PME Contributed Session on Research by Undergraduates

Game of Life on Penrose Tiling : Robinson Triangle

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Part 01

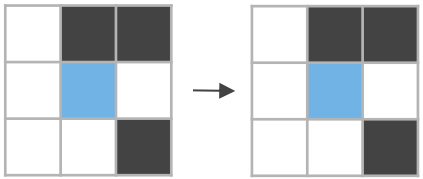
Key Concepts

01

Conway’s Game of Life

A mathematical game starting with initial configuration of live cells and observing how it evolves.

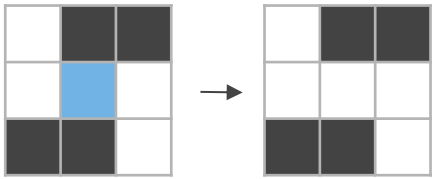
Survival



The cell **survives** when it has:

- Two or three alive neighbors

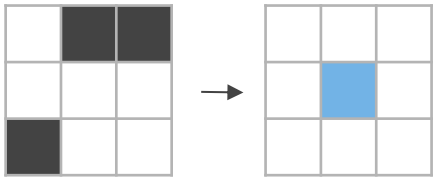
Death



The cell **dies** when it has:

- Four or more alive neighbors
- One or no alive neighbors

Birth



The cell has **birth** when it has:

- Three alive neighbors

01

Penrose Tiling

Sets of tiles with two different shapes that tile only nonperiodically, by mathematician Roger Penrose

Tiling

A **tiling (of the plane)** is a collection of subsets of the plane, i.e. tiles, which cover the plane without gaps or overlaps.

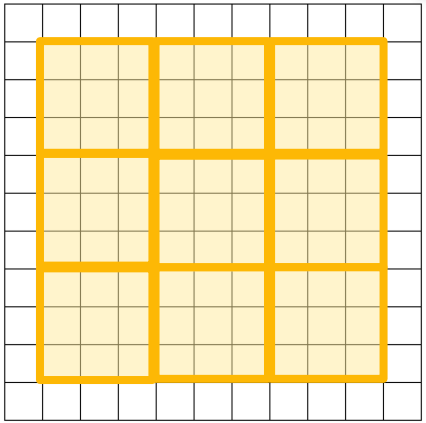
Periodic and Nonperiodic Tiling

A **periodic tiling** is one on which you can outline a region that tiles the plane by translation, that is, by shifting the position of the region without rotating or reflecting it. A **nonperiodic tiling** is not periodic.

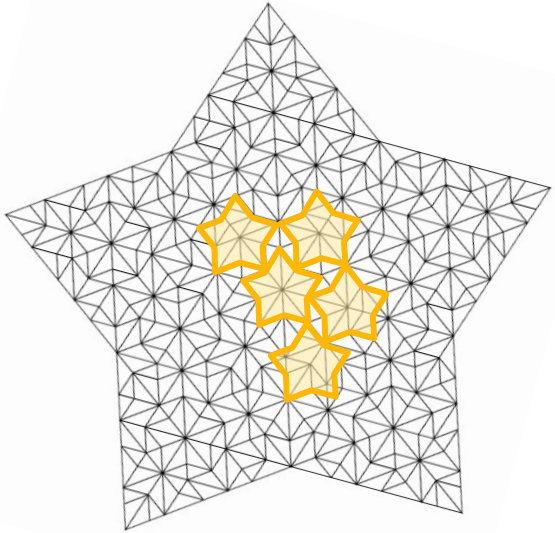
01

Periodic and Nonperiodic Tiling

A periodic tiling is one on which you can outline a region that tiles the plane by translation, by shifting the position of the region without rotating or reflecting it. A nonperiodic tiling is not periodic.



Periodic Tiling

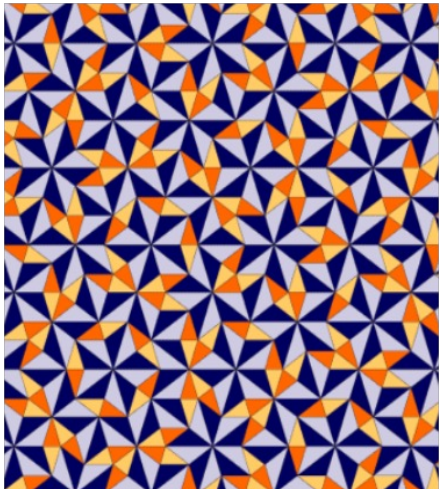


Nonperiodic Tiling

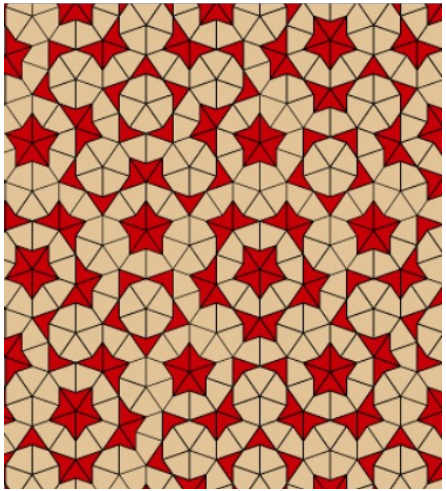
01

Variations of Penrose Tiling

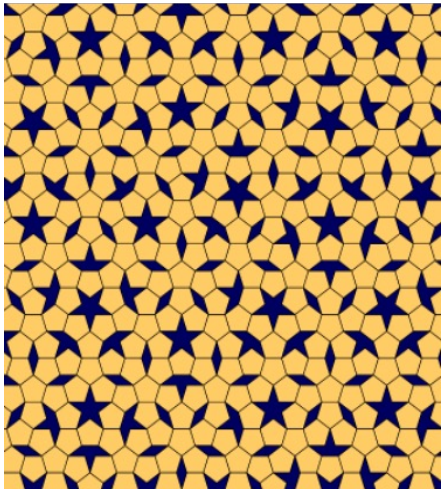
Different version of Penrose tiling (Image from Bielefeld Tiling Encyclopedia)



01.
Robinson Triangle



02.
Kite and Dart



03.
Pentagon Boat Star

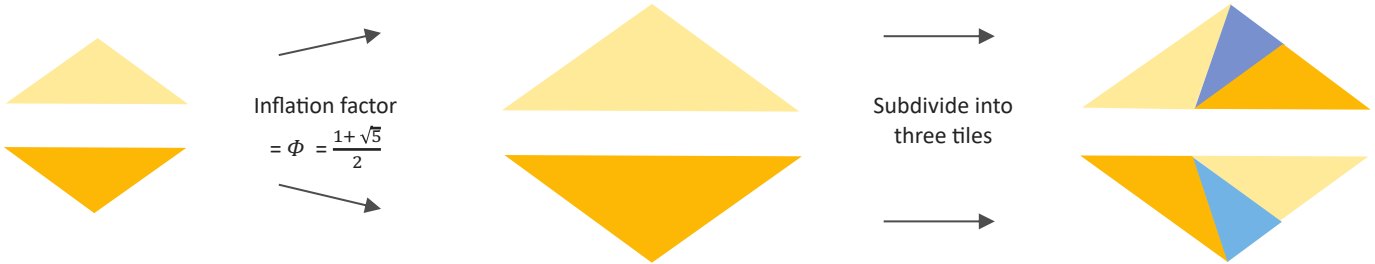
Part 02

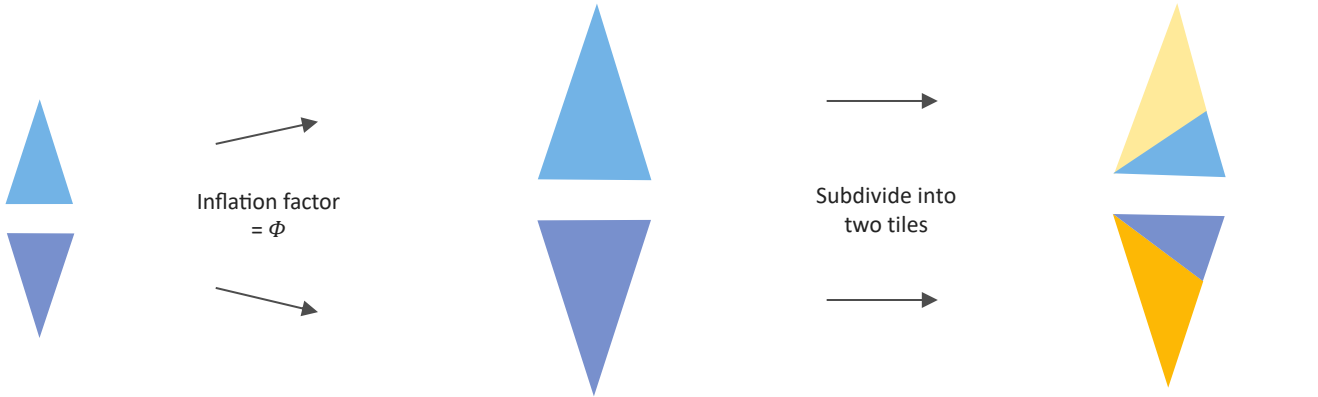
Game of Life Algorithms

02

Tiling using substitution method

There are four tiles in the tiling and four substitution (inflate & subdivide) rules to apply

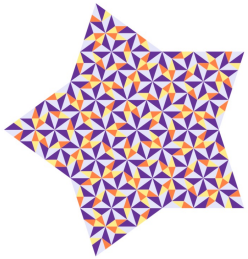


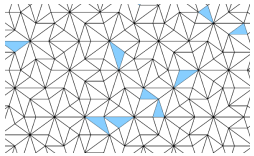


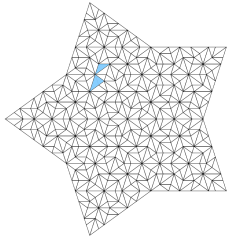
02

Three steps to implement Game of Life on Robinson Triangle Tiling

Create multiple function using *Julia*







1. Generate Tiling using Substitution Method

Initial configuration `[tile type, position, orientation]`, number of substitutions, and side length of a tile is given.

2. Implement Game of Life on Robinson Triangle Tiling

Using neighboring tile's information, apply Game of Life rules to each tile when initial live cell list is given as an input.

3. Get graphics of each generation and make animations

Using live cell list of each generation, get a graphic of Game of Life. Using those graphics, generate animations.

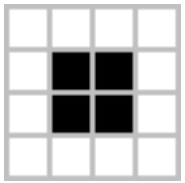
Part 03

Finding Patterns

Terms for Patterns from Game of Life

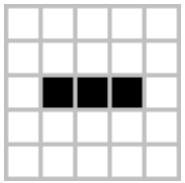
Still Life, Oscillator, Spaceship (Images from Wikipedia-Conway's Game of Life)

Still life



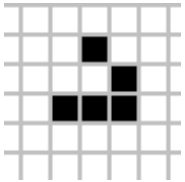
- Configuration does not change from one generation to the next.
- No death or birth.

Oscillators



- Returns to initial configuration after a finite number of generations.

Spaceships

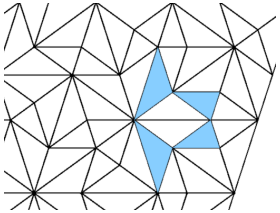
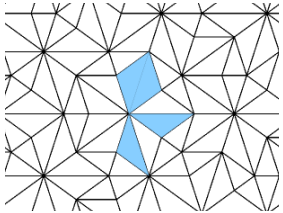
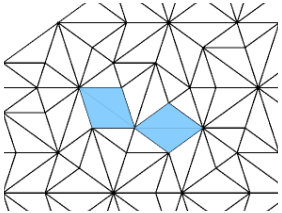
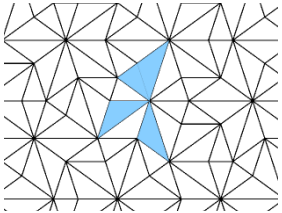
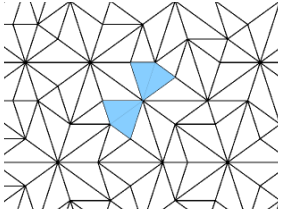
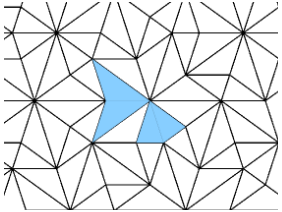
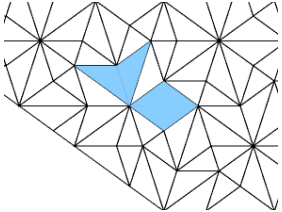
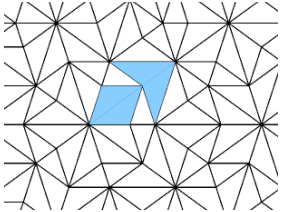


- Each configuration reappears after a certain number of generations in the same orientation but in a different position.

03

Finding patterns: still life

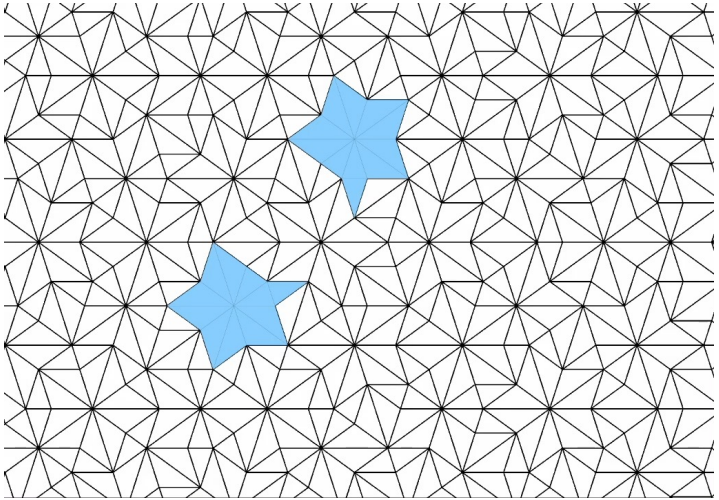
Playing Game of Life multiple times until we find interesting still life patterns

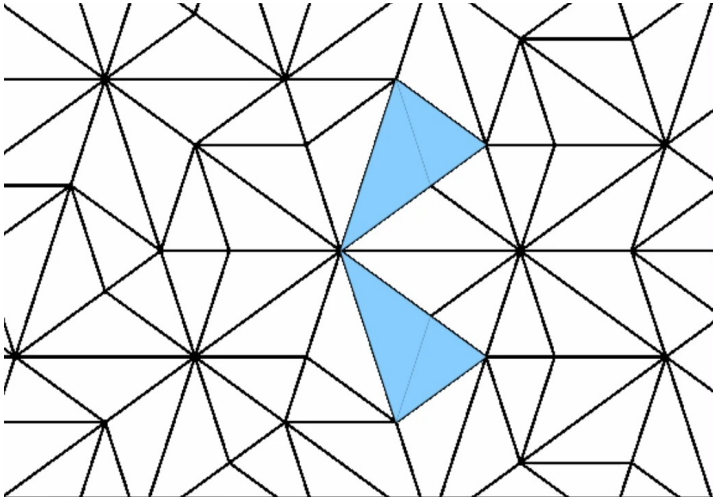
03

Finding patterns: oscillator

Playing Game of Life multiple times until we find interesting oscillator patterns



period 14 oscillator



period 4 oscillator

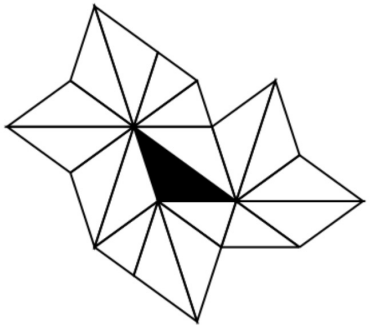
Part 04

Classification

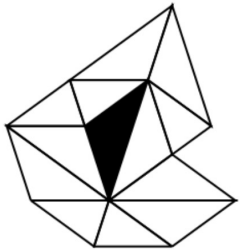
04

Classification

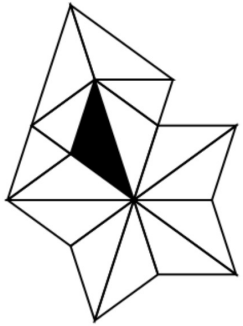
Identify distinct neighborhoods and find four-cell still life in each neighborhoods



18 neighbors



12 neighbors

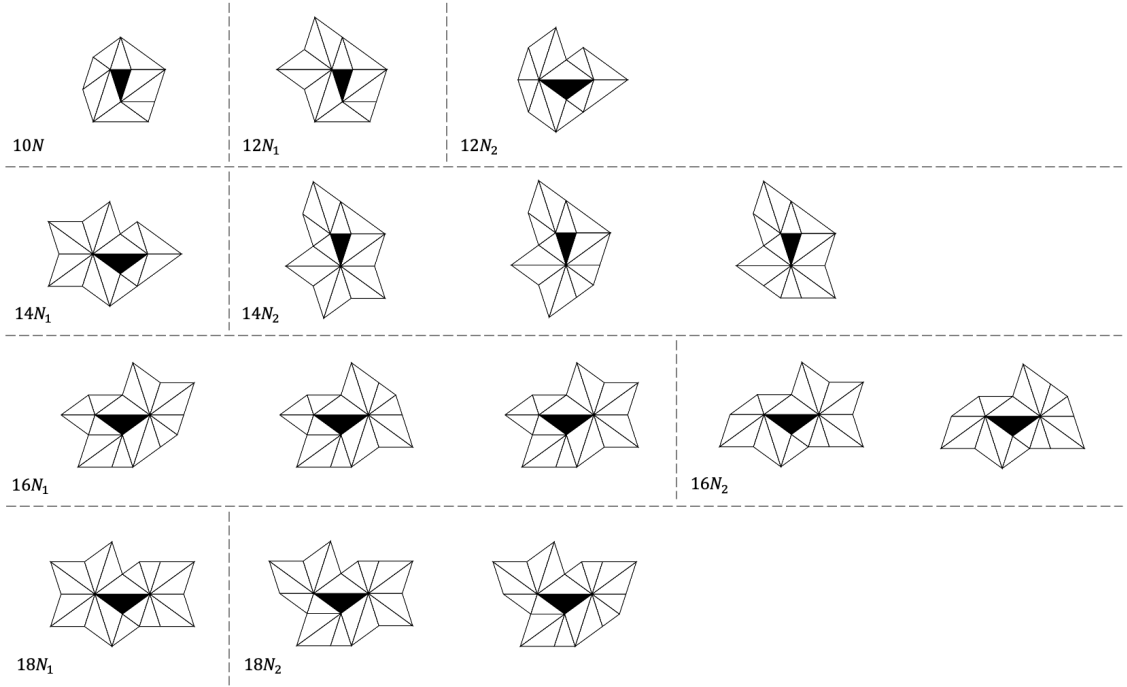


14 neighbors

04

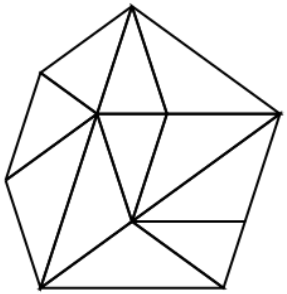
Neighborhoods

Nine distinct neighborhoods in Robinson triangle tiling



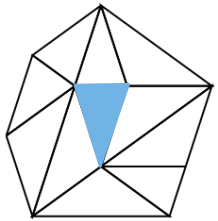
Classification of four-cell still life

Create two algorithms to find valid four-cell still life configuration in each neighborhoods



$10N$

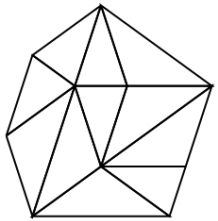
Case 1



- **Alive center cell + 3 neighbors**

Among $\binom{10}{3} = 120$ configurations,
use four-layered algorithm that finds/get rid of configurations with birth.
Remaining configurations are four-cell still life.

Case 2



- **Dead center cell + 4 neighbors**

Among $\binom{10}{4} = 210$ configurations,
use five-layered algorithm that finds/get rid of configurations with birth **and death**.
Remaining configurations are four-cell still life.

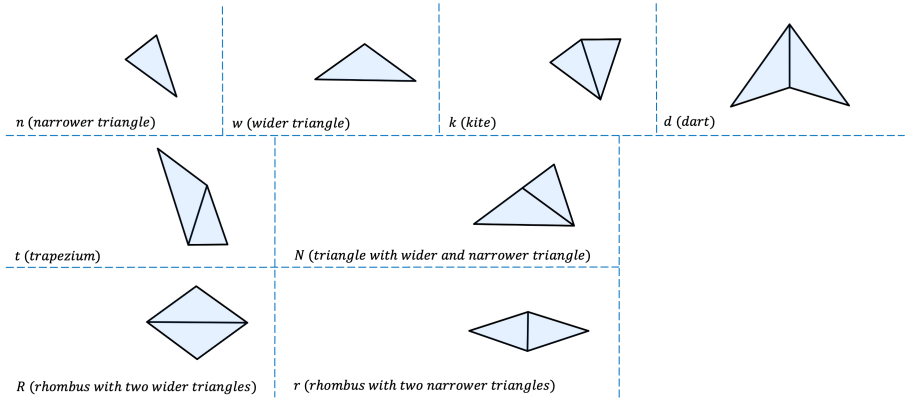
04

Classification of four-cell still life

Categories of all four-cell still life in Robinson triangle tiling

Neighborhood	Number of valid still life
$10N$	7
$12N_1$	42
$12N_2$	42
$14N_1$	154
$14N_2$	154
$16N_1$	159
$16N_2$	189
$18N_1$	301
$18N_2$	194

Classified 1242 four-cell still life into 31 categories with labels



$d-d$

$d-k$

$d-N$

$d-r$

$d-R$

$d-t$

$d-w-w$

$k-k$

$k-N$

$k-R$

$k-w-w$

$n-d-w$

$n-k-w$

$N-N$

$n-n-d$

$n-n-R$

$n-n-w-w$

$N-R$

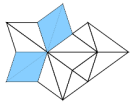
$N-t$

$n-w-N$

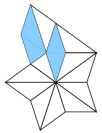
$n-w-R$

$n-w-w-w$

$r-R$



$R-R$



$r-t$

$R-t$

$w-w-N$

$w-w-r$

$w-w-R$

$w-w-t$

$w-w-w-w$

Classification of four-cell still life 31 categories

Reference

Wikipedia. Game of Life.
https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life

Kathryn Lindsey. What is Tiling? http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page1.htm#:~:text=The%20Mathematical%20Definition%20of%20Tilings&text=In%20mathematics%2C%20a%20tiling%20(of,plane%20without%20gaps%20or%20overlaps.

Siobhan Roberts. The lasting lessons of john conway's game of life.https://www.nytimes.com/2020/12/28/science/math-conway-game-of-life.html?searchResultPosition=1, accessed June 1, 2022.

Martin Gardner. Mathematical games: The fantastic combinations of john conway'snew solitaire game "life". Scientific American, 223(4):120–123, 1970.

Martin Gardner. Mathematical games: Extraordinary nonperiodic tiling that enrichesthe theory of tiles. Scientific American, 236(1):110–121, 1977.

Colin Adams. The tiling book: An introduction to the mathematical theory of tilings.unpublished, N.D.

Bielefeld University. Penrose pentagon boat star. https://tilings.math.uni-bielefeld.de/substitution/penrose-pentagon-boat-star/, accessed June18, 2022.

Bielefeld University. Penrose kite dart. https://tilings.math.uni-bielefeld.de/substitution/penrose-kite-dart/, accessed June 18, 2022.

Bielefeld University. Penrose rhomb. https://tilings.math.uni-bielefeld.de/substitution/penrose-rhomb/, accessed June 18, 2022.

Bielefeld University. Robinson triangle. https://tilings.math.uni-bielefeld.de/substitution/robinson-triangle/, accessed June 18, 2022.

Bielefeld University. Substitution. https://tilings.math.uni-bielefeld.de/glossary/substitution/, accessed June 18, 2022.

Nick Owens and Susan Stepney. The Game of Life Rules on Penrose Tilings: StillLife and Oscillators, pages 331–378. Springer London, London, 2010.

PowerPoint design references: Peedori's PowerPoint, Saebyeol's PowerPoint

THANK YOU

Q & A