Local Search

CS472/CS473 — Fall 2005

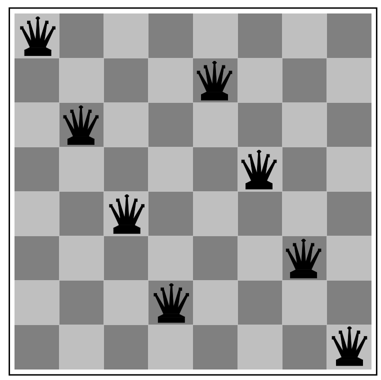
Scaling Up

- So far, we have considered methods that systematically explore the full search space, possibly using **principled** pruning (A* etc.).
- The current best such algorithms (RBFS / SMA*) can handle search spaces of up to 10^{100} states
  $\rightarrow \sim 500$ binary valued variables.
- But search spaces for some real-world problems might be much bigger — e.g. $10^{30,000}$ states.
- Here, a completely different kind of search is needed.
  $\rightarrow$ *Local Search Methods*

Optimization Problems

- We're interested in the Goal State — not in how to get there.
- Optimization Problem:
  - State: vector of variables
  - Objective Function: $f : state \rightarrow \Re$
  - Goal: find state that minimizes objective function
- Examples: VLSI layout, job scheduling, map coloring, N-Queens.

Example



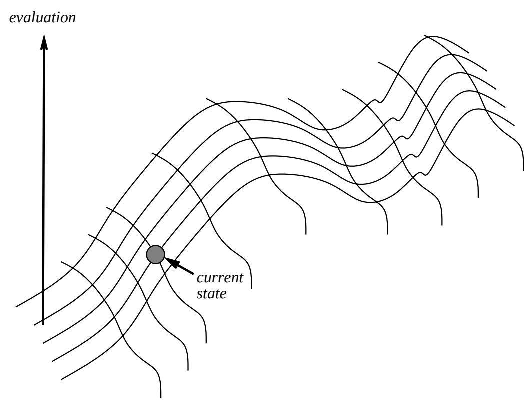
Local Search Methods

- Applicable to optimization problems.
- Basic idea:
  - use a single **current state**
  - don't save paths followed
  - generally move only to successors/neighbors of that state
- Generally require a **complete state description**.

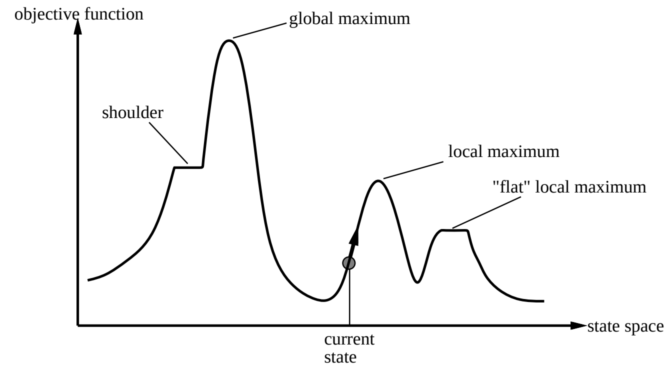
Hill-Climbing Search

```
function HILL-CLIMBING(problem) returns a solution state
  inputs: problem, a problem
  static: current, a node
          next, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
     next ← a highest-valued successor of current
     if VALUE[next] < VALUE[current] then return current
     current ← next
  end
```



Hill Climbing Pathologies



Improvements to Basic Local Search

Issue: How to move more quickly to successively higher plateaus and avoid getting "stuck" / **local minima**.

Idea: Introduce uphill moves ("noise") to escape from long plateaus (or true local minima).

Strategies :

- Multiple runs from randomly generated initial states
- Random-restart hill-climbing
- Tabu search
- Simulated Annealing
- Genetic Algorithms

Variations on Hill-Climbing

1. **random restarts:** simply restart at a new random state after a pre-defined number of local steps.
2. **tabu:** prevent returning quickly to same state.
   Implementation: Keep fixed length queue ("tabu list"): add most recent step to queue; drop "oldest" step.
   Never make step that's currently on the tabu list.

Demo (CS473 Project by Matt Taylor):
$http : //www.cs.cornell.edu/selman/Demos/index.html$

Simulated Annealing

Idea:
Use conventional hill-climbing techniques, but occasionally take a step in a direction other than that in which the rate of change is maximal.

As time passes, the probability that a down-hill step is taken is gradually reduced and the size of any down-hill step taken is decreased.

Kirkpatrick *et al.* 1982; Metropolis *et al.* 1953.

Simulated Annealing Algorithm

$current \leftarrow$ initial state
for $t \leftarrow 1$ to inf do

$T \leftarrow schedule[t]$
if $T = 0$ then return $current$
$next \leftarrow$ randomly selected successor of $current$
$\Delta E \leftarrow f(next)$ - $f(current)$
if $\Delta E > 0$ then $current \leftarrow next$
else $current \leftarrow next$ only with probability $e^{\Delta E/T}$

Genetic Algorithms

- Approach mimics *evolution.*
- Usually presented using a rich (and different) vocabulary:
  - fitness, populations, individuals, genes, crossover, mutations, etc.
- Still, can be viewed quite directly in terms of standard **local search**.

Features of evolution

- High degree of parallelism
- New individuals ("next state / neighboring states"):
  derived from "parents" ("crossover operation")
  genetic mutations
- Selection of next generation:
  based on survival of the fittest

Genetic Algorithms

Inspired by biological processes that produce genetic change in populations of individuals.

Genetic algorithms (GAs) are local search procedures that usually the following basic elements:

- A Darwinian notion of **fitness**: the most fit individuals have the best chance of survival and reproduction.
- Mating operators:
  - Parents are selected.
  - Parents pass their genetic material to children.
  - Mutation: individuals are subject to random changes in their genetic material.

General Idea

- Maintain a population of individuals (states / strings / candidate solutions)
- Each individual is evaluated using a **fitness function**, i.e. an objective function. The fitness scores force individuals to compete for the privilege of survival and reproduction.
- Generate a sequence of generations:
  - From the current generation, select **pairs** of individuals (based on fitness) to generate new individuals, using **crossover**.
- Introduce some noise through random **mutations**.
- Hope that average and maximum fitness (i.e. value to be optimized) increases over time.

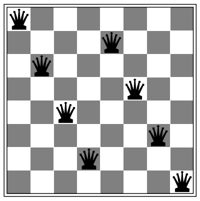
Genetic algorithms as search

- Genetic algorithms are local heuristic search algorithms.
- Especially good for problems that have large and poorly understood search spaces.
- Genetic algorithms use a randomized parallel beam search to explore the state space.
- You must be able to define a good fitness function, and of course, a good state representation.

Binary string representations

- Individuals are usually represented as a string over a finite alphabet, usually bit strings.
- Individuals represented can be arbitrarily complex.
- E.g. each component of the state description is allocated a specific portion of the string, which encodes the values that are acceptable.
- Bit string representation allows crossover operation to change multiple values in the state description. Crossover and mutation can also produce previously unseen values.

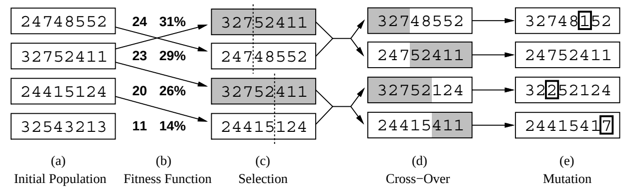
8-queens State Representation



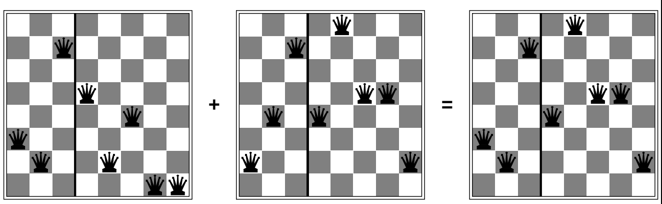
option 1: 86427531

option 2: 111 101 011 001 110 100 010 000

GA: High-level Algorithm



Crossover Example



Another Example

World championship chocolate chip cookie recipe.

	flour	sugar	salt	chips	vanilla	fitness
1	4	1	2	16	1	
2	4.5	3	1	14	2	
3	2	1	1	8	1	
4	2.2	2.5	2.5	16	2	
5	4.1	2.5	1.5	10	1	
6	8	1.5	2	8	2	
7	3	1.5	1.5	8	2	

generation 1

GA$(Fitness, Fitness\_threshold, p, r, m)$

- $P \leftarrow$ randomly generate p individuals
- For each i in P, compute $Fitness(i)$
- While $[\max_i Fitness(i)] < Fitness\_threshold$
  1. Probabilistically **select** $(1-r)p$ members of P to add to P_S.
  2. Probabilistically choose $\frac{r \cdot p}{2}$ pairs of individuals from P. For each pair, $\langle i_1, i_2 \rangle$, apply **crossover** and add the offspring to P_s
  3. **Mutate** $m \cdot p$ random members of P_s
  4. $P \leftarrow P_s$
  5. For each i in P, compute $Fitness(i)$
- Return the individual in P with the highest fitness.

Selecting Most Fit Individuals

Individuals are chosen probabilistically for survival and crossover based on **fitness proportionate selection**:

$$\Pr(i) = \frac{Fitness(i)}{\sum_{j=1}^{p} Fitness(i_j)}$$

Other selection methods include:

- **Tournament Selection:** 2 individuals selected at random. With probability p, the more fit of the two is selected. With probability $(1 - p)$, the less fit is selected.
- **Rank Selection:** The individuals are sorted by fitness and the probability of selecting an individual is proportional to its rank in the list.

Crossover Operators

Single-point crossover:

Parent A:	**1**	**0**	**0**	**1**	**0**	**1**	**1**	**1**	**0**	**1**
Parent B:	0	1	0	1	1	1	0	1	1	0
Child AB:	**1**	**0**	**0**	**1**	**0**	1	0	1	1	0
Child BA:	0	1	0	1	1	**1**	**1**	**1**	**0**	**1**

Two-point crossover:

Parent A:	**1**	**0**	**0**	**1**	**0**	**1**	**1**	**1**	**0**	**1**
Parent B:	0	1	0	1	1	1	0	1	1	0
Child AB:	**1**	**0**	**0**	**1**	1	1	0	**1**	**0**	**1**
Child BA:	0	1	0	1	**0**	**1**	**1**	1	1	0

Uniform Crossover

Uniform crossover:

Parent A:	**1**	**0**	**0**	**1**	**0**	**1**	**1**	**1**	**0**	**1**
Parent B:	0	1	0	1	1	1	0	1	1	0
Child AB:	**1**	1	**0**	**1**	1	1	**1**	1	**0**	**1**
Child BA:	0	**0**	0	1	**0**	**1**	0	**1**	1	0

Mutation

Mutation: randomly toggle one bit

Individual A:	1	0	0	1	0	1	1	1	0	1
Individual A':	1	0	0	**0**	0	1	1	1	0	1

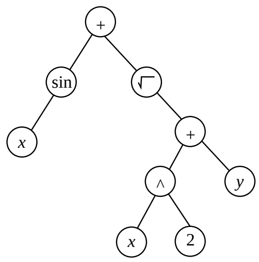
Mutation

- The **mutation** operator introduces random variations, allowing solutions to jump to different parts of the search space.
- What happens if the mutation rate is too low?
- What happens if the mutation rate is too high?
- A common strategy is to use a high mutation rate when search begins but to decrease the mutation rate as the search progresses.

Genetic Programming

In **Genetic Programming**, programs are evolved instead of bit strings. Programs are represented by trees. For example:

$$\sin(x) + \sqrt{x^2 + y}$$



Remarks on GA's

- In practice, several 100 to 1000's of strings.
- **Crowding** can occur when an individual that is much more fit than others reproduces like crazy, which reduces diversity in the population.
- In general, GA's are highly sensitive to the representation.
- Value of crossover difficult to determine (so far) ($\rightarrow$ local search).

Local Search — Summary

Surprisingly efficient search method.

Wide range of applications.
    any type of optimization / search task
Handles search spaces that are too large
    (e.g., 10^{1000}) for systematic search
Often best available algorithm when
    lack of global information.
Formal properties remain largely elusive.
Research area will most likely continue to thrive.