trade (in increments of one unit of good 1) and continue as long as trade benefits both. As soon as the marginal exchange reduces the utility of either agent, trade stops. At that time, a second agent is selected and the procedure is repeated. The selection of the initial trading agent is made without replacement, that is, once agent i is selected to be the initiating agent (the one who engages in search) he is not picked as the searching agent until all 500 agents have had the opportunity to search and trade. However, an agent can select any partner. So, if agent i is selected as the first trader, he finds a partner and trades. A different agent, agent j, is then selected to initiate trade, but if agent i is offering the best price to j, then j will trade with i. Thus, while agent i is selected as one who initiates search and trade only once per round, he may participate in trade numerous times if selected by other traders during the round. These rules insure that all agents will have the opportunity to trade at least one time in each round, although they may choose to abstain. It is also possible for an agent to pass in one round (decline to trade) but then engage in trade at a later time if prices have changed to make trade sufficiently enticing.

Equilibrium is defined as a point of rest, when agents cannot find trading opportunities that benefit both individuals. Feldman (1973) studied the equilibrium characteristics of welfare-improving bilateral trade and showed that as long as all agents possess some non-zero amount of one of the commodities (all agents have some g_1 or all agents have some g_2) then the pairwise optimal allocation is also a Pareto optimal allocation. In this experiment, all agents are initially endowed with a positive amount of both goods, thus equilibria are Pareto optimal.

2. Experimental Design

This model economy is simulated in a C++ program and with repeated simulations, or experiments, we explore the effects of different trading rules on the efficiency of the system. Artificial autonomous agents carry out trade at a micro level, that is, agents trade directly with other agents. It is neither a Walrasian world nor a *tatonnement* process. Agents search for trade partners under a set of rules and their behavior defines a trade network for the economy. Our purpose is to study the performance of such networks. Much of the emphasis on networks in the field of economics is concerned with network formation, such as in Jackson and Wolinsky (1996) and Kirman (1986, 1983). Similarly, McFadzean and Tesfatsion (1999) and Ioannides (1997) construct a model with autonomous agents in which trade networks evolve as agents learn.

In this study, we do not explore network formation. Instead, a series of trade networks are imposed on a population of artificial agents. We let each society trade and then compare the dynamic behavior and equilibrium characteristics of those markets. Of particular interest are the changes that arise when a network becomes increasingly decentralized and how efficiently decentralized markets equate aggregate supply and demand. With an animated economy of autonomous agents, we can replicate the initial conditions for each set of simulations and observe the same agents under different circumstances. Using fifty different populations, we observe each population’s trading behavior under different trading regimes. Each regime starts the simulation with agents who possess the same attributes, preferences, wealth, and objectives, as the agents in the other trade regimes. The only change is the set of potential trading partners available for search. After observing one population’s behavior in four different trade environments, a second population of agents is generated and the behavior of this new population is observed in each environment.

The four trade networks explored are: a Global Network, a Local Disconnected Network, a Local Connected Network, and a Small-world. It is useful to envision our economy as existing on a ring lattice with the agents spread around the rim of a circle as shown in Figure 1. The Global Network allows every agent to trade

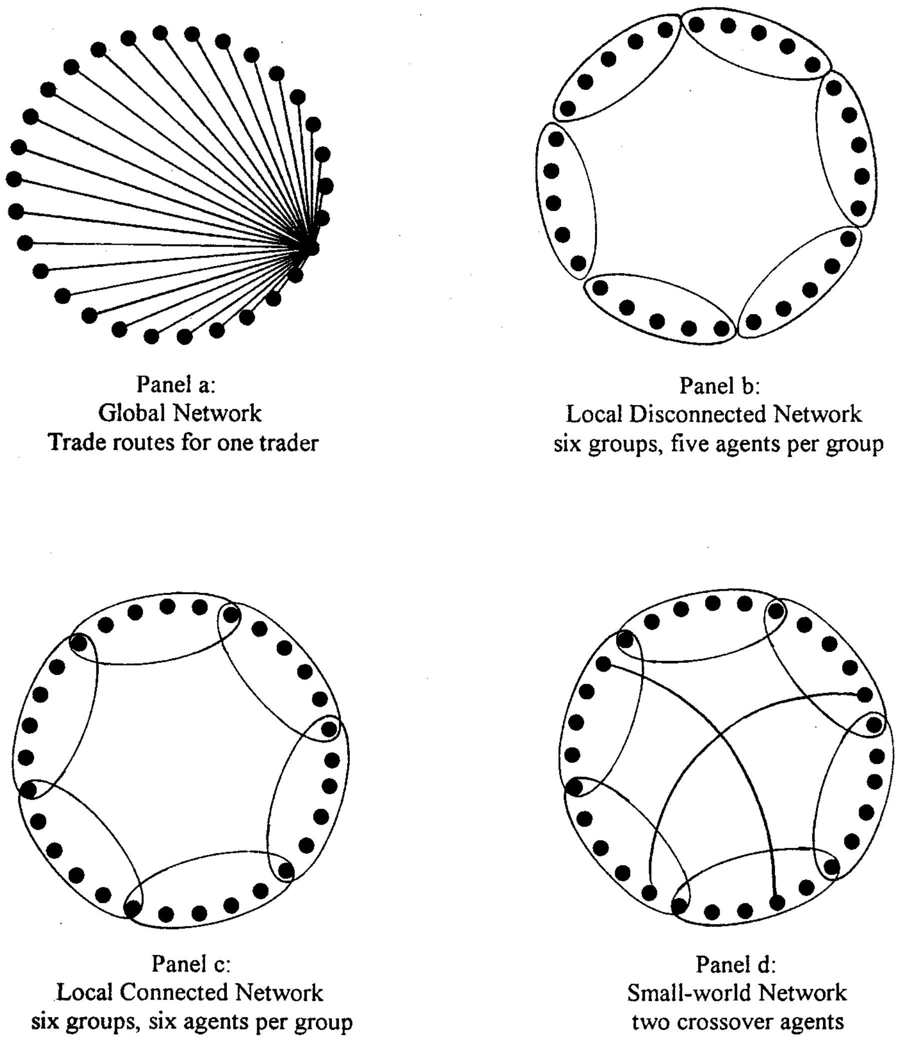

Figure 1. Sketches of four networks (population: 30 agents).

with any other agent in the system. Panel a of Figure 1 shows the potential trading partners of one agent in this global system.

The second regime is a Local Disconnected Network, which places each agent into a distinct group. There are no overlapping group members. More precisely, each agent is assigned to a subset, M_k (containing m members) of the population N (containing n members) such that $m < n$. In the Local Disconnected Network $M_k \cap M_l = \varnothing$ so there is no exchange between trade groups. Panel b of Figure 1 gives a visual image of such a disconnected network.

The other trade networks possess global and local attributes. For example, the Local Connected Network differs from the Local Disconnected Network in that

$M_k \cap M_l \neq \emptyset$. These trade groups overlap. When aligned on a ring, each group shares one trader with each neighboring group. All trade is still local, in the sense that it is carried out with a subset of the population (your neighbors) but through repeated trade, goods can be distributed around the entire ring. Panel c shows an example of a local connected network consisting of six trade groups with six members in each group. Our experiments use 50 groups with 11 members each.

The fourth model studies a Small-world Network. It is constructed by taking the local, connected market of the previous example and adding a few additional trade routes that connect one local market to another distant, local market. These trade routes are established by randomly selecting a trader $m_i \in M_k$ and making him a member of another trade group, M_l, as well. In this experiment, these random connections are subject to two restrictions: (i) the selected trader is not one of the traders that already links neighboring trade groups and (ii) this new link cannot connect two groups that already have a common trader. An illustration of such a network appears in Panel d of Figure 1.

To initialize these trade networks in the C++ program, four parameters are set by the user. They are (i) the number of trade groups, (ii) the number of agents within each group, (iii) whether or not the trade groups overlap and (iv) the number of crossover agents in the system. The following settings were used to establish the trade networks studied in this paper.

Global network:

(i)	number of groups:	1
(ii)	agents in each group:	500
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

Local disconnected network

(i)	number of groups:	50
(ii)	agents in each group:	10
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

Local connected network:

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	0

Small-world network

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	5

3. Experimental Results

While the individual economic agents pursue the same objective function in each regime, the restrictions placed on the agents ability to trade are expected to affect the system's performance. This study is particularly interested in how different trade rules affect the following issues.

Table I. Average equilibrium characteristics. [a]

	Prices (standard deviation)	Rounds	Total trades	Total searches
Global network	1.0046 (0.00168)	8.08	1953.38	2,015,960
Local disconnected network	1.0396 (0.2771)	7.02	1727.7	31,590
Local connected network	1.0048 (0.0146)	497.14	93,975.72	2,734,270
Small-world network	1.0045 (0.00724)	242.54	45,944.56	1,236,954

[a] Averages calculated from 50 simulations of each network configuration.

H1: Price convergence: Is there a significant difference in the dispersion of prices across each trade network?

H2: Speed of convergence: Do the different trade networks require a significantly different number of rounds of trading to reach their steady state?

H3: Number of trades: Is there a difference in the number of trades it takes for each network to reach its steady state?

H4: Search: Is there a difference in the amount of search and negotiation in each trade network?

Table I presents the average equilibrium characteristics of 50 different populations. Clearly, each regime is unique. In the Global Network every agent can negotiate and trade with any other agent in the population. Thus, the most advantageous trades occur first and a global equilibrium is reached quickly. On average, there was little variation around the equilibrium price in the global networks. Specifically, the average standard deviation was 0.00168 for an equilibrium price of 1.0046. This price was reached quickly, in 8.08 rounds of trading, on average, and required about 1953 trades.[5] This speed of convergence, however, comes at a cost. In this open network each trade follows an extensive search involving all 500 agents, that is, every agent negotiates a unique price with every other agent in the economy. This search and bargaining occurs even if the agent does not trade because he must discover whether gains to trade exist, or not. Thus, while the populations studied here found a market clearing price after a mere 8.08 rounds of trading, that entailed $500 \times 499 \times 8.08$ or more than 2 million searches and rounds of price negotiation.

In the second system, the Local Disconnected Network, equilibrium is reached quickly, after only 1727 trades, but prices vary significantly from trader to trader. On average the standard deviation of equilibrium prices is 0.2771 which is more than 150 times as large as the price deviation in the Global Network. The number of searches falls drastically as each agent need confer with only nine other agents, the rest of his or her group, before trading. And, once a local market reaches its equilibrium, those agents do not need to search in subsequent periods even if other local markets are trading. It seems that rules suppressing trade succeed in suppressing trade.[6] Each isolated local market converges on its own price so the equilibrium price vector contains a wide range of prices. Clearly, there are opportunities for mutually beneficial exchange between groups that go wanting.

While these first two artificial societies behaved as intuition would suggest, the global network collapsed on a uniform price and convergence was absent in the disconnected network, it is perhaps surprising that these equilibrium price vectors emerged so rapidly when all trade was one on one. The dynamics of this price activity can be displayed on a case by case basis and a representative example appears Figure 2.[7] Panel a of Figure 2 shows price convergence in the Global Network and after only three rounds of negotiation much of the price variation is gone and the steady state price is reached after eight rounds. Panel b shows the price dynamics in the Disconnected Network and while each subgroup independently arrives at its own local equilibrium, the groups do not converge on a single, global price.

Our central interest lies in economies between these extremes, markets that have local and global attributes. In the Local Connected Network, for example, all trade is local, carried out with members belonging to your trade group (which is a fraction of the total population), but goods can flow around the entire economy through a series of successive trades.

To initiate trade in this model, an agent is selected from one group and he trades as before although his search excludes individuals outside his group. After trading, another agent is selected from a different group. This selection procedure continues until every group has had one agent initiate trade. Then a second agent is selected from the first group, and so on. Since some goods must be shipped around a significant portion of the circle, more exchanges are required to reach equilibrium. In general, this network takes much longer to reach equilibrium, 497 rounds of trade with 94,000 separate exchanges, on average. Prices are much more uniform than in the disconnected population but not as uniform as the global model. Finally, Table I also shows that, on average, the total number of searches needed to reach equilibrium is higher in the Local Connected Network. Restricting the number of individuals available for trade creates two offsetting forces acting on the total number of searches. On the one hand it reduces the number of searches occurring in each round of trade, but on the other hand it raises the number of rounds required to reach equilibrium. In our simulations, the latter effect outweighed the former

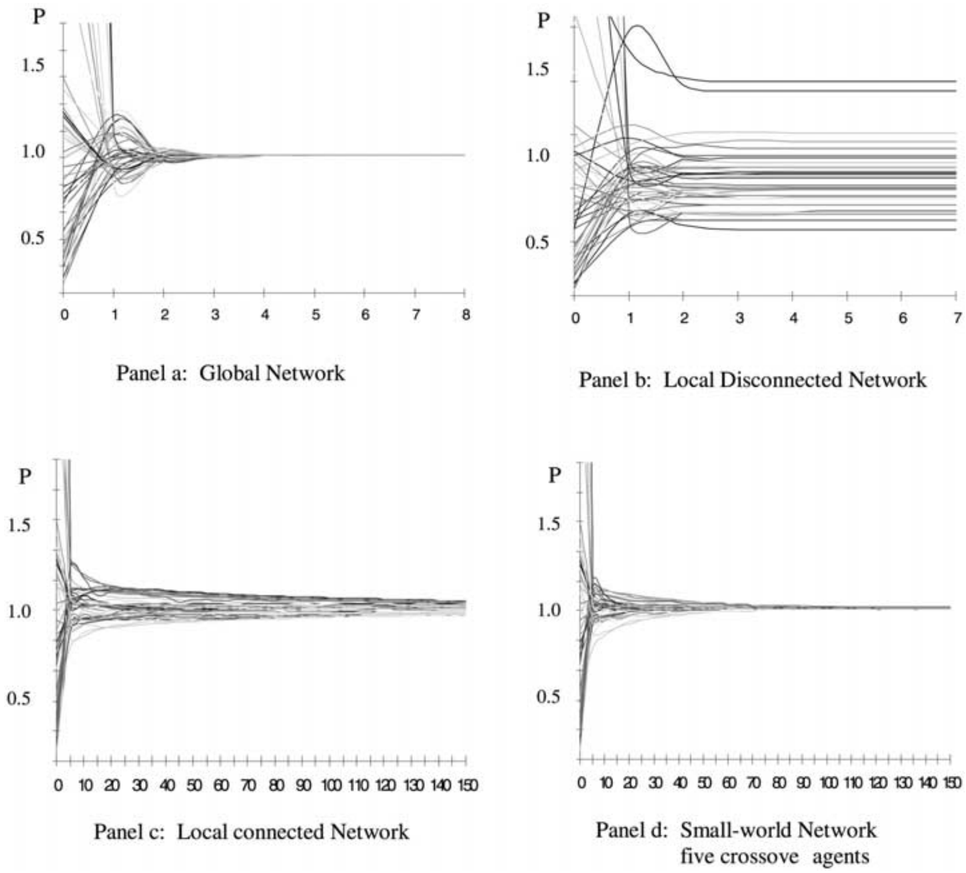

Figure 2. Price convergence in four networks.

and the total number of searches rose from about 2 million in the global network to 2.7 million in the local connected network.

While this local network eventually reaches a global equilibrium price, it takes almost fifty times as many exchanges, and 700,000 more searches than the global network. Search and exchange are free in this simulation, but they are not in markets and the dramatic rise in search, negotiation, and exchange implies a considerable increase in the cost of reaching equilibrium. Consequently, the restriction of trade to local partners makes it more expensive to clear markets. At some point, this expense would reduce the volume of trade and society would lose some of the welfare gains of voluntary exchange.

The fourth model is a Small-world Network with five crossover agents. Again, all trade is local in the sense that individuals trade exclusively members of their sub-group, but with successive trades, goods can be distributed globally. The addition of just a few cross-traders has a remarkable impact on the performance of the system. The last row of Table I shows the equilibrium results for Small-world networks with five crossover agents out of a total population of 500. Equilibrium is reached in 242 rounds, on average, which is far more than the global system that converged in 8.08 rounds but far less than the local connected world that took 497 rounds of trade. The impact on search costs was even more dramatic. While the global model reached equilibrium in 8.08 rounds of trade, each round involved about 250,000 searches. In the small-world example, each trader searches a fragment of the entire population, thus the number of searches needed to reach the global equilibrium price is reduced by 40% compared to the global model and is less than one-half of the searches required by local connected network. Furthermore, the standard deviation of this equilibrium price is smaller than any of the other configurations, save the global network. Indeed, on average the standard deviation of the small-world network is half the local connected network and only a fraction (about 3/100) of the disconnected network even though initial endowments and utility functions of the agents are identical.

Panels c and d of Figure 2 show examples of the dynamics of price convergence in these last two models. After 150 rounds of trading, the local connected trade network still had prices that ranged from around 0.9 to 1.1 but the small-world has almost reached its equilibrium. Small-world trade networks appear to incorporate the benefits of local trade (reduced search and negotiation costs) while assimilating global resource allocations fairly rapidly. Because of this ability to economize search and exchange costs, we look more closely at the attributes of such networks.

4. Small-World Networks

Watts and Strogatz (1998) show that the neural network of *Caenorhabditis elegans* (a worm), the power grid of the western United States, and the collaboration of film actors in motion pictures are small-world networks. In each case information, or power, or acquaintance, can be spread quickly over the entire network even though each node is connected to a relatively small number of partners. We see these characteristics, local exchange and global reach, in markets. Small-world networks balance the costs of these characteristics.

Two attributes of small-world networks are of particular importance: path length, p, and group size, z. Path length is defined as the minimum number of exchanges required for an agent to trade with every other agent in the population, averaged over all agents. Path length grows as a population is divided into additional groups. At one extreme, the global network explored in Figure 1a above has a path length equal to one as any agent can directly trade with any other agent. At the other extreme, the disconnected local network, Figure 1b, has an undefined or infinitely long path. Agents in a disconnected group cannot send goods to or receive goods from another group. Between lie the networks on which we wish to concentrate. Consider a population divided into two groups with some portion of the members belonging to both groups. Let A be a set of agents belonging to one group, B the set of agents belonging to the other and AB the set of agents

Table II. Impact of crossover agents on path length and group size.

Numbers of crossovers	Path length[a]	Group size
0	12.50	11
1	10.45	11.04
2	8.88	11.09
3	7.48	11.13
4	6.86	11.18

[a] Averges calculated from 50 populations.

belonging to both, $AB = A \cap B$. Agents who are members of *AB* can trade with any other agent in a single step ($p = 1$). Agents who belong exclusively to A $\{i \in A;\ i \notin AB\}$ or exclusively to B $\{i \in B;\ i \notin AB\}$ can trade with others in their own set in a single step ($p = 1$). To trade with members outside their group they must make an intermediate trade with an agent in *AB*, and for those agents $p = 2$. Thus, a partitioning of the population increases path length and consequently increases the number of trades necessary to reach equilibrium.[8]

Group size, z, affects the amount of search and negotiation that takes place before an exchange. Agents search and negotiate with all members of their group before selecting a trading partner; thus, agents in larger groups incur greater costs. For the group as a whole, however, the increase is not linear, as the number of searches in a group of size m is $m^2 - m$. All else equal, as a population is divided into smaller groups, search and negotiation costs fall.

Consequently, as a population is divided into more trade groups, the number of exchanges needed to reach a global equilibrium increases, but the amount of search and price negotiation falls. Importantly, the tradeoff is unequal and small-world networks economize by cutting path length more rapidly than it increases group size. Table II presents the average path length and group size parameters for the fifty populations used in this study. Notice that while each crossover agent dramatically reduces path length, practically cutting the networks total length in half with just four crossover agents, the impact on group size is barely noticeable.

For an intuitive example imagine a single crossover agent who constructs a trade route across the middle of the ring lattice (say from 3 to 9 on a clock face). This bridge adds a single agent to the connected groups and leaves all other groups untouched, thus, average group size is virtually undisturbed. However, this new trade route has the potential of shortening the path length for almost every agent in the network. For example, the agent at the 2 o'clock position can exchange goods with the agent at 9 o'clock almost as quickly as he can trade with the agent at 3 o'clock. Before the bridge he had to pass goods through a series of groups to move around the rim of the lattice.

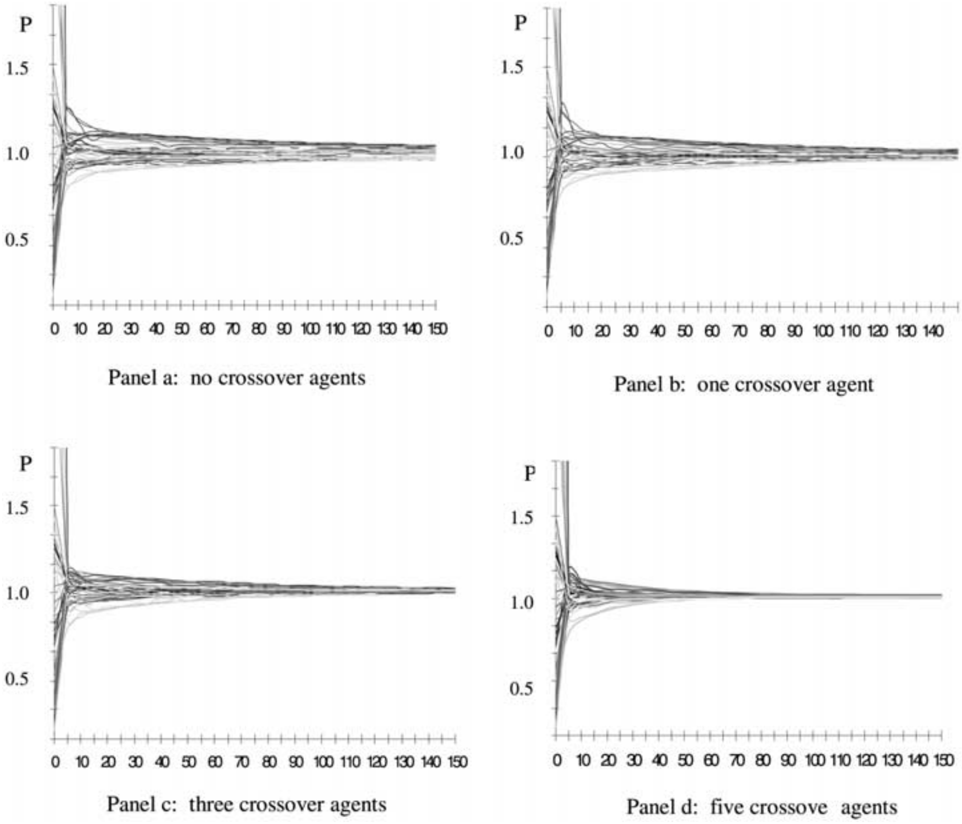

Figure 3. Dynamic price adjustment: adding crossover agents.

The effects on trade are dramatic. Figure 3 shows the dynamics of the price convergence for our one of our test populations. We start with 50 groups each containing 11 agents such that one agent in each group is included in the next group. Each panel in Figure 3 shows the path of fifty prices during the first 150 rounds of trading. We start with no crossover agents (Panel a) and see that after the 150 rounds (that is after all 500 agents have had the opportunity to initiate trade 150 times) prices are converging, but an equilibrium has not been achieved. In this example, prices still range from 0.9 to almost 1.1. Panel b shows the effect of a single crossover agent. It is dramatic. Equilibrium is not achieved after 150 rounds, but the convergence is much quicker and after 150 rounds the range of prices has narrowed to a low of about 0.96 and a high of 1.05. The subsequent panels show that additional crossover agents continue to speed the rate of convergence. With five crossover agents, equilibrium is reached in 167 rounds of trading. Furthermore, most of that price variation dissipates in half of that time.

Small-world networks seem to offer a structure that can reduce the transactions cost of exchange. They also mimic a familiar world in which most trading is local but resources are distributed globally. And, further inspection suggests there may

Table III. Wealth of crossover agents.[a]

	Agent A	Agent B	Agent C	Agent D	Agent E
0 crossovers	1929.61	887.73	1267.45	670.246	2059.95
1 crossover	**1997.17**	874.167	1269.23	675.557	2059.83
2 crossovers	1990.07	**1385.97**	1267.61	677.555	2058
3 crossovers	1984.94	1388.68	**1318.54**	677.591	2062.74
4 crossovers	1991.28	1389.3	1308.88	**985.451**	2066.52
5 crossovers	1981.82	1376.47	1307.04	981.252	**2105.27**

[a] The boldfaced number is the point at which that particular agent established a bridge to another trade group. Wealth from population #23, seed number 18847.

be micro-level incentives for such networks to evolve. In his exploration of bilateral exchange, Axtell suggests that exchange processes face an evolutionary challenge and that, 'the [process] ultimately selected would be the most efficient' (1998, p. 4). So, natural selection might pick small-world networks as efficient structures when search and negotiation accounts for a non-trivial portion of transaction costs.

But there is a stronger reason to expect such evolution. By creating more efficient trade routes, small-world networks offer an opportunity for individual traders to amass greater wealth. Once again using the example population from Figure 3, Table III shows the final wealth ($w^i = p(g_1^i) + g_2^i$ where p is the equilibrium price) of the five potential crossover agents. The first row is the final wealth of the potential crossover agents in a network with no crossovers. The second row is their final wealth in an identical network when agent A has established a bridge to a noncontiguous trade group. The third row has two crossover agents, A and B, and so forth.

Two results are apparent from Table III. First, being a crossover agent enhances your wealth. In every case, when an agent became part of a crossover bridge that allowed goods to be carried to a noncontiguous group, the wealth of that agent increased. In Table III, the boldfaced entry shows the establishment of a bridge. The wealth in the boldfaced cell is greater, and often much greater, then the wealth in the cell directly above which represents the same agent's wealth from a system in which he had not established a trade bridge. While Table III displays the results of a single experiment, the tendency of crossover agents to raise their wealth was prevalent in all these simulations. Specifically, in the 250 cases studied here (fifty populations with five agents per population) the crossover agents' wealth increases 228 times or more than 90% of the time.

A second observation is that competition from other trade routes tends to erode this wealth. Referring once again to Table III, compare the wealth of each agent in the boldface cell with the cells directly below. The lower cells contain the final wealth of the crossover agents as additional trade bridges are established elsewhere in the network. In most cases (7 out of 10 in this example) the entry of a competing crossover trader reduces the final wealth of the agents who had previously established a bridge. Thus competition reduces the market power of a crossover agent. This result was also generally true in the fifty populations studied where approximately 60% of the time the addition of new crossover agents reduced the wealth of the existing crossover agents.

However, there are several cases in which the addition of a crossover agent increases the wealth of an existing crossover agent. For example, in Table III Agent B has an increase in his wealth when the third crossover (Agent C) is added to the system (his wealth rises from 1385 to 1388). Usually additional trade routes compete with one another and reduce the wealth of agents who 'own' preexisting bridges. However, depending on the location of a particular trade route, a new crossover bridge can actually funnel additional goods from a distant part of the network to the vicinity of an existing bridge. Then the previously established bridge, and its controlling agent, become intermediaries for some of the goods crossing the new bridge. Thus, while the random addition of additional trade routes generally reduces the market power of existing bridges through competition, advantageous (and in this experiment accidental) coalitions can, and do, arise.

In conclusion, Small-world Networks have several characteristics that suggest they may be of general interest in economics. First, all trade is local, that is, trade occurs among relatively small sub-groups of the population. Since the establishment and maintenance of exchange relationships can be an expensive proposition, local trade or trade with agents with whom you have past knowledge, can economize on these costs. Second, when a few overlapping traders belong to distant trade groups, goods can be traded between any particular pair of agents with relatively few exchanges. Thus, markets can quickly assimilate global resource constraints. Third, these markets seem familiar; we recognize that our own purchases are locally executed and yet have a global reach. Fourth, while this paper does not directly investigate the emergence of networks, it appears that agents who construct crossover routes and create small-world networks are rewarded for their effort. So there is a private incentive to build such networks. Thus, while this paper is based on a simplistic economy consisting of two-goods and bilateral exchange, it suggests that small-world networks have the ability to strike a balance between the advantages and costs of having many trade partners. It seems likely that more complex markets would gain from small-world structures as well.

Notes

[1] Details of the C++ program can be found at http://cas.uah.edu/wilhitea/papers/traders/tradersabs.html.

[2] Requiring increments of a whole unit of good 1 adds some rigidity (and realism) to the model. The effects of this rigidity were explored by altering the aggregate initial endowment.

[3] In the reported simulations endowments range from 10 to 1500 units of each good.

[4] Two other pricing rules, the arithmetic mean and the geometric mean, were explored, but they had little effect on our central conclusions.

[5] This result supports Bell's (1998) conclusion that decentralized systems clear rapidly.

[6] If local markets contained a single agent, the endowment would be the equilibrium -reached with no trade.

[7] To make the picture less muddy, only 50 of the 500 prices are tracked.

[8] The proportion of agents who can trade across groups also affects path length. Consider a population divided into two equal groups. If half of the members in A and B are also in AB, path length is 1.25 ($p = 1$ for half the population and equals 1.5 for the other half). On the other hand if only 2% of the total population belongs to AB, $p = 0.02(1) + 0.98(1.5) = 1.49$.

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