Time series analysis on racial hate crimes in the United States:

Regression with ARIMA errors, spectral analysis, GARCH, and ARFIMA

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Introduction

Often, it is hard to understand the reality of life-threatening risks that minorities face. Sometimes people's bias is painfully mundane and subtle, but other times it is grand, horrifying, and unexplainably personal. For example, discrimination and bias against certain racial groups led to tragic, traumatic incidents, such as the death of George Floyd and the Atlanta spa shootings. Recently, hate crimes motivated by the offender's bias against different races are increasing, with Blacks or African Americans and Asians becoming victims most frequently. Acknowledging the urgency to further understand the trend of the racial hate crimes, this project focuses on finding explanatory variables that could help explain the variability of racial hate crimes and ultimately find the best predictive models using various time series methods. We produce different time series models for all racial hate crimes using spectral analysis and regressions with ARIMA errors with explanatory variables. In addition, as the mid-semester project showed how difficult it is to find a model that fits anti-Asian hate crime data, we implemented GARCH and ARFIMA. Finally, we find the best predictive models of racial hate crimes, comparing all models.

Methods

1. Data

Data Acquisition and Biases

For this project, we use datasets from the Federal Bureau of Investigation (FBI), the United States Bureau of Labor Statistics, and the national centers for environmental information (NOAA). Our response variable, the number of racial hate crime cases comes from the FBI Crime Data Explorer. It provides data of hate crimes from 1991 to 2020. Additionally, we get non-racial hate crimes[1] data to be used as our explanatory variable. The U.S. Bureau of Labor Statistics has data related to seasonally unadjusted

[1] Hate crimes motivated by offenders' bias against religion, sexual orientation, disability, gender, and gender identity.

monthly unemployment rate and consumer price index (CPI). The NOAA has the monthly average temperature of the U.S. We use these three variables as our explanatory variables. Data from three sources are publicly available in CSV format.

Since this project uses the same hate crime data from the mid-semester project, we have the same concern with the underreporting derived from various factors such as different levels of police training and psychological factors that affected victims to underreport (Hong, 2022). Even though we have potential concerns, the FBI data is built based on data collected from multiple independent agencies. Thus, we can consider data to be random but we focus on providing the lower bound of racial hate crimes when making predictive models. Once we have more complete datasets that overcome the issue with underreporting, we can have better models that could be generalized.

The unemployment rate is calculated by the Census Bureau through the Current Population Survey (CPS) and it uses a probability-selected sample of approximately 60,000 eligible households (U.S. Census Bureau, 2019). Therefore, the data could be considered a random sample of the U.S. population. Each month, prices for about 80,000 goods and services in 75 urban areas across the nation are collected to generate the CPI. Urban areas consist of about 6,000 housing units and approximately 23,000 retail establishments (U.S. Bureau of Labor Statistics, 2021). Since multiple sources collect data to generate CPI through the robust method used by the U.S. Bureau of Labor Statistics, we assume the CPI data to be random. However, the CPI only considers prices of goods and services in urban areas and we need to be cautious that the data are not representative of the whole population.

The average temperature data is based on observations that are already adjusted to address the artificial effects affecting the climate record such as changes in instrument, location of observation, and different levels of training of observers which could affect the data (NOAA, 2022). Thus, this data also possesses randomness. However, the average temperature includes data from all states. Depending on the geographical location, the climate differs and using the average of all states could be a vague generalization. Therefore, future research could narrow down to state-level analysis on racial hate crimes to address such issues.

2. Analysis

Assumption and Validation

For the purpose of statistical analysis, we assume that each data point for our response variables and explanatory variables are independent. Also, by excluding multiple-biased hate crimes from our hate crime data, we make the assumption that each hate crime is motivated by single bias. To validate each model, we check whether the model satisfy the stationarity and independence condition. To find the best predictive model, we compare Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), R-squared value, of each model. Also, we divide each data into two subsets and check the R-squared value of different models and calculate normalized RMSE. Once we find the best predictive model, we predict racial hate crimes for two years, 2021 and 2022.

Models

From the mid-semester project, we generated function of time and SARIMA models for three time series. To find the best predictive models for three categories of racial hate crimes, we implement four additional methods. First, we create function of time models using the spectral analysis. The spectral analysis finds the most significant frequencies that could explain the variability of the given time series. Second, we fit regressions with ARIMA errors with explanatory variables. Often, a response variable is associated with different explanatory variables as well as the past of itself and its error. Thus, regressions with ARIMA errors help identify statistically significant variables among those variables which could explain the given time series. Third, we utilize GARCH. When a given time series has burst in variability after a certain period of time, we can fit ARMA to the variance. Finally, we use ARFIMA. When the ACF shows non-exponential decay in lags, applying first-order difference to the given time series could lead to over-differencing. In this case, we apply fractional differencing with ARFIMA. With four additional methods, we fit our time series data and compare model performance of all models we have so far.

We use *Python 3* to collect and clean data (Van Rossum & Drake, 2009) and *R v4.1.3* (R Core Team, 2022) for generating predictive models.

Results and Conclusions

1. Spectral Analysis

When decomposing all three time series, we can see that the trend is complicated to be detrended in a parametric way (Figure A1). Therefore, we detrend each time series nonparametrically and get periodograms to find the most significant periods and frequencies (Figure A2). The spectral analysis suggests the 12 months, yearly cycle as the most significant frequency for all racial hate crimes and anti-Black of African American hate crimes and the 90 months cycle as the most significant frequency for anti-Asian hate crimes. Using these frequencies, we fit function of time model to each time series. All three model show that two independent variables are statistically significant ($p < 0.05$). However, all models have low adjusted R-squared[2] with a little concern remaining with linearity of the residuals (Figure A3). Thus, function of time models using frequencies detected from the spectral analysis are not good for the prediction.

2. Regression with ARIMA errors with explanatory variables

All racial hate crimes

We check if our response variable and explanatory variables[3] have a linear relationship (Figure A4). Then we generate a stepwise regression model to find which variables to include in our regression with ARIMA errors. The stepwise regression suggests four explanatory variables, unemployment rate, CPI, temperature, and non-racial hate crimes. Using selected variables, we generate a regression model and check the ACF and PACF of the residuals and fit ARMA to the errors. Among five different models, the model equation for the best model is as follows:

$$HC_t = 140.055 + 1.744 temp_t + 17.396 unemp_t - 1.198 cpi_t + 1.334 non\ racial\ HC_t + \eta_t$$

$$\eta_t = 0.961\eta_{t-1} + \varepsilon_t - 0.7289\varepsilon_{t-1}$$

The racial hate crime of the current month t (HC_t) is positively correlated with the average temperature of current month ($temp_t$), the unemployment rate of current month ($unemp_t$), and the non-racial hate crime of current month ($non\ racial\ HC_t$). However, it is negatively correlated with the CPI of current month (cpi_t). Also, the result suggests ARMA(1,1) for the residuals. Thus, the residuals of the current month (η_t) is associated with the residuals of the last month (η_{t-1}), the error term of the residual of the current month (ε_t), and the error term of the residuals of the past month (ε_{t-1}). The time series plot of the residuals of our final model looks stationary, passes four unit root tests, and has no significant autocorrelation identified from ACF and PACF (Figure A5). Also, this model has the best AICc, BIC, and R-squared values (3987.95, 4018.62, 0.7150). Thus, we can use this model for prediction. The residuals look normal and we can trust forecast intervals (Figure A5).

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[2] Three function of time models with one sine and cosine variables have adjusted R-squared values of 0.153, 0.143, 0.029 meaning 15.3%, 14.3%, and 2.9% variability of each category of hate crime could be explained by the significant frequency.

[3] Unemployment rate, CPI, detrended CPI, average temperature, time, and non-racial hate crimes.

Anti-Black or African American hate crimes

We follow the same procedure, checking the linear relationship between variables and generating a stepwise regression model to find statistically significant explanatory variables. The stepwise regression suggests four variables, unemployment rate, CPI, temperature, and non-racial hate crimes. Using selected variables, we generate a regression model and check the ACF and PACF of the residuals to fit ARMA to the errors. Among five different models, the model equation for the best model is as follows:

$$Black\ HC_t = -4.148 + 1.325 temp_t + 5.414 unemp_t + 0.239 cpi_t + 0.282 non\ racial\ HC_t + \eta_t$$

$$\eta_t = 1.352\eta_{t-1} - 0.362\eta_{t-2} + \varepsilon_t - 0.853\varepsilon_{t-1}$$

The anti-Black or African American hate crime of the current month *t* ($Black\ HC_t$) is positively correlated with the average temperature of current month ($temp_t$), the unemployment rate of current month ($unemp_t$), the CPI of current month (cpi_t), and the non-racial hate crime of current month ($non\ racial\ HC_t$). Also, the model suggests ARMA(2,1) for the residuals. Thus, the residuals of the current month (η_t) is associated with the residuals of the last month, two months ago (η_{t-1}, η_{t-2}), the error term of the residual of the current month (ε_t), and the error term of the residuals of the past month (ε_{t-1}). The time series plot of the residuals of our final model looks stationary, passes four unit root tests, and has no significant autocorrelation identified from ACF and PACF (Figure A6). Also, this model has the best AICc, BIC, and R-squared values (3602.76, 3637.22, 0.6725). Thus, we can use this model for prediction. The residuals have concerns remaining with the outliers so we cannot trust forecast intervals (Figure A6).

Anti-Asian hate crimes

Following the similar procedure from above, a stepwise regression model found CPI, temperature, non-racial hate crimes, and time are statistically significant to include in the model. Using these four variables, we create a regression model and check the ACF and PACF of the residuals to fit ARMA to the errors. Among five different models, the model equation for the best model is as follows:

$$Asian\ HC_t = 59.282 + 0.047 temp_t - 0.336 cpi_t + 0.037 non\ racial\ HC_t + 0.082\, t + \eta_t$$

$$\eta_t = 1.159\eta_{t-1} - 0.197\eta_{t-2} + \varepsilon_t - 0.816\varepsilon_{t-1}$$

The anti-Asian hate crime of the current month t ($Asian\,HC_t$) is positively correlated with the average temperature of current month ($temp_t$), the non-racial hate crime of current month ($non\,racialHC_t$), and time (t). However, our response variable is negatively correlated with the CPI of current month (cpi_t). The model suggested ARMA(2,1) for the residuals. Thus, the residuals of the current month (η_t) is associated with the residuals of the last month, two months ago (η_{t-1}, η_{t-2}), the error term of the residual of the current month (ε_t), and the error term of the residuals of the past month (ε_{t-1}). The time series plot of the residuals of our final model looks stationary, passes four unit root tests, and has no significant autocorrelation identified from ACF and PACF (Figure A7). Also, this model has the best AICc, BIC, and R-squared values (2277.47, 2311.93, 0.5492). Thus, we can use this model for prediction. The residuals look normal and we can trust forecast intervals (Figure A7).

3. GARCH

The time series plot of the first-order differenced, log transformation applied all racial hate crimes and anti-Black or African American hate crimes showed constant variance over time. However, the time series plot of the first-order differenced, log transformation applied anti-Asian hate crime seems to have increased variability after a certain period of time (Figure A8). Using the information from the ACF and PACF, we fit ARMA to the return of the anti-Asian hate crime. However, the residuals did not have significant autocorrelation remaining. Therefore, fitting GARCH to anti-Asian hate crimes was not appropriate.

4. ARFIMA

ACF plots of three time series show non-exponential decay in lags (Figure A9). However, checking *d*, which is a parameter to estimate the degree of differencing, we decide to fit ARFIMA to anti-Asian hate crimes[4]. Checking the residuals of the fractionally differenced time series, the time series plot shows constant mean and variance. Also, it passed three stationarity tests (ADF, PP, ERS). The ACF and PACF shows no significant autocorrelation remaining. Recalling back to the mid-semester project, the anti-Asian hate crime was difficult to find the best SARIMA models when we applied first-order differencing to the data. After trying ARFIMA and checking the model results, the problem from the mid-semester project could be due to the over-differencing of the data. To confirm if ARFIMA could be a

[4] All racial hate crimes (*d*=0.4757504), anti-Black or African American hate crimes (*d*=0.4911473), and anti-Asian hate crimes (*d*=0.3988466).

good choice, we fit auto.arima() to the first-order differenced data. However, it is suggested to fit ARMA (2,2) which is a complicated model with four additional parameters.

As we observe seasonal patterns from the ACF of two time series, all racial hate crimes and anti-Black or African American hate crimes, we also check if the ACF of seasonal differenced data shows evidence to fit ARFIMA. However, the ACF of the seasonally differenced data (period of 12 months) was in the borderline to choose between fitting AR or ARFIMA. Checking the parameter to estimate the degree of the differencing, we fitted ARFIMA to all racial hate crimes[5]. However, we identify evidence to fit seasonal ARMA to the residuals which we could not find sources to learn how to fit such a model. Therefore, we conclude only anti-Asian hate crimes could be fitted by ARFIMA.

5. Comparing models

So far, we have function of time models based on the frequencies found by spectral analysis, SARIMA models (from mid-semester project), and regression with ARIMA errors for three time series. In addition, for anti-Asian hate crimes, we also have the ARFIMA model. Since function of time models do not have linear residuals, we cannot use them for prediction. Thus, we focus on finding the best model among SARIMA and regression with ARIMA errors models for all racial hate crimes and anti-Black or African American hate crimes using AIC, BIC, and cross-validated R-squared. For anti-Asian hate crimes, we compare AIC and BIC of SARIMA and regression with ARIMA errors models. Since we cannot compare ARFIMA with other models using AIC and BIC, we calculate cross-validated R-squared for all three models.

For all racial hate crimes and anti-Black or African American hate crimes, regression with ARIMA errors has better AIC and BIC (Table A1). For anti-Asian hate crimes, regression with ARIMA errors has the best AIC but the SARIMA model had better BIC. Also, we calculate R-squared and normalized RMSE to compare models. For all racial hate crimes and anti-Asian hate crimes, regression with ARIMA errors has larger R-squared and smaller normalized RMSE. However, for anti-Black or African American hate crimes, SARIMA models have larger R-squared and slightly smaller normalized RMSE (Table A1).

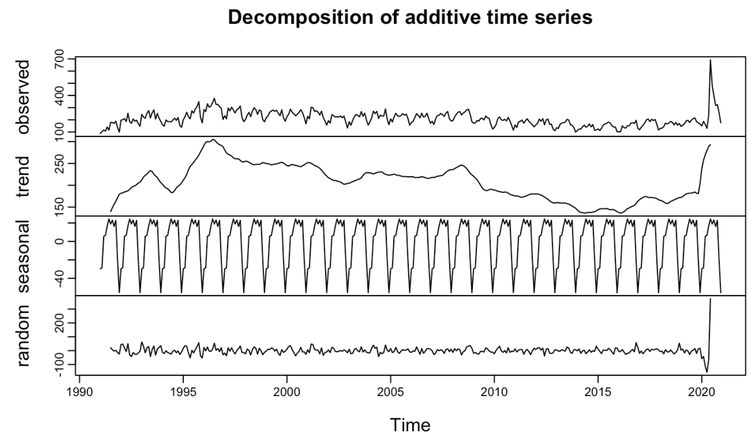
Additionally, we cross-validate our models. We divide each time series data into two subsets; data from 1991-2016 and 2017-2020. We fit SARIMA, regression with ARIMA errors models, and ARFIMA using the first subset. Then we make predictions on 2017 to 2020 using above models to compare predicted values and the actual data (Table A2).

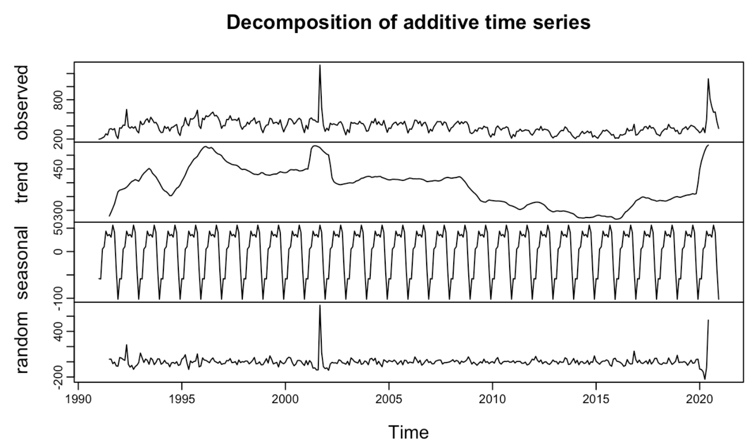
[5] All racial hate crimes (d=0.3878510), and anti-Black or African American hate crimes (d=0.4749521)

For all racial hate crimes, the SARIMA model has slightly better predictive performance when calculating the R-squared (0.1606) compared to the regression with ARIMA errors (0.1113). For anti-Black or African American hate crimes the regression with ARIMA errors has better predictive performance when calculating the R-squared (0.2034) compared to SARIMA (0.1173). For anti-Asian hate crimes, the ARFIMA makes better predictions with the highest R-squared (0.3028) followed by the regression with ARIMA errors (0.1798) and SARIMA model (0.0336).

Even though SARIMA had less error when predicting all racial hate crimes, the difference between two R-squared values are comparably small. Overall, AIC and BIC were better in regression with ARIMA errors with less concern with normal residuals, we make predictions on 2021 and 2022 using three regression models (Figure A10). Also, for anti-Asian hate crimes, we make predictions on 2021 and 2022 using ARFIMA (Figure A10).

Appendix





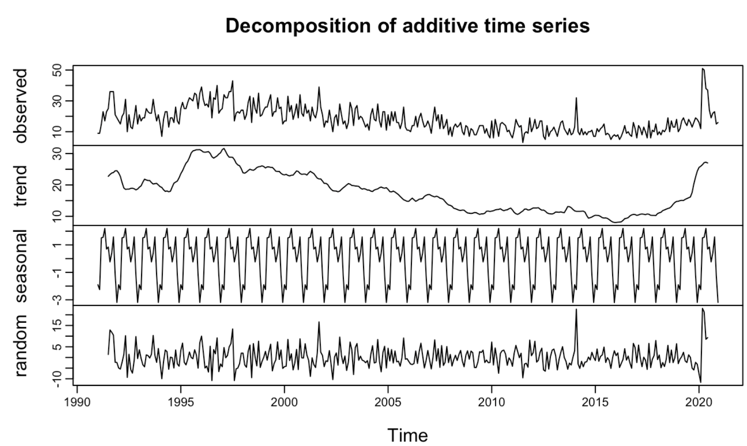


Figure A1. Decomposing three time series: all racial hate crimes, anti-Black or African American hate crimes, and anti-Asian hate crimes (from the top to bottom).

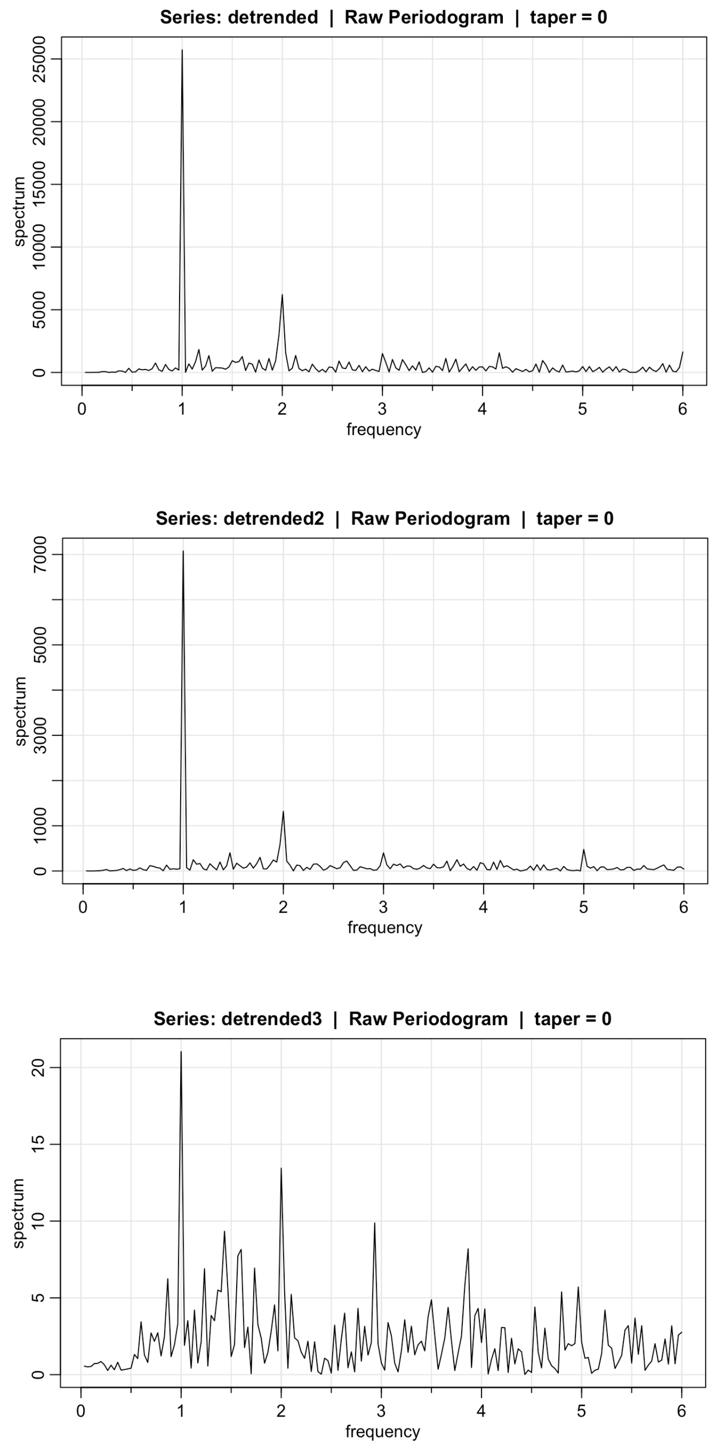


Figure A2. Periodogram using detrended time series: all racial hate crimes, anti-Black or African American hate crimes, and anti-Asian hate crimes (from the top to bottom).

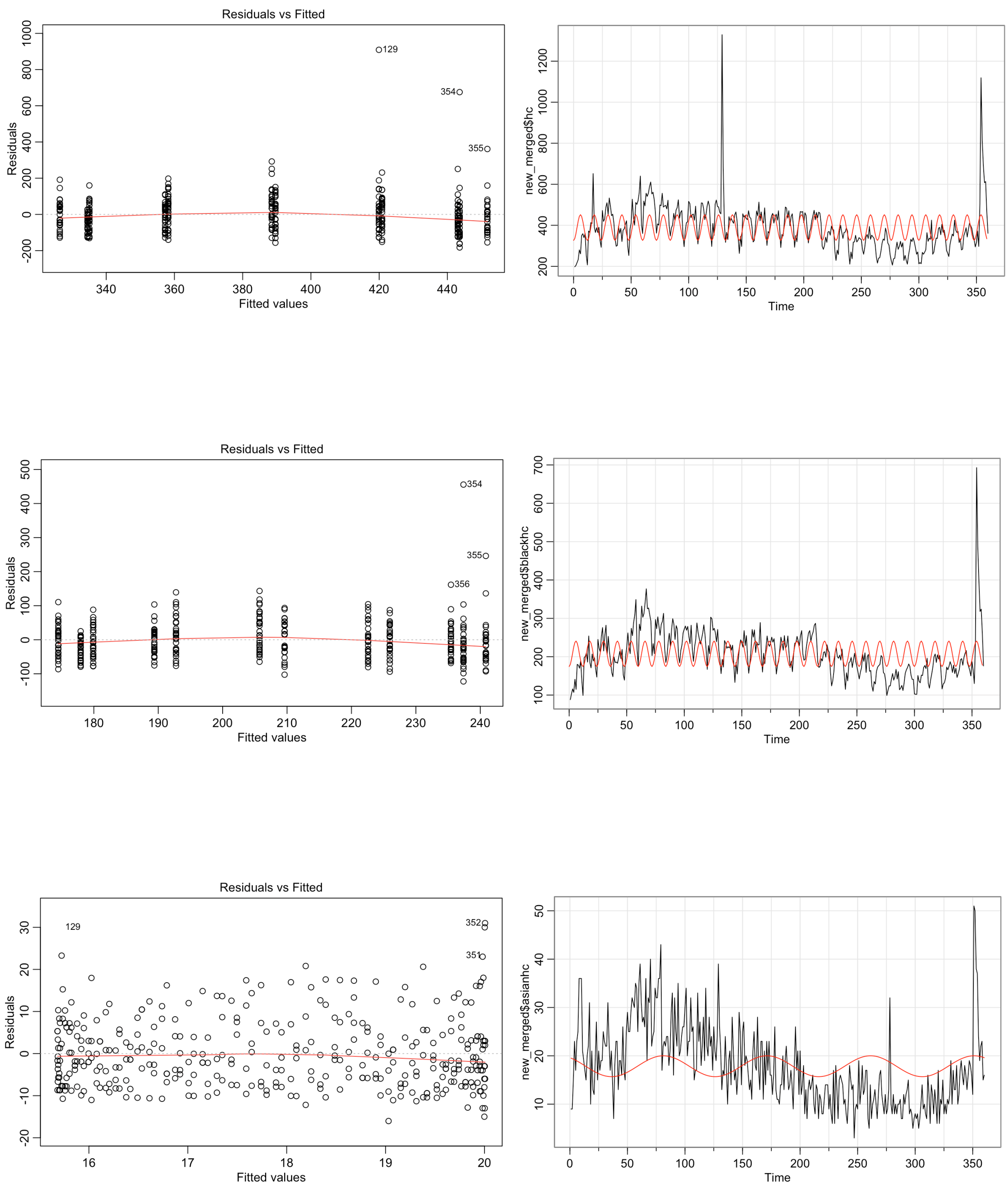


Figure A3. Residuals and fitted plot of function of time models using frequencies identified from the spectral analysis: all racial hate crimes, anti-Black or African American hate crimes, and anti-Asian hate crimes (from the top to bottom).

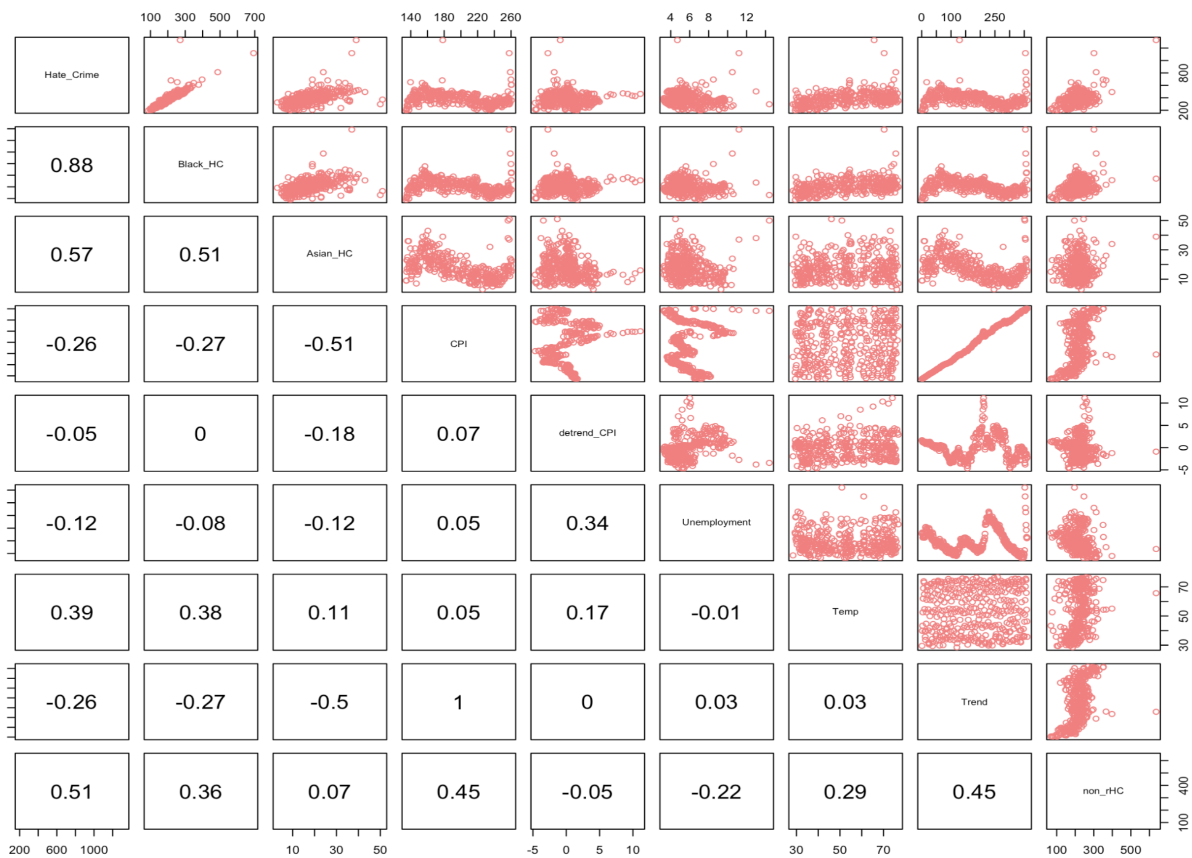


Figure A4. Observing relationships between variables.

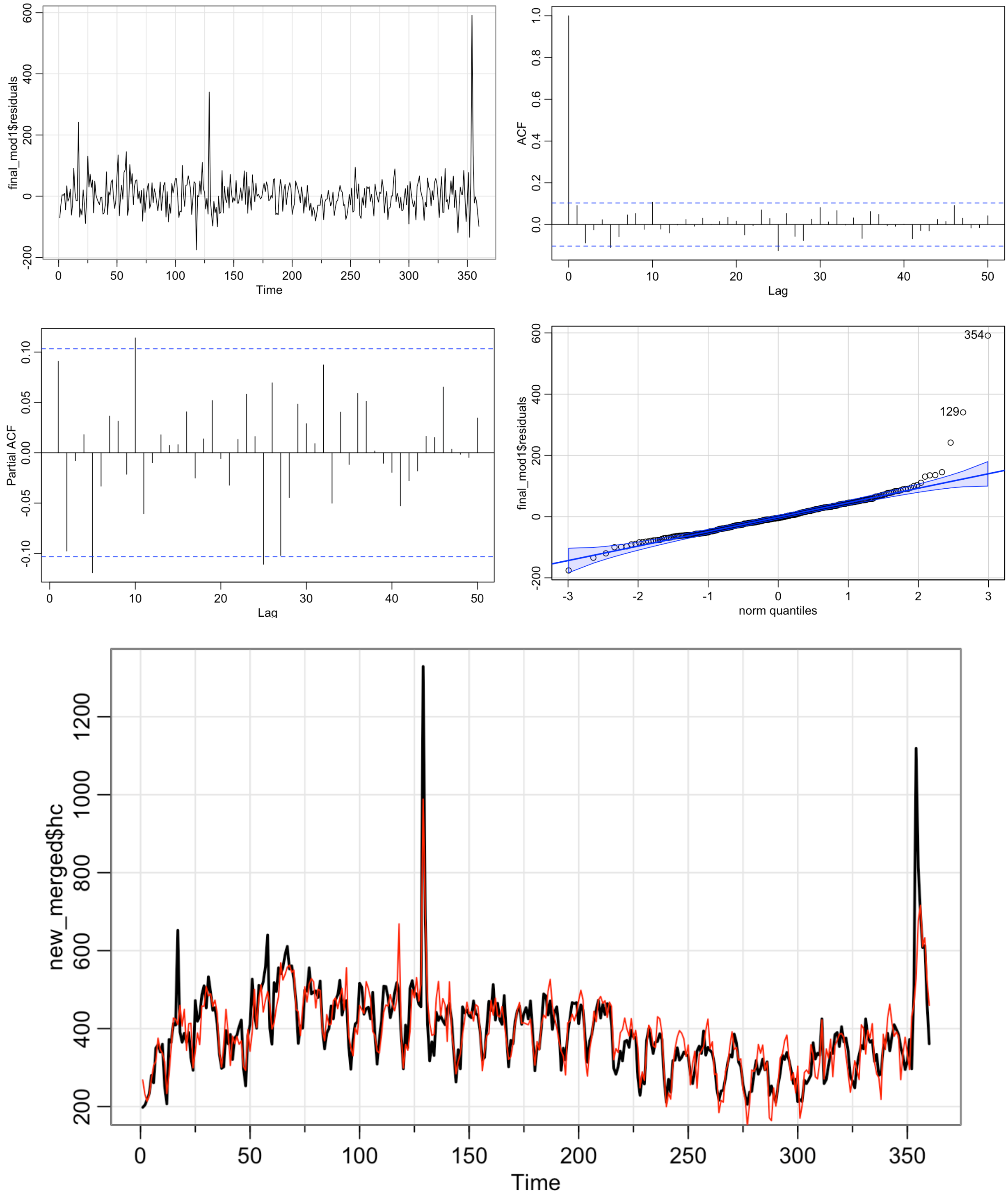


Figure A5. Residuals of the regression with ARIMA errors model using all racial hate crimes as the response variable. Also the last plot shows how the model fits the data.

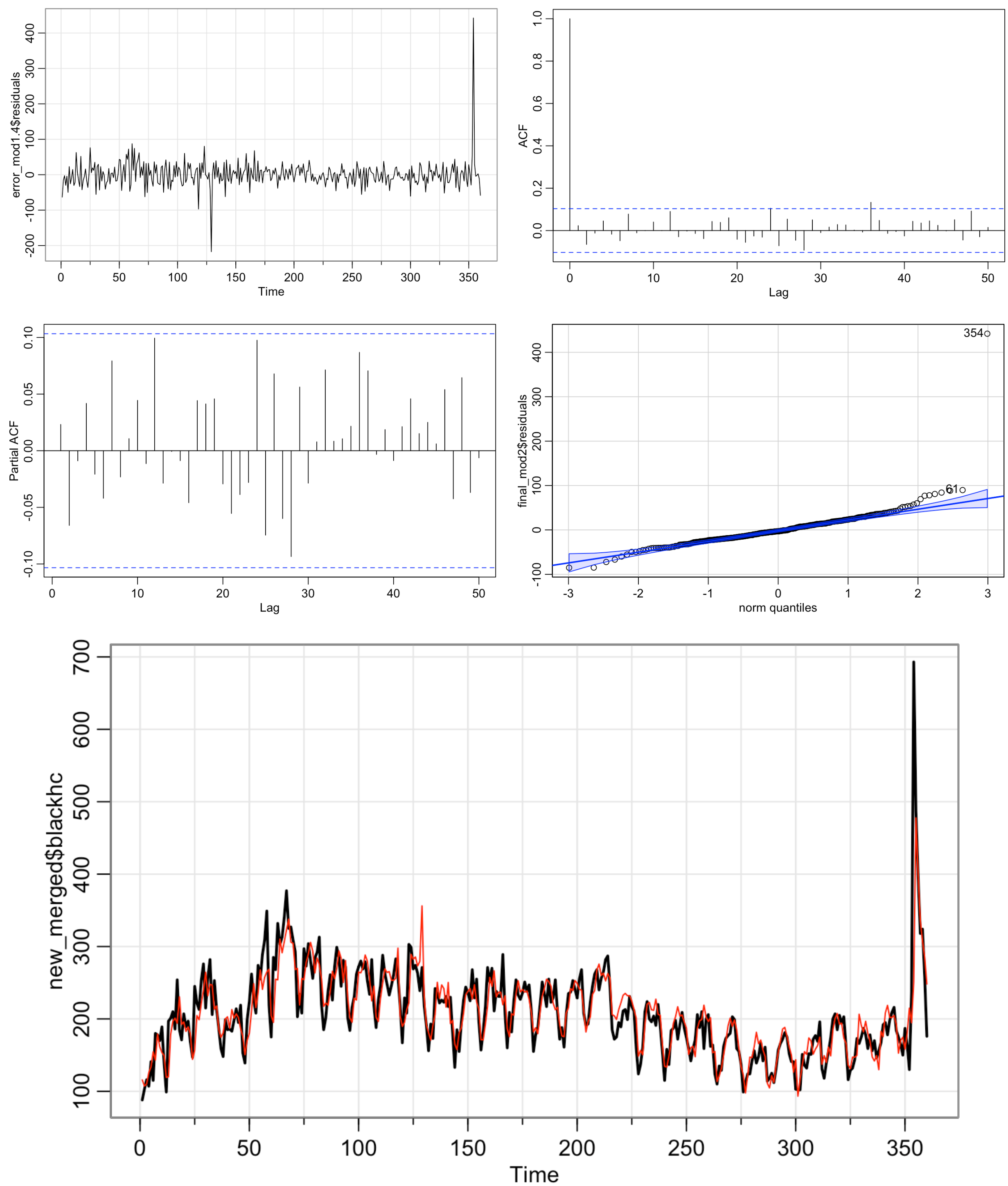


Figure A6. Residuals of the regression with ARIMA errors model using anti-Black or African American hate crimes as the response variable. Also the last plot shows how the model fits the data.

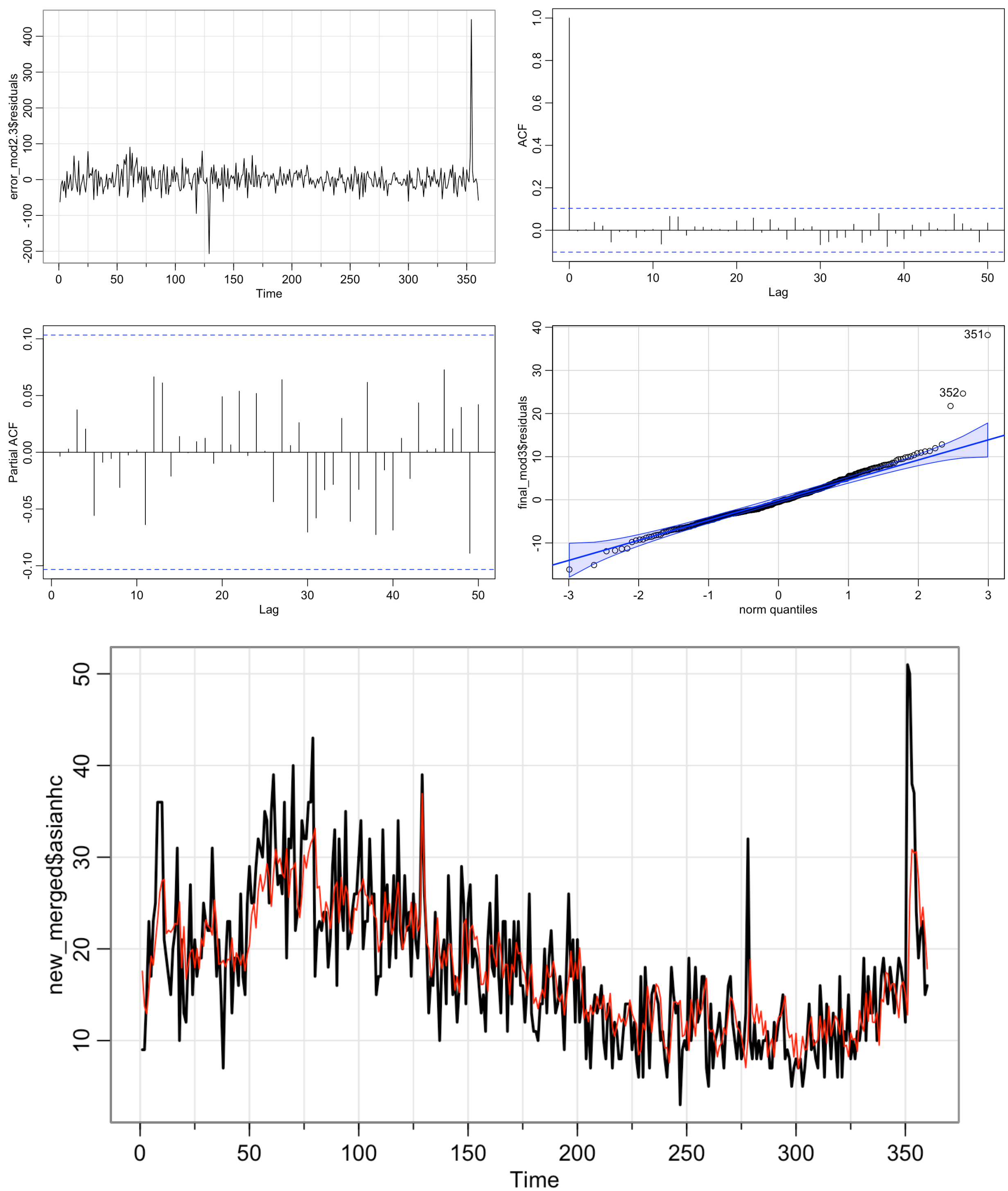


Figure A7. Residuals of the regression with ARIMA errors model using anti-Asian hate crimes as the response variable. Also the last plot shows how the model fits the data.

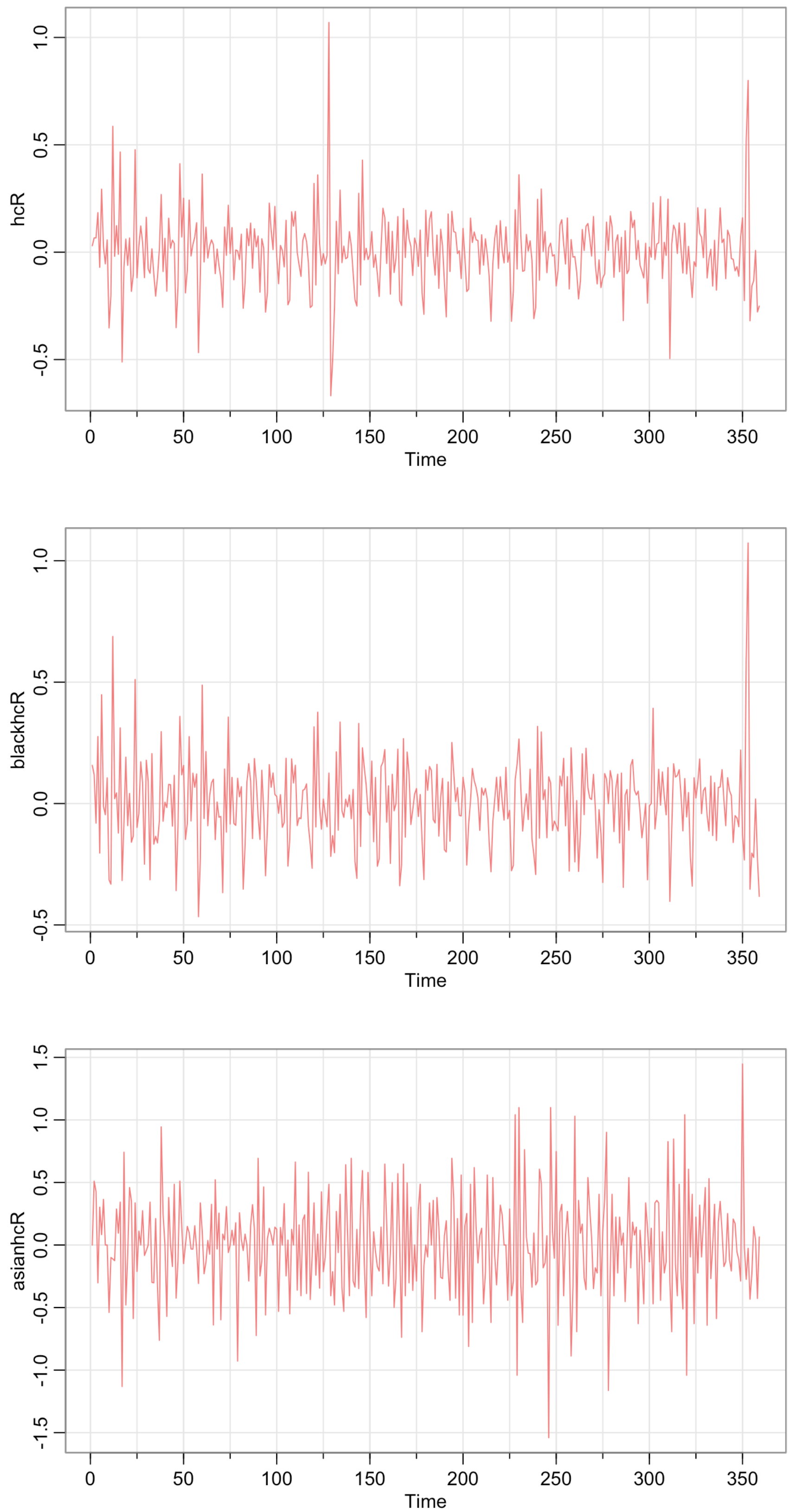


Figure A8. Checking GARCH behavior of the return of three time series.

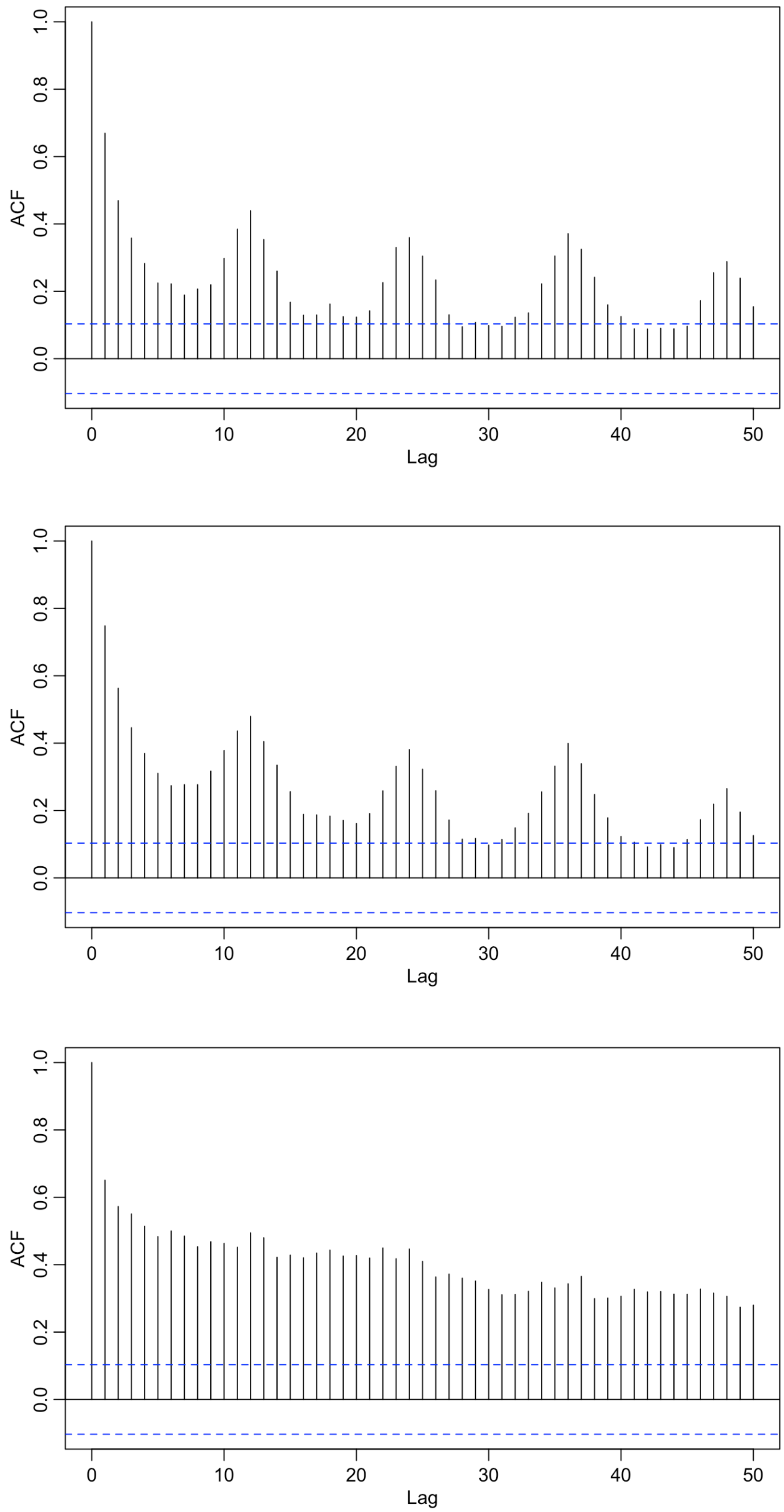


Figure A9. ACF of all racial hate crimes, anti-Black or African American hate crimes, and anti-Asian hate crimes (from the top to bottom).

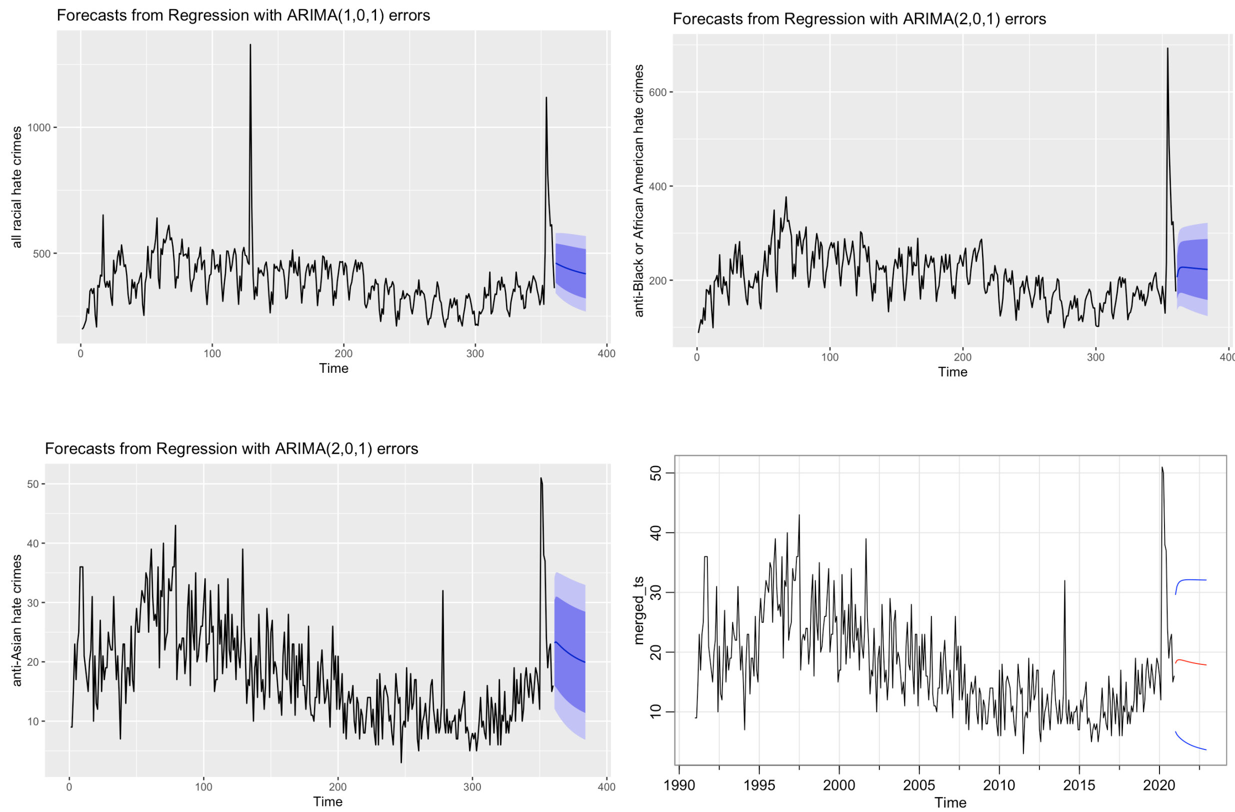


Figure A10. Prediction using regression with ARIMA errors models (first three plots). We can trust forecast intervals for all racial hate crimes and anti-Asian hate crimes. The last plot is a prediction on anti-Asian hate crimes using the ARFIMA.

Time Series	Model	**AIC**	**BIC**	**R²**	**Normalized RMSE**
All racial hate crimes	SARIMA*	4017.1	4036.36	0.5787597	0.065
	Regression with ARIMA errors	3987.95	4018.62	0.714976	0.053
Anti-African American or Black	SARIMA*	3478.41	3493.81	0.6875213	0.057
	Regression with ARIMA errors	3602.76	3637.22	0.6727	0.058

Anti-Asian	SARIMA*	2285.7	2305.12	0.52753	0.119
	Regression with ARIMA errors	2277.47	2311.93	0.5492432	0.116

Table A1. Comparing AIC, BIC, R^2, and normalized RMSE of the function of time model and the SARIMA model. SARIMA models are from the mid-semester project.*

Time Series	Model	**R^2**
All racial hate crimes	SARIMA*	0.1606
	Regression with ARIMA errors	0.1113
Anti-African American or Black	SARIMA*	0.1173
	Regression with ARIMA errors	0.2034
Anti-Asian	SARIMA*	0.0336
	Regression with ARIMA errors	0.1798
	ARFIMA	0.3028

Table A2. Cross-validation: SARIMA models are from the mid-semester project.* However, we use different subset for cross-validation.

References

Federal Bureau of Investigation. (n.d.). *Hate Crime* [Dataset]. Federal Bureau of Investigation Crime Data Explorer. https://crime-data-explorer.fr.cloud.gov

Hong, S.H.M. (2022). *Time series analysis on racial hate crimes in the United States.* Unpublished

R Core Team (2022). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Shumway, R., & Stoffer, D. (2019, May 20). *Time Series (Chapman & Hall/CRC Texts in Statistical Science)* (1st ed.). Routledge.

NOAA National Centers for Environmental information, *Climate at a Glance: National Time Series*, published December 2022, retrieved on December 15, 2022 from https://www.ncei.noaa.gov/access/monitoring/climate-at-a-glance/national/time-series

U.S. Bureau of Labor Statistics. (2021). *Design : Handbook of Methods: U.S. Bureau of Labor Statistics*. https://www.bls.gov/opub/hom/cpi/design.htm

U.S. Bureau of Labor Statistics. (n.d.). *CPI for All Urban Consumers (CPI-U)* [Dataset]. https://data.bls.gov/timeseries/CUUR0000SA0?years_option=all_years

U.S. Bureau of Labor Statistics. (n.d.). *Labor Force Statistics from the Current Population Survey: Unemployment Rate* [Dataset]. https://data.bls.gov/timeseries/LNS14000000

U.S. Census Bureau, Current Population Survey Design and Methodology Technical Paper 77, October 2019.

Van Rossum, G., & Drake, F. L. (2009). *Python 3 Reference Manual*. Scotts Valley, CA: Create Space.