EVOLUTIONARY OPTIMIZATION ALGORITHMS

Biologically-Inspired and Population-Based Approaches to Computer Intelligence

Solution Manual

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CHAPTER 1

Introduction

There are no problems for this chapter.

CHAPTER 2

Optimization

Written Exercises

2.1 Consider the problem $\min f(x)$, where

$$f(x) = 40 + \sum_{i=1}^{4} x_i^2 - 10\cos(2\pi x_i).$$

Note that $f(x)$ is the Rastrigin function – see Section C.1.11.

a) What are the independent variables of $f(x)$? What are the decision variables of $f(x)$? What are the solution features of $f(x)$?

b) What is the dimension of this problem?

c) What is the solution to this problem?

d) Rewrite this problem as a maximization problem.

Solution:

a) The independent variables, the decision variables, and the solution features are the same: x_1, x_2, x_3, and x_4.

b) This is a four-dimensional problem.

c) The solution is $x_1 = x_2 = x_3 = x_4 = 0$.

d) $\min f(x) \Longleftrightarrow \max g(x)$, where

$$g(x) = -f(x) = -40 - \sum_{i=1}^{4} x_i^2 + 10\cos(2\pi x_i).$$

2.2 Consider the function $f(x) = \sin x$.

a) How many local minima does $f(x)$ have? What are the function values at the local minima, and what are the locally minimizing values of x?

b) How many global minima does $f(x)$ have? What are the function values at the global minima, and what are the globally minimizing values of x?

Solution:

a) $f(x)$ has an infinite number of local minima: $x^* = 3\pi/2 + 2k\pi$ for any integer k, and $f(x^*) = -1$.

b) The answer to this problem is the same as the answer to part (a).

2.3 Consider the function $f(x) = x^3 + 4x^2 - 4x + 1$.

a) How many local minima does $f(x)$ have? What are the function values at the local minima, and what are the locally minimizing values of x?

b) How many local maxima does $f(x)$ have? What are the function values at the local maxima, and what are the locally maximizing values of x?

c) How many global minima does $f(x)$ have?

d) How many global maxima does $f(x)$ have?

Solution:

a) $f(x)$ has one local minima: $x^* = (-4 + 2\sqrt{7})/3 \approx 0.43$, and $f(x^*) \approx 0.10$.

b) $f(x)$ has one local maxima: $x^* = (-4 - 2\sqrt{7})/3 \approx -3.10$, and $f(x^*) \approx 22.05$.

c) $f(x)$ does not have any global minima because it decreases without bound as $x \to -\infty$.

d) $f(x)$ does not have any global maxima because it increases without bound as $x \to \infty$.

2.4 Consider the same function as in Problem 2.3, $f(x) = x^3 + 4x^2 - 4x + 1$, but with the constraint $x \in [-5, 3]$.

a) How many local minima does $f(x)$ have? What are the function values at the local minima, and what are the locally minimizing values of x?

b) How many local maxima does $f(x)$ have? What are the function values at the local maxima, and what are the locally maximizing values of x?

c) How many global minima does $f(x)$ have? What is the function value at the global minimum, and what is the globally minimizing values of x?

d) How many global maxima does $f(x)$ have? What is the function value at the global maximum, and what is the globally maximizing values of x?

Solution:

a) $f(x)$ has two local minima: $x_1^* = (-4 + 2\sqrt{7})/3 \approx 0.43$ with $f(x_1^*) \approx 0.10$, and $x_2^* = -5$ with $f(x_2^*) = -4$.

b) $f(x)$ has two local maxima: $x_1^* = (-4 - 2\sqrt{7})/3 \approx -3.10$ with $f(x_1^*) \approx 22.05$, and $x_2^* = 3$ with $f(x_2^*) = 52$.

c) $f(x)$ has one global minimum: $x_2^* = -5$ with $f(x_2^*) = -4$.

d) $f(x)$ has one global maximum: $x_2^* = 3$ with $f(x_2^*) = 52$.

2.5 Recall that Figure 2.4 shows the Pareto front for a two-objective problem in which the goal is to minimize both objectives.

a) Sketch a possible set of points in the (f, g)-plane and the Pareto front for a problem in which the goal is to maximize $f(x)$ and minimize $g(x)$.

b) Sketch a possible set of points in the (f, g)-plane and the Pareto front for a problem in which the goal is to minimize $f(x)$ and maximize $g(x)$.

c) Sketch a possible set of points in the (f, g)-plane and the Pareto front for a problem in which the goal is to maximize both $f(x)$ and $g(x)$.

Solution: See Figure P2.1.

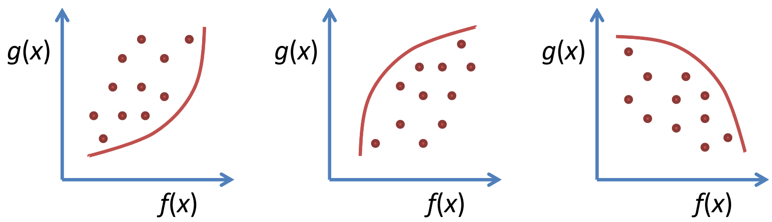

Figure P2.1 Problem 2.5 solution. The figure on the left shows the Pareto front for a problem in which the goal is to maximize $f(x)$ and minimize $g(x)$. The figure in the center shows the Pareto front for a problem in which the goal is to minimize $f(x)$ and maximize $g(x)$. The figure on the right shows the Pareto front for a problem in which the goal is to maximize both $f(x)$ and $g(x)$.

2.6 How many unique closed paths exist through N cities? By *unique* we mean that the starting city does not matter, and the direction of travel does not matter. For example, in a four-city problem with cities A, B, C, and D, we consider route $A \to B \to C \to D \to A$ equivalent to routes $D \to C \to B \to A \to D$ and $B \to C \to D \to A \to B$.

Solution: There are $N!$ possible permutations of N cities. However, if the starting city does not matter, then the number of possible routes reduces to $N!/N = (N-1)!$. If the direction of travel does not matter, then the number of possible routes further reduces to $(N-1)!/2$.

2.7 Consider the closed TSP with the cities in Table 2.2.

City	x	y
A	5	9
B	9	8
C	-6	-8
D	9	-2
E	-5	9
F	4	-7
G	-9	1

Table P2.1 TSP coordinates of cities for Problem 2.7.

a) How many closed routes exist through these seven cities?

b) Is it easy to see the solution by looking at the coordinates in Table 2.2?

c) Plot the coordinates. Is it easy to see the solution from the plot? What is the optimal solution? This problem shows that looking at a problem in a different way might help us find a solution.

Solution:

a) If we consider routes with different beginning and ending cities as unique routes, and we consider routes with different travel directions as unique, then there are $7! = 5,040$ closed routes exist through these seven cities. If we do not consider routes with different starting cities and different travel directions as different routes (see Problem 2.6), then there are $6!/2 = 360$ unique closed routes through these seven cities.

b) No, it is not easy for most people to see the solution by looking at the coordinates in Table 2.2.

c) See Figure P2.2 for a plot of the coordinates (`TSPProb1.m`). It is easy to see the optimal solution from the plot: $A \to B \to D \to F \to C \to G \to E \to A$.

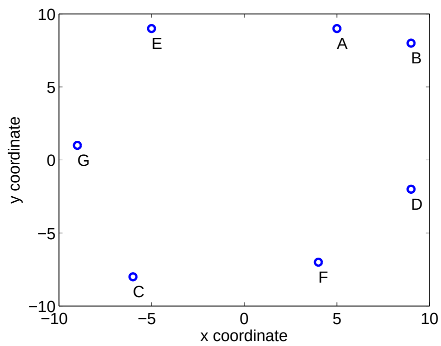

Figure P2.2 Problem 2.7(c) solution.

2.8 Given an arbitrary maximization problem $f(x)$ and a random initial candidate solution x_0, what is the probability that the steepest ascent hill climbing algorithm will find a value x' such that $f(x') > f(x_0)$ after the first generation?

Solution: In the absence of additional information, the probability of improvement due to randomly changing a single solution feature is 1/2. The probability of *not* improving due to any of n random, independent changes is $(1-1/2)^n = 2^{-n}$. Steepest ascent provides $n+1$ chances for improvement during each generation, where n is the dimension of x, so the probability of improving after the first generation is $1 - 2^{-(n+1)}$.

Computer Exercises

2.9 Plot the function of Problem 2.4 with the local and global optima clearly indicated.

Solution: See Figure P2.3 (`OptCompMin3.m`).

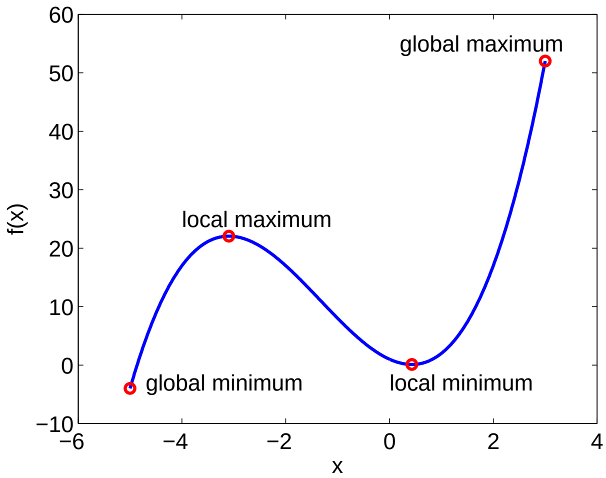

Figure P2.3 Problem 2.9 solution.

2.10 Consider the multi-objective optimization problem $\min\{f_1, f_2\}$, where

$$f_1(x_1, x_2) = x_1^2 + x_2, \text{ and } f_2(x_1, x_2) = x_1 + x_2^2$$

and x_1 and x_2 are both constrained to $[-10, 10]$.

a) Calculate $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ for all allowable integer values of x_1 and x_2, and plot the points in (f_1, f_2) space (a total of $21^2 = 441$ points). Clearly indicate the Pareto front on the plot.

b) Given the resolution that you used in part (a), give a mathematical description of the Pareto set. Plot the Pareto set in (x_1, x_2) space.

Solution:

a) See Figure P2.4 (`ParetoProb1.m`).

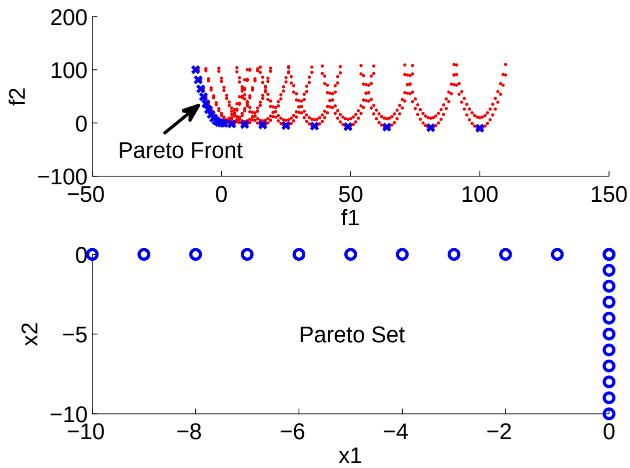

Figure P2.4 Problem 2.10 solution.

b) Examining the Pareto plot shows that the Pareto set can be described as

$$\{x_1 = 0 \text{ and } x_2 \leq 0, \text{ or } x_2 = 0 \text{ and } x_1 \leq 0\}.$$

Note that this is a correct description for the resolution specified in Problem 2.10, but not for smaller resolutions.

2.11 Adaptive hill climbing.

a) Run 20 Monte Carlo simulations of the adaptive hill climbing algorithm, with 1,000 generations per Monte Carlo simulation, for the two-dimensional Ackley function. Record the minimum value achieved by each Monte Carlo simulation, and compute the average. Do this for 10 different mutation rates, $p_m = k/10$ for $k \in [1, 10]$, and record your results in Table P2.2. What is the best mutation rate?

b) Repeat part (a). Do you get the same, or similar, results? What do you conclude about the number of Monte Carlo simulations that you need to get reproducible results for this problem?

c) Repeat part (a) for the 10-dimensional Ackley function. What do you conclude about the relationship between the optimal mutation rate and the problem dimension?

Solution: The solution was obtained with `AdaptiveHillClimbing.m` and `MonteAdaptiveHill.m`.

a) See the second column of Table P2.2. It appears that $p_m = 0.5$ gives the best performance. Your results may vary due to randomness in the hill climbing algorithm.

b) See the third column of Table P2.2. It appears that $p_m = 0.6$ gives the best performance. Results are similar to part (a), but not identical because of randomness in the hill climbing algorithm. This shows that 20 Monte Carlo simulations is not enough to make definite conclusions about this problem.

c) See the fourth column of Table P2.2. It appears that small values of p_m give better performance. Results are completely different than parts (a) and (b). This shows that the optimal mutation rate strongly depends on the problem dimension.

p_m	Problem 2.11(a) Result	Problem 2.11(b) Result	Problem 2.11(c) Result
0.1	3.2	2.5	4.8
0.2	1.8	1.7	5.5
0.3	1.5	1.2	7.6
0.4	1.3	1.3	9.8
0.5	1.0	1.4	12.0
0.6	1.4	1.1	13.6
0.7	1.3	1.2	15.2
0.8	1.8	1.6	16.5
0.9	2.2	2.4	17.2
1.0	3.6	4.1	17.7

Table P2.2 Problem 2.11 solution.