NordDesign 2008
August 21 – 23, 2008
Tallinn, Estonia

Application of evolutionary methods for solving optimization problems in engineering

J. Majak*, R. Küttner*, M. Pohlak*, M. Eerme*, K. Karjust*

(*) Department of Machinery, Tallinn University of Technology,
Ehitajate tee 5, 19086 Tallinn, Estonia

jmajak@staff.ttu.ee, rein.kyttner@ttu.ee, meelisp@staff.ttu.ee,
eerme@staff.ttu.ee, Kristo@staff.ttu.ee

Abstract

This paper is focused on solving engineering optimization problems, which contain often real and integer variables, a number of local extremes, multiple optimality criteria and disciplines. In latter case the conventional approaches based on traditional gradient technique fail or perform poorly. In the current study, an optimization approach that integrates meta-modeling and hybrid genetic algorithm (HGA) is developed. The methodology proposed is validated on following practical examples: optimal design of composite bathtub (large composite plastics), design of car frontal protection system.

Keywords: Meta-modeling, global optimization, HGA.

1 Introduction

During last decades, the efficiency of the different architectures of evolutionary algorithms in comparison to other heuristic techniques has been tested in various engineering design problems. These algorithms are based on stochastic search techniques simulating mechanisms of natural selection, genetics and evolution. An overview on evolutionary computing (EC) techniques in structural engineering can be found in [1, 2]. Most known evolutionary computing techniques are genetic algorithms (GA), evolution strategies (ES) and evolutionary programming (EP). These approaches differ in the types of generation - to - generation alterations and on computer representation of population. The fourth general approach is genetic programming (GP), developed recently for automated creating of a computer program [3]. GP represents individuals as executable trees of code.

Evolutionary algorithms including GA have property to avoid the local extreme and have a better global perspective than the traditional gradient based methods [4]. Certain class of optimal design problems contains multiple global extremes. Desirably all or as many as possible global extremes should be found. Obviously, in latter case the algorithms manipulating with population instead of single solution are preferred.

However, manipulating with population instead of single solution has also some drawbacks – numerous evaluations of candidate solutions are necessary. For complex engineering problems, such evaluations are time consuming (capacious FEA, tests, etc.). Latter problem is solved most commonly by using meta-models. Various techniques including regression and interpolation tools (splines, least square regression, artificial neural network, Kriging, etc) can be utilized for building surrogate models [5]. An accuracy and computational cost are basic characteristics, which must be considered in selection of the appropriate meta-models [6].

GAs have been developed rapidly during last decades as an effective and simple optimization technique. One of the drawbacks of the traditional GA is a ratchet effect (crossover cannot introduce new gene values). In order to overcome the drawbacks of the traditional GA a large number of improvements have been provided (CHC GA, adaptive GA [7], niche GA and hybrid GA [8-9], etc.). In order to achieve higher accuracy, the real-coded GA operators are used in engineering design instead of traditional binary operators (more efficient for operating with real numbers, the chromosome is implemented by a vector of floating-point numbers) [10-11]. Development of evolutionary algorithms for multi-objective and multi-disciplinary optimization problems [12-13] is another actual topic in engineering design.

In the current study artificial neural networks (ANN) and HGA are used for performing meta-modeling and search for global extreme, respectively. Thus, the number of function evaluations is reduced and convergence to global extreme can be expected. In order to speed up algorithm, the GA is combined with gradient method (steepest descent). In this hybrid GA algorithm the global search is performed by the use of real-coded (or binary coded) GA and local search by the use of gradient method. Some modifications to hybrid GA algorithm are mode depending on the character of particular optimization problem solved.

The structural analysis of the car frontal protection system and composite bathtub is performed by the use of FEM software packages LS-DYNA and HyperWorks, respectively. For optimal design of the composite bathtub the multistage optimization procedure has been developed. The optimal thickness distribution is determined with free size optimization on local level by the use of HyperWorks. The final properties of the part are determined by minimizing the cost and production time simultaneously (multi-criteria optimization, global level). In the case of first problem considered (design of car frontal protection system) an alternative numerical approach is developed by the use of finite element optimization package LS-OPT and the obtained numerical results are validated against experimental test results [14].

2 Meta-modeling and hybrid GA

The stress-strain analysis of the considered systems is performed by use of FE software packages LS-DYNA (both, explicit and implicit solvers are used), HyperWorks and ANSYS. The results of stress-strain analysis are validated against corresponding experimental results and are used in meta-modeling. In response surface method (RSM) the design surface is fitted to the response values using regression analysis. Least squares approximations are used for this purpose most commonly. In the current paper, the generalized regression neural networks (NN) are used for the surface fitting. The output data obtained from FE analysis are treated as response values. Let us proceed from predetermined set of designs. The surface constructed by the use of NN does not normally contain the given response values (similarity with least-squares method in this respect). An approach proposed is based on the use of the MATLAB Neural Network Toolbox. In MATLAB NN Toolbox, a two-layer network is generated by the use of function *newgrnn*. The first layer has *radbas* neurons and the second layer has *purelin* neurons. The response surfaces are generated simultaneously (with one call to *newgrnn*) for all response quantities. In order to calculate outputs for a concurrent set of values of the design variables, a network simulation function *sim* is utilized. Similar two-layer (one hidden layer) network is generated also in optimization software package LS-OPT for composing response surface.

Note that in the current study the meta-modeling technique is applied not only for building objective (fitness) functions, but also for building some constraint functions (needed to be evaluated from FEA/experiments). In order to determine the minimal value of the objective function the hybrid GA algorithm containing local and global level search has been developed. The global and local level search has been performed by the use of GA and

steepest descent methods respectively. In order to achieve higher accuracy the real-coded algorithm is used. The best individual (solution) of the population generated by GA is used as an initial value of the gradient method (local level search). In the cases where elite population (set of solutions obtained by fitness-based selection rule) contains individuals, which chromosomes (parameters) differ substantially it is reasonable to perform local search for all these individuals. Thus, the number local searches necessary depend on result of global search. The local search may be considered as design improvement, since the global search realized by the use GA may convergence to solution close to global optimum not exactly to optimum, also the gradient method is less time consuming. The final solution is determined by comparison of the results of all local searches performed (selection is based on value of objective function). In the case of problems where a number of solutions i.e. fitness functions values which are close to each other belong to the same sub-domain, the local search can be also performed by the use of GA (two stage GA).
The solution is implemented in MATLAB code. Note that the 2D array “population” should be sorted using the values of the fitness function given in array “scores” before selection of the elite population (initially unsorted). An alternative solution for design of car frontal protection system) is realized by the use of FE software package LS-OPT [15]. Latter solution is based on the use of leap-frog algorithm.

3 Optimal design of composite bathtub

The objective is the optimization of structure and manufacturing processes of the composite plastic bathtub. The structural analysis of the product is performed with Finite Element Analysis. The optimal thickness distribution is determined with free size optimization. The final properties of the part are determined by minimizing the cost and production time simultaneously.

3.1 Problem formulation

The current paper is concentrated on design of derivative products of a product family. For finding out optimal technology route we have to cut down the structure of the technology process into different process segments, meaning that we have to solve different sub systems, like finding out the optimal vacuum forming technology, the technology for post-forming operations (trimming, drilling the slots and cut-outs into the part, decoration, printing, etc), strengthening (reinforcing) and assembly. The bathtub is produced in two stages – in the first stage the shell is produced by vacuum forming, and in the second stage the shell is strengthened by adding glass-fiber-epoxy layer on the one side. The current study is focused on strengthening of the shell by adding glass-fiber-epoxy layer and the first stage, vacuum forming process, is described briefly.
The part thinning in vacuum forming process has been analyzed with different materials like ABS, PMMA, polycarbonate and acrylic FF0013 Plexiglas. In the following, the acrylic FF0013 Plexiglas formed at the temperature 320-340˚C is considered (heating time 6 min and cooling time 2 min). The sample of vacuum formed part and the final assembled product are shown in Figure 1.

a) b)

Figure 1. The sample vacuum formed part (a) and the final assembled product (b)

In vacuum forming the thinning is a natural consequence of the deformation conditions. The thickness variations are potentially large for a part. Therefore, it is often important to control the thickness variations in order to meet functional requirements of the part. The values of thinning of the plastic sheet in the forming operations can be determined from experience, special tests or simulations. The experimental tests have been performed in order to analyze the wall thickness reduction in certain materials. The results of analysis for Plexiglas are given in Figure 2.

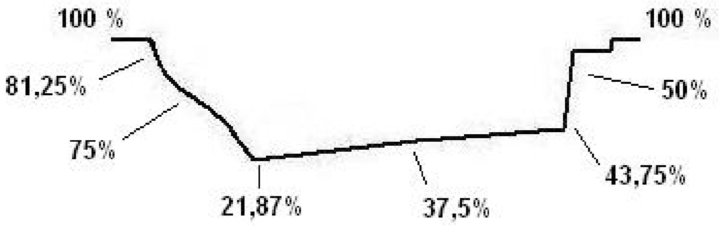

Figure 2. Wall thickness reduction in the 3.2 mm thick blank

It can be seen from Figure 2 that the thickness reduction is maximal in bottom area. Obviously, the strengthening of the shell is necessary and it can be performed in both stages of manufacturing process. In the following the detailed attention is paid to reinforcement of the shell (adding glass-fiber-epoxy layer) since the stiffness of the reinforcement layer is significantly higher than acrylic layer.

3.2 Multi stage optimization model

The reinforcement problem of the bathtub shell can be formulated as a multi-objective optimization problem and expressed in mathematical form as

$$\begin{aligned} &\min F(x) = (F_1(x), F_2(x)), \\ &F_1(x) = C(x_1, x_2, \dots, x_n), \\ &F_2(x) = T(x_1, x_2, \dots, x_n), \end{aligned} \tag{1}$$

subjected to linear and nonlinear constraints. In (2) $C(x)$ and $T(x)$ are cost of the glass-fiber-epoxy layer and manufacturing time, respectively and x is a vector of design variables. The linear and nonlinear constraints proceed from technological (maximum layer thickness), exploitation (displacement limit) and safety (stress limit) considerations.

Since the units used to measure the objectives $F_1(x)$ and $F_2(x)$ are different (cost and time), it is reasonable to represent the objectives in terms of relative deviation i.e.

$$f_1(x) = \frac{\max F_1(x) - F_1(x)}{\max F_1(x) - \min F_1(x)}, \quad f_2(x) = \frac{\max F_2(x) - F_2(x)}{\max F_2(x) - \min F_2(x)}. \tag{2}$$

The optimization problem posed above is a quite complicated including topology optimization, meta-modeling, experimental study, etc. and is divided into the following subtasks (stages):

- evaluation of the objective functions $f_1(x)$ and $f_2(x)$ for given vector of design variables x (includes FEA);
- meta-modeling (response surface modeling);
- global optimization using multiple criteria analysis techniques discussed in details below.

3.3 Results and discussion

The values of the objective function corresponding to weighted summation technique are pointed out in Figure 3, where dependence on maximum thickness of the reinforcement layer is shown. The values of the weight w_1 corresponding to the first criteria (cost) are varied from 0.2 to 0.8. As it can be seen from Figure 3, the shape of the curves describing objective function depend on the values of the weights, but the extreme value of the objective is reached in the case of same value of the maximum thickness of the reinforcement layer. The objective decreases in same range where the material volume decreases, after that the material volume approaches to constant value, but the objective increases significantly. Latter fact is caused due to additional drying expenses (layer-wise covering technology is used due to technological limits on maximal layer thickness in one-time layer setup, thus, larger total thickness means that larger number of sub-layers should be used). Similar values of the objective function are obtained in the case of compromise programming technique (omitted for conciseness sake).

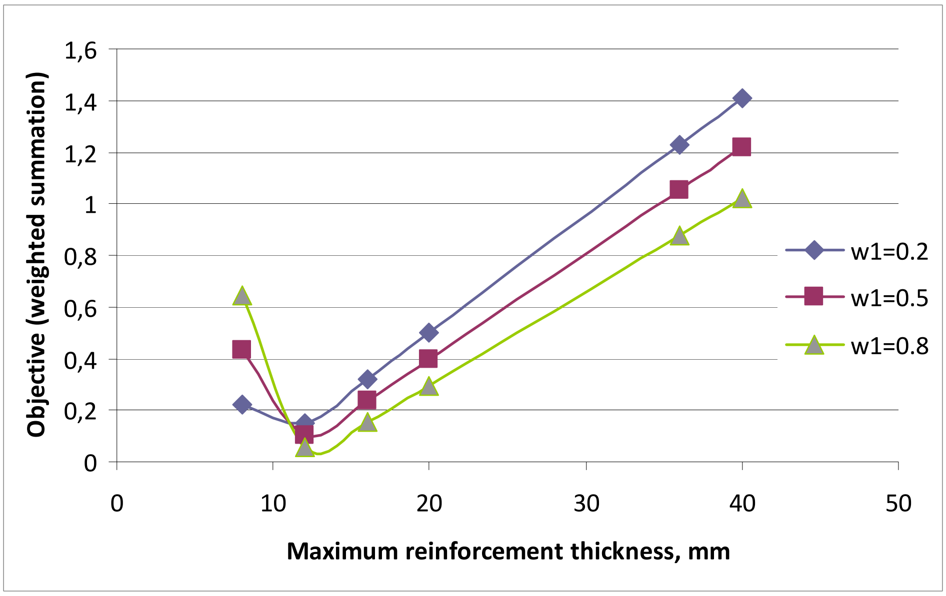

Figure 3. Objective function (weighted summation) vs. maximum thickness of the reinforcement layer

The bathtub with optimal thickness distribution of reinforcement layer corresponding to extreme value of the objective function (compromise programming and weighted summation techniques) is shown in Figure 4.

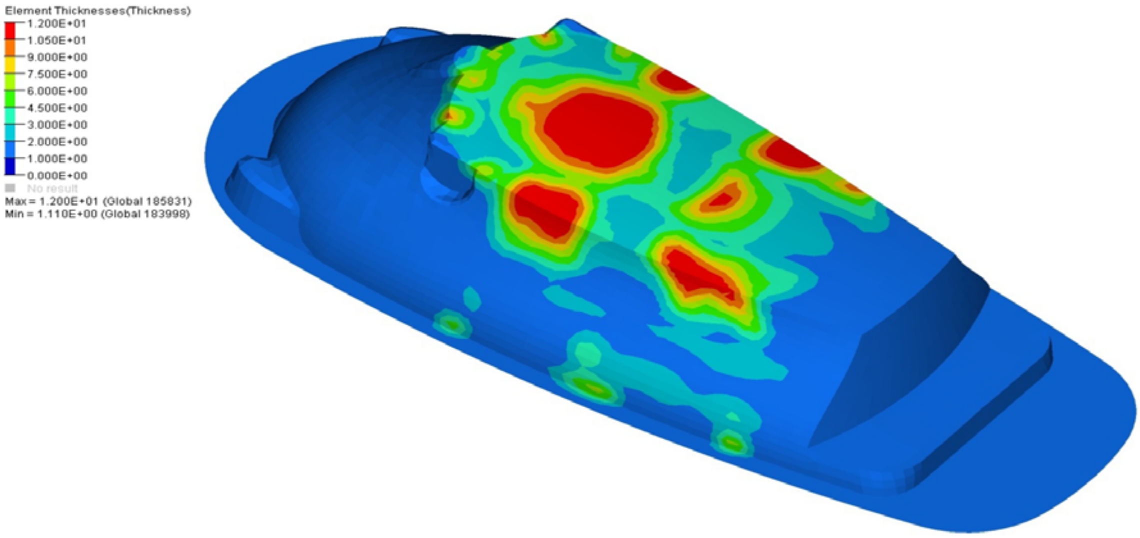

Figure 4. The optimal thickness distribution of reinforcement

It appears that the reinforcement layer is the thickest in areas where the local loading is applied (at the middle of the bottom area) and bottom-wall transitional areas (see Figure 4).

4. Optimal design of car frontal protection system

The main attention is paid to optimal design of brackets. Preliminary configuration of the bracket is given by the manufacturer. An analysis of car-pedestrian collision situation is performed by the use of LS-DYNA explicit solver and the stiffness analysis with LS-DYNA implicit solver.

4.1 Problem formulation

The European directive 2005/66/EC defines several different tests for frontal protection system. As it can be seen, the tubular extra accessories that are mounted to the front of vehicle will worsen considerably the situation for pedestrian in case of accident, so only minimum requirements can be met without adding sophisticated systems (like airbags, etc). Minimum test is lower legform impact test. Upper legform test is required for systems with height over 500 mm. In the current study, it is assumed that the height of the car frontal protection system designed is less than 500 mm and main attention is paid to the safety requirements proceeding from lower legform test (see Figure 5).

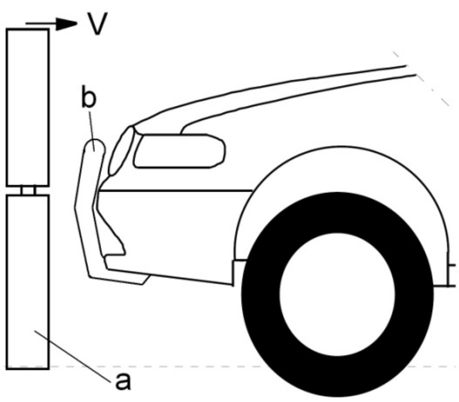

Figure 5. Lower legform impact testing (a – Legform impactor, b – Frontal protection system, V – velocity of impactor)

In the test the impactor (a in Figure 5) has been shot at the speed of 11.1 m/s at the frontal protection system of the vehicle. There are three types of sensors mounted inside the impactor: acceleration sensor, bending angle sensor and shear displacement sensor. According to the directive 2005/66/EC:

- the maximum dynamic knee bending angle shall not exceed 21.0°;
- the maximum dynamic knee shearing displacement shall not exceed 6.0 mm;
- the acceleration measured at the upper end of the tibia shall not exceed 200g.

It is assumed above that the total permissible mass of the vehicle is less than 2500kg. In the case where the total permissible mass of the vehicle exceeds 2500kg, the corresponding maximum values of the knee bending angle, knee shearing displacement and acceleration measured at the upper end of the tibia are 26.0°, 7.5 mm and 250g, respectively.

With bending angle and shear displacement it is easier to fit between the limits, with acceleration limit the situation is more complicated.

In the literature, different kinds of energy absorbing structures (rings, thin walled members, laminates, honeycombs, etc.) can be found, materials vary from solid metals to composites and cellular materials [16-18]. Unfortunately, most of structures absorb energy in an unstable manner. Two principally different types of energy absorbing structures are classified as follows: type I structure with a flat-topped load-displacement curve and type II structure with a high peak of reaction force when impact loading starts followed by smaller peaks or more constant level of reaction forces. More desirable situation would be if the reaction force increased steadily to some predefined level and would remain constant on this level (Lu 2003 et al.). In the current study the energy absorbing structure of type I (bracket) has been redesigned by changing geometry, adding cutouts, folds and performing parameters design. The resulting bracket belongs to energy absorbing structure of type II. In order to decrease the acceleration, optimal design of tubular parts and brackets has to be addressed.

The current study is focused on the design of brackets. The main energy absorbing component is shown in Figure 6. Initial design of the energy absorbing component was given by the manufacturer. Thus, the topology is predefined in certain extent by the manufacturer and main task was to search for the optimal set of design variables a, b, c, d and e (see Figure 6). However, some corrections in topology are available (for example the fold: form, location; etc.). The properties of the tubes are selected as appropriate as technologically possible (light structure, thin walls, etc), detailed design of tubes is omitted.

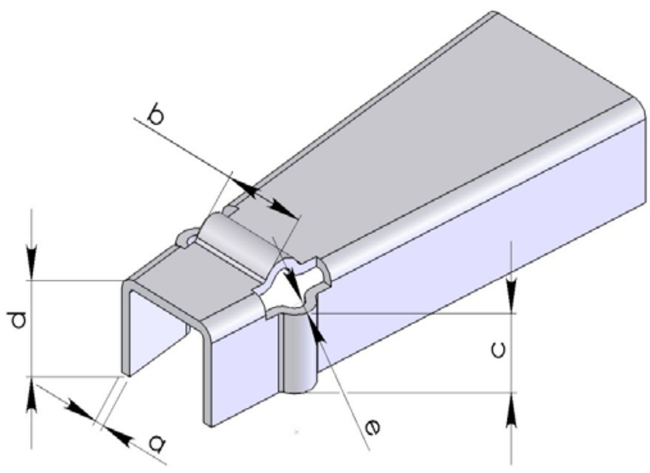

Figure 6. Energy absorbing component (a, b, c, d and e are design variables)

In the current study, two different optimality criteria are discussed. The objective functions corresponding to these criteria can be expressed as

– minimization of the peak force F (peak acceleration)

$$f_1(\bar{x}) = \max_t F(t,\bar{x}); \tag{3}$$

– minimization of the difference between maximal and minimal force

$$f_2(\bar{x}) = \max_t F(t,\bar{x}) - \min_t F(t,\bar{x}). \tag{4}$$

In (3)-(4) t stands for time, $\bar{x} = (x_1, x_2, ..., x_n)$ is a vector of independent design variables and $F(t,\bar{x})$ stands for axial (frontal) force component.

In order to cover both criteria the multi-criteria optimization problem is formulated and solved applying the weighted summation and compromise programming analysis techniques.

4.2 Finite element analysis

LS-DYNA software was utilized for numerical analysis. Fully integrated shell elements are used. The stress-strain behavior is modeled with multi-linear approximation. In order to consider plastic anisotropy the Hill's second order yield criterion is employed. The FEA is performed separately for crash simulation and stiffness analysis. The total number of simulations depends on the number of design variables and on grid density, fixed in the stage of simulation data design. The dynamic and static analysis is performed with the same sets of the simulation data in order to get complete set of output data. The output data used in further optimization procedure contains extreme values of the frontal force component (obtained from the dynamic analysis) and displacements in y-z plane.

In order to validate the FEA models the experimental study was carried out. Several versions of component shown in Figure 6 were tested (the number of design variables used in the case of different approaches was from 4 up to 8). The preliminary estimates of the force components and deformation modes are obtained from the compression tests of the brackets performed on universal testing equipment. In Figure 7 the load displacement curves obtained from experimental tests and FEA are compared. The design parameters values are a=1.6 mm, b=12 mm, c=6 mm and d=10 mm (see Figure 6). The folds with triangular shape (instead of convex arc) are considered and instead of the design parameter e given in Figure 6, the bend angle with the value 5 degrees is used.

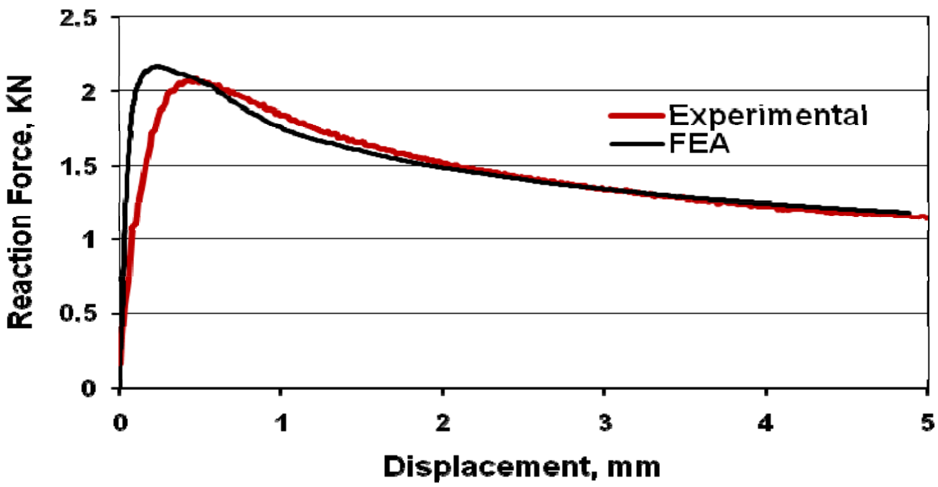

Figure 7. Load-displacement curves: experimental and FEA.

It can be seen from Figure 7 that the experimental and FEA results are found to be in good agreement, the peak values of the reaction force and also the shapes of the curves are close.

4.3. Numerical and experimental results

The limitation on acceleration (or corresponding force component) appears to be the most critical. For that reason f_1 is considered as dominating term in optimality criterion. As a result of design process, the maximal value of the frontal force component $F(t,\bar{x})$ is reduced more than 4 times in comparison with reference solution. The reference solution was chosen with reserve since the predicting of the value of y-z displacement u_c (constraint) corresponding to certain set of design variables is extremely complicated. In Figure 8 the frontal force component $F(t,\bar{x})$, corresponding to the initial (reference) and optimal sets of design variables, is given, respectively. All constraints are fulfilled in the case of both designs. Note that energy absorption is twice higher in the case of initial design. Latter fact can be explained with reduced dimensions of the component.

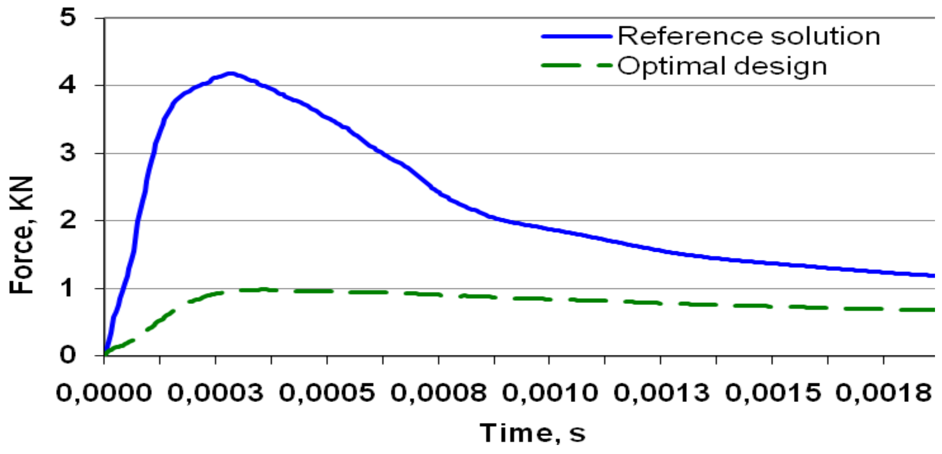

Figure 8. Force - Time diagram: the reference solution and the optimal design.

It can be seen from Figure 8 that the shape of the force curve corresponding to the optimal design is quite similar with the shape of curve corresponding to energy absorber of type II, described above.

Conclusions

The global optimization algorithm based on the use of meta-modeling and hybrid genetic algorithm is developed and applied for solving a number of quite different engineering design problems including design of car frontal protection system, design of composite bathtub, etc. It can be concluded that the optimization algorithm proposed has been shown good performance with respect to convergence to global extreme (responsibility of the global level search, GA) and accuracy (responsibility of the local level search, gradient method). Certain adaption of the algorithm may be necessary depending on character of optimization problem considered (GA operators used, constraint handling, parameters tuning).
Both case studies considered coming from industry and the optimal designs developed found practical application.

Acknowledgements

The work has been supported by Estonian Science Foundation grant G6835.

References

[1] Kicinger, R. Arciszewski, T. De Jong. K.A, Evolutionary Computation and Structural Design: a *Survey of the State of the Art. Computers & Structures*, 2005, 83(23-24), pp. 1943-1978.

[2] Koza, J. R., Genetic programming: on the programming of computers by means of natural selection. Cambridge, Mass.: MIT Press. 1992.

[3] Spall, J.C., Introduction to stochastic search and optimization. Wiley-Interscience, 2003.

[4] Deb, K. and Tiwari, S., Multi-objective optimization of a leg mechanism using genetic algorithms. *Engineering Optimization*, 2005, 37(4), pp. 325-350.

[5] Bhattacharya, M., Surrogate based Evolutionary Algorithm for Design Optimization. *Proc. of world academy of science, eng. and tech.*, 2005, 10, pp. 52-57.

[6] Jin, Y. Olhofer, M. Sendhoff, B., A Framework for Evolutionary Optimization with Approximate Fitness Functions. *IEEE Transactions on Evolutionary Computation*, 2002, 6(5), pp. 481-494.

[7] Srinivas, M., Patnaik, L.M., Adaptive probabilities of crossover and mutations in GAs, *IEEE Trans. Syst. Man Cyber*. 1994, 24, pp. 656–667.

[8] Yuan, Q., He, Z. Leng, H., A hybrid genetic algorithm for a class of global optimization problems with box constraints. *App. Math. Comp*. 2008, 197, pp. 924–929.

[9] Kao, Y.T. Zahara, E., A hybrid genetic algorithm and particle swarm optimization for multimodal functions. *Applied Soft Computing*. 2008, 8, pp. 849–857.

[10] Kumar, S. Naresh, R., Efficient real coded genetic algorithm to solve the non-convex hydrothermal scheduling problem. *Electrical Power and Energy Systems* (in press).

[11] Kim, J.W. Kim, S.W., New encoding/converting methods of binary GA/real coded GA. *IEICE Trans. Fund*., 2005, E88A, 6.

[12] Deb, K. Multi-objective optimization using evolutionary algorithms. Chichester, New York: John Wiley & Sons. 2002.

[13] Coello Coello, C.A., An updated survey of GA-based multiobjective optimization techniques. *ACM Computing Surveys*, 2000, 32(2), pp. 109-143.

[14] Pohlak, M. Majak, J. Eerme, M., Optimization study of car frontal protection system. *ASMDO: First international Conference on Multidisciplinary Optimization and Applications.* 17 - 20 April 2007, Besancon. Edited by D.H. Bassir.

[15] Stander, N. Roux, W. Eggleston , T. Craig, K., LS-OPT user's manual. Livermore Software Technology Corporation. 2006.

[16] Alghamdi, AAA., Collapsible impact energy absorbers: an overview. *In Thin-Walled Structures*, 2001, 39, pp. 189–213.

[17] De Kanter, J., Energy absorption of monolithic and fibre reinforced aluminium cylinders. Delft University of Technology, PhD Thesis, 2006

[18] Lu, G. Yu, T.X., Energy absorption of structures and materials, Woodhead Publishing Limited, Cambridge, England. 2003.