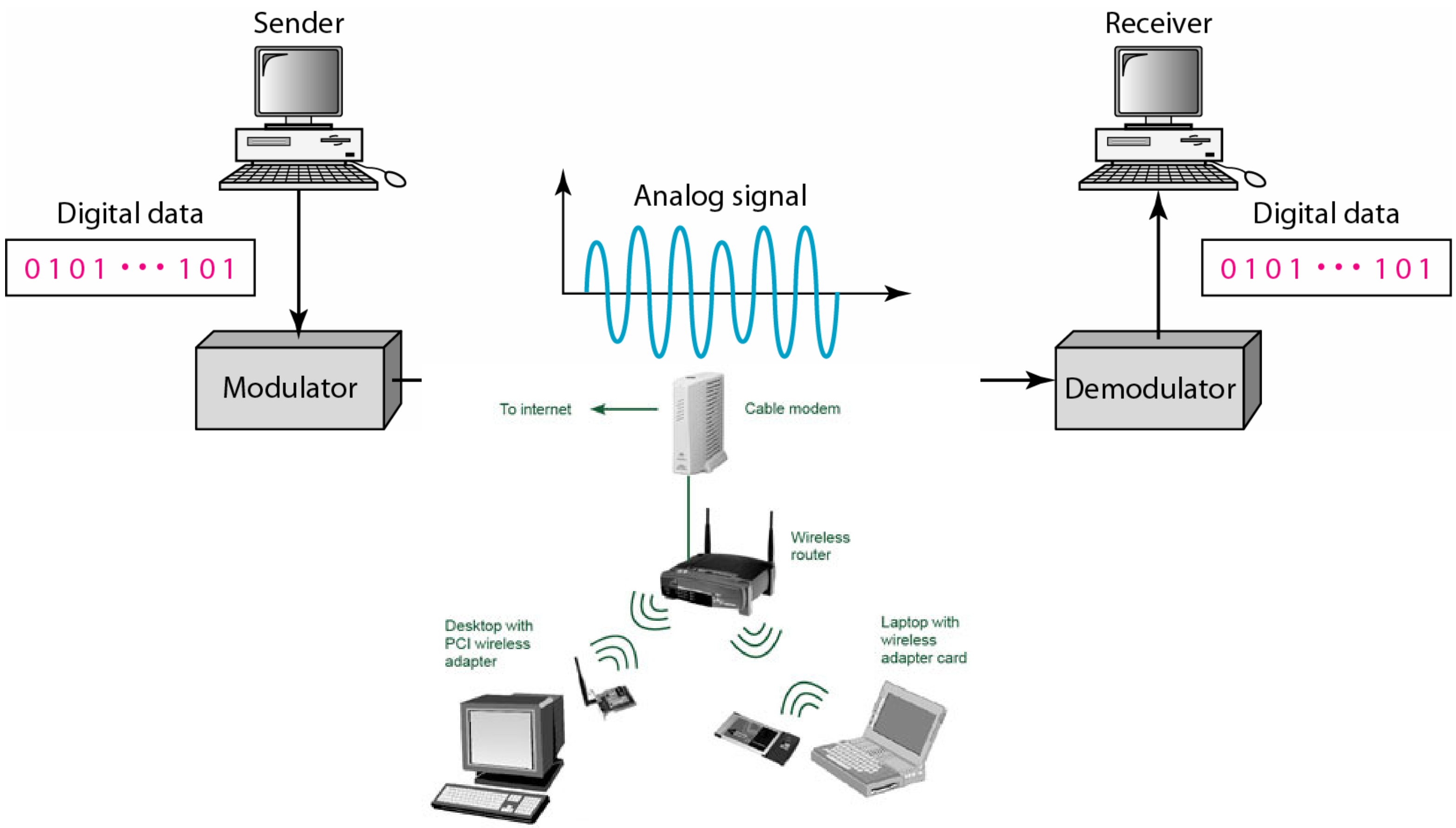
Analog Transmission of Digital Data:

ASK, FSK, PSK, QAM

Required reading:
Garcia 3.7

CSE 3213, Fall 2010
Instructor: N. Vlajic

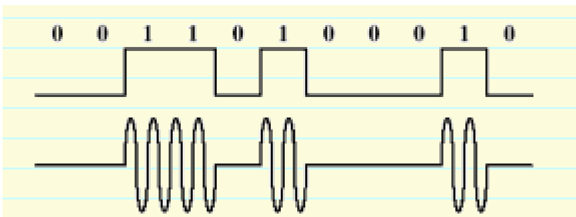
Why Do We Need Digital-to-Analog Conversion?!



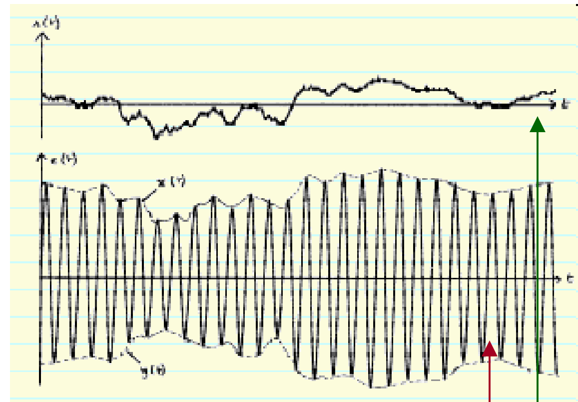
1) **The medium/channel is band pass, and/or**
2) **Multiple users need to share the medium.**

Modulation of Digital Data

Modulation – process of converting digital data or a low-pass analog to band-pass (higher-frequency) analog signal



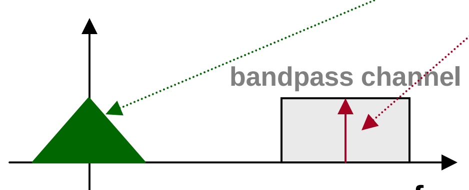
Digital-to-analog modulation.



Analog-to-analog modulation.

Carrier Signal – aka carrier freq. or modulated signal - high freq. signal that acts as a basis for the information signal

- information signal is called modulating signal

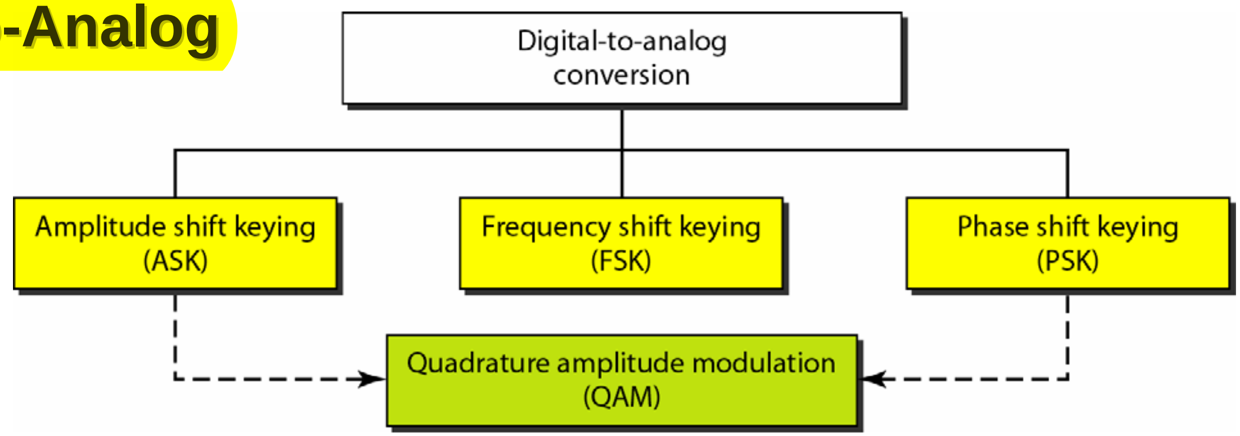


Modulation of Digital Data (cont.)

Digital-to-Analog Modulation – **process of changing one of the characteristic of an analog signal (typically a sinewave) based on the information in a digital signal**

- **sinewave is defined by 3 characteristics (amplitude, frequency, and phase) ⇒ digital data (binary 0 & 1) can be represented by varying any of the three**
- **application: transmission of digital data over telephone wire (modem)**

Types of Digital-to-Analog Modulation



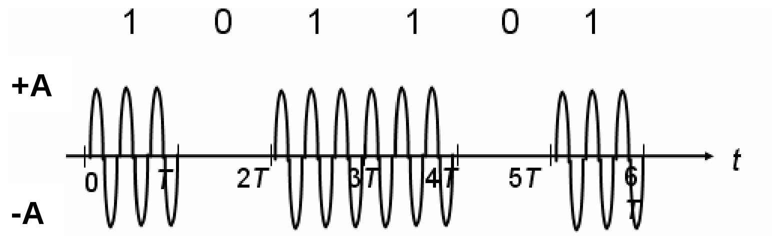
Modulation of Digital Data: ASK

ASK – strength of carrier signal is varied to represent binary 1 or 0

- **both frequency & phase remain constant while amplitude changes**
- **commonly, one of the amplitudes is zero**

$$s(t) = \begin{cases} A_0 \cos(2\pi f_c t), & \text{binary } 0 \\ A_1 \cos(2\pi f_c t), & \text{binary } 1 \end{cases}$$

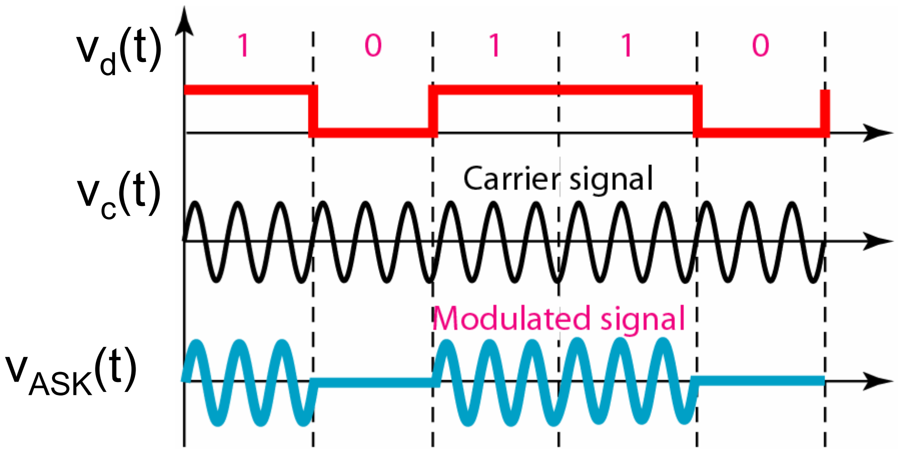
Is this picture, from the textbook, entirely correct?!

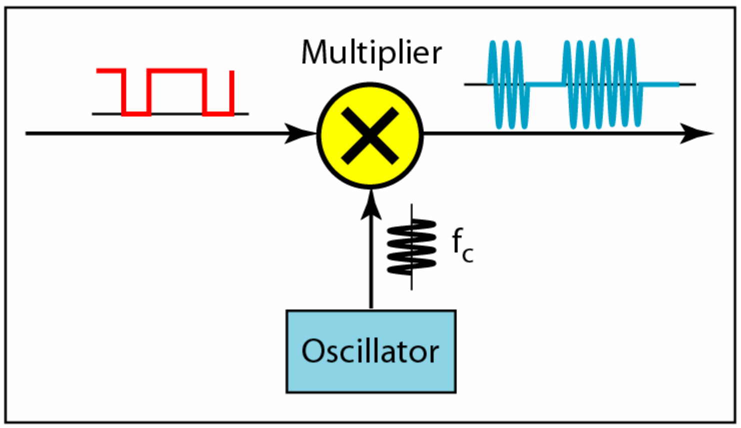


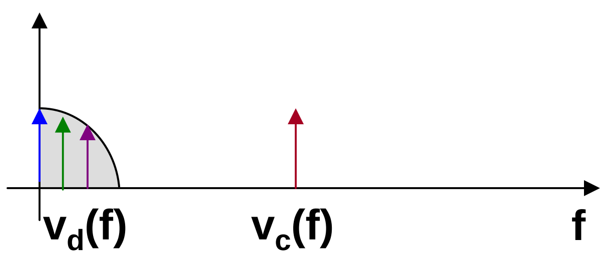
- **demodulation: only the presence or absence of a sinusoid in a given time interval needs to be determined**
- **advantage: simplicity**
- **disadvantage: ASK is very susceptible to noise interference – noise usually (only) affects the amplitude, therefore ASK is the modulation technique most affected by noise**
- **application: ASK is used to transmit digital data over optical fiber**

Modulation of Digital Data: ASK (cont.)

Example **[ASK]**







How does the frequency spectrum of $v_{ASK}(t)$ look like!?

Modulation of Digital Data: ASK (cont.)

ASK-Modulated Signal: Frequency Spectrum

$$\cos A \cdot \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$$

Carrier signal: $v_c(t) = \cos(2\pi f_c t) = \cos(\omega_c t)$, **where $2\pi f_c = \omega_c$**

Digital signal:
(unipolar!!!)

$$v_d(t) = A \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right]$$

Modulated signal:

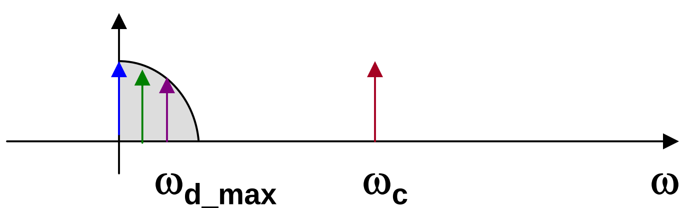
$$v_{ASK}(t) = v_c(t) \cdot v_d(t) =$$

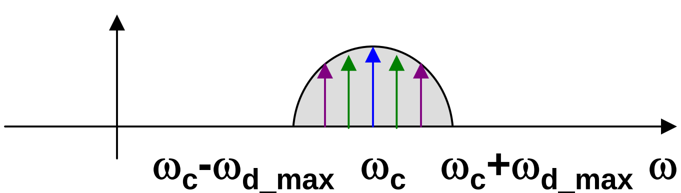
$$= \cos\omega_c t \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] =$$

$$= \frac{1}{2}\cos\omega_c t + \frac{2}{\pi}\boxed{\cos\omega_c t \cdot \cos\omega_0 t} - \frac{2}{3\pi}\boxed{\cos\omega_c t \cdot \cos 3\omega_0 t} + \ldots =$$

$$= \frac{1}{2}\cos\omega_c t + \frac{1}{\pi}\left[\boxed{\cos(\omega_c - \omega_0)t + \cos(\omega_c + \omega_0)t}\right] -$$

$$- \frac{1}{3\pi}\left[\boxed{\cos(\omega_c - 3\omega_0)t + \cos(\omega_c + 3\omega_0)t}\right] + \ldots$$



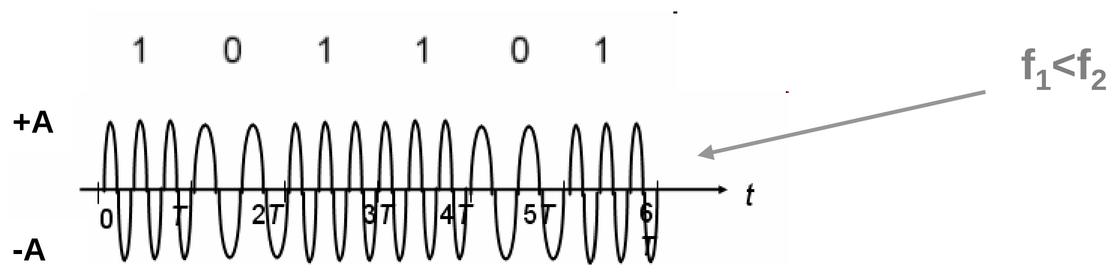


Modulation of Digital Data: FSK

FSK – frequency of carrier signal is varied to represent binary 1 or 0

- **peak amplitude & phase remain constant during each bit interval**

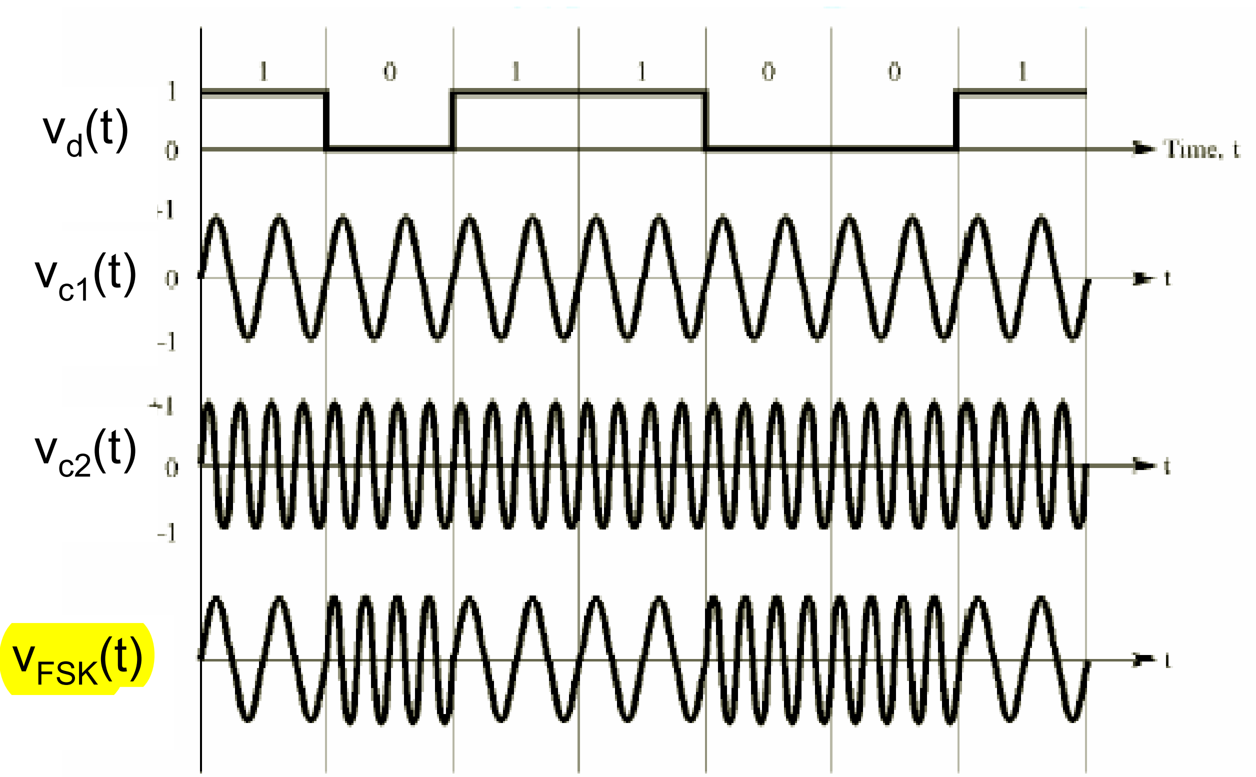
$$s(t) = \begin{cases} A\cos(2\pi f_1 t), & \text{binary } 0 \\ A\cos(2\pi f_2 t), & \text{binary } 1 \end{cases}$$

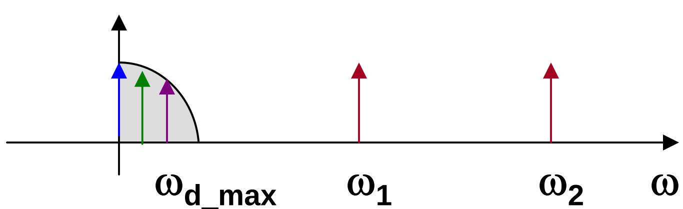


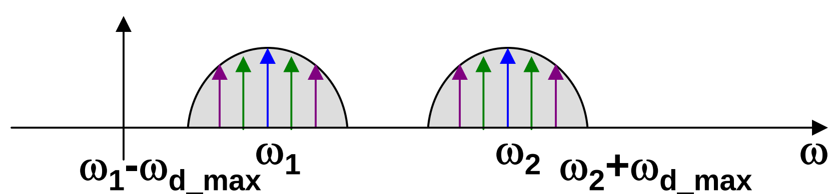
- **demodulation: demodulator must be able to determine which of two possible frequencies is present at a given time**
- **advantage: FSK is less susceptible to errors than ASK – receiver looks for specific frequency changes over a number of intervals, so voltage (noise) spikes can be ignored**
- **disadvantage: <u>FSK spectrum is 2 x ASK spectrum</u>**
- **application: over voice lines, in high-freq. radio transmission, etc.**

Modulation of Digital Data: FSK (cont.)

Example **[FSK]**







Modulation of Digital Data: FSK (cont.)

FSK-Modulated Signal: Frequency Spectrum

Digital signal: $v_d(t)$ **- modulated with ω_1, and**

$v_d'(t) = 1 - v_d(t)$ **- modulated with ω_2**

Modulated signal:

$$v_{FSK}(t) = \cos\omega_1 t \cdot v_d(t) + \cos\omega_2 t \cdot (1 - v_d(t)) =$$

$$= \cos\omega_1 t \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] +$$

$$+ \cos\omega_2 t \cdot \left[\frac{1}{2} - \frac{2}{\pi}\cos\omega_0 t + \frac{2}{3\pi}\cos 3\omega_0 t - \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] =$$

$$= \ldots$$

$$= \frac{1}{2}\cos\omega_1 t + \frac{1}{\pi}\left[\cos(\omega_1 - \omega_0)t + \cos(\omega_1 + \omega_0)t\right] -$$

$$- \frac{1}{3\pi}\left[\cos(\omega_1 - 3\omega_0)t + \cos(\omega_1 + 3\omega_0)t\right] + \ldots +$$

$$\frac{1}{2}\cos\omega_2 t - \frac{1}{\pi}\left[\cos(\omega_2 - \omega_0)t + \cos(\omega_2 + \omega_0)t\right] -$$

$$+ \frac{1}{3\pi}\left[\cos(\omega_2 - 3\omega_0)t + \cos(\omega_2 + 3\omega_0)t\right] + \ldots +$$

Modulation of Digital Data: PSK

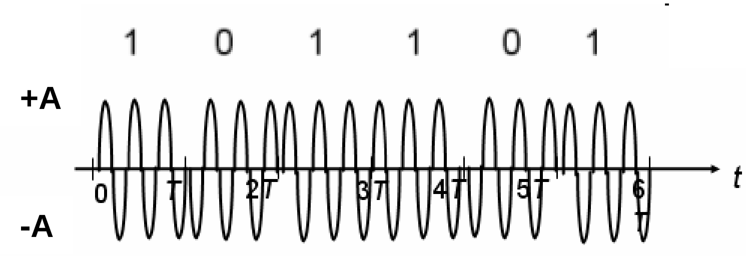
PSK – phase of carrier signal is varied to represent binary 1 or 0

- **peak amplitude & freq. remain constant during each bit interval**
- **<u>example</u>: binary 1 = 0º phase, binary 0 = 180º (πrad) phase**
  $\Rightarrow$ **PSK is equivalent to multiplying carrier signal by +1 when the information is 1, and by -1 when the information is 0**

2-PSK, or Binary PSK, since only 2 different phases are used.

$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary } 1 \\ A\cos(2\pi f_c t + \pi), & \text{binary } 0 \end{cases}$$

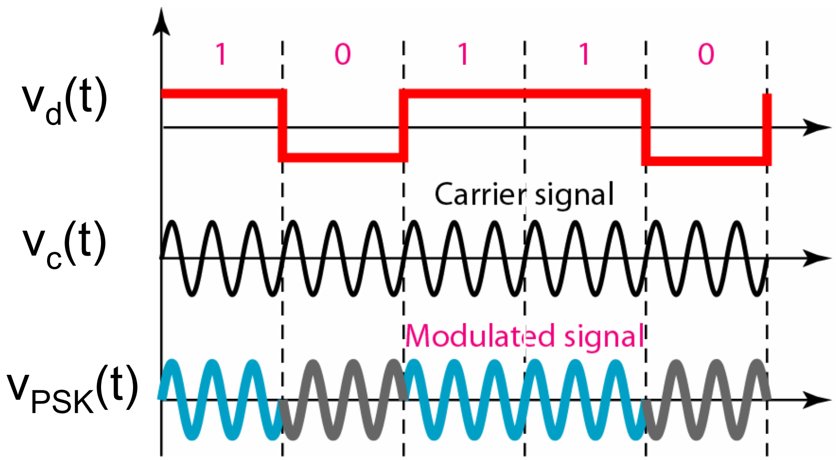
$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary } 1 \\ -A\cos(2\pi f_c t), & \text{binary } 0 \end{cases}$$

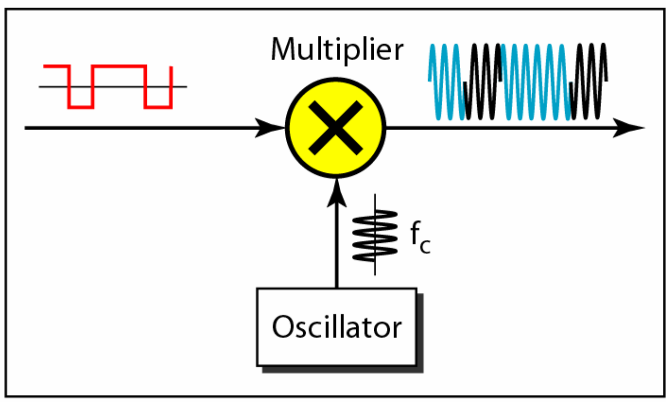


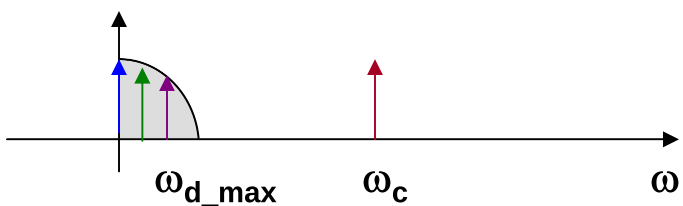
- **demodulation: demodulator must determine the phase of received sinusoid with respect to some reference phase**
- **advantage:**
  - **PSK is less susceptible to errors than ASK, while it requires/occupies the same bandwidth as ASK**
  - **more efficient use of bandwidth (higher data-rate) are possible, compared to FSK !!!**
- **disadvantage: more complex signal detection / recovery process, than in ASK and FSK**

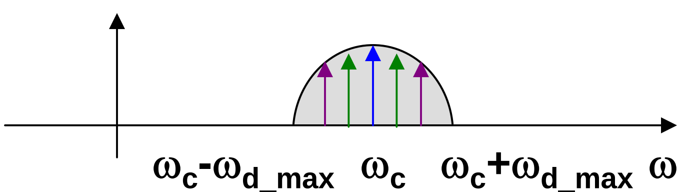
Modulation of Digital Data: PSK (cont.)

Example **[PSK]**









Modulation of Digital Data: PSK (cont.)

PSK Detection

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

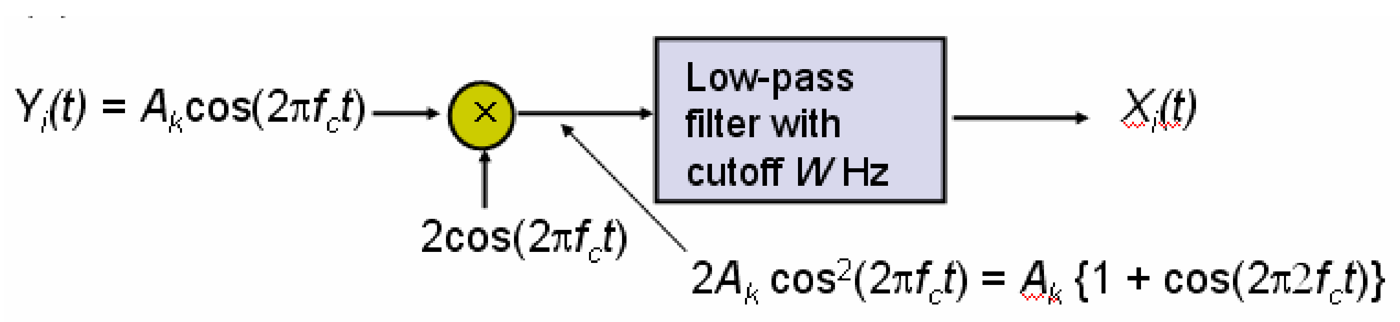
– **multiply the received / modulated signal** $\pm A\cos(2\pi f_c t)$ **by** **$2*\cos(2\pi f_c t)$**

- **resulting signal**

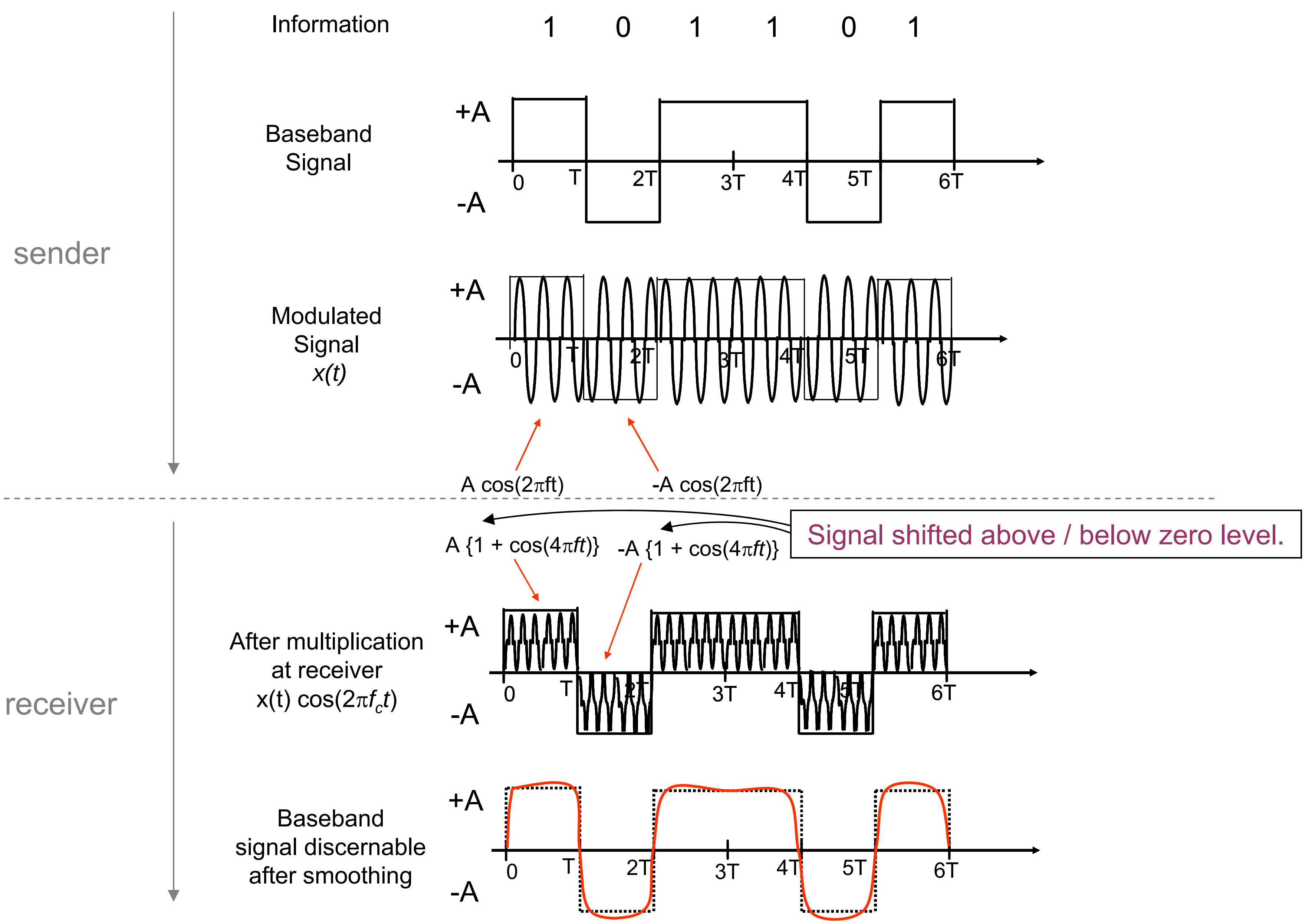
$$2A\cos^2(2\pi f_c t) = A[1 + \cos(\boxed{4\pi f_c} t)], \quad \text{binary } 1$$

$$-2A\cos^2(2\pi f_c t) = -A[1 + \cos(\boxed{4\pi f_c} t)], \quad \text{binary } 0$$

- **by removing the oscillatory part with a low-pass filter, the original baseband signal (i.e. the original binary sequence) can be easily determined**



Modulation of Digital Data: PSK (cont.)



Modulation of Digital Data: PSK (cont.)

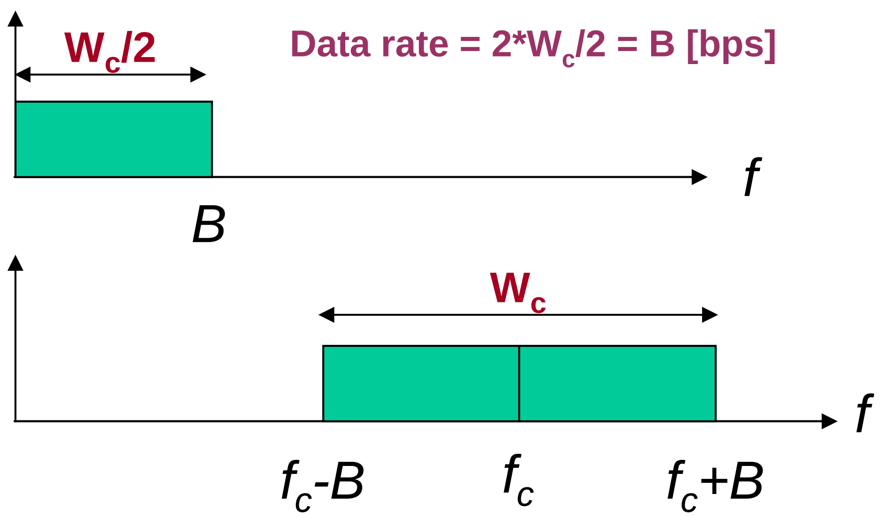
Facts from Modulation Theory

If

Baseband signal *x(t)* with bandwidth $W_c/2$

then

Modulated signal $x(t)\cos(2\pi f_c t)$ has bandwidth W_c Hz



- **If bandpass channel has bandwidth W_c [Hz],**
  - **then baseband channel has $W_c/2$ [Hz] available, so**
  - **modulation system supports $2*(W_c/2) = W_c$ [pulses/second]**
  - **recall Nyqyist Law: baseband transmission system of bandwidth W_c [Hz] can theoretically support 2 W_c pulses/sec**

How can we recover the factor 2 in supported data-rate !?

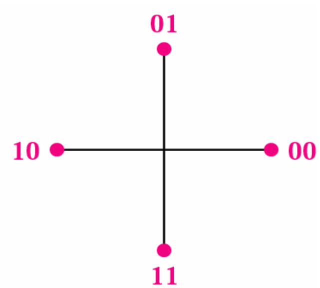
Modulation of Digital Data: PSK (cont.)

QPSK = 4-PSK – PSK that uses phase shifts of 90º=π/2 rad $\Rightarrow$ 4 different signals generated, each representing 2 bits

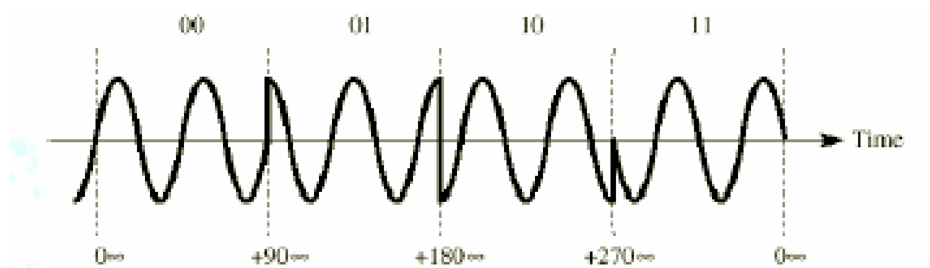
$$s(t)=\begin{cases} A\cos(2\pi f_c t), & \text{binary } 00 \\ A\cos(2\pi f_c t+\frac{\pi}{2}), & \text{binary } 01 \\ A\cos(2\pi f_c t+\pi), & \text{binary } 10 \\ A\cos(2\pi f_c t+\frac{3\pi}{2}), & \text{binary } 11 \end{cases}$$

Dibit	Phase
00	0
01	90
10	180
11	270

Dibit
(2 bits)



Constellation diagram



- **advantage: higher data rate than in PSK (2 bits per bit interval), while bandwidth occupancy remains the same**
- **4-PSK can easily be extended to 8-PSK, i.e. n-PSK**
- **however, higher rate PSK schemes are limited by the ability of equipment to distinguish small differences in phase**

Modulation of Digital Data: QAM

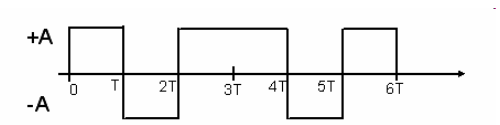
Quadrature Amplitude Modulation (QAM) – **uses "two-dimensional" signalling**

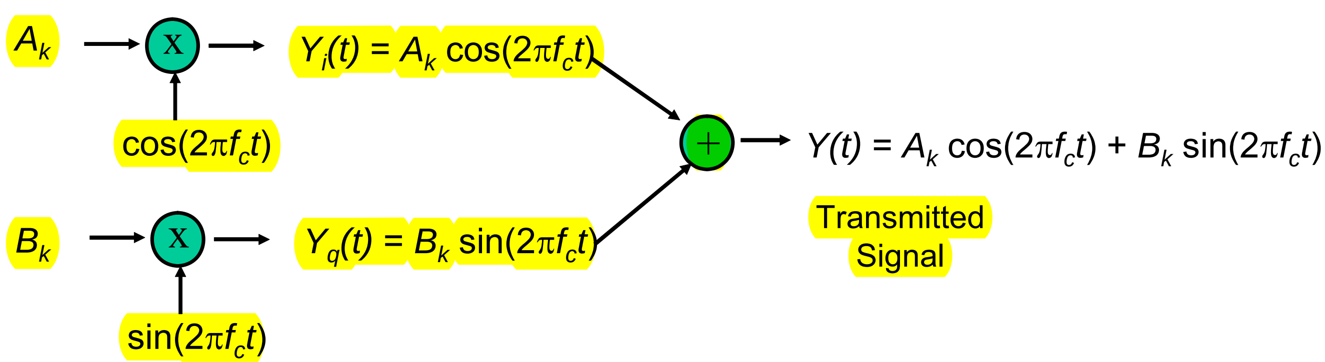
- **original information stream is <u>split into two sequences</u> that consist of odd and even symbols, e.g. B_k and A_k**

1	0	1	1	0	1	…
1	**-1**	**1**	**1**	**-1**	**1**	**…**
B_1	A_1	B_2	A_2	B_3	A_3	**…**

- **A_k sequence (in-phase comp.) is modulated by** $\cos(2\pi f_c t)$
  B_k sequence (quadrature-phase comp.) is modulated by $\sin(2\pi f_c t)$

- **composite signal** $A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t)$ **is sent through the channel**

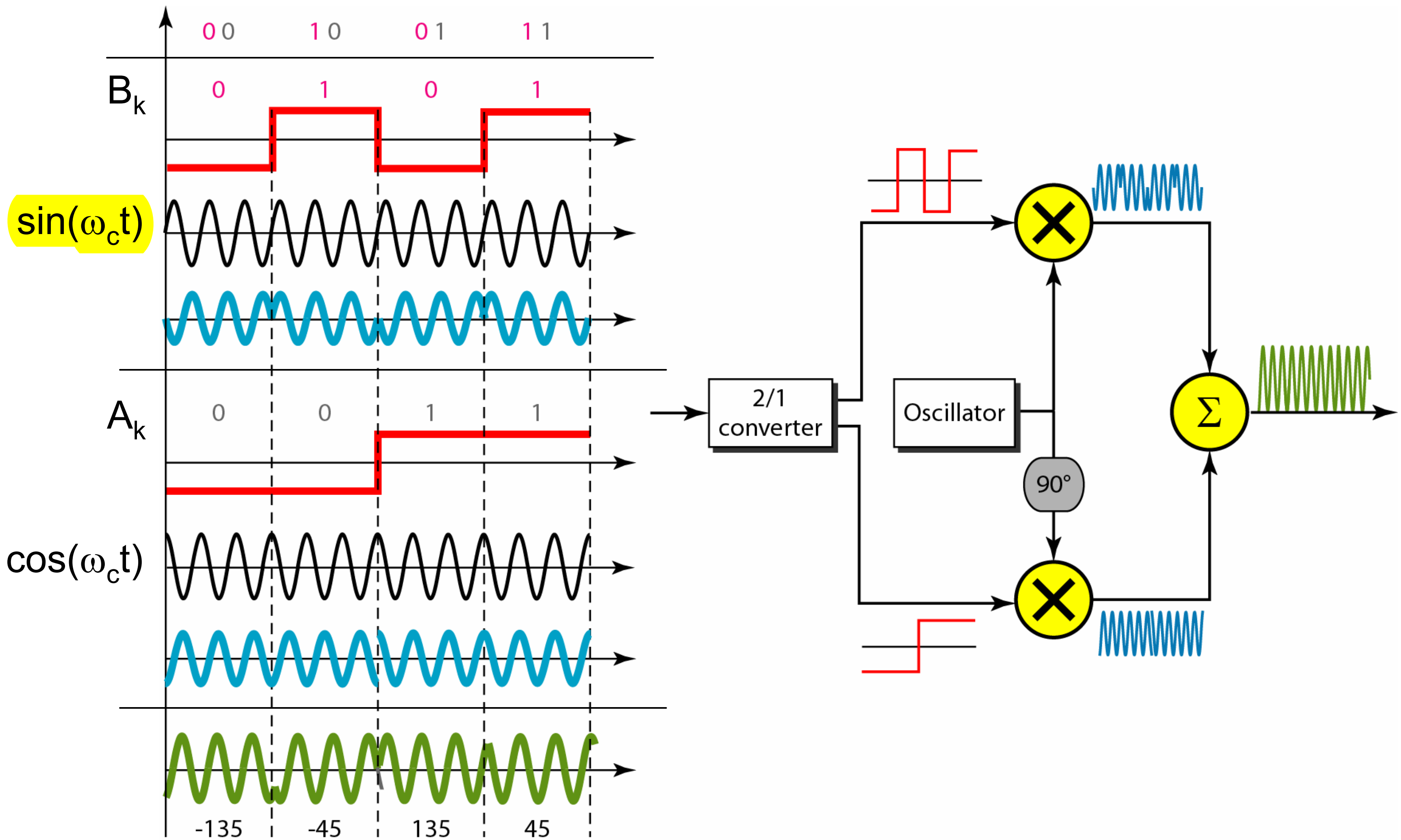




- **advantage: data rate = 2 bits per bit-interval!**

Modulation of Digital Data: QAM (cont.)

Example [QAM]

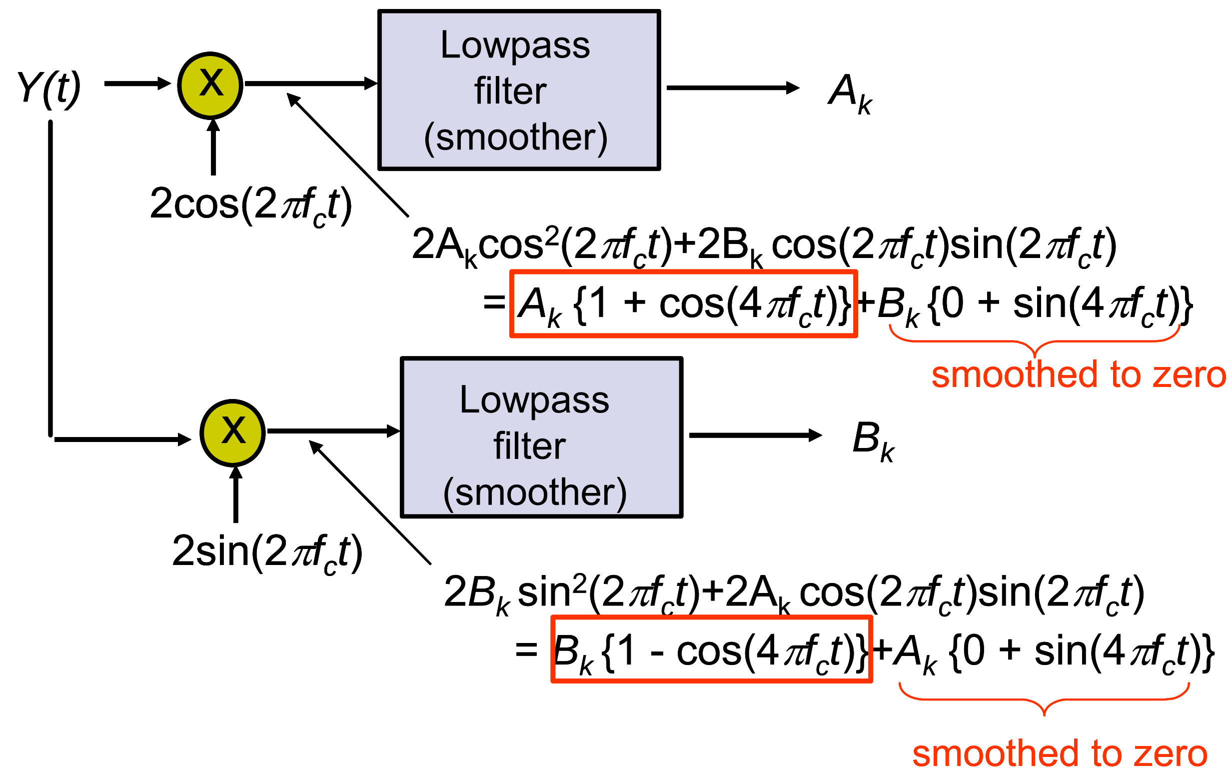


Modulation of Digital Data: QAM

QAM Demodulation

- **by multiplying Y(t) by** $2\cdot\cos(2\pi f_c t)$ **and then low-pass filtering the resultant signal, sequence A_k is obtained**
- **by multiplying Y(t) by** $2\cdot\sin(2\pi f_c t)$ **and then low-pass filtering the resultant signal, sequence B_k is obtained**

$A_k\cos(2\pi f_c t)+B_k\sin(2\pi f_c t)=$ $Y(t)$



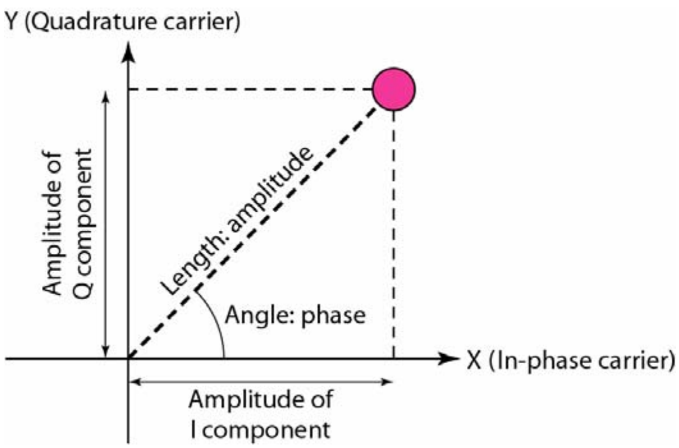
$$\cos^2(A)=\frac{1}{2}(1+\cos(2A))$$

$$\sin^2(A)=\frac{1}{2}(1-\cos(2A))$$

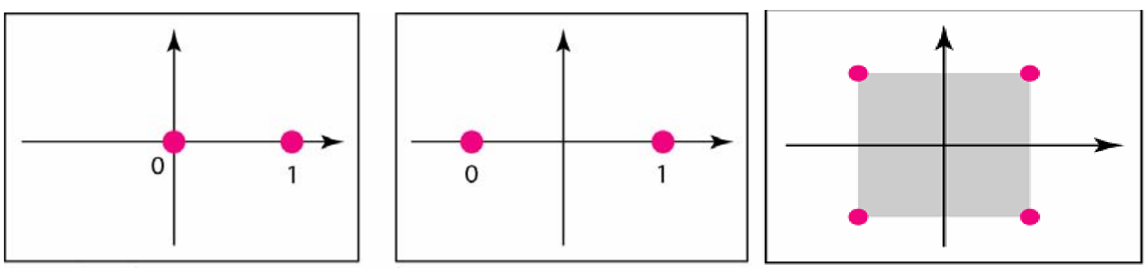
$$\sin(2A)=2\sin(A)\cos(A)$$

Signal Constellation

Constellation Diagram – **used to represents possible symbols that may be selected by a given modulation scheme as points in 2-D plane**



- **X-axis is related to in-phase carrier:** $\cos(\omega_c t)$
  - **the projection of the point on the X-axis defines the peak amplitude of the in-phase component**
- **Y-axis is related to quadrature carrier:** $\sin(\omega_c t)$
  - **the projection of the point on the Y-axis defines the peak amplitude of the quadrature component**
- **the length of line that connects the point to the origin is the peak amplitude of the signal element (combination of X & Y components)**
- **the angle the line makes with the X-axis is the phase of the signal element**



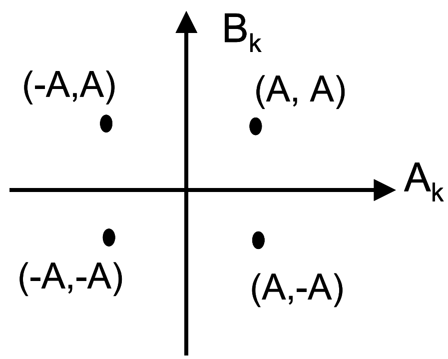
a. ASK (OOK) b. BPSK b. 4-QAM

Modulation of Digital Data: QAM

QAM cont. – **QAM can also be seen as a combination of ASK & PSK**

$$Y(t) = A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t) = \left(A_k^{\,2} + B_k^{\,2}\right)^{\frac{1}{2}} \cos(2\pi f_c t + \tan^{-1}\frac{B_k}{A_k})$$

4-level QAM

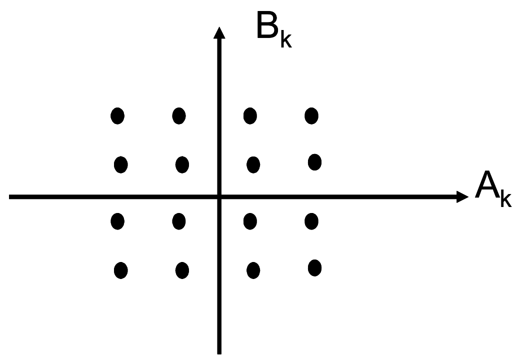


Modulation of Digital Data: QAM

16-level QAM – **the number of bits transmitted per T [sec] interval can be further increased by increasing the number of levels used**

- **in case of 16-level QAM, A_k and B_k individually can assume 4 different levels: -1, -1/3, 1/3, 1**
- **data rate: 4 bits/pulse $\Rightarrow$ 4W bits/second**

$$Y(t) = A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t) = \left(A_k{}^2 + B_k{}^2\right)^{\frac{1}{2}} \cos(2\pi f_c t + \tan^{-1}\frac{B_k}{A_k})$$



A_k and B_k individually can take on 4 different values; the resultant signal can take on (only) 3 different values!!!

In QAM various combinations of amplitude and phase are employed to achieve higher digital data rates.

Amplitude changes are susceptible to noise $\Rightarrow$ the number of phase shifts used by a QAM system is always greater than the number of amplitude shifts.