Winter 2022, CSE342/542
Assignment 1
Deadline: 11:59PM Monday, 7th Feb

Instructions:

- You are required to submit a .py file necessarily for each coding question (you may or may not submit .ipynb files)

- Questions where you need to plot indicate coding questions. Others do not.

Q1:
You are given P(w1)=¼ , P(w2)=¾ and P(x|w1)=N(2,1) , P(x|w2)=N(5,1) where N(a,b) is normal distribution with mean a and variance b .
Plot P(x|w1) vs x, P(x|w2) vs x, P(x|w1)/P(x|w2) vs x. [1]

Also, find the decision boundary which minimizes the error in case of:
i) Zero-one loss. [1]
ii) $\lambda_{12}=2, \lambda_{21}=3, \lambda_{11}=0, \lambda_{22}=0$. [1]
(where λ_{ij} denotes predicting class "i" when the true class is "j")

Would you prefer using the zero-one loss for a task like cancer prediction on a real world dataset? Why, or why not?

Q2 :
X = [X1, X2, X3] be a random vector with $\boldsymbol{u}$ = [5, -5, 6] being the mean vector. The covariance matrix is given by :-

$$\begin{matrix} 5 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 4 \end{matrix}$$

Calculate the mean of
$Y = A^T X + B$
where $A = (2, -1, 2)^T$ and B = 5 [1]

Q3:

Let the conditional densities for two-category one dimensional problem given the following distribution (Cauchy' pdf).

$$Px|w_i) = \frac{1}{\pi b} \cdot \frac{1}{1 + \left(\frac{x - a_i}{b}\right)^2}, \; i = 1, 2$$

A) Compute the optimal or minimum error rate decision boundary for zero one loss. [1]

B) Plot p(w1|x) for a1 = 3, a2= 5 and b= 1 [1]

C) What is the overall error rate? [1]

Q4. a. Find the pdf of x = [a b] a 2-d vector, where a is a Bernoulli random variable and b is a Gaussian random variable. Assume, θ is the parameter for Bernoulli which gives the probability of a = 1, that is p(a=1) = θ. Assume, b follows a Gaussian distribution with mean m and variance σ^2. The covariance of x is [1]

[θ(1-θ) 0;
0 σ^2]

b. Let the pdf of part (a) be p(x). Now assume that there are N iid samples drawn from this pdf. Find θ that maximizes the joint probability q(x) of these N samples. Once you determine q(x), use ln q(x) for computing θ. [2]